pp and $p\bar{p}$ elastic scattering from ISR to Cosmic Ray energies

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Outline

- Introduction
- Forward Scattering
- Dispersion relations
- Stochastic Vaccum Model (SVM) Framework
- Energy Dependence
- Amplitudes in Geometric Space
- p-Air Collisions
- Other models
- Conclusions
- Perspectives

Hadronic Collider Experiments

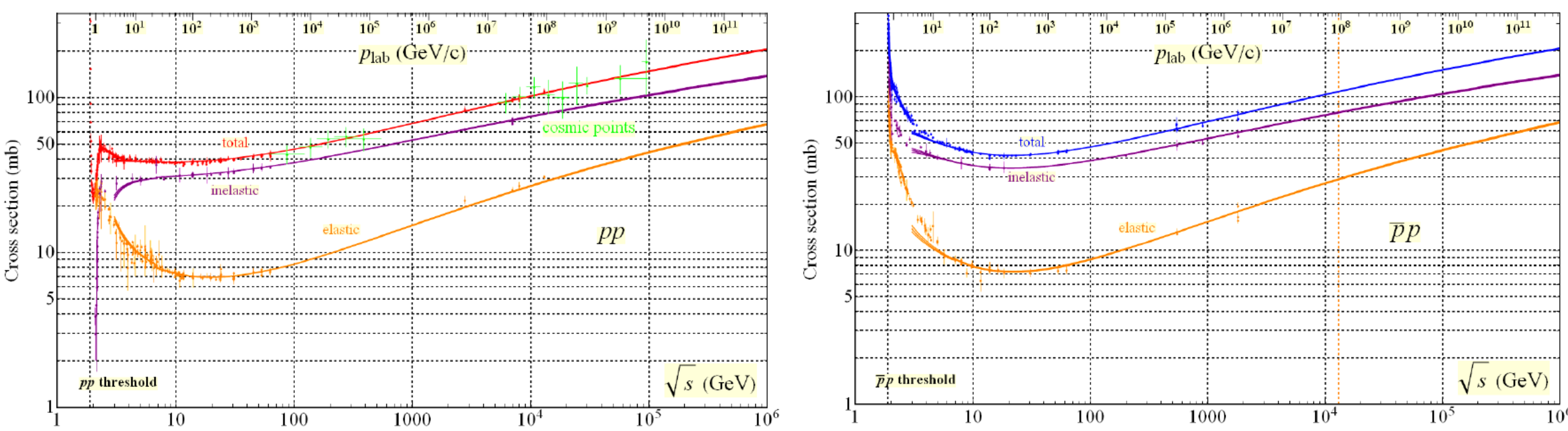
Intersecting Storage Rings-CERN, 1971–1984

Proton-Antiproton Collider(SPS)-CERN, 1981–1991

Tevatron-Fermilab, 1987–2011

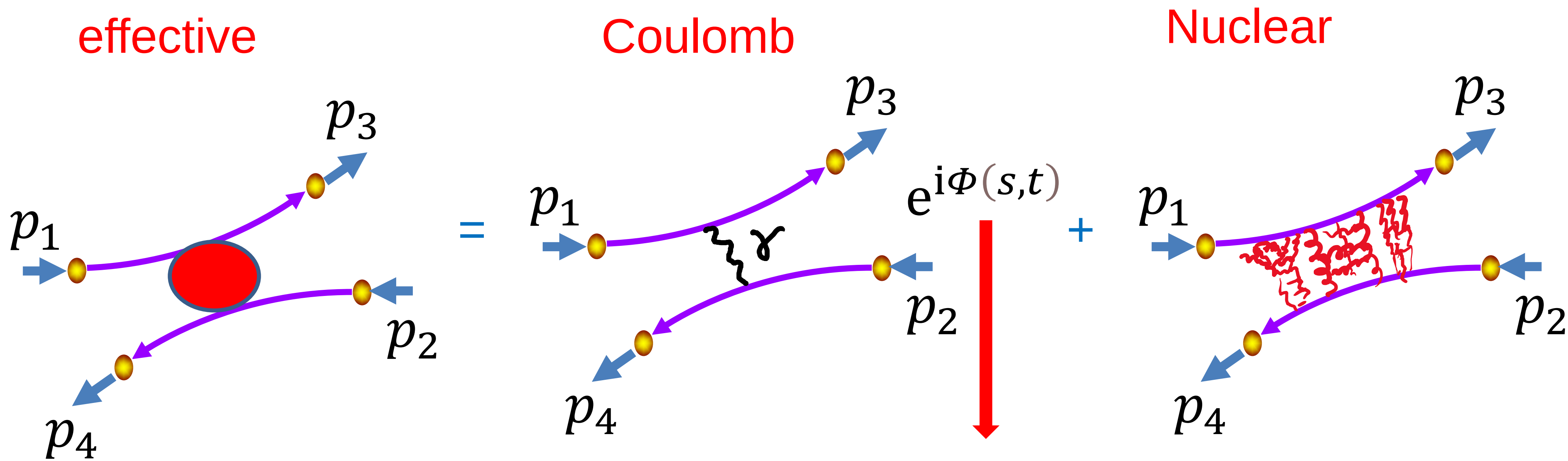
Relativistic Heavy Ion Collider-BNL, 2000–...

Large Hadron Collider-CERN, 2009–...



P.A. Zyla et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020)

Relativistic Elastic Scattering



Coulomb phase

L.D. Solov'ev, *JETP* **22**, 205 (1966) 26;
H. Bethe, *Ann. Phys. (N.Y.)* **3**, 190 (1958) 27;
G.B. West, D.R. Yennie, *Phys. Rev.* **172**(5), 1413 (1968);
V. Kundr□at and M. Lokaj□□cek, *Phys. Lett. B* **611** (2005) 102 ;
R. Cahn, *Z. Phys. C* **15** (1982) 253.

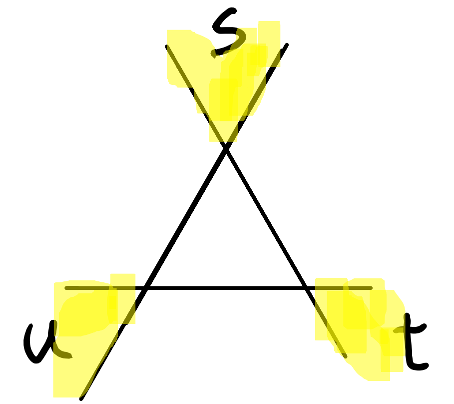
Mandelstam variables

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Assumptions



Mandelstam plane

Analytic nuclear amplitude $A(s,t,u)$

Singularities have a physical meaning

Crossing symmetric amplitudes $A_{pp}(s,t,u) = A_{p\bar{p}}(u,t,s)$

Unitarity of S matrix $ss^{\dagger} = 1$

Theorems

Optical theorem $\sigma_T = \frac{1}{2|p|\sqrt{s}} \operatorname{Im} A(s,t)$

Froissart theorem/bound $\sigma_T(s) \leq C \log^2\left(\frac{s}{s_0}\right) \qquad s \to \infty$

Pomeranchuck theorem $\frac{\sigma_T^{pp}(s)}{\sigma_T^{p\bar{p}}(s)} \to 1 \qquad s \to \infty$

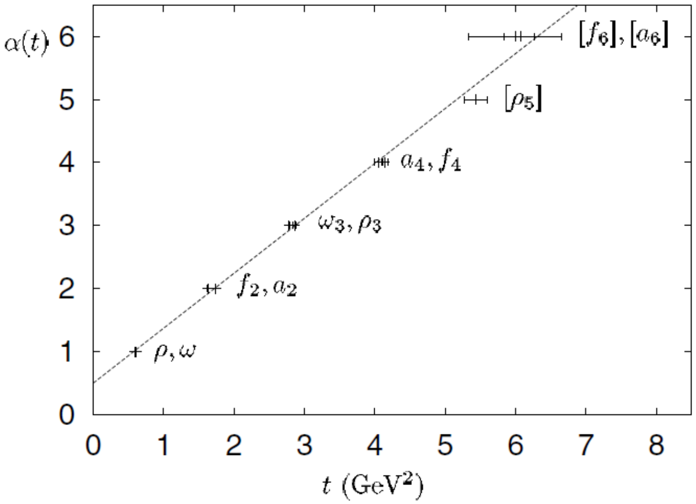
Regge Theory

Partial wave expantion $$A(s,t) = \sum_{l=0}^{\infty}(2l+1)f_l(s)P_l(z_t) \qquad z_t = 1 + \frac{2s}{t-4m^2}$$

Large energies $$A^{\pm}(s,t) = \sum_i \beta_i^{\pm}(t)\Gamma\left(-\alpha_i^{\pm}(t)\right)\left(1 \pm \mathrm{e}^{-i\pi\alpha_i^{\pm}(t)}\right)s^{\alpha_i^{\pm}(t)}$$

Reggeons - baryon and meson trajectories

S. Donnachie, H. G. Dosch, P. Landshoff and O. Nachtmann, *Pomeron Physics and QCD* , Cambridge Univ. Press 2002.



From ISR energies (20 GeV) and beyond (Pomeron trajectory)

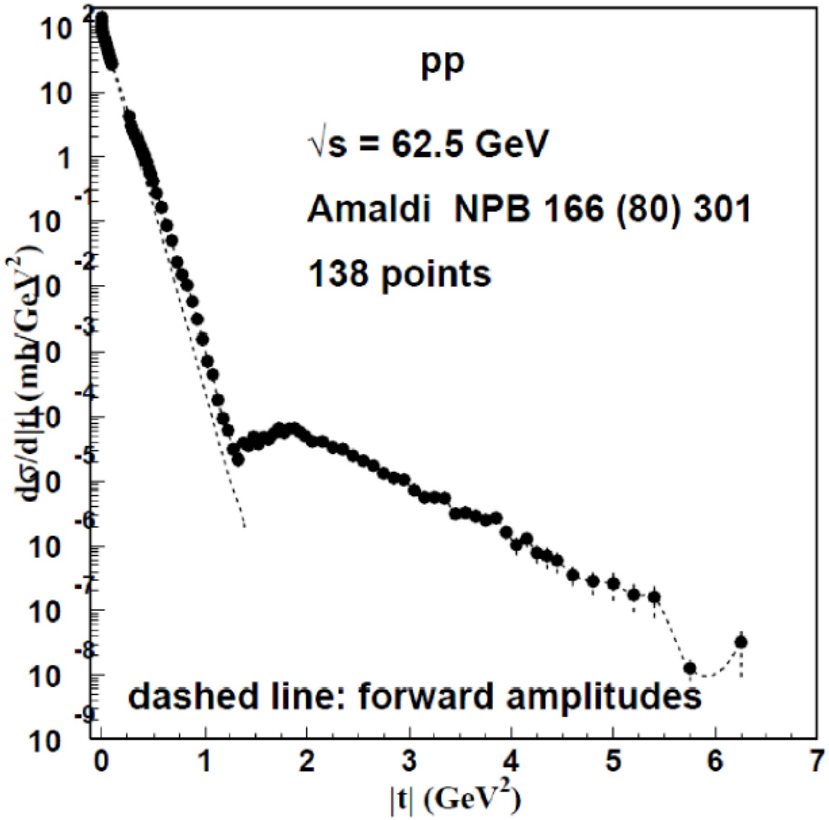
$$\alpha_{\mathbb{P}}(t) = 1 + \epsilon_0 + \alpha' t$$

$\epsilon_0 = 0.08086$ old

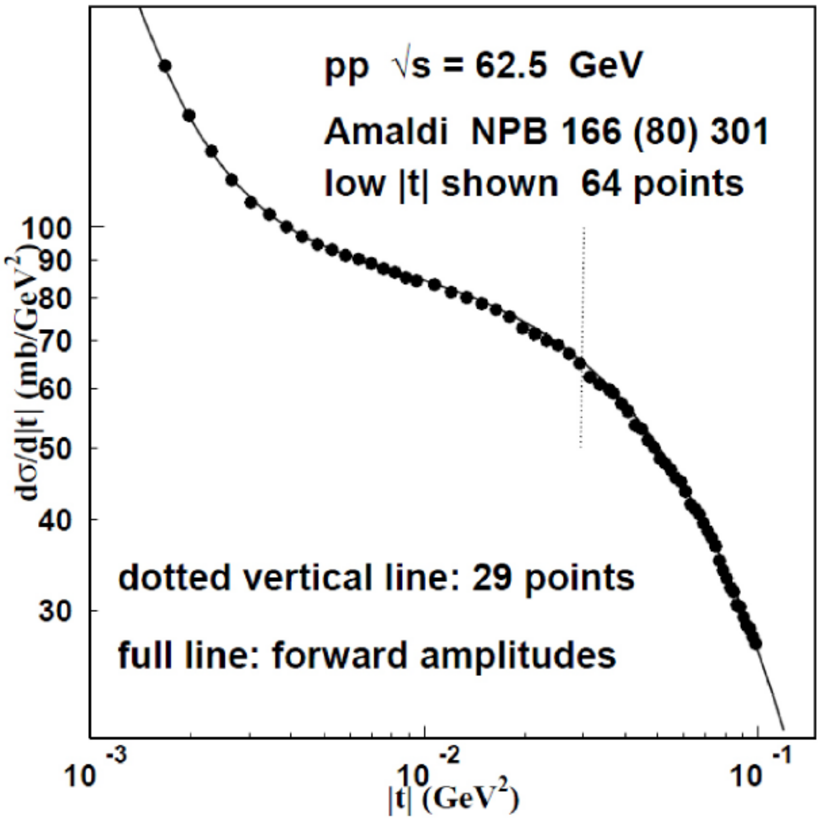
$\epsilon_0 = 0.096$ new

from phenomenology

Experimental data – differential cross section



full t range



forward t range

Minimum model dependent phenomenology of forward scattering

Nuclear and Coulomb amplitude

$$T(s,t) = T^N(s,t) + T^C(t)e^{\alpha\Phi}$$

$$T^N(s,t) \approx T^N_R(s,0)e^{B_R t/2} + iT^N_I(s,0)e^{B_I t/2}$$

$$T^C(t) = \mp\frac{2\alpha}{|t|}F^2_{\text{proton}}(t)e^{i\alpha\Phi(s,t)}$$

$$F_{\text{proton}} = (0.71/(0.71+|t|))^2$$

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Forward quantities

Optical theorem $\sigma = 4\pi(\hbar c)^2 T_I^N(s,0)$

Ratio of real and imaginary amplitudes $\rho = \frac{T_R^N(s,0)}{T_I^N(s,0)}$

Usual slope definition $\frac{d\sigma}{dt} = \left|\frac{d\sigma}{dt}\right|_{t=0} e^{Bt}$

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Differential cross section

$$\frac{d\sigma}{dt} = \pi(\hbar c)^2\left\{\left[\frac{\rho\sigma}{4\pi(\hbar c)^2}\,e^{B_R t/2} + F^C(t)\cos(\alpha\Phi)\right]^2 + \left[\frac{\sigma}{4\pi(\hbar c)^2}\,e^{B_I t/2} + F^C(t)\sin(\alpha\Phi)\right]^2\right\}$$

West-Yennie phase $\Phi(s,t) = \mp\left[\ln\left(-\frac{t}{s}\right) + \int_{-4p^2}^{0}\frac{dt'}{|t'-t|}\left[1-\frac{T^N(s,t')}{T^N(s,t)}\right]\right]$

Because $B_R \neq B_I$ we have a different phase $\Phi(s,t) = \mp\left[\ln\left(-\frac{t}{s}\right) + \frac{1}{c^2+1}[c^2 I(B_R) + I(B_I)]\right]$

where $I(B) = E_1\left[\frac{B}{2}(4p^2+t)\right] - E_i\left[-\frac{Bt}{2}\right] + \ln\left[\frac{B}{2}(4p^2+t)\right] - \ln\left[-\frac{Bt}{2}\right] + 2\gamma$ and $c \equiv \rho e^{B_R - B_I}t/2$

Minimum model dependent phenomenology of forward scattering

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$$T^N(s,t) \approx T_R^N(s,0)e^{B_R t/2} + iT_I^N(s,0)e^{B_I t/2}$$

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Optical theorem $\sigma = 4\pi(\hbar c)^2 T_I^N(s,0)$

Ratio of real and imaginary amplitudes $\rho = \dfrac{T_R^N(s,0)}{T_I^N(s,0)}$

Usual slope definition ~~$\dfrac{d\sigma}{dt} = \left|\dfrac{d\sigma}{dt}\right|_{t=0} e^{Bt}$~~ The measured slope is $B = \dfrac{\rho^2 B_R + B_I}{1+\rho^2}$

Differential cross section

$$\frac{d\sigma}{dt} = \pi(\hbar c)^2 \left\{ \left[\frac{\rho\sigma}{4\pi(\hbar c)^2} e^{B_R t/2} + F^C(t)\cos(\alpha\Phi)\right]^2 + \left[\frac{\sigma}{4\pi(\hbar c)^2} e^{B_I t/2} + F^C(t)\sin(\alpha\Phi)\right]^2 \right\}$$

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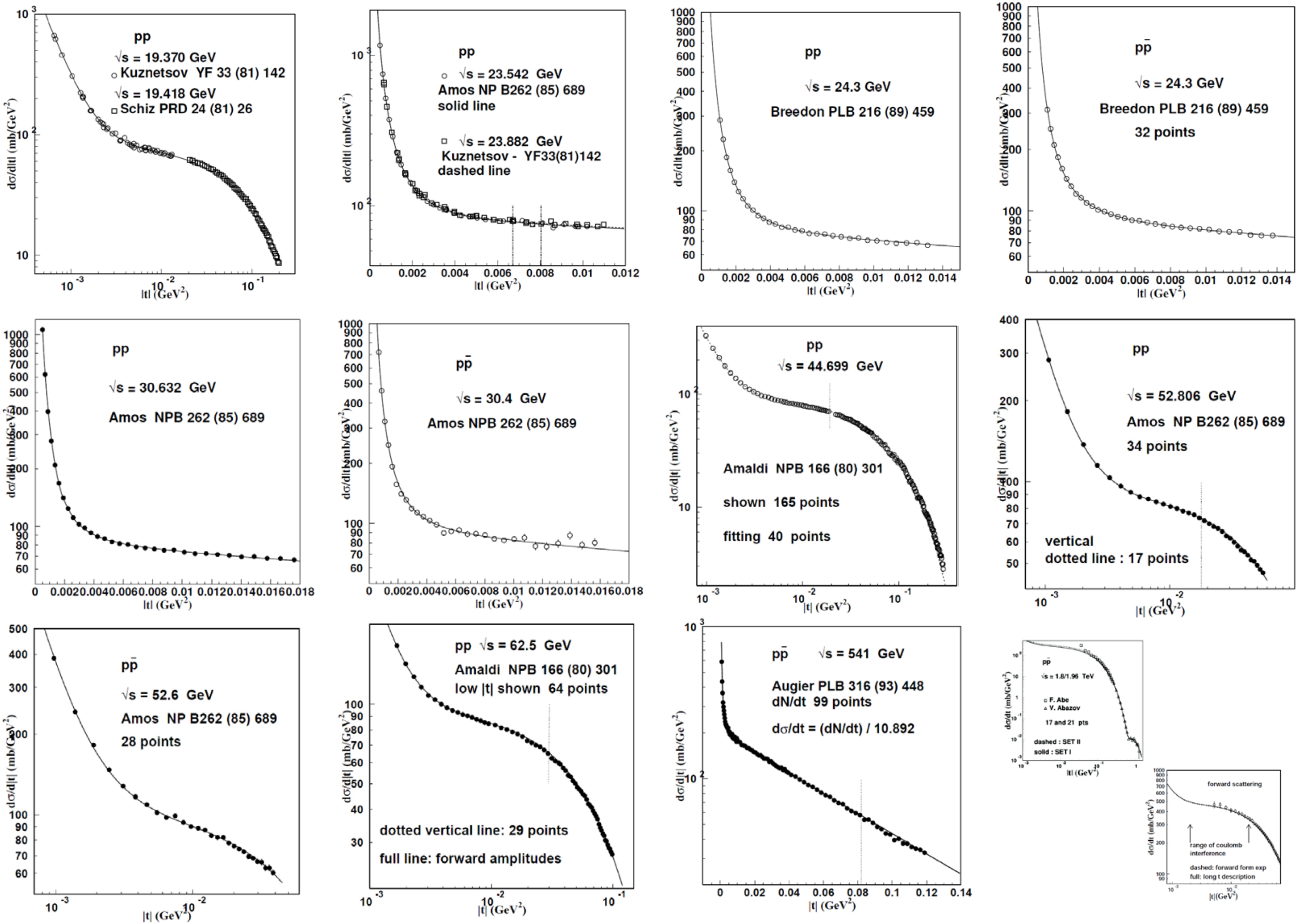
Forward scattering differential cross sections at several energies for pp and $p\bar{p}$



Dispersion relations

Particle Data Group total cross section representation

$$\sigma^{p^{\mp}p}(s) = P + H\log^2\left(\frac{s}{s_0}\right) + R_1\left(\frac{s}{s_0}\right)^{-\eta_1} \pm R_2\left(\frac{s}{s_0}\right)^{-\eta_2}$$

even and odd amplitudes

$$F_+(s,u) = \left[F_{p\bar{p}\to p\bar{p}}(s,u) + F_{pp\to pp}(s,u)\right]/2$$
$$F_-(s,u) = \left[F_{p\bar{p}\to p\bar{p}}(s,u) - F_{pp\to pp}(s,u)\right]/2$$

