Probing flow fluctuations through factorization breaking in heavy-ion collision

based on P. Bozek and R. Samanta, 2021- arXiv: 2109.07781

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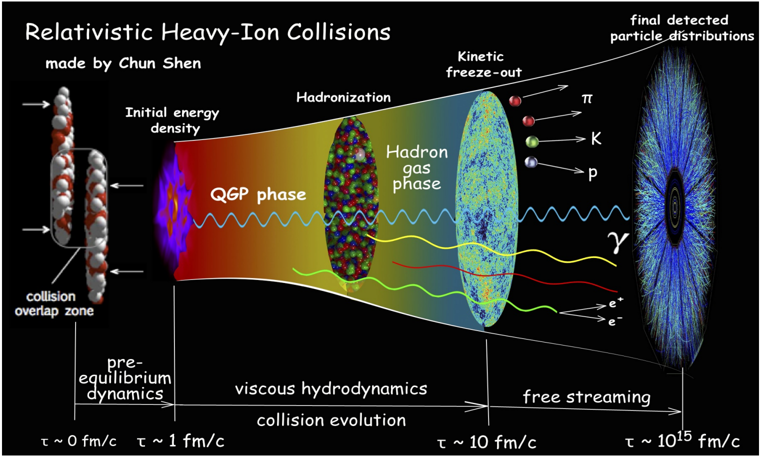
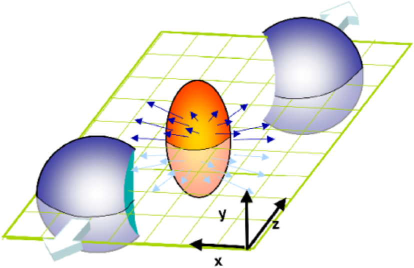
BIAŁASÓWKA HEP Seminar
December 10, 2021

The Little Bang



Shen, Heinz, arXiv:1507.01558

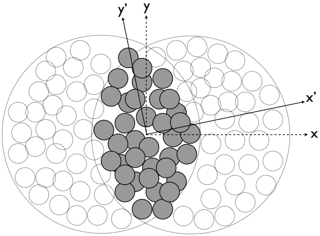
Collective flow in HI Collisions



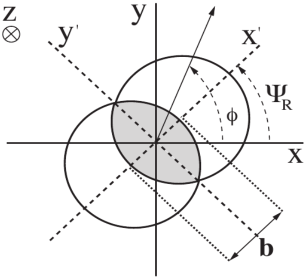
U. Heinz, arXiv:0810.5529

Asymmetry in source distribution $\Longrightarrow$ **Collective expansion of fireball** $\Longrightarrow$ **Momentum anisotropy**

Momentum anisotropy



PHOBOS arXiv:0711.3724

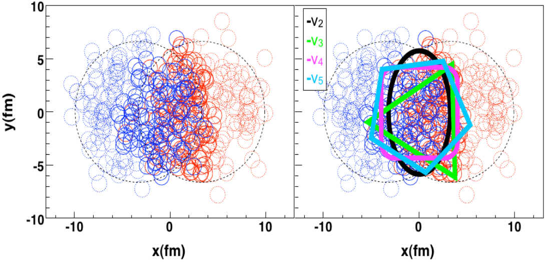


Bilandzic, Snellings and Voloshin arXiv:1010.0233

Momentum anisotropy as fourier expansion of flow harmonics

$$\frac{dN}{dpd\phi} = \frac{dN}{2\pi dp}\left(1 + \sum_{n=1}^{\infty} v_n(p) cos\left[n\left(\phi - \Psi_n\right)\right]\right)$$

Harmonic flow coefficients

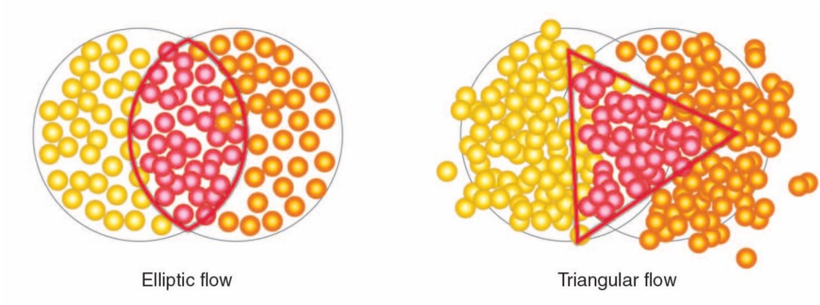


Alver, Baker, Loizides, Steinberg arXiv:0805.4411

Coefficients of harmonic flow

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos\left[2(\phi - \Psi_2)\right] + 3v_3 \cos\left[3(\phi - \Psi_2)\right] + \ldots$$

Elliptic and Triangular flow



Elliptic flow

- $V_2 \longrightarrow$ Elliptic flow
- $V_2 \propto \mathcal{E}_2$
- $\mathcal{E}_2 \longrightarrow$ Elliptic deformation in source

Triangular flow

- $V_3 \longrightarrow$ Triangular flow
- $V_3 \propto \mathcal{E}_3$
- $\mathcal{E}_3 \longrightarrow$ Triangular deformation in source

Collective flow fluctuates from event to event

Fluctuations of harmonic flow

- Theoretically, $V_n = v_n\, e^{i\, n\Psi_n}$, $v_n =$ flow magnitude and $\Psi_n =$ flow angle

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- Cumulant method is used to measure the flow coefficients
- The two particle cumulant, $v\{2\} = \sqrt{\langle V_n V_n^* \rangle}$, where $\langle \ldots \rangle$ denote the event average

Measuring moments of the flow harmonics

Cumulant method

- Experimentally, $q_n = \frac{1}{N}\sum_{i=1}^{N} e^{i\ n\phi_i}$,

  N = the number of particles
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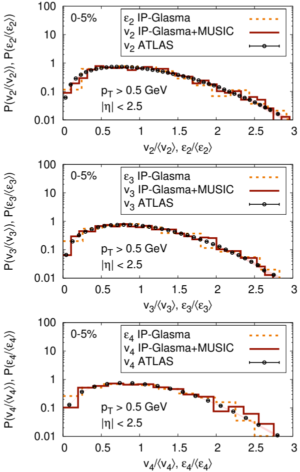
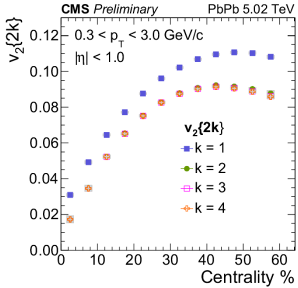
Only even moments of the flow can be measured !

Distribution of the flow harmonics



$$v_2\{2\} > v_2\{4\} \simeq v_2\{6\} \simeq v_2\{8\}$$



CMS Nucl. Phys. A937 (2017) 401

Gale, Jeon, Tribedy, Venugopalan arXiv: 1210.5144

Mapping the flow fluctuation

How can we map the flow fluctuations ?

