On the shape of the Bose-Einstein correlation function

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Outline

Introduction to femtoscopy

1. Two-particle correlations
2. On the variable of the correlation function
3. Final state interaction
4. The Levy parametrization and its possible interpretations

Results from RHIC and SPS

1. 0-30%, cent. dep., $\sqrt{s_{NN}}$ dep., 3D, 3 particle Au+Au from PHENIX
2. Be+Be and Ar+Sc from NA61
3. 0-30% Au+Au from STAR

The discussion of the results

Femtoscopy – general remarks

Originates from radio astronomy

- Hanbury-Brown and Twiss observed intensity correlation
- In high energy physics, Goldhaber, Goldhaber, Lee and Pais

Technique to access the spatio-temporal structure of the particle emitting source

$$C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1) N_1(p_2)}$$

where we can use the Yano-Koonin formula to relate the mom. dists. to the source:

$$N_2(p_1, p_2) = \int dx_1 dx_2 S(x_1, p_1) S(x_2, p_2) \left| \Psi_2^{(p_1, p_2)}(x_1, x_2) \right|^2$$

S: source function, Ψ_2 two-particle wavefunction

Femtoscopy – two approaches

$$N_2(p_1, p_2) = \int dx_1 dx_2 S(x_1, p_1) S(x_2, p_2) |\Psi_2(x_1, x_2)|^2$$

Assume the source shape: S ~ **Gaussian**

Measure in a clean environment, e. g. in pp

Learn about the final state interactions hidden in the wave function

Program in ALICE:

$p-K, p-p, p-\Lambda, \Lambda-\Lambda, p-\Xi, p-\Omega,$

$p-\Sigma, p-\phi, N-\Sigma, N-\Lambda$

Assume the **wave function**: free planewave

$$|\Psi_2|^2 = 1 + \cos\left((p_1 - p_2)x\right)$$

Not to realistic: Coulomb (and strong) FSI

What is the interacting wave function?

$$\Psi_2 \sim \frac{\Gamma(1+i\eta)}{e^{\frac{\pi\eta}{2}}}\left[e^{i\boldsymbol{kr}} F\left(-i\eta, 1, i(kr - \boldsymbol{kr})\right)\right]$$

$$+\boldsymbol{r} \rightarrow -\boldsymbol{r}$$

(more complicated with strong interaction)

Learn about the source size and shape

Femtoscopy – the core-halo model

Usually pions, kaons, protons are measured

Resonance contributions are considerable: core-halo model

$$S(x,p) = \sqrt{\lambda}\, S_{core}(x,p) + \left(1-\sqrt{\lambda}\right) S_{halo}^{R_h}(x,p)$$

Let's introduce the pair source function as

$$D_{AB}(x,p) = \int d^3R\ \ S_A\left(R + \frac{x}{2}, p\right) S_B\left(R - \frac{x}{2}, p\right)$$

With this the pair source function in the core-halo model:

$$D(x,p) = \lambda D_{cc}(x,p) + \underbrace{2\sqrt{\lambda}\left(1-\sqrt{\lambda}\right) D_{ch}(x,p) + \left(1-\sqrt{\lambda}\right)^2 D_{hh}(x,p)}_{\text{Notation: } D_{(h)}/(1-\lambda)}$$

Femtoscopy – general form

With $K = 0.5(p_1 + p_2)$ and $Q = p_1 - p_2$! Also assume that $p_1 \approx p_2$

$$C_2(Q,K) \approx \lambda \int d^3r D_{cc}(r,K) \left|\Psi_2^{(Q)}(r)\right|^2 + (1-\lambda) \int d^3r D_{(h)}(r,K) \left|\Psi_2^{(Q)}(r)\right|^2$$

If we take the $R_h \to \infty$ limit the Bowler-Sinyukov formula is given:

$$\boxed{C_2(Q,K) \approx 1 - \lambda + \lambda \int d^3r D_{cc}(r,K) \left|\Psi_2^{(Q)}(r)\right|^2}$$

The simple planewave case (i.e. no FSI):

$$C_2^{(0)}(Q,K) = 1 + \lambda \frac{\widetilde{D}_c(Q,K)}{\widetilde{D}_c(Q=0,K)}$$

On the **3D** variable of the correlation function

$$C_2(Q,K) \approx 1-\lambda+\lambda\int d^3r D_{cc}(r,K)\left|\Psi_2^{(Q)}(r)\right|^2$$

The K dependence is much smoother than the Q dependence

Use the Q as a variable and the measure the K dep. of the params.

$$Q\cdot K=(p_1-p_2)(p_1+p_2)=p_1^2-p_2^2=0\rightarrow Q_0=\vec{Q}\,\frac{\vec{K}}{K_0}$$

$C_2(Q)$ can be transformed to $C_2\left(\vec{Q}\right)$

Go to LCM system where $\vec{Q}=\left(Q_{out},Q_{side},Q_{long}\right)$

On the **1D** variable of the correlation function

What about in 1D? Could be necessary due to the lack of statistics

Usual choice: $q_{inv} = \sqrt{-Q^{\mu}Q_{\mu}}$, arguable choice!

$$q_{inv} = (1 - \beta_t^2)Q_{out}^2 + Q_{side}^2 + Q_{long}^2$$

But q_{inv} could be very small even if $Q_{out}^2 \approx Q_{side}^2 \approx Q_{long}^2 \neq 0$

It is also known that the source approximately spherical at RHIC

1D variable!

$$q_{inv} = |q_{PCMS}| \quad \Rightarrow \quad \boxed{Q = |q_{LCMS}|}$$

Here, sphericity preserved, so Q independent of the direction of q_{LCMS}

Final state interactions

Like-charged pions → Coulomb correction

Strong final state interaction may play a role

Effect of the resonances: core-halo model

- Long-lived resonances contribute to the halo
- In-medium mass modifications could cause specific m_T dependence

Partially coherent particle production (core-halo model)

Aharonov-Bohm like effect: the hadron gas acts as a background field, the correlated bosons paths are the closed loop

Levy parametrization of the C_2

Generalized Gaussian – Levy distribution

$$\mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3q \; e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

$\alpha = 2$: Gaussian, $\alpha = 1$: Cauchy, $0 < \alpha \leq 2$: Levy

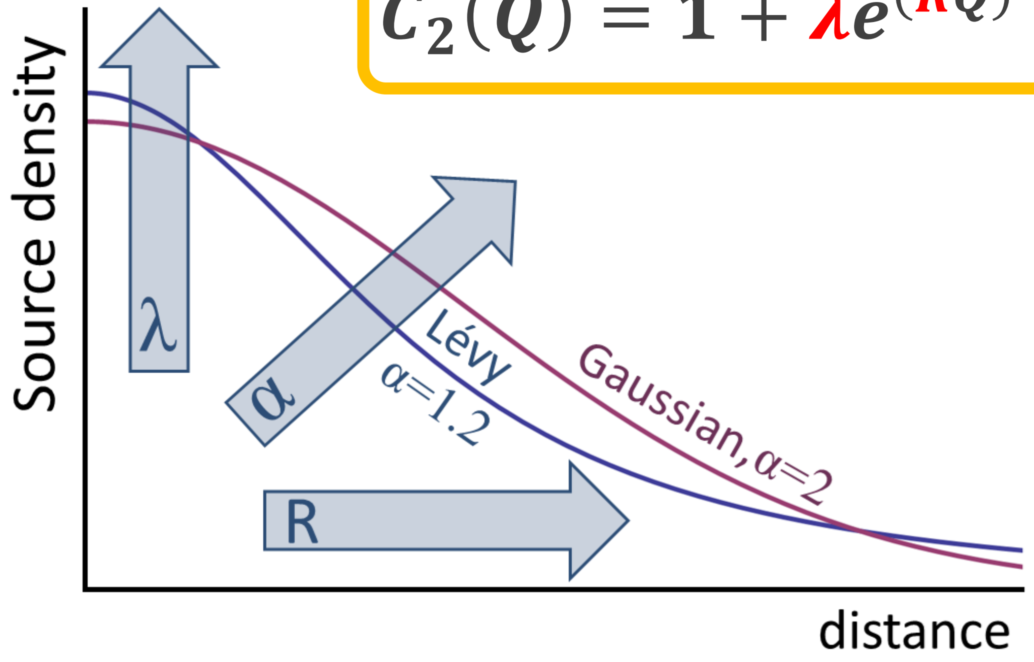
Assume the source to be Levy!

$$\boxed{\boldsymbol{C_2(Q) = 1 + \lambda e^{(RQ)^\alpha}}}$$

$\lambda(K)$: core-halo parameter

$R(\mathrm{K})$: Levy-scale parameter

$\alpha(K)$: Levy index of stability



Physics in the parameters

Possible interpretations of the λ:

1. Specific m_T suppression linked to in-medium mass modification of η'
2. Measuring two- and three particle correlations could shed light on partially coherent particle production (see core-halo model):

$$f_c(K) = \frac{N_c(K)}{N(K)} \quad \text{and} \quad p_c(K) = \frac{N_c^p(K)}{N_c(K)}$$

$$\lambda_2 = f_c^2[(1-p_c)^2 + 2p_c(1-p_c)]$$
$$\lambda_3 = 2f_c^3[(1-p_c)^3 + 3p_c(1-p_c)^2] + 3f_c^2[(1-p_c)^2 + 2p_c(1-p_c)]$$

$$\kappa_3 = \frac{(\lambda_3 - 3\lambda_2)}{2\sqrt{\lambda_2^3}}$$

Independent from f_c

Physics in the parameters

Possible interpretation of the R:

- Important: $R_{Levy} \neq R_{Gauss}$
- Is it related to the size? Check hydro-like scaling: $\frac{1}{R^2} = A\, m_T + B$
- Seen in Gaussian parametrizations