Dispersion relations for amplitudes

$$\text{Re}\ F_+(s,u) = K + \frac{2}{\pi}s^2\mathbf{P}\int_{2m^2}^{\infty}\frac{\text{Im}\ F_+(s')}{s'(s'^2-s^2)}\,ds' \qquad \text{Re}\ F_-(s,u) = \frac{2}{\pi}s\mathbf{P}\int_{2m^2}^{\infty}\frac{\text{Im}\ F_-(s')}{s'^2-s^2}\,ds'$$

with a common principal value integral $I(n,\lambda,x) = \mathbf{P}\int_1^{+\infty}\frac{x'^{\lambda}\ \log^n(x')}{[x'^2-x^2]}dx'$ with $x = E/m \approx s/2m^2$

Instead of PV we use exact derivative dispersion relations (DDR)

R.F. Ávila and M.J. Menon, Nucl. Phys. **A744** (2004) 249; Braz. J. Phys. **37**, 358 (2007)
E. Ferreira and J. Sesma *J. Math. Phys.* **49**, 033504 (2008); *J. Math. Phys.* 54, 033507 (2013)

We show a new representation of exact DDR

conditions: $x > 1$, n zero or positive integer and $\Re(\lambda) \le 1$

$$I(n,\lambda,x) = -\frac{\pi}{2x^2}\frac{\partial^n}{\partial\lambda^n}\left[x^{1+\lambda}\cot\left(\frac{\pi}{2}(1+\lambda)\right)\right] + \frac{(-1)^n}{x^2}2^{-(n+1)}n!\ \Phi\left(\frac{1}{x^2}, n+1, \frac{1+\lambda}{2}\right)$$

E. Ferreira, A. K. Kohara and J. Sesma, *Phys. Rev.* **C 97** (2018) 1, 014003

Hurwitz Lerch transcendents

$$\frac{1}{2^N}\frac{1}{x}\Phi\left(\frac{1}{x^2}, N, \frac{1+\lambda}{2}\right) = \frac{x^{-1}}{(1+\lambda)^N} + \frac{x^{-3}}{(3+\lambda)^N} + \frac{x^{-5}}{(5+\lambda)^N} + \ldots$$

Interesting properties of the transcendents

$$\frac{\partial}{\partial\lambda}\Phi(z,N,\frac{1+\lambda}{2}) = -\frac{N}{2}\,\Phi(z,N+1,\frac{1+\lambda}{2}) \qquad \frac{\partial I(0,\lambda,x)}{\partial\log(x)} + (1-\lambda)I(0,\lambda,x) = -\frac{1}{x^2-1}$$

$$\frac{\partial I(n,\lambda,x)}{\partial\lambda} = I(n+1,\lambda,x) \qquad \frac{\partial I(n,\lambda,x)}{\partial\log(x)} + (1-\lambda)I(n,\lambda,x) = nI(n-1,\lambda,x)$$

E. Ferreira, A. K. Kohara and J. Sesma; *Frac. Calc. and App. Analysis*, **23**, 2 (2020)

Exact DDR for real amplitudes

$$\sigma\rho\begin{pmatrix}\mathrm{pp}\\ \mathrm{p\bar{p}}\end{pmatrix} = \frac{K}{s} + H\pi\log\left(\frac{s}{s_0}\right) + \frac{4m^2}{s\pi}\left(P + H\left[\log^2\left(\frac{s_0}{2m^2}\right) + 2\log\left(\frac{s_0}{2m^2}\right) + 2\right]\right) + R_1\left[-\left(\frac{s}{s_0}\right)^{-\eta_1}\tan\left(\frac{\pi\eta_1}{2}\right) + \left(\frac{s_0}{2m^2}\right)^{\eta_1}\frac{2m^2}{s}\left(\frac{2/\pi}{1-\eta_1}\right)\right]$$

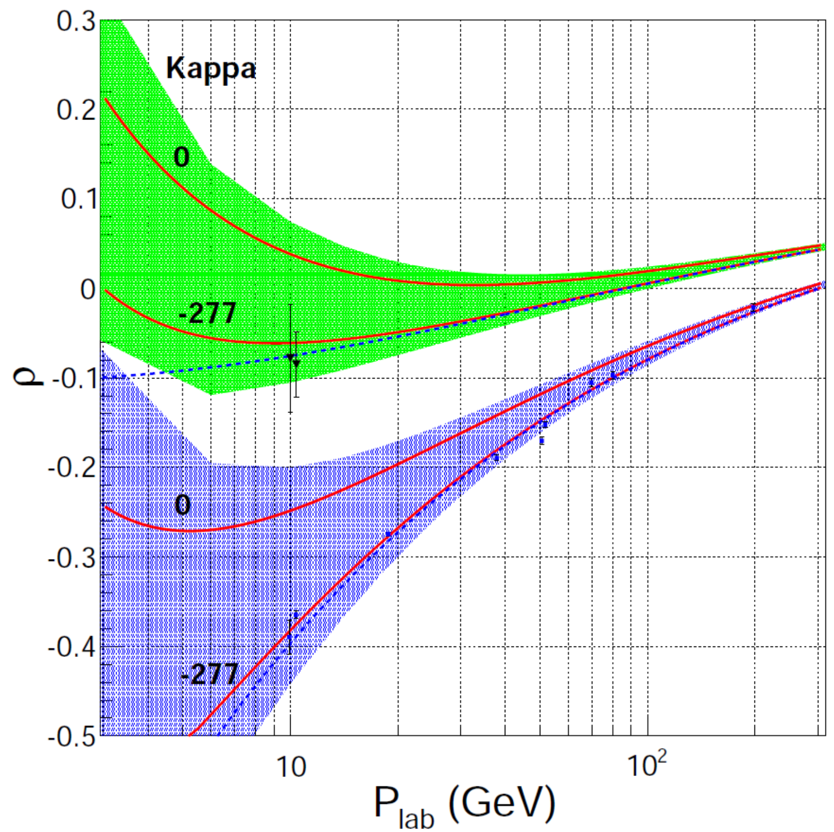
$$R_2\left(\frac{s}{s_0}\right)^{-\eta_2}\cot\left(\frac{\pi\eta_2}{2}\right) + R_2\left(\frac{s_0}{2m^2}\right)^{\eta_2}\left(\frac{2m^2}{s}\right)^2\left(\frac{2/\pi}{2-\eta_2}\right)$$

PDG uses approximated DR forms of

$$\sigma\,\rho^{a\mp b} = \left[\pi H\log\left(\frac{s}{s_M^{ab}}\right) - R_1^{ab}\left(\frac{s}{s_M^{ab}}\right)^{-\eta_1}\tan\left(\frac{\eta_1\pi}{2}\right) \pm R_2^{ab}\left(\frac{s}{s_M^{ab}}\right)^{-\eta_2}\cot\left(\frac{\eta_2\pi}{2}\right)\right]$$

K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

Exact form and PDG approximation



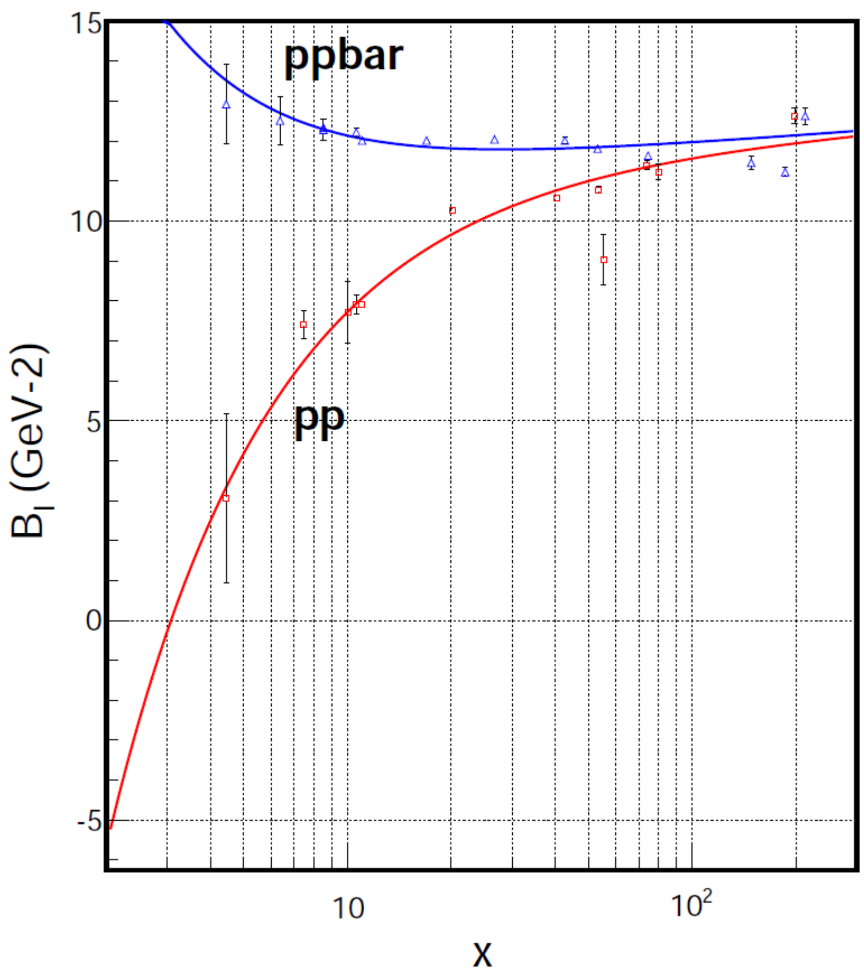
Imaginary slope

Extension of forward imaginary amplitude of PDG

$$\exp[-B_I t/2]$$

$B_I(s)$ parametrized as $$B_I\binom{\mathrm{pp}}{\mathrm{p\bar{p}}}(x) = b_0 + b_1\log(x) + b_2\log^2(x) + b_3 x^{-\eta_3} \pm b_4 x^{-\eta_4}$$

Parametrization of very low energy data for pp and ppbar



Dispersion Relation for slopes

E. Ferreira, *Int. J. Mod. Phys. E* **16**, 2893 (2007)

$$\frac{\partial \mathrm{Re}\, F_+(x,t)}{\partial t}\Big|_{t=0} = \frac{2}{\pi}\; s\; x\; \frac{1}{2}\; \mathbf{P}\int_1^{+\infty} \left\{ \frac{[P + H\log^2(x'/x_0) + R_1(x'/x_0)^{-\eta_1}]}{[x'^2 - x^2]}\; [b_0 + b_1\log(x') + b_2\log^2(x') + b_3 x'^{-\eta_3}] + \frac{R_2(x'/x_0)^{-\eta_2}}{[x'^2 - x^2]} b_4 x'^{-\eta_4} \right\} dx'$$

$$\frac{\partial \mathrm{Re}\, F_-(x,t)}{\partial t}\Big|_{t=0} = \frac{2}{\pi}\; s\; \frac{1}{2}\; \mathbf{P}\int_1^{+\infty} \left\{ \frac{x'[P + H\log^2(x'/x_0) + R_1(x'/x_0)^{-\eta_1}]}{[x'^2 - x^2]}\; b_4\; x'^{-\eta_4} + \frac{x' R_2(x'/x_0)^{-\eta_2}}{[x'^2 - x^2]}[b_0 + b_1\log(x') + b_2\log^2(x') + b_3 x'^{-\eta_3}] \right\} dx'$$

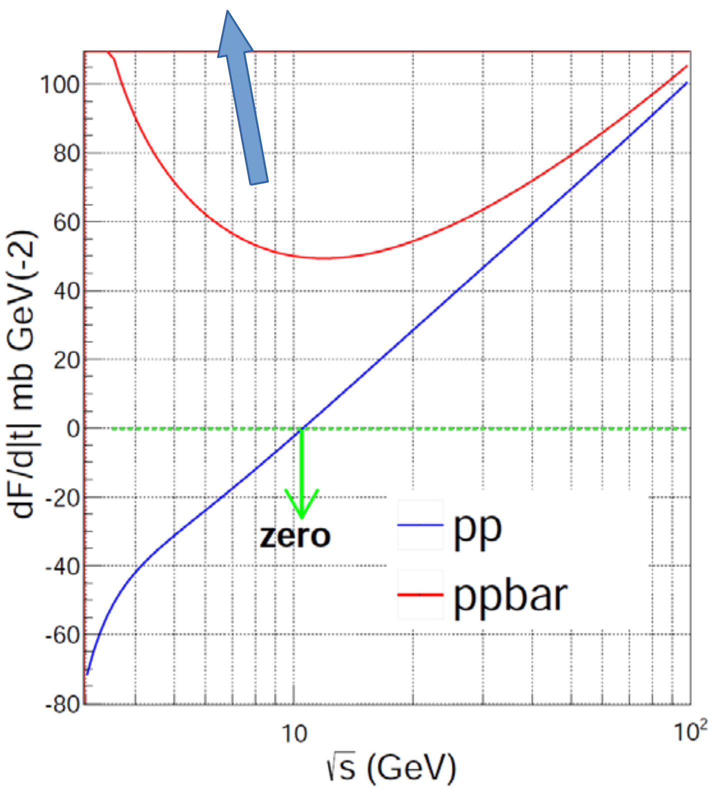
We write

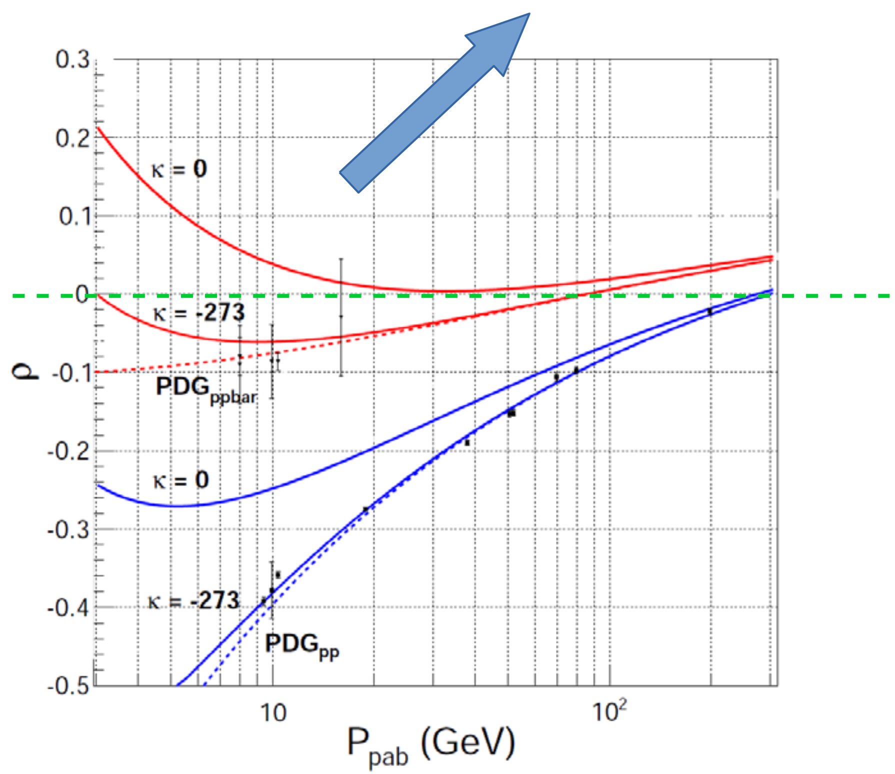
$$\frac{\partial \mathrm{Re}\, F_+(x,t)}{\partial t}\Big|_{t=0} = \frac{1}{2}\left[(\sigma\rho B_R)\;(\mathrm{p\bar{p}}) + (\sigma\rho B_R)\;(\mathrm{pp})\right]$$

$$\frac{\partial \mathrm{Re}\, F_-(x,t)}{\partial t}\Big|_{t=0} = \frac{1}{2}\left[(\sigma\rho B_R)\;(\mathrm{p\bar{p}}) - (\sigma\rho B_R)\;(\mathrm{pp})\right]$$

No zero for ppbar

Two zeros for ppbar





Linear t correction in forward scattering

$$T_I^N(t) = \frac{1}{4\sqrt{\pi}(\hbar c)^2}\sigma(1-\mu_I t)e^{B_I t/2}$$

$$T_R^N(t) = \frac{1}{4\sqrt{\pi}(\hbar c)^2}\sigma(\rho-\mu_R t)e^{B_R t/2}$$

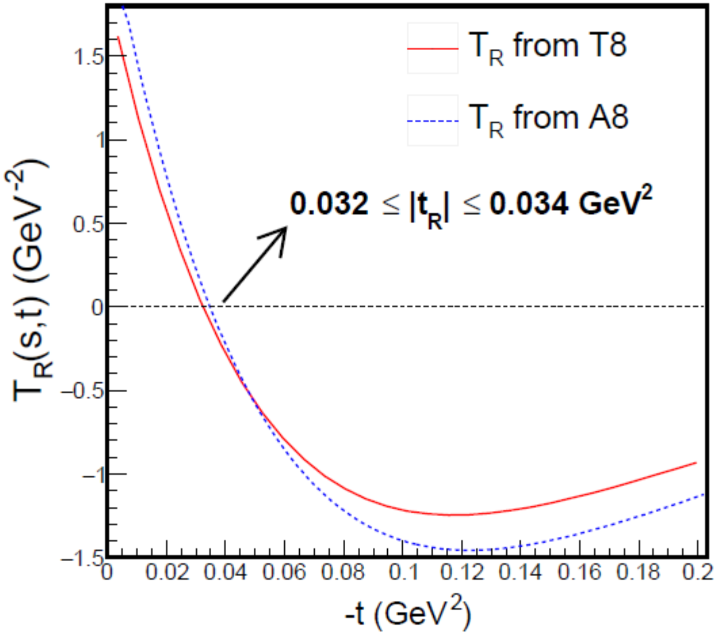
real and imaginary nuclear amplitudes

A. K. K., E. Ferreira, T. Kodama and M. Rangel, *Eur. Phys. J. C* **77**, (2017) 12, 877

$\sqrt{s}$ (GeV)	dataset	$\Delta\|t\|$ range (GeV^2)	N points	Ref.	σ (mb)	B_I (GeV^{-2})	ρ
7	Totem T7	0.005149-0.3709	87	1	98.6±2.2	19.9±0.3	0.14 (fix)[a]
7	Atlas A7	0.0062-0.3636	40	2	95.35 ± 0.38	19.73 ± 0.14	0.14 (fix)[b]
8	Totem T8	0.000741-0.19478	60	3	103.0 ± 2.3	19.56 ± 0.13	(0.12 ± 0.03)[c]
8	Atlas A8	0.0105-0.3635	39	4	96.07 ± 0.18	19.74 ± 0.05	0.1362 (fix)[d]

