X(3872) as bound state or correlation: Ultralow energy properties from ultrahigh energy processes

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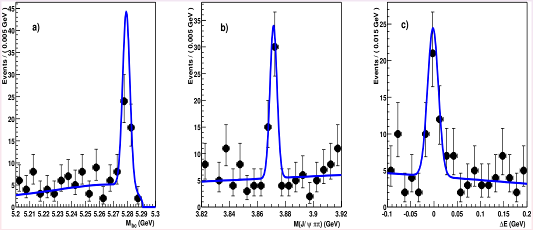
Motivation

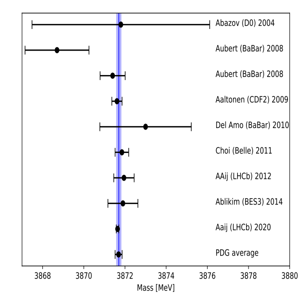
The QCD spectrum

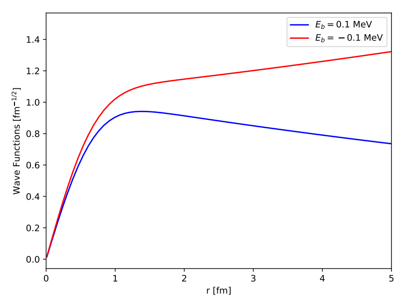
- Individual single state determination
- Collective state determination (thermodynamics,level density)
- Dashen and Kane (1975): Two types of hadrons depending on their contribution within the typical $SU(3)$ splitting.
- Impact on proliferation of X,Y,Z states
- X(3872) is a $J^{PC} = 1^{++}$ with mass $B_X = M_D + M_{\bar{D}*} - M_X = 0.00(18)$MeV



X(3872) summary

- A compilation of values does not indicate clearly if the state is bound or not
- What is the nature of the state ? $c\bar{c}$, Diquark-antidiquark , mixture
- Long distances dominate





To count or not to count

- The best way to look for missing states is to count them

$$1, 2, 3 \ldots$$

- Thermodynamics is a way of counting
- The partition function of QCD counts

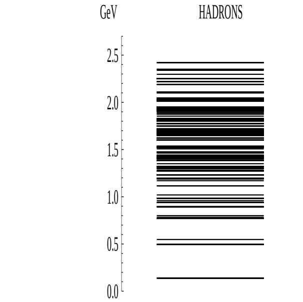
$$Z_{\rm QCD} = \sum_n e^{-E_n/T} \qquad H_{\rm QCD}\psi_n = E_n\psi_n$$

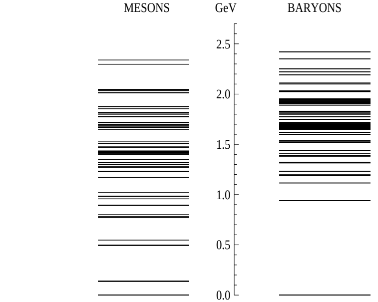
- Spectrum of QCD $\to$ Thermodynamics
- Colour singlet states (hadrons + ...???)
- Do we see quark-gluon substructure BELOW the “phase transition” ?
- Completeness relation in Hilbert space $\mathcal{H}_{\rm QCQ}$

$$\mathbf{1} = \sum_n |\Psi_n\rangle\langle\Psi_n| \approx \underbrace{\sum_n |\bar q q; n\rangle\langle \bar q q; n|}_{\rm mesons} + \underbrace{\sum_n |qqq; n\rangle\langle qqq; n|}_{\rm baryons} + \underbrace{\sum_n |\bar q q g; n\rangle\langle \bar q q g; n|}_{\rm hybrids} + \ldots$$

- Given H, is there a sum rule involving ALL resonances ?.