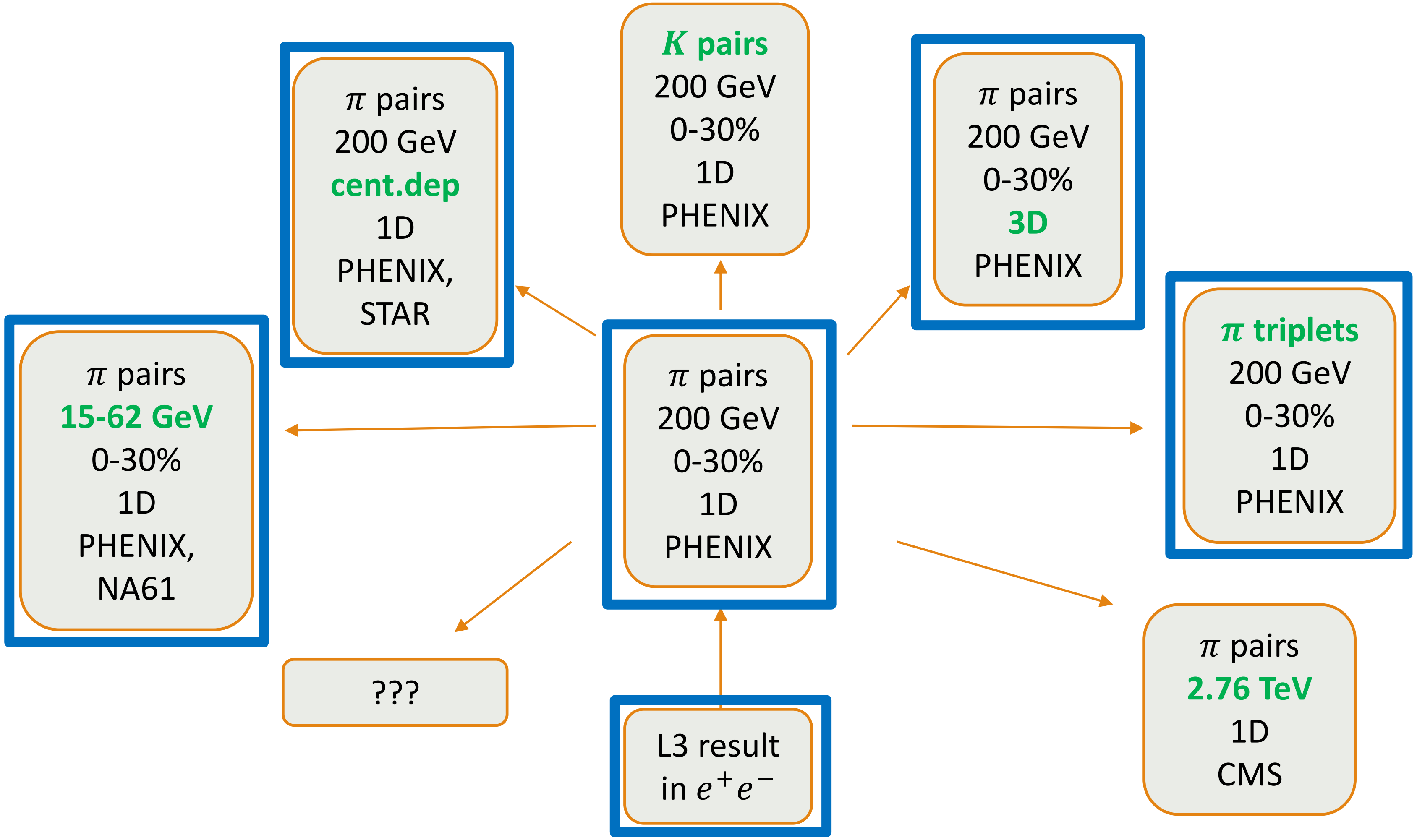
Possible interpretation of the α:

- Surprising similarity with the critical exponent of the spatial correlation in 3D

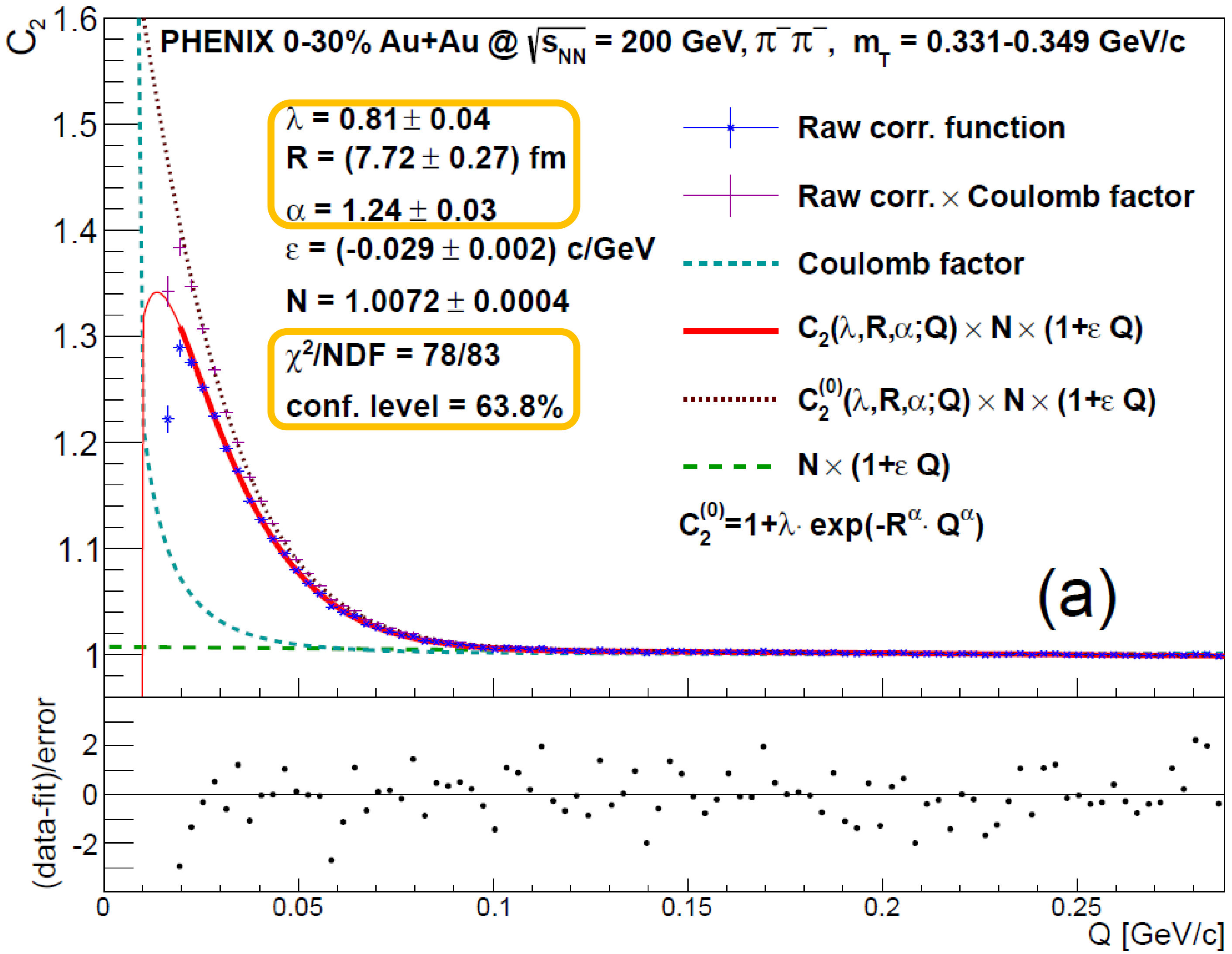
$$\text{spatial corr.} \sim r^{-1-\eta} \qquad \text{symm. Levy dist.} \sim r^{-1-\alpha}$$

- Sudden change in α could be a sign for critical behavior
- Could be the sign of anomalous diffusion or jets

The tree of the Levy analyses



The first results – PHENIX 0-30% Au+Au



- Measured correlation function in 31 m_T bin with 0-30% cent.
- Coulomb correction incorporated into the fit function
- $\alpha \neq 2$ nor $\alpha \neq 1$
- The fits are acceptable in terms of confidence level and χ^2/NDF
- Gaussian parametrization cannot describe the data

