Rapidity space entanglement and high energy process

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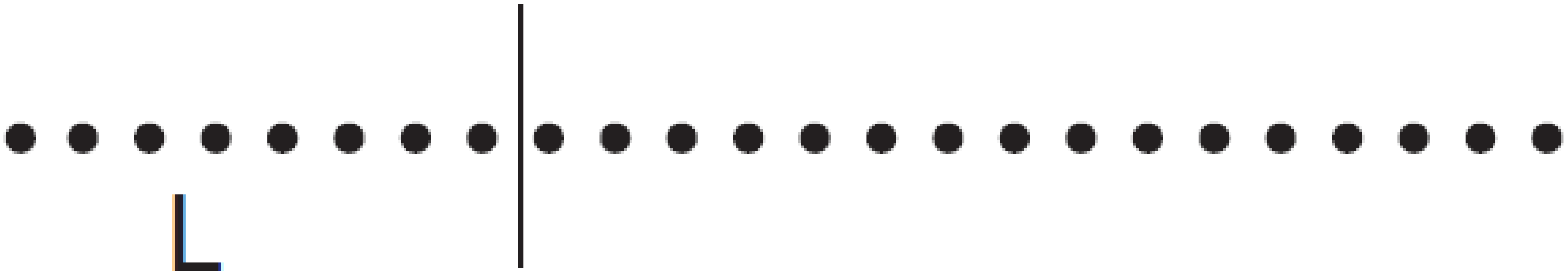
- **The talk is based on recent work 2203.00739 and 2202.02612.**

Outline

- Introduction to Quantum Entanglement
- High-energy process: large rapidity limit
- Rapidity space entanglement : sub-critical system
- Rapidity space entanglement : critical system and rapidity evolution
- Rapidity space entanglement: string picture
- Conclusion

Introduction to Quantum Entanglement

- $\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_1$, full density matrix ρ.
- Reduced density matrix : $\rho_0 = tr_1 \rho$.
- Entanglement entropy: $S = -tr \rho_0 \ln \rho_0$. Roughly, about
    1. How measurement in 0 affects 1.
    2. How generic the state is.
    3. How many channels between subsystems.
- Reflects deep laws in quantum many body system, in particular, QFT.



Introduction to Quantum entanglement

- The real space entanglement focuses on a segment in position space with length L.

Introduction to Quantum Entanglement

- Example: real space entanglement in **1D.**

1. Generic state: $S = \mathcal{O}(L) = \ln \dim \mathcal{H}$. All 2^L sates contribute.
2. Short range interaction: Area law, $S \leq \mathcal{O}(\ln \mathrm{L})$.

- Gapless system, $\mathrm{c(L)} = L\frac{dS}{dL} = \mathrm{c} + \mathcal{O}(\frac{1}{\mathrm{L}})$.
- Gapped system, $\mathrm{c(L)} = L\frac{dS}{dL} = \mathcal{O}(e^{-mL})$ at large distance.

Introduction to Quantum Entanglement

- Example: real space entanglement in 1D.

3. Non-trivial even for free-system.

- Macroscopic: replica trick, CFT based methods.
- Microscopic: Fisher-Hartwig conjecture.

  Open questions remain.

4. Relation to matrix-product states, entanglement renormalization group and string duality.

High-energy process: large rapidity limit

1. High-energy experiment: presence of a large rapidity gap Y.

- DIS at small-x : $Y = \ln\frac{1}{x}$.
- Forward scattering with $s \gg -t, Y = \ln\frac{s}{m^2}$.

3. Very non-trivial asymptotic behavior in Y.

- Perturbative evolution equations and saturation conjecture.
- Insights from String-Gauge duality.

High-energy process: large rapidity limit

4. Nontrivial “conspiracy” between fast and slow degrees freedom.
5. How many entanglement between fast and slow degrees of freedom?

