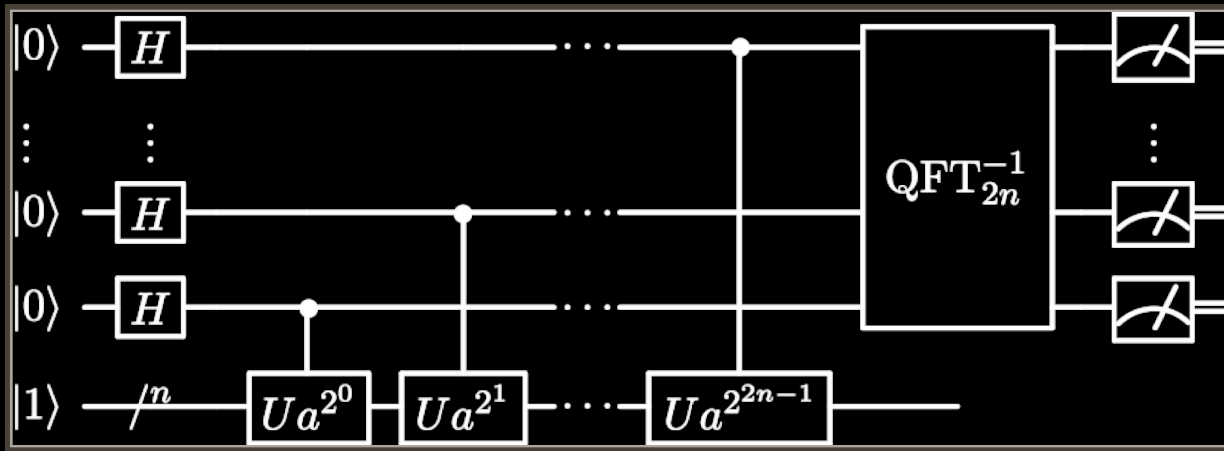


Wstęp do obliczeń kwantowych

Paweł Szumniak

Wydział Fizyki i Informatyki Stosowanej Akademia Górniczo-Hutnicza w Krakowie

Fundusze Europejskie Inteligentny Rozwój
Rzeczpospolita Polska
FNP Fundacja na rzecz Nauki Polskiej
Unia Europejska Europejski Fundusz Rozwoju Regionalnego

Historia

- **1970s quantum information theory**
- Alexander Holevo (n>n) , Charles H. Bennett (reversibly), R. P. Poplavskii (infeasibility)
- **Stanisław Ingarden** publishes a seminal paper entitled
  "Quantum Information Theory" Reports on Mathematical Physics, 10, 43–72, **1976**.
- **1980s**
- **Paul Benioff** - describes quantum mechanical Hamiltonian models of computers
- **Yuri Manin** briefly motivates the idea of quantum computing

- **1981**
  Richard Feynman "*it appeared to be impossible in general to simulate an evolution of a quantum system on a classical computer in an efficient way*". He proposes a **basic model for a quantum computer** that would be capable of such simulations

- **1982** Wootters, William; **Zurek, Wojciech** (**1982**). "A Single Quantum Cannot be Cloned". Nature. 299 (5886): 802–803.

- **1985 David Deutsch**, describes the **first universal quantum computer**.

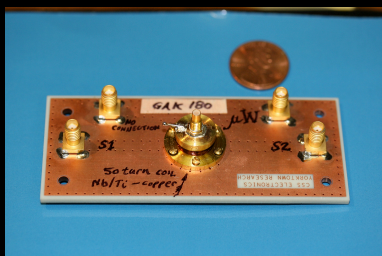
- **1994 Peter Shor**, Bell Labs, discovers an very important algorithm that allows a quantum computer to factor large integers quickly. It solves both the factoring problem and the discrete log problem. Shor's algorithm can theoretically break many of the cryptosystems in use today. Its invention sparked a tremendous interest in quantum computers.

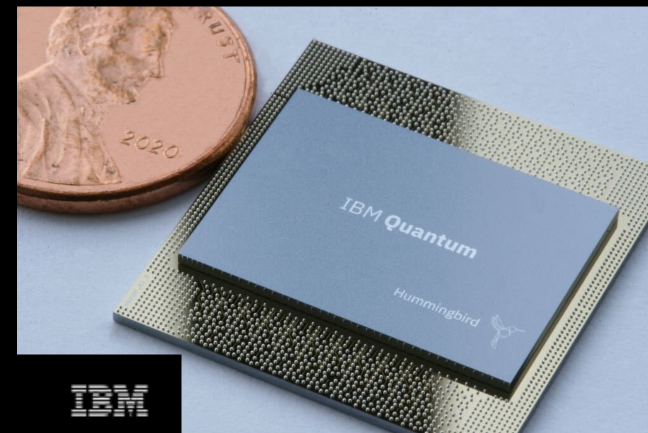
https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication

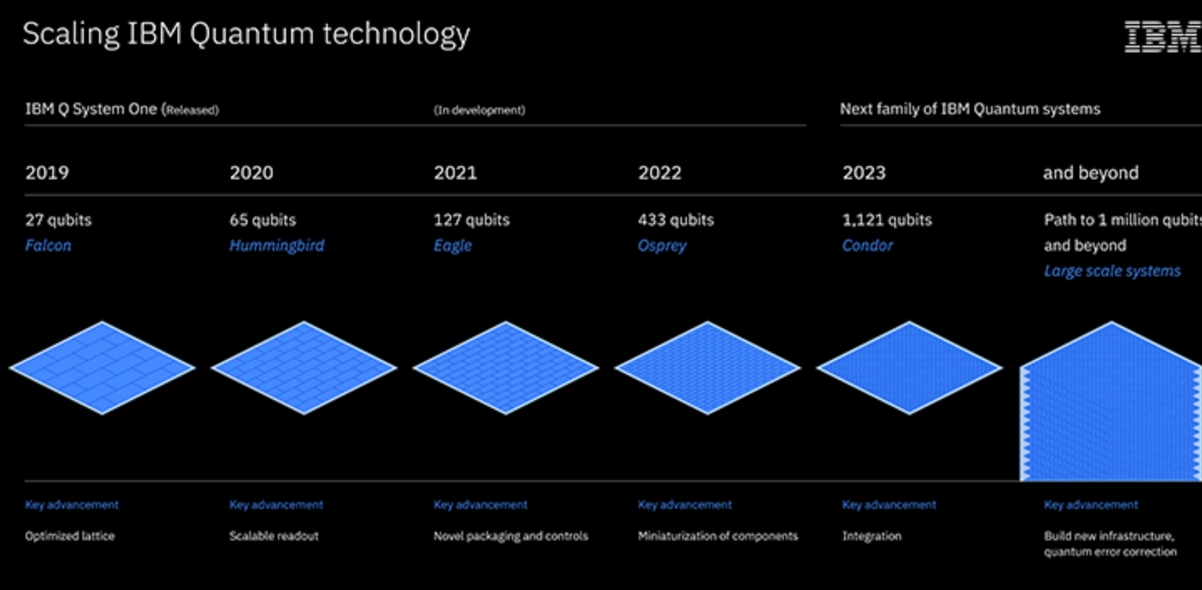
Komercyjny komputer kwantowy IBM 2019

https://newsroom.ibm.com/2019-01-08-IBM-Unveils-Worlds-First-Integrated-Quantum-Computing-System-for-Commercial-Use