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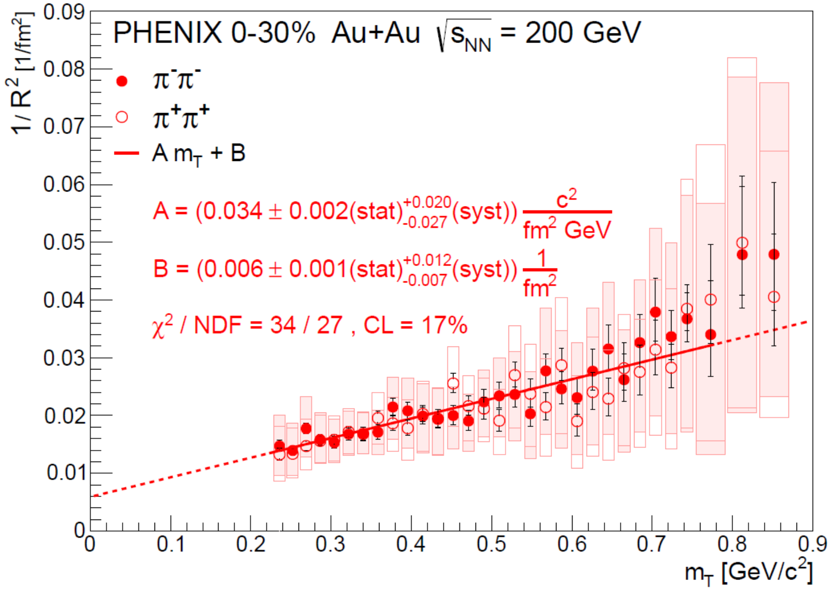
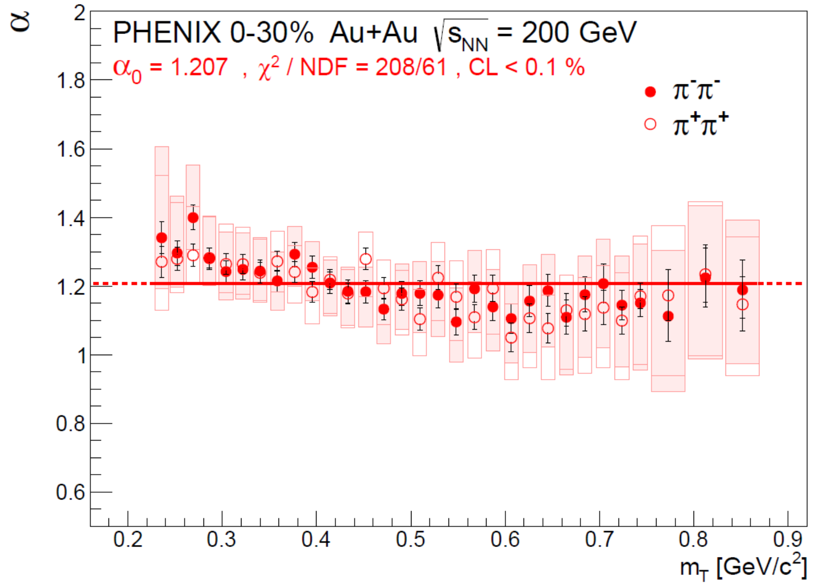
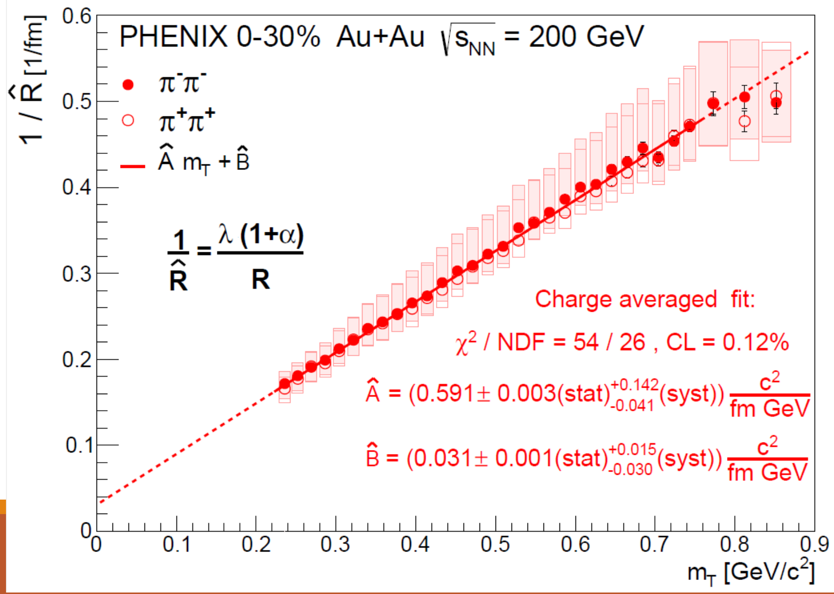
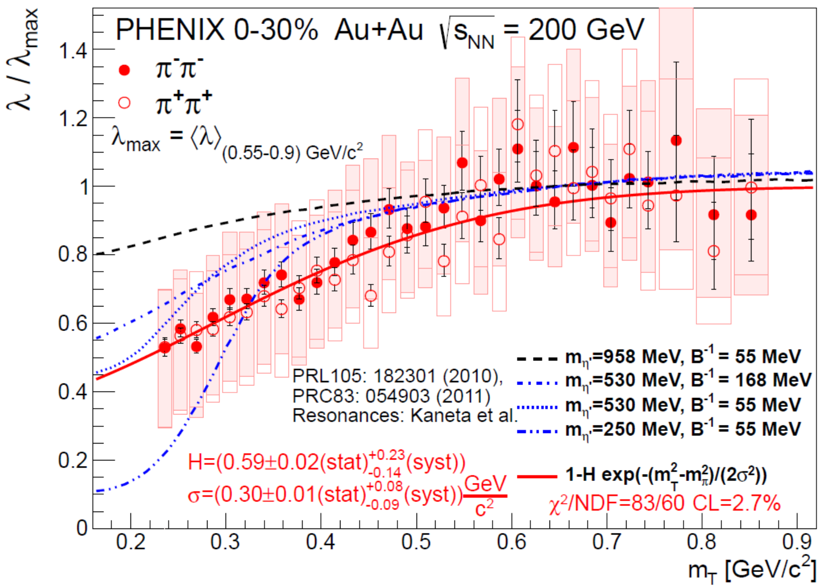






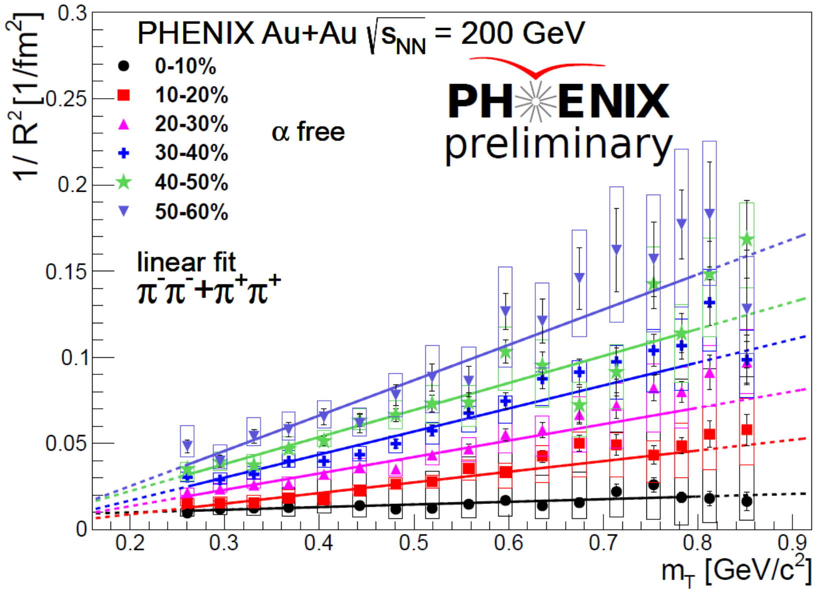


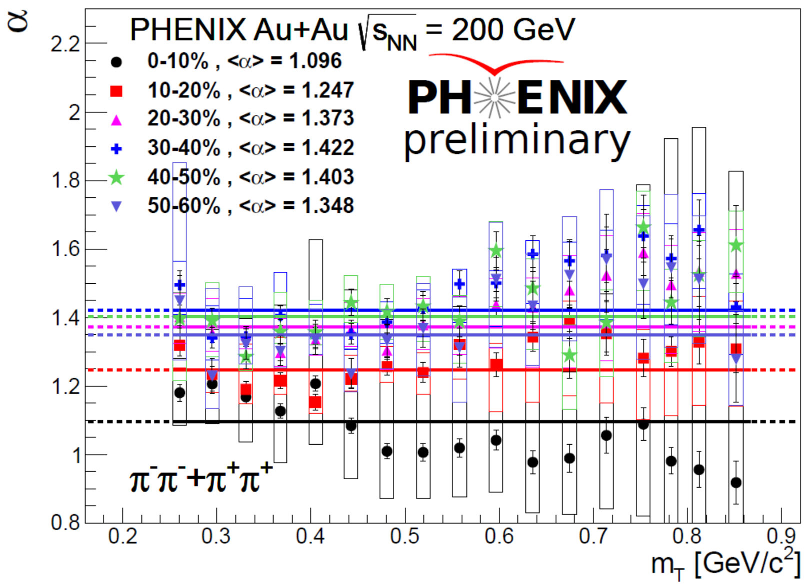
- R exhibits hydro scaling
- $1 < \alpha < 2$, $\langle\alpha\rangle \approx 1.2$
- $\lambda(m_T)$ suppressed which compatible with modified η' mass in the medium (compared with a resonance model)

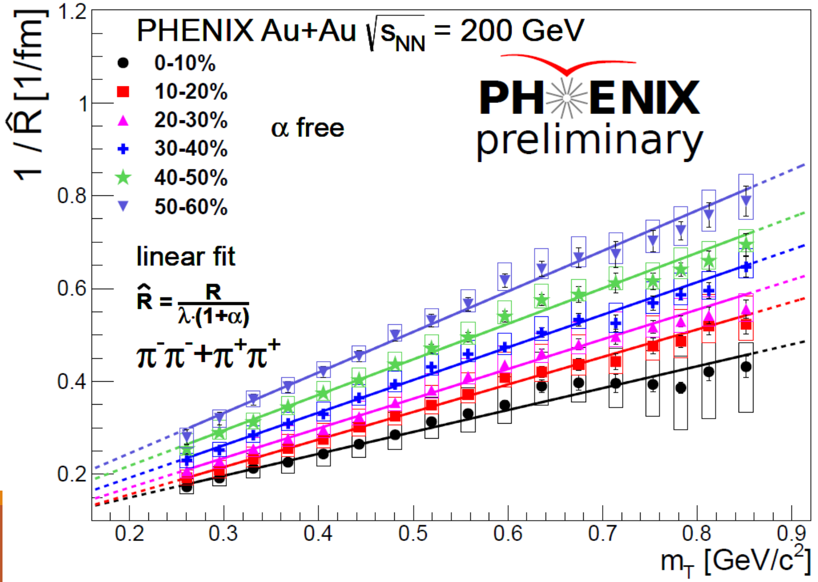
- New scaling parameter
  - Interpretation?
- Interpretation of α ?

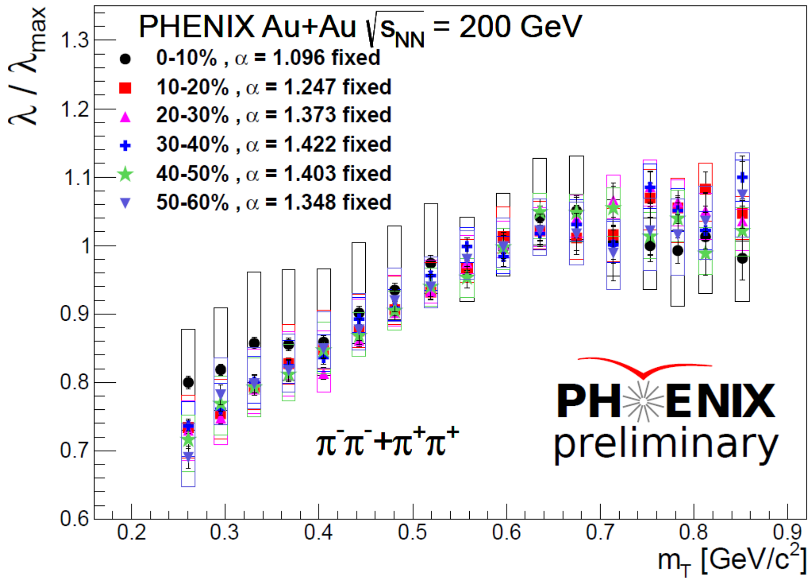
- Let's see the N_{part} and $\sqrt{s_{NN}}$ dependence