The states in the Particle Data Group (PDG) book

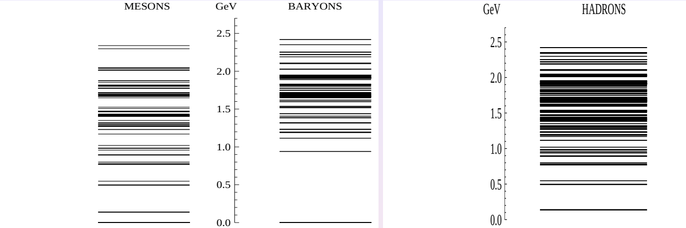




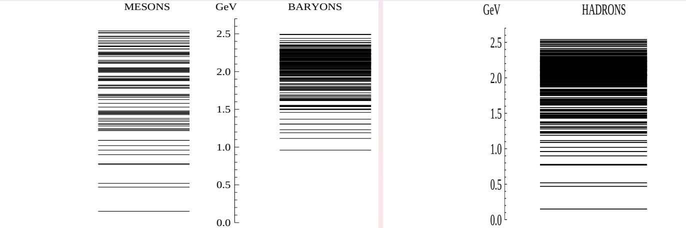
- No data (is a compilation)
- No particles (resonances)
- No book
- Which particles enter PDG ?

Hadron Spectrum (u,d,s)

- Particle Data Group (PDG) compilation 2016



- Relativized Quark Model (RQM) Isgur, Godfrey, Capstik, 1985

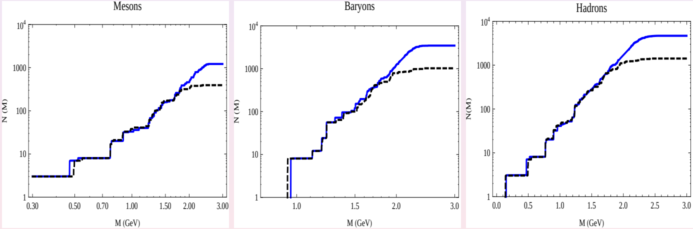


Cumulative number of states

- Compare H_{QCD}, H_{PDG}, H_{RQM} with staircase function

$$N(M) = \sum_n \theta(M - M_n)$$

- Which states count ?
- Is $N_{\mathrm{QCD}}(M)$ accessible ?



$N_{q\bar{q}} \sim M^6$ $\quad N_{qqq} \sim M^{12}$ $\quad N_{qD} \sim M^6$ $\quad N_{\bar{q}q\bar{q}q} \sim M^{18}$ $\quad N_{\mathrm{hadrons}} \sim e^{M/T_H}$

$T_H \sim 150\mathrm{MeV}$= Hagedorn temperature

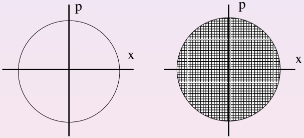
How to count classical states

- Classical particles

$$E = \frac{p^2}{2m} + V(x)\,; \qquad \frac{dE}{dt} = 0 \to m\frac{d^2x}{dt^2} = -V'(x)\,,$$

- A state is given by the initial conditions $x_0 = x(0)$ and $p_0 = mx'(0)$ which we represent in phase space
- How many states have energy smaller than E ?
- Case: Harmonic oscilator $V(x) = m\omega^2 x^2/2$. Constant energy states are ellipses with $a = \sqrt{2E/m}/\omega$ and $b = \sqrt{2Em}$.

The number of states is n corresponds to the number of points inside the ellipse$= \infty$!!.
We regularize the divergence by considering experimental accuracy Δx y Δp and count one by one



Take $h = \Delta x \Delta p$ the elementary square surface and then

$$n = \sum_{x,p} \frac{\Delta x \Delta p}{h} \sim S = \pi ab = 2\pi\omega E \implies E = nh\nu\,, \ \mathrm{Planck}(1900)$$

In general, θ-step function

$$n = \int \frac{dxdp}{h}\theta(E - \frac{p^2}{2m} + V(x)) \implies nh = \int_{x_-}^{x_+} p(x)dx\ , \mathrm{Bohr-Sommerfeld}(1917)$$

