### $B_c^+ \to \tau^+ \nu_{\tau}$ and $B^+ \to \tau^+ \nu_{\tau}$ at FCC-ee BEACH 2022, Kraków June 09, 2022

Clement Helsens, Donal Hill, Markus Klute, Stephane Monteil, Xunwu Zuo



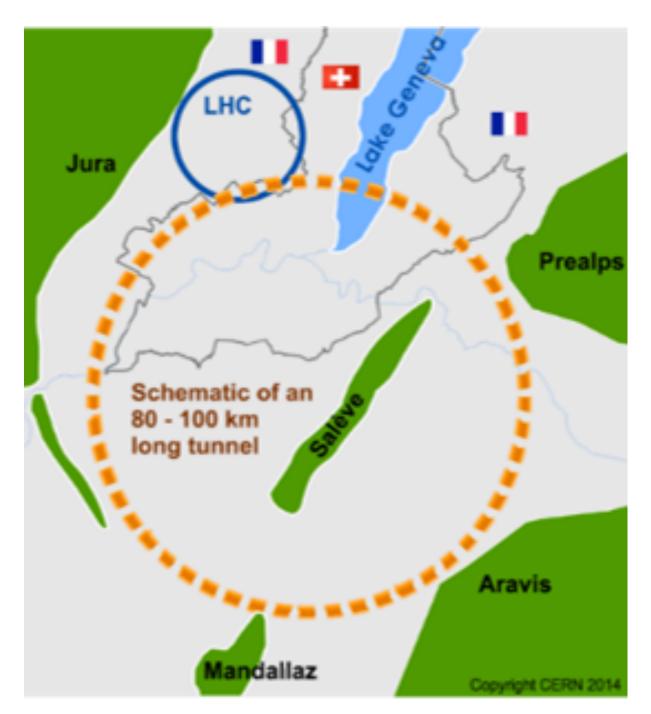
### Outline

- This set of slides is based on 2 FCC-ee performance studies
  - $B_c^+ \rightarrow \tau^+ \nu_{\tau}$ : <u>https://link.springer.com/article/10.1007/JHEP12(2021)133</u>
  - $B^+/B_c^+ \rightarrow \tau^+ \nu_{\tau}$ : paper draft in preparation
- Table of content
  - Introduction
  - Common analysis workflow
  - $B_c^+ \rightarrow \tau^+ \nu_{\tau}$  results
  - $B^+/B_c^+ \rightarrow \tau^+ \nu_\tau$  results

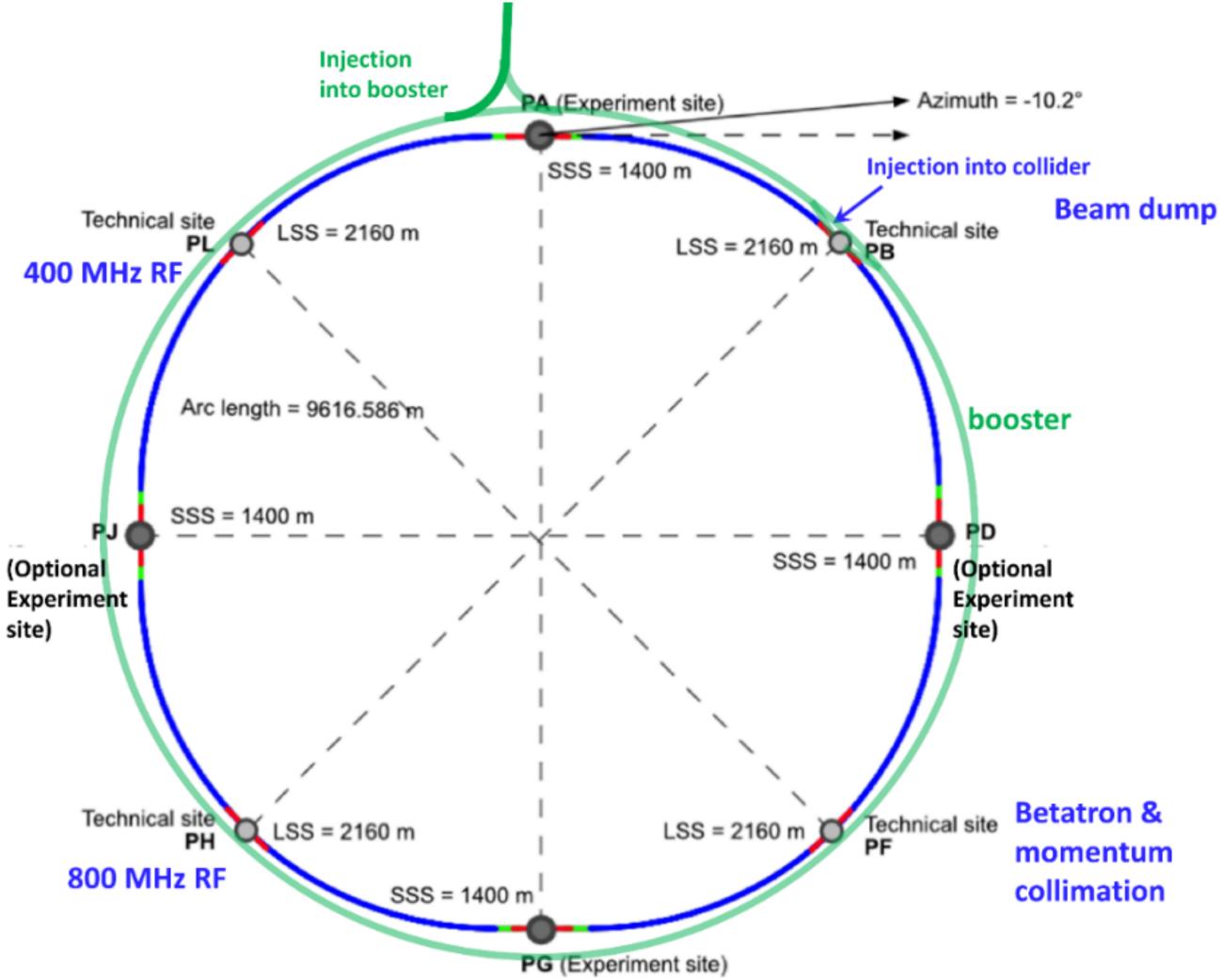


### FCC-ee

- 91km ring near CERN
- Possibility for 4 experiment sites
- Operate at Z, WW, ZH, tt energies
- ~15 years of operation starting 2045(?)



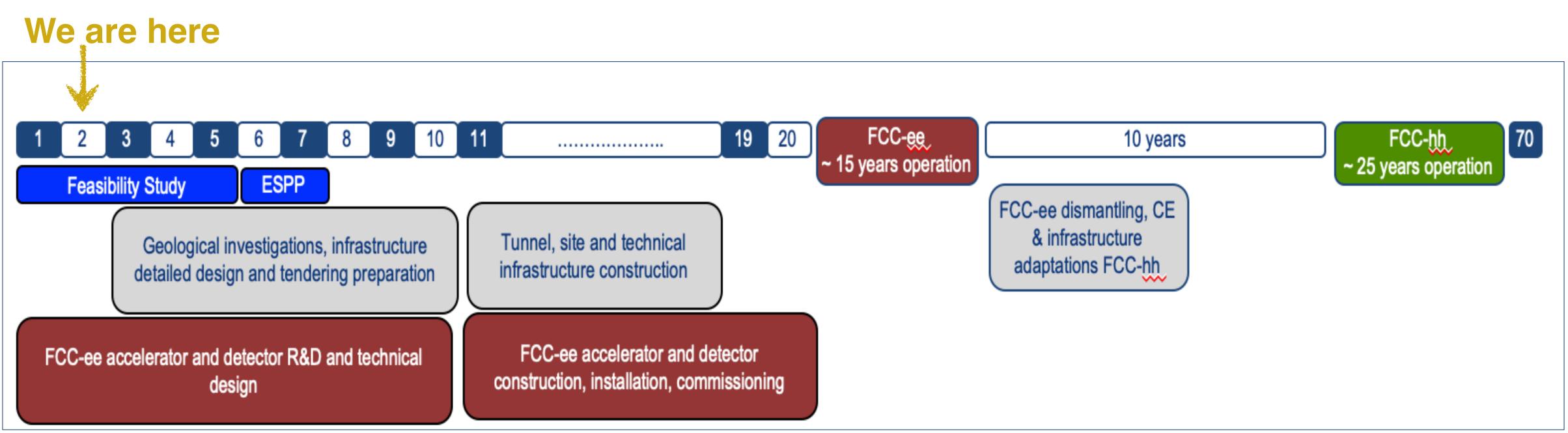






## Feasibility study of FCC-ee

- Feasibility study: 2021 2025
  - Infrastructure requirements & environmental impact
  - Accelerator design
  - Physics performance & detector design



### xunwu.zuo@cern.ch







### FCC-ee as flavor factory

- FCC-ee expects to operate at Z-pole for 4 years, producing a total of  $5 \times 10^{12}$  Z bosons
  - 1M times LEP dataset  $5 \times 10^6$  Z bosons
  - Unparalleled opportunity for all flavor physics
- About 13x as many  $B^0/B^+$  as at Belle II (50 ab<sup>-1</sup>)
- All species of b-hadrons are produced
- Decay products significantly boosted
- In particular for  $B_c^+ \to \tau^+ \nu_{\tau}$  and  $B^+ \to \tau^+ \nu_{\tau}$ :
  - Not possible to identify at LHCb
  - Not enough energy for  $B_c^+$  at Belle II  $\Upsilon(4S)$





Phase	Run duration	Center-of-mass	Integrated	
	(years)	Energies (GeV)	Luminosity (ab	
FCC-ee-Z	4	88-95	150	
FCC-ee-W	2	158-162	12	
FCC-ee-H	3	240	5	
FCC-ee-tt	5	345-365	1.5	

Attribute	$\Upsilon(4S)$	pp	$Z^0$
All hadron species		1	1
High boost		1	1
Enormous production cross-section		1	
Negligible trigger losses	~		1
Low backgrounds	1		1
Initial energy constraint	1		(1





