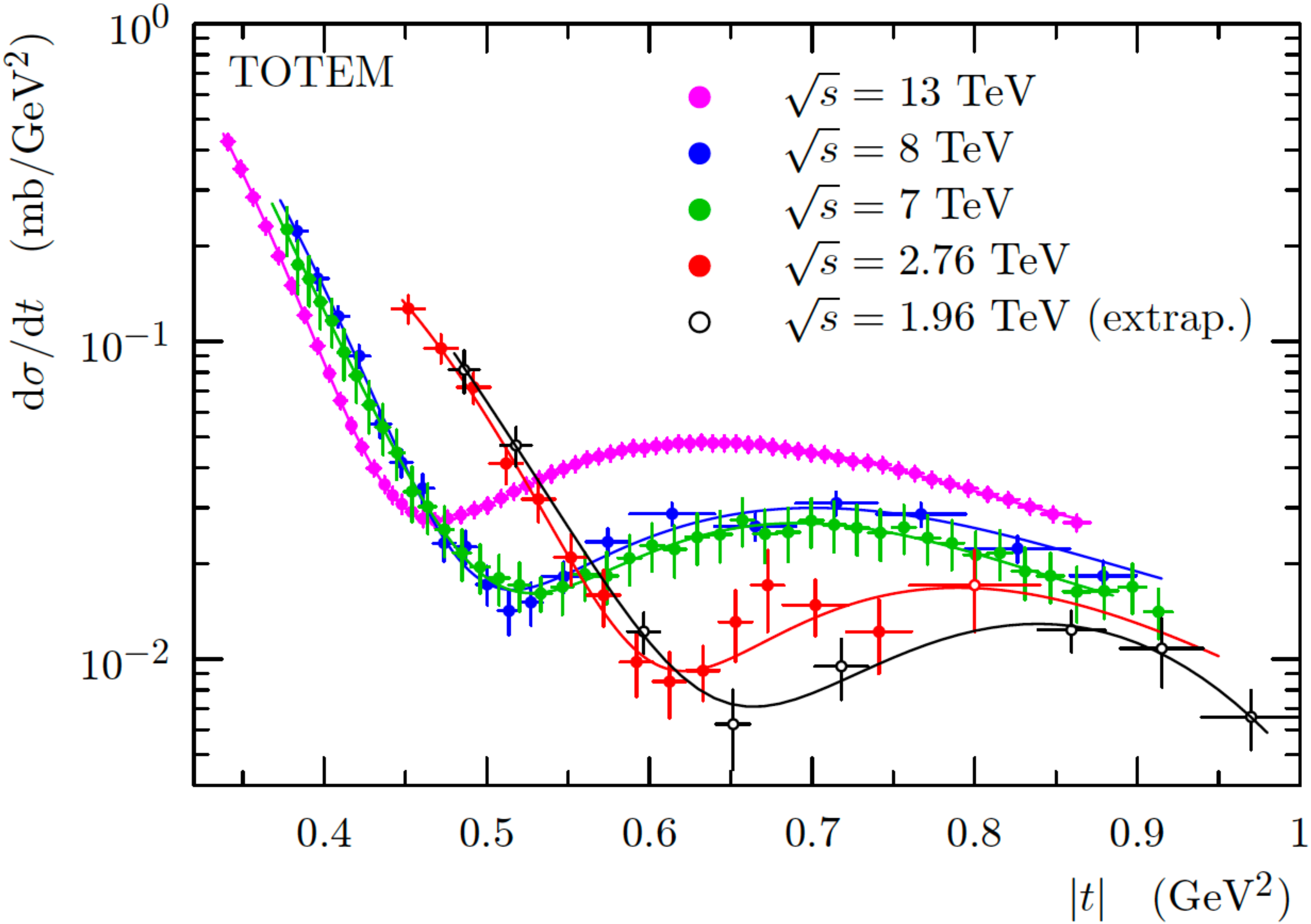
Scaling properties of elastic pp ($p\bar{p}$) cross-section.

Michał Praszałowicz

“Białasówka” 7.10.2022.



V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

Introduction

Impact parameter space (Barone, Predazzi):

$$\sigma_{\text{el}} = \int d^2\boldsymbol{b} \left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right|^2,$$

$$\sigma_{\text{tot}} = 2\int d^2\boldsymbol{b} \, \text{Re}\left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right],$$

$$\sigma_{\text{inel}} = \int d^2\boldsymbol{b} \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right].$$

Cross-sections

Impact parameter space (Barone, Predazzi):

$$\sigma_{\rm el} = \int d^2\boldsymbol{b}\ \overbrace{\left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right.}^{-i\,A_{\rm el}}\left.\right|^2 ,$$

$$\sigma_{\rm tot} = 2\int d^2\boldsymbol{b}\ {\rm Re}\left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right] ,$$

$$\sigma_{\rm inel} = \int d^2\boldsymbol{b}\ \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right] .$$

A bit of history

Nuclear Physics B59 (1973) 231–236 North-Holland Publishing Company

GEOMETRIC SCALING, MULTIPLICITY DISTRIBUTIONS AND CROSS SECTIONS

J DIAS DE DEUS

The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

Received 8 March 1973

Abstract From a geometric picture of hadrons as extended objects we arrive at some universal features of high energy collisions In this approach the mean multiplicity, as a function of s and the KNO scaling function are universal and asymptotically the ratio $\sigma_{\text{elastic}}/\sigma_{\text{total}}$ is expected to be the same for all processes

Geometric scaling

$$\Omega(s,b) = \Omega\left(b/R(s)\right)$$

Opacity is a function of one varible,
and $R(s)$ grows with energy. Changing variable

$$\boldsymbol{b} \to \boldsymbol{B} = \boldsymbol{b}/R(s)$$

$$\sigma_{\text{inel}} = R^2(s) \underbrace{\int d^2\boldsymbol{B} \left[1 - \left|e^{-\Omega(B)}\right|^2\right]}_{\text{constant}}$$