Counting hadronic mass states

- In a arbitrary dimensional space with $H(p_1, \dots p_n; x_1, \dots, x_n)$

$$N(E) = \int \prod_{i=1}^{n} \frac{d^3 x_i d^3 p_i}{h^3} \theta(E - H(p,x))$$

- For a $\bar{q}q$ system in Center of mass

$$H(p,r) = 2\sqrt{p^2 + m^2} + \sigma r \implies N = \int \frac{d^3 x d^3 p}{h^3} \theta(E - H(p,r)) \sim M^6$$

- For a qqq system in Center of mass

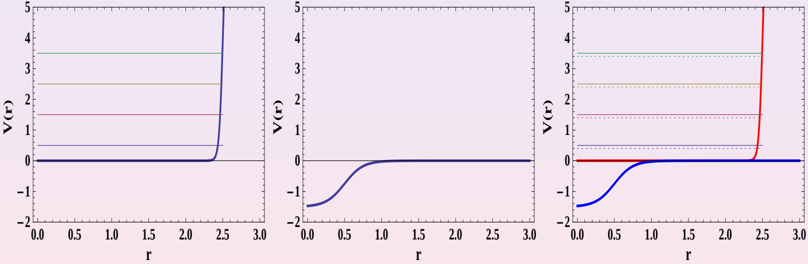
$$H(p,r) = \sum_i \sqrt{p_i^2 + m^2} + \sum_{i<j} \sigma r_{ij} \implies N = \int \prod_{i=1}^{3} \frac{d^3 x_i d^3 p_i}{h^3} \theta(E - H(p,r)) \sim M^{12}$$

How to count in the continuum

(Beth-Uhlenbeck, 1937)

- Two particles in a spherically symmetric potential

$$-\frac{\hbar^2}{2\mu}\nabla^2\Psi + V(r)\Psi = E\Psi \implies \Psi(\vec{x}) = \frac{u_l(r)}{r}Y_{lm_l}(\theta,\phi) \implies -\frac{\hbar^2}{2\mu}u_l'' + \frac{l(l+1)}{r^2}u_l + V(r)u_l = Eu_l$$



1. Confining potential: All states are bound
2. Short range potential: Finite number of bound states + Continuum (scattering states)
3. Short range + Confining : All states are bound but with en energy shift

How to count in the continuum

Positive energy solutions

$$E = \frac{\hbar^2 k^2}{2\mu} > 0\,, \quad V(r) = \frac{\hbar^2}{2\mu} U(r)\,, \quad -u_l'' + \frac{l(l+1)}{r^2} u_l + U(r) u_l = k^2 u_l \underbrace{\Longrightarrow}_{r \to 0} u_l(r) \to r^{l+1}$$

1. Particle in a large box $U(r) = 0$

$$u_l(r) \to \sin\left[kr - \frac{l\pi}{2} + \right]\,, \quad u_l(R) = 0 \Longrightarrow kR - \frac{l\pi}{2} = n\pi$$

2. Scattering positive energy solutions: phase-shifts

$$E = \frac{\hbar^2 K^2}{2\mu} > 0\,, \quad -u_l'' + \frac{l(l+1)}{r^2} u_l + U(r) u_l = K^2 u_l \underbrace{\Longrightarrow}_{r \gg a} u_l(r) \to \sin\left[Kr - \frac{l\pi}{2} + \delta_l(K)\right]$$

3. Finite box boundary condition $R \gg a$

$$u_l(R) = 0 = \sin\left[KR - \frac{l\pi}{2} + \delta_l(K)\right] \Longrightarrow KR - \frac{l\pi}{2} + \delta_l(K) = n\pi$$

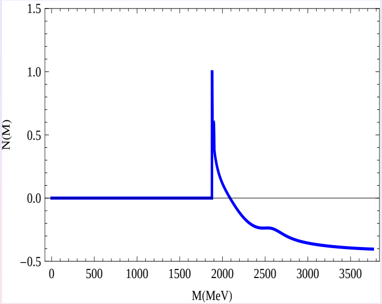
There is a shift in the wavenumber k due to the interaction $\delta_l(k)$

$$K = k - \delta_l(k)/\pi R + \mathcal{O}(1/R^2)$$

This implies a shift in the level density which depends on the interaction

Who counts ?