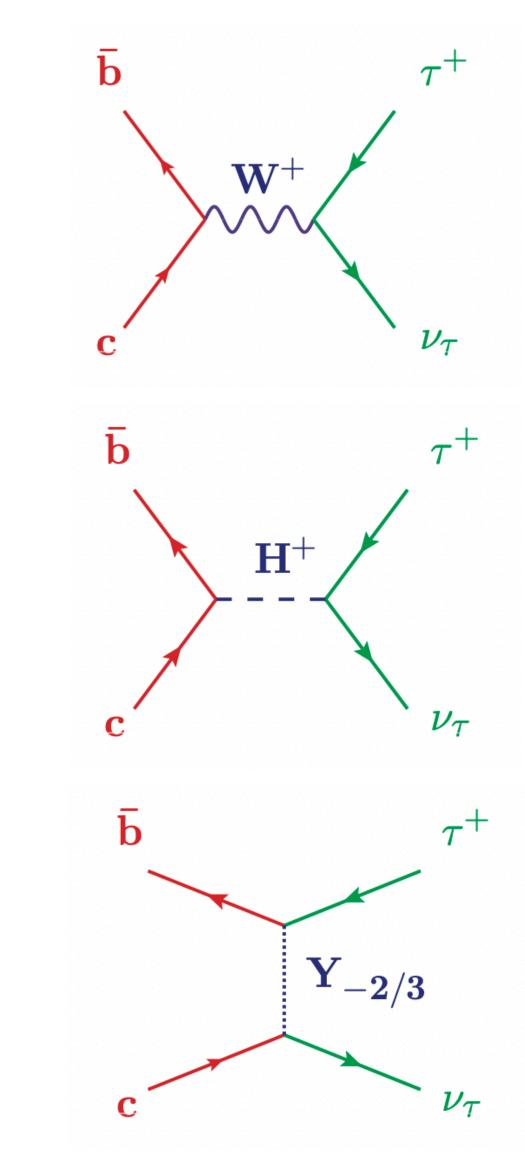


 $B_c^+/B^+ \to \tau^+ \nu_\tau$  decays

- Clean probes to measure  $|V_{cb}|$  and  $|V_{ub}|$
- Sensitive to BSM physics, like charged Higgs and leptoquarks
- $B_c^+ \to \tau^+ \nu_{\tau}$  and  $B^+ \to \tau^+ \nu_{\tau}$  are helicity and CKMsuppressed.
  - $f(B_c^+) \approx 0.04 \%$ ,  $\mathscr{B}(B_c^+ \to \tau^+ \nu_{\tau}) \approx 1.94 \%$
  - $f(B^+) \approx 43\%$ ,  $\mathscr{B}(B^+ \to \tau^+ \nu_{\tau}) \approx 1.09 \times 10^{-4}$
- In the  $5 \times 10^{12}$  Z events scenario of FCC-ee
  - $B_c^+ \rightarrow \tau^+ \nu_{\tau} \; (\pi^+ \pi^+ \pi^- \bar{\nu}_{\tau}) \approx 1 \mathrm{M}$
  - $B^+ \rightarrow \tau^+ \nu_{\tau} \; (\pi^+ \pi^+ \pi^- \bar{\nu_{\tau}}) \approx 6 \mathrm{M}$

### xunwu.zuo@cern.ch





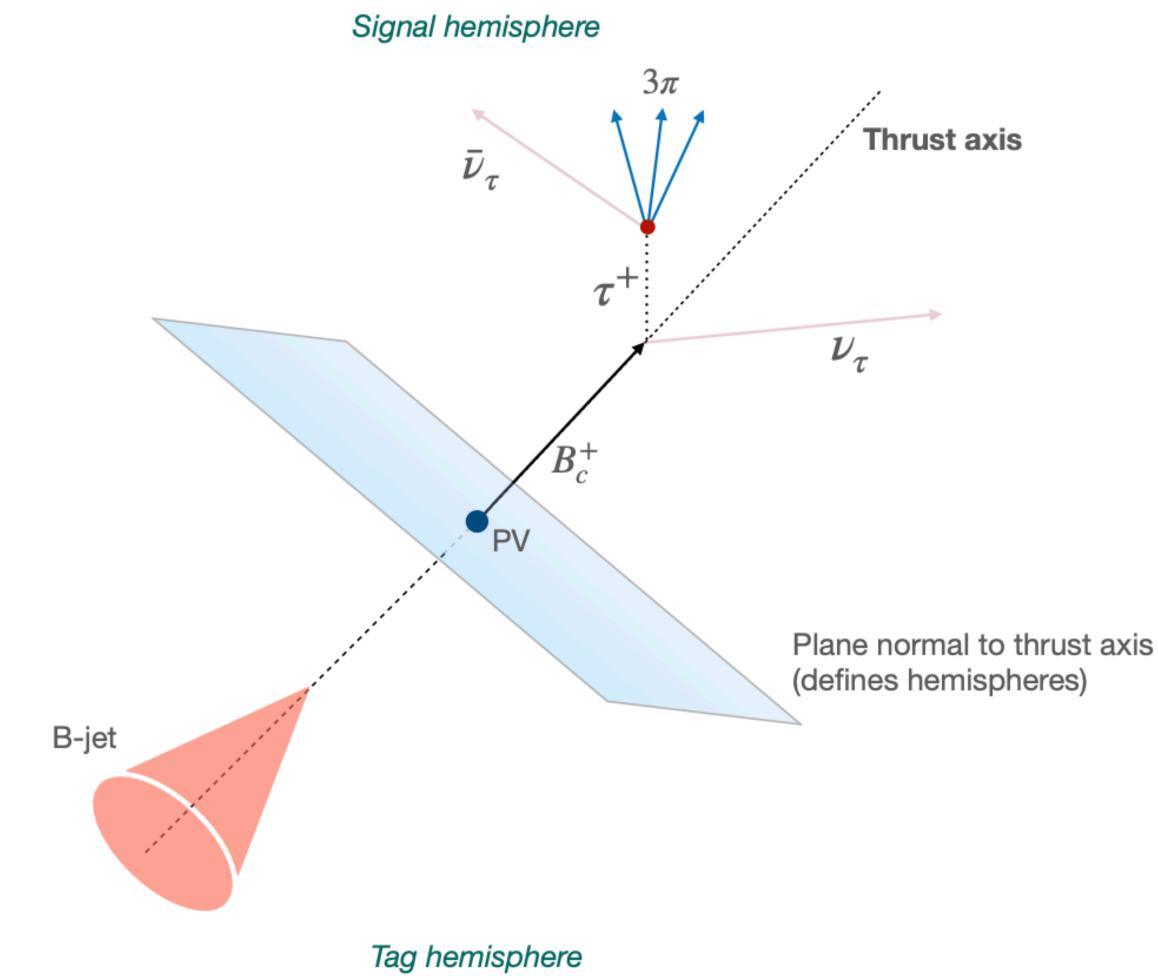
## b-decay hemispheres

Thrust axis defined as the axis that aligns the most with particle momenta.

$$T_i = \frac{\sum_i |\overrightarrow{p_i} \cdot \hat{n}|}{\sum_i |\overrightarrow{p_i}|}$$

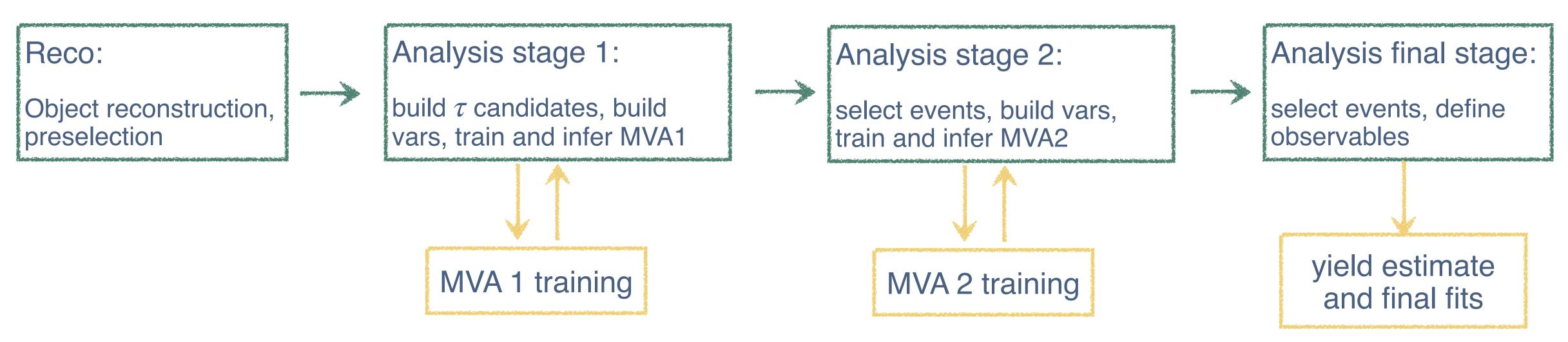
- Measures the decay axis of  $Z \rightarrow b\bar{b}$
- Due to high missing energy in the signal decays
  - The thrust axis would be skewed in signal events
  - The two hemispheres would have very different energy distributions.





### Analysis workflow

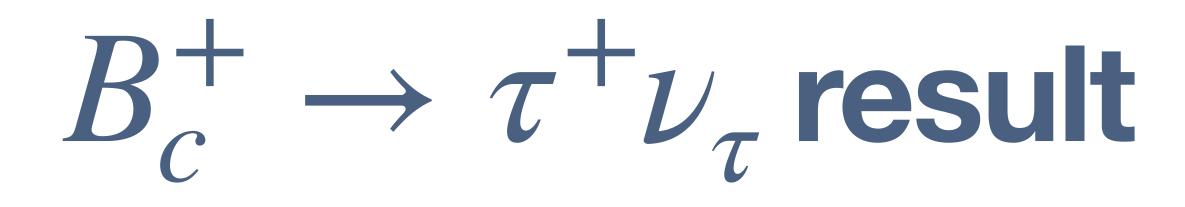
- Focus on three-prong  $\tau^+ \to \pi^+ \pi^- \bar{\nu}_{\tau}$  decay
  - Reconstruct secondary vertex, measure  $B_c^+/B^+ + \tau^+$  combined flight distance
  - 9% decay ratio, sufficient amount of signal events
- Use two stages of MVA to optimize signal purity
  - Keep the training sample at a reasonable size, O(1M)



Fully based on common FCC software from EMD4hep to FCCAnalysis framework





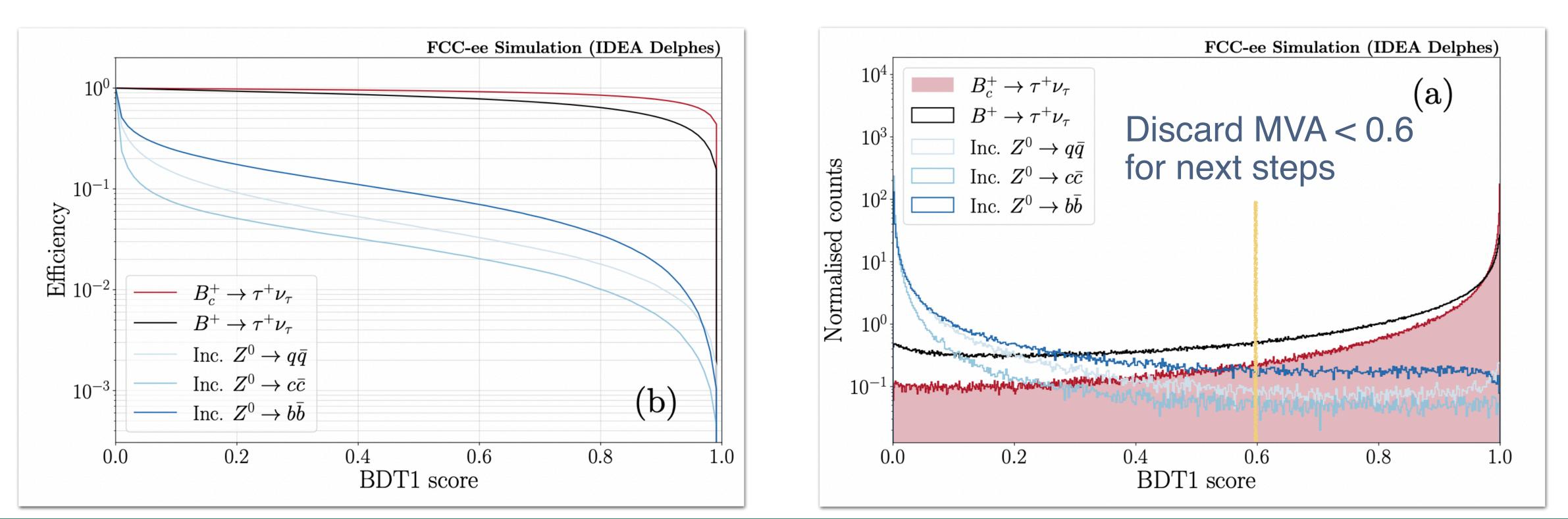


xunwu.zuo@cern.ch



## Stage1 MVA

- Goal: use event-level variables to remove "easy" backgrounds from  $Z \rightarrow c\bar{c}, q\bar{q}$  processes
- Signal:  $B_c^+ \to \tau^+ \nu_{\tau}$ , background: inclusive  $Z \to b\bar{b}, c\bar{c}, q\bar{q}$  decays
- $Z \rightarrow b\bar{b}$  least rejected since it is the most similar to Signal
- $B^+ \rightarrow \tau^+ \nu_{\tau}$  not used in training but separated from  $B_c^+ \rightarrow \tau^+ \nu_{\tau}$  to some extent.