N_{part} dependence – PHENIX Au+Au





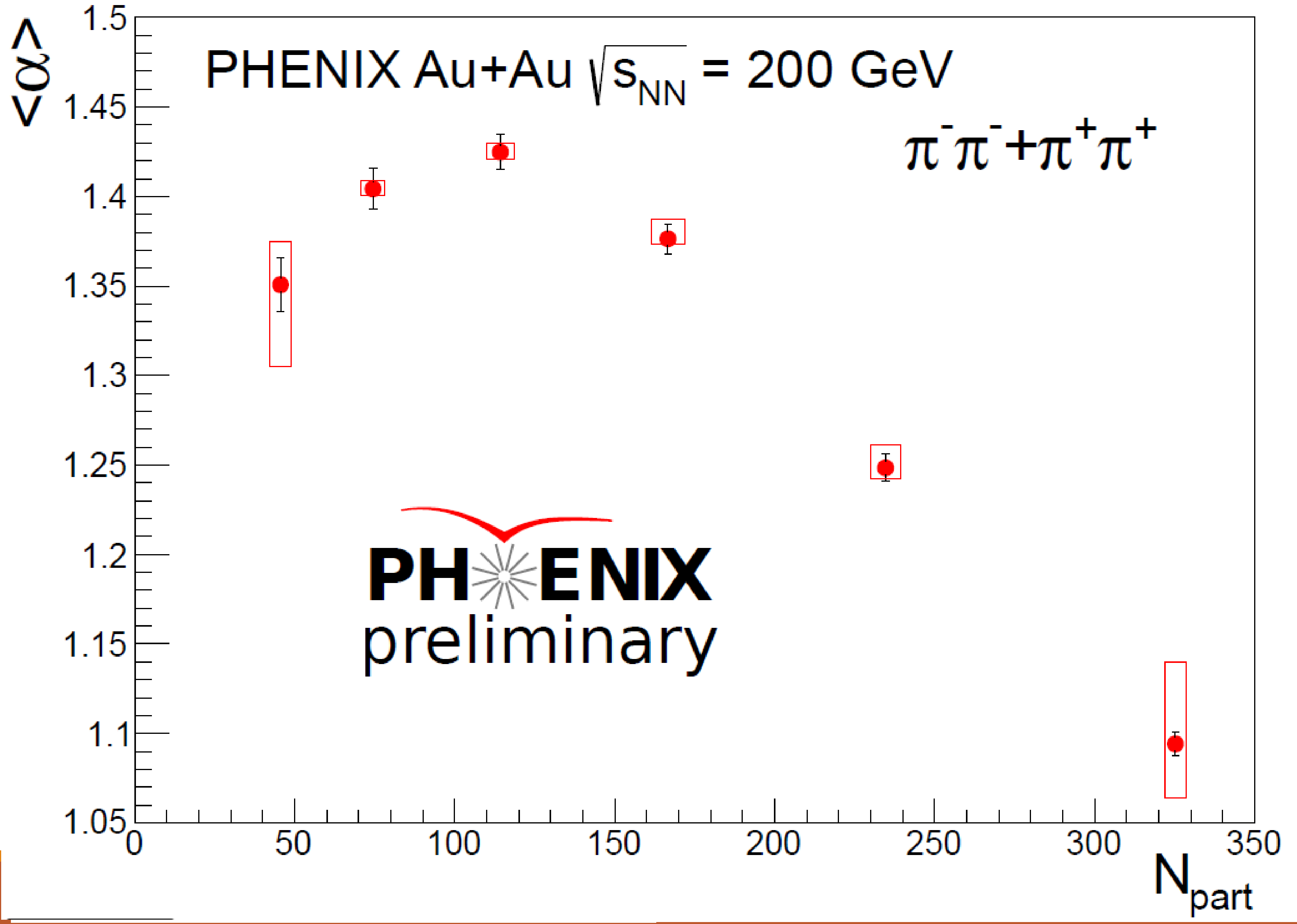




- R exhibits hydro scaling
- $1 < \langle\alpha\rangle < 2$
- $\langle\alpha\rangle$ depends on N_{part}
- $\lambda(m_T)$ suppressed
- The suppression doesn't depend on centrality
- Models can be ruled out

- Preliminary results!
- Improved, final results are on the way (Christmas plan to write the paper finally)

N_{part} dependence – PHENIX Au+Au



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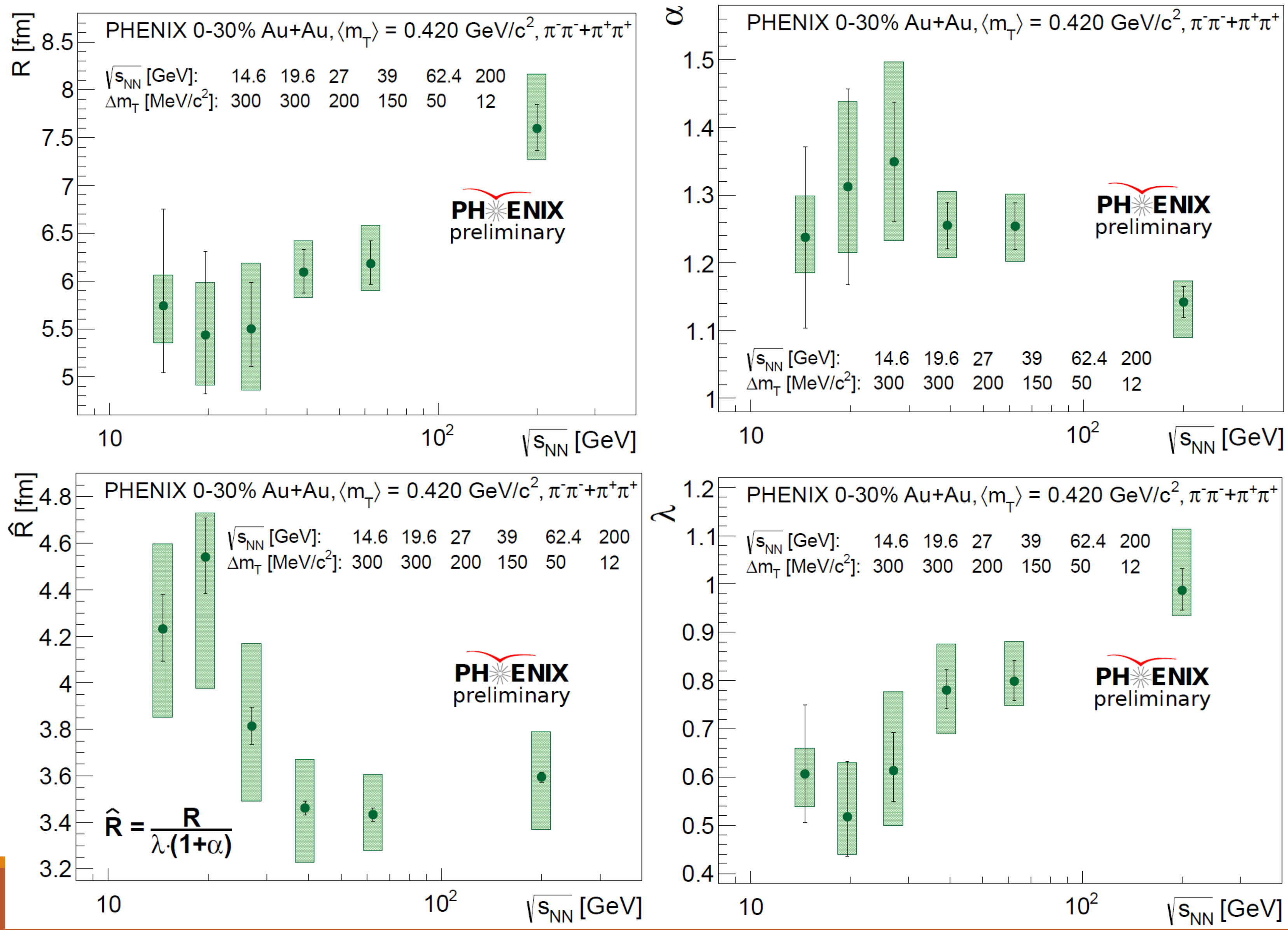
$\sqrt{s_{NN}}$ dependence – PHENIX Au+Au



- Integrated in m_T due to the lack of statistics
- α does not really depend on $\sqrt{s_{NN}}$
- Non-monotonic behavior of $\hat{R}$ observed
  - Interpretation?

- For $\sqrt{s_{NN}} \geq 39$ GeV there are m_T dependent analysis but the trends are not clear

Partial conclusions and critiques

Gaussian parametrization clearly not acceptable in terms of χ^2/NDF and CL

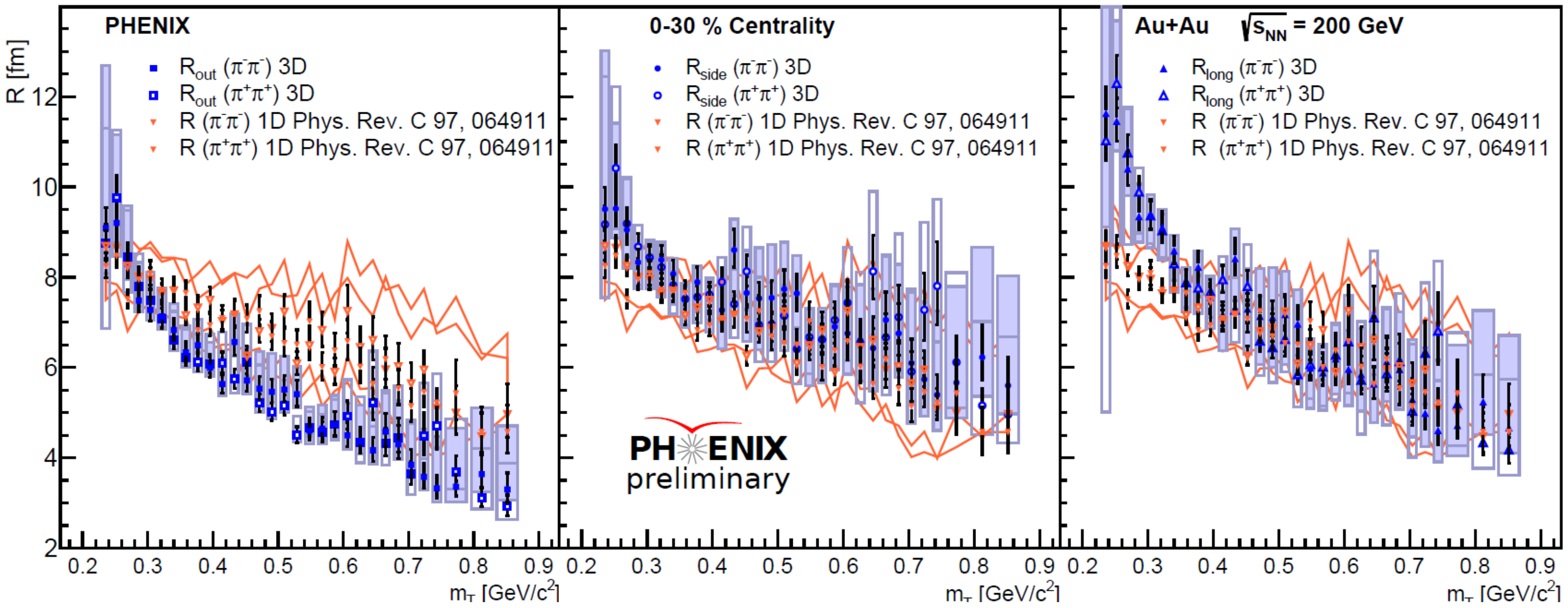
Levy gives satisfactory description of the measured 1D data at RHIC BES 1 energies in Au+Au collisions