- In general, the harmonic flow coefficients depend on the momentum(p) and pseudorapidity (η).

$$\frac{dN}{d\phi dp d\eta} \propto 1 + 2V_2(p,\eta)e^{-i\ 2\phi} + 3V_3(p,\eta)e^{-i\ 3\phi} + \ldots$$

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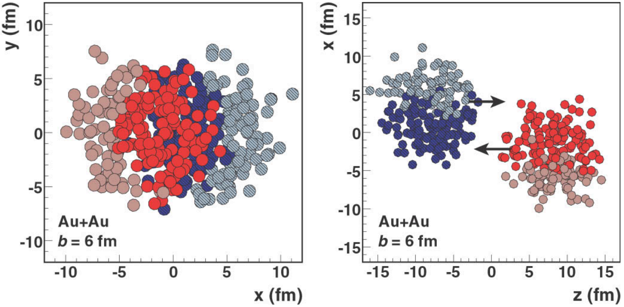
- Can we map the moments of the flow $V_n(p,\eta)$? Yes !
- We could map the covariance : $\langle V_n(p_1,\eta_1)V_n^*(p_2,\eta_2)\rangle$

or the correlation : $\frac{\langle V_n(p_1,\eta_1)V_n^*(p_2,\eta_2)\rangle}{\sqrt{\langle V_n(p_1,\eta_1)V_n^*(p_1,\eta_1)\rangle\langle V_n(p_2,\eta_2)V_n^*(p_2,\eta_2)\rangle}}$

Forward-backward asymmetry



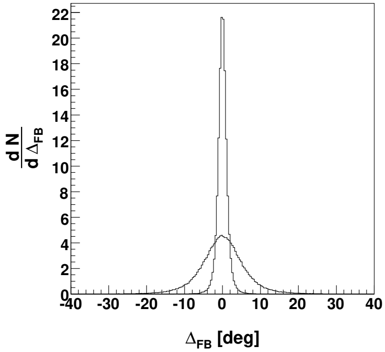
Ann.Rev.Nucl.Part.Sci. 57 (2007) 205

Forward-backward asymmetry and twist

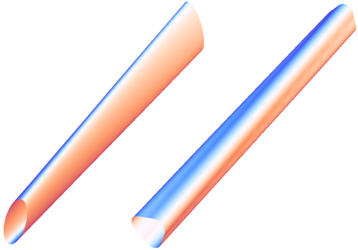
- Initial model: Glauber Monte Carlo ⟶ different distributions for forward and backward going participants
- Different event plane angles in forward and backward rapidities(space-time); result in a 'Twist' ⟶ Torqued fireball .

Visualizing FB asymmetry

Forward-backward flow angle decorrelation



Bozek, Broniowski, Moreira arXiv: 1011.3354



Twisted event plane angles

- Event plane angle fluctuates event by event
- Orientations of the twisted principal axes are random in forward an backward rapidities.

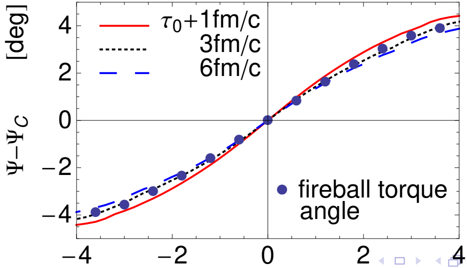
3+1D hydro evolution of the torqued fireball Bozek, Broniowski, Moreira

Evolution of the torqued fireball

- The initial density for the twisted profile :

$$s(x, y, \eta) \propto \rho_+(Rx, Ry) f_+(\eta_{||}) + \rho_-(R^T x, R^T y) f_-(\eta_{||})$$

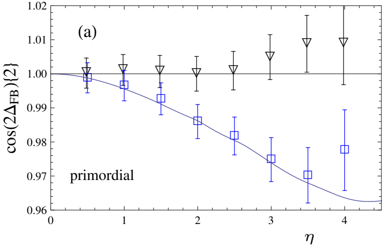
- Forward and backward going participants rotate the principal axis in the opposite direction on transverse plane; causing a twist or torque.
- The twist survives the hydrodynamic evolution.



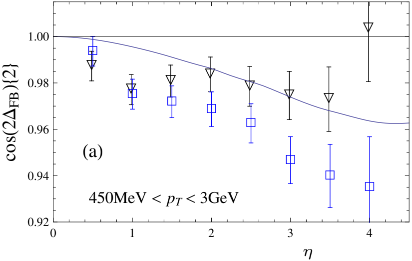
2-bin correlation observable

correlation coefficient, factorization breaking coefficient

primordial particles, torque events + notorque events



charged particles, torque events + notorque events



FB angle correlations

$$cos(n\Delta\Psi_n) \simeq \frac{\langle \frac{1}{n_F n_B} \sum_{i\in F, j\in B} cos\left[n(\phi_i - \phi_j)\right]\rangle}{\sqrt{< v_n^2(F) >}\sqrt{< v_n^2(B) >}}$$

- Substantial nonflow contribution is present
- 2-bin observables in η are dominated by nonflow !

Flow decorrelation in transverse momenta: p_T-bin

Correlations between two different p_T bins : First moment

- Flow vector factorization coefficient:

$$r_n(p_1, p_2) = \frac{\langle V_n(p_1) V_n^*(p_2) \rangle}{\sqrt{\langle V_n(p_1) V_n^*(p_1) \rangle \, \langle V_n(p_2) V_n^*(p_2) \rangle}}$$

can be measured experimentally $r_n(p_1, p_2) \leq 1$; correlation coefficient : $\rho(V_n(p1), V_n(p2))$

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- Flow magnitude correlation :

$$r_n^{v_n}(p_1, p_2) = \frac{\langle |V_n(p_1)||V_n(p_2)|\rangle}{\sqrt{\langle |V_n(p_1)|^2\rangle \ \langle |V_n(p_2)|^2\rangle}}$$

cannot be measured experimentally !

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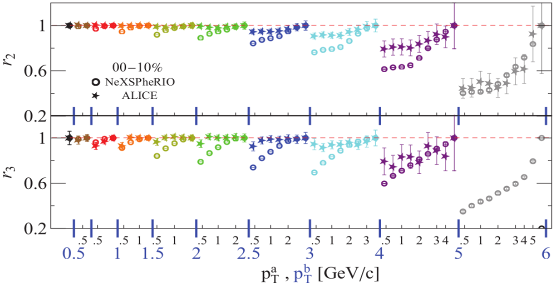
- Angle correlation :

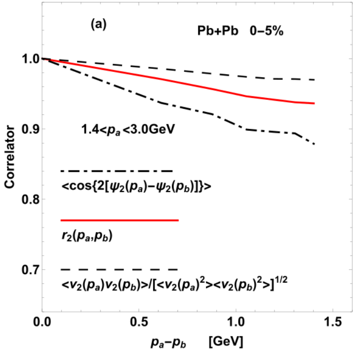
$$r_n^{\Psi_n}(p_1, p_2) = \frac{\langle |V_n(p_1)||V_n(p_2)| cos[n(\Psi_n(p1) - \Psi_n(p2))] \rangle}{\langle |V_n(p_1)||V_n(p_2)| \rangle}$$

which again cannot be measured experimentally !