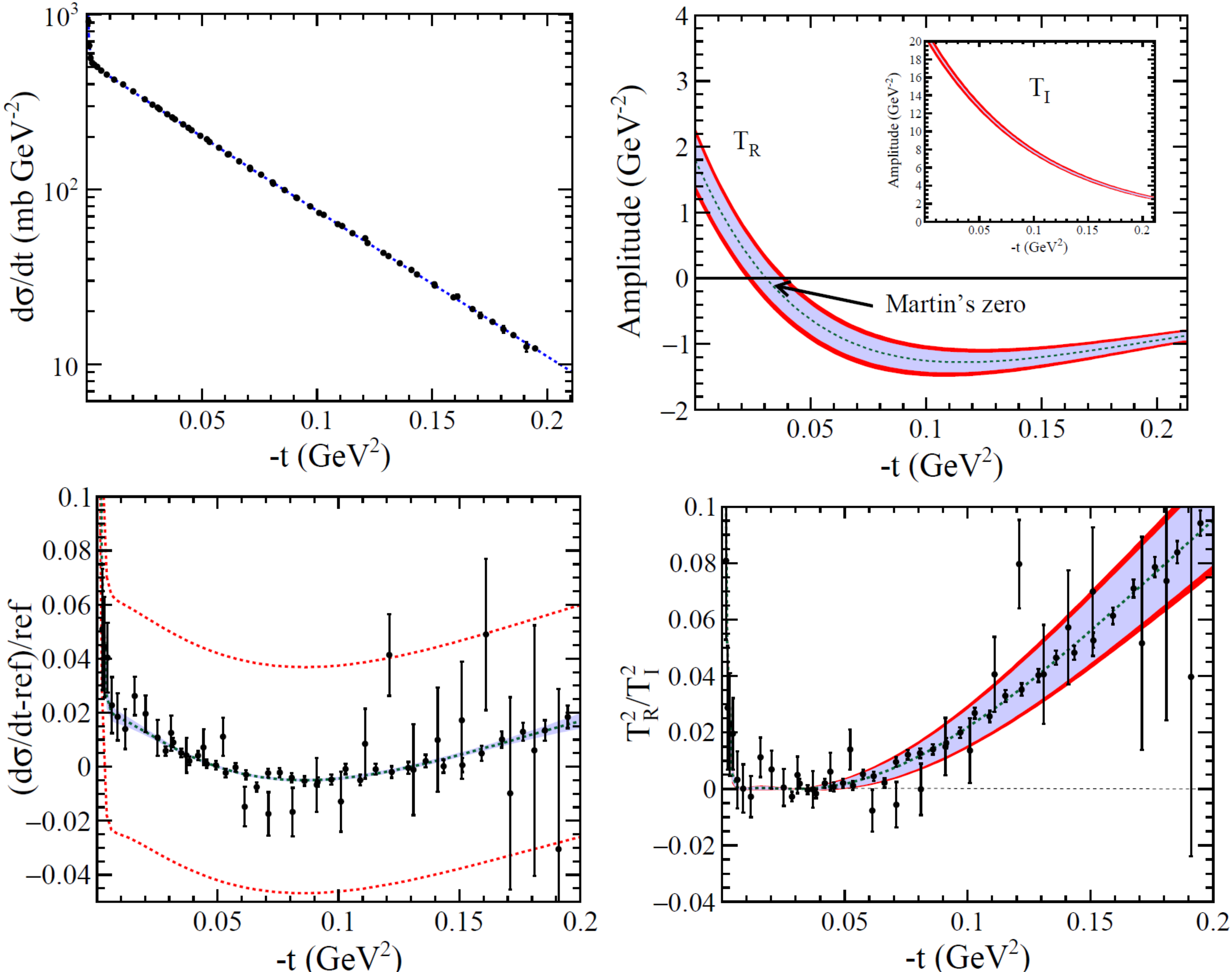
Experimental LHC/TOTEM data points at 7 and 8 TeV

$$t_R = \rho/\mu_R$$

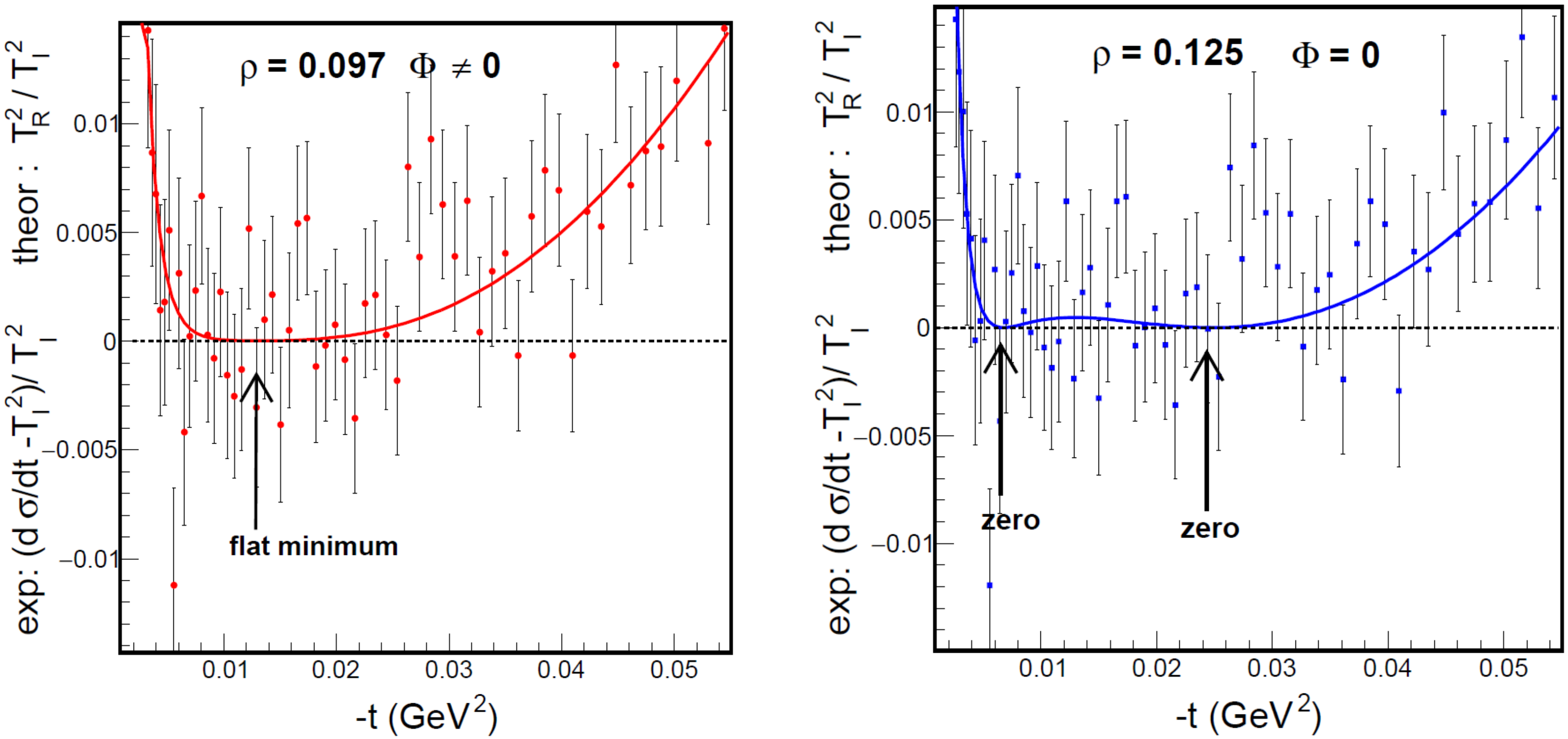


Zero of the real amplitude goes to the origin faster than the zero of imaginary part for large energies

A. Martin, *Lett. Nuovo- Cim.* **7**, 811 (1973)


dσ/dt (mb GeV-2)
-t (GeV2)
TR
TI
Amplitude (GeV-2)
Martin's zero
(dσ/dt-ref)/ref
TR2/TI2

T8 analysis



Totem data at 13 TeV

A. K. Kohara., E. Ferreira, and M. Rangel, *Phys. Lett. B* **789** (2019) 1-6

The first zero in the second plot is the interplay between real and Coulomb amplitude!!!

Simple model for zeros, crossing and 'scaling'

Considering an scaled amplitude
Non-crossing symmetric

$$F^N(E,t) \sim \mathrm{i}CE\log^2(E)\, f(\tau) \qquad \tau = t\log^2 E$$

A. Martin, *Lett. Nuovo Cimento* **7** (1973) 811

Crossing symmetric amplitude

$$F^N(E,t) \sim \mathrm{i}CE\left(\log E - \mathrm{i}\frac{\pi}{2}\right)^2 f(\tau') \qquad \tau' = t\left(\log E - \mathrm{i}\frac{\pi}{2}\right)^2$$

Inspired in the above we define

$$F^N_{\mp}(s,t) = F^N_{\mp}(s) f(\tau') = \left[F^R_{\mp}(s) + \mathrm{i}F^I_{\mp}(s)\right] f(\tau')$$

$$f(\tau') = \mathrm{e}^{\tau'} = \mathrm{e}^{\Omega'_R(s,t) + \mathrm{i}\Omega'_I(s,t)}$$

$$F^R_{\mp}(s) = s[\beta(P_1 + 2H\log s) - R_1 s^{-\eta_1}\sin(\eta_1\beta) \mp R_2 s^{-\eta_2}\cos(\eta_2\beta)]$$

$$F^I_{\mp}(s) = s[P + P_1\log s + H(\log^2 s - \beta^2) + R_1 s^{-\eta_1}\cos(\eta_1\beta) \pm R_2 s^{-\eta_2}\sin(\eta_2\beta)]$$

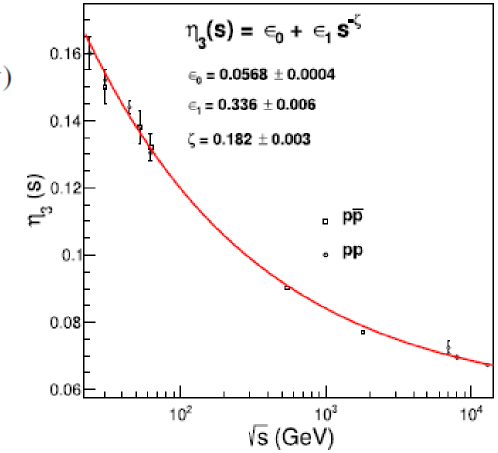
$$\Omega_{\mathrm{R}}(s,t) = [b_0 + b_1\log s + b_2(\log^2 s - \beta^2) + b_3 s^{-\eta_3}\cos(\eta_3\beta)]t$$

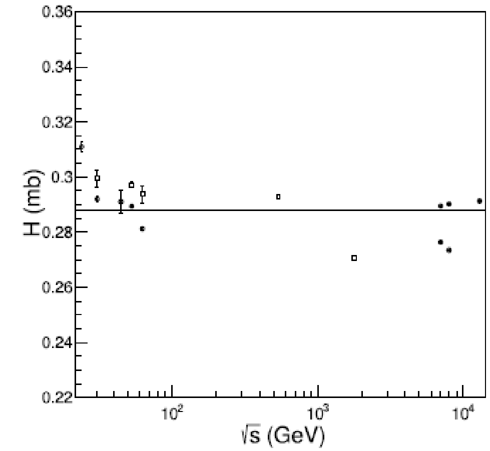
$$\Omega_{\mathrm{I}}(s,t) = -[b_1\beta + 2b_2\beta\log s - b_3 s^{-\eta_3}\sin(\eta_3\beta)]t$$

A. K. Kohara, *J. Phys. G: Nucl. Part. Phys.* **46** (2019) 125001

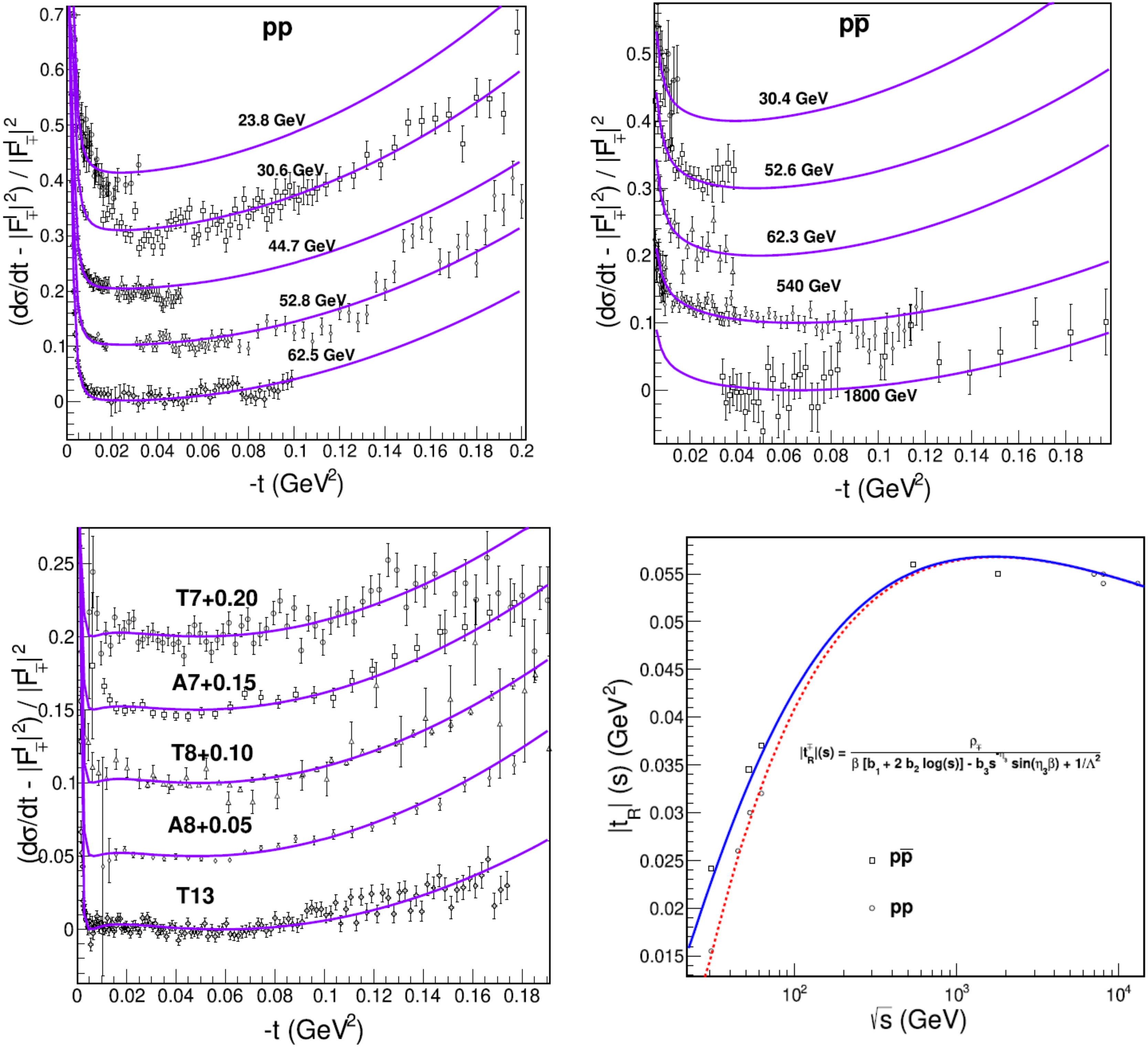
$$F^R_{\mp}(s,\,t) = F^I_{\mp}(s)\left[-\sin\Omega_I(s,\,t) + \frac{F^R_{\mp}(s)}{F^I_{\mp}(s)}\cos\Omega_I(s,\,t)\right]e^{\Omega_R(s,t)}$$

$$F^I_{\mp}(s,\,t) = F^I_{\mp}(s)\left[\cos\Omega_I(s,\,t) + \frac{F^R_{\mp}(s)}{F^I_{\mp}(s)}\sin\Omega_I(s,\,t)\right]e^{\Omega_R(s,t)}.$$





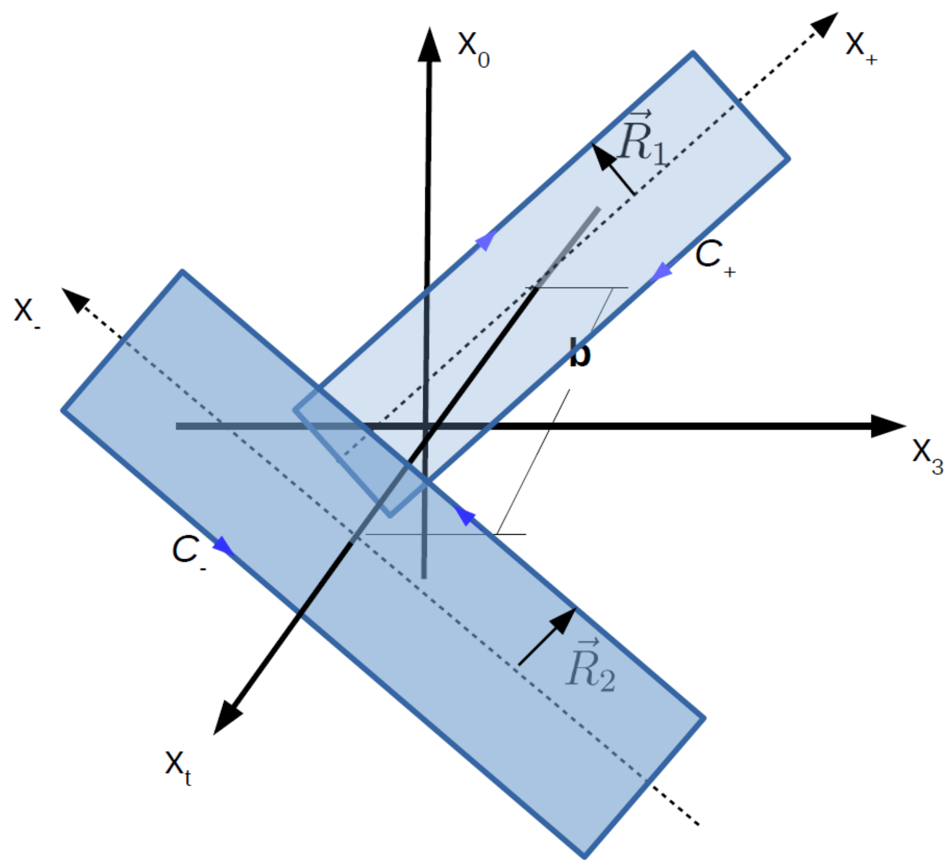
$\sqrt{s}$ (GeV)	Parameters: H (mb)	Parameters: η_3	Derived quantities: σ (mb)	ρ	B (GeV^{-2})	$\sigma_{elas.}$ (mb)	χ^2/ndf
pp							
23.882	0.311 ± 0.002	0.16 (fix)	39.57	0.034	11.77	7.27	90.7/62
30.6	0.292 ± 0.001	0.1522 ± 0.001	39.79	0.049	12.23	7.03	94.1/68
44.7	0.291 ± 0.0004	0.144 (fix)	41.51	0.077	13.12	7.08	87.3/67
52.8	0.2894 ± 0.0003	0.1383 ± 0.0003	42.41	0.088	13.26	7.30	245/88
62.5	0.2812 ± 0.0005	0.1304 ± 0.0004	42.76	0.092	13.15	7.49	111.1/62
200*	0.2887 (fix)	0.106 (fix)	52.05	0.133	14.61	9.94	—
900*	0.2887 (fix)	0.085 (fix)	68.38	0.145	16.43	15.15	—
2760*	0.2887 (fix)	0.076 (fix)	84.04	0.143	18.23	20.51	—
7000	0.2895 ± 0.0003	0.0735 ± 0.0002	99.51	0.138	20.39	25.57	74.4/59
7000	0.2764 ± 0.0002	0.0707 ± 0.0001	95.43	0.136	19.90	24.11	42.9/33
8000	0.2903 ± 0.0001	0.0694 ± 0.0001	102.12	0.137	19.65	27.99	72.5/58
8000	0.2735 ± 0.0001	0.0698 ± 0.0001	96.74	0.135	20.11	24.51	28.8/25
13 000	0.2913 ± 0.0001	0.0673 ± 0.0001	111.43	0.134	20.99	31.11	149.4/126
14 000*	0.2887 (fix)	0.067 (fix)	112.65	0.132	21.13	31.15	—
57 000*	0.2887 (fix)	0.063 (fix)	141.59	0.123	24.19	42.93	—
p$\bar{p}$							
30.4	0.299 4 ± 0.003	0.15 (fix)	41.32	0.076	12.05	7.73	22.4/25
52.6	0.2971 ± 0.001	0.138 (fix)	43.44	0.102	13.24	7.69	29.9/27
62.3	0.293 8 ± 0.003	0.132 ± 0.004	44.13	0.107	13.32	7.89	19.9/15
540	0.293 0 ± 0.0004	0.0901 ± 0.0003	62.95	0.145	15.65	13.52	164.9/97
1800	0.270 6 ± 0.001	0.0771 ± 0.0004	73.71	0.141	17.19	16.78	43.8/53


pp
23.8 GeV
30.6 GeV
44.7 GeV
52.8 GeV
62.5 GeV
(dσ/dt - |F^I_∓|^2) / |F^I_∓|^2
-t (GeV^2)
pp̄
30.4 GeV
52.6 GeV
62.3 GeV
540 GeV
1800 GeV
T7+0.20
A7+0.15
T8+0.10
A8+0.05
T13
|t_R^∓|(s) = ρ_∓ / β [b_1 + 2 b_2 log(s)] - b_3 s^-η_3 sin(η_3 β) + 1/Λ^2
pp̄
pp
|t_R| (s) (GeV^2)
√s (GeV)

QCD inspired model in the framework of Stochastic Vacuum model

Wilson loop is defined $W[C] \equiv \text{ tr } Pe^{-ig \oint_{C(x,x)} dz^\mu A_\mu(z)} = \text{ tr } V[C(x,x)]$

loop-loop scattering amplitudes $J(\vec{b}, \vec{R}_1, \vec{R}_2) = Z_\psi^{-2} \langle \text{ tr } [V[C_+] - 1] \text{ tr } [V[C_-] - 1] \rangle$



Stochastic Vacuum Model (SVM) framework:

Wilson loop expectation value

H. G. Dosch, *Phys. Lett. B* **190**, 177 (1987)

$$\langle W[C]\rangle \approx \exp\left[-\frac{g^2}{2^2 2!}\int_S dS^{\mu_1\nu_1}(z_1)dS^{\mu_2\nu_2}(z_2)\langle \text{ tr } F_{\mu_1\nu_1}[z_1, C(w,z_1)]F_{\mu_2\nu_2}[z_2, C(w,z_2)]\rangle\right]$$

Expanding the Wilson loop

$$J(\vec{b},\vec{R}_1,\vec{R}_2) = -(-ig)^4(\frac{1}{2}!)^2 \text{ tr } [\tau_{C_1}\tau_{C_2}] \text{ tr } [\tau_{D_1}\tau_{D_2}] \int_{S_1}\prod_{i=1}^{2} dS^{\mu\nu}(x_i)\int_{S_2}\prod_{j=1}^{2} dS^{\alpha\beta}(y_j)$$

$$\times\frac{1}{N_C^2}\langle F^{C_1}_{\mu_1\nu_1}(x_1,w)F^{C_2}_{\mu_2\nu_2}(x_2,w)F^{D_1}_{\alpha_1\beta_1}(y_1,w)F^{D_2}_{\alpha_2\beta_2}(y_2,w)\rangle + \text{ (higher order)}$$

SVM factorization

$$\langle F^{C_1}F^{C_2}F^{D_1}F^{D_2}\rangle = \langle F^{C_1}F^{C_2}\rangle\langle F^{D_1}F^{D_2}\rangle + \langle F^{C_1}F^{D_1}\rangle\langle F^{C_2}F^{D_2}\rangle + \langle F^{C_1}F^{D_2}\rangle\langle F^{C_2}F^{D_1}\rangle$$

Dimensionless hadron-hadron amplitudes

$$J_{H_1H_2}(\vec{b},S_1,S_2) = \int d^2\vec{R}_1\int d^2\vec{R}_2\ J(\vec{b},\vec{R}_1,\vec{R}_2)\ |\psi_1(\vec{R}_1)|^2\ |\psi_2(\vec{R}_2)|^2$$

H.G. Dosch, E. Ferreira and A. Kramer, *Phys. Lett. B* **289**, 153 (1992); *Phys. Lett. B* **318**, 197 (1993) ;*Phys. Rev. D* **50**, (1994)

