

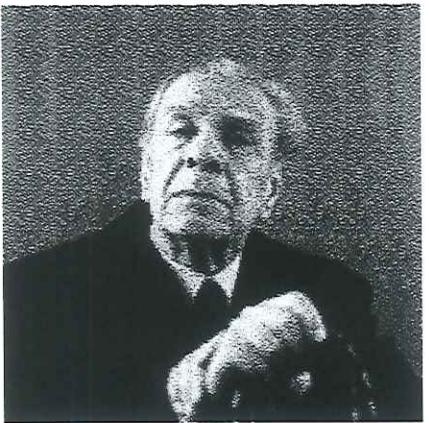
Heavy-light hadrons: Old laces & new pieces

Maciej A. Nowak

Mark Kac Complex Systems Research Center,
Institute of Theoretical Physics,
Zagłębianie Univ

March 4th 2022, "Kawiory"

(NCN Grant UMO-2017/27/B/ST2/01139)



"Solomon saith: There is no new thing upon the earth. So that as Plato had an imagination, that all knowledge was but remembrance; so Solomon giveth his sentence, **that all novelty is but oblivion.**

Francis Bacon: Essays, LVIII"

— Jorge Luis Borges, The Aleph and Other Stories

Outline:

- **Old Laces** - Chiral dynamics, summarized in e.g. "Nuclear Chiral Dynamics"
(M. Rho, I. Zahed, MAN; Singapore 1996)
- **Two revolutions** (exp. and theoretical)
- **New Pieces** - Holographic HL Chiral Effective Action
Y. Liu, I. Zahed; PRD 95 (2017) 056022 , PRD 95 (2017) 116012
Y. Liu, MAN, I. Zahed ; PRD 100 (2019) 126023 ; PRD .. (2022) in press (Tetraquarks)
Y. Liu, MAN, I. Zahed ; PRD 104 (2021), 114021 ; PRD 104 (2021) 114022 (Pentaquarks)
Y. Liu, K. Mamo, MAN, I. Zahed ; PRD 104 (2021) 114023 . (Photoproduction of Pentaquarks)

Heavy and light.

2 (3) quarks (u, d, s)

$$m_u/d \ll \Lambda_{QCD}, m_s \sim \Lambda_{QCD}$$

3 heavy quarks (c, b, t)

$$m_h \gg \Lambda_{QCD}$$

- Light sector: $SB \times S$ $SU_L(n_c) \times SU_R(n_c) \rightarrow SU(n_c)$

Effective chiral actions (G-model, NJL, X.P.T..)

- Heavy sector: heavy spin decouples as $m_h \rightarrow \infty$
Then $(0^-, 1^-)$ becomes degenerated in the $m_h \rightarrow \infty$ limit

$$H = \frac{1+\not{x}}{2} \left(i \gamma_5 D + \gamma^\mu D_\mu \right)^*$$

- Expansion in $1/N_c$

Light sector:

- Goldstone particles (pions)

$$\mathcal{L}_G \sim \frac{f\pi^2}{4} \text{Tr } D_g U D^\mu U + \dots$$

$$U = e^{i/f\pi \vec{\tau} \cdot \vec{\Pi}}$$

- What about baryons? "Ergonomic solution": Skyrme
Baryon is a soliton, Baryon number is topological (winding)

$$\vec{\Pi} = f(r) \frac{\vec{r}}{r}, \quad \mathcal{L} = \mathcal{L}_G + \mathcal{L}_{\text{Skyrme}} + \mathcal{L}_{\text{WZW}} \xrightarrow[\text{stabilizer}]{} (\pi^0 \rightarrow rr)$$

- Such classical solution has zero modes (rotations $U \rightarrow A(t) U A^\dagger(t)$)
- Quantization of collective coordinates in moduli space (Witten, Adkins, Kappeler)

$$H = M_0 + \frac{J^2}{2\Omega_{\text{sol}}} = M_0 + \frac{J^2}{2I_{\text{sol}}}$$