IBM roadmap







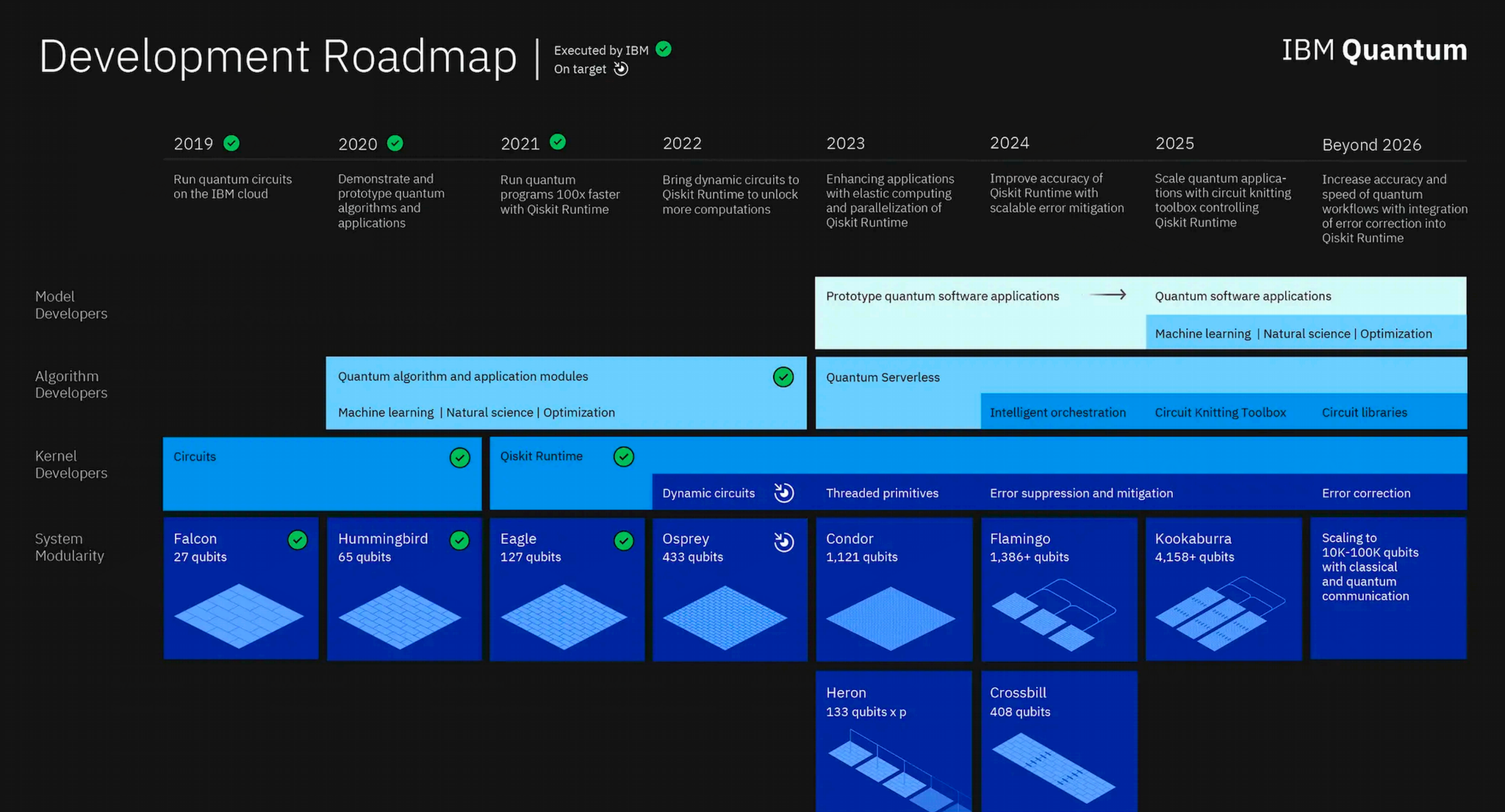
$$10^3 - 10^4$$

$$10^6 - 10^8$$

20 milion qubits to break RSA 2048, for gate error 10^{-3}

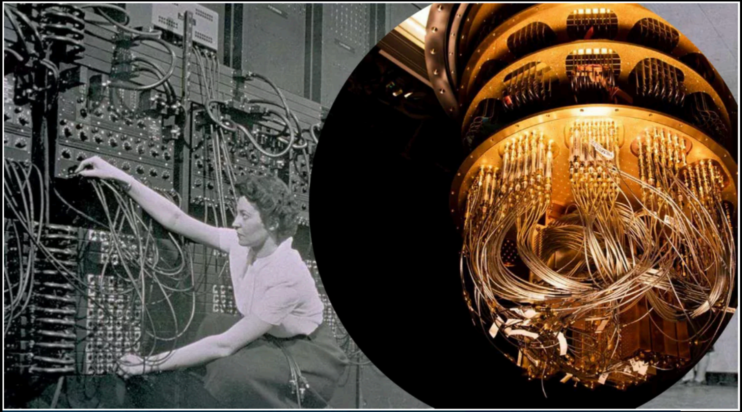
https://research.ibm.com/blog/ibm-quantum-roadmap

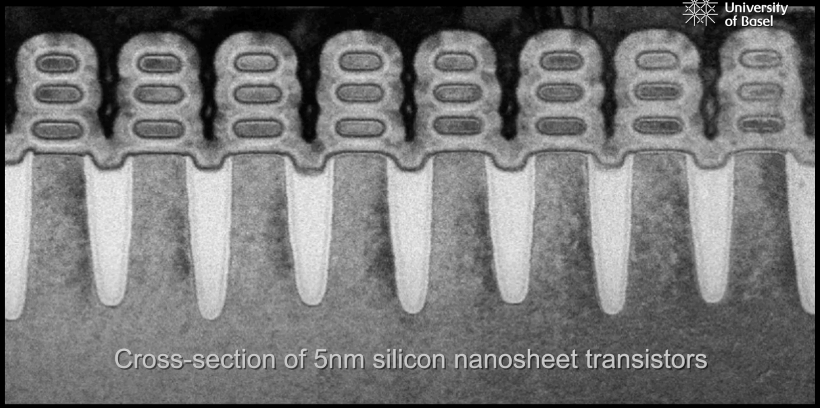
IBM roadmap

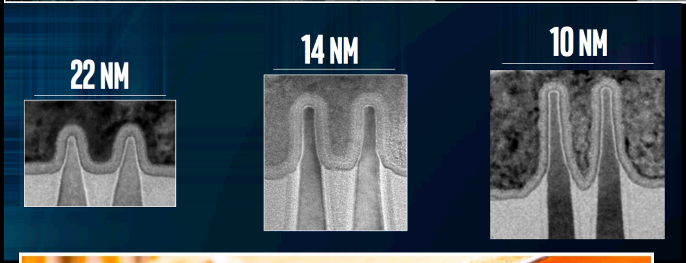


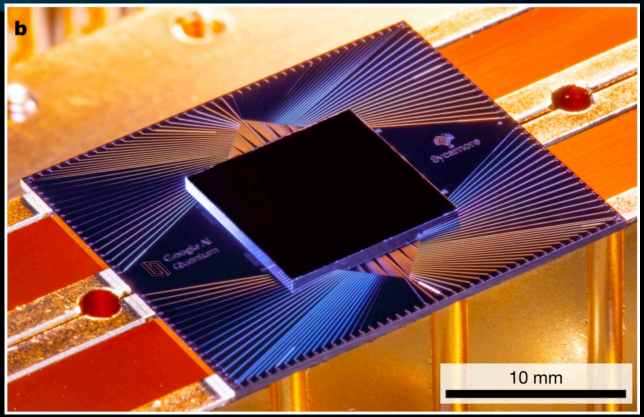
https://research.ibm.com/blog/ibm-quantum-roadmap-2025

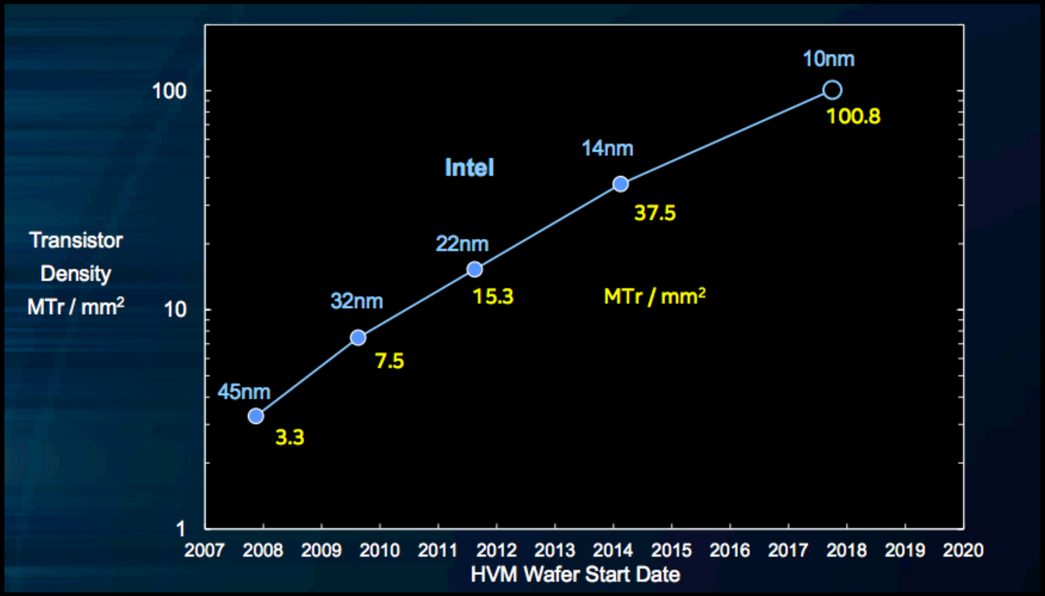
Technologia: komputery kwantowe vs klasyczne problem skalowalności





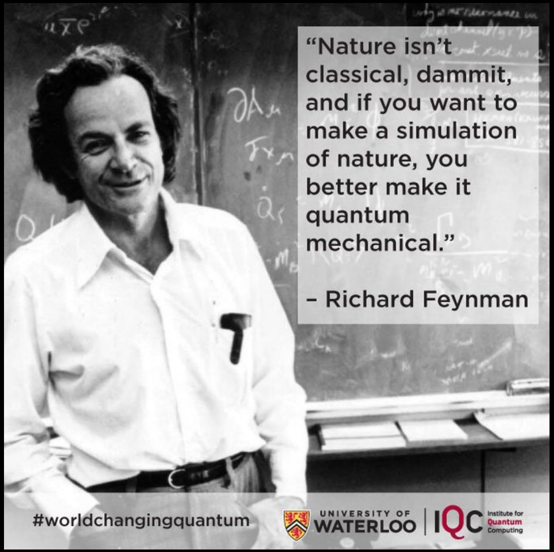






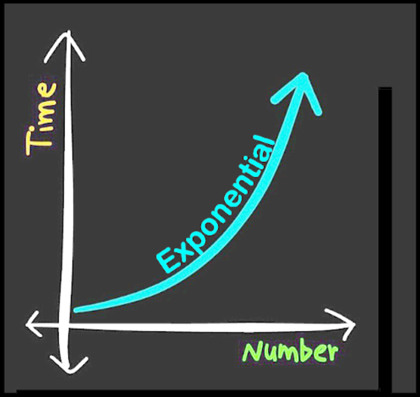
Motywacja

- Symulacja układów (1981 r.) kwantowych (Feynman)

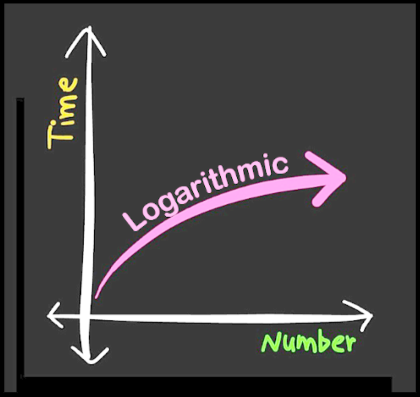


Richard Feynman

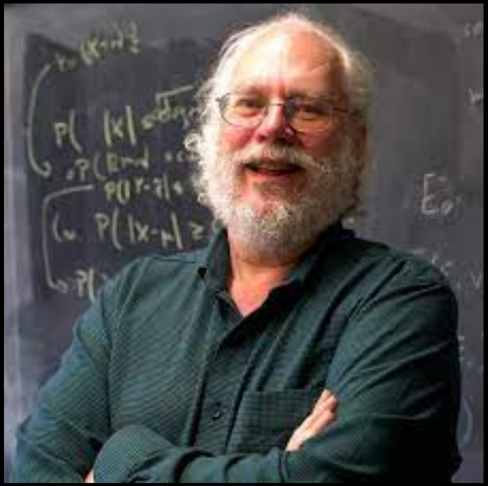
- Łamanie kryptosystemu RSA
  faktoryzacja liczb pierwszych (QFT) (1994 r.)