Results

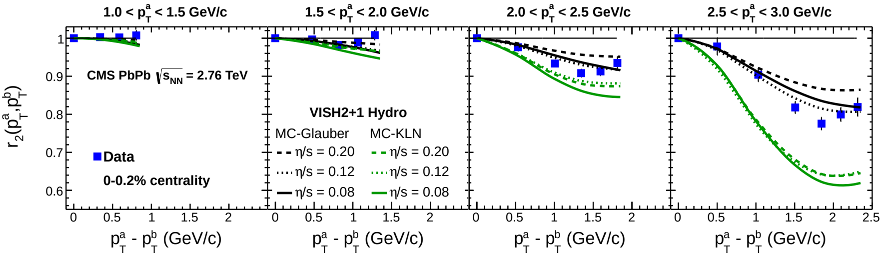


Gardim, Grassi, Luzum, Ollitrault arXiv: 1211.0989



P. Bozek arXiv: 1808.04248

Experimental vs theoretical results



CMS arXiv: 1503.01692

Observation

- Results are qualitatively described by models
- Can provide constraints on initial conditions

Flow vector, angle and magnitude decorrelation

First moment

Let's consider two flow vectors : $\vec{X}$ and $\vec{Y}$

- flow vector decorrelation(factorization breaking coefficient)

$$r = \frac{\langle \vec{X}\vec{Y} \rangle}{\sqrt{\langle X^2 \rangle \ \langle Y^2 \rangle}} = \frac{\langle X \ \ Y \ \ cos(\Delta\Psi) \rangle}{\sqrt{\langle X^2 \rangle \ \langle Y^2 \rangle}}$$

measures both the magnitude and the angle decorrelation.
(can be measured experimentally)

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- In experiment we can only measure the scalar product of two vectors

$$\vec{X} \cdot \vec{Y}$$

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- So, we look for the second moment of correlations coefficient

Flow vector, angle and magnitude decorrelation

Second moment

- flow vector decorrelation(factorization breaking coefficient)

$$r^2 = \frac{\langle 2(\vec{X}\vec{Y})^2 - X^2Y^2\rangle}{\sqrt{\langle X^4\rangle \ \langle Y^4\rangle}} \ or \ \frac{\langle \vec{X}^2\vec{Y}^{2*}\rangle}{\sqrt{\langle X^4\rangle \ \langle Y^4\rangle}} = \frac{\langle X^2 \ Y^2 \ cos(2\Delta\Psi)\rangle}{\sqrt{\langle X^4\rangle \ \langle Y^4\rangle}}$$

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- flow magnitude decorrelation :

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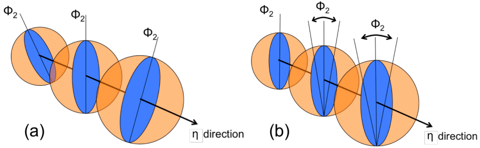
(can be extracted from the two quantities above !)

- That's why one is interested in the even moments of factorization coefficient during model study.

Flow decorrelation in longitudinal direction : correlation in η



Jia, Huo arXiv: 1402.6680

Correlations in η bins

- Flow correlation in η can be constructed using 3-bin or 4-bin correlator

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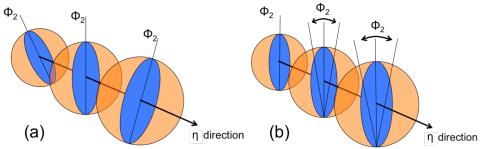

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- Flow vector decorrelation (angle + magnitude):

$$r_{n,k}(\eta) = \frac{\langle V_n^k(-\eta) V_n^{*k}(\eta_{ref})\rangle}{\langle V_n^k(\eta) V_n^{*k}(\eta_{ref})\rangle} \quad (3-bin\ corr.)$$

$$= \frac{\langle v_n^k(-\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(-\eta) - \Psi_n(\eta_{ref}))]\rangle}{\langle v_n^k(\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(\eta) - \Psi_n(\eta_{ref}))]\rangle}$$

$$\simeq 1 - 2F_{n,k}^{asy} - 2F_{n,k}^{twi}$$

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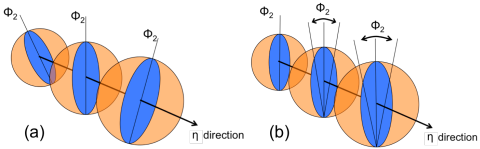

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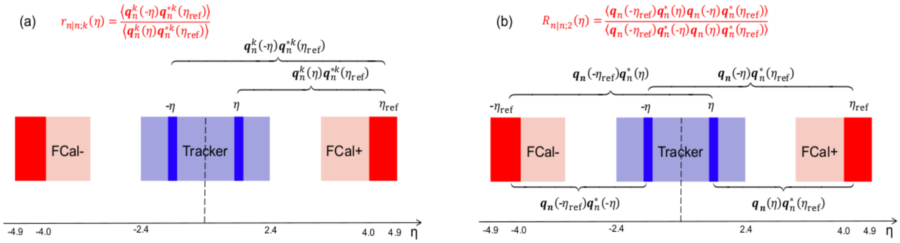
$$= \frac{\langle v_n^k(-\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(-\eta) - \Psi_n(\eta_{ref}))]\rangle}{\langle v_n^k(\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(\eta) - \Psi_n(\eta_{ref}))]\rangle}$$

$$\simeq 1 - 2F_{n,k}^{asy} - 2F_{n,k}^{twi}$$

- $F_{n,k}^{asy} \longrightarrow$ FB v_n asymmetry $\longrightarrow$ magnitude decorrelation

Correlations in η bins

- $2F^{twi}_{n,k} \longrightarrow$ FB event plane twist $\longrightarrow$ angle decorrelation



Correlations in η bins

- $2F_{n,k}^{twi} \longrightarrow$ FB event plane twist $\longrightarrow$ angle decorrelation
- Also, one can construct

$$R_{n,2}(\eta) = \frac{\langle V_n(-\eta_{ref}) V_n^*(\eta) V_n(-\eta) V_n^*(\eta_{ref}) \rangle}{\langle V_n(-\eta_{ref}) V_n^*(-\eta) V_n(\eta) V_n^*(\eta_{ref}) \rangle} \quad (4 - bin\ corr.)$$

$$= \frac{\langle v_n(-\eta_{ref}) v_n(\eta) v_n(-\eta) v_n(\eta_{ref}) cos[n(\Psi(-\eta_{ref}) - \Psi(\eta) + \Psi(-\eta) - \Psi(\eta_{ref}))]\rangle}{\langle v_n(-\eta_{ref}) v_n(-\eta) v_n(\eta) v_n(\eta_{ref}) cos[n(\Psi(-\eta_{ref}) - \Psi(-\eta) + \Psi(\eta) - \Psi(\eta_{ref}))]\rangle}$$

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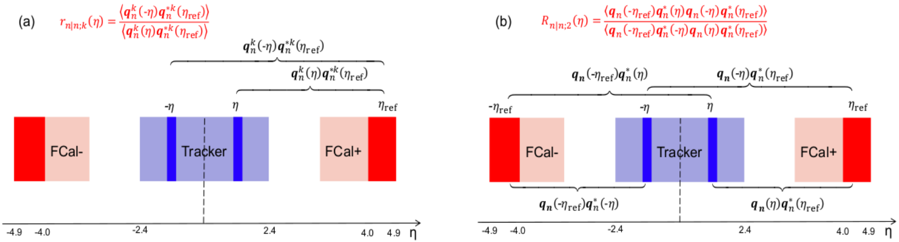

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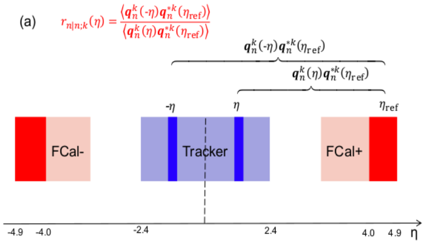