- Two parameters, good phenomenology

Strange sector

- $SU(2) \rightarrow SU(3)$: Mazur, MAN, Presnovitz \rightarrow "elegant disaster" (1984)

- Why-out: m_s is too heavy:

Born-Oppenheimer approx: fast vibrations of kaon in $SU(2)$ solitonic background. Then slow rotation of the bound state happens in presence of Berry phase from kaon

$$H = H_0 + \text{binding} + \frac{(\bar{J} - (1 - c_K) \text{tr } K \bar{I} K^+)^2}{2 \Omega_{\text{sol}}}$$

(isospin-spin transmutation)



versus



Callan-Klebanov-Hombosted 1985

- Successful phenomenology (also, no pentaquark)

Heavy sector:

One has to remember about heavy spin symmetry

- $$H = M_0 + \text{binding} + \frac{[(\bar{J} - \bar{S}_H) - (1 - c_D) \text{tr } \bar{D} \bar{I} D^+ - (1 - c_{D^*}) \text{tr } \bar{D}^* \bar{I} D^{*+}]^2}{2 S_{\text{sol}}}$$

- When $m_h \rightarrow \infty$, phases cancel

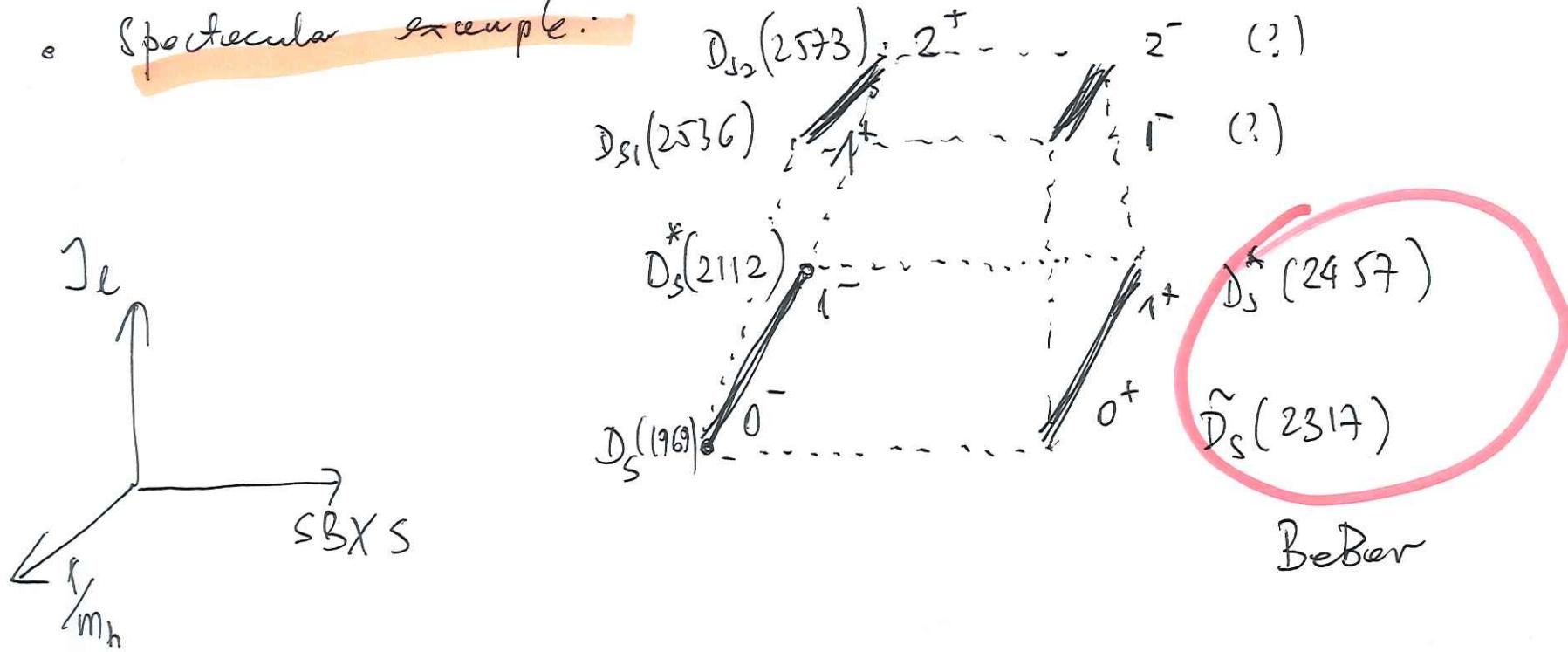
$$\frac{(\bar{J} - \bar{S}_H)^2}{2 S_{\text{sol}}} = \frac{I^2}{2 S_{\text{sol}}}$$