Rapidity space entanglement : sub-critical system

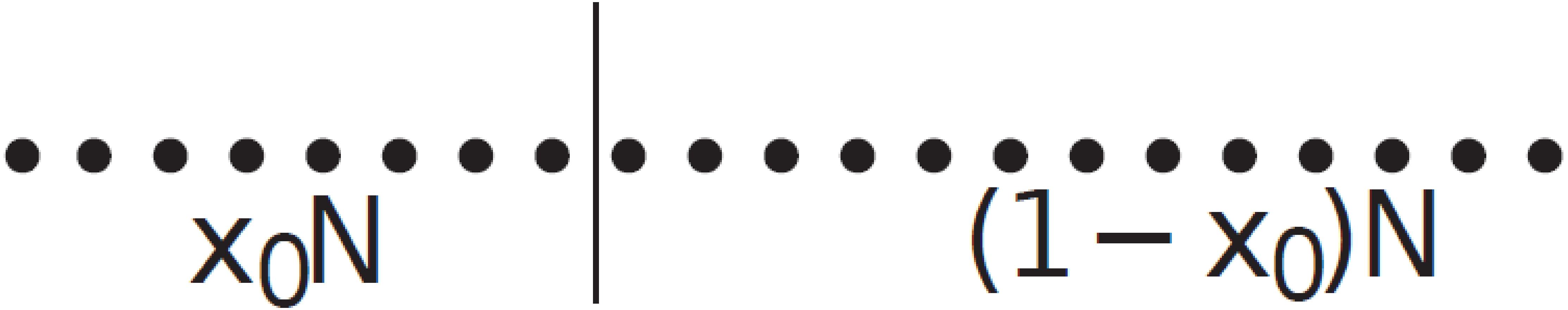
- Meson state in the 2D QCD in large N_c limit.

1. Light-front wave function: $|m\rangle = \frac{1}{\sqrt{N}}\sum\phi(x)|x,\bar{x}\rangle$.

- $0 < x < 1$: longitudinal momentum fraction.
- N: total number of digits.

2. Rapidity space entanglement between sub-systems:

$$[0,1]\mathrm{N} = [0, x_0]N \cup [x_0, 1]N.$$



Rapidity space entanglement : sub-critical system

- In the light-front formulation of QFT, there are N total digits in longitudinal momentum fraction. We consider entanglement between the first $x_0 N$ and the rest.

Rapidity space entanglement : sub-critical system

3. $S(x_0) = c(x_0) \ln x_0 N + b(x_0)$. Area law satisfied.
4. $c(x_0)$ and $b(x_0)$ expressed in terms of **quark parton distribution functions (PDFs)**.
5. For small x_0, $c(x_0) \sim x_0^{2\beta+1}$ and $b(x_0) \sim x_0^{2\beta+1} \ln \frac{1}{x_0}$

- The same asymptotic coefficients for forward scattering: $A(s) \sim s^{-2\beta}$.

Rapidity space entanglement : sub-critical system

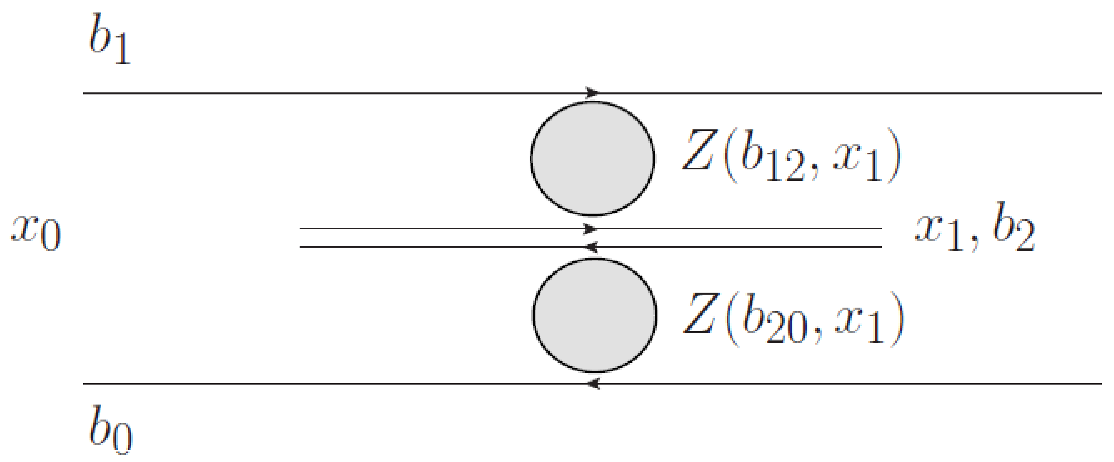
- $S(x_0) = c(x_0)\ln x_0 N + b(x_0)$.