CLASSICAL



QUANTUM



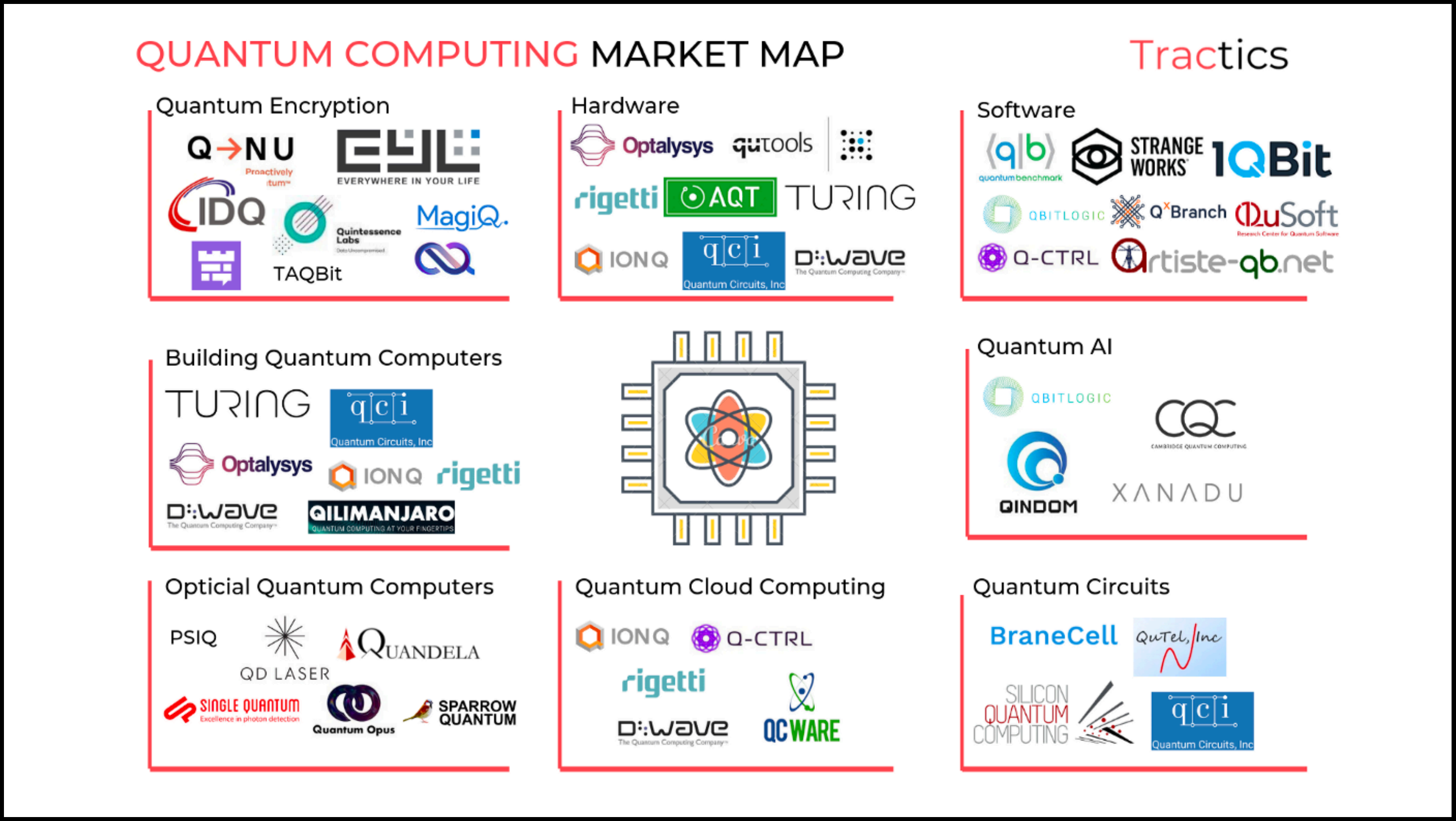
Peter Shor
https://youtu.be/dONacVnW1Ng

Informatyka kwantowa

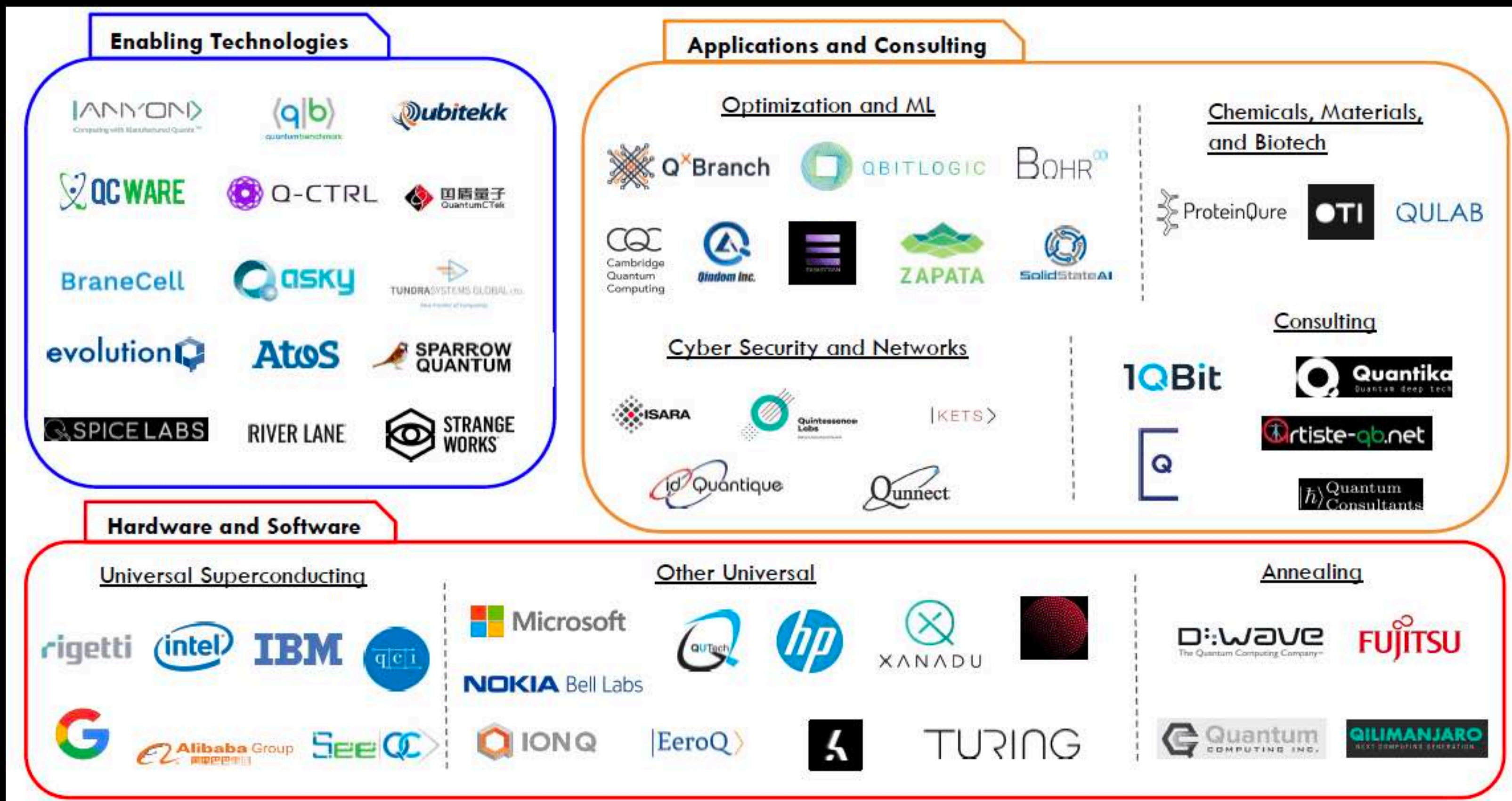
- Symulacje kwantowe
- Obliczenia kwantowe
- Kryptografia kwantowa
- Kwantowa metrologia
- Sieci kwantowe i komunikacja kwantowa (Kwantowy Internet)
- Informacja kwantowa – zagadnienia fundamentalne

John Preskill: Ph CS 219A Lecture 1 Introduction (https://youtu.be/w08pSFsAZvE)

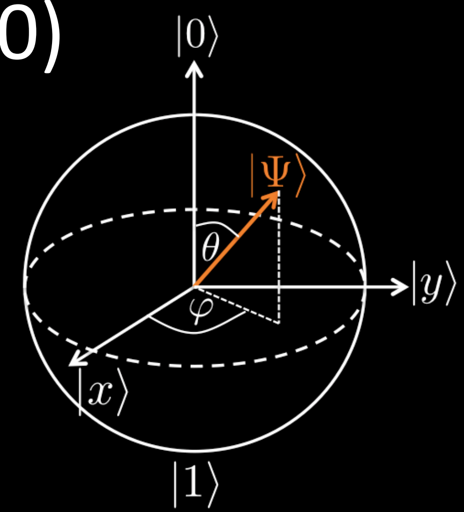
Industry


QUANTUM COMPUTING MARKET MAP
Tractics
Quantum Encryption
Q→NU
Proactively quantum
EYL
EVERYWHERE IN YOUR LIFE
IDQ
Quintessence Labs
MagiQ.
TAQBit
Hardware
Optalysys
qutools
rigetti
AQT
TURING
IONQ
qci
Quantum Circuits, Inc
D-Wave
The Quantum Computing Company
Software
qb
quantum benchmark
STRANGE WORKS
1QBit
QBITLOGIC
Q^x Branch
QuSoft
Q-CTRL
artiste-qb.net
Building Quantum Computers
TURING
qci
Quantum Circuits, Inc
Optalysys
IONQ
rigetti
D-Wave
The Quantum Computing Company
QILIMANJARO
QUANTUM COMPUTING AT YOUR FINGERTIPS
Quantum AI
QBITLOGIC
CQC
CAMBRIDGE QUANTUM COMPUTING
QINDOM
XANADU
Optical Quantum Computers
PSIQ
QD LASER
QUANDELA
SINGLE QUANTUM
Excellence in photon detection
Quantum Opus
SPARROW QUANTUM
Quantum Cloud Computing
IONQ
Q-CTRL
rigetti
D-Wave
The Quantum Computing Company
QC WARE
Quantum Circuits
BraneCell
QuTel, Inc
SILICON QUANTUM COMPUTING
qci
Quantum Circuits, Inc

Industry


Enabling Technologies
IANYOND
(q|b)
Qubitekk
QC WARE
Q-CTRL
BraneCell
asky
evolutionQ
AtoS
SPARROW QUANTUM
SPICE LABS
RIVER LANE
STRANGE WORKS
Applications and Consulting
Optimization and ML
Q^x Branch
QBITLOGIC
BOHR
Cambridge Quantum Computing
Qindom Inc.
ZAPATA
SolidStateAI
Chemicals, Materials, and Biotech
ProteinQure
OTI
QULAB
Cyber Security and Networks
ISARA
|KETS⟩
id Quantique
Qunnect
Consulting
1QBit
Quantika
Artiste-qb.net
Q
Quantum Consultants
Hardware and Software
Universal Superconducting
rigetti
intel
IBM
qci
Alibaba Group
SeeQC
Other Universal
Microsoft
NOKIA Bell Labs
IONQ
|EeroQ⟩
XANADU
TURING
Annealing
D-Wave
The Quantum Computing Company
FUJITSU
Quantum Computing Inc.
QILIMANJARO

Kryteria fizycznej implementacji komputerów kwantowych: DiVicenzo (2000)



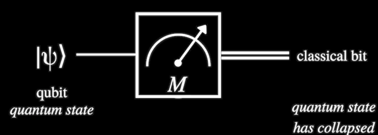
1. **Skalowalny** układ fizyczny z dobrze zdefiniowanymi kubitami

2. Możliwość ustawiania stanów kubitów na początku algorytmu kwantowego w precyzyjnym stanie (**inicjalizacja**) $|0\rangle^{\otimes N} = |0\ldots0\rangle$