- Which are the complete set of states in the PDG ?
- Should X,Y,Z's or the deuteron or ^{208}Pb enter as multiquark states ?



- The cumulative number in a given channel in the continuum with threshold $M_{\rm th}$

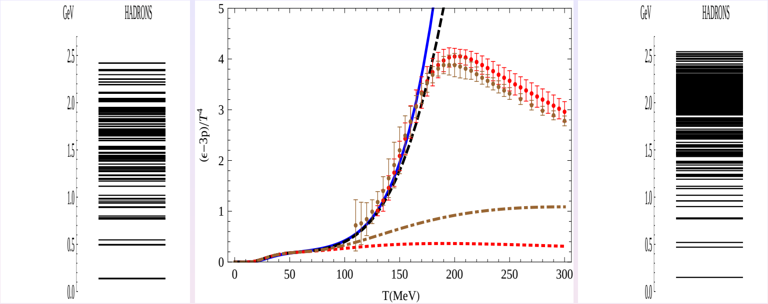
$$N(M) = \sum_n \theta(M - M_n) + [\delta(M) - \delta(M_{\rm th})]/\pi$$

- Levinson's theorem

$$N(\infty) = n_B + [\delta(\infty) - \delta(M_{\rm th})]/\pi = 0$$

- Deuteron doesn't count

QCD Spectrum and Trace anomaly



- PDG is thermodynamically equivalent to RQM !!

$$\mathcal{A}_{\rm HRG}(T) \equiv \frac{\epsilon - 3P}{T^4} = \frac{1}{T^4} \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{E_n(p) - \vec{p}\cdot\vec{\nabla}_p E_n(p)}{e^{E_n(p)/T} + \eta_n} ,$$

$$E_n(p) = \sqrt{p^2 + M_n^2} \qquad \eta_n = \pm 1$$

- Non-interacting Hadron-Resonance Gas works for $T < 0.8 T_c$
- Spectrum $\rightarrow$ Thermodynamics

Fluctuations of conserved charges

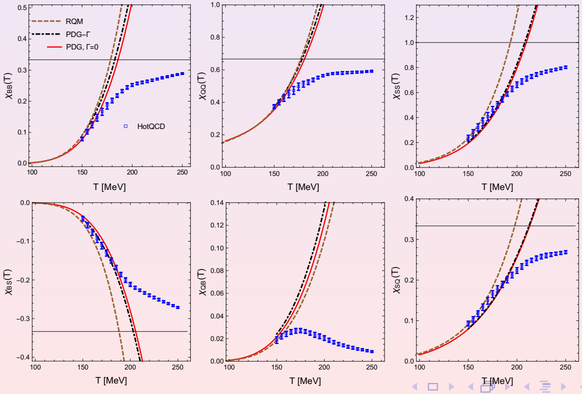
- Conserved charges

$$\langle N_B \rangle_T = 0 \qquad \langle N_Q \rangle_T = 0 \qquad \langle N_S \rangle_T = 0$$

- Vacuum Fluctuations

$$\chi_{BB}(T) = \langle N_B^2 \rangle_T \to \frac{1}{N_c} \qquad \chi_{QQ}(T) = \langle N_Q^2 \rangle_T \to \sum_{i=1}^{N_f} q_i^2$$

$$\chi_{SS}(T) = \langle N_S^2 \rangle_T \to 1 \qquad C_{BS}(T) = -3 \frac{\langle N_S N_B \rangle_T}{\langle N_S^2 \rangle_T} \to 1$$