Hadrons wave functions

$$\psi_H(R) = \sqrt{(2/\pi)}\frac{1}{S_H}e^{-R^2/S_H^2}$$

Asymptotic forms at large *b/a* ,

(a is the vacuum correlation lenght)

$$J(b/a) = \exp\left(-\frac{3\pi}{8}\frac{b}{a}\right)\left[\frac{A_1(S_H/a)}{b/a} + \frac{A_2(S_H/a)}{(b/a)^2} + \ldots\right]$$

F. Pereira and E. Ferreira, *Phys. Rev. D* **55**, 130 (1997)

Small and intermediate values of *b/a* need Gaussian form of $J(b/a)$

Suggested form $J(b/a) = J(0)\left[e^{-b^2/a_1} + a_2\ A_\gamma(b)\right]$ with $A_\gamma(b) = \frac{e^{-\rho_4\sqrt{\gamma^2+b^2}}}{\sqrt{\gamma^2+b^2}}(1 - e^{\rho_4\gamma-\rho_4\sqrt{\gamma^2+b^2}})$

Fourier transform gives analytical closed form

$$T(s,t) = is[\langle g^2 FF\rangle a^4]^2 a^2\pi\left\{J(0)\left[a_1 e^{-a^2|t|a_1/4} + 2a_2\ A_\gamma(t)\right]\right\}$$ with $$A_\gamma(t) = \frac{e^{-\gamma\sqrt{\rho^2+a^2|t|}}}{\sqrt{\rho^2+a^2|t|}} - e^{\gamma\rho}\frac{e^{-\gamma\sqrt{4\rho^2+a^2|t|}}}{\sqrt{4\rho^2+a^2|t|}}$$

$$\int_0^\infty J_0(\beta v)\,\frac{e^{-\lambda\sqrt{1+v^2}}}{\sqrt{1+v^2}}\,v\,dv = \frac{e^{-\sqrt{\lambda^2+\beta^2}}}{\sqrt{\lambda^2+\beta^2}}$$

analytical integral

KFK ***t*** **– space amplitude**

$$T_I(s,t) = \alpha_I \mathrm{e}^{-\beta_I|t|} + \lambda_I\Psi_I(\gamma_I,t) \qquad \Psi_I(\gamma_I,t) = 2\ \mathrm{e}^{\gamma_I}\left[\frac{\mathrm{e}^{-\gamma_I\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - \mathrm{e}^{\gamma_I}\ \frac{\mathrm{e}^{-\gamma_I\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}}\right]$$

Real amplitude with the same form and different parameters

Energy dependence of KFK amplitudes

Elastic differential cross section $\frac{d\sigma}{dt} = (\hbar c)^2 \; |T_R(s,t) + iT_I(s,t)|^2$

Real and imaginary amplitudes $T_K^N(s,t) = \alpha_K(s) e^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t) + \delta_{K,R} R_{ggg}(t)$

$$\Psi_K(\gamma_K(s),t) = 2\, e^{\gamma_K} \left[\frac{e^{-\gamma_K \sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - e^{\gamma_K} \, \frac{e^{-\gamma_K \sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}} \right]$$

with 8 parameters to be determined at each energy.

tri-gluon exchange

$$R_{ggg}(t) \equiv \pm 0.45 \; t^{-4}(1 - e^{-0.005|t|^4})(1 - e^{-0.1|t|^2})$$

A. Donnachie, P. Landshoff, *Zeit. Phys. C* **2**, 55 (1979); *Phys. Lett. B* **387**, 637(1996).

KFK forward quantities

$$\sigma(s) = 4\sqrt{\pi}\,(\hbar c)^2 \; (\alpha_I(s) + \lambda_I(s))$$ Optical theorem

$$\rho(s) = \frac{T_R^N(s,t=0)}{T_I^N(s,t=0)} = \frac{\alpha_R(s) + \lambda_R(s)}{\alpha_I(s) + \lambda_I(s)}$$ Real/Imaginary

$$B_K(s) = \frac{2}{T_K^N(s,t)} \frac{dT_K^N(s,t)}{dt}\Big|_{t=0} = \frac{1}{\alpha_K(s) + \lambda_K(s)} \left[\alpha_K(s)\beta_K(s) + \frac{1}{8}\lambda_K(s) a_0 \Big(6\gamma_K(s) + 7 \Big) \right]$$

Real and Imaginary slopes

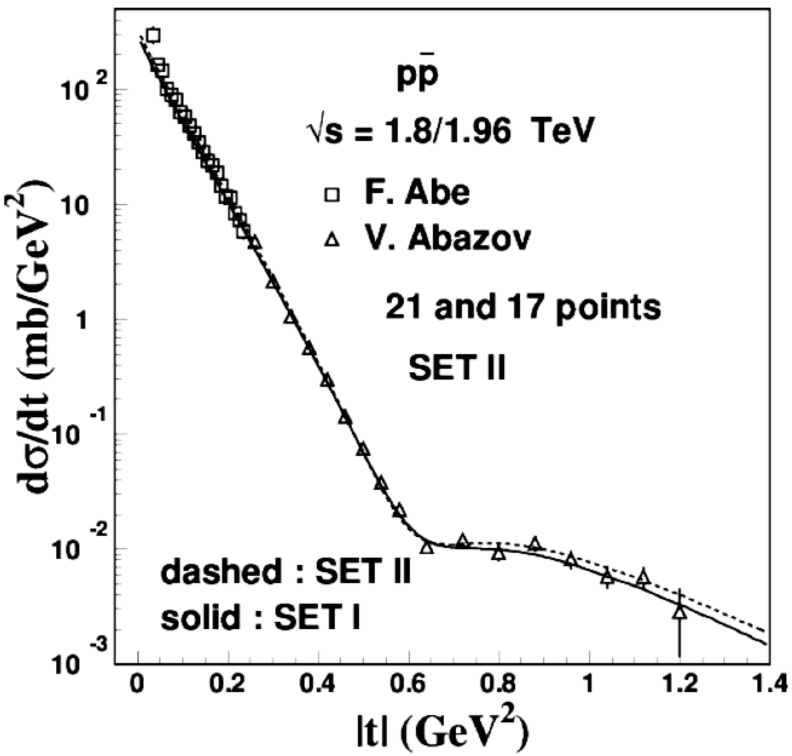
1.8 TeV $p\bar{p}$

Two contradictory measurements

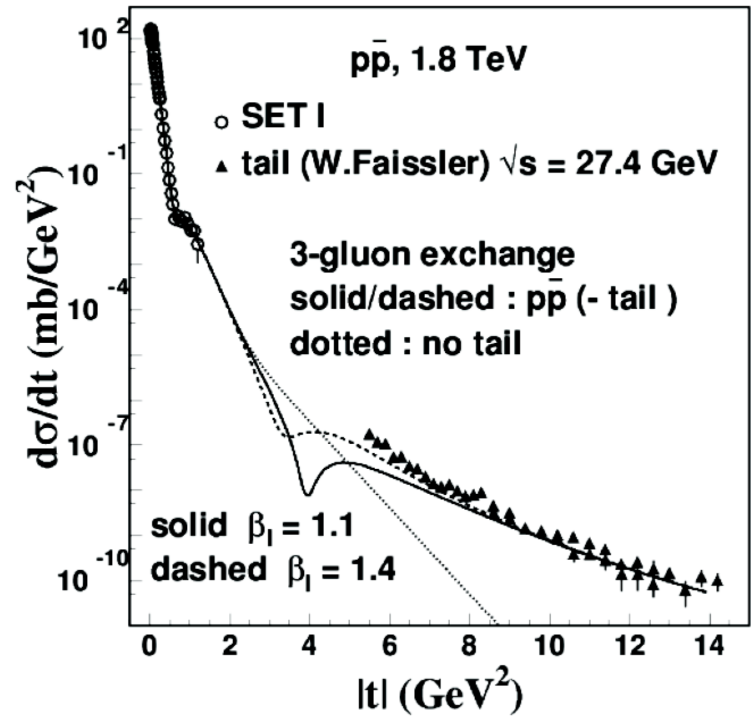
E-710: $\sigma = 80.03 \pm 2.24$ mb

CDF: $\sigma = 72.8 \pm 3.1$ mb

We compatibilize the analysis



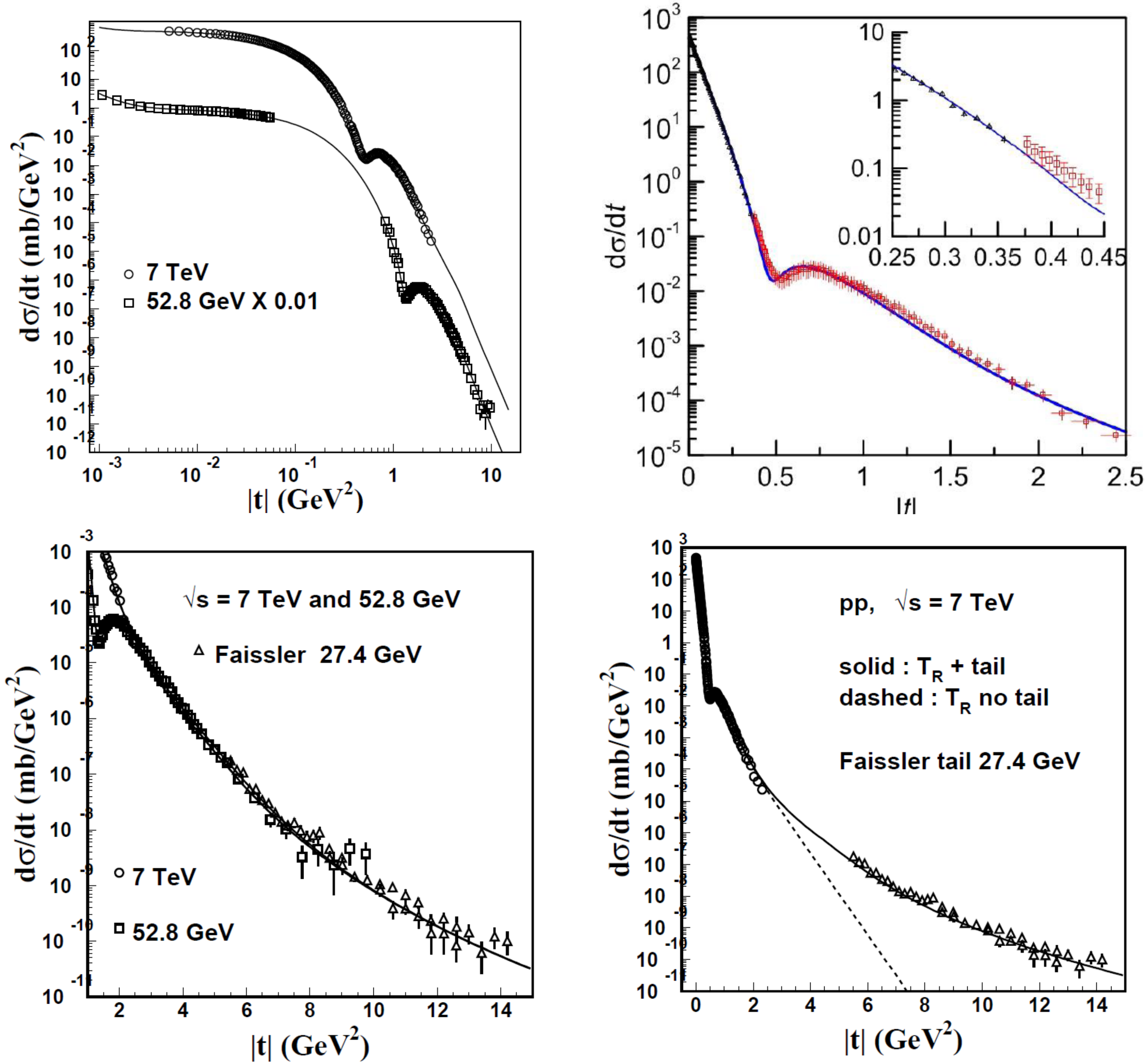
Prediction of a second dip due to tri-gluon exchange



A. Kendi Kohara, E. Ferreira and T. Kodama, *Eur. Phys. J. C* , **73**, 2326 (2013)

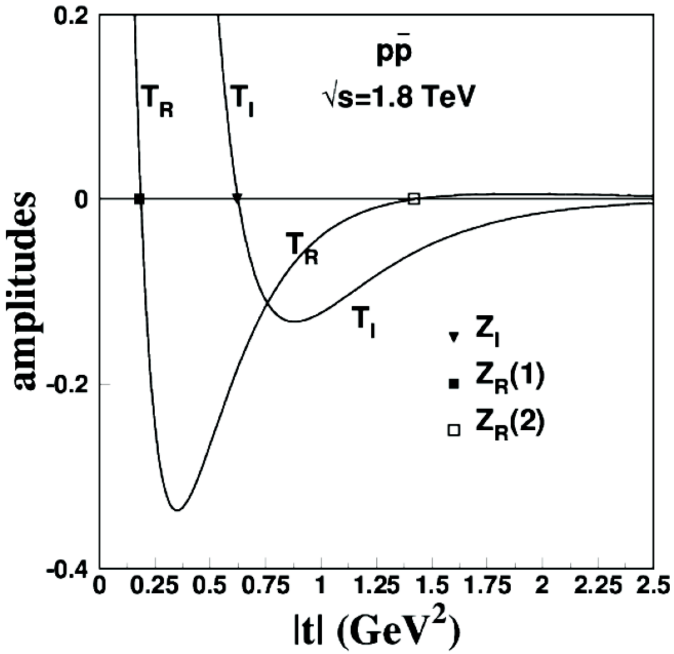
SET	N points	β_I GeV^{-2}	λ_R GeV^{-2}	$\|t\|_{\rm infl}$ GeV^2	$(d\sigma/dt)_{\rm infl}$ mb/GeV^2	σ(el) mb	σ mb	$\langle\chi^2\rangle$
I	52	3.7785 ± 0.0078	3.6443 ± 0.0093	0.745	0.01013	16.67	72.76 ± 0.13	0.7661
II	38	3.5686 ± 0.0186	3.8645 ± 0.0093	0.727	0.01114	18.92	77.63 ± 0.44	1.4961
III	78	3.7441 ± 0.0080	3.6784 ± 0.0096	0.741	0.01029	17.02	73.54 ± 0.20	2.6591

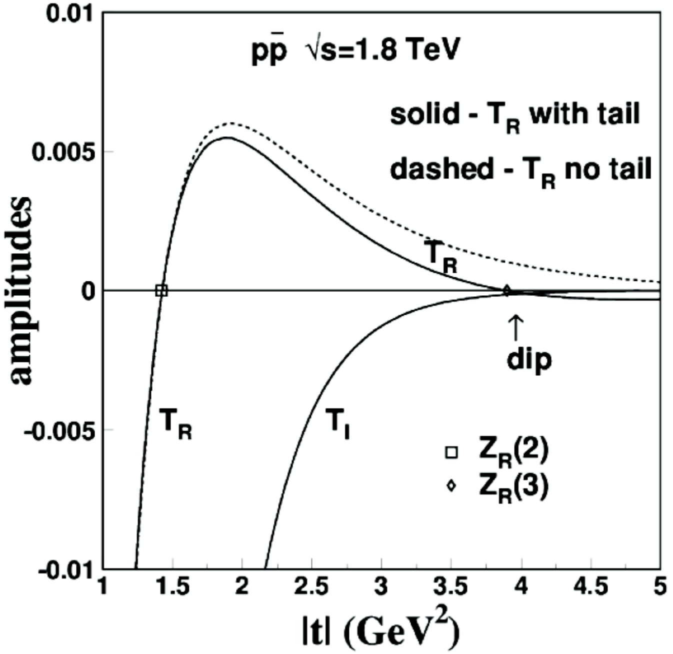
pp at 52 GeV (ISR-CERN) and pp at 7 TeV (TOTEM-CERN)

