SU(3) flavor symmetry breaking in $B \to D\bar{D}$ decays

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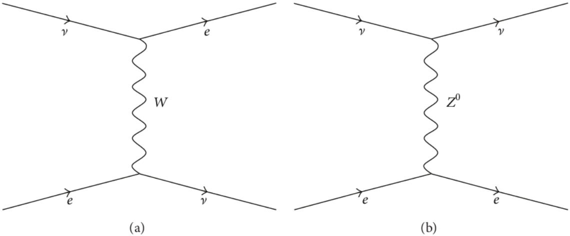
The Weak Interaction in Standard Model (SM)

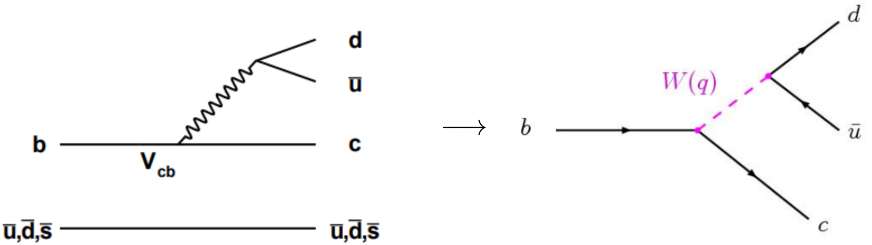
- Mediated by $W^{\pm}$, Z_0 bosons
- Two type of vertices :
  - Charged-Current (CC): ($W^{\pm}$)
  - Neutral-Current (NC): (Z_0)



Vertex :$\sim \frac{-ig_W}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$ Weak-propagator: $\sim \frac{1}{q^2-M_X^2}$ (X= W,Z)

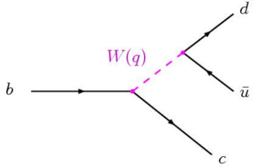
B meson decay as weak interaction

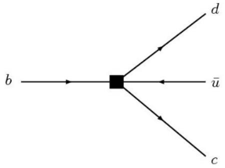
- depends on the energy/length scale.
- for weak b-decays : length scale $\delta x \sim \frac{1}{M_W}$: large M_W and small x
- e.g. for $B^- \to D^0\pi^-$ decay,
  at the quark level : $(b\bar{u}) \to (c\bar{u})(d\bar{u}) \Longrightarrow b \to cd\bar{u}$



Effective Theory : Fermi Model

Full theory (SM) $\rightarrow$ Fermi model





g^2 × current × W-propagator × current $\xrightarrow{|q| \ll M_W}$ effective coupling × local 4-quark operator

- b quark mass much smaller than W boson's mass : $|q| \leq m_b \ll M_W$
- So, the propagator approximates at this scale as : $\frac{1}{q^2 - M_W^2} \sim \frac{1}{M_W^2}$
- Effect of large scale M_W in effective Fermi coupling constant:

$$\frac{g_W^2}{8 M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.166639 \cdot 10^{-5}\, GeV^{-2}$$

Effective Hamiltonian for $b \to c d\bar{u}$

After the Operator Product Expansion(OPE) at short distance,

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu) \mathcal{O}_i + h.c \qquad (b \to c d\bar{u})$$

$$Eff.\ Coupl. \times CKM \times OPE$$

$C_i(\mu) \longrightarrow Wilson's\ coefficients$ $\qquad \mathcal{O}_i \longrightarrow Effective\ operator$

- **Current-Current Operators**: ($b \to c d\bar{u}$, analogously for $b \to q q' q"$)

$$\mathcal{O}_1 = (\bar{d}_L^a \gamma_\alpha u_L^b)(\bar{c}_L^b \gamma^\alpha b_L^a)$$
$$\mathcal{O}_2 = (\bar{d}_L^a \gamma_\alpha u_L^a)(\bar{c}_L^b \gamma^\alpha b_L^b)$$

- The Wilson Coefficients $C_i(\mu)$ contains all information about **Short-Distance Physics** $\equiv$ Dynamics above a Scale μ