3. **Długie czasy koherencji**
(w porównaniu z czasami działa bramek kwantowych)

4. Uniwersalny zestaw **bramek kwantowych** $\hat{U}(t,t_0) = \exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} dt'\,\hat{H}(t')\right]$

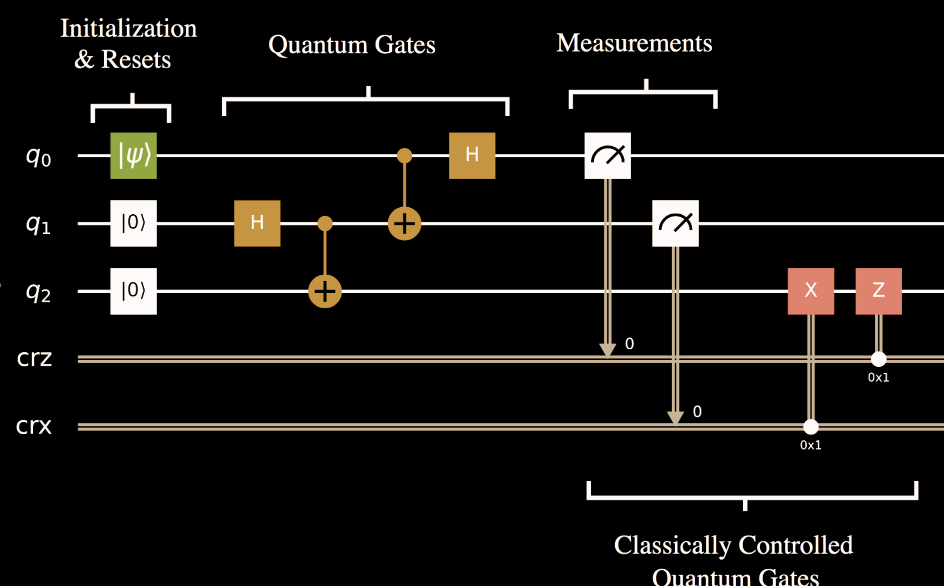
5. Musi istnieć wydajna procedura służąca do **pomiaru** stanu kubitów na końcu realizacji zadanego algorytmu kwantowego



Pozostałe dwa kryteria konieczne do realizacji kwantowej komunikacji

1. Możliwość konwertowania informacji kwantowej ze stacjonarnych kubitów do mobilnych kubitów

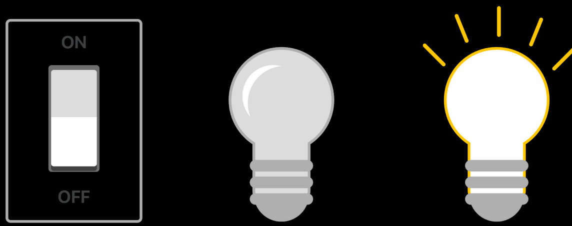
2. Możliwość przesyłania mobilnych kubitów między zadanymi miejscami



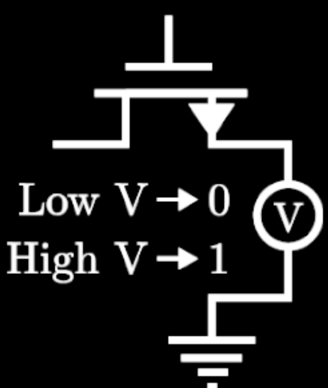
Quantum

Jednostka informacji kwantowej – Bit kwantowy

Bit klasyczny



0 1



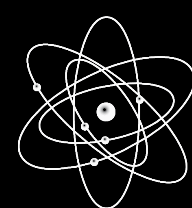
Bit kwantowy - kubit

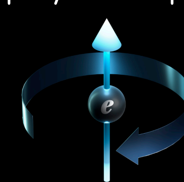
$$|\Psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$\alpha_0, \alpha_1 \in \mathbb{C} \qquad |\alpha_0|^2 + |\alpha_1|^2 = 1$$

$$|\Psi\rangle \sim e^{i\varphi}|\Psi\rangle \qquad \hat{H}|\Psi\rangle = E|\Psi\rangle$$

$$E_1 \quad |1\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$





$$E_0 \quad |0\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

N vs 2^N

Fizyczna implementacja

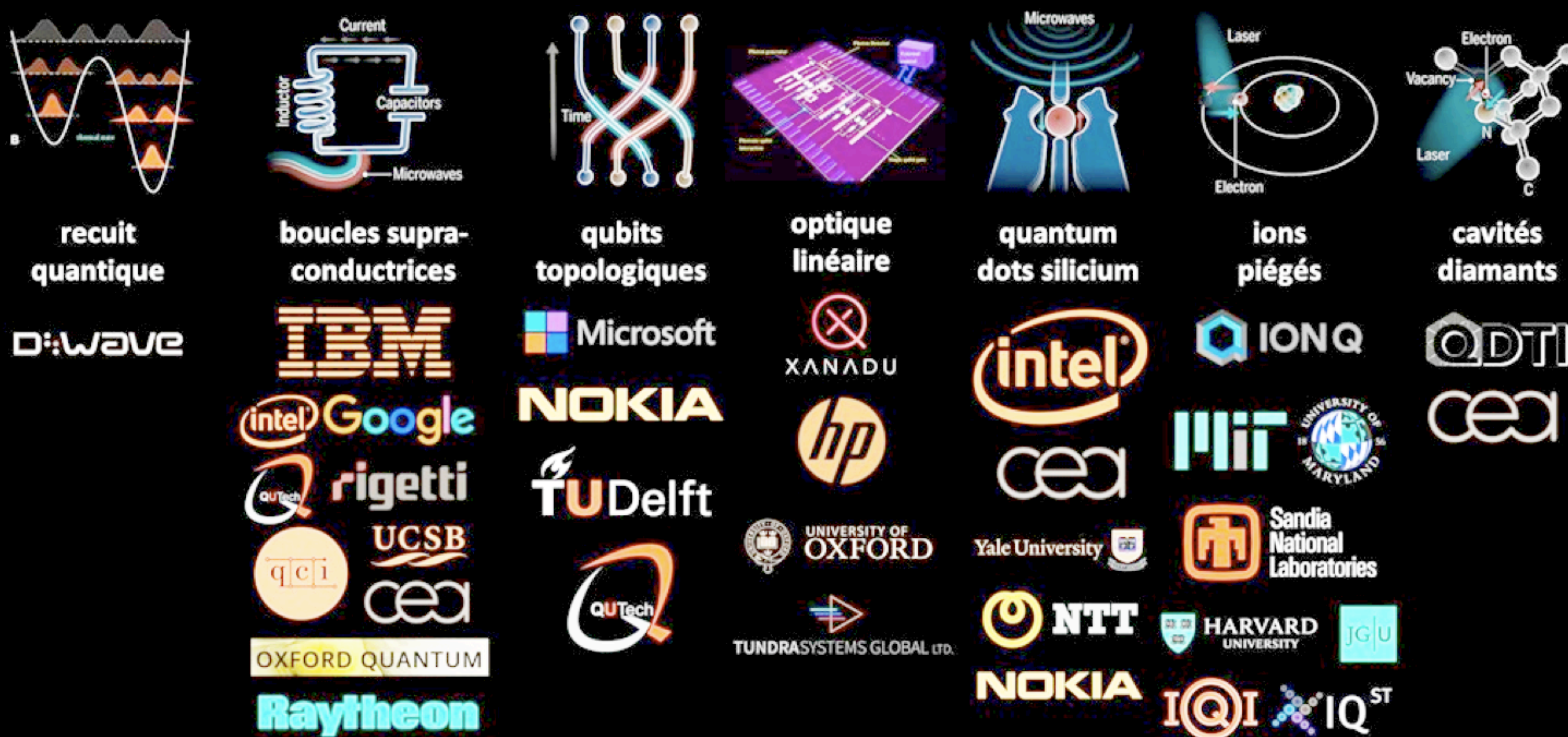
A bit of the action

In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.

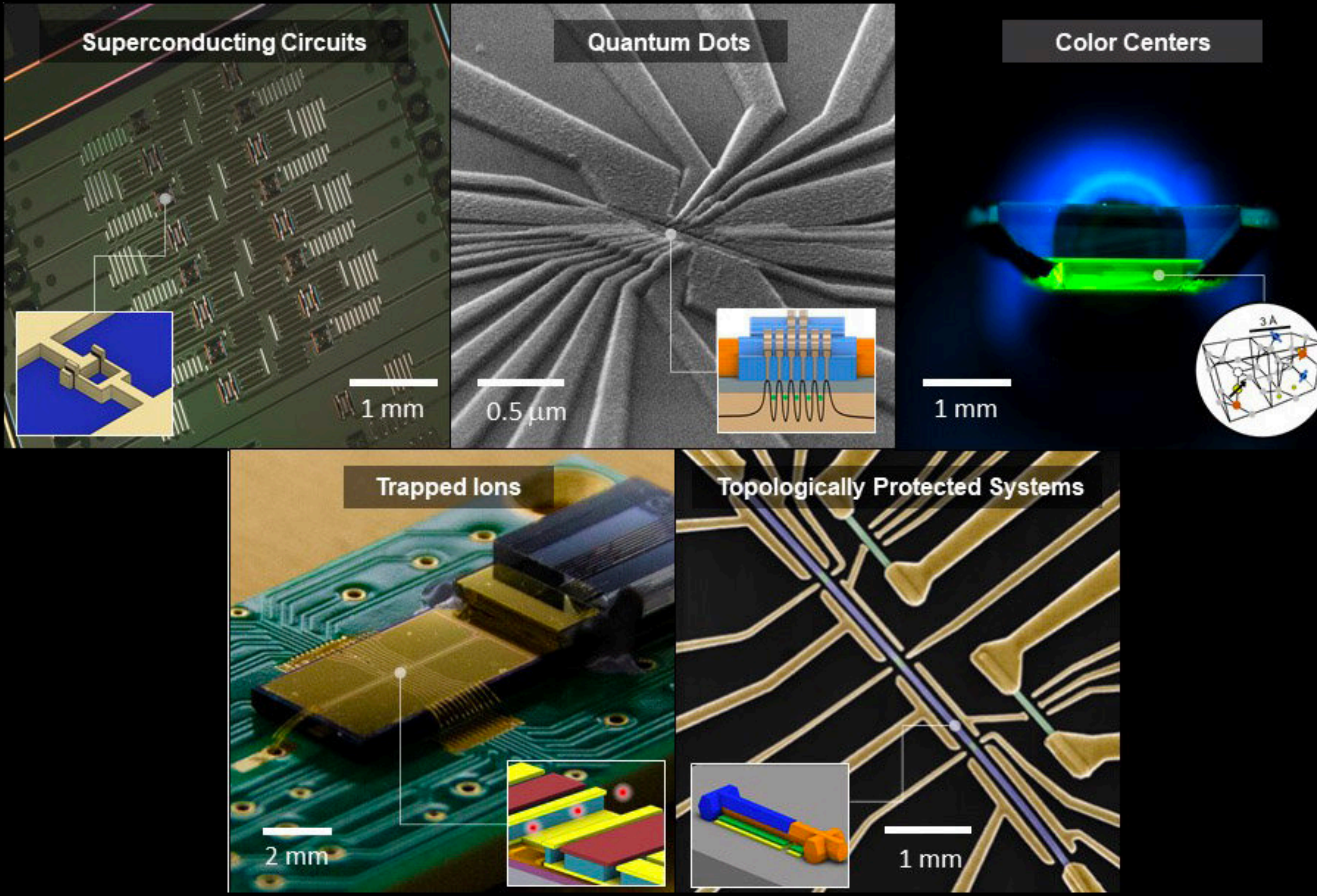
	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into superposition states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