Occupation number in the continuum

- Quantum virial expansion
- Single mass state

$$\bar{n} = \frac{\langle N \rangle_T}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{g}{e^{\sqrt{k^2+m^2}/T} + \eta} \qquad \eta = \pm \quad \text{(Bosons/Fermions)}$$

- Effects of interactions (scattering phase shifts and resonance)

$$n(T) = \int \frac{d^3p}{(2\pi)^3} dm \frac{g}{e^{\sqrt{p^2+m^2}/T} + \eta} \rho(m) \,,$$

- Level density in the continuum

$$\rho(m) = \sum_n \delta(m - m_n) + \frac{1}{\pi} \frac{d\delta}{dm} \,, \qquad m = \sqrt{s} \text{ (CM energy)}$$

- Example Resonance: $\rho \to \pi\pi$ (contribution)

$$\delta(m) = \tan^{-1} \left[\frac{m - m_R}{\Gamma_R} \right] \to \frac{1}{\pi} \delta'(m) = \frac{1}{\pi} \frac{\Gamma_R}{((m - m_R)^2 + \Gamma_R^2}$$

- Example Weakly bound state: $d \to pn$ (no-contribution)

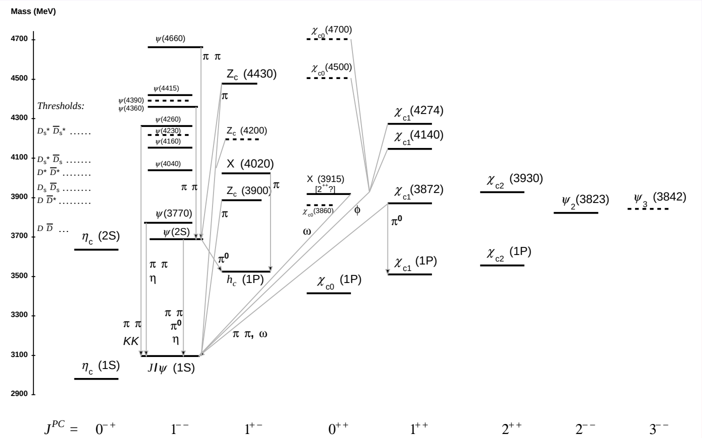
$$p \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \dots \qquad m = \sqrt{p^2 + M^2}$$

Charmonium and X,Y,Z

92. Charmonium system **1**

92. Charmonium System

Updated August 2019.



$X(3872)$ Bound state in a continuum

The Salamanca model

$$|\Psi\rangle = |c\bar{c}\rangle + |D\bar{D}^*\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle + \sum_\beta \chi_\beta(P)|\phi_A \phi_B \beta\rangle,$$

This model assumes that the transition operator is

$$\begin{aligned} T &= -3\sqrt{2}\gamma' \sum_\mu \int d^3p d^3p' \, \delta^{(3)}(p+p') \times \\ &\times \left[\mathcal{Y}_1\left(\frac{p-p'}{2}\right) b_\mu^\dagger(p) d_\nu^\dagger(p') \right]^{C=1, I=0, S=1, J=0}, \end{aligned}$$

Matrix elements

$$\langle \phi_A \phi_B \beta | T | \psi_\alpha \rangle = P \, h_{\beta\alpha}(P) \, \delta^{(3)}(\vec{P}_{\rm cm}).$$

$$\sum_{\beta} \int \left(H_{\beta' \beta}(P', P) + V^{\rm eff}_{\beta' \beta}(P', P) \right) \times \times \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P'), \tag{1}$$

where $H_{\beta' \beta}$ is the Resonating Group Method (RGM) Hamiltonian for the two-meson states obtained from the $q\bar{q}$ interaction. The effective potential $V^{\rm eff}_{\beta' \beta}$ encodes the coupling with the $c\bar{c}$ bare spectrum, and can be written as

$$V^{\rm eff}_{\beta' \beta}(P', P; E) = \sum_{\alpha} \frac{h_{\beta' \alpha}(P') h_{\alpha \beta}(P)}{E - M_{\alpha}}, \tag{2}$$

