# Properties of Quarkyonic Matter

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Mass and radii of observed neutron stars and data from neutron star collisions give an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a phase transition
Relativistic degrees of freedom appear to be important

After a short review, will discuss a field theoretical method to include both quark and nucleon degrees of freedom in a consistent field theoretical formalism

# High Temperature and Low Baryon Density
# Thermal Matter

Useful to think in the large number of of colors limit

$$N_c \rightarrow \infty$$

In this limit, quark loops are suppressed by one over Nc

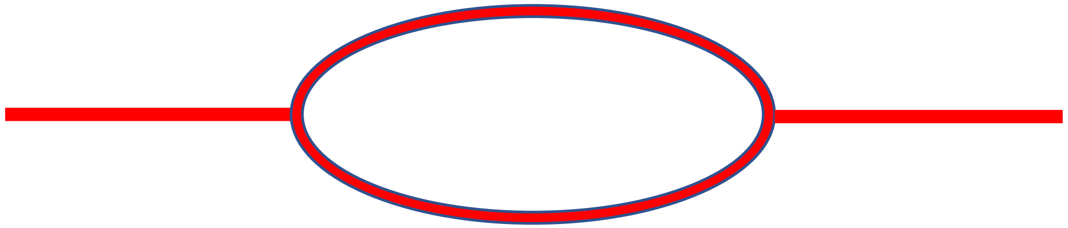
Quark pair production cannot short out the long range linear potential, so there is confinement.

Baryons are heavy and made of N_c quark so their mass is of order N_c

Mesons are weakly interacting, with interaction strength of order 1/N_c

High Temperature World at Zero Baryon Density

The confining force is cutoff at a screening length associated with polarization of thermal gluons



$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At some temperature the Debye length is less than the confinement scale, the potential can no longer become linear

$$T_D \sim \Lambda_{QCD}$$

Can also imagine it from thinking about hadrons

At low temperatures there is a gas of very weakly interacting hadrons, since if

$$T << \Lambda_{QCD}$$

The density of mesons is of order one in powers of N_c because the mesons are color singlet

If there was a gas of gluons, the density would be of order

$$N_c^2$$

This happens because there is an exponentially growing density of mesons that have cross sections of order 1/N_c^2 whose interactions become big when the density is of order N_c^2

What about finite density:

Quarkyonic Matter:

Confinement at finite temperature disappears because the Debye screening length become shorter than the confinement scale. Gluons give

$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At finite density, there is a typical Fermi momentum or chemical potential associated with quark density

$$\mu_Q$$

The Debye screening length associated with quarks is very large

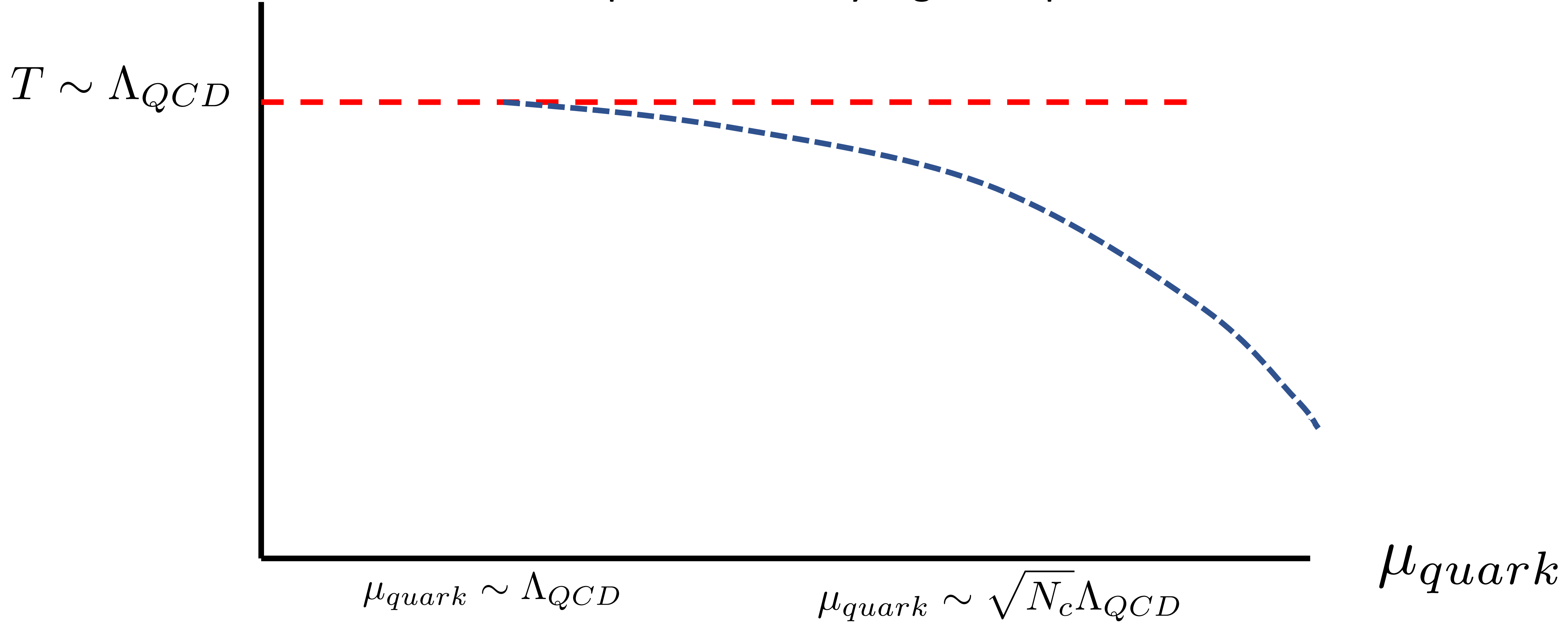
In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