(Heavy spin symmetry at the baryonic level)

- Soliton can capture more than one meson (exotic), Savage-Wise symmetry
- Same happens (a priori) for χ doublets

Chiral doublers scenario (MAN, M.Rho, I.Zahed (1992), Bordeon-Hill (1993))

- Requirement of simultaneous presence of both symmetries enforces the emergence of opposite to h_1 , multiplet $G = \frac{1+\sqrt{5}}{2} (\tilde{D} + \gamma^\mu \gamma_5 \tilde{D}_\mu^*)$
- Doublers communicate via axial current
- $m_G - m_h \sim O(\mathcal{I}_{\text{light}}) \sim 350 \text{ MeV}$
- Reorganization, not doubling the hadronic states
- Spectacular example:



Meson-Baryon "supersymmetry"

- QQ  \rightarrow Diquark with color $\bar{3}$, alike \bar{Q}
- Such symmetry relates spin-splittings of $\bar{H}L$ to HHL
- $(\Delta M_{HHL} = \frac{3}{4} \Delta M_{\bar{H}L})$ Jorgenson & Wise (1990)
- Hermitian duality $HH \leftrightarrow \bar{H}$ leads to further correspondence
 - e.g. $HLL \rightarrow \overline{HH} LL$ (heavy baryon - doubly heavy tetraquark)

New Zoo... (alike quark chemistry)

- soliton captures H meson
- soliton \sim G meson
- soliton captures \bar{H} (heavy pentagonal)
- soliton \sim \bar{G} (darker of h. pentagonal)
- soliton captures more than one ...
- etc ..

Exotica

$\bar{c}du\bar{s}$
 $c\bar{c}q\bar{q}$
 $c\bar{c}u\bar{d}$
 $c\bar{c}u\bar{s}$
 $c\bar{c}s\bar{s}$
 $c\bar{c}c\bar{c}$
 $b\bar{b}u\bar{d}$
 $c\bar{c}uud$
 $c\bar{c}uds$
 $cc\bar{u}\bar{d}$

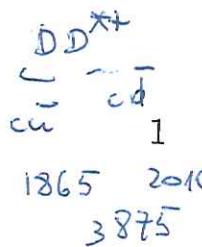
States

2866 2909

- $\rightarrow X_0(2900), X_1(2900)$ [21, 22] $\left. \begin{array}{l} 2007 \\ D^* \\ \end{array} \right\} \begin{array}{l} 1864.7 \\ D^0 \\ \end{array}$ but wider
 $\rightarrow \chi_{c1}(3872)$ [6] threshold ≈ 120 keV
 $\left. \begin{array}{l} Z_c(3900) \\ Z_c(4020) \\ Z_c(4050) \\ Z_c(4200) \\ Z_c(4430) \\ Z_{cs}(3985) \\ Z_{cs}(4000) \\ Z_{cs}(4220) \end{array} \right\}$ [23, 24–25, 26, 27, 28–32, 31, 33] [29–32]
 $\left. \begin{array}{l} \chi_{c1}(4140) \\ X(4630) \\ X(4685) \\ X(4740) \end{array} \right\}$ [35–38], [34], [39] [35–38], [34], [39]
 $X(6900)$ [14]
 $Z_b(10610), Z_b(10650)$ [40]