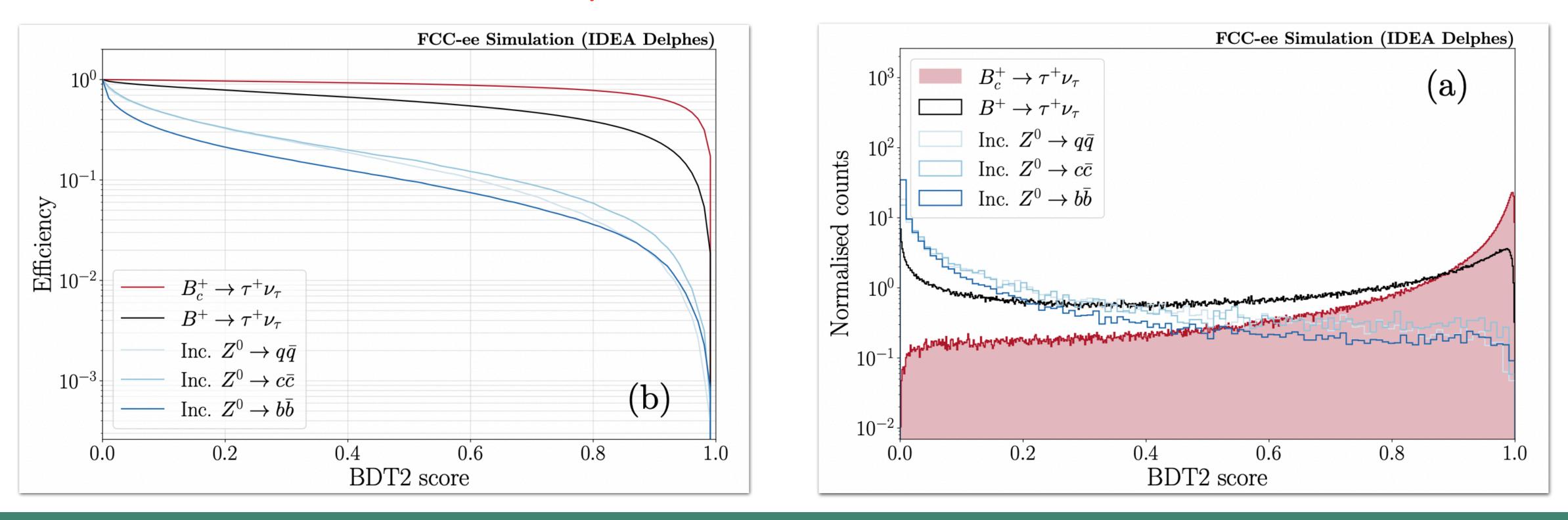






## Stage2 MVA

- Goal: use full kinematic property of  $\tau^+ \to \pi^+ \pi^- \bar{\nu}_{\tau}$  decay to maximize signal purity.
- Signal:  $B_c^+ \to \tau^+ \nu_{\tau}$ , background: inclusive  $Z \to b\bar{b}, c\bar{c}, q\bar{q}$  decays
- Very good rejection of all backgrounds
- High level of rejection of  $B^+ \to \tau^+ \nu_{\tau}$  even if it is not used in the training







### Final selection

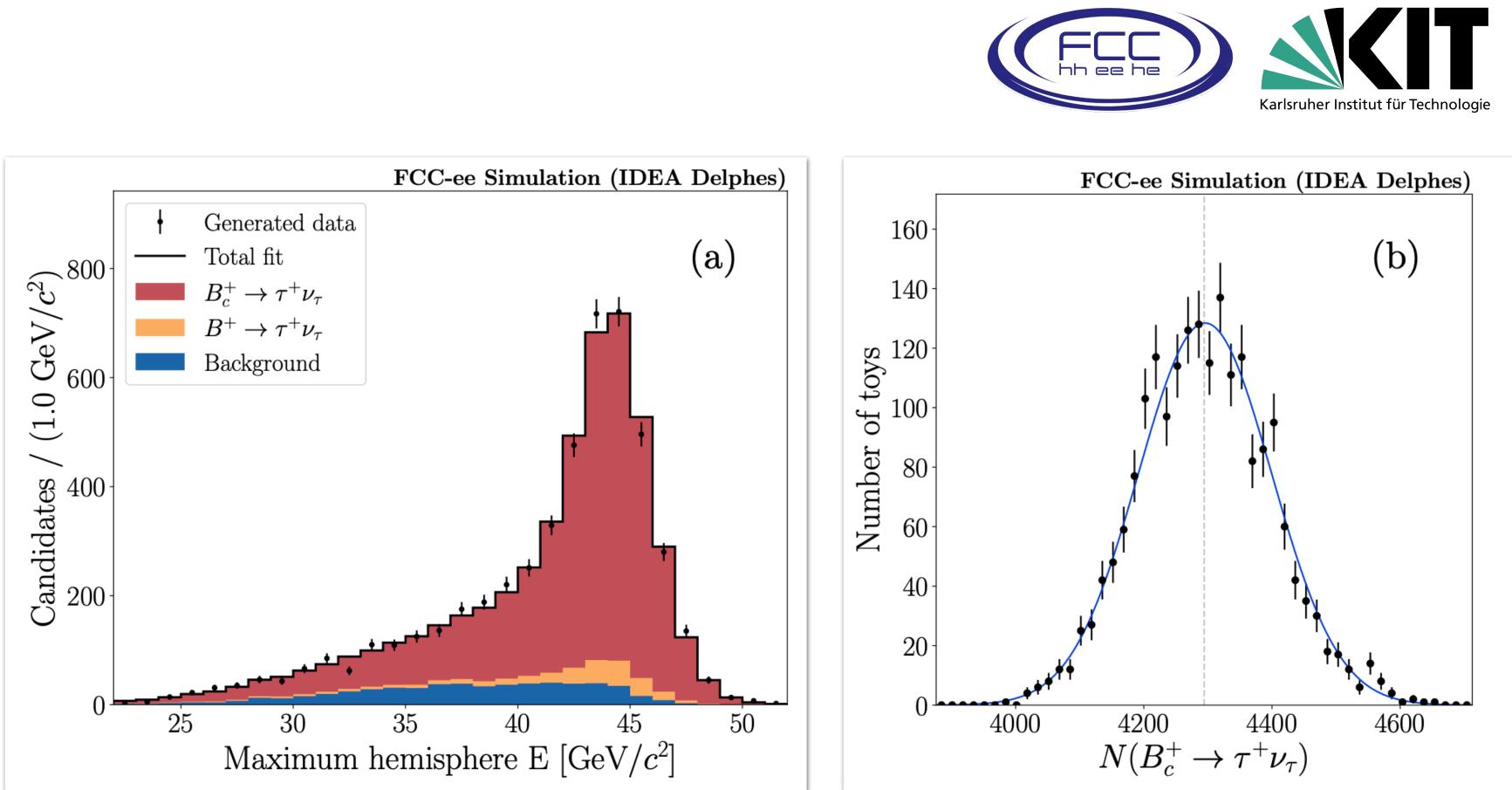
- After the Stage2 MVA,  $Z \rightarrow c\bar{c}$  and  $Z \rightarrow q\bar{q}$  are reduced by a factor more than  $10^9$ 
  - All simulation events are rejected (Do not consider for further steps)
- Inclusive  $Z \rightarrow b\bar{b}$  largely rejected, not enough events to estimate efficiency at high MVA scores.
  - Use a set of exclusive  $B^0, B^+, B^0_s, \Lambda^0_h$  samples to estimate background yields.
- Estimate method:
  - N<sup>bkg</sup> Bkg yield after baseline MVA cuts estimated from inclusive samples
  - $\epsilon$ (MVA1 cut | baseline): Bkg cut efficiency relative to baseline estimated from exclusive samples
  - $\epsilon$ (MVA2 cut | baseline): Bkg cut efficiency relative to baseline estimated from exclusive samples
  - Assuming MVA1 and MVA2 are uncorrelated,  $N_{\text{final}}^{\text{bkg}} = N_{\text{baseline}}^{\text{bkg}} \times \epsilon(\text{MVA1 cut} | \text{baseline}) \times \epsilon(\text{MVA2 cut} | \text{baseline})$

### xunwu.zuo@cern.ch



## Template fit

- From 2D scan of (MVA1, MVA2) cuts, the optimal yields are:
  - $B_c^+ \rightarrow \tau^+ \nu_{\tau}$ : 4295
  - $B^+ \rightarrow \tau^+ \nu_{\tau}$ : 285
  - Total bkg: 448



- Template fit on the total energy in the hemisphere with more energy
- 2000 pseudo-experiments performed with Asimov dataset.
- Statistical uncertainty on signal is about 2.4%