The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

Cabbibo

Kobayashi

Maskawa

- Weak interaction of quarks are different from weak interaction of leptons !
- The CKM matrix transforms mass-eigenstates to flavor or weak-eigenstates :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates | CKM matrix | Mass Eigenstates

- **The Wolfenstein parametrization**

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-\iota\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-\iota\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

B $\to$ D$\bar{\text{D}}$ decay

- Quark content :

$$B \equiv (B^+, B^0, B_s^0) = (\bar{b}u, \bar{b}d, \bar{b}s) \qquad D \equiv (D^0, D^+, D_s^-) = (c\bar{u}, c\bar{d}, c\bar{s})$$

- $B \to D\bar{D}$ decay at quark level:

$$(\bar{b}q) \to (c\bar{q})(\bar{c}q) \equiv b \to c\bar{c}\bar{q} \text{ where } q = (d, s)$$

- Under Effective Field Theory (EFT), the quark level Effective Hamiltonian :

$$H^q_{eff} = \frac{G_F}{\sqrt{2}}\{V_{cb}V^*_{cq}(C_1 O_1^{q,c} + C_2 O_2^{q,c}) + V_{ub}V^*_{uq}(C_1 O_1^{q,u} + C_2 O_2^{q,u}) - V_{tb}V^*_{tq}\sum_{i=3}^{6} C_1 O_i^q\}$$

  where, by the unitarity of CKM matrix elements, $V_{ub}V^*_{uq} + V_{cb}V^*_{cq} + V_{tb}V^*_{tq} = 0$ and the operators are given by,

$$O_1^{q,c} = (\bar{c}_j b_i)_{V-A}(\bar{q}_i c_j)_{V-A}, \qquad O_2^{q,c} = (\bar{c}b)_{V-A}(\bar{q}c)_{V-A},$$
$$O_1^{q,u} = (\bar{u}_j b_i)_{V-A}(\bar{q}_i u_j)_{V-A}, \qquad O_2^{q,u} = (\bar{u}b)_{V-A}(\bar{q}u)_{V-A},$$
$$O_3^q = (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A}, \quad O_4^q = (\bar{q}_i b_j)_{V-A}\sum_{q'}(\bar{q}'_j q'_i)_{V-A},$$

- Operators are generated from all the diagrams (or processes) that contribute to the decay channel by renormalization in effective field theoretical approach

Decay amplitude

- The decay amplitude for $B \to D\bar{D}$ decays :

$$A(q) = \left\langle D\bar{D}\right| H_{eff}^{q} \left|B\right\rangle$$

- Inserting H_{eff}^{q}, and using the unitarity of CKM elements,

$$A(q) = \frac{G_F}{\sqrt{2}}(V_{cb}V_{cq}^{*}T(q) + V_{tb}V_{tq}^{*}P(q))$$

Where, $T(q)$ = Tree level amplitudes from all the Tree level diagrams and $P(q)$ = Penguin amplitudes from all the Penguin diagrams

- Calculation of $T(q)$ and $P(q)$ $\longrightarrow$ matrix elements of those operators in different models: Lattice QCD !
- four-quark level interaction $\longrightarrow$ Hadronic interaction : Calculation of Form Factors! $f(q^2)$
- **Alternative : SU(3) flavor symmetry approach !**

SU(3) flavor symmetry

- Symmetry between the 3 lightest quarks of Standard Model : u ,d and s
- Symmetry transformation belongs to Special Unitary group of dimension 3; **SU(3)**:

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

- Now in Lie group theory , $\hat{U} = e^{\iota \vec{\alpha} \cdot \vec{T}}$
  where, $\alpha =$ infinitesimal transformation and
  $\vec{T}$ is the generator of the group ; $\vec{T} = \frac{1}{2}\vec{\lambda}$
- For a group of **N** dimension there are N^2 **-1** number of generators:
  –there are **8** $\lambda' s \longrightarrow$ **8** Gell-Mann matrices