$P_c(4312)$ [41], $P_c(4380)$ [42], $P_c(4440)$, $P_c(4457)$ [41],
 $P_c(4357)$ [43] 36 \weakend
 $P_{cs}(4459)$ [44]

T_{cc}^\dagger



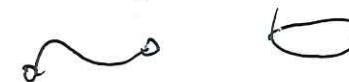
1865 2010
3875

④ Gravity / Gauge duality (holography, AdS/CFT)

In the 70'ties:

QCD
Fundamental theory of quarks & gluons

strings - flux tubes



effective, like LUND model

Maldacena

Gauge theory in 4 dim \simeq String theory in higher dimensions

Witten:

Application of duality to QCD - pure YM in 3+1
at large N and $\lambda = g_{YM}^2 N$

Surprising similarity of spectrum of glueballs to large N lattice

Sakai & Sugimoto 2005

- Adding N_f massless fermions: $SU_L(N_f) \times SU_R(N_f) \rightarrow SU_V(N_f)$ SBKS

- Low energy limit $S = S_{\text{YM}} + S_{\text{CS}}$

$$S_{\text{YM}} = \infty \int d^4x dz \text{Tr} \left(\frac{1}{2} k(z)^{-1/3} F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \quad k(z) = 1 + z^2$$

Mode expansion:

$$A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu) \Psi_n(z)$$

$$A_5(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu) \Phi_n(z).$$

- Keeping only $\varphi^{(0)}$ gives $\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{\text{Skyrme}} + \mathcal{L}_{WZ}$, i.e. Skyrme model
- $B_\mu^{(1)} \sim g$, $B_\mu^{(2)} \sim \omega_1$ give hidden gauge model
- Successful phenomenology with very few parameters.

From S. Sugimoto talk

mass

mass	ρ	a_1	ρ'	(a'_1)	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
our model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

↑
input ($M_{KK} \simeq 949$ MeV)

coupling

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{KK}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{KK}^2$	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{KK}$	4.63 GeV	$2.8 \sim 4.2$ GeV

Baryon in Sakai & Sugimoto

4 dim pion

Skyrmion (static solution)

5 dim gauge field

↓
BPST instanton in x_1, x_2, x_3, z in flavor

Topological number = baryon number

Direct realization of 1989 Atiyah-Manton idea

$$U(\vec{x}) = P \exp \left(i \int dz A_z(\vec{x}, z) \right)$$

8 zero modes ($SU(2), \vec{x}, z, S$) lead to moduli space

quantization

$$M = M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3} (n_g + n_z + 1)} \right) M_{KK}$$

$$l = 2I = 2J = 1, 3, 5 \dots$$

From S. Sugimoto talk

- If we choose $M_{KK} \simeq 500$ MeV and use nucleon mass ($\simeq 940$ MeV) to fix the constant M_0 , (we only consider the mass difference), we obtain

(n_ρ, n_z)	(0, 0)	(1, 0)	(0, 1)	(1, 1)	(2, 0)/(0, 2)	(2, 1)/(0, 3)	(1, 2)/(3, 0)
$N(l=1)$	$[940]^+$	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta(l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

States appeared in the Skyrme model (\pm : parity)

- $I = J$ states from Particle Data Group look like....

(? : not found, * : evidence of existence is poor)

S. Sagimoto talk (Honolulu)

Table 2: $\langle r^2 \rangle_{I=0}$, $\langle r^2 \rangle_{I=1}$ and $\langle r^2 \rangle_A$ are isoscalar, isovector and axial mean square radii, respectively. $g_{I=0}$ and $g_{I=1}$ are isoscalar and isovector g-factors. g_A is the axial coupling.

	our model	experiment
$\langle r^2 \rangle_{I=0}$	$(0.74 \text{ fm})^2$	$(0.81 \text{ fm})^2$
$\langle r^2 \rangle_{I=1}$	$(0.74 \text{ fm})^2$	$(0.94 \text{ fm})^2$
$\langle r^2 \rangle_A$	$(0.54 \text{ fm})^2$	$(0.67 \text{ fm})^2$
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