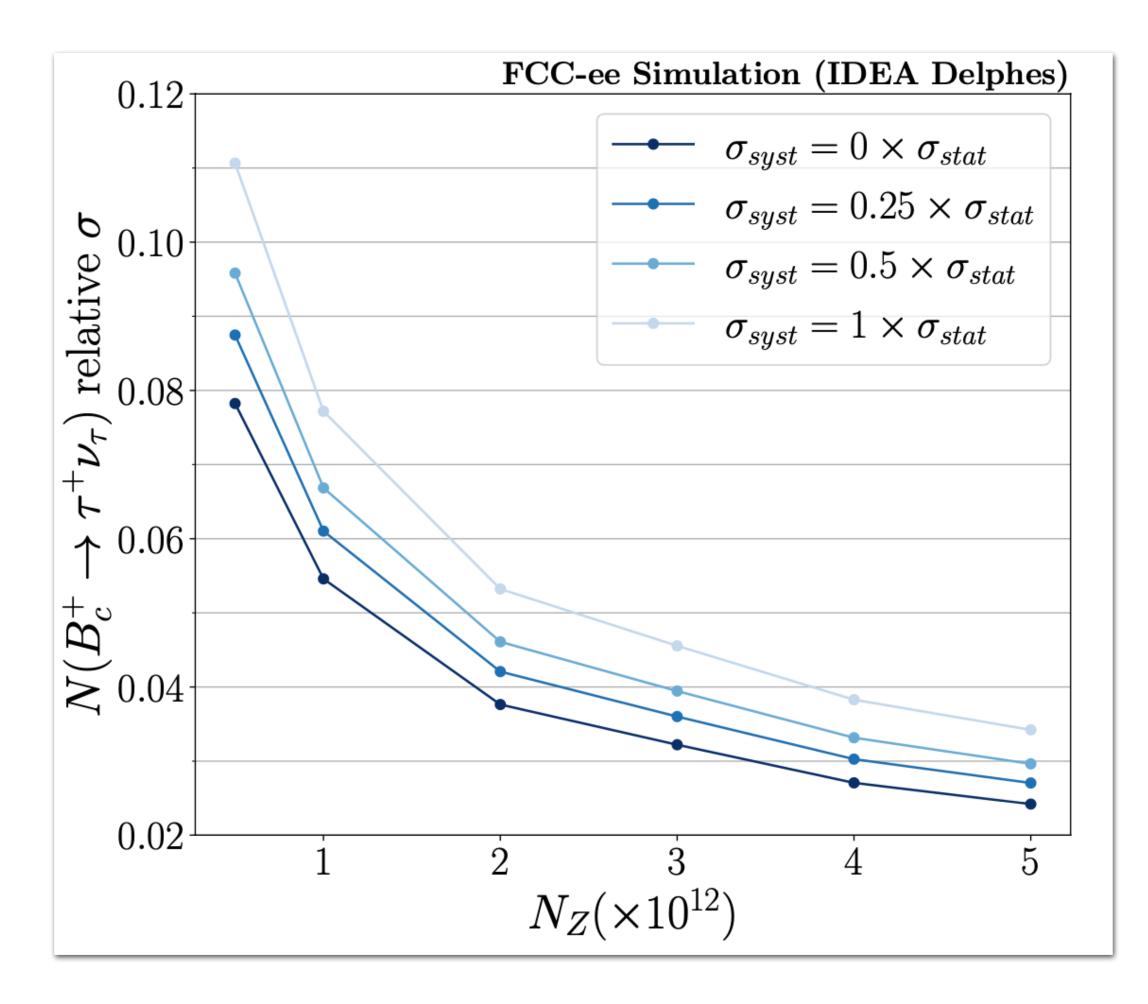




## Systematic uncertainty

- By design, syst. uncert. at FCC-ee is expected to be constrained to the level comparable to stat. uncert. of EW precision measurements, and not a major concern for this result.
- Current analysis relies on strong assumptions in the background estimate method. Hard to estimate the uncertainty from these assumptions.
- Consider a few scenarios  $\sigma_{syst} = [0, 0.25, 0.5, 1.0] \times \sigma_{stat}$









xunwu.zuo@cern.ch



## Precision on $\mathscr{B}(B_c^+ \to \tau^+ \nu_{\tau})$

• Using  $B_c^+ \rightarrow j/\psi \mu^+ \nu_\mu$  as a normalization mode, measure the ratio

$$\begin{split} R_{c} &= \frac{\mathscr{B}(B_{c}^{+} \to \tau^{+}\nu_{\tau})}{\mathscr{B}(B_{c}^{+} \to J/\psi\mu^{+}\nu_{\mu})} \\ &= \frac{N(B_{c}^{+} \to \tau^{+}\nu_{\tau})}{N(B_{c}^{+} \to J/\psi\mu^{+}\nu_{\mu})} \times \frac{\varepsilon(B_{c}^{+} \to J/\psi\mu^{+}\nu_{\mu})}{\varepsilon(B_{c}^{+} \to \tau^{+}\nu_{\tau})} \times \frac{\mathscr{B}(J/\psi \to \mu^{+}\mu^{-})}{\mathscr{B}(\tau^{+} \to \pi^{+}\pi^{+}\pi^{-}\bar{\nu_{\tau}})} \end{split}$$

- $R_c$  has good experimental precision ~4%
  - Decoupled from  $|V_{ch}|$  and  $f(B^+)$
  - Highly sensitive to BSM
- Can be translated to a measurement on  $\mathscr{B}(B_c^+ \to \tau^+ \nu_{\tau})$ 
  - of  $\mathscr{B}(B_c^+ \to J/\psi \mu^+ \nu_\mu)$  $\sim c$   $\mu$





• Use current best  $|V_{cb}|[1]$ , and form-factors calculated from lattice QCD [2, 3] for the SM value





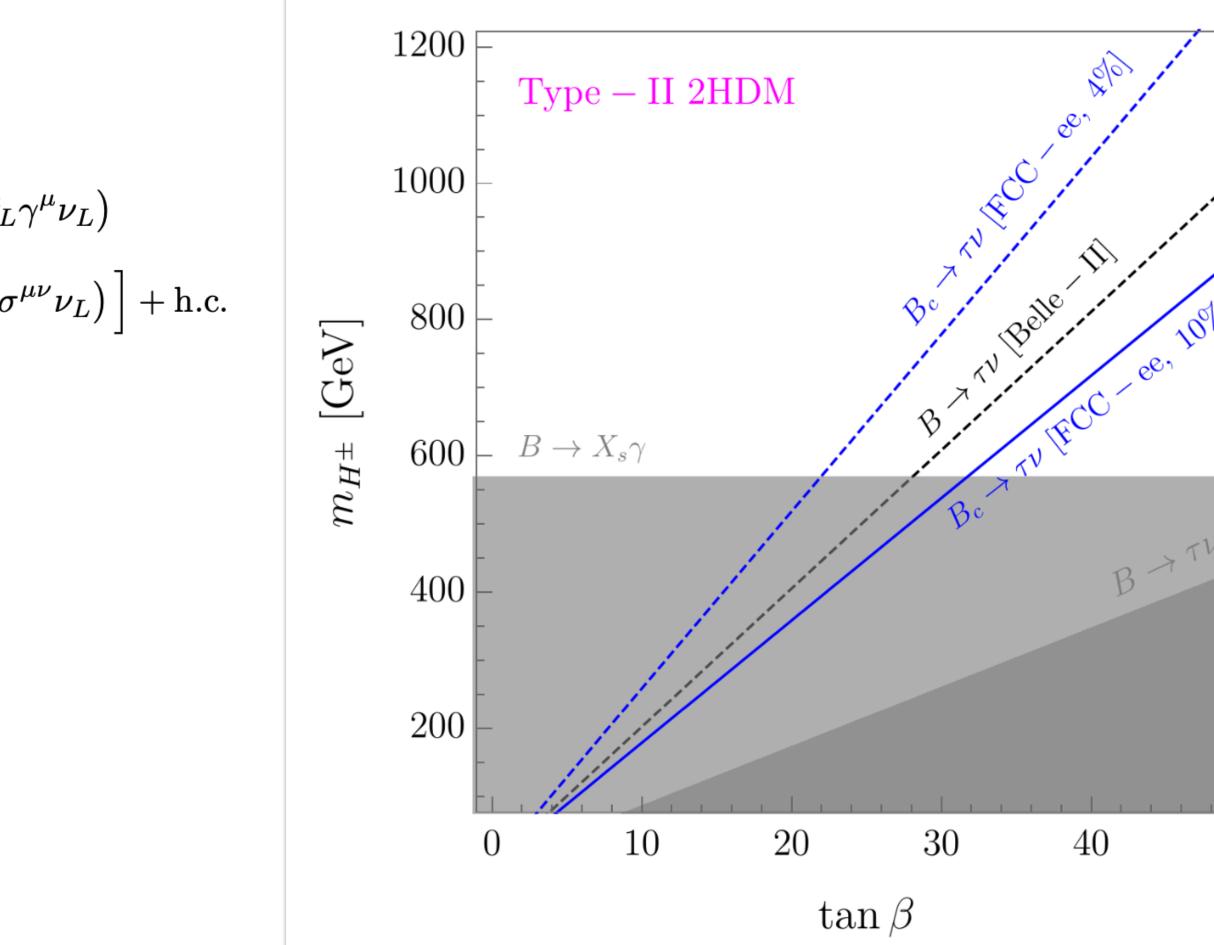
### Probe of 2HDM

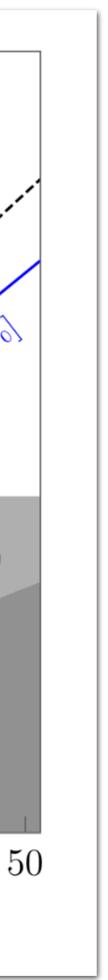
• Consider effective Hamiltonian for SM + BSM  $b \rightarrow c \tau \nu_{\tau}$  transition

$$egin{aligned} \mathcal{H}_{ ext{eff}} &= 2\sqrt{2}G_F V_{cb} \Big[ ig(1+g_{V_L}ig) ig(ar{c}_L \gamma_\mu b_Lig) ig(ar{ au}_L \gamma^\mu 
u_Lig) + g_{V_R} ig(ar{c}_R \gamma_\mu b_Rig) ig(ar{ au}_L 
u_Lig) + g_{S_R} ig(ar{c}_L b_Rig) ig(ar{ au}_R 
u_Lig) + g_T ig(ar{c}_R \sigma_{\mu
u} b_Lig) ig(ar{ au}_R \sigma_\mu
u_Lig) ig(ar{ au}_R \sigma_\mu
u_Lig) &= g_{S_R} ig(ar{c}_L b_Rig) ig(ar{ au}_R 
u_Lig) + g_T ig(ar{c}_R \sigma_{\mu
u} b_Lig) ig(ar{ au}_R \sigma_\mu
u_Lig) ig(ar{ au}_R \sigma_\mu
u_Lig) &= g_{S_R} ig(ar{u}_L 
u_Lig) ig(ar{ au}_R 
u_Lig) + g_T ig(ar{u}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) &= g_{S_R} ig(ar{u}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) &= g_{S_R} ig(ar{u}_R 
u_Lig) ig(ar{ au}_R 
u_Lig) ig(au_L 
u_Lig)$$

- Computed in two scenarios where  $\Gamma(B_c^+ \to \tau^+ \nu_{\tau}) / |V_{cb}|^2$  is 10% and 4%.
- Can be interpreted in Type-II 2HDM where the effective coefficients are determined by  $m_{H^{\pm}}$  and  $tan\beta$