Immediate consequences

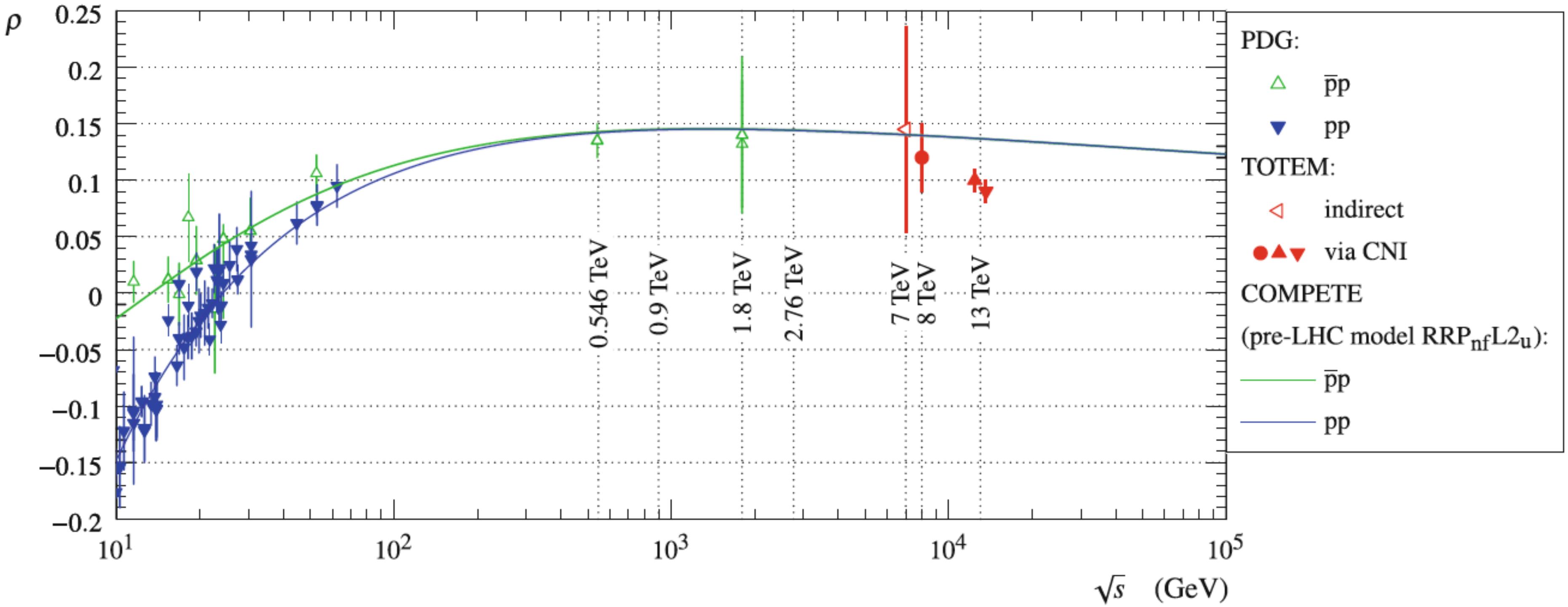
$$\sigma_{\rm el} = \int d^2\boldsymbol{b}\,\left|1 - e^{-\Omega(s,b)+i\chi(\cancel{s,b})}\right|^2,$$

$$\sigma_{\rm tot} = 2\int d^2\boldsymbol{b}\,{\rm Re}\left[1 - e^{-\Omega(s,b)+i\chi(\cancel{s,b})}\right],$$

$$\sigma_{\rm inel} = \int d^2\boldsymbol{b}\,\left[1 - \left|e^{-\Omega(s,b)}\right|^2\right].$$

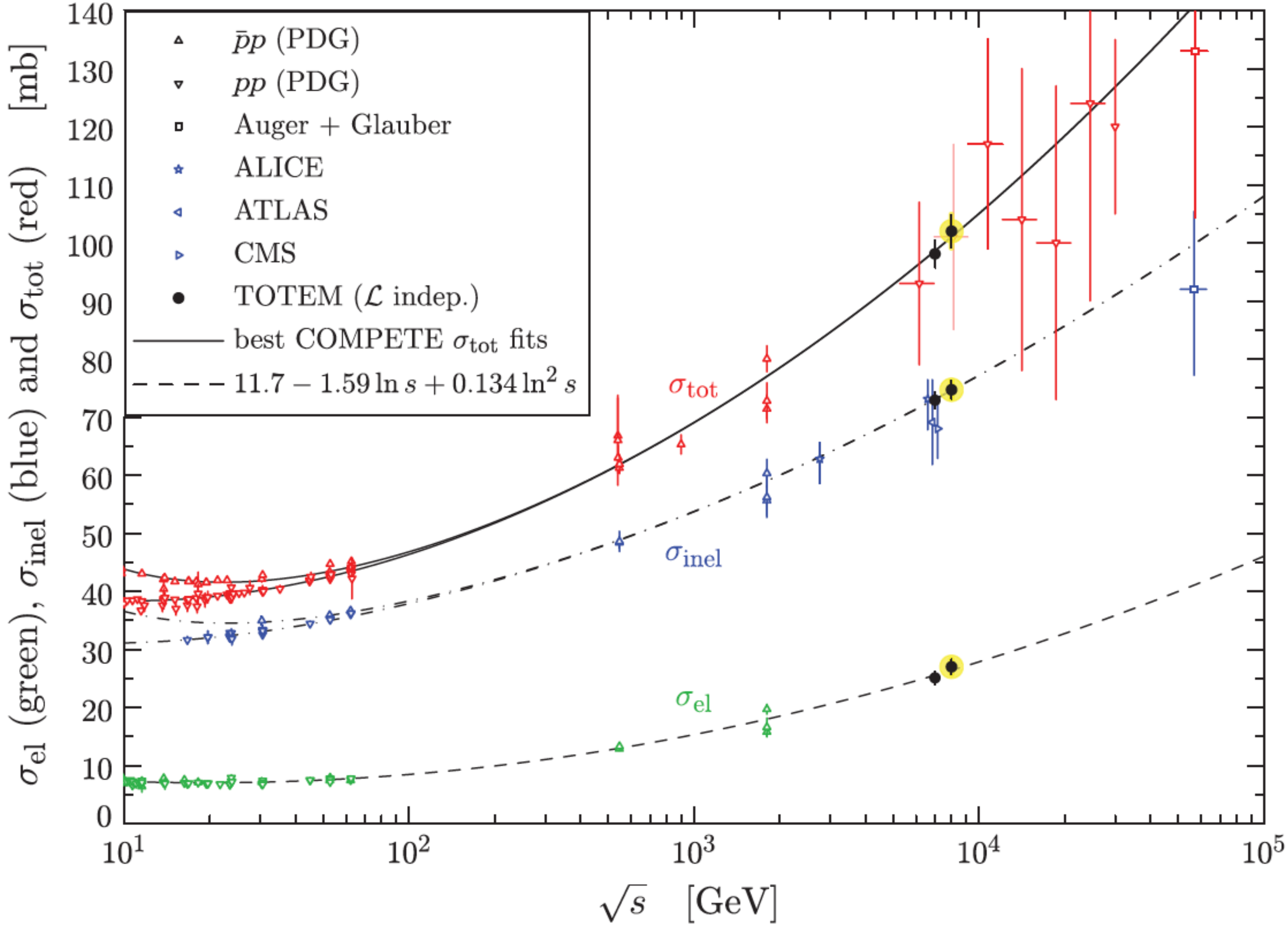
If we neglect χ (indeed ρ parameter is small), then all cross-sections have the same energy dependence.