Holographic H-L effective theory (Liu, Zohreh; 2017)

- "CK"-like scheme based on adding ∞ -heavy flavor ($m_H = \infty$)

$$M = M_0 + (N_Q + N_{\bar{Q}}) m_H + \left[\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2 \left(1 - \frac{15(N_Q - N_{\bar{Q}})}{4N_c} + \frac{5(N_Q - N_{\bar{Q}})^2}{3N_c^2} \right)^2 \right]^{1/2} M_{KK} + \sqrt{\frac{2}{3}(n_f + n_z + 1)} M_{KK}$$

- Various combinations of q -numbers give all types of H-L hadrons.

$N_Q = 1 \quad N_{\bar{Q}} = 0$ baryons HLL

$N_Q = 2 \quad N_{\bar{Q}} = 0$ - - - HHL

$N_Q = N_{\bar{Q}} = 1$ Pentaquarks $H\bar{H}LLL$

$n_z \neq 0$ excited, $n_g \neq 0$ odd parity - -

- Three parameters: M_{KK}, M_0, m_H

Adding spin-effects (subleading terms in m_H^{-1})

Liu, Mamo, MANI Zahed (2021)

- 3 parameters $M_0 \rightarrow m_N$, $M_{K\bar{K}} \rightarrow m_{\Lambda_c}$, $m_n = M_D$ for c
 $m_H = M_B$ for b
- 3 Pentaquarks $\frac{1}{2} \frac{1}{2}^-$, $\frac{1}{2} \frac{1}{2}^-$, $\frac{1}{2} \frac{3}{2}^-$ Ξ_c^{\pm}
 $s=1$ $s=0$ $s=1$
 $\left(\frac{1}{2} \frac{5}{2}^+ \text{ ruled out} \right)$, consistent with $P_c(4312, 4440, 4457)$ (LHCb)
- Recently reported $P_c(4337)$ at 3 σ significance is not supported
- Open and hidden decay widths Liu, Mamo, MANI Zahed (2021) b
e.g. $P_c \rightarrow \Lambda_c + \bar{D}$ $\Gamma(s=0, J=\frac{1}{2}) : \Gamma(s=1, J=\frac{1}{2}) : \Gamma(s=1, J=\frac{3}{2}) = \frac{1}{2} : \frac{5}{6} : \frac{1}{3}$
- Formfactors (Liu, Mamo, MANI Zahed; 2022), consistent with recent GLUEX result.
 $r_p \rightarrow \underbrace{\gamma/\epsilon}_{P_c^+} p$

TABLE I. Charm baryons and Pentaquarks

B	IJ^P	l	n_ρ	n_z	N_Q	$N_{\bar{Q}}$	Mass-MeV	Exp-MeV
Λ_c	$0\frac{1}{2}^+$	0	0	0	1	0	2286	2286
Σ_c	$1\frac{1}{2}^+$	2	0	0	1	0	2557	2453
	$1\frac{3}{2}^+$	2	0	0	1	0	2596	2520
Λ_c^*	$0\frac{1}{2}^-$	0	0	1	1	0	2683	2595
	$0\frac{1}{2}^+$	0	1	0	1	0	2726	2765
Σ_c^*	$1\frac{1}{2}^-$, $1\frac{3}{2}^-$	2	0	1	1	0	[2947/2986]	–
	$1\frac{1}{2}^+$, $1\frac{3}{2}^+$	2	1	0	1	0	[2948/2995]	–
P_c	$\frac{1}{2}\frac{1}{2}^-$, $\frac{1}{2}\frac{3}{2}^-$	1	0	0	1	1	[4340/4360/4374] [4312/4440/4457]	
P_c^*	$1\frac{1}{2}^-$, $1\frac{3}{2}^-$	1	0	1	1	1	[4732/4752/4767]	–
	$1\frac{1}{2}^+$, $1\frac{3}{2}^+$	1	1	0	1	1	[4725/4746/4763]	–