Generators of SU(3) : Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Tensor decomposition under flavor SU(3)

- Under flavor SU(3) u, d, s transforms as a triplet (3), denoted by ,

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \; d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \; s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- all other quarks c, t, b transforms as singlet (1)
- For $B \to D\bar{D}$ ($\bar{b} \to c\bar{c}\bar{q}$) decays:

$$\begin{array}{ccccc} \bar{b} & \longrightarrow & c & \bar{c} & \bar{q} \\ \bar{1} & \longrightarrow & 1 & \bar{1} & \bar{3} \end{array}$$

- So, the quark level interaction Hamiltonian transform as:

$$\bar{1} \otimes 1 \otimes \bar{1} \otimes \bar{3} = \bar{3} \qquad \textit{Tensor product}$$

$\longrightarrow$ we get $H(\bar{3})^q$ with q = d and s

SU(3) reduced amplitude for $B \to D\bar{D}$ *decays*

- Under SU(3) flavor symmetry the decay amplitude becomes :

$$A(q) = \left\langle D\bar{D} \right| H^q_{eff} \left| B \right\rangle = \frac{G_F}{\sqrt{2}} (V_{cb} V^*_{cq} T(q) + V_{tb} V^*_{tq} P(q))$$

with

$$T(q) = A^T_{DD} (B_i H^{q^i})(D_j \bar{D}^j) + B^T_{DD} (B_i \bar{D}^i)(D_j H^{q^j})$$

and

$$P(q) = A^P_{DD} (B_i H^{q^i})(D_j \bar{D}^j) + B^P_{DD} (B_i \bar{D}^i)(D_j H^{q^j})$$

- q=d correspond to the $\Delta S = 0$ Cabibbo-suppressed decays and q=s correspond to the $\Delta S = 1$ Cabibbo-favored decays with,

$$H(\bar{3})^d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \textit{and} \ \ H(\bar{3})^s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- The coefficients A^T_{DD}, B^T_{DD}, A^P_{DD} and B^P_{DD} are the $SU(3)$ invariant complex amplitudes. given by:

$$X^{T,P}_{DD} = \mathrm{Re}(x^{T,P}_{\mathrm{DD}}) + \iota \ \mathrm{Im}(x^{T,P}_{\mathrm{DD}}) \, , \ \textit{with} \ X = (A, B) \ \textit{and} \ x = (a, b)$$

Breaking of SU(3) flavor symmetry

- The flavor SU(3) symmetry between u,d and s quark is not exact ! because of their mass difference !

$$m_u \sim 2\ MeV,\ m_d \sim 4\ MeV,\ and\ m_s \sim 100\ MeV\ !!$$

$$m_s \gg m_u, m_d$$

- In reality SU(3) flavor symmetry is **badly broken** in nature
- To include this breaking we add a $s\bar{s}$ in our interaction: Hamiltonian

$$s\bar{s} = 3 \otimes \bar{3} = 8 \oplus 1$$

- We contract the octet part with the unbroken Hamiltonian, given by W:

$$W(8) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Amplitudes under SU(3) breaking in $B \to D\bar{D}$ *decays*

- After symmetry breaking decay amplitude becomes :

$$A(q) = \left\langle D\bar{D}\right| H^q_{eff} \left|B\right\rangle = \frac{G_F}{\sqrt{2}}(V_{cb}V^*_{cq}\{T(q) + \mathbf{\Delta T(q)}\} + V_{tb}V^*_{tq}\{P(q) + \mathbf{\Delta P(q)}\})$$

- $\Delta T(q)$ and $\Delta P(q)$ correspond to the inclusion of breaking!
- **Approximations** :
  - Breaking associated to the Tree and Penguin amplitudes are of the same order.
  - Broken amplitudes having same origin from unbroken contraction are assigned same parameters
- With this we have : $\Delta T(q) \simeq \Delta P(q) = \Delta_{DD}(q)$

$$\Delta_{DD}(q) = C_{DD}(B_i H^{q^i})(D_j W^j_k \bar{D}^k) + D_{DD}(B_i \bar{D}^i)(D_j W^j_k H^{q^k}) + C_{DD}(B_i W^i_j H^{q^j})(D_k \bar{D}^k) + D_{DD}(B_i W^i_j \bar{D}^j)(D_k H^{q^k})$$