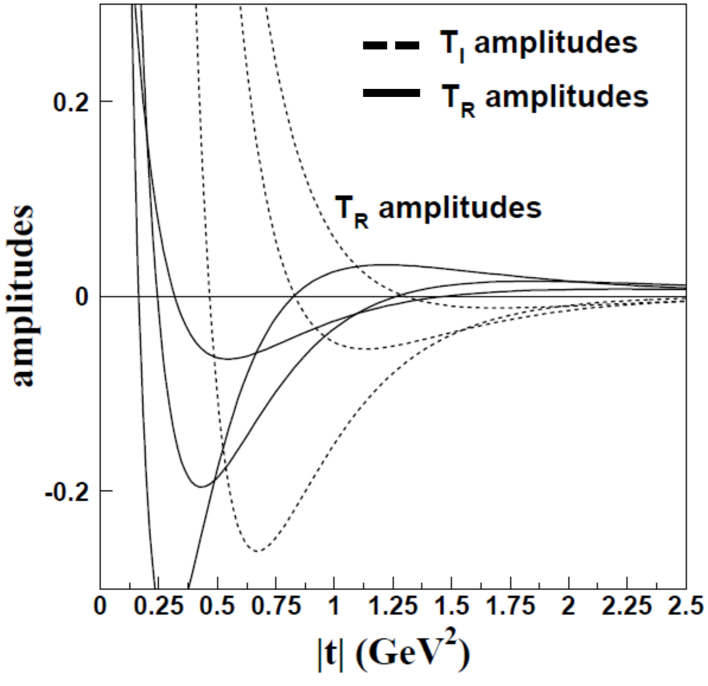


A. Kendi, E. Ferreira and T. Kodama , *Phys.Rev. D* **87**, 054024 (2013)

Real and imaginary amplitudes

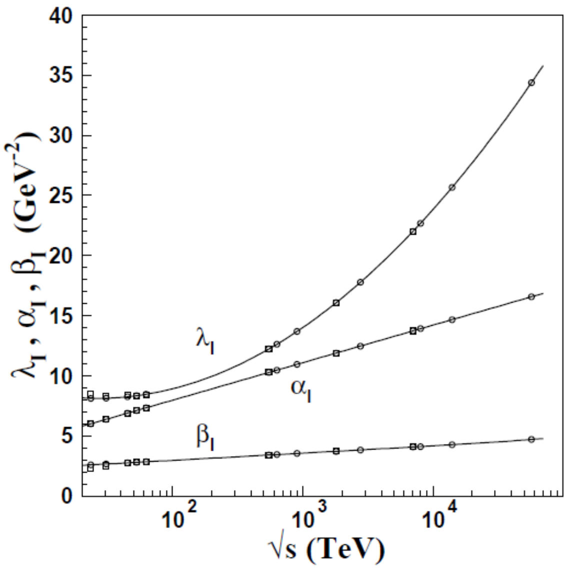


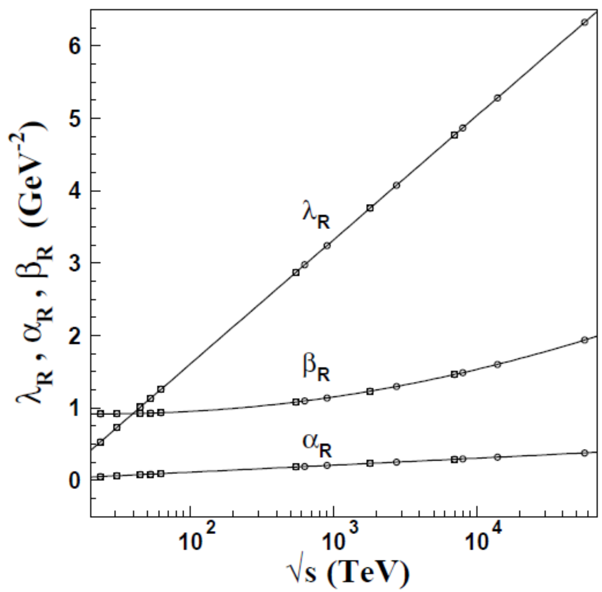


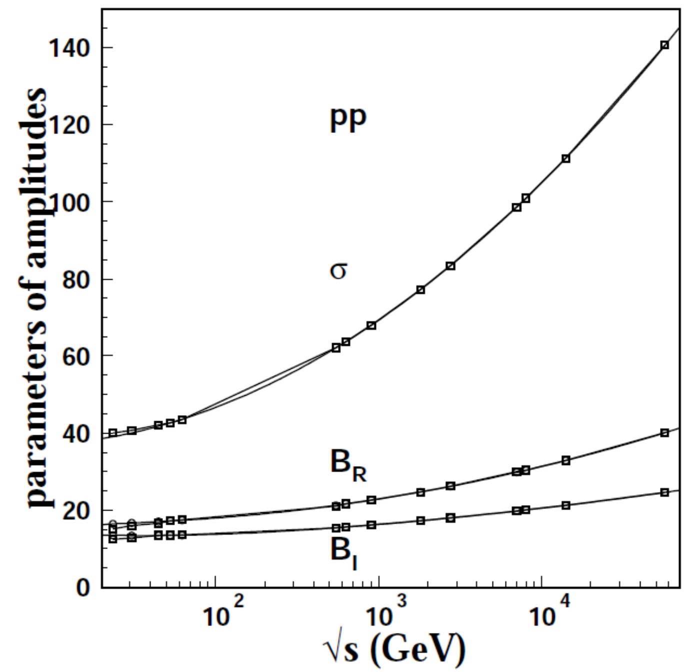


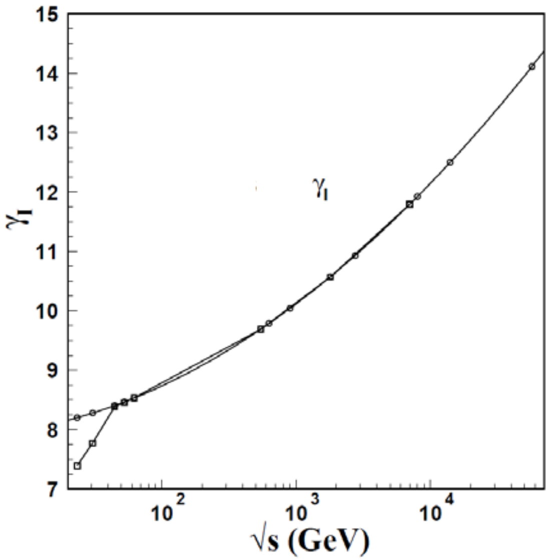
Energy dependence of parameters

forward and integrated quantities

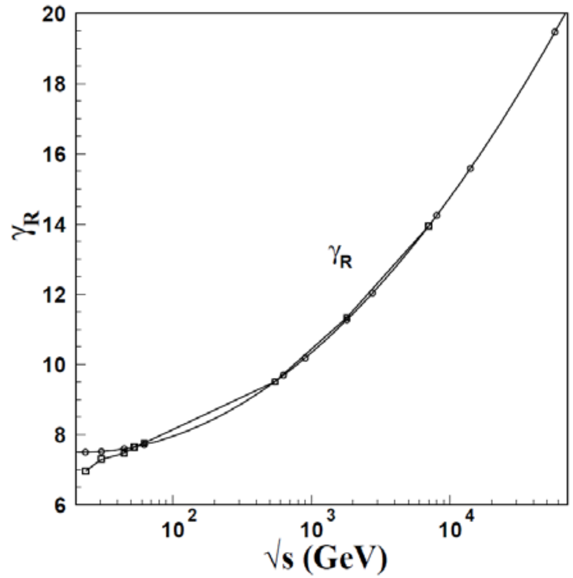






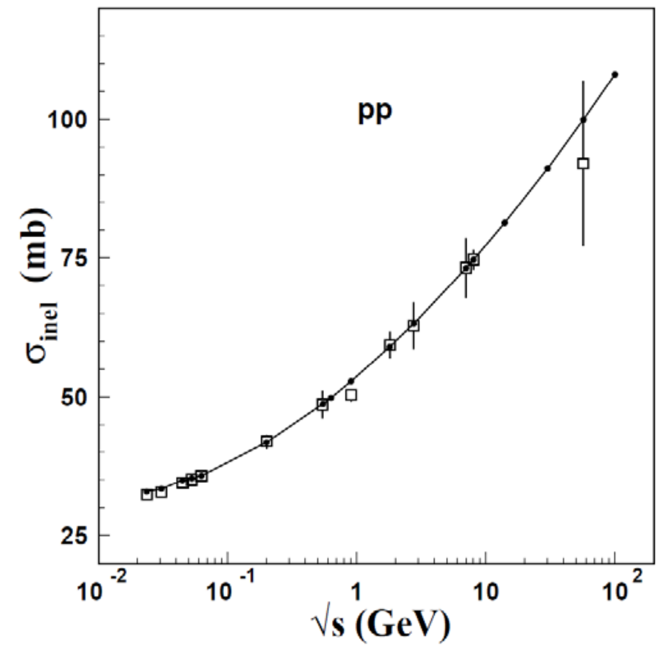


Imaginary



Real

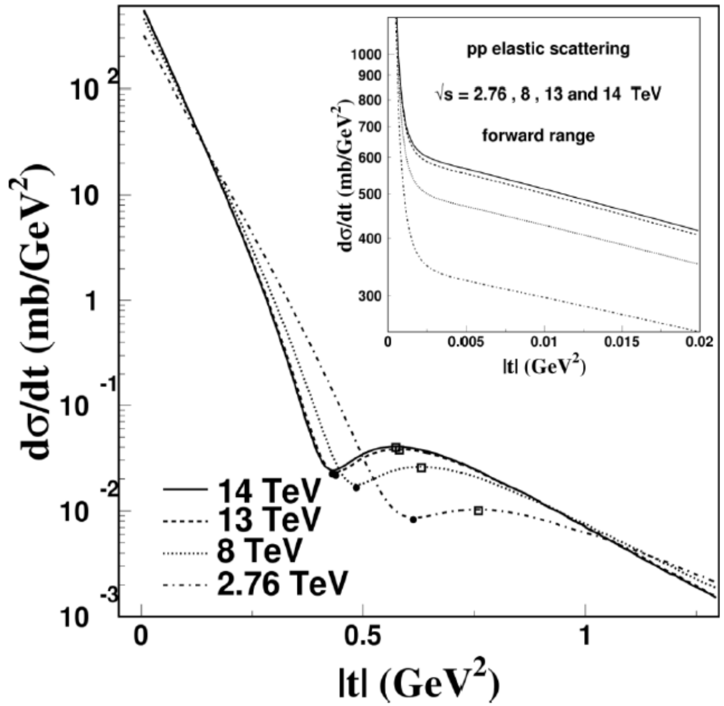
pp inelastic cross section

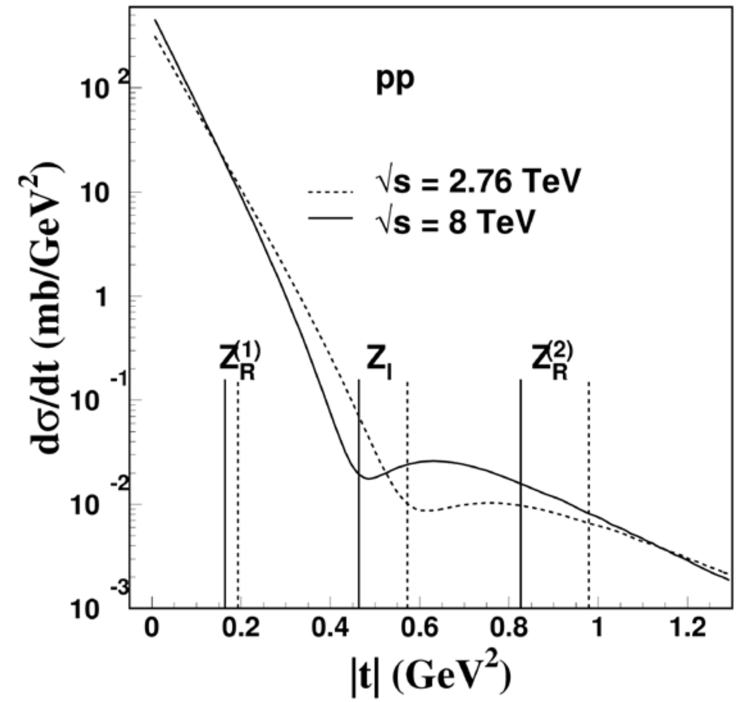


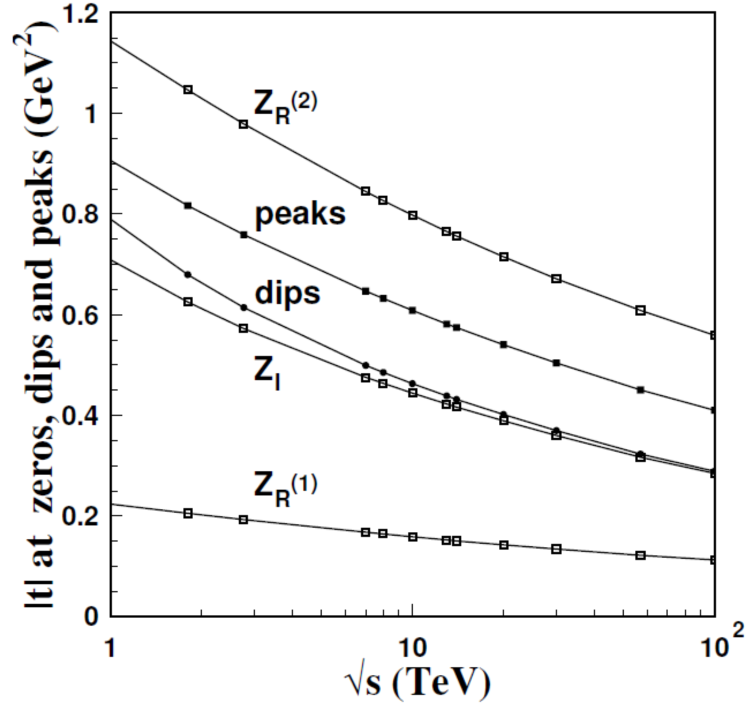
simple logarithmic forms!

Predictions for LHC enegies (zeros, dips, bumps and ratios)

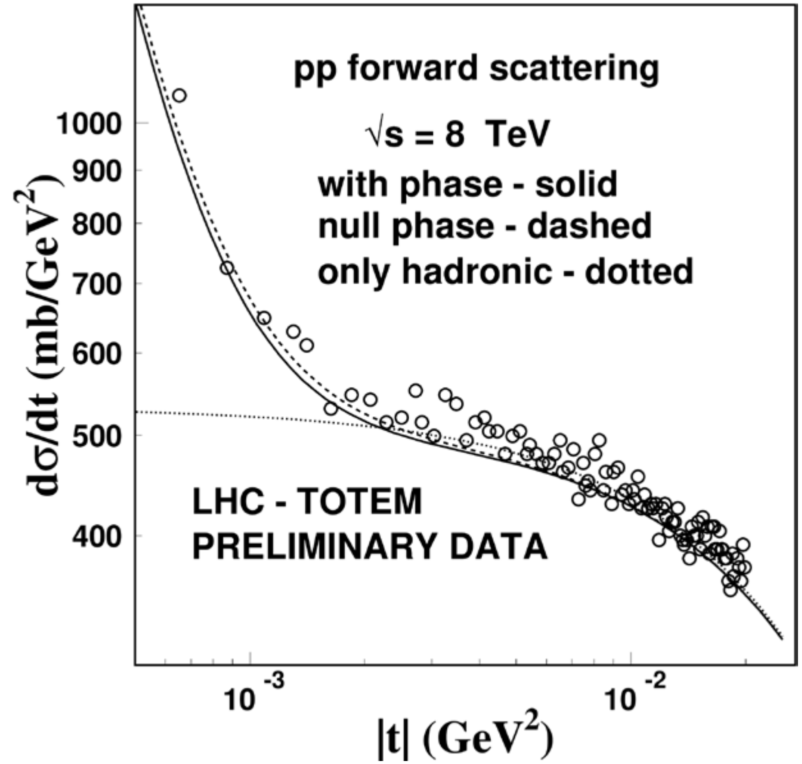
A. Kendi Kohara, E. Ferreira and T. Kodama, *Eur. Phys. J. C* , **74**, 3175 (2014)

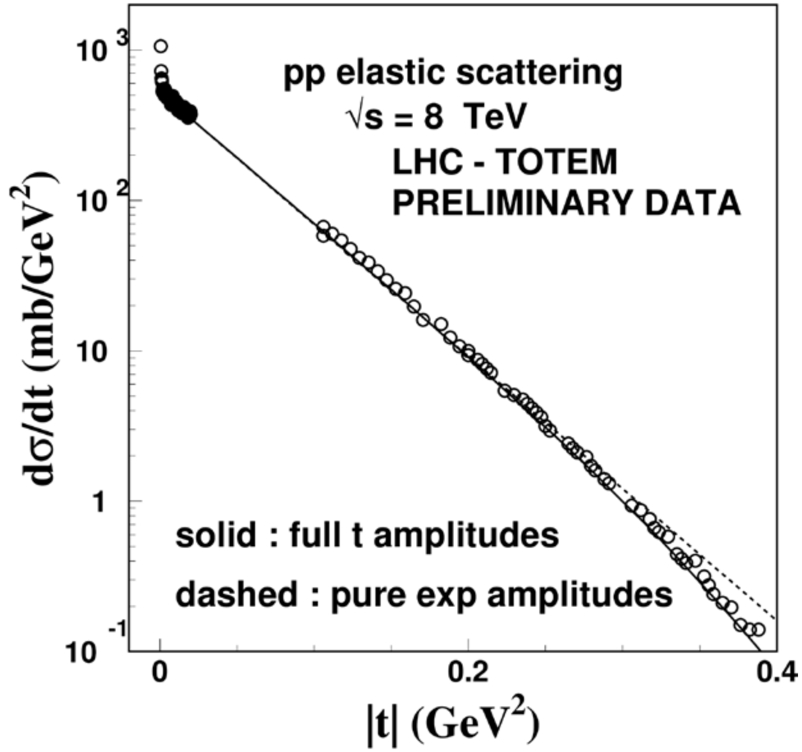


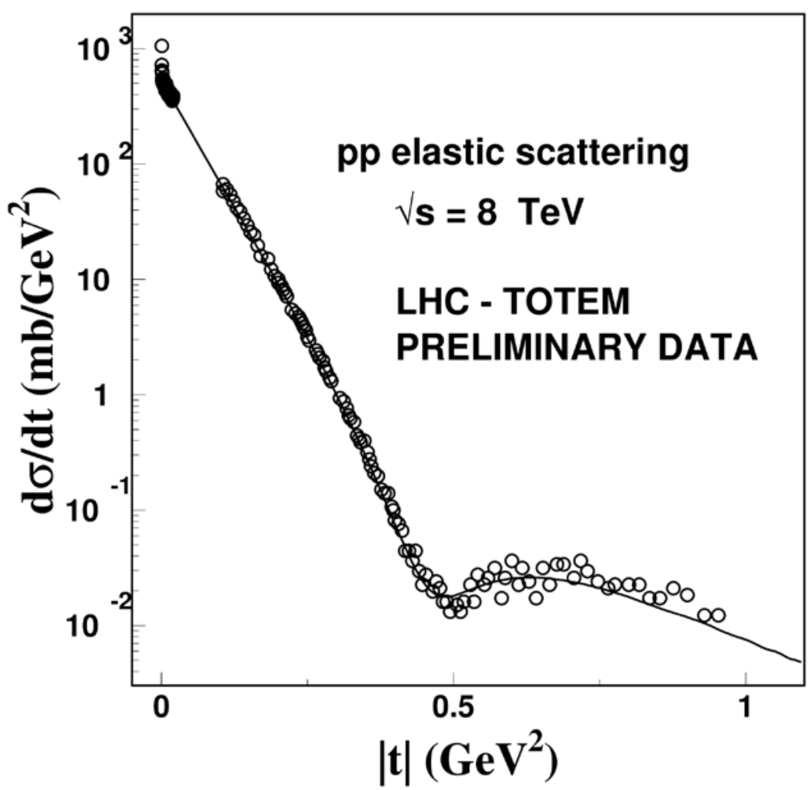


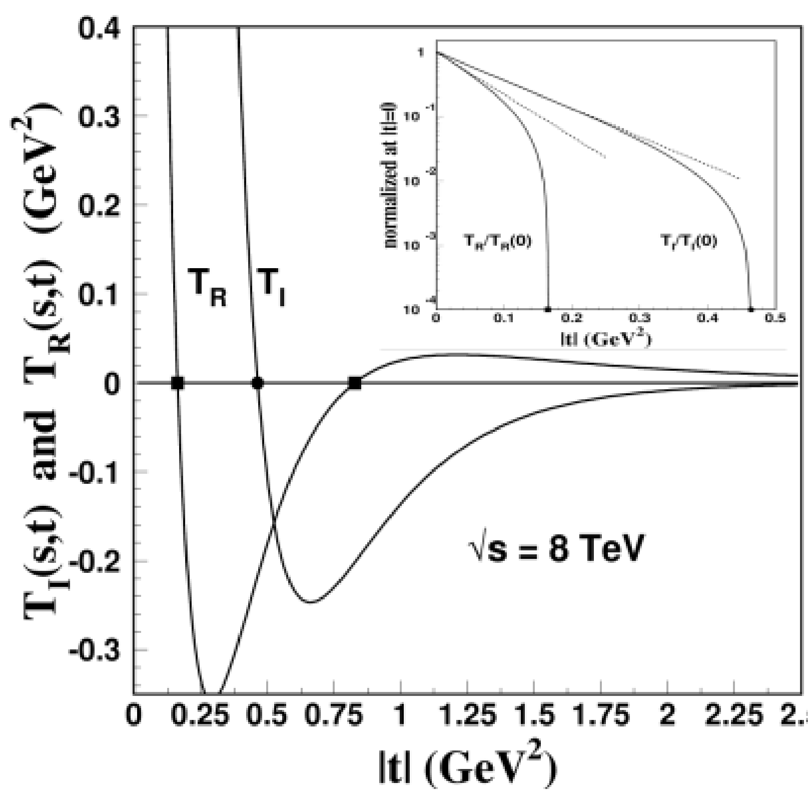


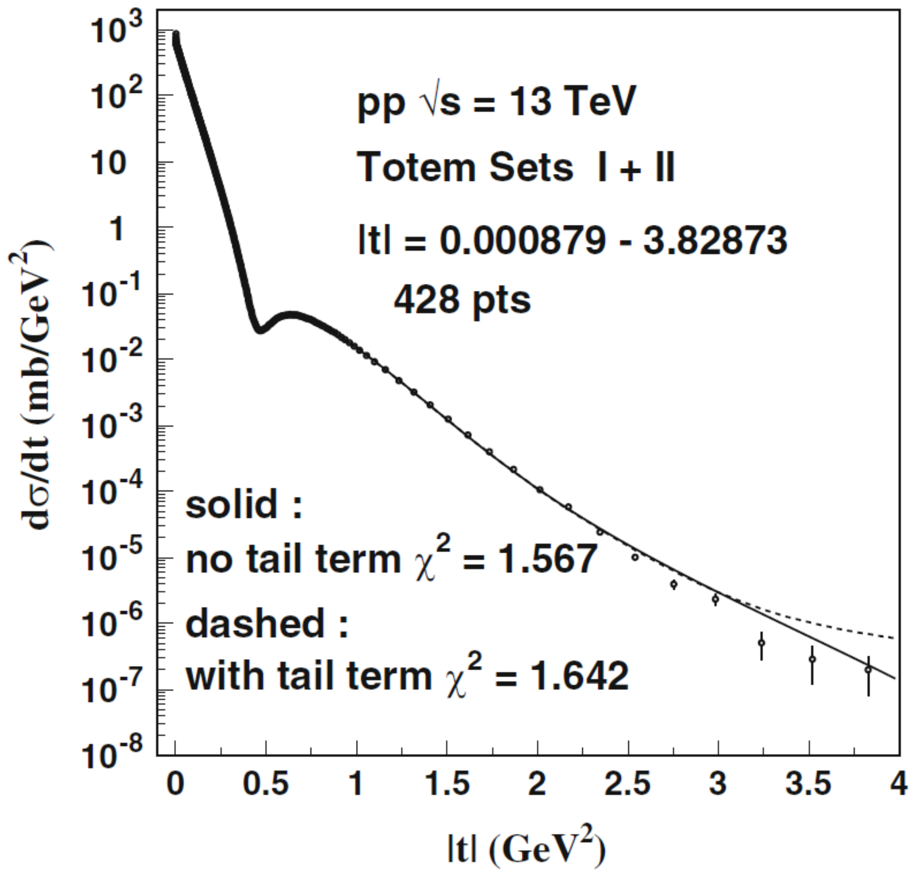
$$A + \frac{1}{a + b\log\sqrt{s} + c\log^2\sqrt{s}}$$


pp forward scattering
√s = 8 TeV
with phase - solid
null phase - dashed
only hadronic - dotted
LHC - TOTEM
PRELIMINARY DATA
dσ/dt (mb/GeV²)
|t| (GeV²)
1000
900
800
700
600
500
400
10⁻³
10⁻²


pp elastic scattering
√s = 8 TeV
LHC - TOTEM
PRELIMINARY DATA
solid : full t amplitudes
dashed : pure exp amplitudes
dσ/dt (mb/GeV²)
|t| (GeV²)
10³
10²
10
1
10⁻¹
0
0.2
0.4


pp elastic scattering
√s = 8 TeV
LHC - TOTEM
PRELIMINARY DATA
dσ/dt (mb/GeV²)
|t| (GeV²)
10³
10²
10
1
10⁻¹
10⁻²
0
0.5
1


T_I(s,t) and T_R(s,t) (GeV⁻²)
|t| (GeV²)
T_R
T_I
√s = 8 TeV
normalized at |t|=0
T_R/T_R(0)
T_I/T_I(0)
|t| (GeV²)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5


pp √s = 13 TeV
Totem Sets I + II
|t| = 0.000879 - 3.82873
428 pts
solid :
no tail term χ2 = 1.567
dashed :
with tail term χ2 = 1.642
dσ/dt (mb/GeV2)
|t| (GeV2)
0
0.5
1
1.5
2
2.5
3
3.5
4
10-8
10-7
10-6
10-5
10-4
10-3
10-2
10-1
1
10
102
103