To fix the parameters for the charmed heavy baryons, we choose $M_D = 1.87$ GeV for the D-meson mass in (7) and fix $M_{KK} = 0.475$ GeV to reproduce the $M_{\Lambda_c} = 2.286$ GeV. This low value of M_{KK} is consistent with the value used to reproduce the nucleon

TABLE II. Bottom baryons and Pentaquarks

B	IJ^P	l	n_ρ	n_z	N_Q	$N_{\bar{Q}}$	Mass-MeV	Exp-MeV
Λ_b	$0\frac{1}{2}^+$	0	0	0	1	0	5608	5620
Σ_b	$1\frac{1}{2}^+$	2	0	0	1	0	5962	5810
	$1\frac{3}{2}^+$	2	0	0	1	0	5978	5830
Λ_b^*	$0\frac{1}{2}^-$	0	0	1	1	0	5998	5912
	$0\frac{1}{2}^+$	0	1	0	1	0	6029	(6072)
Σ_b^*	$1\frac{1}{2}^-$, $1\frac{3}{2}^-$	2	0	1	1	0	[6351/6367]	–
	$1\frac{1}{2}^+$, $1\frac{3}{2}^+$	2	1	0	1	0	[6344/6367]	–
P_b	$\frac{1}{2}\frac{1}{2}^-$, $\frac{1}{2}\frac{3}{2}^-$	1	0	0	1	1	[11155/11163/11167]	–
P_b^*	$1\frac{1}{2}^-$, $1\frac{3}{2}^-$	1	0	1	1	1	[11544/11553/11556]	–
	$1\frac{1}{2}^+$, $1\frac{3}{2}^+$	1	1	0	1	1	[11532 /11543/11579]	–