- The broken coefficients, C_{DD} and D_{DD} have similar complex form as unbroken ones

Observables of interest

- **Branching Ratio** :

$$BR(B_i \to D_j D_k) = \frac{\Gamma(B_i \to D_j D_k)}{\Gamma(B_i)}, \quad \Gamma = Deacy\ width$$

Decay width: $\Gamma(B_i \to D_j D_k) = \frac{p^*}{32\pi^2 m_{B_i}^2} \int |A(B_i \to D_j D_k)|^2 d\Omega$

- **Ratio of the BR's**: $\frac{BR(B_i \to D_j D_k)}{BR(B_l \to D_m D_n)}$
- **CP violating observables** : $\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f})$
  - **Direct CP violation**:

$$A_{CP} = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}$$

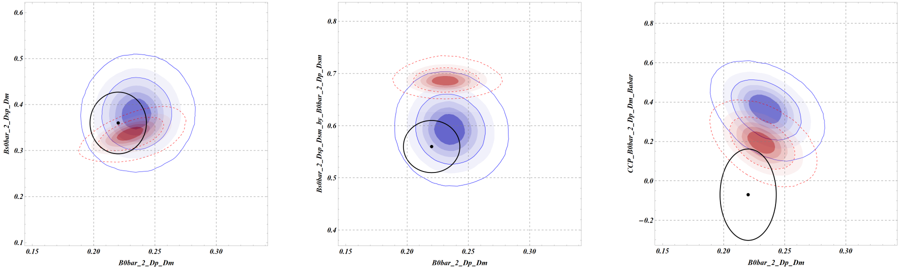
  - **CP violation in mixing**: $\bar{B}^0 \to f \leftrightarrow B^0 \to f$

$$C_{CP} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad S_{CP} = \frac{2 Im\lambda}{1 + |\lambda|^2}, \quad \lambda = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

Fit results : Frequentist analysis

- The observables are calculated with the SU(3) reduced amplitude for both the exact and broken SU(3).
- As a fit to the experimental data of these observables (HFLAV, Belle, BaBar, LHCb), frequentist analysis has been performed
- Exact SU(3):
    - No. of SU(3) parameters : 7
    - $\chi^2 = 12.42$
    - P value = 0.332
- Broken SU(3):
    - No. of SU(3) parameters : 11
    - $\chi^2 = 5.49$
    - P value = 0.599
- The P value is significantly improved with the broken SU(3) symmetry description of the decay
- χ^2 analysis $\rightarrow$ still inconclusive! due to the lack of enough data

Bayesian Analysis : Preliminary!

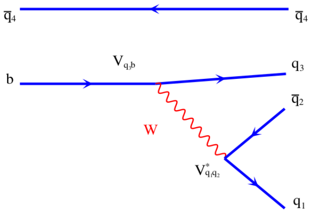


Fit results : Bayesian analysis

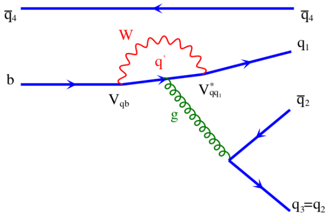
- The correlation between the observables approach to the data after SU(3) breaking!
- This analysis also tells the trend of the future data : Prediction for more precise measurements !
- **More data required for more sophisticated and concrete analysis !**

SU(3) Topology of $B \to D\bar{D}$ decays

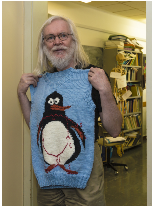
There are mainly two types of diagrams that contribute to B decays topology: Tree level diagram and Penguin or Loop level diagram