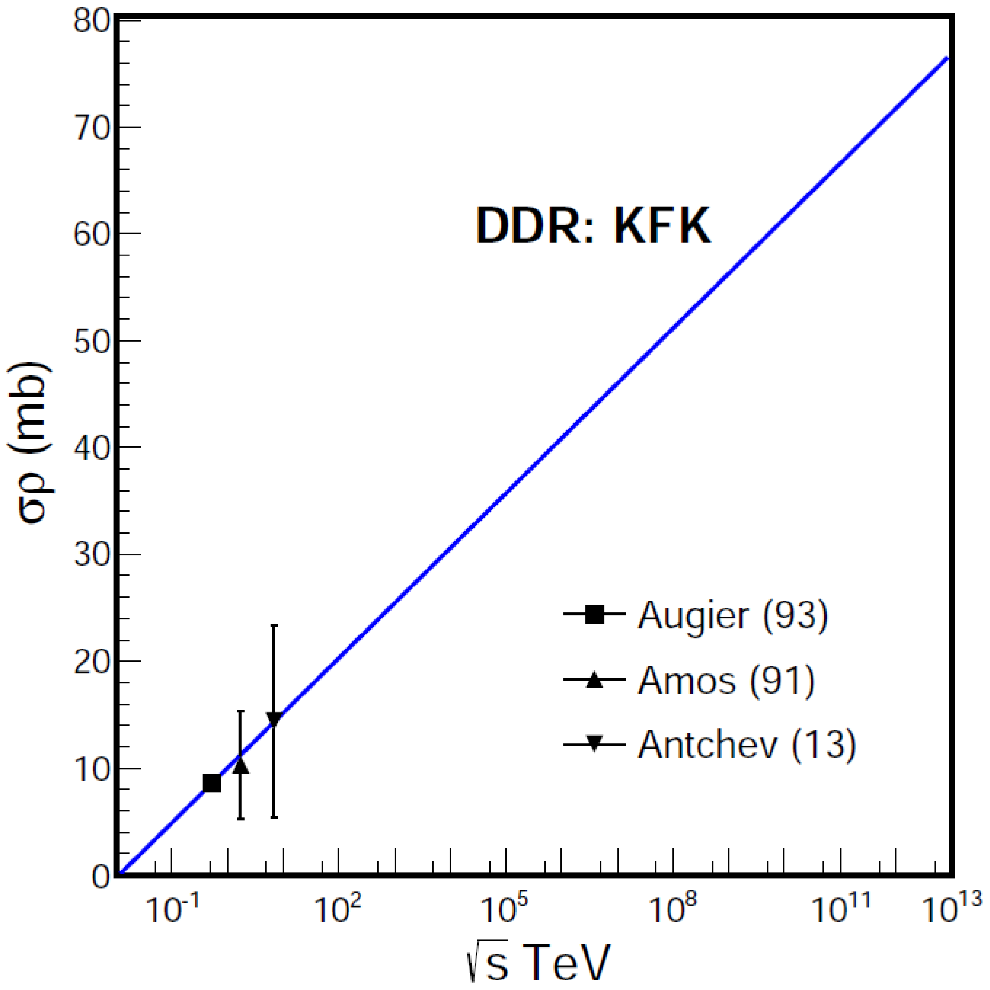
Dispersion relations applied to KFK amplitudes at high energies

Double product from DDR
$$\sigma\rho = (\hbar c)^2 4\pi^{3/2}\left[\frac{\bar{\alpha}_{I1}+\bar{\lambda}_{I1}}{2} + \bar{\lambda}_{I2}\log x\right]$$

Double product from KFK
$$\sigma\rho = 4\sqrt{\pi}\,(\hbar c)^2\left[\bar{\alpha}_{R0}+\bar{\lambda}_{R0}+\left(\bar{\alpha}_{R1}+\bar{\lambda}_{R1}\right)\log x\right]$$

Collecting terms we obtain the correspondence

DDR	KFK
-1.741716	-1.676787
0.404716	0.392804



We compare the collected terms obtained for triple product

DDR	KFK
-18.897808	-15.715451
6.726881	4.286346
-0.262464	-0.292835
0.017041	0.016234

Connection with soft Pomeron

Total cross section in Pomeron framework $\sigma_{\rm Pom} = A + B\ (s/s_0)^{0.096}$

KFK total cross section $\sigma = C_0 + C_1 \ln\sqrt{\frac{s}{s_0}} + C_2 \ln^2\sqrt{\frac{s}{s_0}}$

After a little algebra $\sigma = C_0 - \frac{1}{2}\frac{C_1}{C_2} + \frac{1}{2}\frac{C_1^2}{C_2}\left(1 + x + \frac{1}{2}x^2\right)$ with $x = \frac{C_2}{C_1}\ln\frac{s}{s_0}$

As far as $x \ll 1$ (KFK case $\sqrt{s} < 10^4$ TeV) → $e^x \sim 1 + x + x^2/2$

and we write $\sigma \simeq C_0 - \frac{1}{2}\frac{C_1^2}{C_2} + \frac{1}{2}\frac{C_1^2}{C_2}\left(\frac{s}{s_0}\right)^{C_2/C_1}$

with $C_0 = 69.3286,\quad C_1 = 12.6800,\quad C_2 = 1.2273$

$$\sigma(s) \simeq\ 3.8259 + 65.5026\ (s/s_0)^{0.09679}$$

same intercept of soft Pomeron

b-space (geometric space)

Fourier transform of KFK amplitude $i\sqrt{\pi}\,\left(1-e^{i\chi(s,\vec{b})}\right) \equiv \widetilde{T}(s,\vec{b}) = \widetilde{T}_R(s,\vec{b}) + i\widetilde{T}_I(s,\vec{b})$

Eikonal formalism → $\chi(s,\vec{b}) = \chi_R(s,\vec{b}) + i\chi_I(s,\vec{b})$

with $\widetilde{T}_K(s,\vec{b}) = \dfrac{\alpha_K}{2\beta_K} e^{-b^2/4\beta_K} + \lambda_K \widetilde{\psi}_K(s,b)$ and $\widetilde{\psi}_K(s,b) = \dfrac{2e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}}{a_0\sqrt{\gamma_K^2 + b^2/a_0}}\left[1 - e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}\right]$

The unitarity conditions imposes $\dfrac{\widetilde{T}_R^2}{\pi} \leq e^{-2\chi_I(s,\vec{b})} \leq 1$ or $0 \leq \chi_I \leq -\dfrac{1}{2}\log(\widetilde{T}_R^2/\pi)$

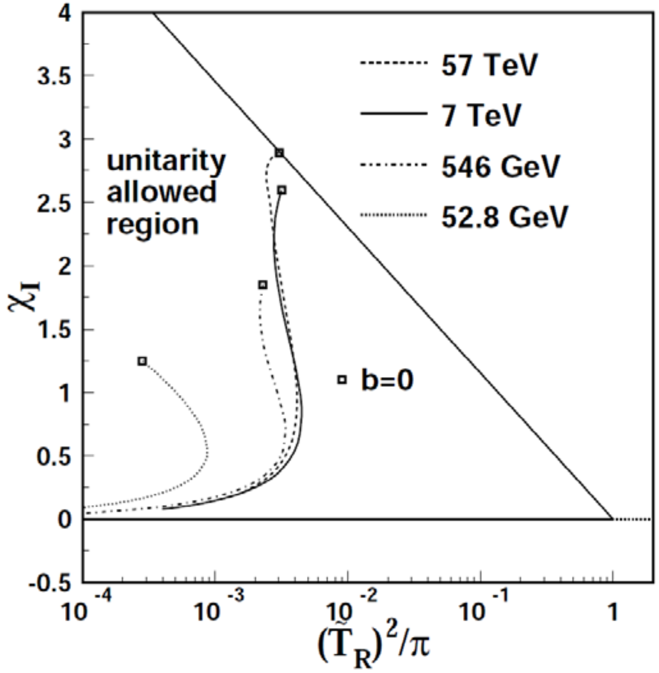
Physical cross sections are written

$$\sigma_{\text{el}}(s) = \frac{(\hbar c)^2}{\pi}\int d^2\vec{b}\,|\widetilde{T}(s,\vec{b})|^2 \equiv \int d^2\vec{b}\,\frac{d\widetilde{\sigma}_{\text{el}}(s,\vec{b})}{d^2\vec{b}} \qquad \frac{d\widetilde{\sigma}_{\text{el}}(s,\vec{b})}{d^2\vec{b}} = 1 - 2\cos\chi_R e^{-\chi_I} + e^{-2\chi_I}$$

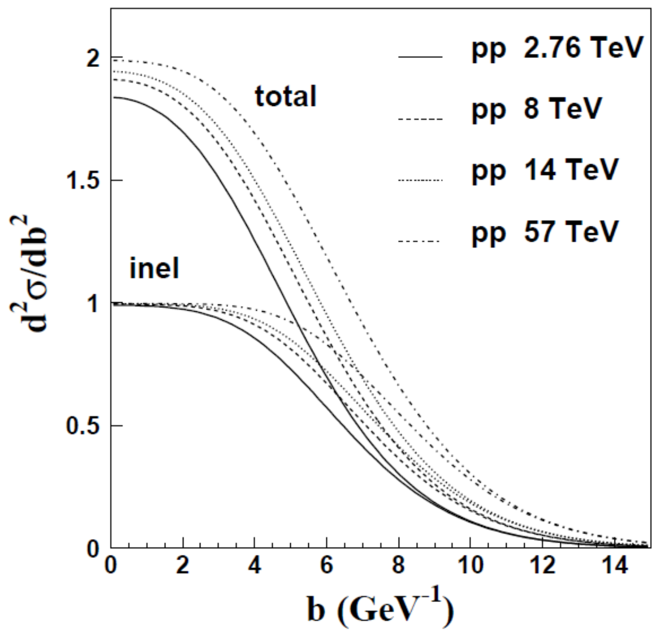
$$\sigma(s) = \frac{2}{\sqrt{\pi}}(\hbar c)^2\int d^2\vec{b}\,\widetilde{T}_I(s,\vec{b}) \equiv \int d^2\vec{b}\,\frac{d\widetilde{\sigma}_{\text{tot}}(s,\vec{b})}{d^2\vec{b}} \qquad \frac{d\widetilde{\sigma}(s,\vec{b})}{d^2\vec{b}} = 2\left(1 - \cos\chi_R e^{-\chi_I}\right)$$

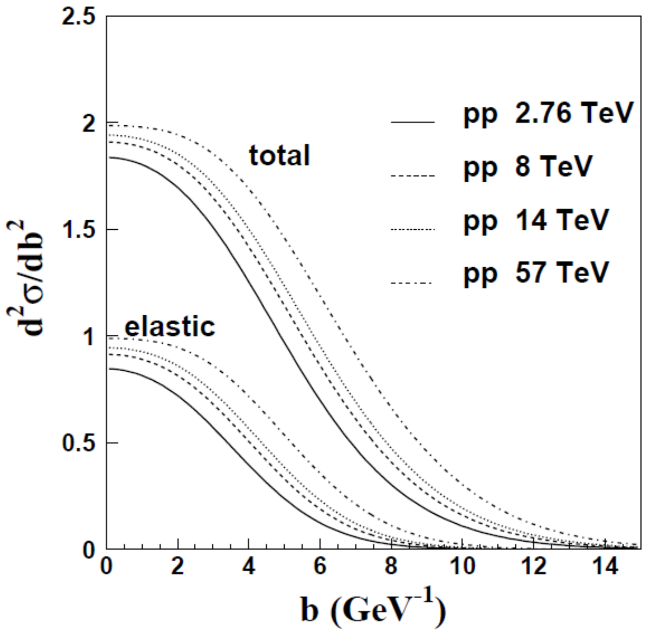
$$\sigma_{\text{inel}} = (\hbar c)^2\int d^2\vec{b}\left(\frac{2}{\sqrt{\pi}}\widetilde{T}_I(s,\vec{b}) - \frac{1}{\pi}|\widetilde{T}(s,\vec{b})|^2\right) \equiv \int d^2\vec{b}\,\frac{d\widetilde{\sigma}_{\text{inel}}(s,\vec{b})}{d^2\vec{b}} \qquad \frac{d\widetilde{\sigma}_{\text{inel}}(s,\vec{b})}{d^2\vec{b}} = 1 - e^{-2\chi_I}$$

Unitarity constraint bound → real amplitude

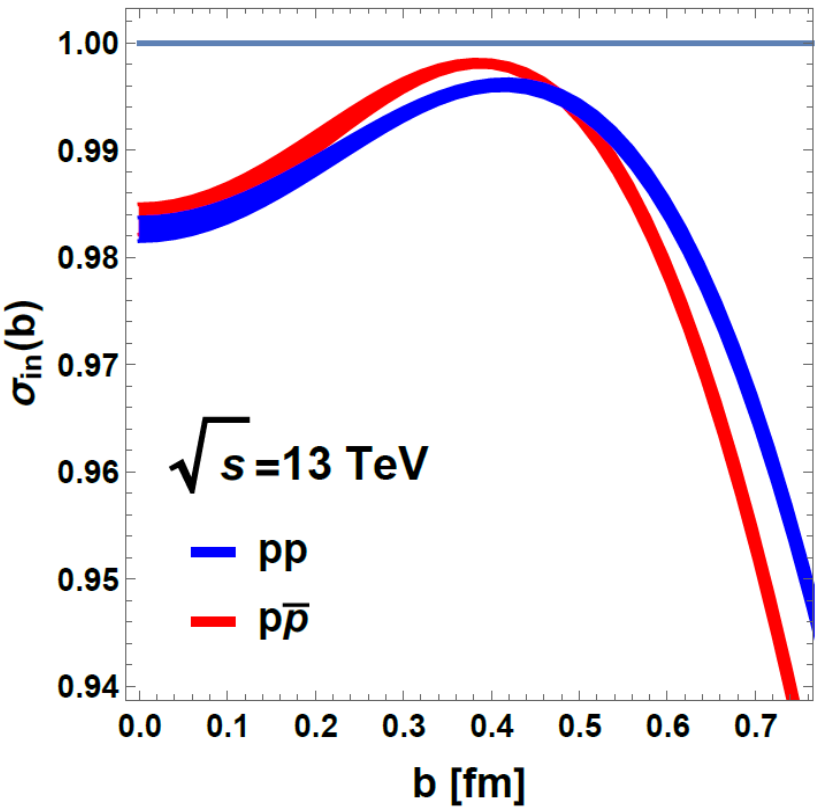


Monotonic results for elastic differential 'cross sections'

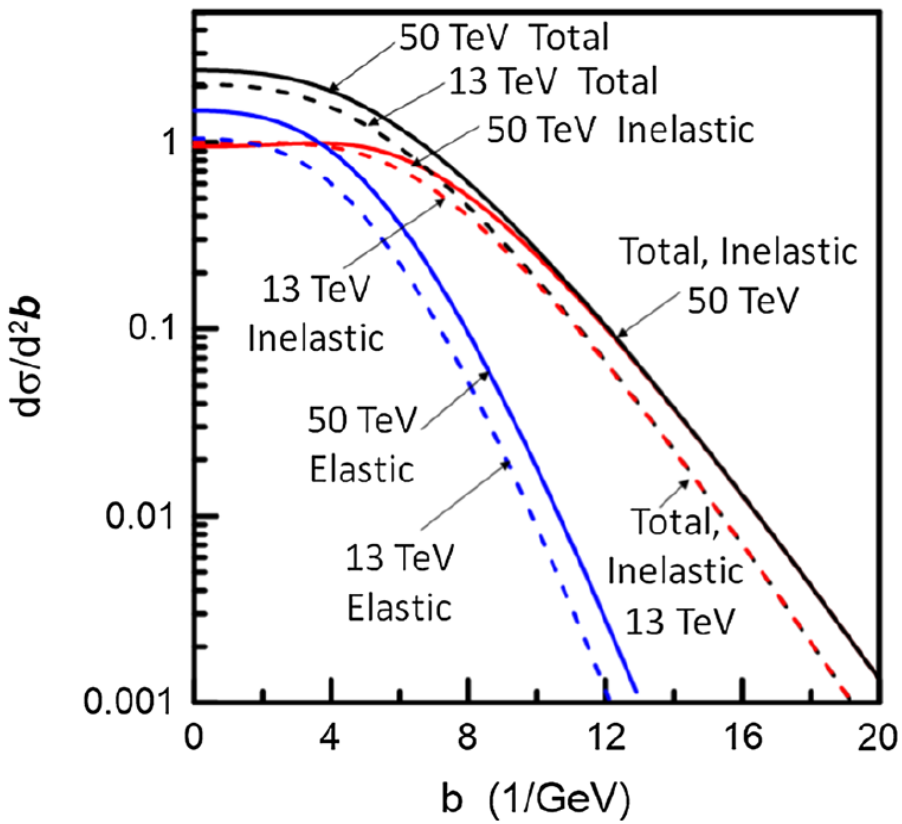




Something is going on at 13 TeV

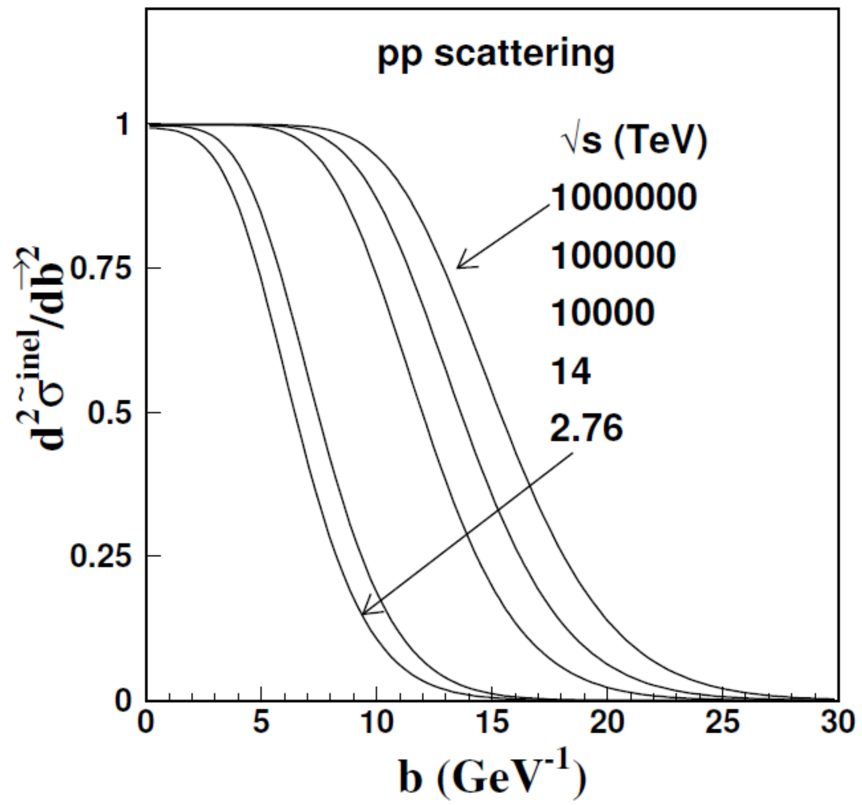


W. Broniowski, L. Jenkovszky, E. R. Arriola, I. Szanyi, *Phys. Rev. D* **98**, 074012 (2018); E. R. Arriola, W. Broniowski, *Few Body Syst.* **57** (2016) 7, 485-490; *Phys. Rev. D* **95** (2017) 7, 074030

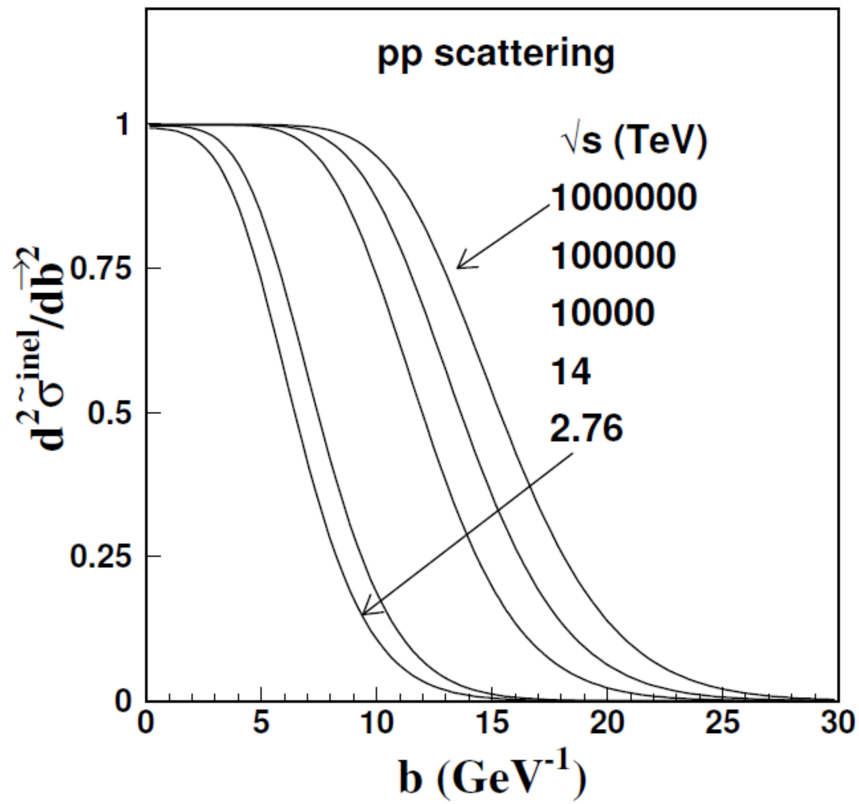


E. Ferreira, A. K. Kohara and T. Kodama; *Eur. Phys. J. C* **81** (2021) 4, 290

Extrapolating to high energies we have a travelling wave



...and this good behavior continues until asymptotic energies



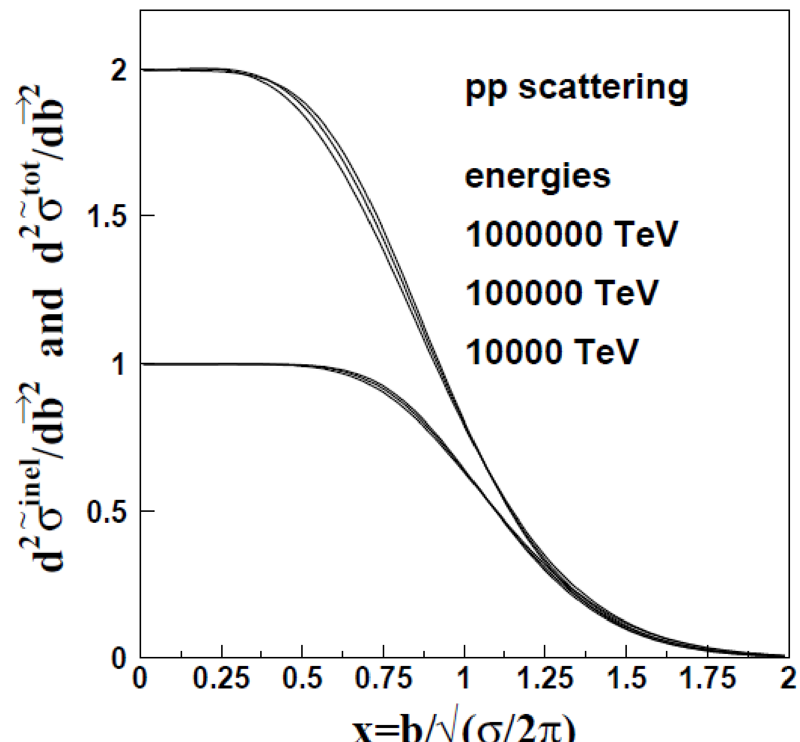
$d\sigma/d^2\vec{b}$ as function of *b* has a long tail.