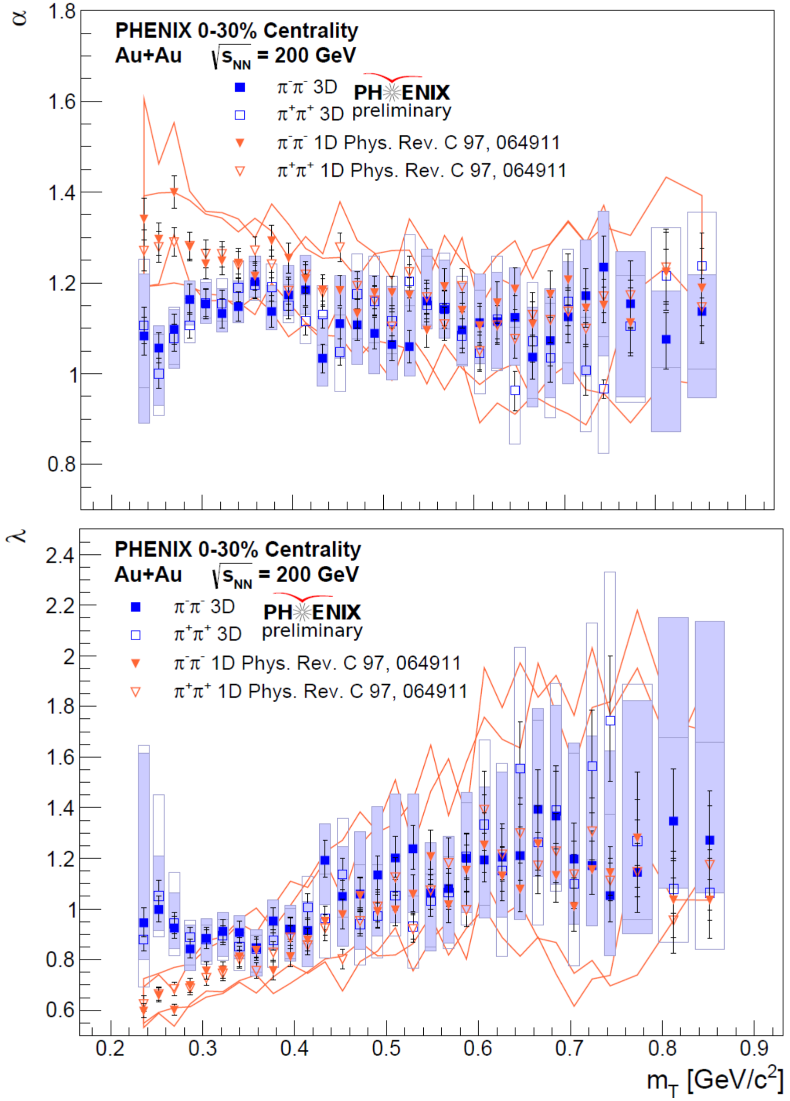
$1 < \alpha < 2$, doesn't depend on m_T strongly but centrality dependent

Why? Two main explanation besides the aforementioned:

- We use 1D variable which has an influence. In 3D it would be Gaussian!
- We measure the average of many Gaussian correlation functions with different width so the average is not Gaussian

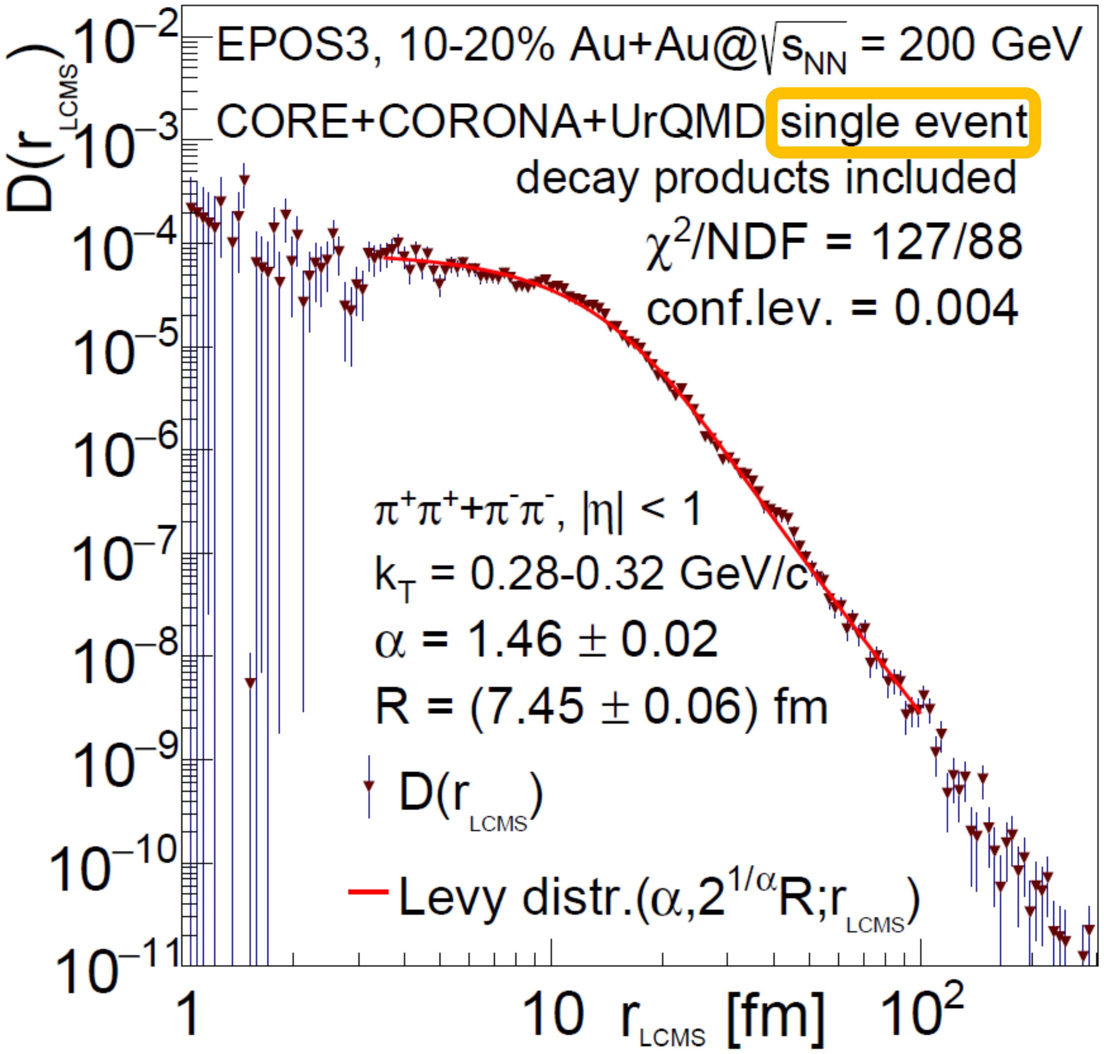
3D correlation – PHENIX 0-30% Au+Au





- 3D measurement gives very similar results compared to 1D
- The source appears to be spherical
- λ suppression is there in 3D too, with small discrepancy
- Preliminary data!

EPOS simulation – event-by-event correlation



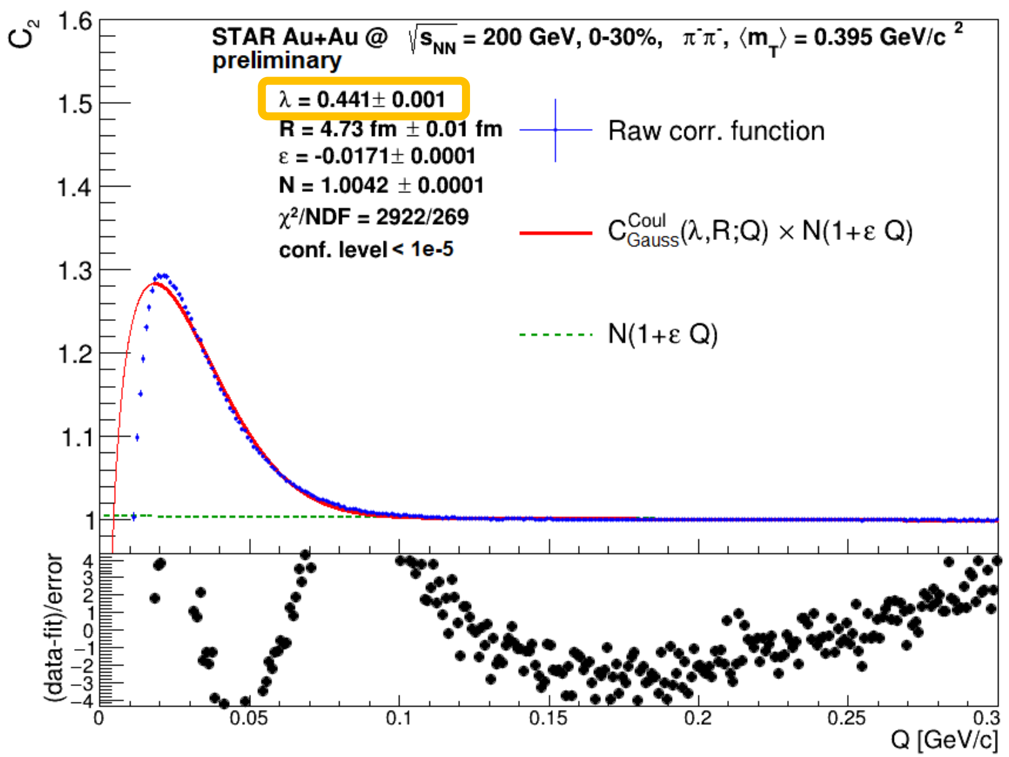
- Core-halo picture is included
- UrQMD for the hadronic cascade