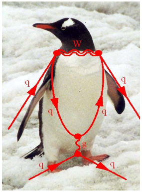
Tree level diagram



Penguin(QCD) diagram

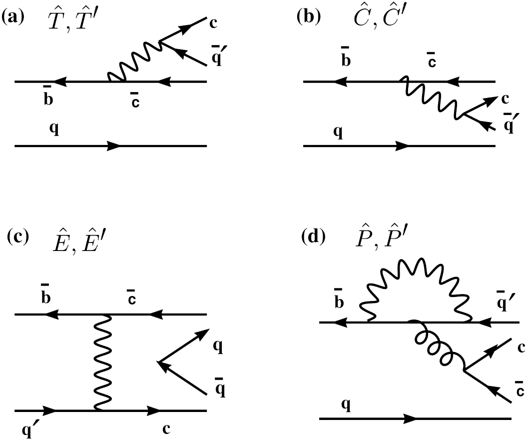


John Ellis (1977)



Topological diagrams with exact SU(3) symmetry

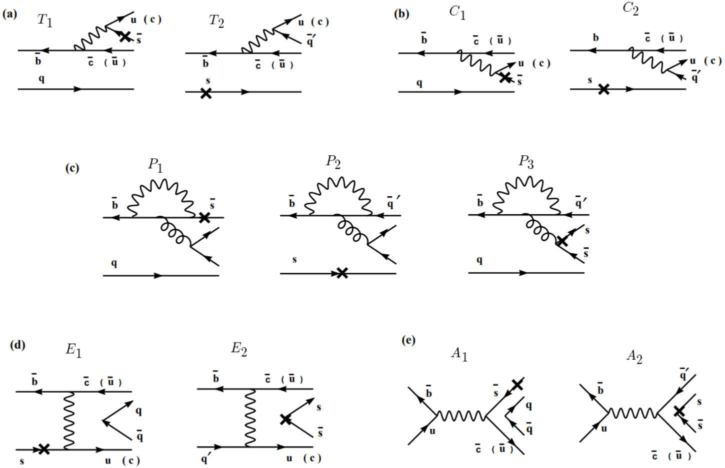
color favored tree   color suppressed tree



Exchange   QCD penguin

Diagrams with broken SU(3) symmetry

To include SU(3) breaking effect in the topological diagram a "X" mark has been introduce on the s- quark line.



Topological amplitude with unbroken and broken SU(3) symmetry for $B \to D\bar{D}$ *decays*

Decay Modes	Unbroken	Broken
$\mathcal{A}(B^- \to D^0 D_s^-)$	$\hat{T}' + \hat{P}'$	$\hat{T}' + \hat{P}' + T_1' + P_1'$
$\mathcal{A}(\bar{B}_s \to D_s^+ D_s^-)$	$\hat{T}' + \hat{P}' + \hat{E}'$	$\hat{T}' + \hat{P}' + \hat{E}' + T_1' + T_2' + P_1' + P_2' + E_1' + E_2'$
$\mathcal{A}(\bar{B}_s \to D^+ D^-)$	$\hat{E}'$	$\hat{E}' + E_1'$
$\mathcal{A}(\bar{B}_s \to D^+ D_s^-)$	$\hat{T}' + \hat{P}'$	$\hat{T}' + \hat{P}' + T_1' + P_1'$
$\mathcal{A}(\bar{B}_s \to D^0 \bar{D}^0)$	$\hat{E}'$	$(\hat{E}' + E_1')$
$\mathcal{A}(\bar{B}^0 \to D^+ D^-)$	$\hat{T} + \hat{P} + \hat{E}$	$\hat{T} + \hat{P} + \hat{E}$
$\mathcal{A}(\bar{B}^0 \to D^0 \bar{D}^0)$	$-\hat{E}$	$-\hat{E}$
$\mathcal{A}(\bar{B}_s \to D_s^+ D^-)$	$\hat{T}' + \hat{P}'$	$T' + \hat{P}' + T_2' + P_2'$
$\mathcal{A}(\bar{B}^0 \to D_s^+ D_s^-)$	$\hat{E}'$	$\hat{E}' + E_2'$
$\mathcal{A}(B^- \to D^0 D^-)$	$\hat{T} + \hat{P}$	$\hat{T} + \hat{P}$