1. The "c function": $c(x_0) = \int_0^{x_0} q(x) + \bar{q}(x)\, dx \leq 1$.
2. Expected to generalized to multi-parton wave functions in sub-critical system.

Rapidity space entanglement : critical system and rapidity evolution

- In QCD, a famous example is the soft gluon wave function of a quarkonium system.

1. Emission of small-x gluon generates rapidity divergences in light-front wave functions and their norm squares.
2. Leading divergences resumed into closed equation in planar limits.
3. Generates BFKL, BK-like equations in various limits/approximations.



$$Z(b_{10}, x_0, x_{\rm min}, u) = S(b_{10}, \frac{x_0}{x_{\rm min}}) + \frac{\alpha_s C_F}{\pi^2} \int_{x_{\rm min}}^{x_0} \frac{dx_1}{x_1} S(b_{10}, \frac{x_0}{x_1}) \int db_2^2 \frac{b_{10}^2}{b_{12}^2 b_{20}^2} u(x_1, b_2) Z(b_{12}, x_1, x_{\rm min}, u) Z(b_{20}, x_1, x_{\rm min}, u) \; .$$

Rapidity space entanglement : critical system and rapidity evolution

- Emission of the hardest soft gluon splits the original dipole into two dipoles in which softer gluons emit independently.

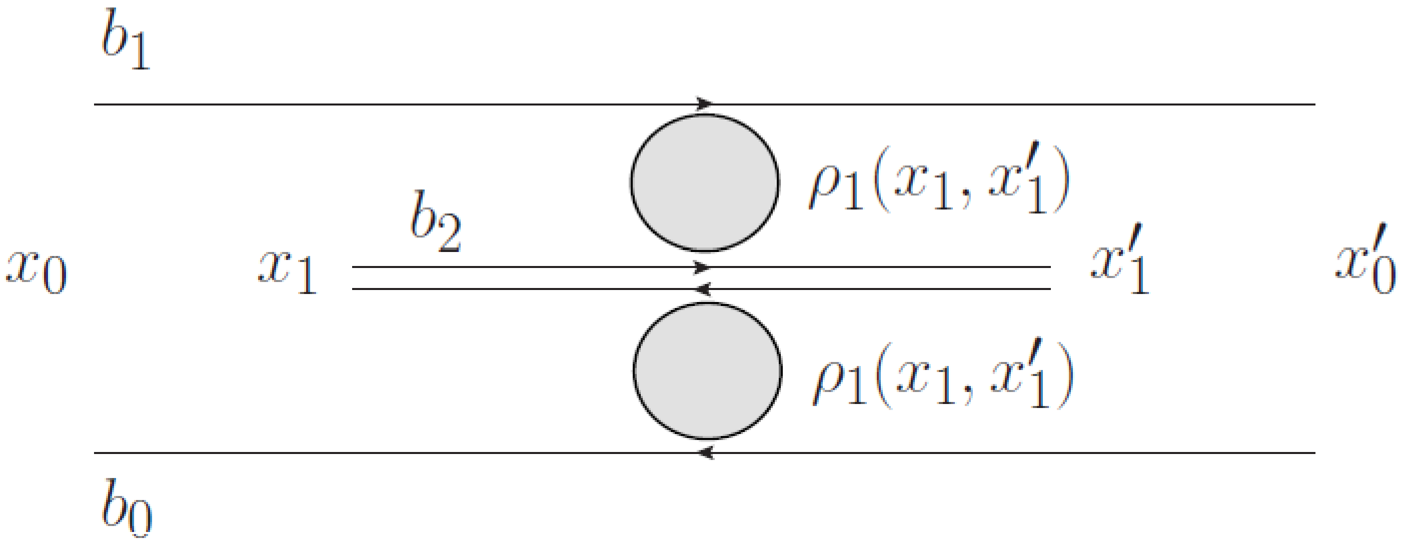
Rapidity space entanglement : critical system and rapidity evolution

- Entanglement in Mueller's dipole:

1. At order α_s, rapidity divergence leads to enhanced logarithmic behavior: $S \sim \alpha_s \ln^2 N + \alpha_s \ln N$.
2. Generally, to order k : $S \sim \alpha_s^k \ln^{k+1} N + \alpha_s^k \ln^k N$.
3. Needs re-summation.
4. Evolution equation for reduced density matrix can be written out.