- Levy gives the good description

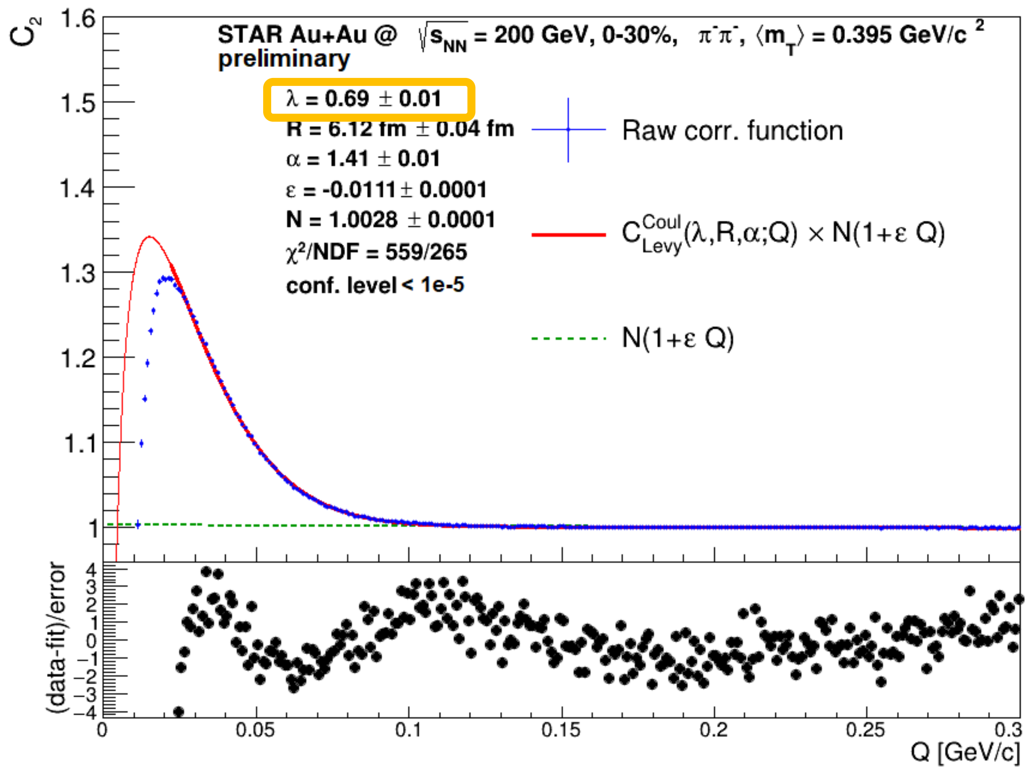
- It is a single event!
- This analysis support that the origin of the Levy shape could be explained only with the experimental averaging
- This analysis also support the role of the resonances, i.e., the anomalous diffusion

- With this confidence let's look at other experiments

STAR 0-30% Au+Au

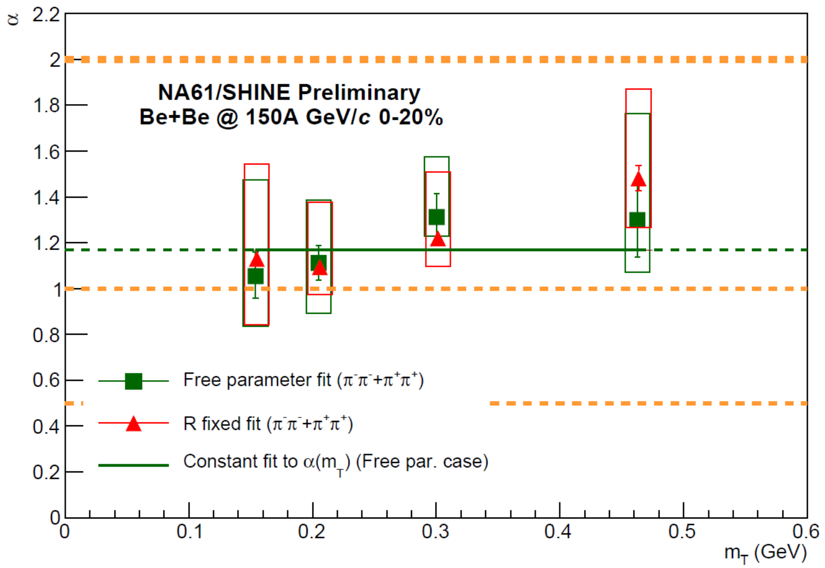


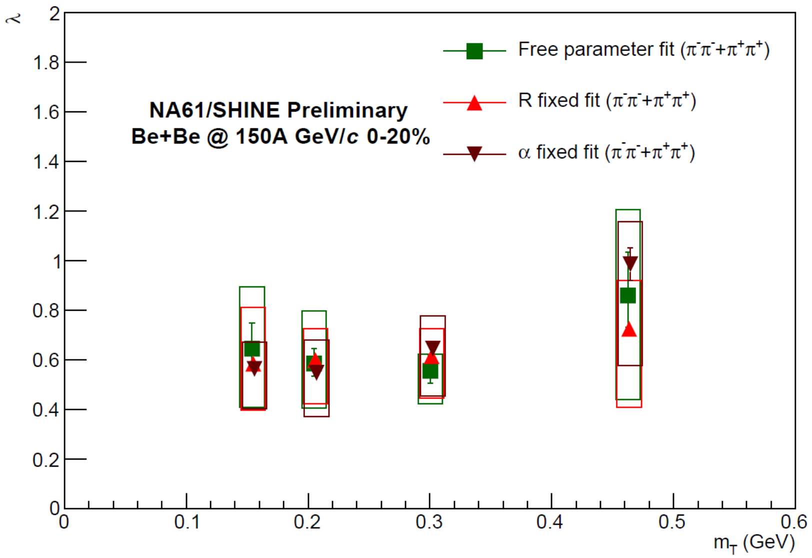
Gaussian



Levy

NA61 Ar+Sc and Be+Be at 150 AGeV

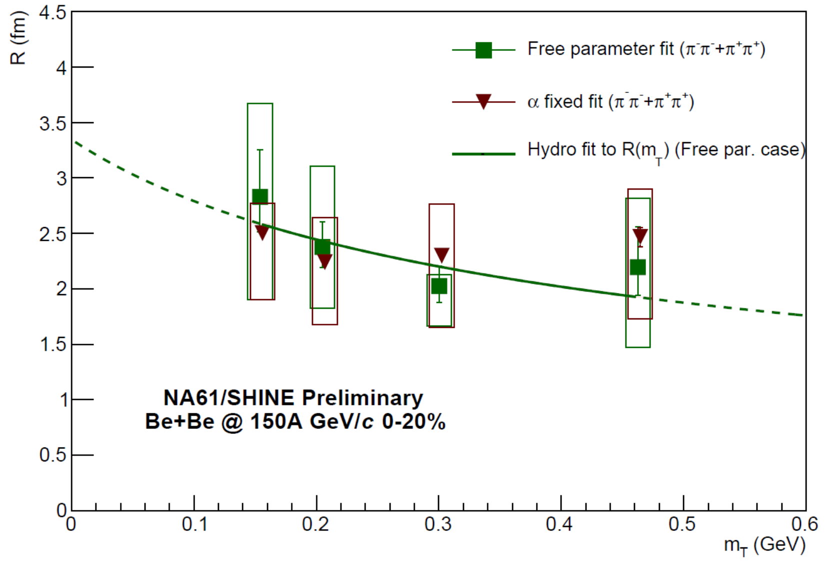




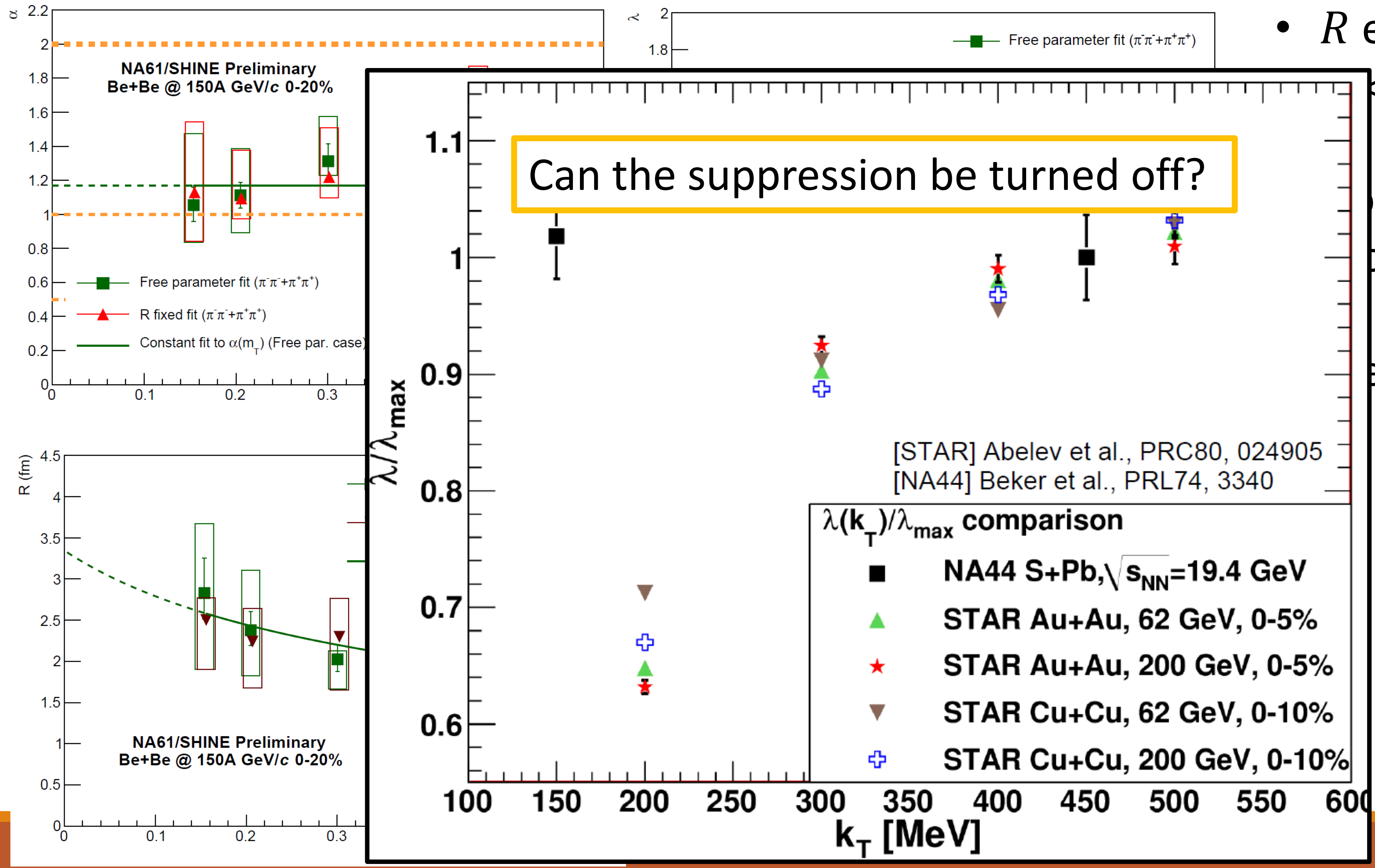
- R exhibits hydro scaling
- $1 < \alpha < 2$

- No suppression observed
- Models can be ruled out

- Preliminary results!