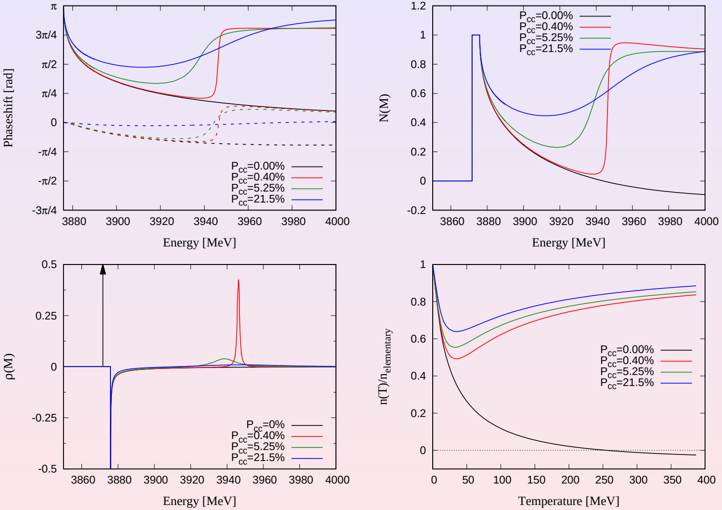
where M_{α} are the masses of the bare $c\bar{c}$ mesons.

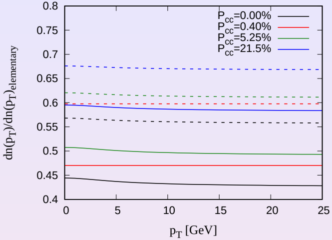
$$S^{J1} = \begin{pmatrix} \cos \epsilon_j & -\sin \epsilon_j \\ \sin \epsilon_j & \cos \epsilon_j \end{pmatrix} \begin{pmatrix} e^{2i\delta^{1j}_{j-1}} & 0 \\ 0 & e^{2i\delta^{1j}_{j+1}} \end{pmatrix} \times \begin{pmatrix} \cos \epsilon_j & -\sin \epsilon_j \\ \sin \epsilon_j & \cos \epsilon_j \end{pmatrix}. \tag{3}$$

From here we define the T-matrix

$$S^{JS} = 1 - 2ikT^{JS}, \tag{4}$$

Level density and occupation number



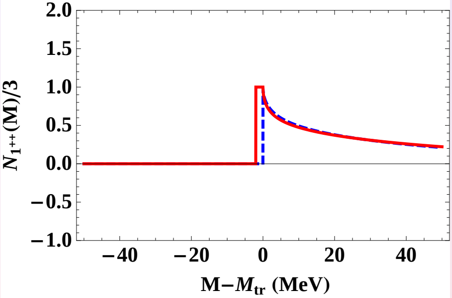


$\gamma(^3P_0)$	$\mathcal{P}_{c\bar{c}}$ [%]	α_0 [fm]	r_0 [fm]	M [MeV]	Γ [MeV]
0.00	0.00	3.14	1.21	3947.43	0.00
0.05	0.40	3.14	1.20	3946.29	1.38
0.10	1.82	3.11	1.17	3943.06	5.88
0.16	5.25	3.05	1.10	3938.56	15.18
0.20	14.25	2.88	0.85	3937.09	37.93
0.23	21.50	2.73	0.63	3947.05	56.03

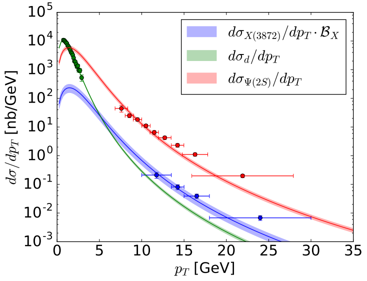
Table: PDG values are $M = 3942(9)$MeV and $\Gamma = 37^{+27}_{-17}$MeV.)