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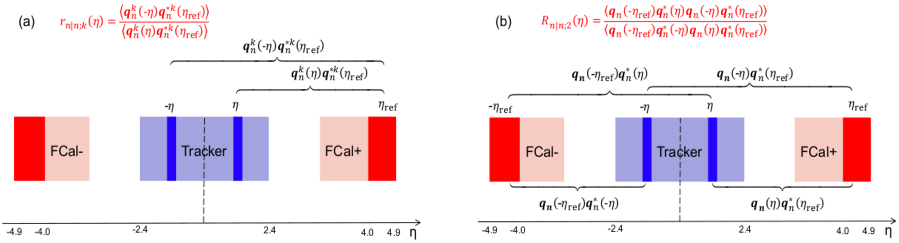
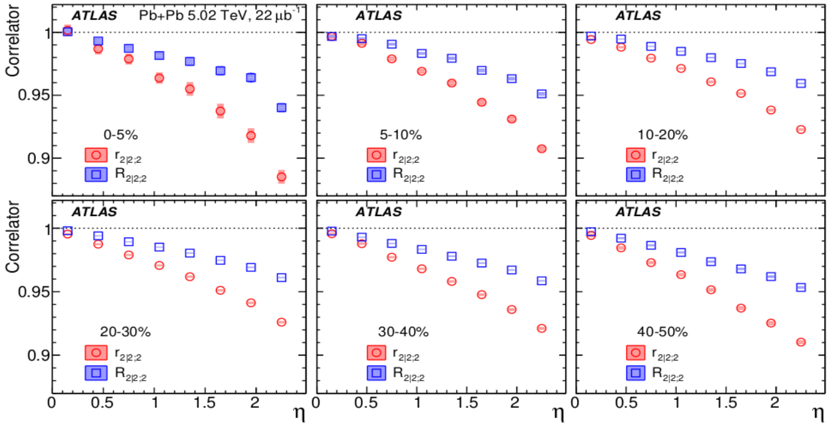
$$\simeq 1 - 2F^{twi}_{n,2}$$

  - effect of asymmetry is same in both numerator and denominator
  - only angle decorrelation due to twist



Correlations in η bins

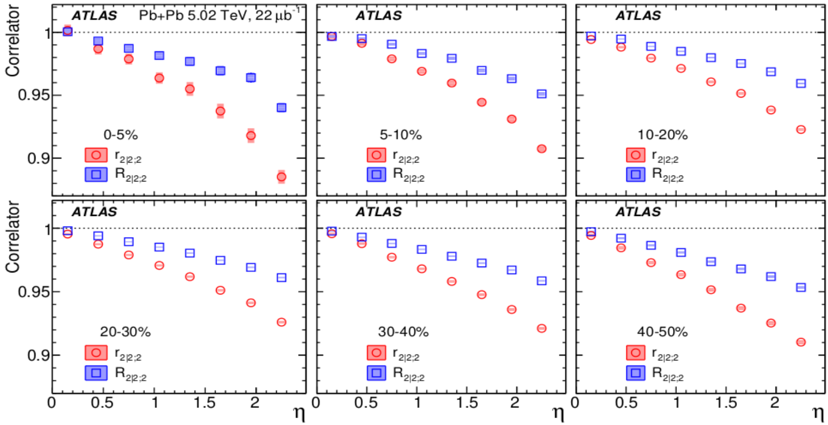
- Measuring $r_{n,k}(\eta)$ and $R_{n,2}(\eta)$, the magnitude and angle decorrelation can be separated.



ATLAS arXiv:1709.02301

Correlations in η bins

- Measuring $r_{n,k}(\eta)$ and $R_{n,2}(\eta)$, the magnitude and angle decorrelation can be separated.
- Moreover we can see, **Magnitude decor. $\simeq$ Angle decor.**



ATLAS arXiv:1709.02301

Why magnitude decorrelation $\simeq$ angle direction ?

Simple model of vector decorrelation Bozek, Mehrabpour

Let's consider two vector: $\vec{X_n} = \vec{V_n} + \vec{\delta_n}$ and $\vec{Y_n} = \vec{V_n} - \vec{\delta_n}$

- It can be shown that, factorization breaking of flow vector:

$$\frac{\langle X_n Y_n^\star \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - 2\frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- factorization breaking of flow magnitude:

$$\frac{\langle X_n Y_n \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- and flow angle decorrelation :

$$\frac{\langle V_n^2 \cos(n\Delta\Psi) \rangle}{\langle V_n^2 \rangle} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

Angle-magnitude correlations in p_T : 2nd moment

- Flow vector **square** factorization coefficient:

$$\frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$$

could be measured experimentally

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Could be measured experimentally ! (unlike the rapidity corr. and the corr. in first moment)

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Could be measured experimentally ! (unlike the rapidity corr. and the corr. in first moment)

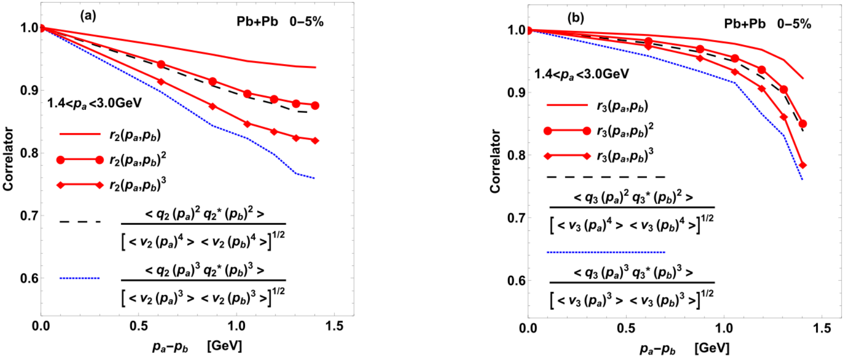
- Angle decorrelation :

$$\frac{flow\ \ vec.\ decor}{flow\ \ mag.\ decorr} = \frac{\langle V_n(p_1)^2 V_n^*(p_2)^2\rangle}{\langle|V_n(p_1)|^2|V_n(p_2)|^2\rangle}$$

$$= \frac{\langle|V_n(p_1)|^2|V_n(p_2)|^2 cos[2n(\Psi_n(p1) - \Psi_n(p2))]\rangle}{\langle|V_n(p_1)|^2|V_n(p_2)|^2\rangle}$$

Could be extracted from the above two !

Results : Elliptic vs Triangular flow



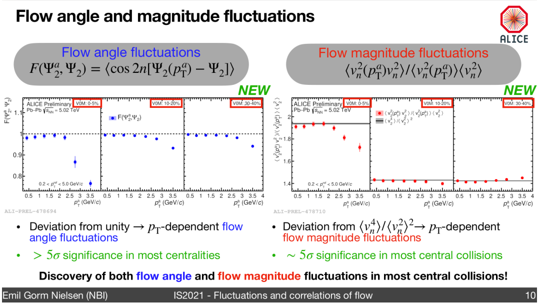
P. Bozek arXiv: 1808.04248

In experiment it is difficult to use above formulas at high pT due to limited statistics !