NA61 Ar+Sc and Be+Be at 150 AGeV



- R exhibits hydro scaling
- $< \alpha < 2$
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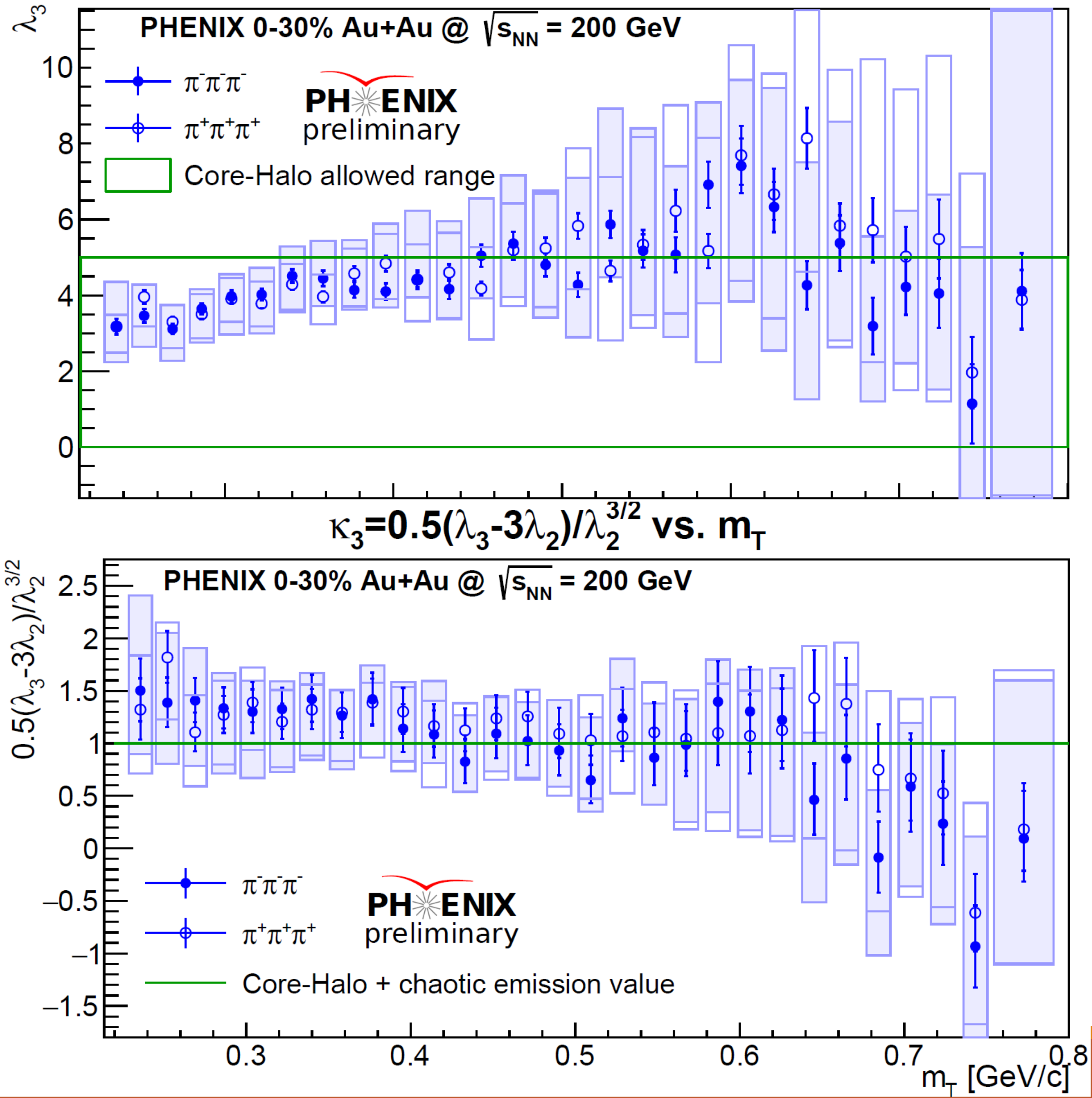
- eliminary results!

3 particle correlation – PHENIX 0-30% Au+Au



$$\kappa_3 = \frac{(\lambda_3 - 3\lambda_2)}{2\sqrt{\lambda_2^3}}$$

- From the definition:
  - No coherence: $p_c = 0 \Rightarrow \kappa = 1$
  - Coherence: $p_c > 0 \Rightarrow \kappa < 1$

- The source seems to be chaotic

Summary and outlook

Levy-type of correlations are measured at different energies, centralities, systems, experiments

Measured with different number of particles

Even models supports the appearance of it

The data favors Levy over Gaussian in all cases

The precise measurements of the parameters are crucial to interpret them

More results on the way and preliminaries will be published with major improvements soon

MERRY CHRISTMAS AND HAPPY NEW YEAR

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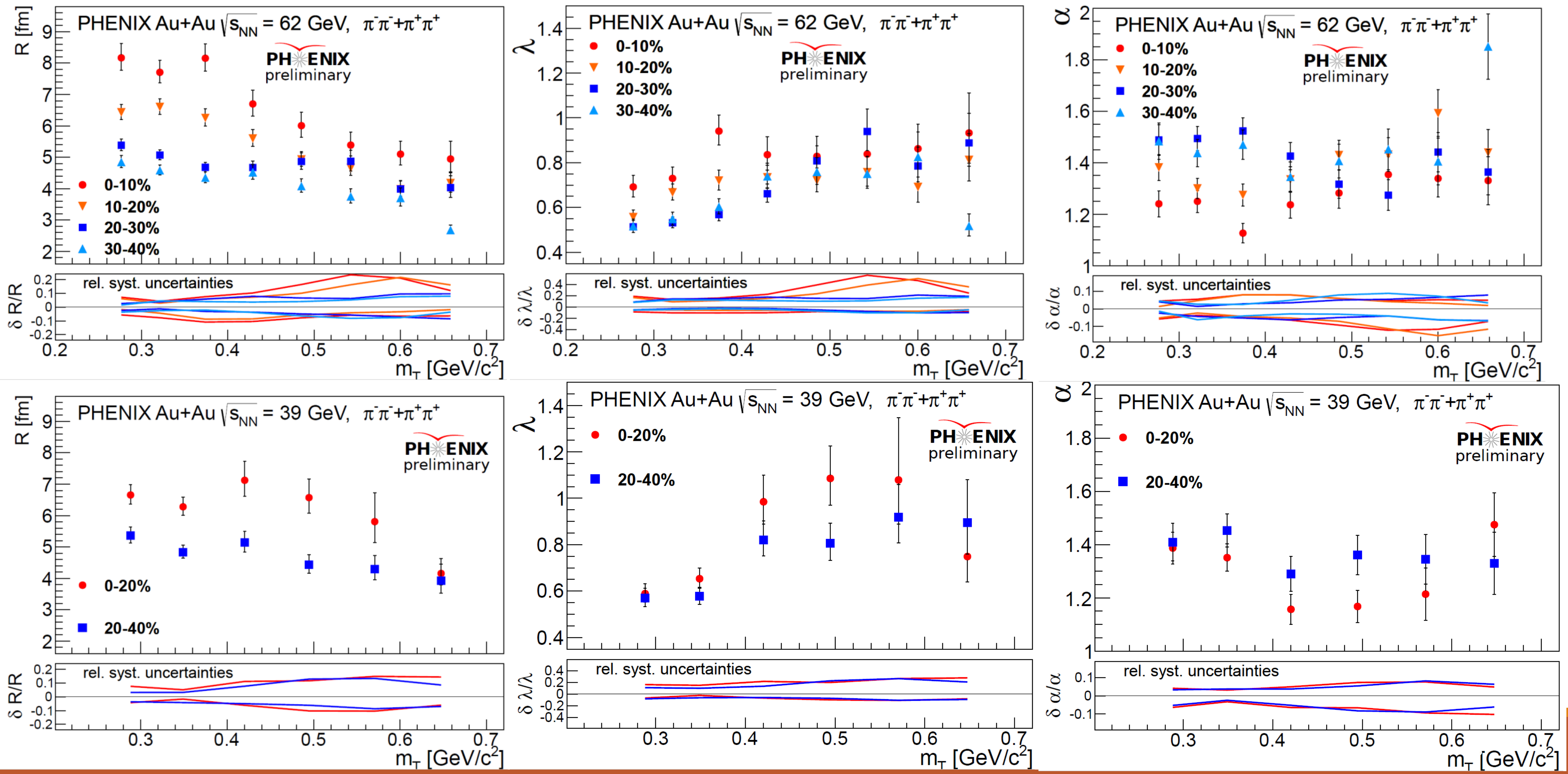
Backup slides