scaling variable definition

$$x = b/\sqrt{\sigma\left(\sqrt{s}\right)/2\pi}$$

This scaled behavior was advocated by J. Dias de Deus a long time ago

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231



A. K. Kohara, E. Ferreira and Kodama, *Eur. Phys. J. C* **74**, 3175 (2014)

Analytical scaled form of KFK amplitude at high energies

Small *t* approximation
$$\Psi_I(\gamma_I(s),t) = 2\, e^{\gamma_I(s)} \left[\frac{e^{-\gamma_I(s)\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - e^{\gamma_I(s)} \frac{e^{-\gamma_I(s)\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}} \right]$$
$$\simeq 2\, e^{\gamma_I(s)} \left[\frac{e^{-\gamma_I(s)(1+\frac{a_0}{2}|t|)}}{(1+\frac{a_0}{2}|t|)} - e^{\gamma_I(s)} \frac{e^{-\gamma_I(s)\ 2\ (1+\frac{a_0}{8}|t|)}}{2(1+\frac{a_0}{8}|t|)} \right]$$

Expanding $(1+a_0|t|/2) \simeq \exp(a_0|t|/2)$ and $(1+a_0|t|/8) \simeq \exp(a_0|t|/8)$

$$\Rightarrow \quad \Psi_I(\gamma_I(s),t) \simeq 2\, e^{-[\gamma_I(s)+1]\ \frac{a_0}{2}|t|} - e^{-[2\gamma_I(s)+1]\ \frac{a_0}{8}|t|}$$

The imaginary amplitude becomes

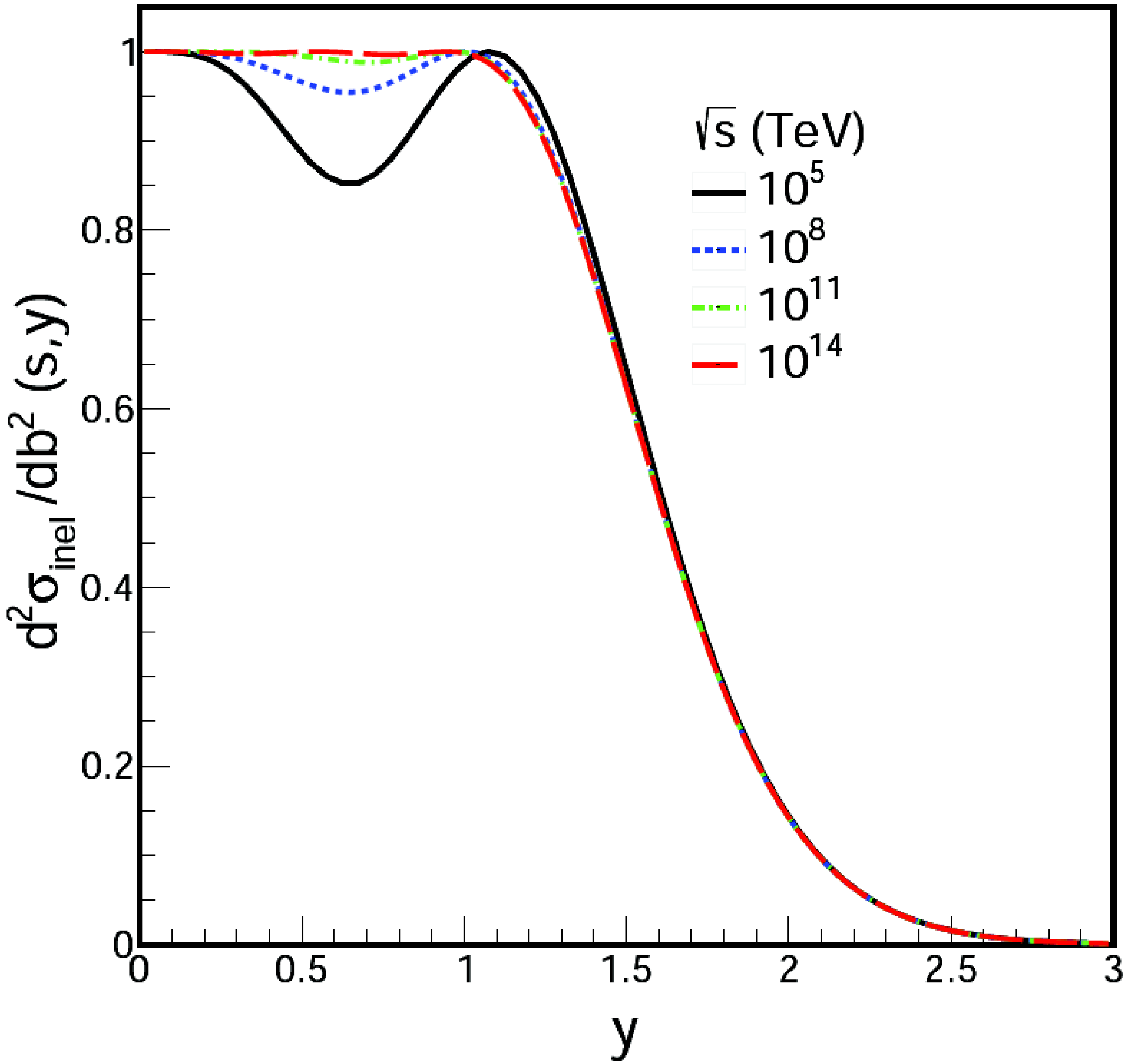
$$\widetilde{T}_I(s,\vec{b}) \simeq \frac{\alpha_I}{2\beta_I} e^{-b^2/4\beta_I} + \frac{2\lambda_I}{a_0} \left[\frac{e^{-b^2/[2(\gamma_I+1)a_0]}}{\gamma_I+1} - 2\frac{e^{-2b^2/[(2\gamma_I+1)a_0]}}{2\gamma_I+1} \right]$$

We truncate the parameters $\alpha_I(s),\ \beta_I(s),\ \lambda_I(s)$ and $\gamma_I(s)$ to the largest log s power

$$\widetilde{T}_I(s,\vec{b}) \approx \frac{\alpha_{I1}}{2\beta_{I1}} e^{-b^2/(4\beta_{I1}\log(\sqrt{s}))} + \frac{2}{a_0}\frac{\lambda_{I2}}{\gamma_{I2}} \left[e^{-b^2/[2\gamma_{I2}\log^2(\sqrt{s})a_0]} - e^{-b^2/[\gamma_{I2}\log^2(\sqrt{s})a_0]} \right]$$

scaling variable definition $y \equiv \dfrac{b}{\sqrt{2\gamma_{I2}a_0}\log\sqrt{s}}$

$$\Rightarrow \quad \widetilde{T}_I(s,y) \approx \frac{\alpha_{I1}}{2\beta_{I1}} e^{-\frac{a_0\gamma_{I2}}{2\beta_{I1}}\log\sqrt{s}\ y^2} + \frac{2}{a_0}\frac{\lambda_{I2}}{\gamma_{I2}} \left[e^{-y^2} - e^{-2y^2} \right]$$



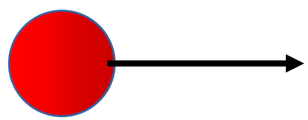
Scaled amplitude

$$\widetilde{T}_I(y) \approx \frac{\alpha_{I1}}{2\beta_{I1}} e^{-\frac{a_0\gamma_{I2}}{2\beta_{I1}} \log\sqrt{s_0}\ y^2} + \frac{2}{a_0}\frac{\lambda_{I2}}{\gamma_{I2}}\left[e^{-y^2} - e^{-2y^2}\right]$$

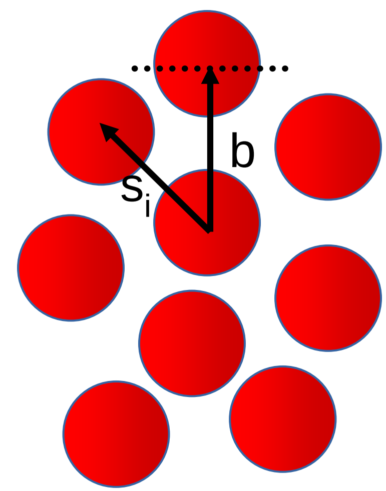
p-air scattering

p-air scattering

p-air cross sections measurements in EAS (extended air showers)

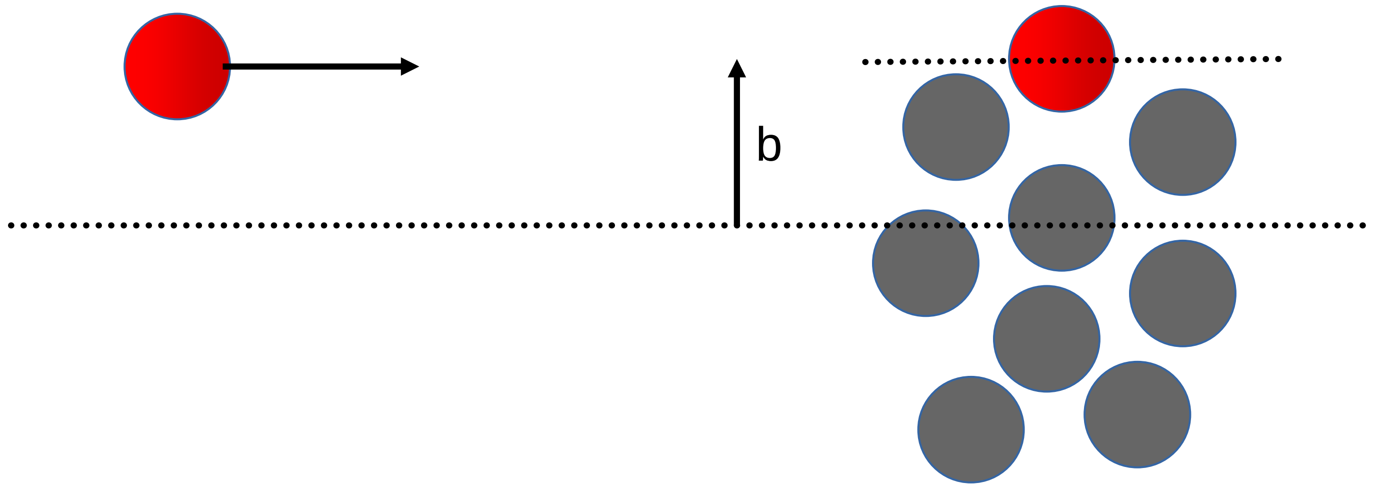


proton from cosmic ray



atom of atmosphere

...considering the nucleus composed by uncorrelated nucleons distributed according to a *s* and *p* harmonic distribution density



Glauber framework

R.J. Glauber, *Phys. Rev.* **100** (1955) 242–248 ;
R.J. Glauber and G. Matthiae, *Nucl. Phys. B* **21** (1970) 135–157

forward amplitudes for pp elastic scattering

$$\widehat{T}_{\rm pp}(s,\vec{b}) = \widehat{T}_R(s,\vec{b}) + i\widehat{T}_I(s,\vec{b}) = \frac{\sigma^{\rm tot}_{\rm pp}}{4\pi(\hbar c)^2}\left[\frac{\rho}{B_R}e^{-\frac{b^2}{2B_R}} + i\frac{1}{B_I}e^{-\frac{b^2}{2B_I}}\right]$$

In terms of eikonal functions

S-matrix in b space

$$-\,i\,\widehat{T}_{\rm pp}(s,\vec{b}) \;=\; 1 - e^{i\chi_{\rm pp}(s,\vec{b})} \;\equiv\; \Gamma_{\rm pp}(s,\vec{b})$$

Optical theorem

$$\sigma^{\rm tot}_{\rm pp}(s) \;=\; 2\,(\hbar c)^2\,\Re\int d^2\vec{b}\,\Gamma_{\rm pp}(s,\vec{b})$$

Analogous optical theorem for p-air

$$\sigma^{\rm tot}_{\rm pA}(s) \;=\; 2\,(\hbar c)^2\,\Re\int d^2\vec{b}\,\Gamma_{\rm pA}(s,\vec{b})$$

Glauber method introduces the p-A amplitude for A independent nucleons

$$\Gamma_{\rm pA}(s,\vec{b},\vec{s}_1,\ldots,\vec{s}_A) = 1 - \prod_{j=1}^{A}\left[1 - \Gamma_{\rm pp}(s,|\vec{b}-\vec{s_j}|)\right]$$

Production cross section $\sigma_{\rm p-air}^{\rm prod} = \sigma_{\rm p-air}^{\rm tot} - (\sigma_{\rm p-air}^{\rm el} + \sigma_{\rm p-air}^{\rm q-el})$

$$T_{\rm p-air}^{fi}(s,q^2) = \frac{1}{2\pi}\int d^2\vec{b}\; e^{ic\vec{q}.\vec{b}} \int \psi_f^*(\vec{r_1},...,\vec{r_A})\,\Gamma_{\rm p-air}(s,\vec{b},\vec{s}_1,...,\vec{s}_A)\;\psi_i(\vec{r_1},...,\vec{r_A})\prod_{j=1}^{A} d^3\vec{r_j}$$

with $$\psi_i^*(\vec{r_1},...,\vec{r_A})\psi_i(\vec{r_1},...,\vec{r_A}) = \prod_{j=1}^{A}\rho_j(\vec{r_j})$$

Nuclear density

$$\sigma_{\rm pA}^{\rm el} + \sigma_{\rm pA}^{\rm q-el} = (\hbar c)^2 \int d^2\vec{b} \int \left| 1 - \prod_{j=1}^{A}\left[1 - \Gamma_{\rm pp}(s,|\vec{b}-\vec{s_j}|)\right]\right|^2 \prod_{k=1}^{A} \rho_k(\vec{r_k})\, d^3\vec{r_k}$$

Diffractive intermediate states according (Good Walker framework with a parameter λ = 0.5)

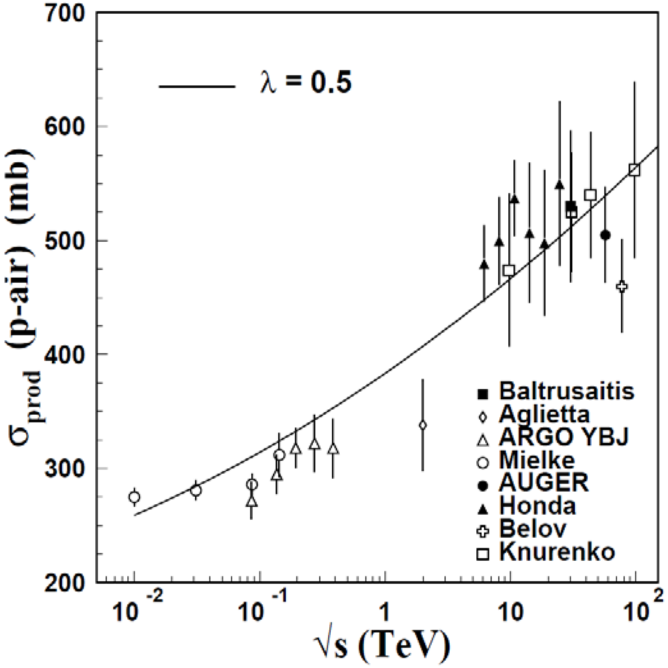
M. L. Good and W. D. Walker, *Phys. Rev.* **120** (1960) 1857

$$\Gamma_{\rm pA}(s,\vec{b},\vec{s}_1,...,\vec{s}_A) = 1 - \frac{1}{2}\prod_{j=1}^{A}\left[1-(1+\lambda)\Gamma_{\rm pp}(\vec{b}-\vec{s}_j)\right] - \frac{1}{2}\prod_{j=1}^{A}\left[1-(1-\lambda)\Gamma_{\rm pp}(\vec{b}-\vec{s}_j)\right]$$

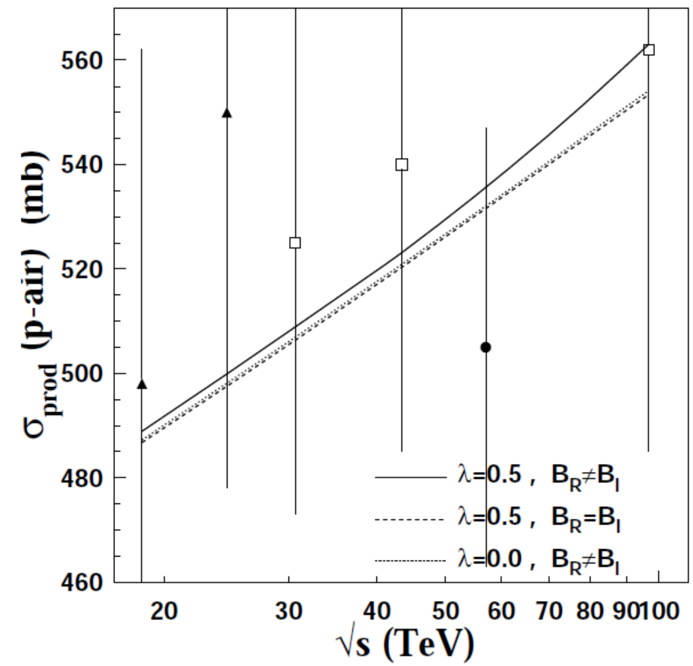
Parametrization of production cross section.

$$\sigma_{\rm p-air}^{\rm prod}(s) = 383.474 + 33.158\log\sqrt{s} + 1.3363\log^2\sqrt{s}$$

Comparison with p-air cosmic ray data

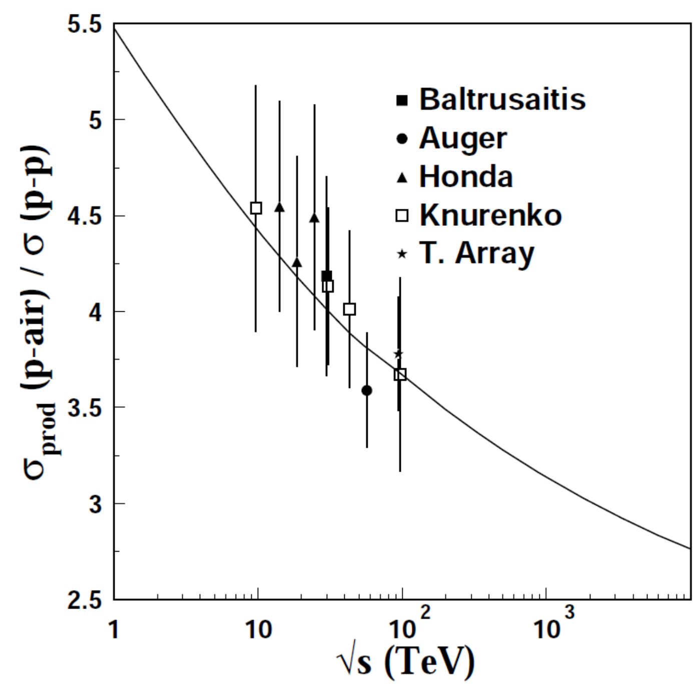


Influence of different slopes



Ratio of pp and p-air

$$\frac{\sigma^{\rm prod}_{\rm p-air}(s)}{\sigma_{\rm pp}(s)} = \frac{383.474 + 33.158 \log \sqrt{s} + 1.3363 \log^2 \sqrt{s}}{69.3286 + 12.6800 \log \sqrt{s} + 1.2273 \log^2 \sqrt{s}}$$