ρ parameter:
real-to-imaginary ratio of the forward amplitude



G. Antchev [TOTEM] EPJ C 79 (2019) 785

ISR (1971 – 1984) pp@ (23.5 – 62.5) GeV



G. Antchev [TOTEM] PRL 111 (2013) 012001

Geometric scaling at the ISR

	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.115}$	$W^{0.109}$	$W^{0.109}$	$W^{0.006}$	$0.02 - 0.095$

Geometric scaling at the ISR

Nuclear Physics B71 (1974) 481–492

SCALING LAW FOR THE ELASTIC DIFFERENTIAL CROSS SECTION IN pp SCATTERING FROM GEOMETRIC SCALING*

A.J. BURAS and J. DIAS de DEUS

The Niels Bohr Institute, University of Copenhagen,
DK-2100 Copenhagen Ø, Denmark

Received 6 December 1973

Abstract: Plots of $(1/\sigma_{in}^2)d\sigma_{el}/d|t| \equiv \Phi(\tau, s)$ as a function of $\tau \equiv |t|\sigma_{in}$ are shown to scale in the NAL-ISR energy region. Such scaling is shown to be a consequence of geometric scaling for the inelastic overlap function $G_{in}(\beta = \pi b^2/\sigma_{in})$ in the limit $\rho = \mathrm{Re}A/\mathrm{Im}A \to 0$ and in the case of $\sigma_{in} \sim (\ln s)^2$ is equivalent to the scaling proposed by Auberson, Kinoshita and Martin. A possible relation to the KNO multiplicity scaling is indicated.