Rapidity space entanglement : critical system and rapidity evolution

$$\rho_1(b_{10}, x_0, x_0', x_{\min}, u) = S^{\frac{1}{2}}\left(b_{10}, \frac{x_0 x_0'}{x_{\min}^2}\right)|0\rangle\langle 0|$$
$$+\frac{\alpha_s C_F}{\pi^2}\int db_2^2 \frac{b_{10}^2}{b_{12}^2 b_{20}^2} \sum_{x_{\min}\leq x_1, x_1' \leq x_0} S^{\frac{1}{2}}\left(b_{10}, \frac{x_0 x_0'}{x_1 x_1'}\right)\frac{|x_1\rangle\langle x_1'|}{\sqrt{x_1 x_1'}} \otimes \rho_1(b_{12}, x_1, x_1', x_{\min}) \otimes \rho_1(b_{20}, x_1, x_1', x_{\min}) \ .$$



Rapidity space entanglement : critical system and rapidity evolution

- A 1D toy model .

$$Z(y,u) = e^{-ay} + aue^{-ay}\int_0^y e^{ay_1} dy_1 Z(y_1,u) Z(y_1,u) \ .$$

1. $Z(y,u) = \sum_{n=0}^{\infty} u^n p_n(y)$
2. Probability of finding n soft gluon: $p_n(y) = e^{-ay}(1-e^{-ay})^n$.
3. At large $y \sim \ln x_0 N$, a very wide width $\sim e^{ay}$ of $p_n(y)$.

Rapidity space entanglement : critical system and rapidity evolution

- $p_n(y) = e^{-ay}(1 - e^{-ay})^n$

4. $\langle n \rangle \sim e^{ay}$: exponential in rapidity gap.
5. $S(y) - y \sim \ln\langle n \rangle = a \ln x_0 N$. Linear but enhanced.
6. $S(y) - y = \ln\langle n \rangle$, the total dipole number $\langle n \rangle$ probed in inclusive process.

Rapidity space entanglement : critical system and rapidity evolution

- Another interesting example: 1+2D QCD

$$Z(b,y,u) = e^{-mby} + um\int_0^y dy_1 e^{-mb(y-y_1)}\int_0^b db' Z(b-b',y_1,u)Z(b',y_1,u) \ ,$$

1. Phase-space constraint: emitted soft gluon must be within the original dipole!
2. Unique to 1+2D. One dimensional transverse direction.

$$\int_{-\infty}^{\infty}\frac{dk^z}{(2\pi)}\frac{e^{ik^z b}}{k^z} = \frac{i}{\pi}\int_0^{\infty}\frac{dk^z}{k^z}\sin(k^z b) = \frac{i}{2}\text{sign}(b) \ ,$$

Rapidity space entanglement : critical system and rapidity evolution

3. Distribution of soft gluons is Poissonian: $p_n = e^{-mby}\frac{(mby)^n}{n!}$.

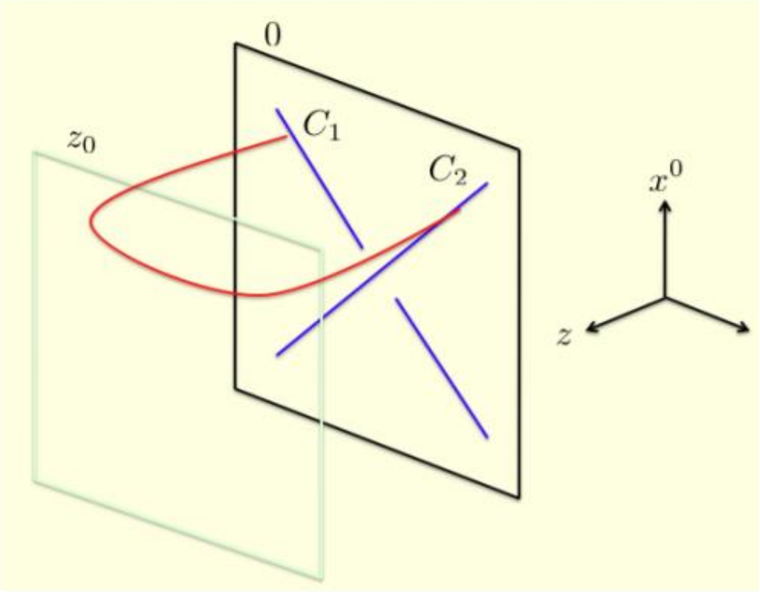
- Peak at $\langle n\rangle = mby$. Linear instead of exponential.
- Much narrower width: $\langle \delta n^2\rangle^{\frac{1}{2}} \sim \sqrt{mby}$.