Rodzaje kubitów i firmy biorące udział w rozwijaniu danych technologii



Fizyczna implementacja



IBM image
Nathalie P. de Leon et al, Materials challenges and opportunities for quantum computing hardware, Science (2021)

Elektronowe i dziurowe kubity spinowe w kropkach kwantowych - teoria - wkład pracowników WFiIS

- **Ultrafast Spin Initialization in a Gated InSb Nanowire Quantum Dot** S. Bednarek, J. Pawłowski, M. Górski, and G. Skowron Phys. Rev. Applied 11, 034012 (2019)
- **All-electric single-electron spin-to-charge conversion**, J. Pawłowski, G. Skowron, M. Górski, and S. Bednarek, Phys. Rev. B 98, 125411 (2018)
- **Valley qubit in a gated MoS2 monolayer quantum dot** J. Pawłowski, D. Żebrowski, and S. Bednarek, Phys. Rev. B 97, 155412 (2018)
- **Spin and valley control in single and double electrostatic silicene quantum dots**, Bartłomiej Szafran and Dariusz Żebrowski, Phys. Rev. B 98, 155305 (2018)
- **Electrical control of a confined electron spin in a silicene quantum dot** Bartłomiej Szafran, Alina Mreńca-Kolasińska, Bartłomiej Rzeszotarski, and Dariusz Żebrowski, Phys. Rev. B 97, 165303 (2018)
- **Electron spin rotations induced by oscillating Rashba interaction in a quantum wire** J. Pawłowski, P. Szumniak, and S. Bednarek, Phys. Rev. B 93, 045309 (2016)
- **Long-distance entanglement of soliton spin qubits in gated nanowires** Paweł Szumniak, Jarosław Pawłowski, Stanisław Bednarek, and Daniel Loss, Phys. Rev. B 92, 035403 (2015)
- **All-electrical control of quantum gates for single heavy-hole spin qubits** P.Szumniak, S. Bednarek, J. Pawłowski, and B. Partoens Phys. Rev. B 87, 195307 (2013)
- and many more ...

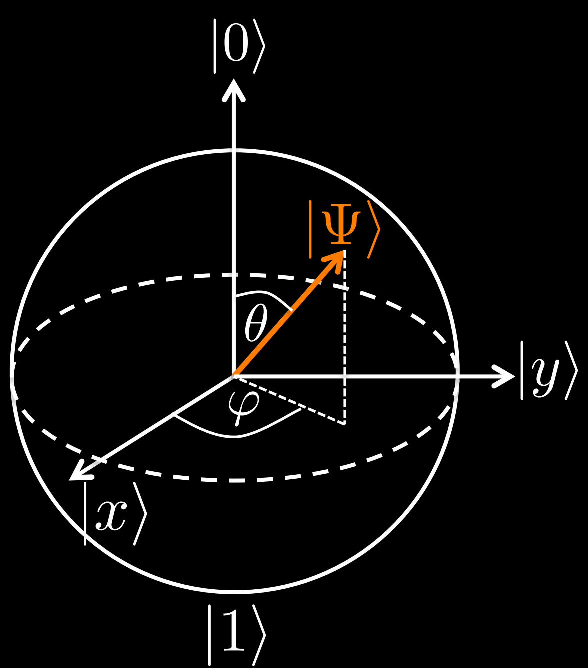
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Notacja Diraca

Notacja	Opis
$z^* \quad (1+i)^* = (1-i)$	Sprzężenie zespolone
$\lvert\psi\rangle = \alpha_0\lvert 0\rangle + \alpha_1\lvert 1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$	Wektor stanu ***ket***
$\langle\psi\rvert = \alpha_0^*\langle 1\rvert + \alpha_1^*\langle 0\rvert = \begin{pmatrix} \alpha_0^* & \alpha_1^* \end{pmatrix}$	Wektor stanu *bra* $\langle\Psi\rvert = \lvert\Psi\rangle^{*^T}$
$\langle\varphi\lvert\psi\rangle = (\beta_0^*\langle 0\rvert + \beta_1^*\langle 0\rvert)(\alpha_0\lvert 0\rangle + \alpha_1\lvert 1\rangle) = \begin{pmatrix} \beta_0^* & \beta_1^* \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \alpha_0\beta_0^* + \alpha_1\beta_1^*$	Iloczyn skalarny (rzutowanie)
$\lvert\varphi\rangle \otimes \lvert\psi\rangle \equiv \lvert\varphi\rangle\lvert\psi\rangle = \begin{pmatrix} \beta_0^*\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \\ \beta_1^*\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \alpha_0\beta_0^* \\ \alpha_1\beta_0^* \\ \alpha_0\beta_1^* \\ \alpha_1\beta_1^* \end{pmatrix}$	Iloczyn tensorowy
$\hat{A}^\dagger = \hat{A}^{*^T}$	Sprzężenie Hermitowskie
$\langle\hat{A}\rangle_{\lvert\Psi\rangle} = \langle\Psi\rvert\hat{A}\lvert\Psi\rangle$	Wartość oczekiwana

Sfera Blocha – służy do łatwej wizualizacji kubitu i operacji na nim wykonywanych



$$|x\rangle = \frac{1}{2}(|0\rangle + |1\rangle)$$

$$|y\rangle = \frac{1}{2}(|0\rangle + i|1\rangle)$$

$$|\Psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$|\langle 0|\Psi\rangle|^2 = |a_0|^2$$

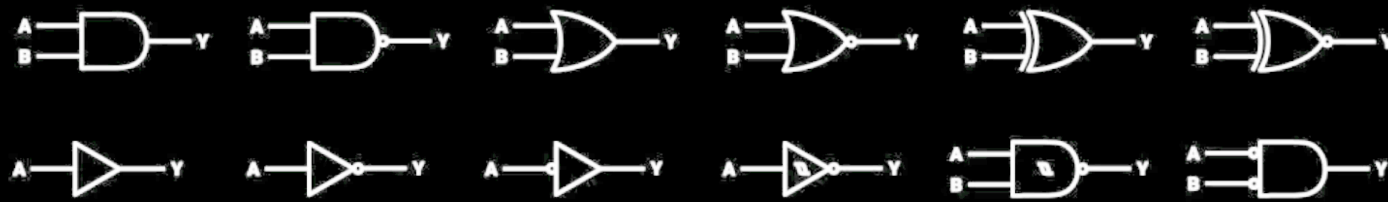
$$|\langle 1|\Psi\rangle|^2 = |a_1|^2$$

$$\langle\Psi|\Psi\rangle = |a_0|^2 + |a_1|^2$$

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

Jednokubitowe kwantowe bramki logiczne

Klasyczne bramki logiczne: NOT, AND, NAND, OR, NOR, XOR, …



Jednokubitowe kwantowe bramki logiczne reprezentowane są przez **liniowy** i **unitarny** operator U (macierz 2x2):

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad UU^\dagger = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix} = I \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad U|0\rangle = a|0\rangle + b|1\rangle$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = c \begin{pmatrix} 1 \\ 0 \end{pmatrix} + d \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad U|1\rangle = c|0\rangle + d|1\rangle$$

Operatory obrotu

$$R_{\vec{n}}(\theta) \equiv \exp\left(-i\vec{n}\cdot\vec{\sigma}\frac{\theta}{2}\right) = I\cos(\theta/2) - i(\vec{n}\cdot\vec{\sigma})\sin(\theta/2)$$

$$R_x(\theta) = \exp(-iX\theta/2) = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

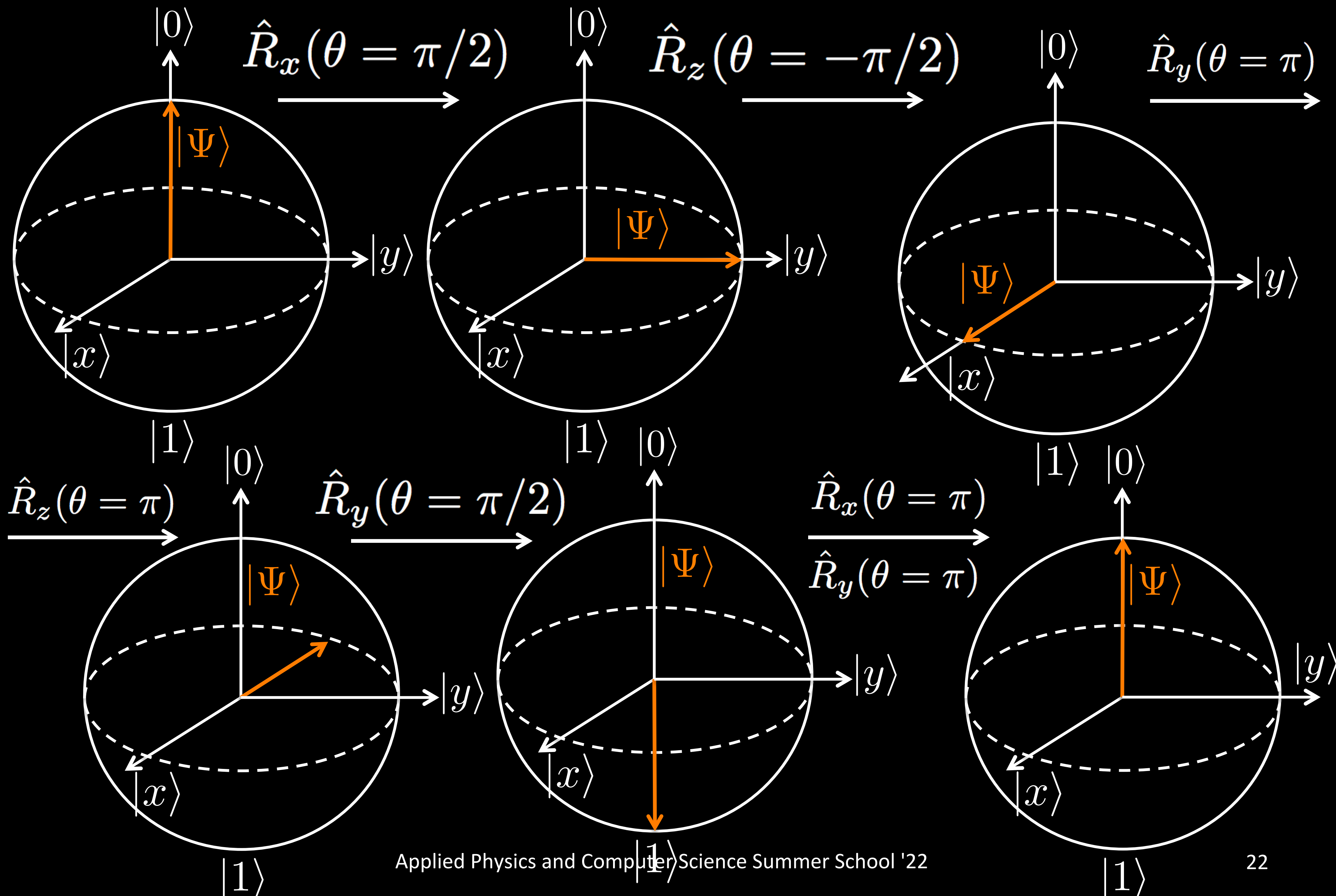
$$R_y(\theta) = \exp(-iY\theta/2) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