- The inclusion of breaking modify the topological amplitudes
- Apparent confusion: some decay modes remain unaffected even after breaking ! $\longrightarrow$ This is not the case ! The SU(3) and topology amplitudes are in different basis

Summary and Outlook

- $(B \to D\bar{D}) \implies$ weak interaction.
- The decay amplitude $\implies$ Heavy Flavor Effective Field Theory $\implies$ Form factor calculation
- Alternative approach: Flavor SU(3) symmetry between u,d and s.
- The breaking of SU(3) gives more accurate description to the observables than exact SU(3) !
- SU(3) breaking effects can also be studied in terms of topological amplitudes
- Future :
  - Run 3 of LHCb, new measurement in Belle and BaBar; More statistics
  - More precise statistical analysis !
- Manuscript under preparation! soon on arXiv: Stay tuned......

References

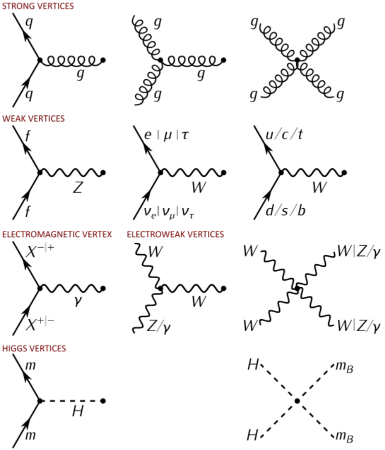
- M. J. Savage and M. B. Wise, "SU(3) Predictions for Nonleptonic B Meson Decays," *Phys. Rev. D*, vol. 39, p. 3346, 1989.
  [Erratum: Phys.Rev.D 40, 3127 (1989)].
- G. Buchalla, A. J. Buras, and M. E. Lautenbacher, "Weak decays beyond leading logarithms," *Rev. Mod. Phys.*, vol. 68, pp. 1125–1144, 1996.
- D. Zeppenfeld, "SU(3) Relations for B Meson Decays," *Z. Phys. C*, vol. 8, p. 77, 1981.
- B. Grinstein and R. F. Lebed, "SU(3) decomposition of two-body B decay amplitudes," *Phys. Rev. D*, vol. 53, pp. 6344–6360, 1996.
- X.-G. He, G.-N. Li, and D. Xu, "SU(3) and isospin breaking effects on $B \to PPP$ amplitudes," *Phys. Rev. D*, vol. 91, no. 1, p. 014029, 2015.
- P. A. Zyla *et al.*, "Review of Particle Physics," *PTEP*, vol. 2020, no. 8, p. 083C01, 2020.
- R. Aaij *et al.*, "Measurement of *CP* violation in $B^0 \to D^+ D^-$ decays," *Phys. Rev. Lett.*, vol. 117, no. 26, p. 261801, 2016.

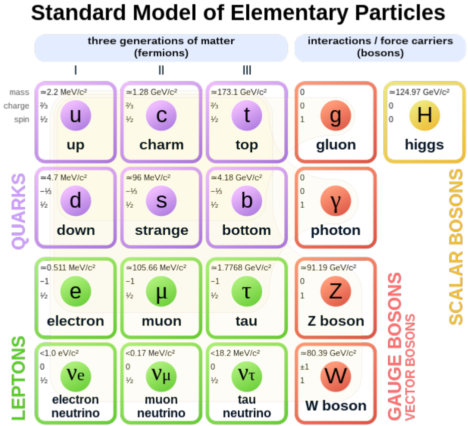
Thank you !

for your attention

Back up

The Standard Model of Particle Physics





Source : Wikipedia

Central Notions : Theoretical Jargon

Factorization

— Separation of scales in Pertirbation Theory

— Simplification of exclusive hadronic matrix elements

$\downarrow$

Operator Product Expansion(OPE)