Flow decorrelation in p_T with global flow

Correlations between global flow and the flow at a fixed pT

- One of the flow at a fixed p_T ($V_n(p)$) and another as global (momentum averaged) (V_n).
- Such factorization breaking coefficients are statistically preferable and accessible !



factorization breaking between V_n^2 and $V_n^2(p)$ can be measured

Model study

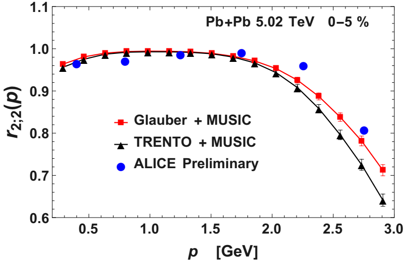
Bozek, Samanta arXiv: 2109.07781

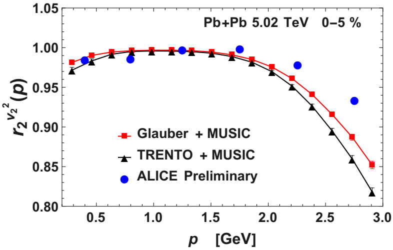
flow vector factorization

$$r_{n;2}(p) = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$

flow magnitude factorization

$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$





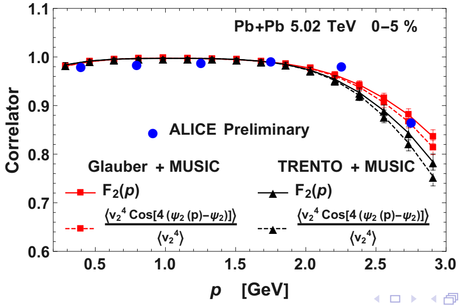
flow angle decorrelation

- Flow angle factorization coefficient,

$$F_n(p) = \frac{flow\ \ vec.\ decor}{flow\ \ mag.\ decor} = \frac{\langle V_n^2 V_n^*(p)^2\rangle}{\langle |V_n|^2 |V_n(p)|^2\rangle}$$

$$= \frac{\langle |V_n|^2 |V_n(p)|^2 cos[2n(\Psi_n - \Psi_n(p))]\rangle}{\langle |V_n|^2 |V_n(p)|^2\rangle} \simeq \frac{\langle |V_n|^4 cos[2n(\Psi_n - \Psi_n(p))]\rangle}{\langle |V_n|^4\rangle}$$

- There exist a strong correlation between flow angle and flow magnitude and they can be factorized.



Can we measure flow angle decorrelation ?

Measuring flow angle decorrelation

- Flow angle factorization coefficient,

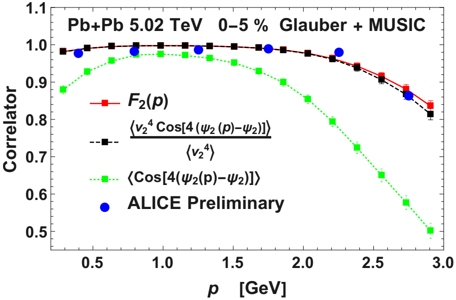
$$F_n(p) = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle}$$

- We can measure v_n^4 weighted angle decorrelation :

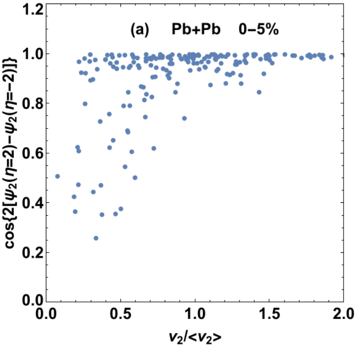
$$\frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle} \simeq \frac{\langle |V_n|^4 cos[2n(\Delta\Psi)] \rangle}{\langle |V_n|^4 \rangle}$$

- But we cannot measure simple angle decorrelation :

$$\frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle} \neq \langle cos[2n(\Delta\Psi)] \rangle$$



Twist angle - flow magnitude correlation in η Bozek, Broniowski arXiv: 1711.03325



Observations

- **Strong correlation** exists between flow magnitude and twist angle
- Correct measure of angle decor.:

$$angdecor. = \frac{flow\ \ vec.\ decor.}{flow\ \ mag.\ decor.} = \frac{\langle v_n^2 cos(n(\Delta\Psi))\rangle}{\langle v_n^2\rangle} \neq \langle cos(n(\Delta\Psi))\rangle$$

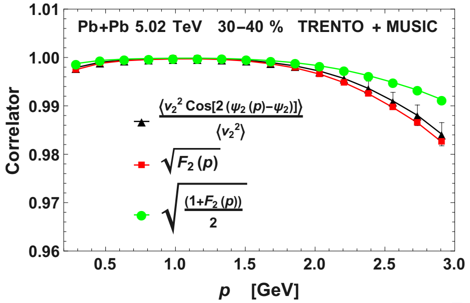
- Weighting correlators by powers of v_n explains the hierarchy.

Extracting the first order flow decorrelation

- We can extract the 1st order flow angle decorrelation from the 2nd order flow angle decorrelation by a simple approximation :

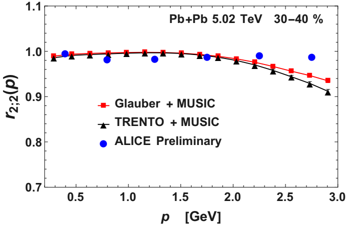
$$\frac{\langle |V_n|^2 cos[n(\Delta\Psi)] \rangle}{\langle |V_n|^2 \rangle} \simeq \sqrt{\frac{\langle |V_n|^4 cos[2n(\Delta\Psi)] \rangle}{\langle |V_n|^4 \rangle}} = \sqrt{F_n(p)}$$

- Meeasuring 2nd order flow fact. coeff. $\longrightarrow$ 1st order flow fact. coeff.
- Upper limit of 1st order flow angle decorrelation $= \sqrt{(1 + F_n(p))/2}$

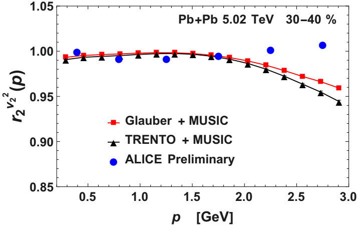


Semi-peripheral collision (30-40 %)

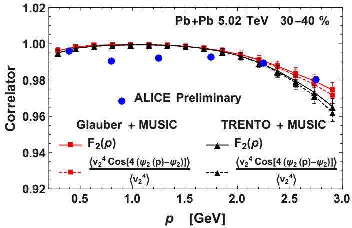
flow vec. decor.



flow mag. decor.



flow angle decor.



Observations

- The flow magnitude decorrelation is **approximately one half** of the flow vector decorrelation:

$$[1 - r_n^{v_n^2}(p)] \simeq \frac{1}{2}[1 - r_{n;2}(p)]$$

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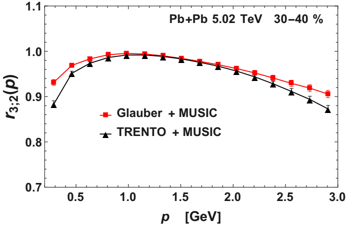
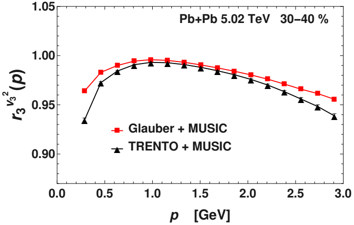
- for central collision our model results reproduce the data.
- For semi-peripheral collision our model results do not reproduce the data
- For 30-40 % the data go slightly above 1 at high $p_T \longrightarrow$ may indicate a significant non-flow contribution.

Same for triangular flow (v_3)

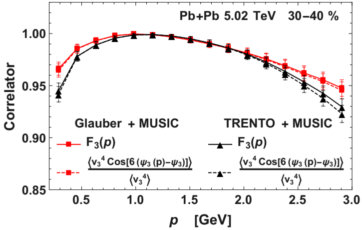
flow vec. decor.



flow mag. decor.



flow angle decor.



Factorization breaking between mixed harmonics non-linear response

Mixed harmonic corr. Qian, Heinz, He, Huo arXiv: 1703.04077

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$$\frac{\langle V_m^*(p) V_k V_n \rangle}{\sqrt{\langle |V_m(p)|^2 \rangle \langle |V_k|^2 |V_n|^2 \rangle}}$$

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- For m = 4 and k = n = 2 , we have V_2^2 - $V_4(p)$ correlation and for m=5, k=2, n=3, we have $V_5(p)$-$V_2 V_3$ correlation and so on.