$$1/r^{2\ quarks}_{Debye} \sim \mu_Q^2/N_c$$

In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

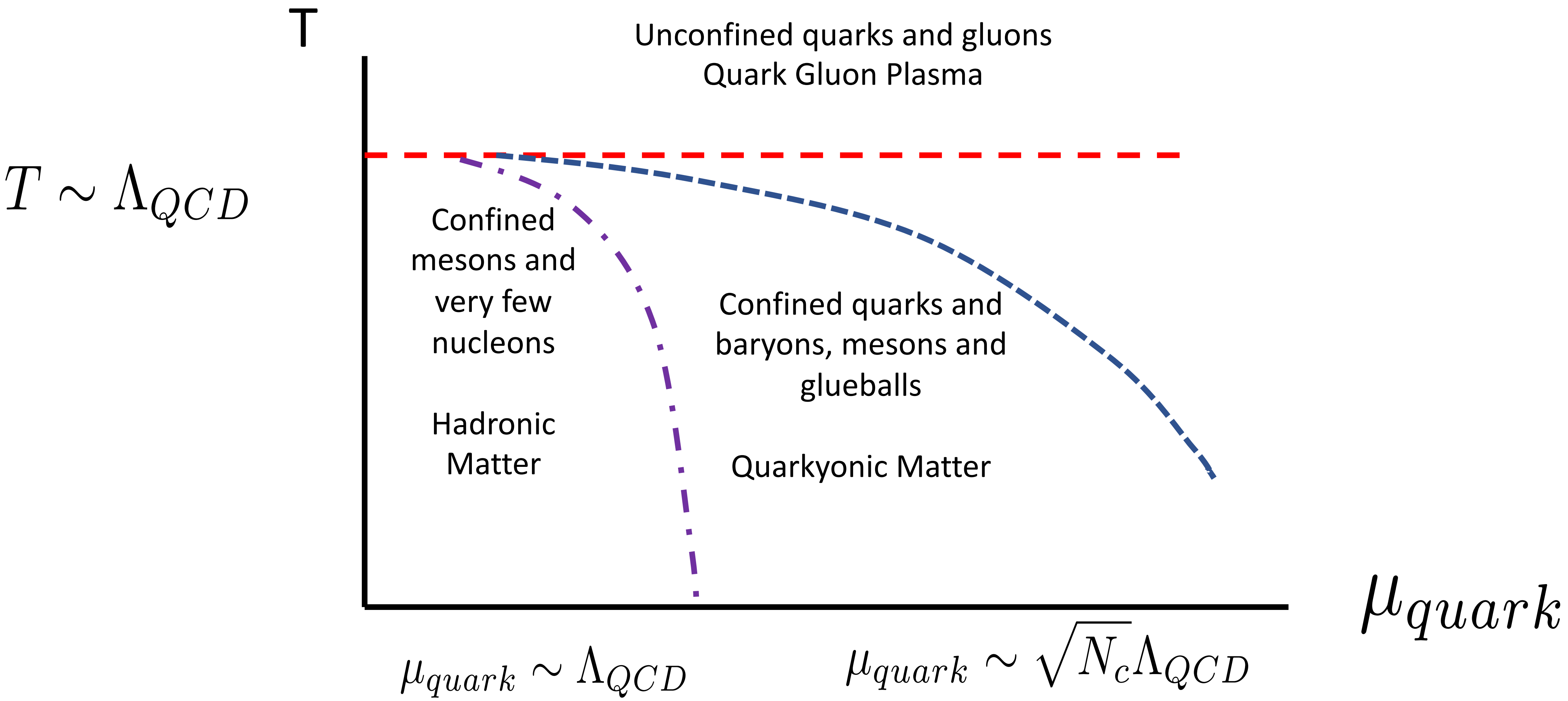
$$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD} >> \Lambda_{QCD}$$