4. The entropy $S - y = \frac{1}{2}\ln(2\pi e\, mby)$.
5. Quenching of phase space, "Kinematic saturation".

Rapidity space entanglement : string picture

- In the string picture, parton-parton scattering depicted by exchange of a minimal surface.

1. World-sheet instanton and thermal entropy S_T.
2. Quantum entropy $S_E = S_T = \ln \langle N \rangle$.
3. $S_E = \frac{1}{6}\frac{\langle \delta b_\perp^2 \rangle}{l_S^2}$ like Bekenstein-Hawkins black-hole.

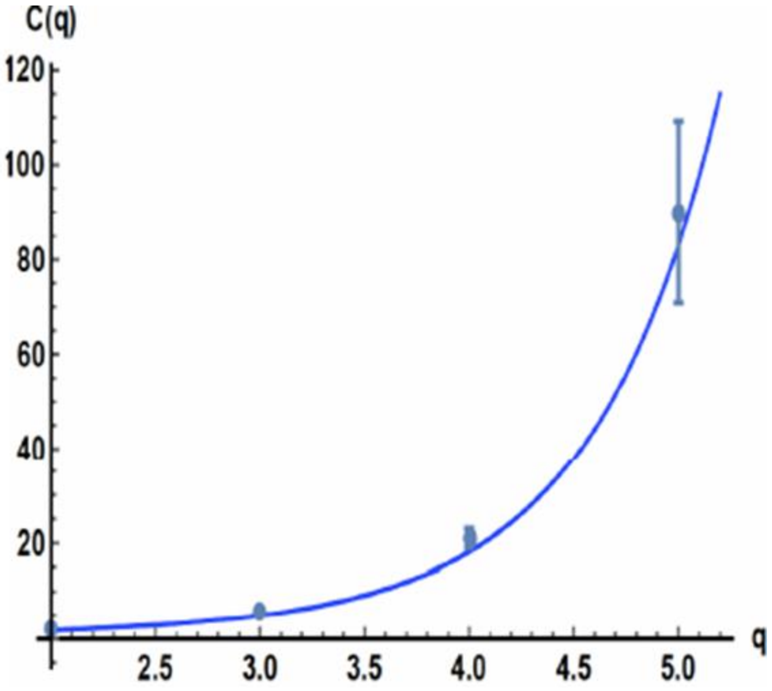


Rapidity space entanglement : string picture

- $\frac{dS_E}{dy} \leq \frac{D_\perp}{6}$: chaos bound saturated.
- Cascade equation for particle multiplicities.

$$p_n(D_\perp) = \frac{(n + D_\perp - 1)!}{n! D_\perp!} e^{-\frac{D_\perp}{6} y} \left(1 - e^{-\frac{1}{6} y}\right)^n ,$$

$$C(q) = \frac{\langle n^q \rangle}{\langle n \rangle^q} = \frac{1}{\langle n \rangle^q (\langle n \rangle - 1)} \mathrm{PolyLog}\left(- q, 1 - \frac{1}{\langle n \rangle}\right) ,$$



Conclusion

- Rapidity space entanglement as a probe of light-front limit of QFT.
- Subcritical system: $S(y) = cy + b$ with $c < 1$.
- Critical system: y^n in perturbative expansion. $S(y) - y = S_1(y)$ with $S_1(y)$ growth with y.
- $1D$ reduction: $S_1(y) = ay$. Expected for 4D QCD. Consistent with the string picture.
- Kinematic saturation in 1+2 QCD. $S_1(y) \sim \frac{1}{2} \ln y$.
- Measured by particle multiplicity.