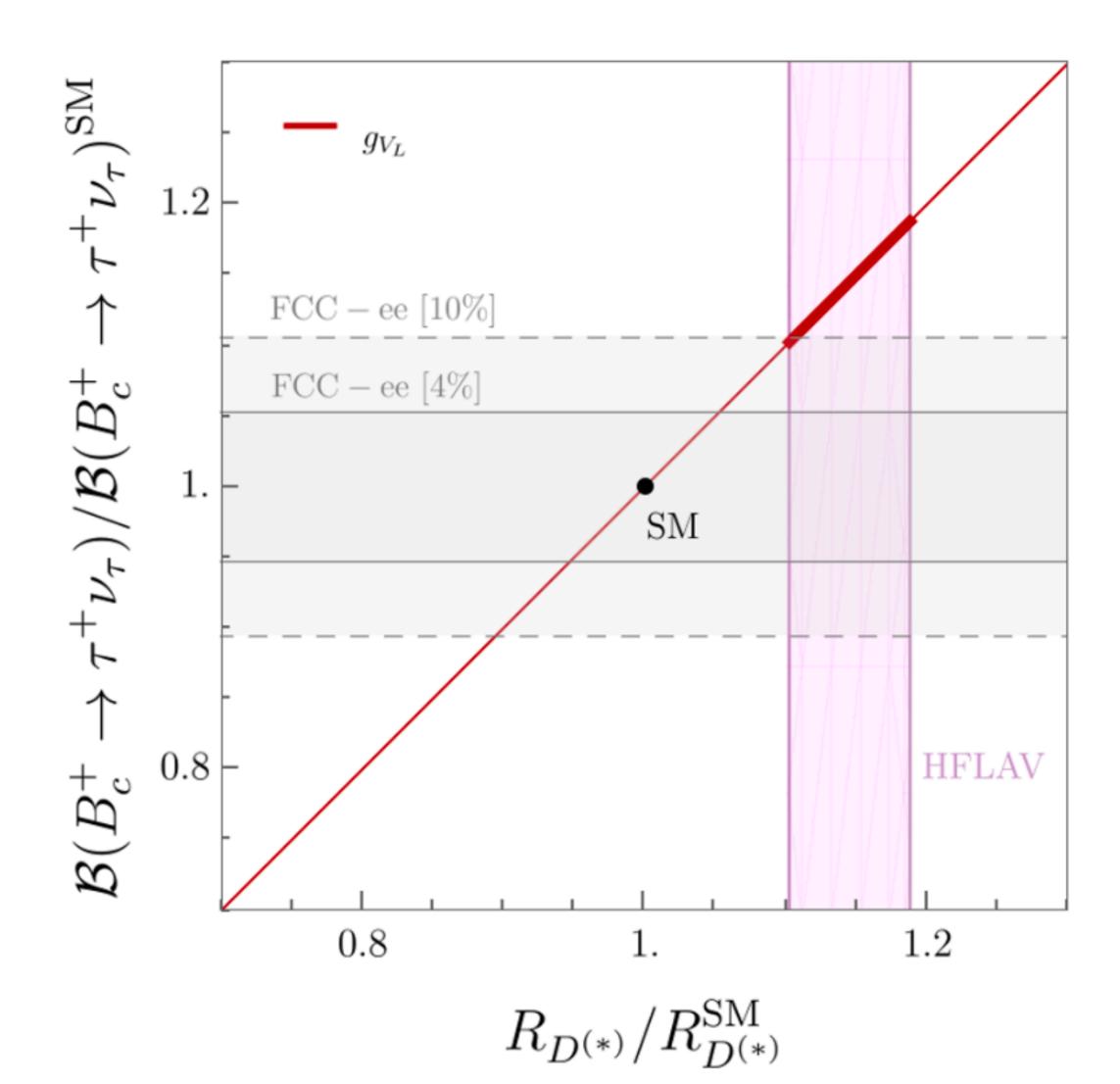




## Probe of leptoquark

- Can also be interpreted with leptoquark model and examine the current deviation of  $R_{D^*}$  from its SM expectations.
  - Leptoquark implies a deviation on  $\Gamma(B_c^+ \to \tau^+ \nu_{\tau}) / |V_{cb}|^2$  larger than 10%. Can be fully examined.







# $B_c^+ \to \tau^+ \nu_{\tau}$ and $B^+ \to \tau^+ \nu_{\tau}$ results

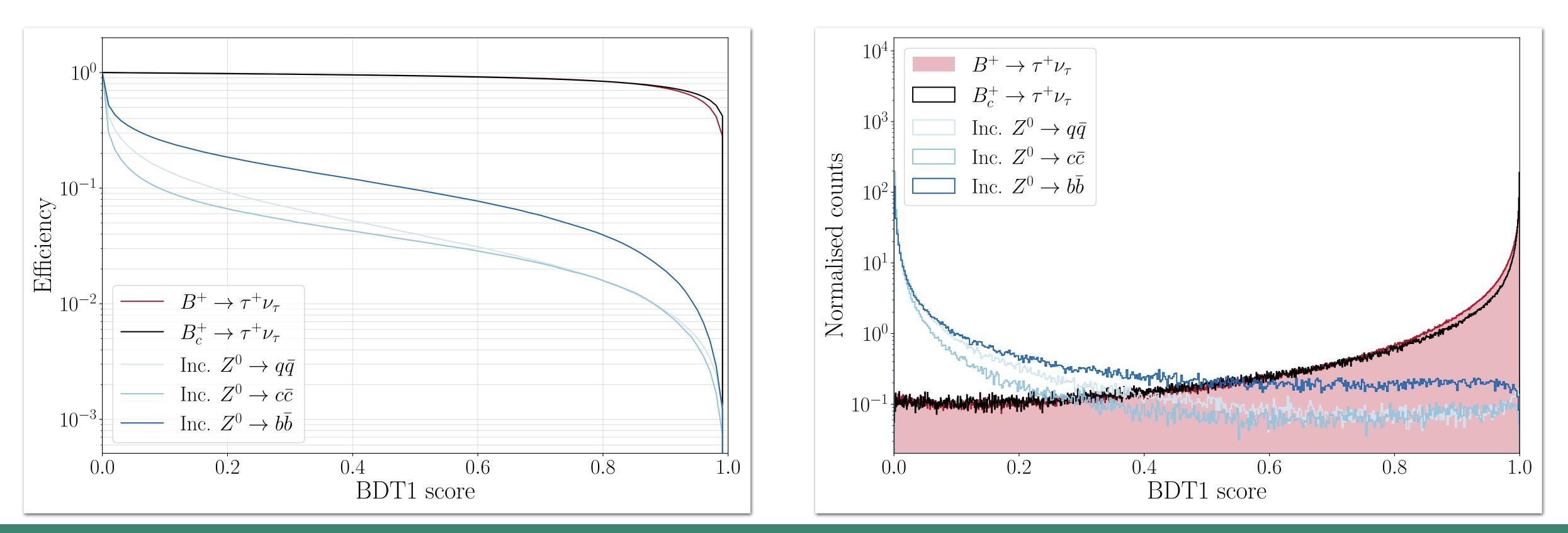
The following results are work in progress

xunwu.zuo@cern.ch



## Stage1 MVA

- Goal: remove "easy" backgrounds (same as in Bc analysis)
- Same setup as in Bc analysis, just include both  $B_c^+ \to \tau^+ \nu_{\tau}$  and  $B^+ \to \tau^+ \nu_{\tau}$  as signals
- Better efficiency for Bu, similar performance on Bc and background as before.