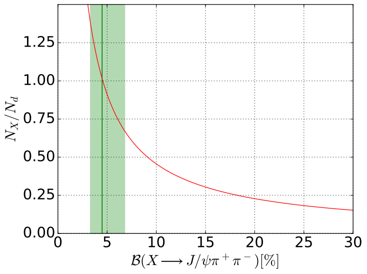
X(3872) vs deuteron

- deuteron is detected by its track *directly*
- $X(3872) \to \rho J/\psi, \omega J/\psi$ decay channels (Resolution is relevant)



Large tail in X production (compact object)





Tsallis distribution for a pure mass state

$$\frac{d^3N}{d^3p} = \frac{gV}{(2\pi)^3}\left(1+(q-1)\frac{E(p)}{T}\right)^{-\frac{q}{q-1}} \xrightarrow{q\to 1} \frac{gV}{(2\pi)^3}e^{-\frac{E(p)}{T}}, \qquad E(p)=\sqrt{m^2+p^2}$$

Production cross section $E(p_T, y) = \sqrt{p_T^2 + m^2}\cosh y$, $d^3N/(d^2p_T dy) \equiv E_p d^3N/d^3p$ with $y = \tanh^{-1}(E_p/p_z)$

$$\frac{1}{2\pi p_T}\frac{d\sigma(m)}{dp_T} = \mathcal{N}\int dy\ E(p_T, y)\left[1+\frac{q-1}{T}E(p_T, y)\right]^{\frac{q}{1-q}}$$

Finite Resolution Δm

- $X(3872) \to \rho J/\psi, \omega J/\psi$ decay channels
- Any state within the resolution $\pm\Delta m/2$ with $J^{PC} = 1^{++}$
- Given $O(m)$ we get

$$O_{\Delta m} \equiv \int_{m-\Delta m/2}^{m+\Delta m/2} dM \rho(M) O(M) \,.$$

where $\rho(M)$ is the density of states, defined as

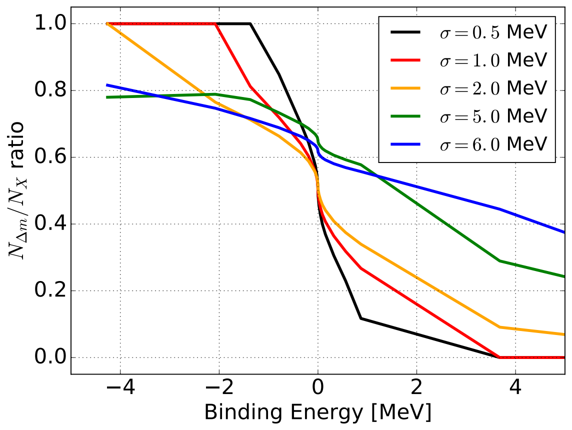
$$\rho(M) = \frac{dN(M)}{dM} = \sum_i \delta(M - M_i^B) + \frac{1}{\pi} \sum_{\alpha=1}^{n} \delta'_\alpha(M) \,, \qquad (5)$$

- Case $\Delta m \gg |B| \equiv |M_B - M_{\rm tr}|$

$$O|_{M^B \pm \Delta m} = O(M^B) + \frac{1}{\pi} \int_{M_{\rm tr}}^{M_{\rm tr}+\Delta m/2} dM \delta'_\alpha(M) O(M) \,.$$

Production ratio

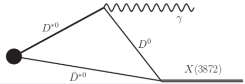
Common Tsallis distribution for $X(3872)$ and deuteron



The relative occupation number *with* and *without* finite resolution $\Delta m = 2\sigma$ as a function of the $X(3872)$ binding energy