Geometric scaling at the ISR

$$\tau = \sigma_{\mathrm{inel}}(s)\,|t| \;=R^2(s)|t| \times \mathrm{const.}$$

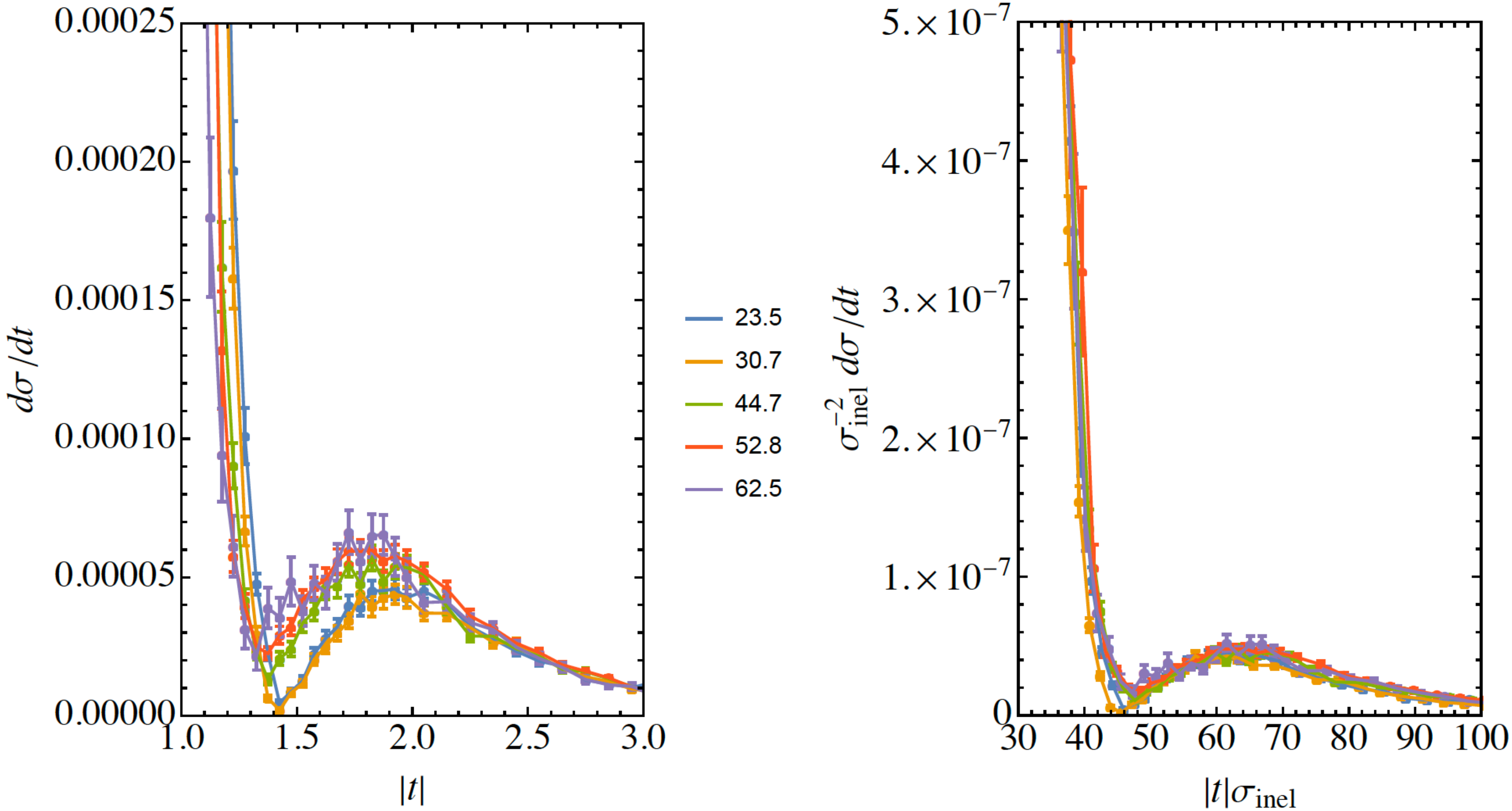
$$\begin{aligned}
\frac{d\sigma_{\mathrm{el}}}{d|t|} &\sim \left| \int_0^\infty db^2 A_{\mathrm{el}}(b^2,s)\, J_0\left(b\sqrt{|t|}\right) \right|^2 \\
&= \left| \sigma_{\mathrm{inel}}(s) \int_0^\infty d\left(b^2/\sigma_{\mathrm{inel}}(s)\right)\; A_{\mathrm{el}}(b^2/\sigma_{\mathrm{inel}}(s))\, J_0\left(\sqrt{\tau}\, b/\sqrt{\sigma_{\mathrm{inel}}(s)}\right) \right|^2 \\
&= \sigma_{\mathrm{inel}}^2(s) \left| \int_0^\infty dB^2 A_{\mathrm{el}}(B^2)\, J_0\left(B\sqrt{\tau}\right) \right|^2 \\
&= \sigma_{\mathrm{inel}}^2(s)\, \Phi(\tau).
\end{aligned}$$

Geometric scaling at the ISR

$$\tau = \sigma_{\mathrm{inel}}(s)\,|t| = R^2(s)|t| \times \mathrm{const.}$$

$$\boxed{\frac{1}{\sigma^2_{\mathrm{inel}}(s)}\frac{d\sigma_{\mathrm{el}}}{d|t|}(s,t) = \Phi(\tau)}$$

Geometric scaling at the ISR



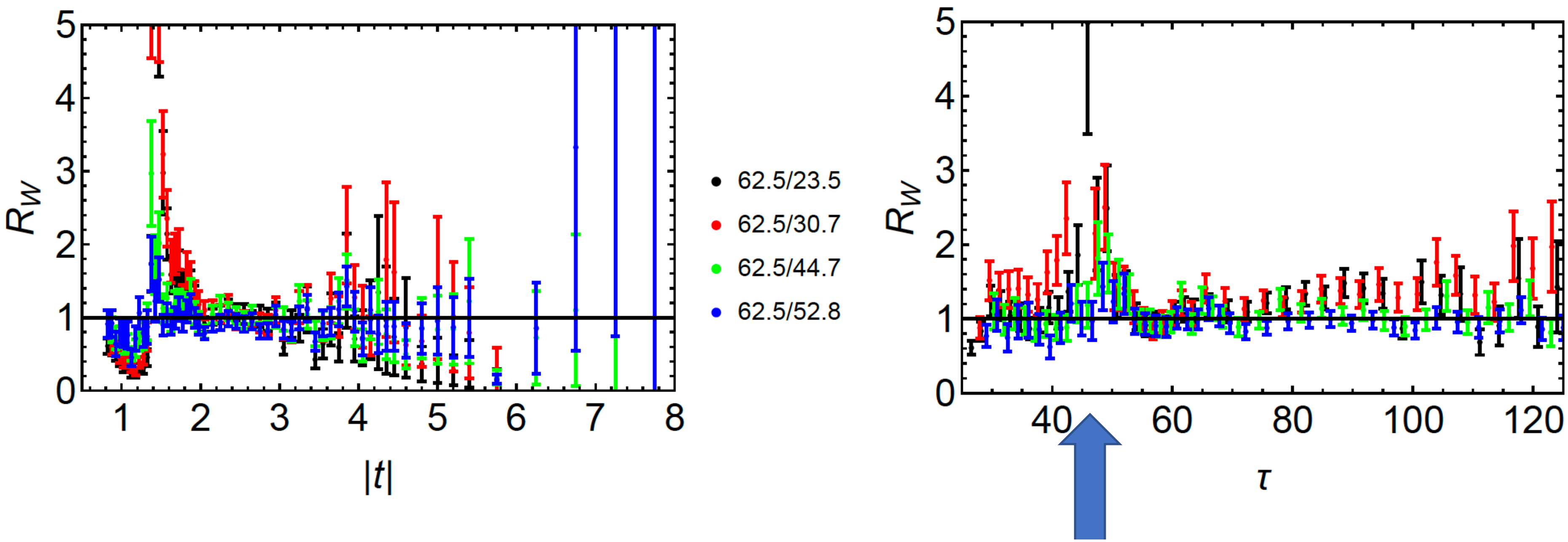
Ratio method

$$\frac{d\tilde{\sigma}_{\mathrm{el}}}{d|t|}(W,\tau_i) = \frac{1}{\sigma^2_{\mathrm{inel}}(W)}\,\frac{d\sigma_{\mathrm{el}}}{d|t|}(W,\tau_i)$$