This is a very interesting result: the system is confined until quark densities that are parametrically high compared to the QCD scale!

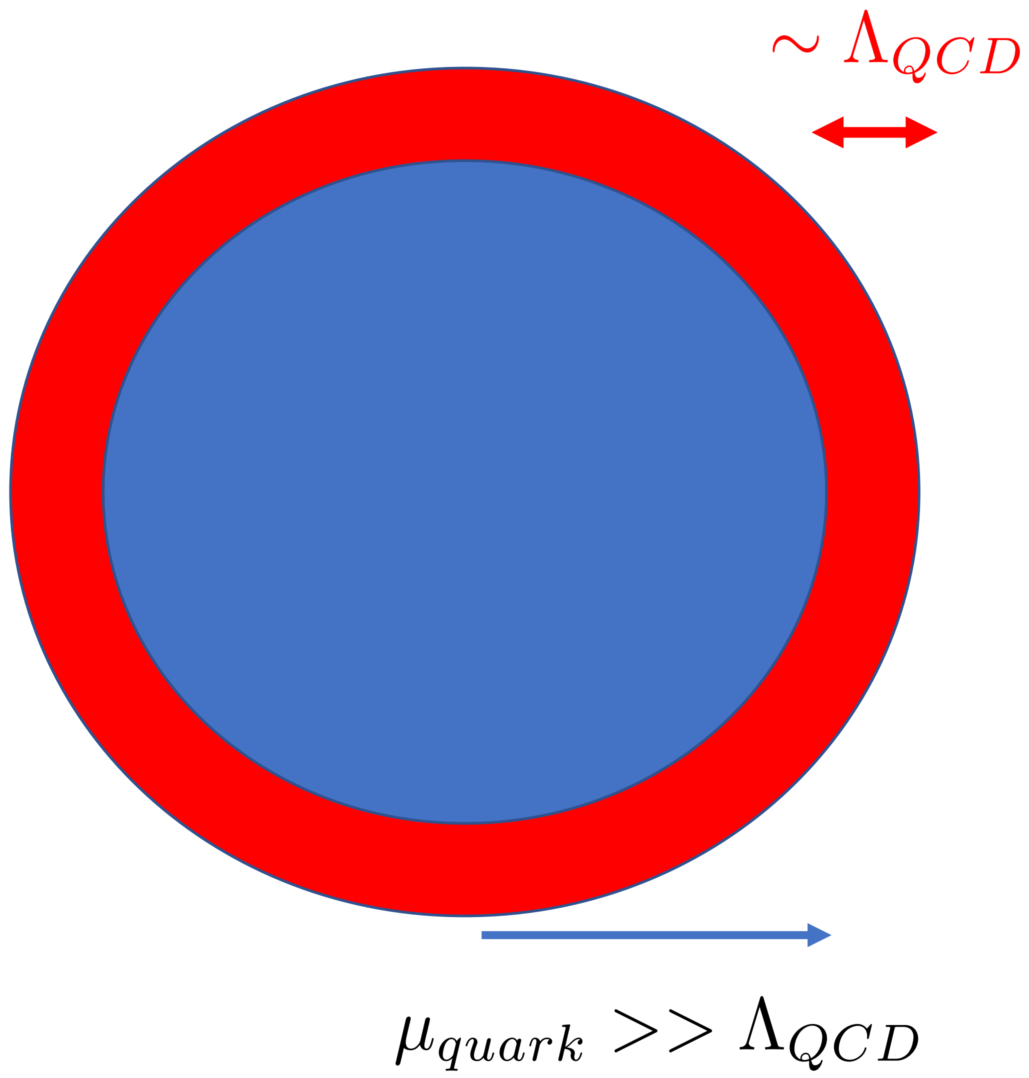


$$n_{baryon} \sim e^{(\mu_B - E)/T} \sim e^{N_c(\mu_q - E_q)/T}$$