$$R_z(\theta) = \exp(-iZ\theta/2) = \begin{pmatrix} \exp(-i\theta/2) & 0 \\ 0 & \exp(i\theta/2) \end{pmatrix}$$

Wyprowadzenie

$$
\begin{aligned}
e^{ia(\hat{n}\cdot\vec{\sigma})} &= \sum_{k=0}^{\infty} \frac{i^k \left[a\left(\hat{n}\cdot\vec{\sigma}\right)\right]^k}{k!} \\
&= \sum_{p=0}^{\infty} \frac{(-1)^p (a\hat{n}\cdot\vec{\sigma})^{2p}}{(2p)!} + i\sum_{q=0}^{\infty} \frac{(-1)^q (a\hat{n}\cdot\vec{\sigma})^{2q+1}}{(2q+1)!} \\
&= I\sum_{p=0}^{\infty} \frac{(-1)^p a^{2p}}{(2p)!} + i(\hat{n}\cdot\vec{\sigma})\sum_{q=0}^{\infty} \frac{(-1)^q a^{2q+1}}{(2q+1)!}
\end{aligned}
$$

Obroty na sferze Bloch’a przykłady



Przykłady jedno-kubitowych bramek kwantowych stosowanych obliczeniach kwantowych bramki Pauliego, sqrt(NOT)

$$X = \sigma_x = \mathrm{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad R_x(\pi) = -iX$$

$$Y = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad R_y(\pi) = -iY$$

$$Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad R_z(\pi) = -iZ$$

$$\sqrt{X} = \sqrt{\mathrm{NOT}} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi/4} & e^{-i\pi/4} \\ e^{-i\pi/4} & e^{i\pi/4} \end{pmatrix}$$

$$R_z(\pi/2)$$

Przykłady jednokubitowych bramek kwantowych stosowanychw obliczeniach kwantowych bramki Pauliego, bramka Hadamarda, sqrt(NOT)

$$X = \sigma_x = \mathrm{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad X(\alpha_0|0\rangle + \alpha_1|1\rangle) = \alpha_1|0\rangle + \alpha_0|1\rangle$$

$$Y = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Y(\alpha_0|0\rangle + \alpha_1|1\rangle) = i(\alpha_1|0\rangle - \alpha_0|1\rangle)$$

$$Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad Z(\alpha_0|0\rangle + \alpha_1|1\rangle) = \alpha_0|0\rangle - \alpha_1|1\rangle$$

$$\sqrt{X} = \sqrt{\mathrm{NOT}} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi/4} & e^{-i\pi/4} \\ e^{-i\pi/4} & e^{i\pi/4} \end{pmatrix}$$

Bramka Hadamarda (interferencja konstruktywna i destruktywna)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \qquad H\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) =$$

$$\frac{1}{2}|0\rangle + \cancel{\frac{1}{2}|1\rangle} + \frac{1}{2}|0\rangle - \cancel{\frac{1}{2}|1\rangle} = |0\rangle \qquad \cancel{\frac{1}{2}|0\rangle} + \frac{1}{2}|1\rangle - \cancel{\frac{1}{2}|0\rangle} + \frac{1}{2}|1\rangle = |1\rangle$$

Bramki zmiany fazy

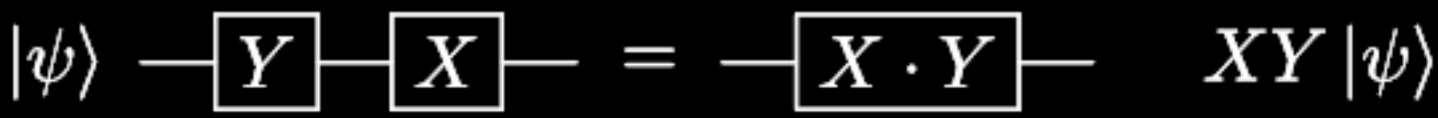
$$P(\varphi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \quad P(\varphi)(\alpha_0|0\rangle + \alpha_1|1\rangle) = \alpha_0|0\rangle + e^{i\varphi}\alpha_1|1\rangle$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = P\left(\frac{\pi}{2}\right) = \sqrt{Z}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} = P\left(\frac{\pi}{4}\right) = \sqrt{S} = \sqrt[4]{Z}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = P(\pi) \qquad Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Obwód kwantowy – szeregowo połączone bramki



$$C = X \cdot Y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = iZ$$

Rejestry Kwantowe – Iloczyn tensorowy

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}, \qquad B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{1,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \\ a_{2,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{2,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,1}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{bmatrix}$$

Rejestry Kwantowe – stany dwukubitowe

$$|\Psi\rangle = c_0|0\rangle_A|0\rangle_B + c_1|0\rangle_A|1\rangle_B + c_2|1\rangle_A|0\rangle_B + c_3|1\rangle_A|1\rangle_B$$

$$|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$$

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\ |01\rangle = |0\rangle \otimes |1\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix},\ |10\rangle = |1\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\\1\\0\end{pmatrix},\ |11\rangle = |1\rangle \otimes |1\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Rejestry Kwantowe – stany dwukubitowe

$$|\langle 00|\Psi\rangle|^2 = |c_0|^2,\ |\langle 01|\Psi\rangle|^2 = |c_1|^2,\ |\langle 10|\Psi\rangle|^2 = |c_2|^2,\ |\langle 11|\Psi\rangle|^2 = |c_3|^2$$

$$\langle\Psi|\Psi\rangle = |c_0|^2 + |c_1|^2 + |c_2|^2 + |c_3|^2$$

$$.P_{|0*\rangle} = |c_0|^2 + |c_1|^2,\ P_{|1*\rangle} = |c_2|^2 + |c_3|^2,\ P_{|*0\rangle} = |c_0|^2 + |c_2|^2,\ P_{|*1\rangle} = |c_1|^2 + |c_3|^2$$

Rejestry Kwantowe – stany N-kubitowe

$$|\Psi\rangle = \alpha_{0\ldots0}|0\ldots0\rangle + \cdots + \alpha_{1\ldots1}|1\ldots1\rangle$$

$$|0\rangle^{\otimes N} = |0\ldots0\rangle \qquad |1\rangle^{\otimes N} = |1\ldots1\rangle$$

$$|\Psi\rangle = \sum_{j=0}^{j=2^N-1} \alpha_j|j\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_{2^N-1}|2^N-1\rangle$$

$$|\alpha_0|^2 + |\alpha_1|^2 + \cdots + |\alpha_{2^N-1}|^2 = 1$$

$$2^N$$

Rejestry Kwantowe – stany rozkładalne

$$|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$$

$$|\Psi_a\rangle = a_0|0\rangle + a_1|1\rangle, \ |\Psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\Psi\rangle \stackrel{?}{=} |\Psi_a\rangle \otimes |\Psi_b\rangle,$$

$$|\Psi_a\rangle \otimes |\Psi_b\rangle = (a_0|0\rangle + a_1|1\rangle) \otimes (b_0|0\rangle + b_1|1\rangle) =$$

$$= a_0b_0|00\rangle + a_0b_1|01\rangle + a_0b_1|10\rangle + a_1b_1|11\rangle,$$

Rejestry kwantowe – stan rozkładalny przykład

$$|\Psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Rejestry kwantowe – stan nierozkładalny – stan splątany – stany Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$$

Rejestry kwantowe – stan nierozkładalny – stan splątany – stany Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$|\Psi_a\rangle_A \otimes |\Psi_b\rangle_B = (a_0|0\rangle_A + a_1|1\rangle_A) \otimes (b_0|0\rangle_B + b_1|1\rangle_B)$$

$$|\Psi_a\rangle_A \otimes |\Psi_b\rangle_B = (a_0b_0|0\rangle_A|0\rangle_B + a_0b_1|0\rangle_A|1\rangle_B + a_1b_0|1\rangle_A|0\rangle_B + a_1b_1|1\rangle_A|1\rangle_B),$$

$$a_0b_0 = \frac{1}{\sqrt{2}},\ a_0b_1 = 0,\ a_1b_0 = 0,\ a_1b_1 = \frac{1}{\sqrt{2}}$$

Sprzeczny warunek => stan nierozkładalny

Stany splatane konsekwencje fizyczne

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

Pomiar cząstki A, otrzymujemy stan: $|0\rangle$

Pomiar stanu odległej cząstki B, pokaże zawsze, że jest ona w stanie $|0\rangle$

Pomiar cząstki A, otrzymujemy stan: $|1\rangle$

Pomiar stanu odległej cząstki B, pokaże zawsze, że jest ona w stanie $|1\rangle$

Pomiar stanu jednej cząstki A determinuje stan drugiej cząstki B, pomimo, że są one odległej nie oddziałują między sobą

Stany splątane eksperyment 12km

Entanglement of two quantum memories via fibres over dozens of kilometres

Yong Yu, Fei Ma, Xi-Yu Luo, Bo Jing, Peng-Fei Sun, Ren-Zhou Fang, Chao-Wei Yang, Hui Liu, Ming-Yang Zheng, Xiu-Ping Xie, Wei-Jun Zhang, Li-Xing You, Zhen Wang, Teng-Yun Chen, Qiang Zhang, Xiao-Hui Bao & Jian-Wei Pan