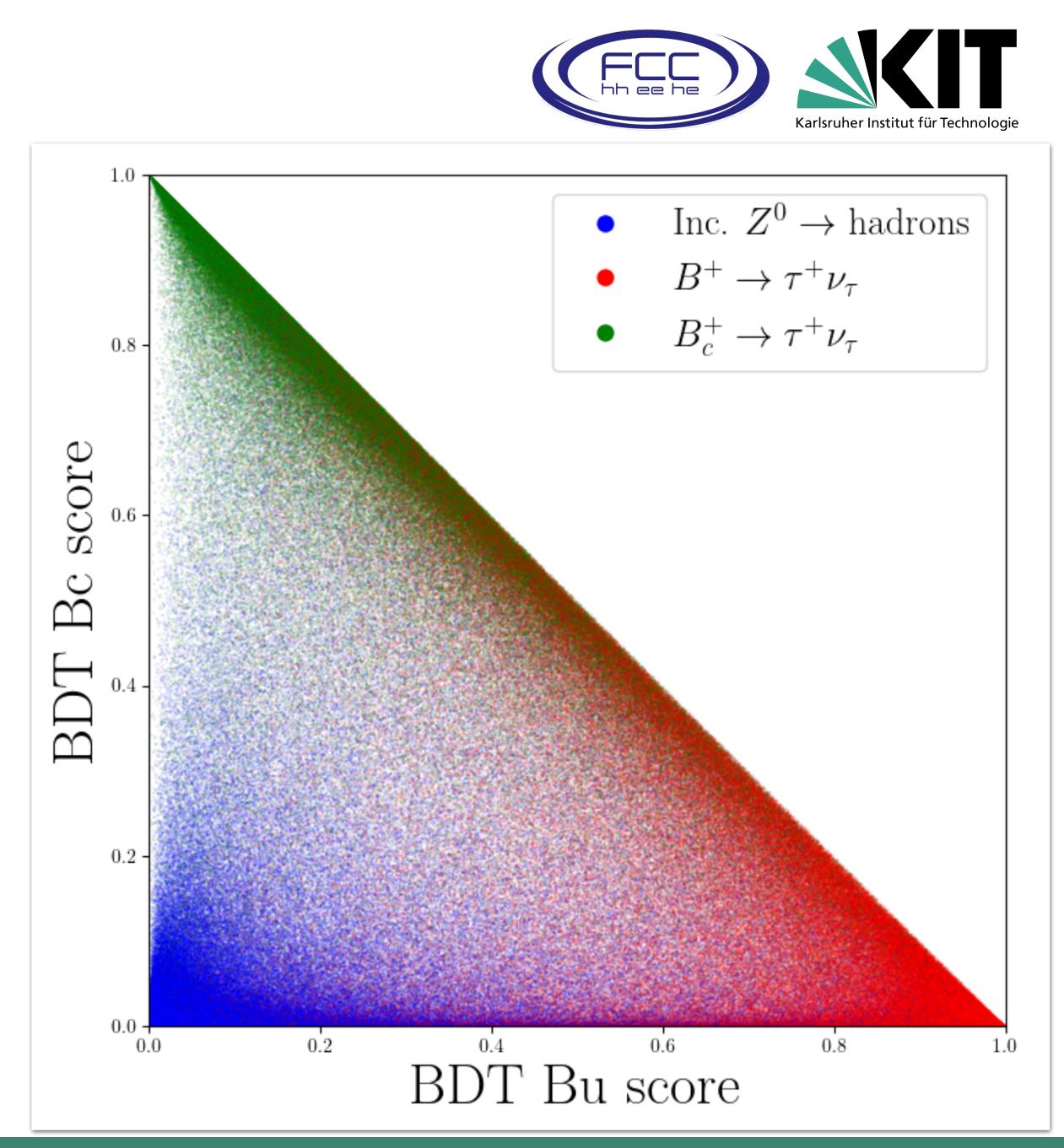






## Stage2 MVA

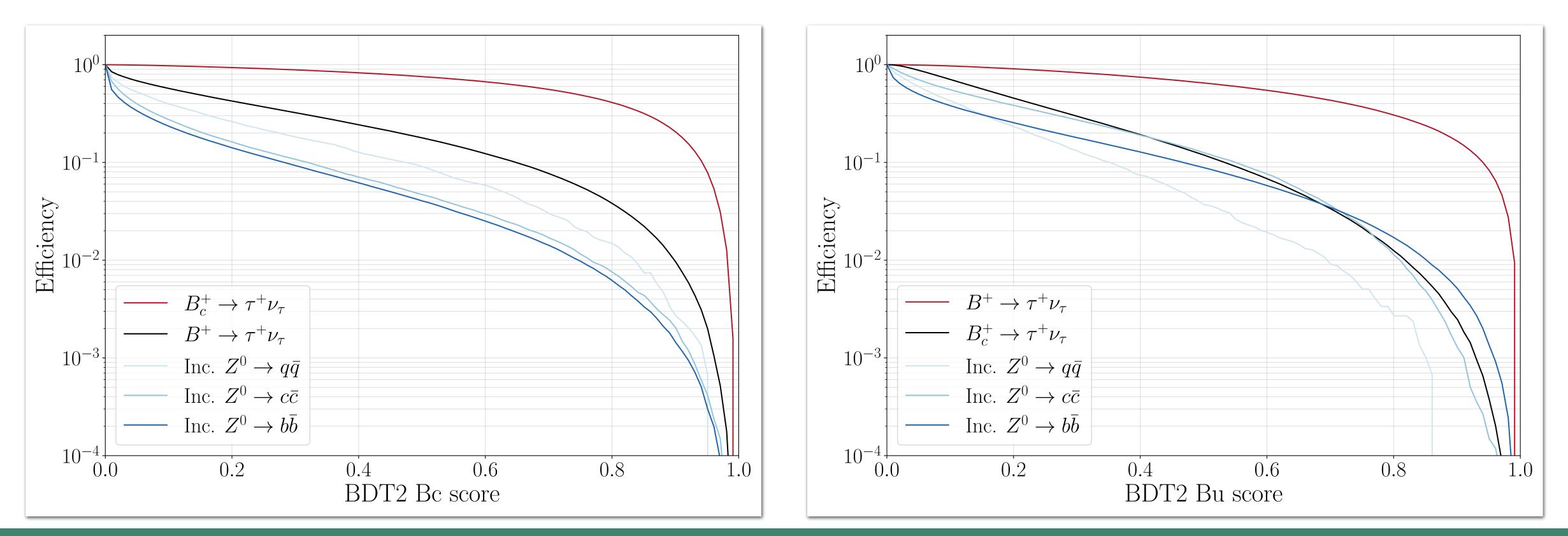
- Goal: separate Bu from background
   while keeping the efficiency for Bc
- Same selection, samples, and input variables as in Bc analysis
- Multiclassifier BDT
  - label 0:  $Z \rightarrow b\bar{b}, c\bar{c}, q\bar{q}$
  - label 1:  $B^+ \rightarrow \tau^+ \nu_{\tau}$
  - label 2:  $B_c^+ \rightarrow \tau^+ \nu_{\tau}$
- Good separation achieved between all 3 processes.





## Stage2 MVA

- High purity of  $B^+ \to \tau^+ \nu_{\tau}$  or  $B_c^+ \to \tau^+ \nu_{\tau}$  in the corresponding high-MVA region
  - Very little cross-contamination
  - Harder to separate  $Z \to b\bar{b}, c\bar{c}, q\bar{q}$  from  $B^+ \to \tau^+ \nu_{\tau}$ , as expected



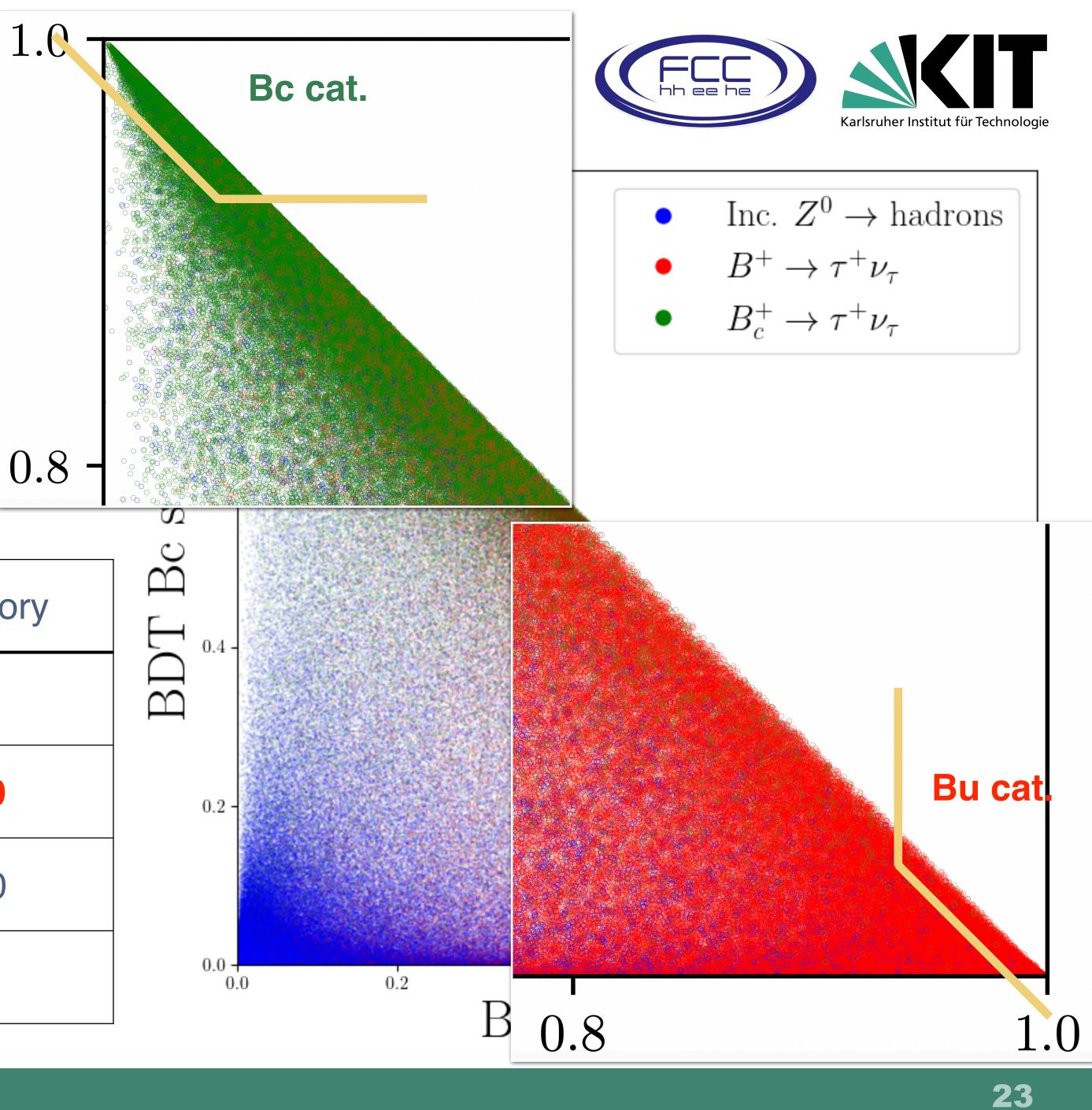




## Categorization (new)

- Select a stripe close to  $MVA_{bkg} \rightarrow 0$
- Orthogonality ensured by selecting different corners.

	Bc category	Bu categor	
Exp. Bc events	5002.2	11.14	
Exp. Bu events	264.6	5115.9	
Exp. bkg events	190.4	1806.0	
Sig. purity	92%	74%	

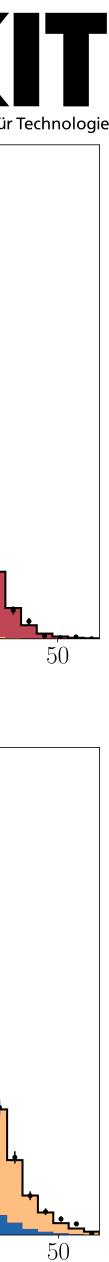


### Template fit

- Simultaneous fit of 2 categories.
- 4 free parameters (yield modifiers)
  - $\mu_{bc}$  correlated across two cats
  - $\mu_{b\mu}$  correlated across two cats
  - $\mu_{bkg}^{Bc\ cat}$ ,  $\mu_{bkg}^{Bu\ cat}$  independent in each category
- Signal strength uncertainty (from 2000 pseudo experiments)

  - $\sigma(\mu_{bc}) = 2.2\%$   $\sigma(\mu_{bu}) = 3.9\%$

800 Generated data Total fit 700  $\rightarrow \tau^+ \nu_{\tau}$ Candidates /  $(0.8 \text{ GeV}/c^2)$  $\rightarrow \tau^+ \nu_{\tau}$ Background Bc cat. 100 45 40 2535 30 Maximum hemisphere E  $[\text{GeV}/c^2]$ 1000Generated data Total fit Candidates /  $(0.8 \text{ GeV}/c^2)$  $\rightarrow \tau^+ \nu_{\tau}$  $B_c^+ \to \tau^+ \nu_{\tau}$ Background Bu cat. 40 45 35 2530 Maximum hemisphere E  $[\text{GeV}/c^2]$ 





## Summary

- Exciting opportunities in flavor physics at FCC-ee
  - Abundant events for rare decay searches
  - Expect great vertexing performance and missing energy measurement
- Good results expected for  $B_c^+ \rightarrow \tau^+ \nu_{\tau}$  and  $B^+ \rightarrow \tau^+ \nu_{\tau}$ 
  - Signal yield precision ~2% for  $B_c^+ \to \tau^+ \nu_\tau$  and ~4% for  $B^+ \to \tau^+ \nu_\tau$
  - Precision on  $\mathscr{B}$  or  $|V_{cb}|$  and  $|V_{\mu b}|$  depends on other inputs
  - Nevertheless expect to set strong constraints on BSM models
- Many other modes to be studied with the same software framework