No baryons for $\mu_{quark} < M_{nucleon}/N_c \sim \Lambda_{QCD}$



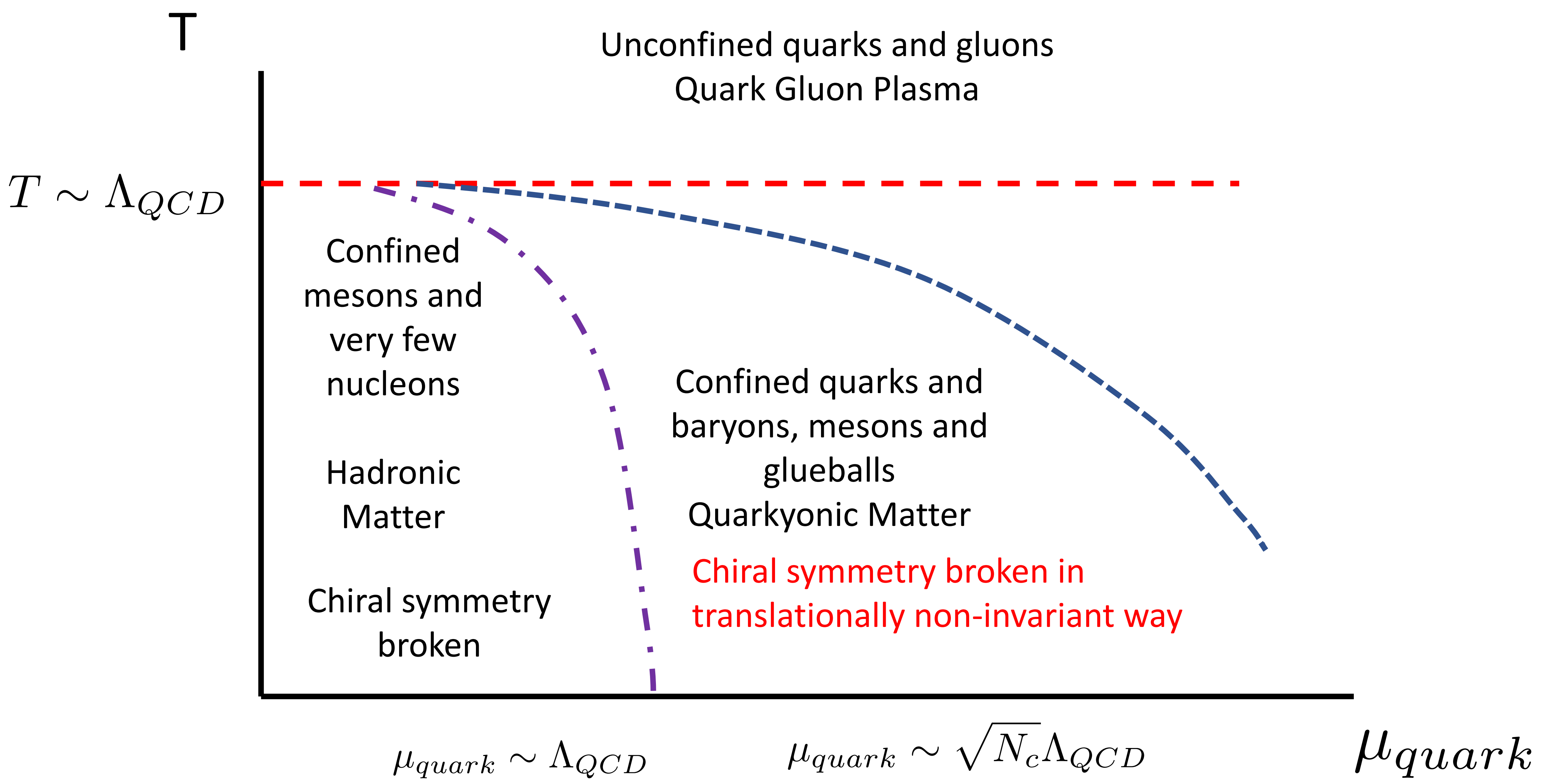
# Fermi Surface is Non-perturbative




Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Chiral symmetry remains broken in quarkyonic matter