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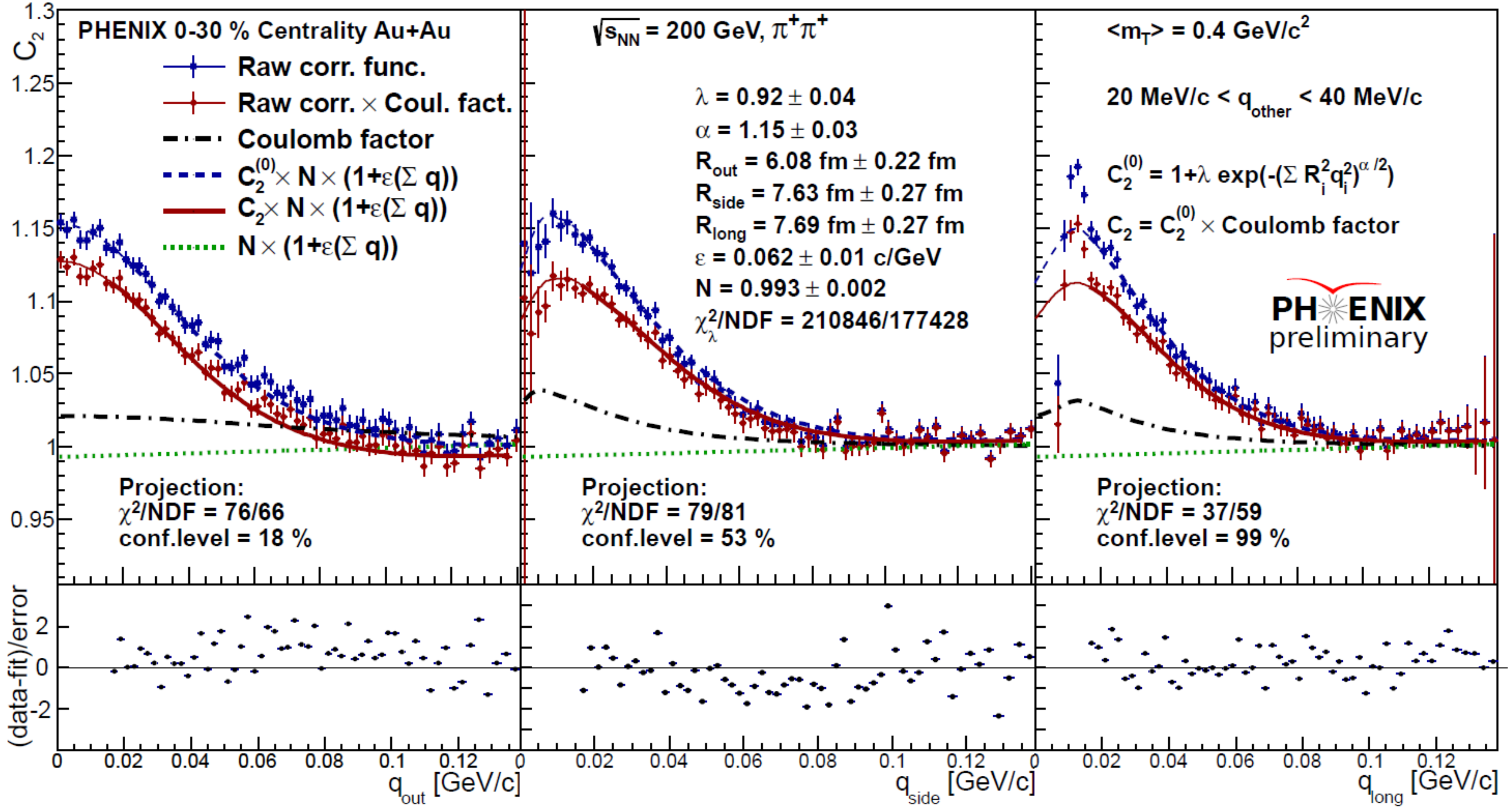

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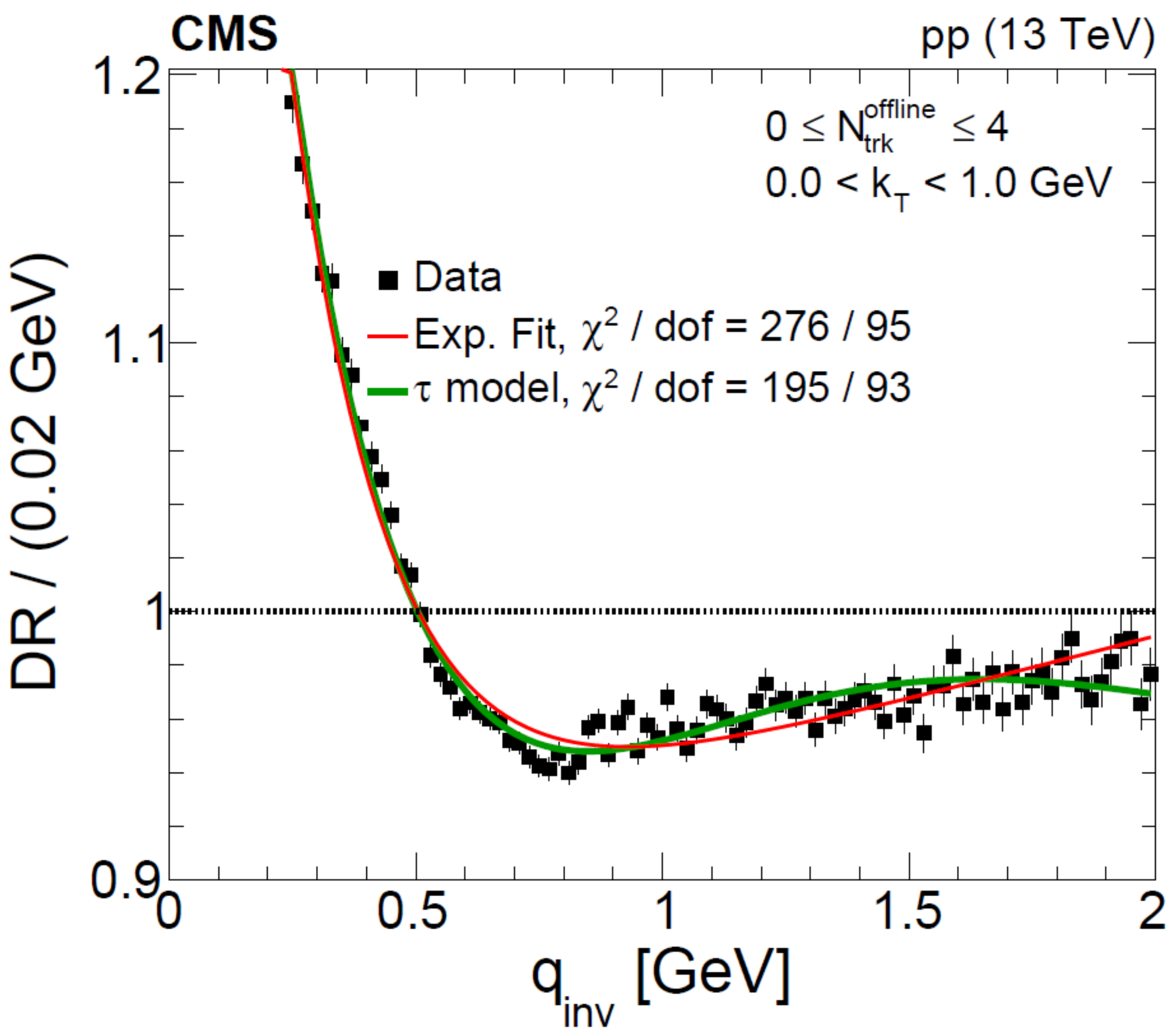

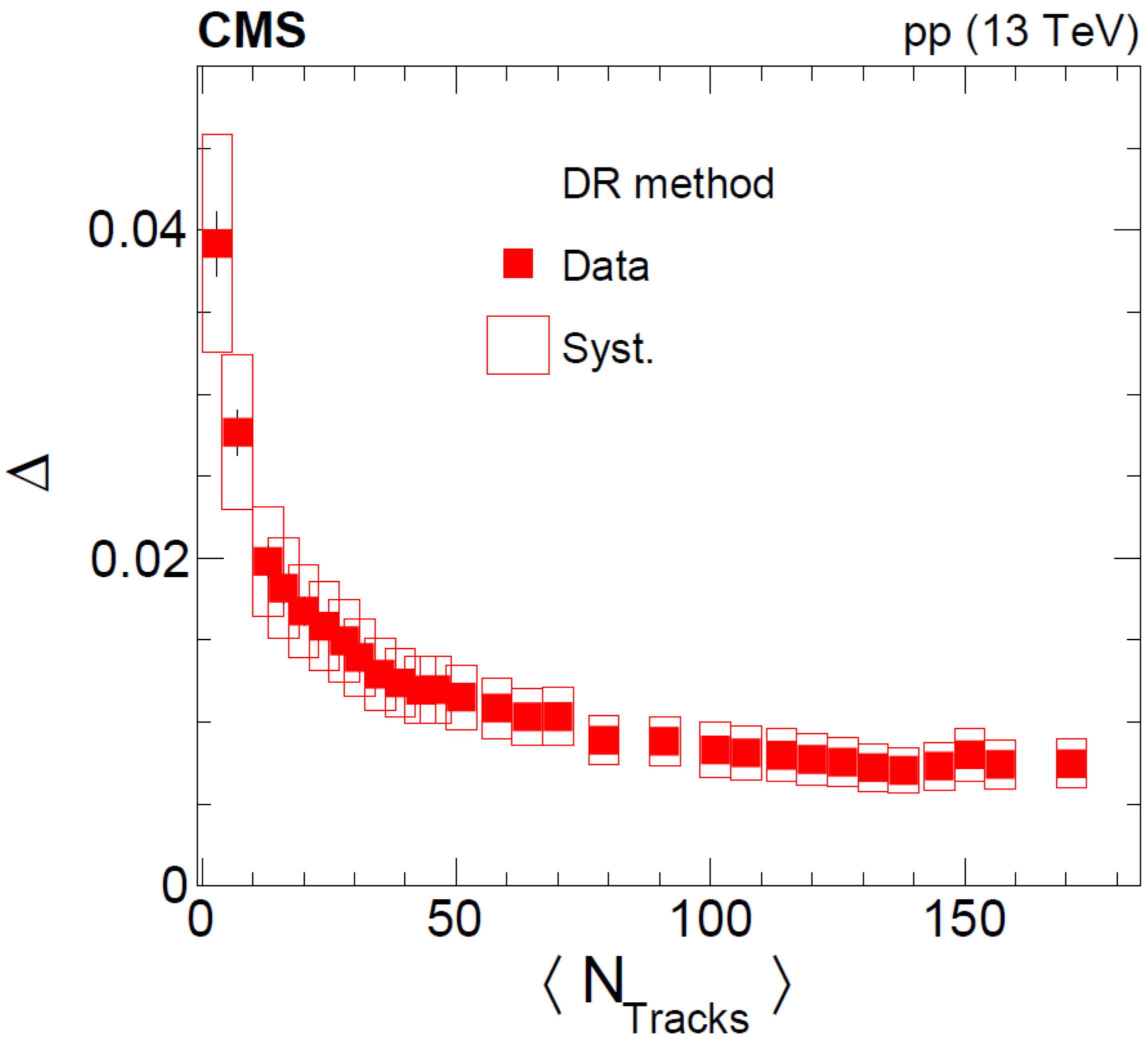

Backup slides

$$Q = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q^2_{\mathrm{long,LCMS}}},$$

$$\text{where} \quad q^2_{\mathrm{long,LCMS}} = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}$$