Short distance expansion ($x \to 0$) of time ordered operator products corresponding to $\left|q^2\right| \to \infty$ in Fourier transform:

$$\int d^4x\ e^{iq.x}\, T(\phi(x)\phi(0)) = \sum_i c_i(q^2)\mathcal{O}_i(0)$$

"Wilson Coefficients" $c_i(q^2)$ *"Effective" Operators* $\mathcal{O}_i(0)$

$\downarrow$

Effective(Quantum) Field Theories

Effective Lagrangian/Hamiltonian :

— Feynman rules $\longrightarrow$ dynamics of low-energy modes

— High-energy(short-distance) information in coefficients/functions

SU(3) amplitudes for decay modes

- Cabbibo-suppressed ($\Delta S = 0$) decay modes:

$$\mathcal{A}(\bar{B}_s \to D^- D_s^+) \to \frac{A\lambda^3 G_F((i\eta - \rho + 1)(\text{Re}(b_{\text{DD}}^P) + i\,\text{Im}(b_{\text{DD}}^P)) - i\,\text{Im}(b_{\text{DD}}^T) - \text{Re}(b_{\text{DD}}^T))}{\sqrt{2}}$$

$$\mathcal{A}(\bar{B}^0 \to D_s^- D_s^+) \to \frac{A\lambda^3 G_F((i\eta - \rho + 1)(\text{Re}(a_{\text{DD}}^P) + i\,\text{Im}(a_{\text{DD}}^P)) - i\,\text{Im}(a_{\text{DD}}^T) - \text{Re}(a_{\text{DD}}^T))}{\sqrt{2}}$$

- Cabbibo-favoured ($\Delta S = 1$) decay modes:

$$\mathcal{A}(\bar{B}_s \to D^- D^+) \to \frac{A\lambda^2 G_F\left(-i\,\text{Im}(a_{\text{DD}}^P) + \left(1 - \frac{\lambda^2}{2}\right)(\text{Re}(a_{\text{DD}}^T) + i\,\text{Im}(a_{\text{DD}}^T)) - \text{Re}(a_{\text{DD}}^P)\right)}{\sqrt{2}}$$

$$\mathcal{A}(\bar{B}_s \to D_s^- D_s^+) \to \frac{G_F}{\sqrt{2}}\left(A\lambda^2\left(1 - \frac{\lambda^2}{2}\right)(i(\text{Im}(a_{\text{DD}}^T) + \text{Im}(b_{\text{DD}}^T)) + \text{Re}(a_{\text{DD}}^T) + \text{Re}(b_{\text{DD}}^T)) - A\lambda^2(i(\text{Im}(a_{\text{DD}}^P) + \text{Im}(b_{\text{DD}}^P)) + \text{Re}(a_{\text{DD}}^P) + \text{Re}(b_{\text{DD}}^P))\right)$$

Broken SU(3) amplitudes

- Cabbibo-suppressed ($\Delta S = 0$) decay modes:

$$\mathcal{A}(\bar{B}_s \to D^- D_s^+) \to A\lambda^3 \frac{G_F}{\sqrt{2}}((i\eta - \rho + 1)(-i\ \mathrm{Im}(d_{\rm DD}) + i\ \mathrm{Im}(b^P_{\rm DD}) - \mathrm{Re}(d_{\rm DD}) + \mathrm{Re}(b^P_{\rm DD})) + i\ \mathrm{Im}(d_{\rm DD}) - i\ \mathrm{Im}(b^T_{\rm DD}) + \mathrm{Re}(d_{\rm DD}) - \mathrm{Re}(b^T_{\rm DD}))$$

$$\mathcal{A}(\bar{B}^0 \to D_s^- D_s^+) \to A\lambda^3 \frac{G_F}{\sqrt{2}}((i\eta - \rho + 1)(-i\ \mathrm{Im}(c_{\rm DD}) + i\ \mathrm{Im}(a^P_{\rm DD}) - \mathrm{Re}(c_{\rm DD}) + \mathrm{Re}(a^P_{\rm DD})) + i\ \mathrm{Im}(c_{\rm DD}) - i\ \mathrm{Im}(a^T_{\rm DD}) + \mathrm{Re}(c_{\rm DD}) - \mathrm{Re}(a^T_{\rm DD}))$$