$$R_W(\tau_i) = \frac{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W_{\mathrm{ref}},\tau_i)}{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W,\tau_i)}$$

$$W_{\mathrm{ref}} = 62.5\ \mathrm{GeV}$$

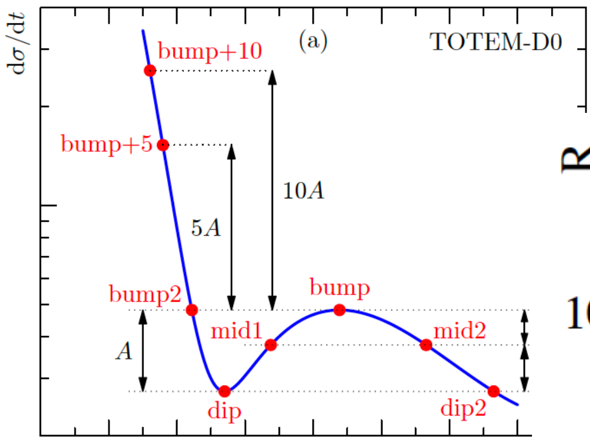
Ratio method

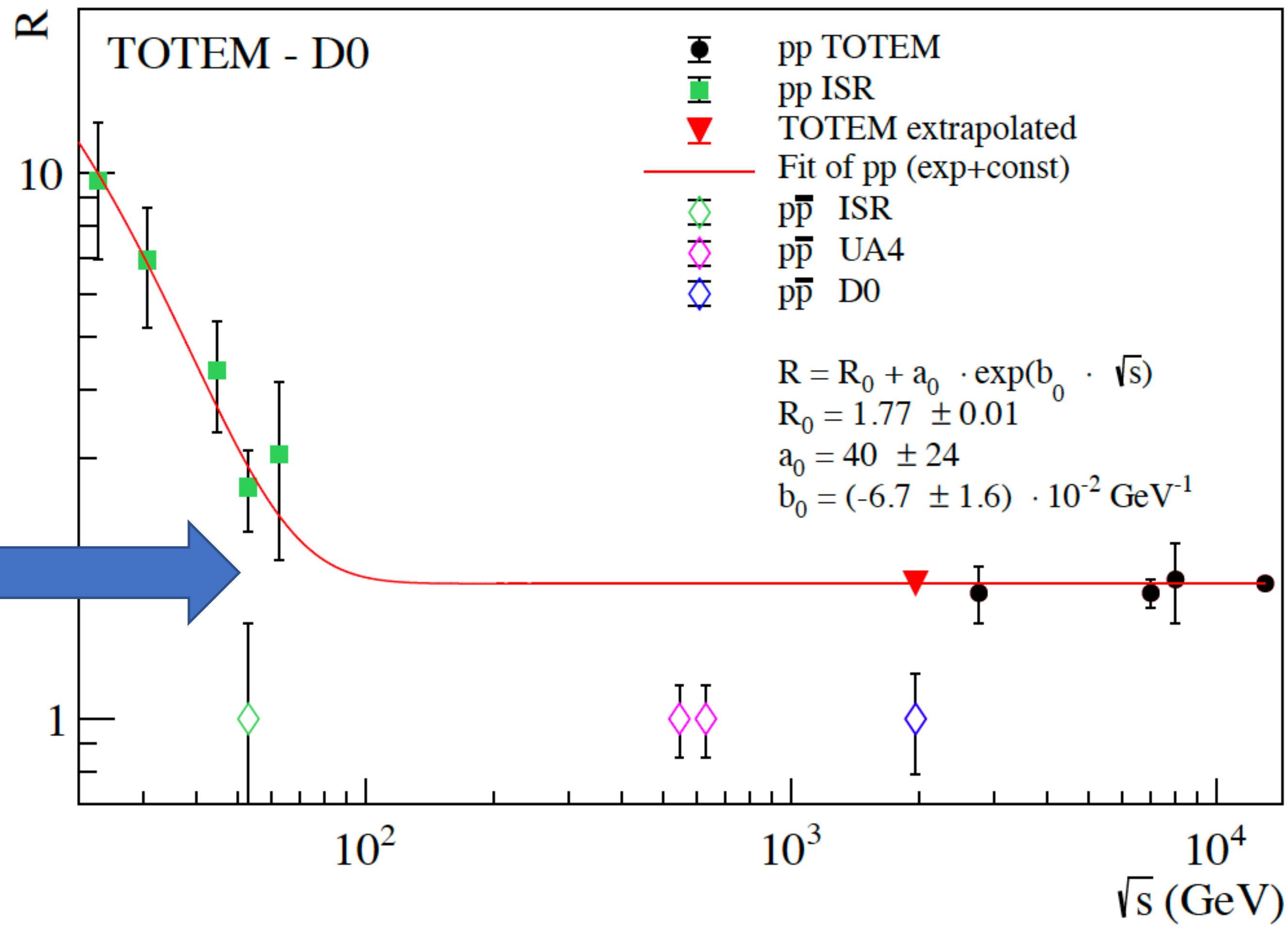


The fact that geometrical scaling is violated in the dip region has been attributed to the vanishing of the imaginary part of the scattering amplitude at $t_{\rm d}$. Whatever small the real part of the amplitude is, it takes over in the vicinity of $t_{\rm d} \pm \Delta t$. For $|t - t_{\rm d}| > |\Delta t|$ it is the imaginary part that dominates, and geometrical scaling is restored [14].