Triangle singularities and $X(3872)$ mass

- Recent proposals to make accurate meassurements of mass (Guo, Braaten;2019)
  $e^+e^- \to X(3872)\gamma$



- Level density in $J^{PC} = 1^{++}$ channel

$$\rho(m) = \delta(m - M_X) + \frac{1}{\pi}\delta'(m). \tag{6}$$

- Detector efficiency

$$R_\sigma(m, M) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(m-M)^2}{2\sigma^2}} \tag{7}$$

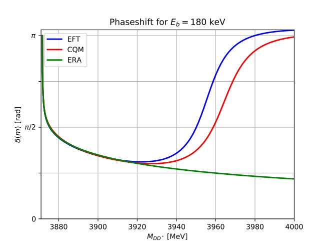
Channel	σ	ΔM	Reference
$J/\psi\pi^+\pi^-$	1.14 ± 0.07	20	Ablikim:2013dyn
$J/\psi\pi^+\pi^-$	3.33 ± 0.08	$6\sigma \approx 20$	Aaij:2011sn
$J/\psi\pi^+\pi^-$	1.2?	18	Choi:2011fc

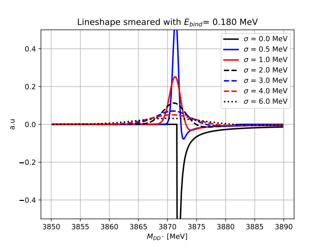
Smearing of level density

- Effective range expansion

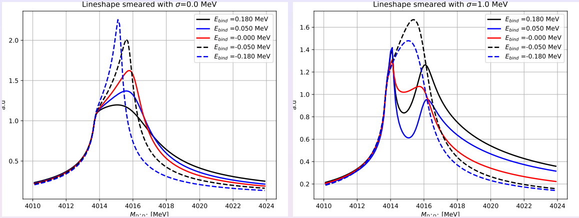
$$k \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 k^2$$

- Comparison between ERA and two coupled-channels models
- $r_0 = 1$ fm $\alpha_0 = \frac{1}{\sqrt{2\mu E_b}} = 10.58$ fm

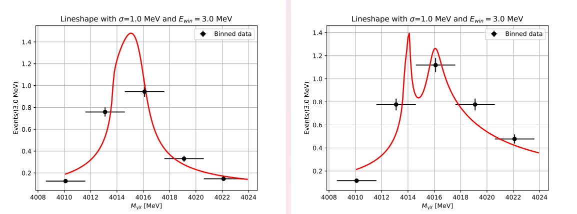




Smearing of production



MonteCarlo simulation $N = 1000$ runs between $E \in [4010, 4020]$



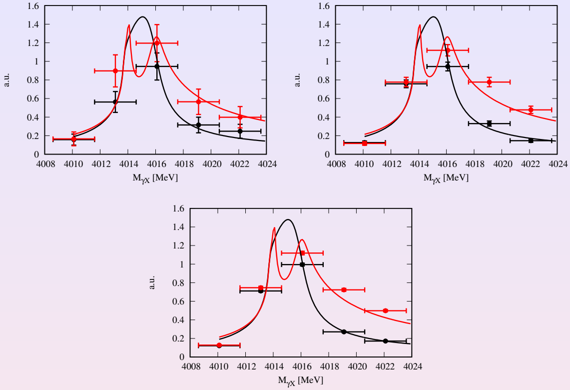


Figure: Binned smeared lineshapes of states for $\sigma = 1$ MeV, $E_w = 3$ MeV and $\Delta M = 20$ MeV for $E_b = 180$ keV (black points) and $E_b = -180$ keV (red points). Full lines show the no-binned smeared lineshape (same color code). The results are done with $N = 100$ (upper left), $N = 1000$ (upper right) and $N = 10000$ (lower left).

Conclusions

- Is the $X(3872)$ a bound state ?
- If not , should it appear in the PDG ?
- Can its mass be determined accurately ?
- Does $X(3872)$ count ?