A. K. Kohara, E. Ferreira and T. Kodama, *J. Phys. G* **41**, 115003 (2014)

p-air in b- space

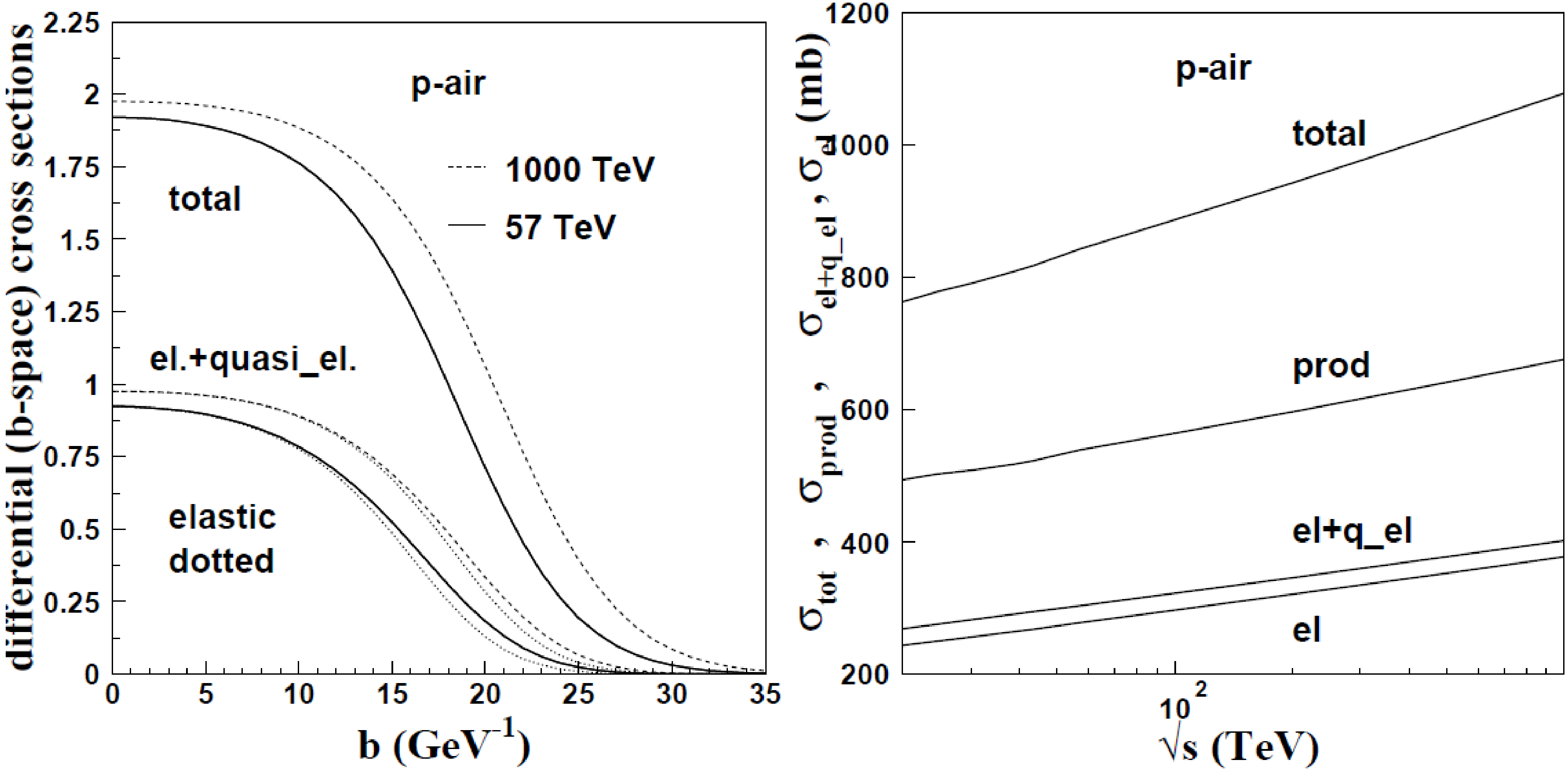
p-air elastic scattering amplitude $$-i\widehat{T}_{\rm pA}(\vec{b}) = 1 - e^{i\chi_{pA}} \simeq 1 - \left\langle \prod_{j=1}^{A} e^{i\chi_{pN_j}} \right\rangle = 1 - \left\langle \prod_{j=1}^{A} \left(1 + i\widehat{T}_{\rm pN}(\vec{b})\right) \right\rangle$$

p-air distributions:

$$\frac{1}{2}\frac{d^2\sigma^{\rm tot}_{pA}}{d^2\vec{b}}(s,\vec{b}) = \left\langle 1 - \prod_{i=1}^{A}\left(1 - \frac{1}{2}\frac{d^2\sigma^{\rm tot}_{pp}}{d^2\vec{b}_i}(s,\vec{b}-\vec{b}_i)\right)\right\rangle$$

$$\frac{d^2\sigma^{\rm el}_{pA}}{d^2\vec{b}}(s,\vec{b}) = \left\langle \left[1 - \prod_{i=1}^{A}(1 - \frac{d^2\sigma^{\rm tot}_{pp}}{d^2\vec{b}_i}(s,\vec{b}-\vec{b}_i))\right]^2 \right\rangle$$

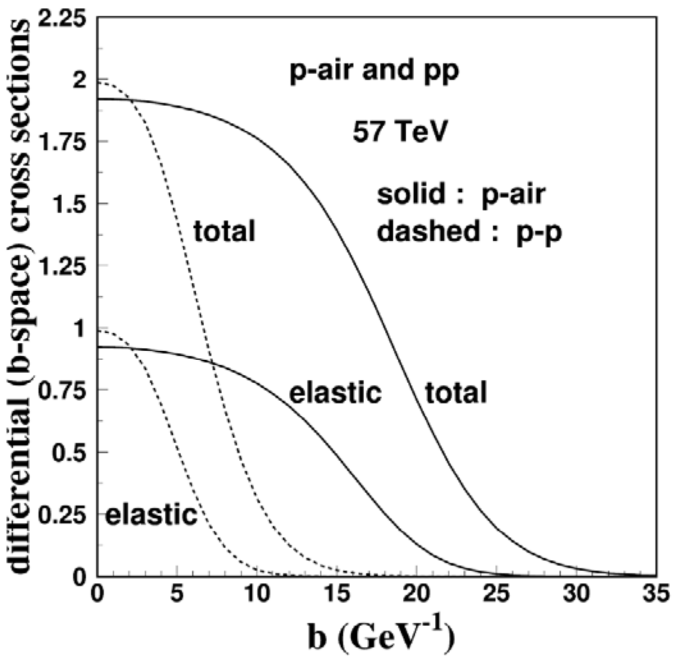
predictions

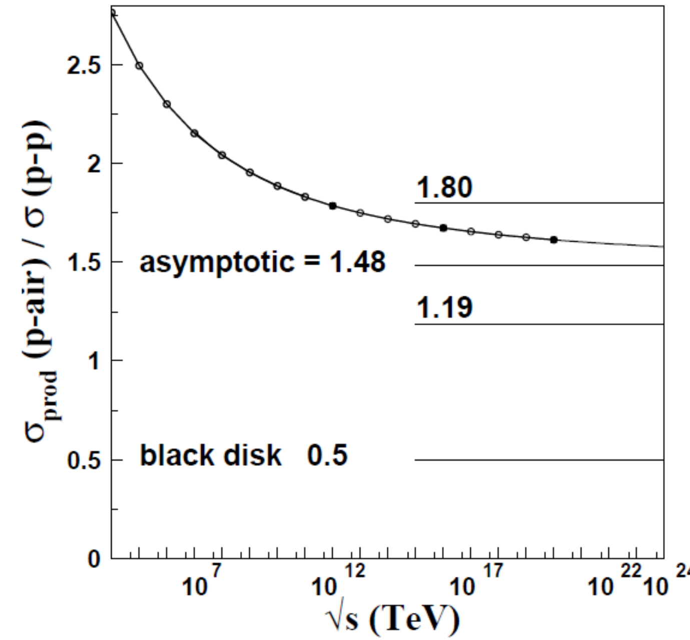


Comparison between p-air and pp

Ratio converges to asymptotic limits

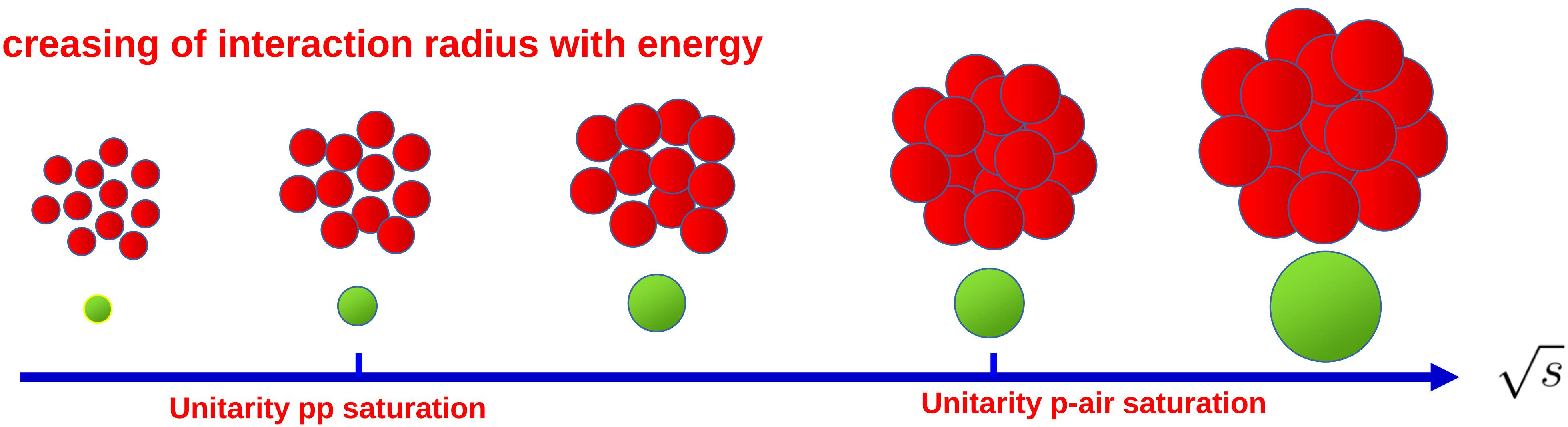
b-space amplitudes of pp and p-air





A. K. Kohara, E. Ferreira and T. Kodama, *J. Phys. G* **41**, 115003 (2014)

Increasing of interaction radius with energy



Comparison of models

Eikonalized models:

Bourrely Soffer and Wu (BSW)

C. Bourrely, J. Soffer, T.T. Wu, Phys. Rev. D 19, 3249 (1979); Nucl. Phys. B 247, 15 (1984); Eur. Phys. J. C 28, 97 (2003)

$$\mathcal{M}(s,t) = \frac{is}{2\pi} \int d^2\vec{b}\, e^{-i\vec{q}\cdot\vec{b}} \left(1 - e^{-\Omega(s,\vec{b})}\right)$$

High Energy General Structure (HEGS)

O.V. Selyugin, Eur. Phys. J. C 72, 2073 (2012)

$$F_H(s,t) = \frac{is}{2\pi} \int d^2\vec{b}\, e^{-i\vec{q}\cdot\vec{b}} \left(1 - e^{\chi(s,\vec{b})}\right)$$

Dynamical Gluon Mass (DGM)

$$A(s,q) = \frac{i}{2\pi} \int d^2\vec{b}\; e^{-i\vec{q}\cdot\vec{b}} (1 - e^{i\chi_{DGM}(s,b)})$$

E. G. S. Luna, A. F. Martini, M. J. Menon, A. Mihara, and A. A. Natale Phys. Rev. D 72, 034019 (2005); D.A.Fagundes, E.G.S. Luna, M.J. Menon and A.A. Natale , Nucl. Phys. A 886, 48 (2012)

Correspondence with KFK amplitudes

$$1 - e^{-\Omega(s,\vec{b})} = 1 - e^{\chi(s,\vec{b})} = 1 - e^{i\chi_{DGM}(s,\vec{b})} = -\frac{i}{\sqrt{\pi}} \left[\tilde{T}_R(s,\vec{b}) + i\,\tilde{T}_I(s,\vec{b})\right]$$

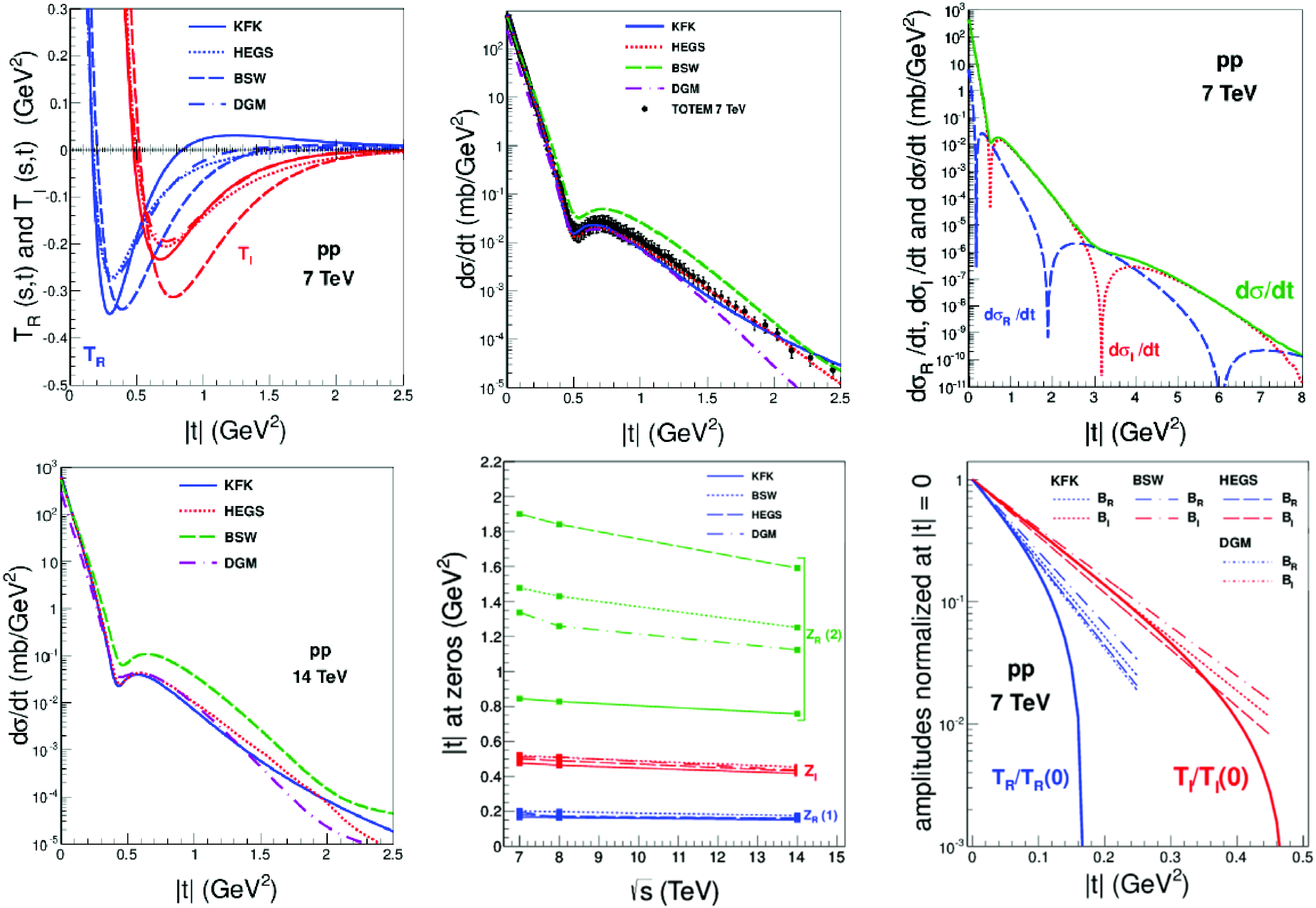
Normalizations

$$\frac{1}{(\hbar c)^2}\frac{d\sigma}{dt} = |T(s,t)|^2 = \frac{\pi}{s^2}|\mathcal{M}(s,t)|^2 = \frac{\pi}{s^2}|F_H(s,t)|^2 = \pi|A(s,t)|^2$$

and

$$\frac{\sigma_{\rm tot}(s)}{(\hbar c)^2} = 4\sqrt{\pi}\,T_I(s,0) = \frac{4\pi}{s}\mathcal{M}_I(s,0) = \frac{4\pi}{s}(F_H)_I(s,0) = 4\pi\; A_I(s,0)$$

Amplitudes, differential cross sections, zeros, slopes and predictions



E. Ferreira, T. Kodama, A. K. Kohara, D. Szilard, *Acta Phys.Polon.Supp.* **8** (2015) 1017

Some Questions

Is it possible to explain the dip structure without odderon?

Should be the scattering amplitudes unique?

Is it possible to constrain the effective proton shape as the energy increases?

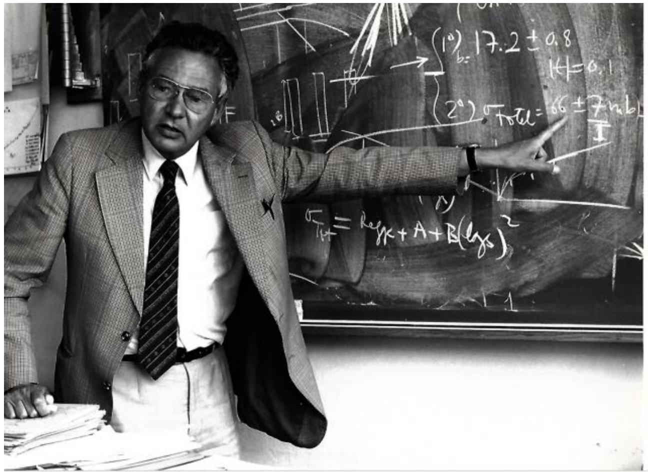
Is there a region at large t where the differential cross section is energy independent?

Can QCD give us any hint of elastic scattering description?

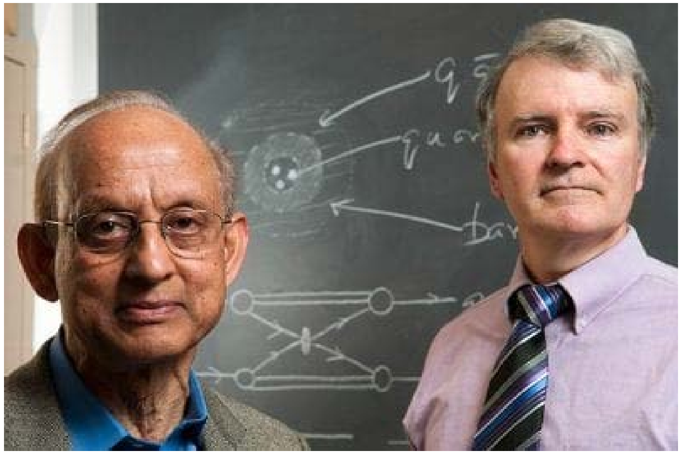
Summary

- Disentanglement of real and imaginary scattering amplitudes
- Use of exact forms of derivative dispersion relations (PDG and KFK)
- Need to consider different slopes in forward scattering
- Predictions all the energies
- Non black disk behavior for asymptotic energies
- Scaling form for asymptotic energies
- Cosmic ray predictions
- Similarities between models shows the need of more attention

In honor of those who recently passed away and made important contributions to the field



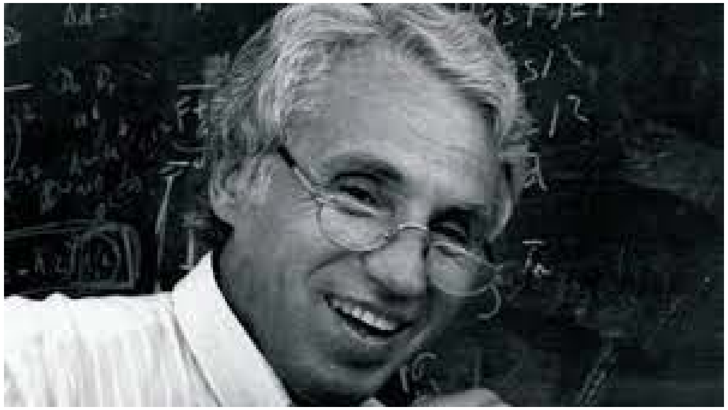
Prof. André Martin (1929 – 2020)



Prof. Munir Islam (? – 2021)

Prof. Milos Vaclav Lokajicek (1923-2019)

Prof. Jorge Dias de Deus (1941 – 2021)



Prof. Jacques Soffer (1940 – 2019)

Thank you!

Back up slides

Theoretical Framework

Amplitudes based on Stochastic Vacuum Model (SVM) framework where gluon condensate and the correlation lenght play an important role.

Wilson loop expectation value

$$\langle W[C]\rangle \approx \exp\left[-\frac{g^2}{2^2 2!}\int_S dS^{\mu_1\nu_1}(z_1)dS^{\mu_2\nu_2}(z_2)\langle \text{ tr } F_{\mu_1\nu_1}[z_1, C(w,z_1)]F_{\mu_2\nu_2}[z_2, C(w,z_2)]\rangle\right]$$

Gluon correlators expectation value

$$\langle g^2 F^a_{\mu\nu}[z_1, C(w,z_1)]F^b_{\alpha\beta}[z_2, C'(w,z_2)]\rangle = \frac{\delta^{a,b}}{12(N_c^2-1)}\langle g^2 FF\rangle\Big[\kappa(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})D(z^2)$$
$$+(1-\kappa)\frac{1}{2}[\frac{\partial}{\partial z_\mu}(z_\alpha g_{\nu\beta} - z_\beta g_{\nu\alpha}) + \frac{\partial}{\partial z_\nu}(z_\beta g_{\nu\alpha} - z_\alpha g_{\nu\beta})D_1(z^2)]\Big]$$

wher e $z = z_1 - z_2, \; \langle g^2 FF\rangle = \langle g^2 F^a_{\mu\nu}(0)F^a_{\alpha\beta}(0)\rangle$ ⟶ gluon condensate

$D(z^2)$ and $D_1(z^2)$ are non-Abelian and Abelian contributions respectively

These functions must fall off with increasing z/a, where a is the correlation lenght defined as

$$\int_0^\infty dz \; D(z^2) = a$$