# Simple large N_c considerations

Near nuclear matter density

$$k_F \sim \Lambda_{QCD}$$

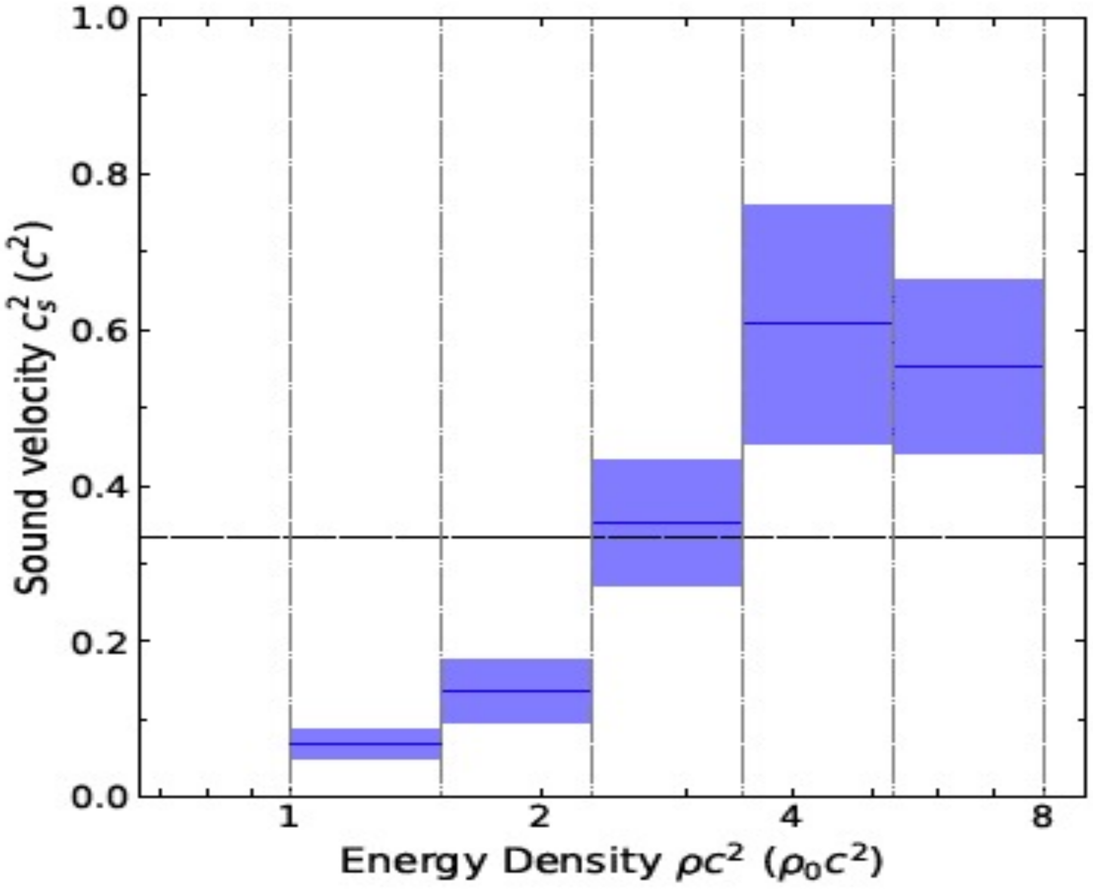
$$\epsilon/n - M_N \sim \Lambda_{QCD}^2/2M_N \sim \Lambda_{QCD}/N_c$$

On the other hand, short distance QCD interactions are of order N_c

$$\epsilon/n - M_N \sim N_c\Lambda_{QCD}$$

But the density of hard cores is also parametrically of order

$$\Lambda_{QCD}^3$$




Y. Fujimoto, K. Fukushima,
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As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \ \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \ MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

How can the density stay fixed if the Fermi momentum goes up.? The nucleon must sit on a shell of varying thickness as the density increases.  The added baryon number has to come in the form of new degrees of freedom: quarks

This can be understood in large N_c arguments:

$k_f \sim \Lambda_{QCD}$ requires $\mu_B - M_N \sim \Lambda_{QCD}/N_c$

Quarks should become important when

$$\mu_Q = \mu_B/N_c \sim \Lambda_{QCD}$$

The hypothesis of quarkyonic matter implies there need be no first order phase transition, The quarkyonic hypothesis requires a transition when the baryon Fermi energy is very close to the nucleon mass, so the transition may in principle occur quite close to nuclear matter density.