xunwu.zuo@cern.ch

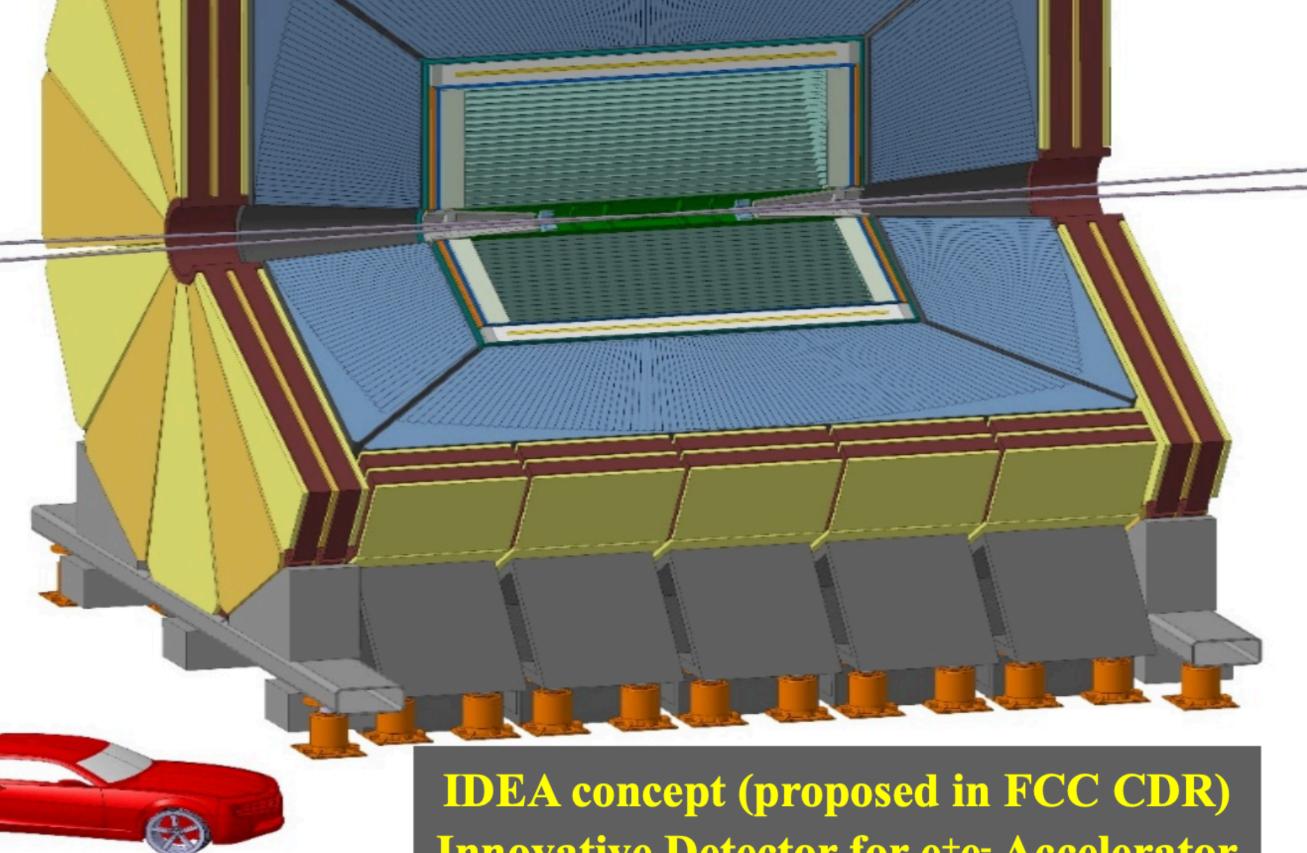


### Detector concept

- Silicon vertex detector
- Short-drift wire chamber
- 2T solenoid magnet
- Duel readout calorimeter
- Iron yoke + muon chambers



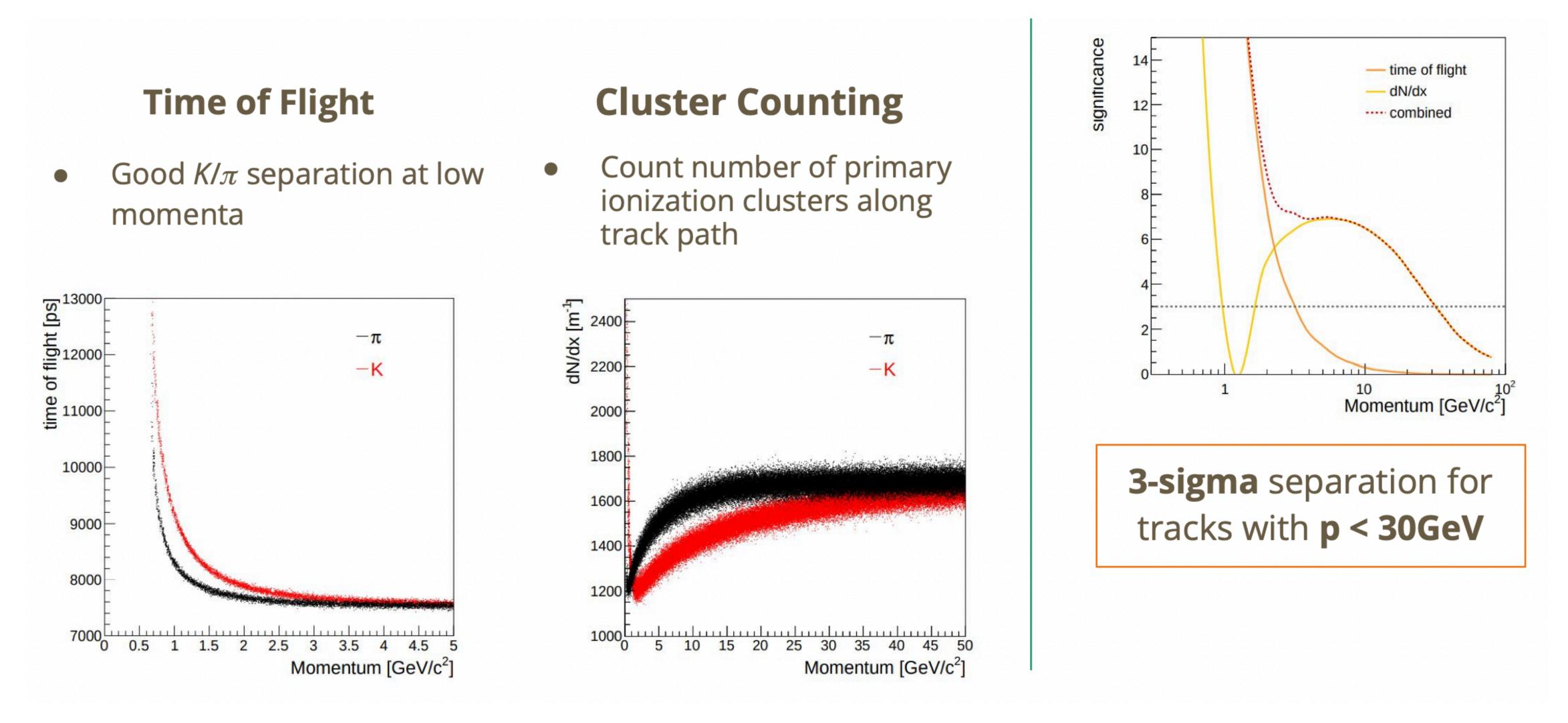








### Kaon vs Pion ID



### xunwu.zuo@cern.ch



### Assume perfect ID in the kinematic region (p < 30 GeV) of study.



### Samples

- $B_c^+ \to \tau^+ \nu_{\tau}$  and  $B^+ \to \tau^+ \nu_{\tau}$  signals, 10M each
- Inclusive  $Z \rightarrow b\bar{b}, c\bar{c}, q\bar{q}$  processes, 1B each
- Exclusive B decays backgrounds, 200M each
- All events generated with Pythia and simulated in DELPHES with IDEA detector



Decay mode	N(expected)	N(generated)	Expected / Generated	Final $\epsilon$
$B^+  o \bar{D}^0 \tau^+ \nu_{ au}$	$5.01  imes 10^9$	$2 \times 10^8$	25.0	$1.46 \times 10^{-9}$
$B^+ \to \bar{D}^{*0} \tau^+ \nu_{\tau}$	$1.22\times10^{10}$	$2 \times 10^8$	61.1	$1.1 \times 10^{-9}$
$B^+  o \bar{D}^0 3\pi$	$3.64  imes 10^9$	$1.9 \times 10^8$	19.2	$1.56 \times 10^{-9}$
$B^+  o {\bar D}^{*0} 3\pi$	$6.7  imes 10^9$	$2  imes 10^8$	33.5	$1.04 \times 10^{-9}$
$B^+ \to \bar{D}^0 D_s^+$	$5.85  imes 10^9$	$2 \times 10^8$	29.3	$2.52\times10^{-10}$
$B^+ \to \bar{D}^{*0} D_s^+$	$4.94 \times 10^9$	$1.75 \times 10^8$	28.2	$2.72\times10^{-10}$
$B^+ \to \bar{D}^{*0} D_s^{*+}$	$1.11 \times 10^{10}$	$2 \times 10^8$	55.6	$2.42\times10^{-10}$
$B^0  o D^- \tau^+ \nu_{ au}$	$7.02  imes 10^9$	$2 \times 10^8$	35.1	$2.69  imes 10^{-9}$
$B^0  o D^{*-} \tau^+ \nu_{\tau}$	$1.02 \times 10^{10}$	$2 \times 10^8$	51.0	$1.25 \times 10^{-9}$
$B^0 \rightarrow D^- 3\pi$	$3.9 \times 10^9$	$2 \times 10^8$	19.5	$3.4 \times 10^{-9}$
$B^0 \rightarrow D^{*-} 3\pi$	$4.69 \times 10^9$	$2 \times 10^8$	23.4	$9.84 \times 10^{-10}$
$B^0 \to D^- D_s^+$	$4.68  imes 10^9$	$2 \times 10^8$	23.4	$3.23\times10^{-10}$
$B^0 \to D^{*-} D_s^+$	$5.2  imes 10^9$	$2  imes 10^8$	26.0	$2.32\times10^{-10}$
$B^0 \to D^{*-} D_s^{*+}$	$1.15\times10^{10}$	$2 \times 10^8$	57.5	$2.35\times10^{-10}$
$B_s^0 \to D_s^- \tau^+ \nu_\tau$	$3.53  imes 10^9$	$2 \times 10^8$	17.6	$3.71 \times 10^{-9}$
$B_s^0  o D_s^{*-} \tau^+  u_{ au}$	$2.35  imes 10^9$	$2  imes 10^8$	11.8	$2.27  imes 10^{-9}$
$B_s^0 \to D_s^- 3\pi$	$8.85  imes 10^8$	$2 \times 10^8$	4.4	$5.53 \times 10^{-9}$
$B_s^0 \to D_s^{*-} 3\pi$	$1.05 \times 10^9$	$2 \times 10^8$	5.2	$3.38  imes 10^{-9}$
$B_s^0 \to D_s^- D_s^+$	$6.39  imes 10^8$	$2  imes 10^8$	3.2	$4.09\times10^{-10}$
$B_s^0 \to D_s^{*-} D_s^+$	$2.02  imes 10^9$	$2 \times 10^8$	10.1	$3.17\times10^{-10}$
$B_s^0 \to D_s^{*-} D_s^{*+}$	$2.09  imes 10^9$	$2 \times 10^8$	10.5	$2.56\times10^{-10}$
$\Lambda_b^0 \to \Lambda_c^- \tau^+ \nu_\tau$	$1.83 \times 10^9$	$2 \times 10^8$	9.1	$1.36 \times 10^{-9}$
$\Lambda_b^0  o \Lambda_c^{*-}  au^+  u_ au$	$1.83  imes 10^9$	$2  imes 10^8$	9.1	$9.44\times10^{-10}$
$\Lambda_b^0  o \Lambda_c^- 3\pi$	$4.31 \times 10^8$	$2 \times 10^8$	2.2	$5.58 \times 10^{-9}$
$\Lambda_b^0  o {\Lambda_c^*}^- 3\pi$	$4.31 \times 10^8$	$2  imes 10^8$	2.2	$9.21\times10^{-10}$
$\Lambda_b^0 \to \Lambda_c^- D_s^+$	$6.15 \times 10^8$	$2 \times 10^8$	3.1	$3.46 \times 10^{-10}$
$\Lambda_b^0  o \Lambda_c^{*-} D_s^+$	$6.15 \times 10^8$	$2 \times 10^8$	3.1	$2.72\times10^{-10}$
$\Lambda_b^0 \to \Lambda_c^{*-} D_s^{*+}$	$6.15 \times 10^8$	$2 \times 10^8$	3.1	$2.5 \times 10^{-10}$

## Stage1 MVA

- Goal: remove "easy" backgrounds from  $Z \rightarrow c\bar{c}, q\bar{q}$  processes
- **Binary BDT classifier** 
  - signal:  $B_c^+ \rightarrow \tau^+ \nu_{\tau}$
  - background: inclusive  $Z \rightarrow b\bar{b}, c\bar{c}, q\bar{q}$  decays
- Pre-selection for training:
  - A primary vertex (PV) is reconstructed
  - At least one  $3\pi$  secondary vertex (SV) is reconstructed
  - At least one  $3\pi$  candidate in the hemisphere with less energy