Nature **578**, 240–245 (2020) | Cite this article

Bramki dwukubitowe - kontrolowane

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad |a, b\rangle \mapsto |a, a \oplus b\rangle$$

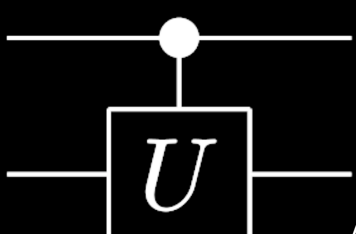
$$\mathrm{C}_U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_{00} & u_{01} \\ 0 & 0 & u_{10} & u_{11} \end{pmatrix}$$

$$|00\rangle \mapsto |00\rangle$$

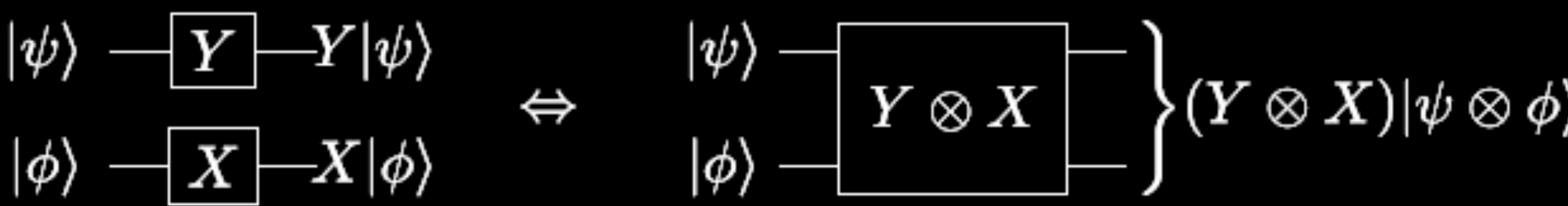
$$|01\rangle \mapsto |01\rangle$$

$$|10\rangle \mapsto |1\rangle C_U|0\rangle = |1\rangle(u_{00}|0\rangle + u_{01}|1\rangle),$$

$$|11\rangle \mapsto |1\rangle C_U|1\rangle = |1\rangle(u_{10}|0\rangle + u_{11}|1\rangle),$$



Bramki dwukubitowe − obwody równoległe



$$C = Y \otimes X = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

Twierdzenie o nieklonowaniu

Załóżmy, że istnieje kwantowa maszyna kopiująca:

$$\hat{U}(|\Psi\rangle|0\rangle) = |\Psi\rangle|\Psi\rangle$$

$$|\Psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$$

Wojciech Żurek

$$\hat{U}(|\Psi\rangle|0\rangle) = 1/2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

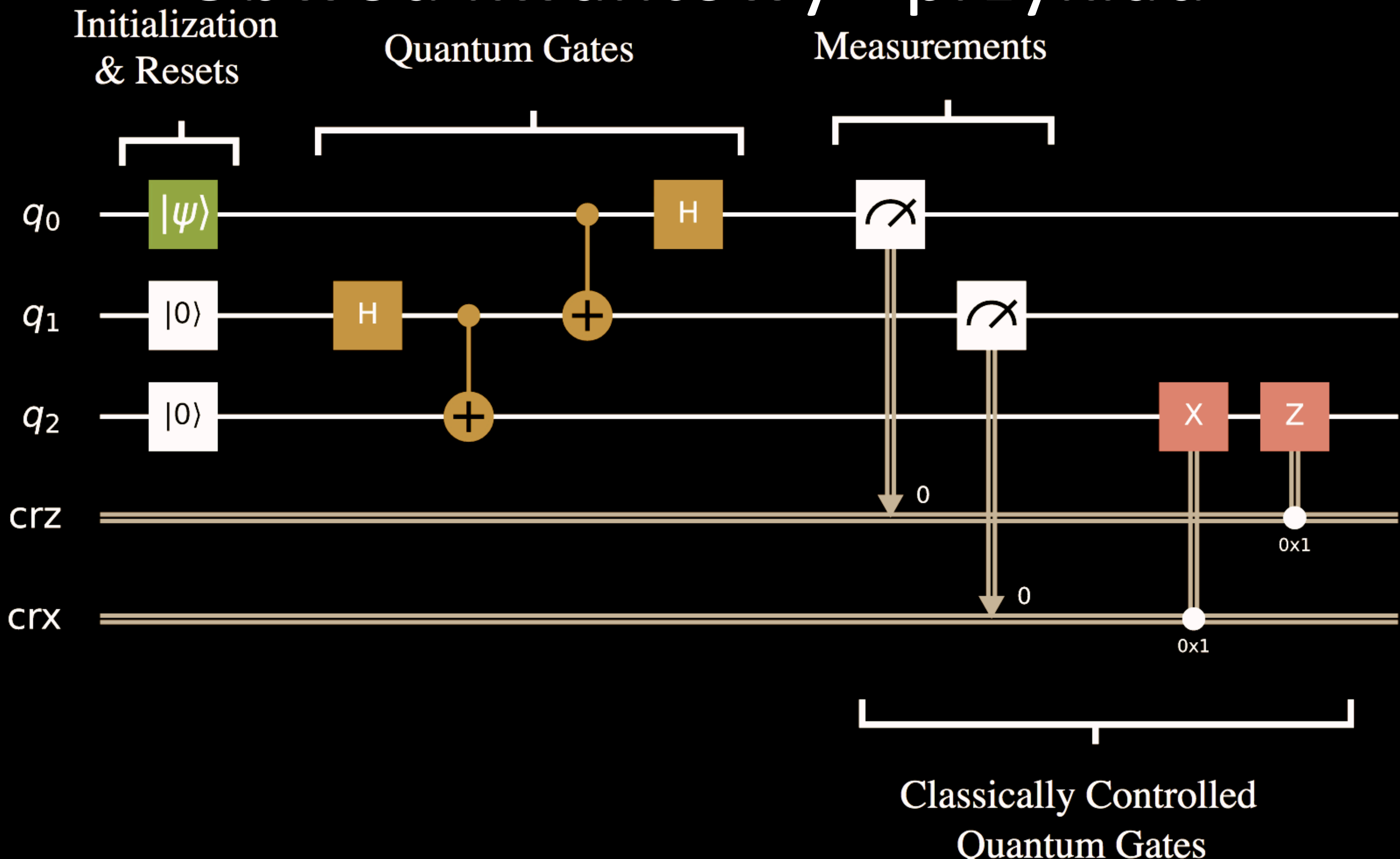
Sprzeczność!!!

Z drugiej strony wykorzystując linowość operatora U

$$1/\sqrt{2}\hat{U}(|0\rangle + |1\rangle)|0\rangle = 1/\sqrt{2}(\hat{U}|0\rangle|0\rangle + \hat{U}|1\rangle|0\rangle)$$

$$1/\sqrt{2}\hat{U}(|0\rangle + |1\rangle)|0\rangle = 1/\sqrt{2}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$

Obwód kwantowy - przykład



Rejestry kwantowe – stan nierozkładalny – stan splątany – stany Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

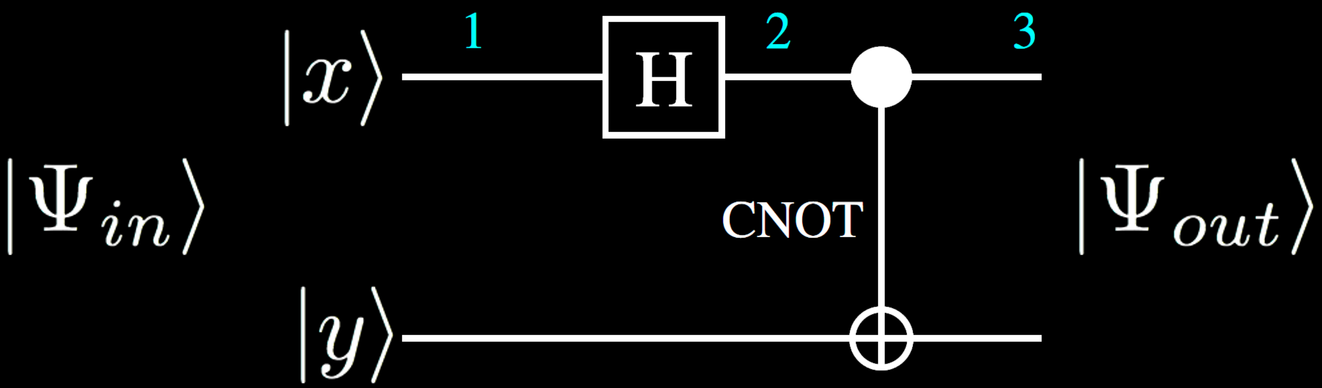
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$$

$$|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}(|0y\rangle + (-1)^x|1\bar{y}\rangle)$$

Obwód kwantowy generowanie stanów splatanych



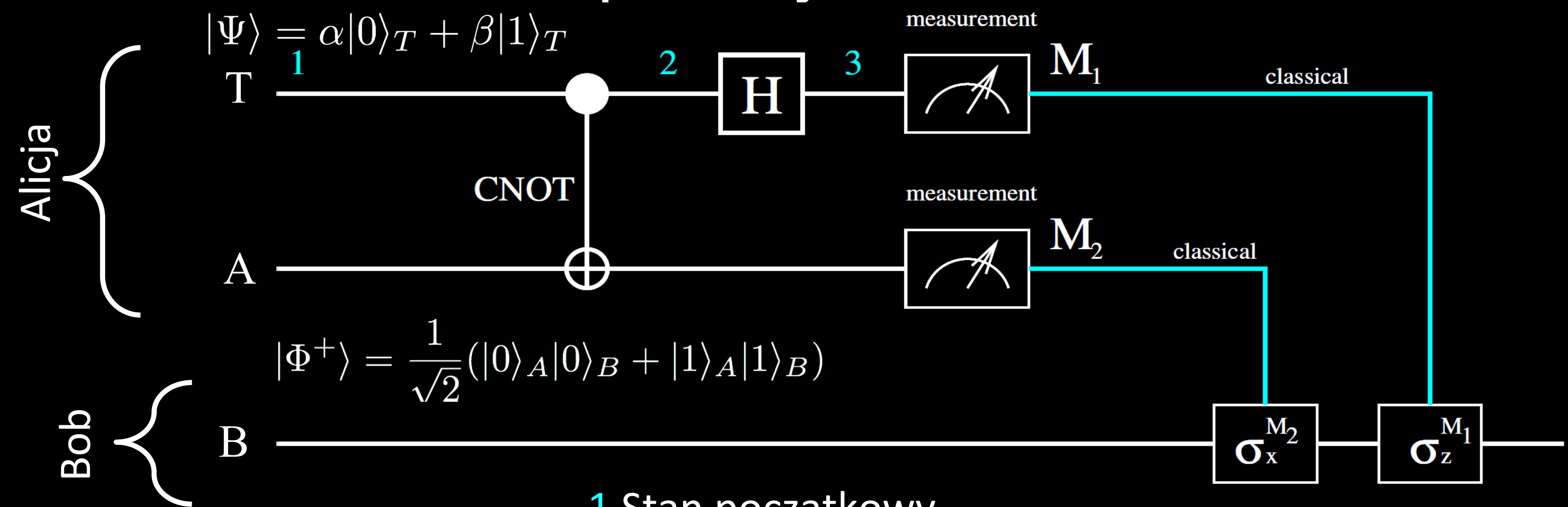
1 $$|\Psi_{in}\rangle = |xy\rangle = |00\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