[14] J. Dias de Deus and P. Kroll, Acta Phys. Polon. B **9**, 157 (1978)

Bump/Dip behaviour



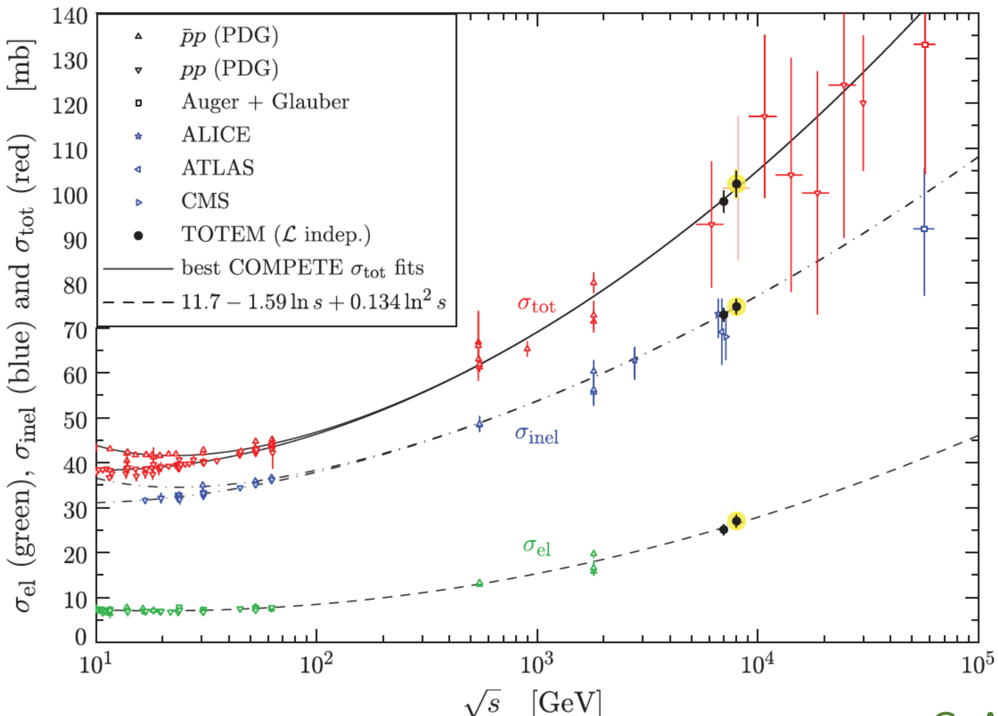


Hope for scaling at the LHC

V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

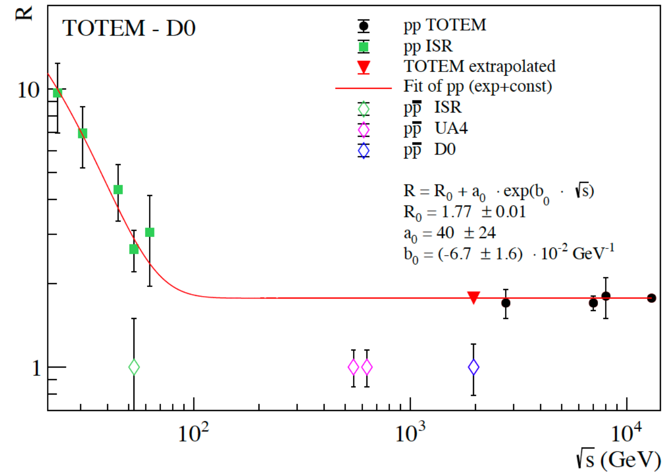
Scaling at the LHC?

	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.115}$	$W^{0.109}$	$W^{0.109}$	$W^{0.006}$	$0.02 - 0.095$
LHC	$W^{0.234}$	$W^{0.153}$	$W^{0.172}$	$W^{0.081}$	$0.15 - 0.10$



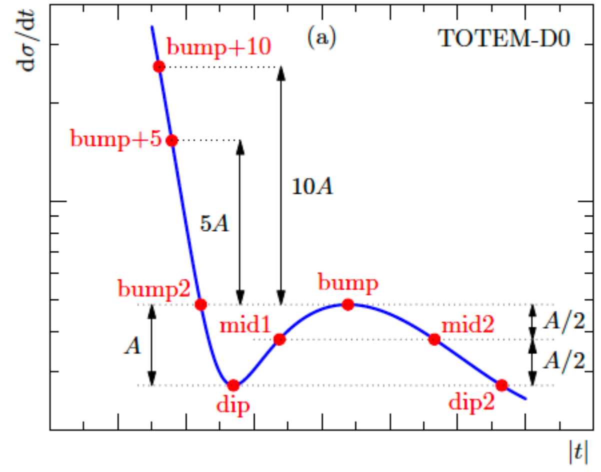
G. Antchev [TOTEM] PRL 111 (2013) 012001

An observation



	W	dip		bump		ratios	
		$\lvert t\rvert_{\mathrm{d}}$	$d\sigma_{\mathrm{el}}/dt$	$\lvert t\rvert_{\mathrm{b}}$	$d\sigma_{\mathrm{el}}/dt$	$t_{\mathrm{b}}/t_{\mathrm{d}}$	$\mathcal{R}_{\mathrm{bd}}$
LHC [TeV]	13.00	0.472	2.73×10^{-2}	0.638	4.80×10^{-2}	1.35	1.78
	8.00	0.525	1.54×10^{-2}	0.706	3.14×10^{-2}	1.37	1.79
	7.00	0.539	1.69×10^{-2}	0.705	2.64×10^{-2}	1.31	1.56
	2.76	0.618	0.87×10^{-2}	0.82	1.71×10^{-2}	1.33	1.97
ISR [GeV]	62.5	1.357	2.54×10^{-5}	1.818	5.99×10^{-5}	1.34	2.36
	52.8	1.393	2.53×10^{-5}	1.836	5.93×10^{-5}	1.32	2.55
	44.7	1.411	1.18×10^{-5}	1.870	5.40×10^{-5}	1.33	4.57
	30.7	1.459	0.20×10^{-5}	1.960	4.22×10^{-5}	1.34	21.11
	23.5	1.472	0.19×10^{-5}	1.964	4.52×10^{-5}	1.33	23.74