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$$V_n = V_n^L + V_n^{NL} \ (n = 4, 5) \ ; \ V_4^{NL} \propto V_2^2 \ and \ V_5^{NL} \propto V_2 V_3$$

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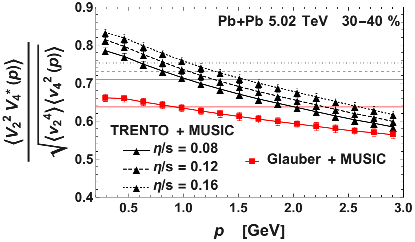
- So, V_2^2-$V_4(p)$ and $V_5(p)$-$V_2 V_3$ correlations measures the non-linear coupling between the higher flow harmonics with lower order.

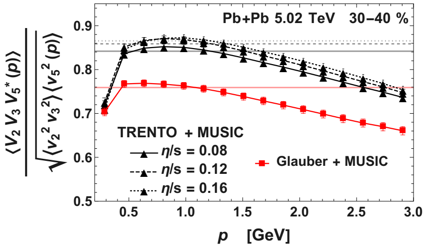
$V_2^2 - V_4(p)$ corr.

$$\frac{\langle V_2^2 V_4^*(p)\rangle}{\sqrt{\langle |V_2|^4\rangle \ \langle |V_4(p)|^2\rangle}}$$

$V_2 V_3 - V_5(p)$ corr.

$$\frac{\langle V_2 V_3 V_5^*(p)\rangle}{\sqrt{\langle |V_2|^2 |V_3^2|\rangle \ \langle |V_5(p)|^2\rangle}}$$





Comment

- But these correlations again can't be measured experimentally !
- So, we go to the next order : V_2^4-$V_4(p)^2$ and $V_2^2 V_3^2$ - $V_5^2(p)$ correlations

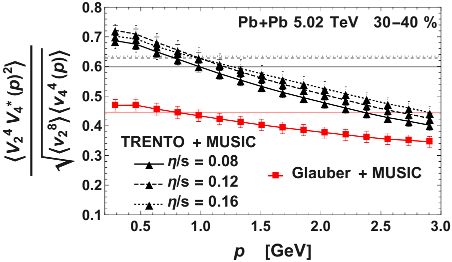
$V_2^4 - V_4^2(p)$ correlations

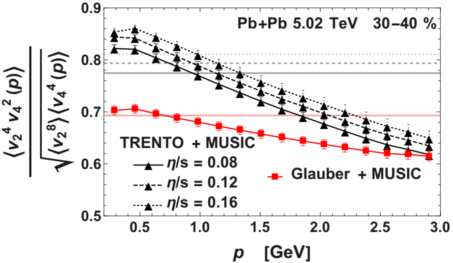
flow vector factorization

$$\frac{\langle V_2^4 V_2^*(p)^2 \rangle}{\sqrt{\langle |V_2|^8 \rangle \, \langle |V_4(p)|^4 \rangle}}$$

flow magnitude factorization

$$\frac{\langle |V_2|^4 |V_4(p)|^2 \rangle}{\sqrt{\langle |V_2(p)|^8 \rangle \, \langle |V_4(p)|^4 \rangle}}$$



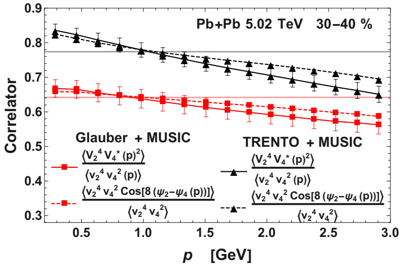


$V_2^4 - V_4^2(p)$ angle decorrelations

Extraction of flow angle decorrelation

- Measuring flow vector and flow magnitude correlation separately lead to the extraction of flow angle decorrelation :

$$\frac{\langle V_2^4 V_4^*(p)^2 \rangle}{\langle |V_2|^4 |V_4(p)|^2 \rangle} = \frac{\langle |V_2|^4 |V_4(p)|^2 cos[8(\Psi_2 - \Psi_4(p))] \rangle}{\langle |V_2|^4 |V_4(p)|^2 \rangle} \simeq \frac{\langle |V_2|^4 |V_4|^2 cos[8(\Psi_2 - \Psi_4(p))] \rangle}{\langle |V_2|^4 |V_4|^2 \rangle}$$



Study of the factorization breaking of mixed flow puts additional constraints on the initial state models !

Summary and outlines

- Flow fluctuations can be probed by studying flow correlation (flow factorization breaking coefficient) in transverse momentum or rapidity.

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  - Flow vec. decor. $= \frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$

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- We study new correlations keeping one flow fixed and another flow as global
- Measurement of mixed flow factorization coefficient (new) measures the non-linear coupling and provides additional constraints to the models

Thank you !

for your attention

Back up slides

FB asymmetry and twist

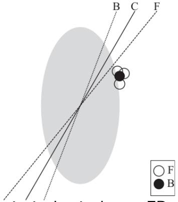
The emission profile

- The emission profile defining the initial density in the (x,y,$\eta_{||}$) has the form:

$$F(\eta_{||}, x, y) \propto \rho_{+}(x, y) f_{+}(\eta_{||}) + \rho_{-}(x, y) f_{+}(\eta_{-}||)$$

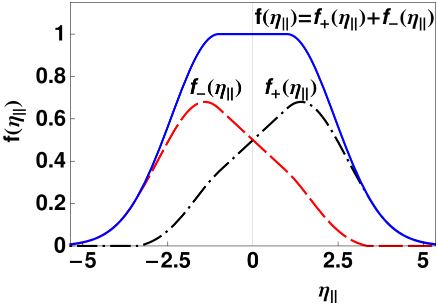
where, $f_{\pm}(\eta_{||}) = f_F(\pm\eta_{||}) f(\eta_{||})$ *with*

$$f(\eta_{||}) = exp[-\frac{(\eta_{||} - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_{||}| - \eta_0)] \ \& \ f_F(\eta_{||}) = \begin{cases} 0, & \eta_{||} \leq -\eta_m \\ \frac{\eta_{||}+\eta_m}{2\eta_m}, & -\eta_m < \eta_{||} < \eta_m \\ 1, & \eta_m \leq \eta_{||} \end{cases}$$



Twist of principal axis due to FB asymmetry

3+1D hydro evolution of the torqued fireball Bozek, Broniowski, Moreira



Evolution of the torqued fireball

- The profile f_+ (f_-) is peaked in the forward (backward) direction in $\eta_{||}$

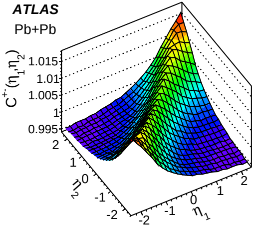
Correlation for other observables

multiplicity-multiplicity

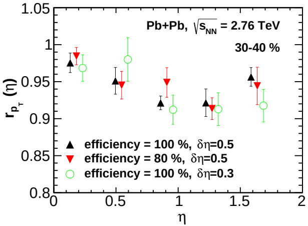
$$\frac{\langle N(\eta_1)N(\eta_2)\rangle - \langle N(\eta_1)\rangle\langle N(\eta_2)\rangle}{\langle N(\eta_1)\rangle\langle N(\eta_2)\rangle}$$

tr. momentum - tr. momentum

$$\frac{Cov([p_T]_F[p_T]_B)}{\sqrt{Var\left([p_T]_F\right)\, Var\left([p_T]_F\right)}}$$



ATLAS arXiv: 1606.08170



Chatterjee, PB arXiv: 1704.02777

measurements in rapidity bins are plagued by non-flow effects !