- Cabbibo-favoured ($\Delta S = 1$) decay modes:

$$\mathcal{A}(\bar{B}_s \to D^- D^+) \to A\lambda^2 \frac{G_F}{2\sqrt{2}} \left(\lambda^2(i\ \mathrm{Im}(c_{\rm DD}) - i\ \mathrm{Im}(a^T_{\rm DD}) + \mathrm{Re}(c_{\rm DD}) - \mathrm{Re}(a^T_{\rm DD})) - 2i\ \mathrm{Im}(a^P_{\rm DD}) + 2i\ \mathrm{Im}(a^T_{\rm DD}) - 2\ \mathrm{Re}(a^P_{\rm DD}) + 2\ \mathrm{Re}(a^T_{\rm DD})\right),$$

$$\mathcal{A}(\bar{B}_s \to D_s^- D_s^+) \to A\lambda^2 \frac{G_F}{\sqrt{2}} \left(\left(1 - \frac{\lambda^2}{2}\right)(-4i(\mathrm{Im}(c_{\rm DD}) + \mathrm{Im}(d_{\rm DD})) + i(\mathrm{Im}(a^T_{\rm DD}) + \mathrm{Im}(b^T_{\rm DD})) - 4\ \mathrm{Re}(c_{\rm DD}) - 4\ \mathrm{Re}(d_{\rm DD}) + \mathrm{Re}(a^T_{\rm DD}) + \mathrm{Re}(b^T_{\rm DD})) + 4i(\mathrm{Im}(c_{\rm DD}) + \mathrm{Im}(d_{\rm DD})) - i(\mathrm{Im}(a^P_{\rm DD}) + \mathrm{Im}(b^P_{\rm DD})) + 4\ \mathrm{Re}(c_{\rm DD}) + 4\ \mathrm{Re}(d_{\rm DD}) - \mathrm{Re}(a^P_{\rm DD}) - \mathrm{Re}(b^P_{\rm DD})\right)$$

Decay width Sum Rules

- One can also construct Sum rules between different decay modes at the decay width level
- Under exact SU(3) :

$$\begin{aligned}
|\mathcal{A}(B^- \to D^0 D_s^-)|^2 &= |\mathcal{A}(\bar{B}^0 \to D^+ D_s^-)|^2, \\
|\mathcal{A}(\bar{B}_s \to D_s^+ D^-)|^2 &= |\mathcal{A}(B^- \to D^0 D^-)|^2, \\
|\mathcal{A}(\bar{B}_s \to D_s^+ D_s^-)|^2 &= |\mathcal{A}(\bar{B}^0 \to D^+ D^-)|^2, \\
|\mathcal{A}(\bar{B}_s \to D^+ D^-)|^2 &= |\mathcal{A}(\bar{B}_s \to D^0 \bar{D}^0)|^2, \\
|\mathcal{A}(\bar{B}^0 \to D^0 \bar{D}^0)|^2 &= |\mathcal{A}(\bar{B}^0 \to D_s^+ D_s^-)|^2.
\end{aligned}$$

- With broken SU(3) symmetry :

$$\begin{aligned}
|\mathcal{A}(B^- \to D^0 D_s^-)|^2 &= |\mathcal{A}(\bar{B}^0 \to D^+ D_s^-)|^2, \\
|\mathcal{A}(\bar{B}_s \to D^+ D^-)|^2 &= |\mathcal{A}(\bar{B}_s \to D^0 \bar{D}^0)|^2,
\end{aligned} \tag{0.1}$$

- Some sum rules are retained after SU(3) symmetry breaking !