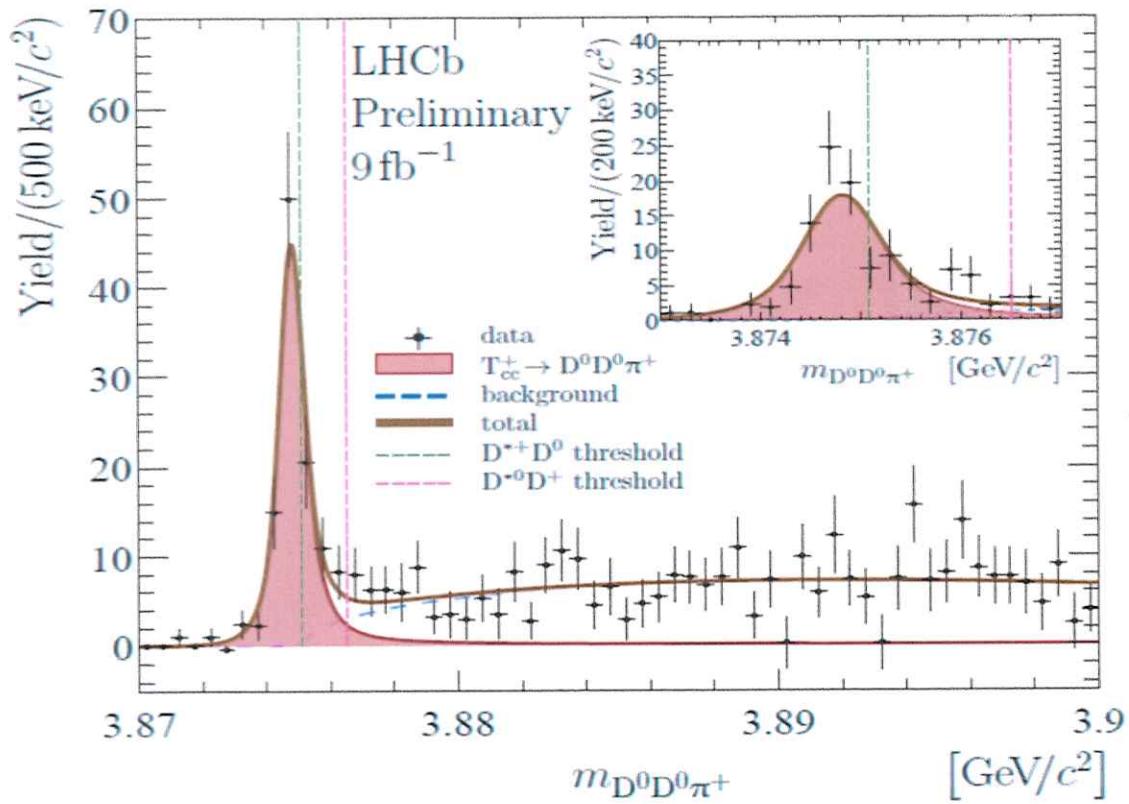


FIG. 1. Newly measured isoscalar charmed T_{cc}^+ tetraquark by LHCb [48].

Tetraquark puzzle

- $\bar{f}H\bar{L}\bar{L}$ — several predictions (positive/negative within $\pm 200 \text{ MeV}$)
- Measurement of $\Xi_{cc}^{++}(3621)$ fixed the normalization for $\bar{b}\bar{b}\bar{u}\bar{d}$ tetraquarks.
(Kerlinker, Rosner 2017, Eichten-Quigg (2017)), bounded up to 200 MeV (!).
- In holographic picture, role of the instanton (baryons, pentaquarks) is played by sphaleron (Baryon # = 0), binding two heavy mesons.
(Liu, MAN, Zahed; 2019) $\bar{b}\bar{b}\bar{q}\bar{q}$ bound at 80 MeV , $\bar{c}\bar{c}\bar{q}\bar{q}$ bound by 90 MeV
- $T_{cc}^+(01^+)$, narrow, discovered at LHCb, bounded at -360 keV , $\Gamma \sim 50 \text{ keV}$
- Normalizing mass to T_{cc}^+ , we (Liu, MAN, Zahed, 2022) predict mass of T_{bc} and T_{bb} , and calculate very narrow width. ($m_H = \infty$)

TABLE I. Binding energies for tetraquarks versus the 't Hooft coupling $\lambda = g_{\text{YM}}^2 N_c$ with $M_{\text{KK}} = 1 \text{ GeV}$

λ	$QQ\bar{q}\bar{q}$ GeV	$bb\bar{q}\bar{q}$ GeV	$bc\bar{q}\bar{q}$ GeV	$cc\bar{q}\bar{q}$ GeV
10	-0.097	-0.088	-0.080	-0.072
15	-0.107	-0.091	-0.077	-0.062
20	-0.108	-0.085	-0.064	-0.041
25	-0.103	-0.073	-0.045	-0.018
30	-0.093	-0.056	-0.024	-0.0016
32	-0.089	-0.048	-0.015	0.00073

TABLE II. Binding energies for tetraquarks versus the 't Hooft coupling $\lambda = g_{\text{YM}}^2 N_c$ with $M_{\text{KK}} = 0.475 \text{ GeV}$.

λ	$QQ\bar{q}\bar{q}$ GeV	$bb\bar{q}\bar{q}$ GeV	$bc\bar{q}\bar{q}$ GeV	$cc\bar{q}\bar{q}$ GeV
10	-0.046	-0.044	-0.042	-0.040
15	-0.051	-0.047	-0.044	-0.040
20	-0.051	-0.046	-0.040	-0.035
25	-0.049	-0.042	-0.035	-0.028
30	-0.045	-0.035	-0.027	-0.019
40	-0.031	-0.018	-0.0076	0.0011

Summary:

- Strongly coupled QCD could be approached via duality from string theory in large N_c and large λ limit, including spectra of heavy-light hadrons
- Few parameters and very restrictive predictions, so models are confutable
- Approach based on confinement, SB χ S and heavy spin symmetry, in the limit of large N_c and large 't Hooft coupling.
- "High brow" theory boils down to relatively simple QM in moduli space.
- Astonishing and deep analogies to "old" physics.

① our table

sofa - sofa

photo