Twist angle and flow magnitude decorrelation
3+1D hydro model Bozek, Broniowski arXiv: 1711.03325

3-bin measures

- flow vector decor:

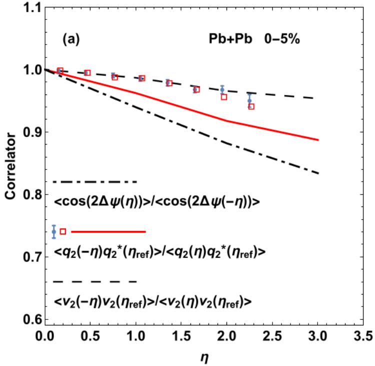
$$r_n(\eta) = \frac{\langle V_n(-\eta) V_n^*(\eta_{ref}) \rangle}{\langle V_n(\eta) V_n^*(\eta_{ref}) \rangle}$$

- flow magnitude decor:

$$r_n^v(\eta) = \frac{\langle v_n(-\eta) v_n(\eta_{ref}) \rangle}{\langle v_n(\eta) v_n(\eta_{ref}) \rangle}$$

- and flow angle decor :

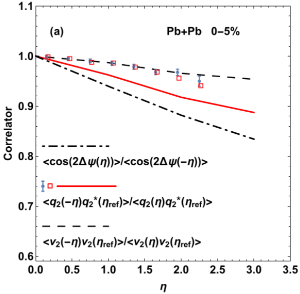
$$r_n^\Psi(\eta) = \frac{\langle cos[n(\Psi_n(-\eta) - \Psi_n(\eta_{ref}))] \rangle}{\langle cos[n(\Psi_n(\eta) - \Psi_n(\eta_{ref}))] \rangle}$$

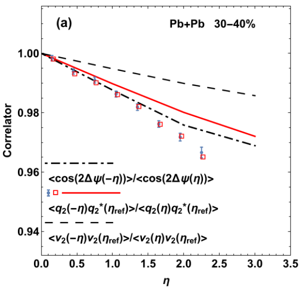


0-5 % : Elliptic flow

Observations

- **Surprising result:** Inverted hierarchy
- **flow mag. decor. $<$ mag $+$ twist angle decor. $<$ twist angle decor.**
- signifies correlation between flow magnitude and twist angle



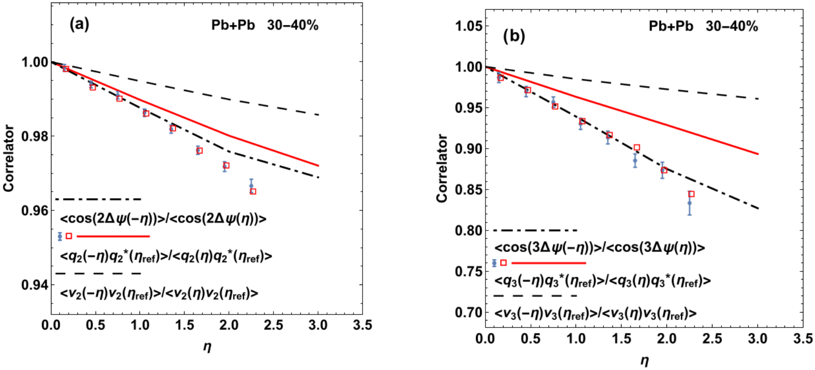


Central vs peripheral collision

Observations

- Inverted hierarchy is **stronger** in central collision
- Large elliptic flow in in semi-central collision $\longrightarrow$ less fluctuations

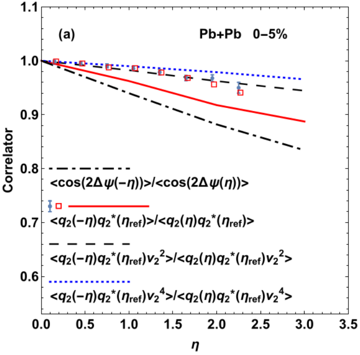
Elliptic vs trianglar flow



Observations

- Inverted hierarchy is **stronger** in triangular flow
- Triangular flow (v_3) is dominated by fluctuations

Correlators weighted by powers of v_n



Elliptic flow

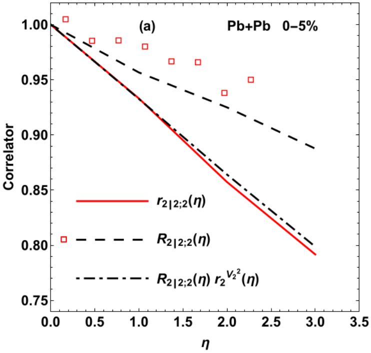
Observations

- hierarchy of correlators matches with the expectation:

$$\frac{\langle V_n(-\eta)V_n^*(\eta_{ref})\rangle}{\langle V_n(\eta)V_n^*(\eta_{ref})\rangle} < \frac{\langle V_n(-\eta)V_n^*(\eta_{ref})v_n^2\rangle}{\langle V_n(\eta)V_n^*(\eta_{ref})v_n^2\rangle} < \frac{\langle V_n(-\eta)V_n^*(\eta_{ref})v_n^4\rangle}{\langle V_n(\eta)V_n^*(\eta_{ref})v_n^4\rangle}$$

- The more power of v_n weighted $\longrightarrow$ the more correlation is found
- Significant correlation exists between flow magnitude and twist angle $\longrightarrow$ also could be measured experimentally

Factorization of factorization breaking



Elliptic flow (2nd moment)

Approximate factorization

- flow magnitude factorization breaking :

$$r_n^{v_n^2}(\eta) = \frac{\langle v_n^2(-\eta) v_n^2(\eta_{ref})\rangle}{\langle v_n^2(\eta) v_n^2(\eta_{ref})\rangle}$$

- twist angle factorization breaking:

$$R_{n,2}(\eta) = \frac{\langle V_n(-\eta_{ref}) V_n^*(\eta) V_n(-\eta) V_n^*(\eta_{ref})\rangle}{\langle V_n(-\eta_{ref}) V_n^*(-\eta) V_n(\eta) V_n^*(\eta_{ref})\rangle}$$

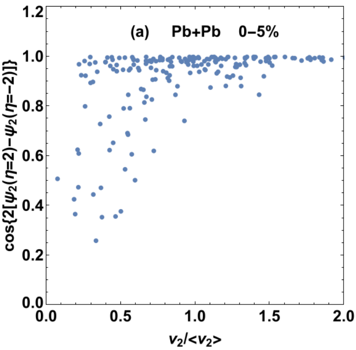
- angle+ magnitude factorization breaking :

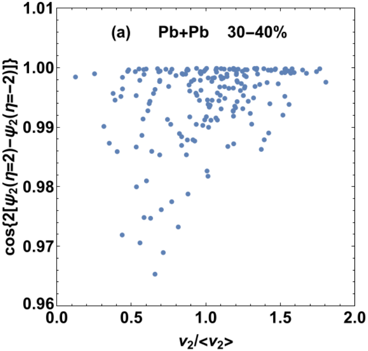
$$r_{n,2}(\eta) = \frac{\langle V_n^2(-\eta) V_n^{*2}(\eta_{ref})\rangle}{\langle V_n^2(\eta) V_n^{*2}(\eta_{ref})\rangle}$$