- Use general event properties as input variables:
  - Energy (charged, neutral, total) in each hemisphere
  - Particle multiplicity (charged, neutral, total) in each hemisphere
  - Number of tracks associated to the PV
  - Number of secondary vertices and their displacements
  - Number of  $3\pi$  candidates











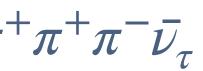




## Stage2 MVA

- Goal: use full kinematic property of  $\tau^+ \rightarrow \pi^+ \pi^- \bar{\nu}_{\tau}$ decay to maximize signal purity.
- **Binary BDT classifier** 
  - signal:  $B_c^+ \rightarrow \tau^+ \nu_{\tau}$
  - background: inclusive  $Z \rightarrow bb, c\bar{c}, q\bar{q}$  decays
- Selection before Stage2
  - Stage1 MVA > 0.6
  - Choose the  $3\pi$  candidate with the least vertex fit  $\chi^2$
  - $m(\pi^+\pi^-)$  compatible with  $\rho^0$ ,  $m(\pi^+\pi^+\pi^-) < m_{\tau}$
  - Significant energy difference between two hemispheres, selected vertex in the hemisphere with less energy





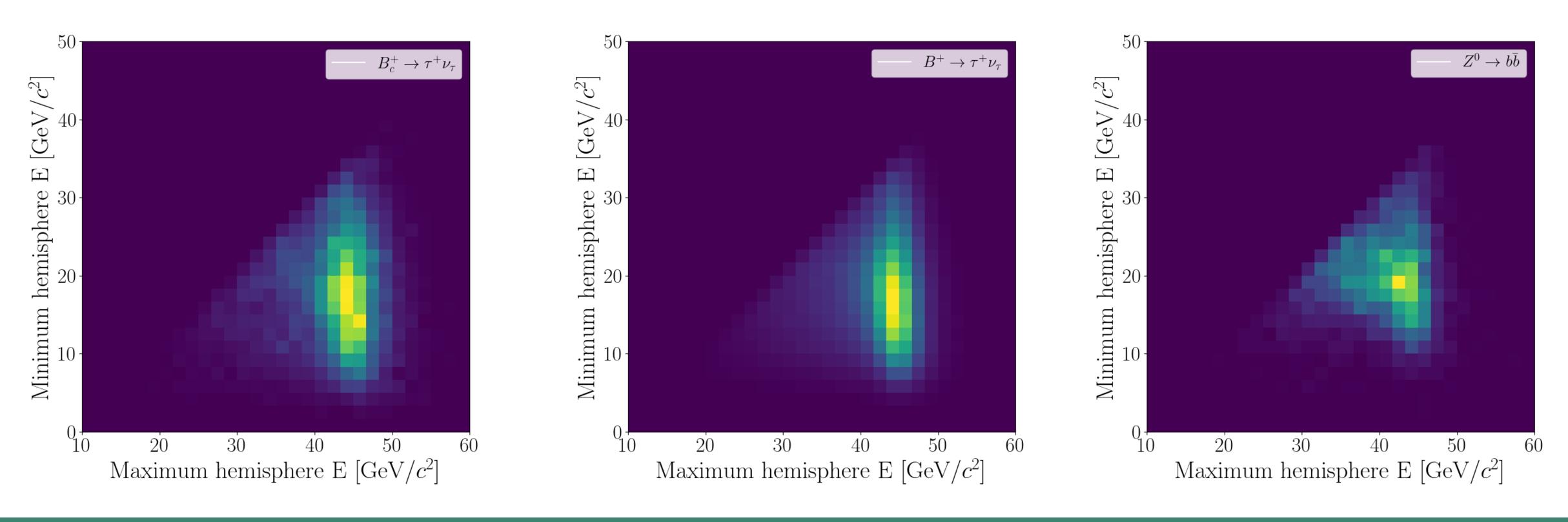
- Input variables:
  - $\tau^+ \rightarrow \pi^+ \pi^+ \pi^- \bar{\nu}_{\tau}$  decay: mass, momentum, impact parameter, angle to the thrust axis
  - Impact parameters of all other secondary vertices
  - Mass of the PV
  - Nominal  $B_c$  energy:  $m_Z$  all reco particles except the  $3\pi$  candidate





### Variables for final fit

No strong correlation between ThrustEmax\_E and ThrustEmin\_E 



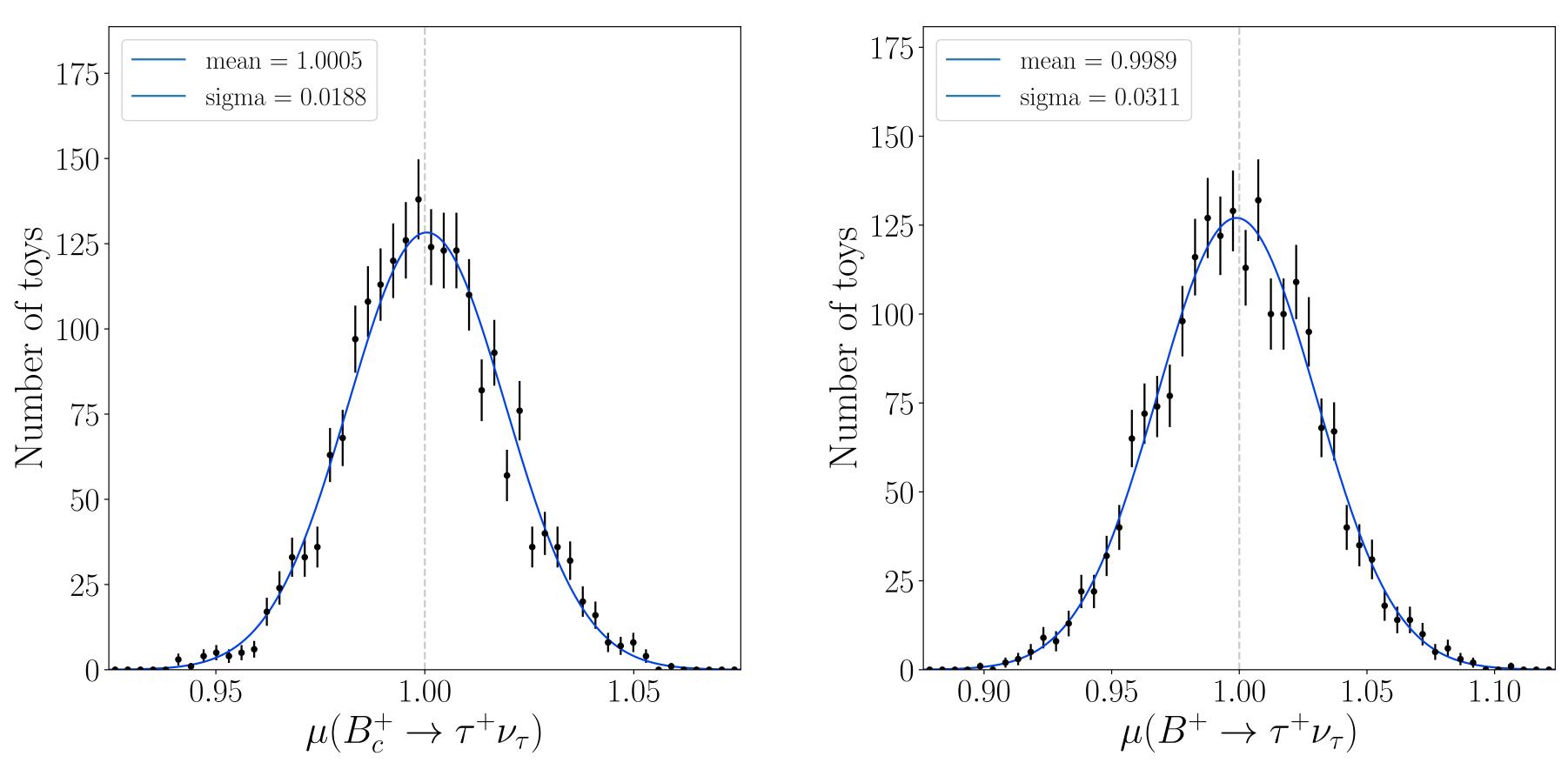




### Fit 2D distribution

• Simultaneous fit of a 2D distribution in 2 categories

- This an aggressive scenario, same binning as in 1D histograms (40 \* 40 bins)
- In practice, due to a lack of statistics, should use more coarse binning, and have worse results.
- To be studies more

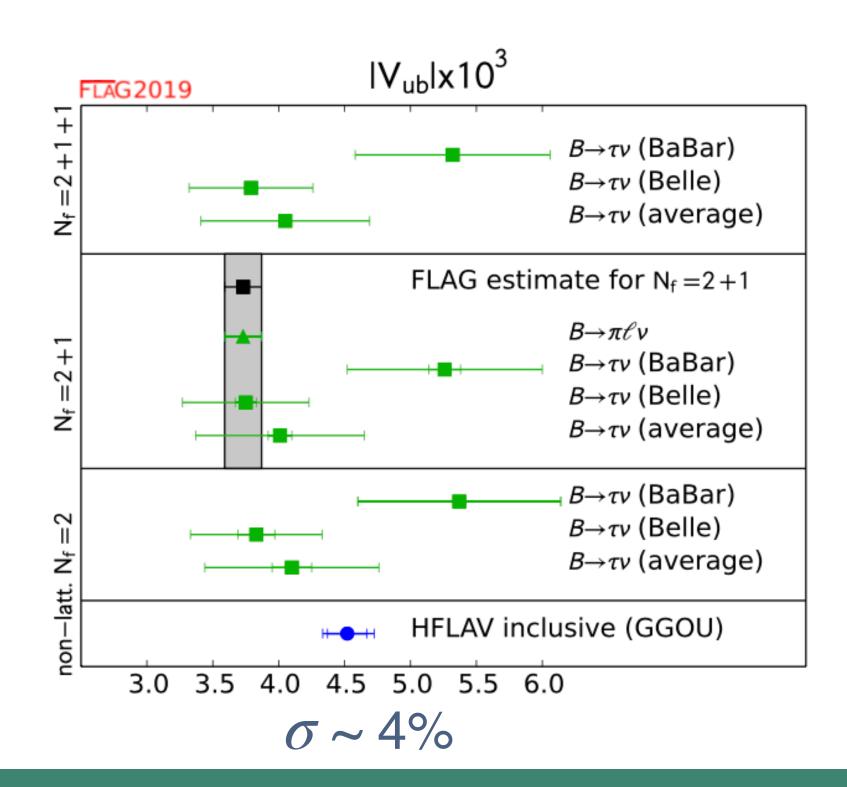






# $|V_{ch}|$ and $|V_{uh}|$

- to determine  $|V_{cb}|$  and  $|V_{\mu b}|$ 
  - Clean measurement with high experimental precision
  - Theoretical uncertainty (from lattice QCD) to be studied



xunwu.zuo@cern.ch



By taking leptonic decay constant  $f_B$  as input, the  $B_c^+ \to \tau^+ \nu_{\tau}$  and  $B^+ \to \tau^+ \nu_{\tau}$  results can be used