2 $$H \otimes I|00\rangle = H|0\rangle \otimes I|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

3 $$|\Psi_{out}\rangle = \mathrm{CNOT}\Big[\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\Big] = \frac{1}{\sqrt{2}}(\mathrm{CNOT}|00\rangle + \mathrm{CNOT}|10\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Teleportacja Kwantowa



1 Stan początkowy

$$|\Psi\rangle|\Phi^+\rangle = \frac{1}{\sqrt{2}}|\Psi\rangle_T\Big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\Big) = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle_T\Big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\Big) + \beta|1\rangle_T\Big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\Big)$$

2 Działamy bramką CNOT na kubit T i A, gdzie T kubit kontrolujący A kubit docelowy

$$.U_{\mathrm{cNOT}} \otimes I\Big(|\Psi\rangle_T|\Phi^+\rangle\Big) = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle_T\Big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\Big) + \beta|1\rangle_T\Big(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\Big).$$

3 Działamy bramką Hadamarda na kubit T

$$.U_{\mathrm{cNOT}} \otimes I\Big(|\Psi\rangle_T|\Phi^+\rangle\Big) = \frac{1}{2}\Big((\alpha|0\rangle_T + \alpha|1\rangle_T)(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B) + \beta(\alpha|0\rangle_T - \alpha|1\rangle_T)(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)\Big).$$

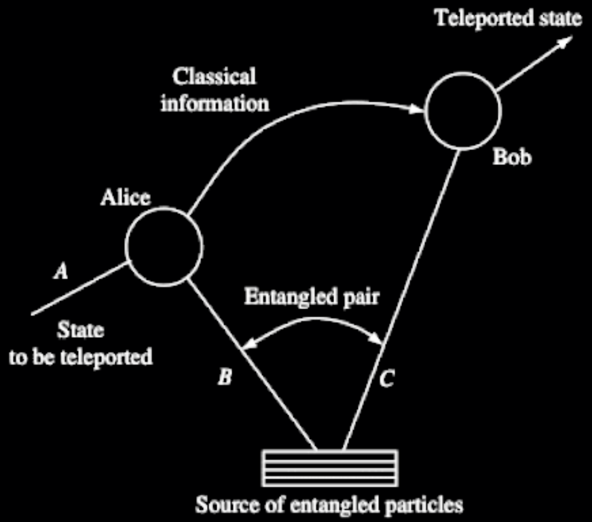
Teleportacja Kwantowa cd

$$|\Psi_{out}\rangle_{TAB} = \frac{1}{2}\Big(\alpha|0\rangle_T|0\rangle_A|0\rangle_B + \alpha|0\rangle_T|1\rangle_A|1\rangle_B + \alpha|1\rangle_T|0\rangle_A|0\rangle_B + \alpha|1\rangle_T|1\rangle_A|1\rangle_B$$
$$+\beta|0\rangle_T|1\rangle_A|0\rangle_B + \beta|0\rangle_T|0\rangle_A|1\rangle_B - \beta|1\rangle_T|1\rangle_A|0\rangle_B - \beta|1\rangle_T|0\rangle_A|1\rangle_B\Big)$$

Grupujemy stany należące do Alicji (T, A) i do Boba (B):

$$\begin{aligned}|\Psi_{out}\rangle_{TAB} = {} & \frac{1}{2}|0\rangle_T|0\rangle_A(\alpha|0\rangle_B + \beta|1\rangle_B) \\ & +\frac{1}{2}|0\rangle_T|1\rangle_A(\beta|0\rangle_B + \alpha|1\rangle_B) \\ & +\frac{1}{2}|1\rangle_T|0\rangle_A(\alpha|0\rangle_B - \beta|1\rangle_B) \\ & +\frac{1}{2}|1\rangle_T|1\rangle_A(-\beta|0\rangle_B + \alpha|1\rangle_B)\end{aligned}$$

Teleportacja Kwantowa cd



$$
\begin{aligned}
|\Psi_{out}\rangle_{TAB} = & \frac{1}{2}|0\rangle_T|0\rangle_A(\alpha|0\rangle_B + \beta|1\rangle_B) \\
& + \frac{1}{2}|0\rangle_T|1\rangle_A(\beta|0\rangle_B + \alpha|1\rangle_B) \\
& + \frac{1}{2}|1\rangle_T|0\rangle_A(\alpha|0\rangle_B - \beta|1\rangle_B) \\
& + \frac{1}{2}|1\rangle_T|1\rangle_A(-\beta|0\rangle_B + \alpha|1\rangle_B)
\end{aligned}
$$

Pomiar Alicji w bazie stanów $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$: tzw pomiar Bella

Bob zależności od rezultatu otrzymanego przez Alicję wykonuje odpowiednią operację oby otrzymać stan identyczny do teleportowanego

$$|0\rangle_T|0\rangle_A \longmapsto I$$

$$|0\rangle_T|1\rangle_A \longmapsto R_x(\pi) = \exp(-i\frac{\pi\sigma_x}{2}) = -iX$$

$$|1\rangle_T|0\rangle_A \longmapsto R_z(\pi) = \exp(-i\frac{\pi\sigma_z}{2}) = -iZ$$

$$|1\rangle_T|1\rangle_A \longmapsto R_y(\pi) = \exp(-i\frac{\pi\sigma_y}{2}) = -iY$$

Teleportacja Kwantowa uwagi końcowe

$$|0\rangle_T|0\rangle_A \longmapsto I$$
$$|0\rangle_T|1\rangle_A \longmapsto R_x(\pi) = \exp(-i\frac{\pi\sigma_x}{2}) = -iX$$
$$|1\rangle_T|0\rangle_A \longmapsto R_z(\pi) = \exp(-i\frac{\pi\sigma_z}{2}) = -iZ$$
$$|1\rangle_T|1\rangle_A \longmapsto R_y(\pi) = \exp(-i\frac{\pi\sigma_y}{2}) = -iY$$

$$I(\alpha|0\rangle_B + \beta|1\rangle_B) = (\alpha|0\rangle_B + \beta|1\rangle_B)$$
$$R_x(\pi)(\beta|0\rangle_B + \alpha|1\rangle_B) = -i(\alpha|0\rangle_B + \beta|1\rangle_B)$$
$$R_z(\pi)(\alpha|0\rangle_B - \beta|1\rangle_B) = -i(\alpha|0\rangle_B + \beta|1\rangle_B)$$
$$R_y(\pi)(-\beta|0\rangle_B + \alpha|1\rangle_B) = -(\alpha|0\rangle_B + \beta|1\rangle_B)$$

Teleportacja Kwantowa uwagi końcowe

Nie można teleportować materii, tylko stany kwantowe
Teleportacja nigdy nie jest związana z transportem materii

Nie ma natychmiastowej transmisji informacji. Alicja musi poinformować Bob'a klasycznym kanałem o wyniku pomiaru na swoich stanach kwantowych. Wtedy dopiero Bob będzie wiedział jaką operację powinien wykonać na swoim stanie. Po wykonaniu tej operacji będzie miał stan identyczny jak ten, który Alicja mu przesyłała na początku.

Teleportacja Kwantowa eksperyment:

Teleportation Systems Toward a Quantum Internet

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Matthew Shaw,[4] Dawn Tang,[1,2] Emma E. Wollman,[4] Christoph Simon,[6] Panagiotis Spentzouris,[3]
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Quantum teleportation is essential for many quantum information technologies, including long-distance quantum networks. Using fiber-coupled devices, including state-of-the-art low-noise superconducting nanowire single-photon detectors and off-the-shelf optics, we achieve conditional quantum teleportation of time-bin qubits at the telecommunication wavelength of 1536.5 nm. We measure teleportation fidelities of $\geq 90\%$ that are consistent with an analytical model of our system, which includes realistic imperfections. To demonstrate the compatibility of our setup with deployed quantum networks, we teleport qubits over 22 km of single-mode fiber while transmitting qubits over an additional 22 km of fiber. Our systems, which are compatible with emerging solid-state quantum devices, provide a realistic foundation for a high-fidelity quantum Internet with practical devices.

Algorytmy kwantowe

Za pomocą zestawu uniwersalnych jedno i dwu-kubitowch kwantowych bramek logicznych można zrealizować dowolny algorytm kwantowy

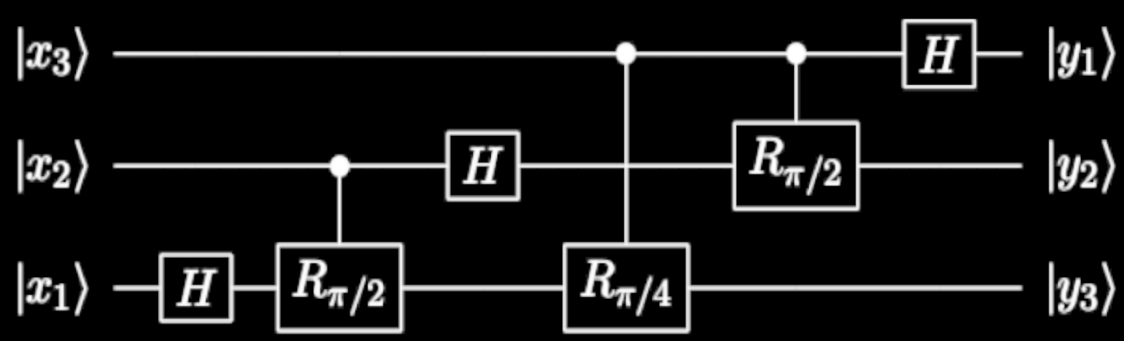
https://quantumalgorithmzoo.org/

Algorytmy bazujące na **kwantowej transformacie Fouriera(QFT)**:

- algorytm Deutscha-Jozsy (odróżniania funkcji zrównoważonej od stałej)
- **algorytm Shora** (faktoryzacji, czyli rozkładu liczb na czynniki pierwsze)
- Algorytm Simona
- …

Algorytmy bazujące na **wzmocnieniu amplitudy prawdopodobieństwa**

- **algorytm Grovera** (przeszukiwanie baz danych)
- …



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  https://www.youtube.com/playlist?list=PL0ojjrEqIyPy-1RRD8cTD_lF1hflo89Iu

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