- approximate factorization relation :

$$r_{n,2}(\eta) \simeq r_n^{v_n^2}(\eta) * R_{n,2}(\eta)$$

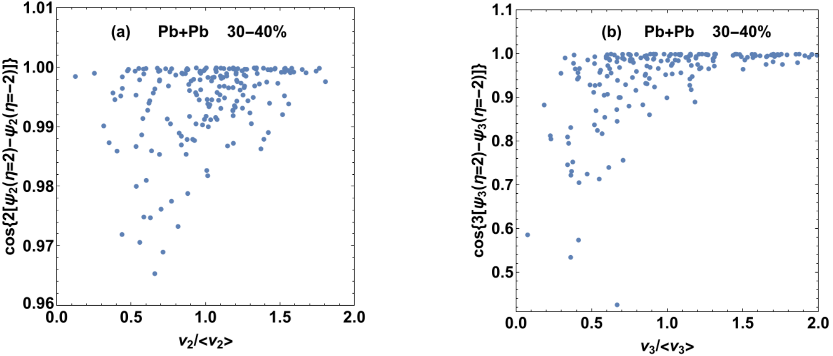
Central vs peripheral





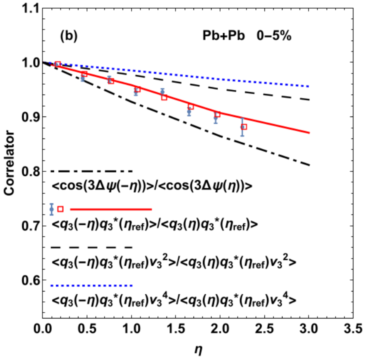
Stronger correlation in central collision

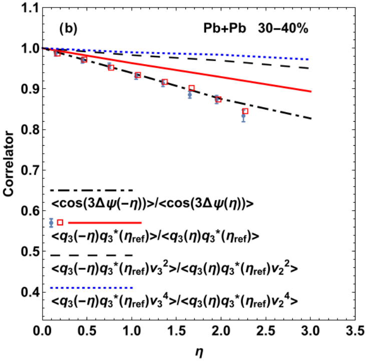
Elliptic vs triangular



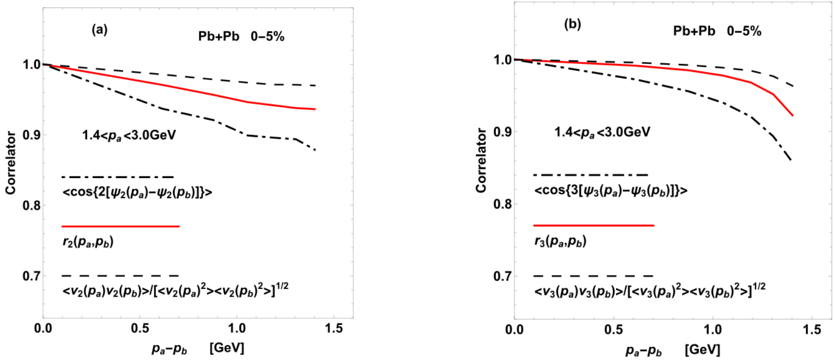
Stronger correlation for triangular flow v_3

Same for triangular flow



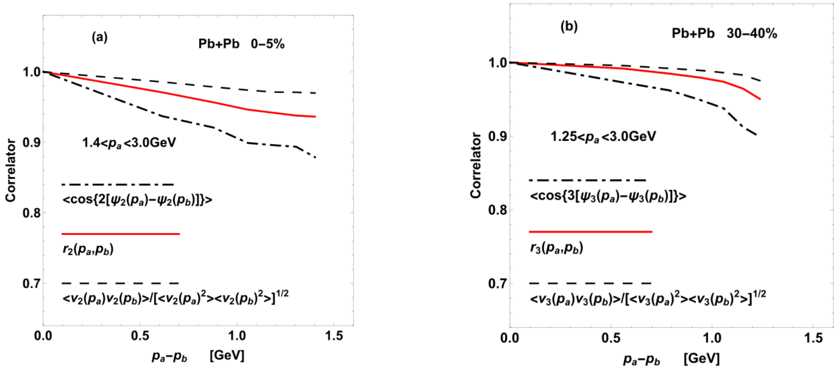


Elliptic vs Triangular flow



P. Bozek arXiv: 1808.04248

Central vs peripheral collision



P. Bozek arXiv: 1808.04248

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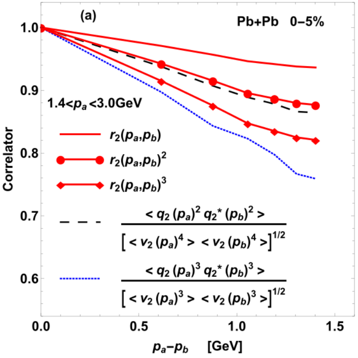
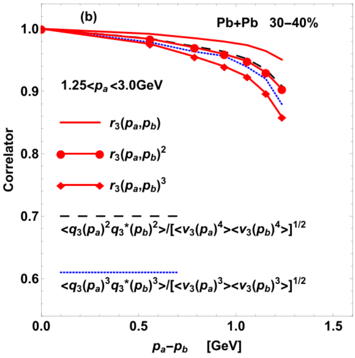




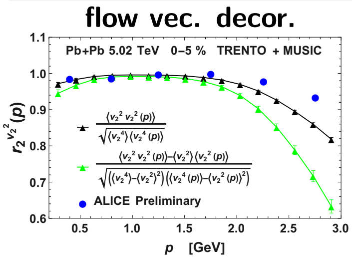
P. Bozek arXiv: 1808.04248

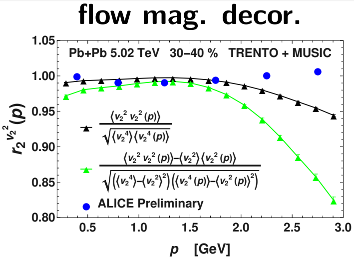
The usual correlation (ρ)

One could use a normalization :

$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 \rangle - \langle |V_n|^2 \rangle \langle |V_n(p)|^2 \rangle}{\sqrt{\left(\langle |V_n|^4 \rangle - \langle |V_n|^2 \rangle^2\right) \left(\langle |V_n(p)|^4 \rangle - \langle |V_n(p)|^2 \rangle^2\right)}}$$

But that gives quite different result !





Equivalence of different normalization

Scaling of data

- The ALICE collaboration use different normalization in their data, namely for vector and magnitude correlation : $\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\langle |V_n|^2\rangle\ \langle |V_n(p)|^2\rangle}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle}{\langle |V_n|^2\rangle\ \langle |V_n(p)|^2\rangle}$ respectively
- We divide them by a factor $\frac{\langle |V_n^4|\rangle}{\langle |V_n^2|\rangle^2}$, the **baseline** of the plots, and we have: $\frac{\langle V_n^2 V_n^*(p)^2\rangle\ \langle |V_n^2|\rangle}{\langle |V_n|^4\rangle\ \langle |V_n(p)|^2\rangle}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle\ \langle |V_n^2|\rangle}{\langle |V_n|^4\rangle\ \langle |V_n(p)|^2\rangle}$
- But we use the definitions: $\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\sqrt{\langle |V_n|^4\rangle\ \langle |V_n(p)|^4\rangle}}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle}{\sqrt{\langle |V_n|^4\rangle\ \langle |V_n(p)|^4\rangle}}$
- The difference between the two normalization is a factor : $\sqrt{\frac{\langle |V_n^4(p)|\rangle \langle |V_n^2|\rangle^2}{\langle |V_n^4|\rangle \langle |V_n^2(p)|\rangle^2}} \simeq 1 \implies \frac{\sqrt{\langle |V_n^4(p)|\rangle}}{\langle |V_n^2(p)|\rangle} \simeq \frac{\sqrt{\langle |V_n^4|\rangle}}{\langle |V_n^2|\rangle}$