$$n_B^n = \frac{2}{3\pi^2} k_n^{F\ 3}$$

Density is of order one in powers of N_c for baryon density computed both by both quark and nucleon degrees of freedom

$$n_B^q = \frac{2}{3\pi^2} k_f^{q\ 3}$$

The problem is that if we take a constituent quark model

$$k_n^F = \sqrt{\mu_B^2 - M_N^2} = \sqrt{N_c^2 \mu_q^2 - N_c^2 M_q^2} = N_c k_q^F$$

Baryon number chemical potential must jump, while density
In nucleons stays constant and density in quarks turns on

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B} \epsilon_N$$

The pressure on the other hand must jump by order N_c squared

$$P \sim \epsilon_q$$

How to construct a theory of with both nucleon and quark degrees of freedom?

Quarks and nucleons exist in differing regions of momentum space. When states are fully occupied, quarks block nucleons. Consider a theory with nucleon, quarks and ghosts:

$$T, \mu_N, \mu_Q, \mu_G$$

The physical nucleonic baryons are basically the N minus the G states

$\mu_N$ is the top of the Fermi sea for nucleons

$\mu_G$ is the bottom of the Fermi sea for nucleons

$\mu_Q$ is the top of the quark Fermi sea

Interactions of N and G field must be identical to cancel the nucleons in the occupied phase space of the quarks. For the quark interactions among themselves there is the action of QCD. Nucleon-quark interactions?

There is a relation between the ghost chemical potential and that of the quarks. In the additive quark parton model

$$\mu_G = N_c \mu_Q$$

With a mean field, in the additive quark parton model

$$\mu_G - gV = N_c \mu_Q$$

More generally, we expect that at the Fermi surface:

$$\frac{dN_G^B}{d^3k} = N_c^3 \frac{dN_Q^B}{d^3k}$$

The factor of $N_c^3$ $\frac{2\pi j N_c}{L}$ $\frac{2\pi (j+1) N_c}{L}$

misses states $\frac{2\pi (j N_c + k)}{L}$

Which are composed of quark states of slightly different momenta.

Such states remain blocked

Finally, there is the quark chemical potential. This is determined by extremizing the pressure, or at zero temperature, the energy per baryon at fixed total baryon number. This means that this chemical potential is dynamically determined. Similar to what occurs in the excluded volume models considered by Duarte, Jeong. Hernandiz-Ortiz and LM.

Advantages of this technique are that one can have an effective field theories for nucleons in combination with underlying dynamics for quarks, and a smooth continuation between such theories. Allows to match onto nuclear mean field theories. Dynamical generation of quarkyonic matter

$$
\begin{aligned}
S_E \;=\; & \int_0^\beta dt \int_V d^3x \left\{ \overline{N}\left(\frac{1}{i}\gamma\cdot\partial - i\mu_N\gamma^0 + M_N\right)N \right. \\
& +\overline{G}\left(\frac{1}{i}\gamma\cdot\partial - i\mu_G\gamma^0 + M_N\right)G \\
& \left. +\overline{Q}\left(\frac{1}{i}\gamma\cdot\partial - i\gamma^0\mu_Q + M_Q\right)Q \right\} .
\end{aligned}
$$

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Kinetic energy term. Can include meson nucleon interactions, and QCD for quarks. Nucleon-quark interactions?

How to find quarkyonic matter:

$$\epsilon(\rho_N = \rho_B + \rho_G - \rho_Q, \rho_G, \rho_Q)$$

Minimize with respect to the quark density at fixed total baryon density.

$$d\rho_G / d\rho_Q = N_c^3$$

The determine:

$$d\epsilon / dn_B = \mu_B$$

And require that the pressure be maximum at any minima found for the energy density, including the end point minimum at zero quark density. If there are two possible values with equal pressure, then do a Maxwell construction

Work in progress:

Solve theory for mean field vector nucleon interaction plus self interacting quarks:

If nucleons have only mean field interactions, and quarks are free with
$M_Q = M_N/N_c$,
then only first order transition between nucleons and quarks and no quarkyonic matter

If $M_Q > M_N/N_c$, first order phase transition from all nucleons to quarkyonic matter

Quark interactions in. plausible low density expansion can convert this to a second order transition to quarkyonic matter. Need to assume

$$\mu_Q = M_Q + \rho_Q/\Lambda^2 + ...$$

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Paper in progress: In general in mean field for vector interaction, in general first order transition. For excluded volume theory can have 2nd order transitions