An observation



	W	dip		bump		ratios	
		$\|t\|_{\mathrm{d}}$	$d\sigma_{\mathrm{el}}/dt$	$\|t\|_{\mathrm{b}}$	$d\sigma_{\mathrm{el}}/dt$	$t_{\mathrm{b}}/t_{\mathrm{d}}$	$\mathcal{R}_{\mathrm{bd}}$
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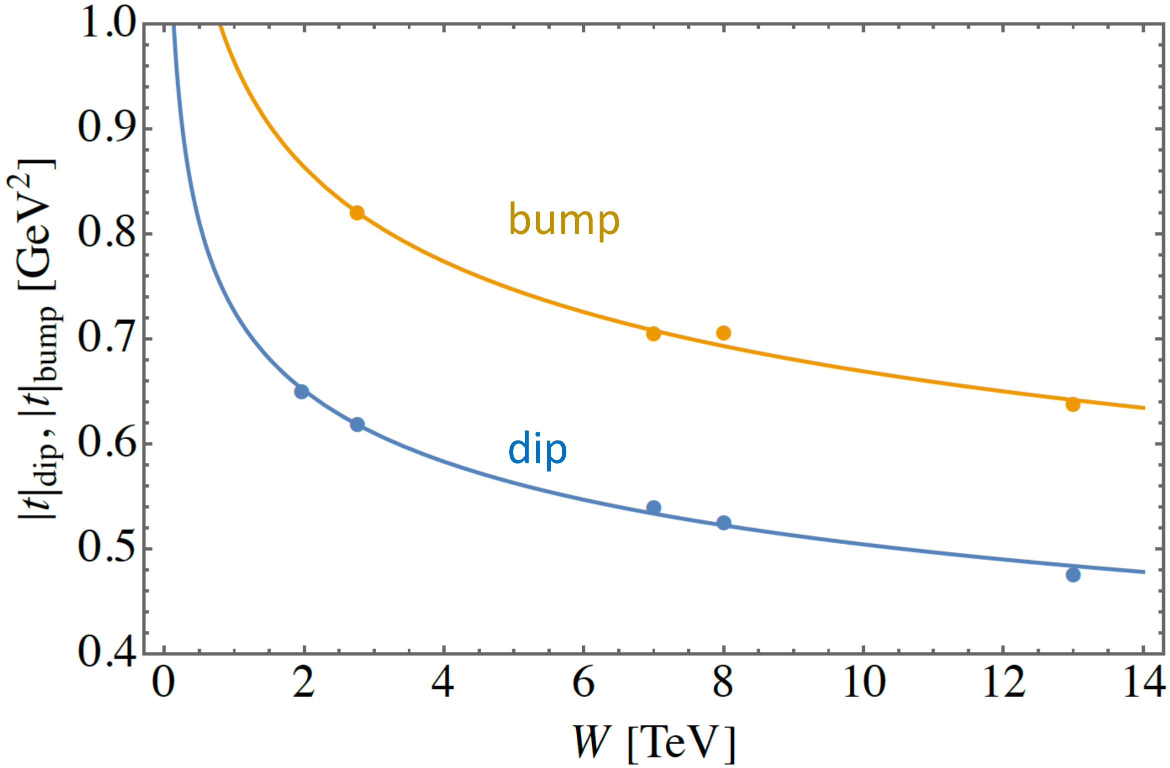
	23.5	1.472	0.19×10^{-5}	1.964	4.52×10^{-5}	1.33	23.74

An observation

The fact that $t_{\text{bump}}/t_{\text{dip}}$ = const. implies: $\tau = f(s)|t|$

$$|t|_{\text{dip}}(W) = 0.726 \times (W/(1\ \text{TeV}))^{-0.159}$$

$$|t|_{\text{bump}}(W) = 0.964 \times (W/(1\ \text{TeV}))^{-0.158}$$



An observation

The fact that $t_{\mathrm{bump}}/t_{\mathrm{dip}}$ = const. implies:

$$|t|_{\mathrm{dip}}(W) = 0.726 \times (W/(1\ \mathrm{TeV}))^{-0.159}$$

$$|t|_{\mathrm{bump}}(W) = 0.964 \times (W/(1\ \mathrm{TeV}))^{-0.158}$$

and therefore the scaling variable

$$\boxed{\tau = (W/(1\ \mathrm{TeV}))^{0.1585}\,|t|}$$

An observation

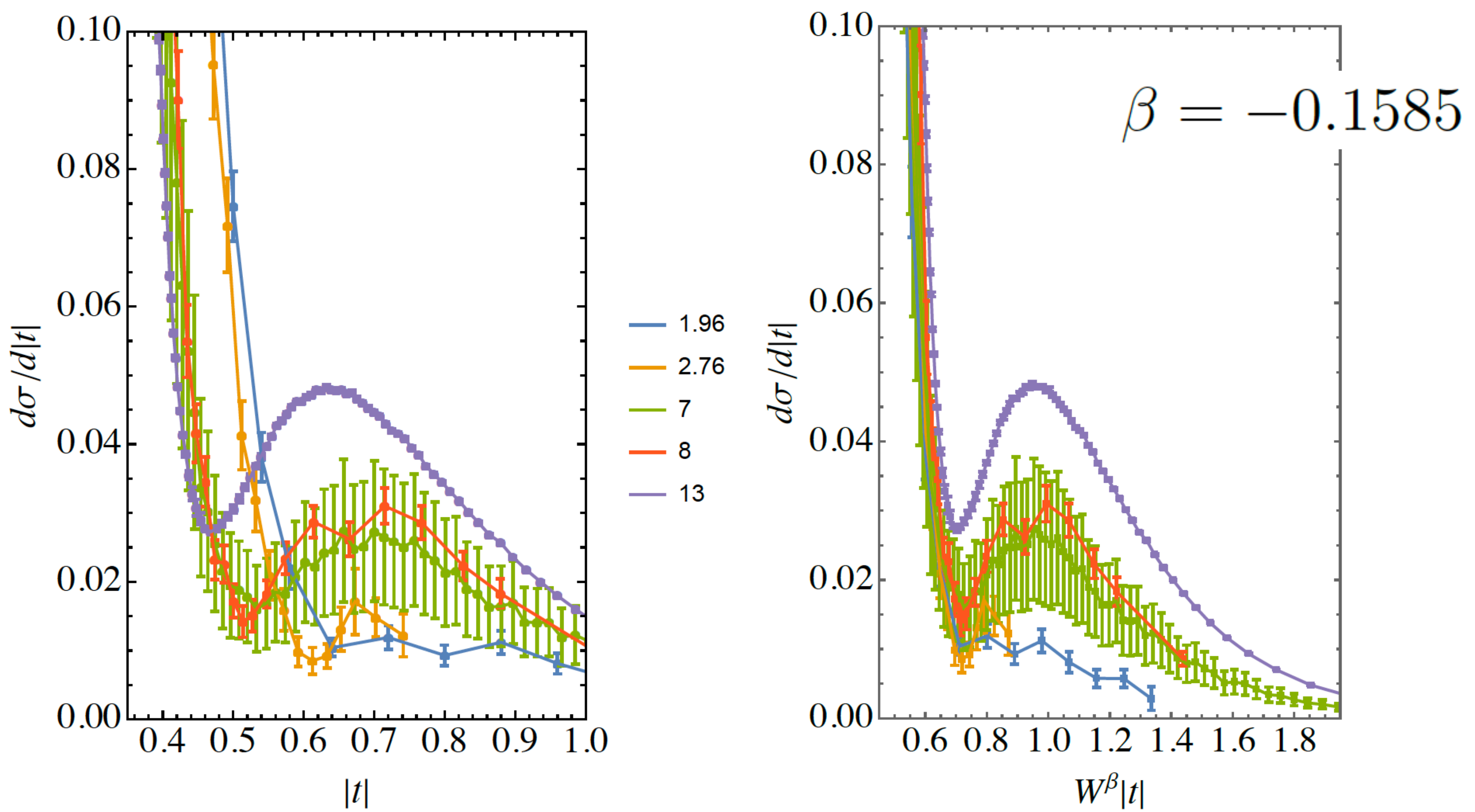
	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.115}$	$W^{0.109}$	$W^{0.109}$	$W^{0.006}$	$0.02 - 0.095$
LHC	$W^{0.234}$	$\boxed{W^{0.153}}$	$W^{0.172}$	$W^{0.081}$	$0.15 - 0.10$

and therefore the scaling variable

$$\boxed{\tau = (W/(1\ \text{TeV}))^{0.1585}\,|t|}$$

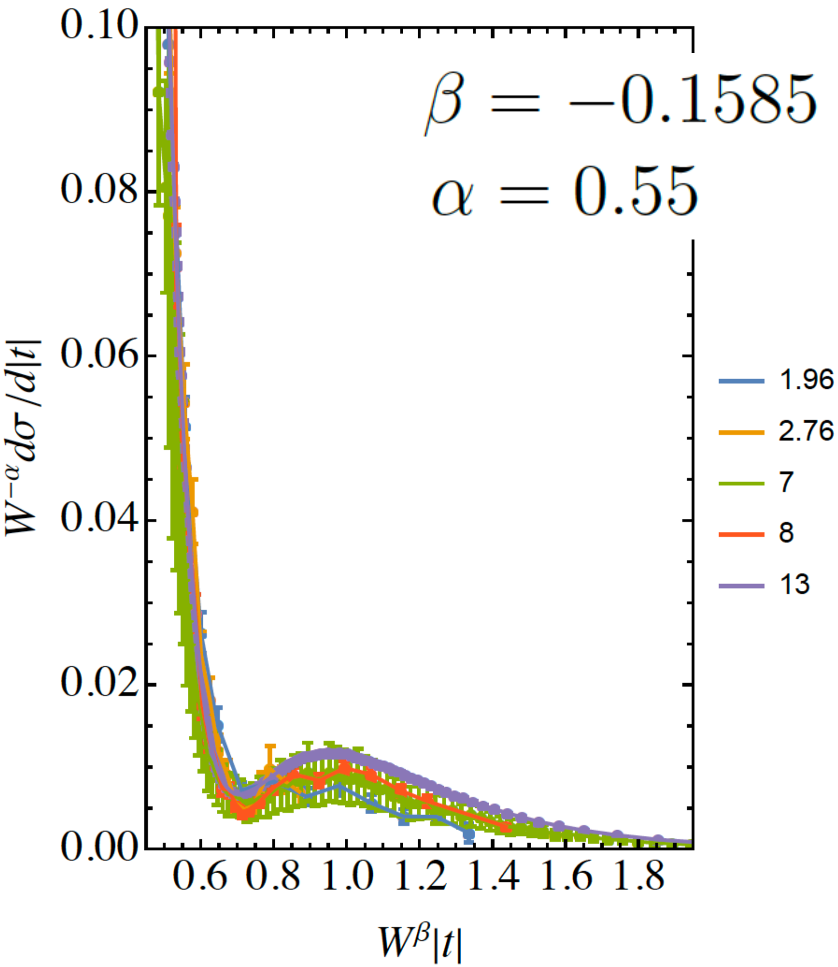
seems to follow the pattern of the ISR!

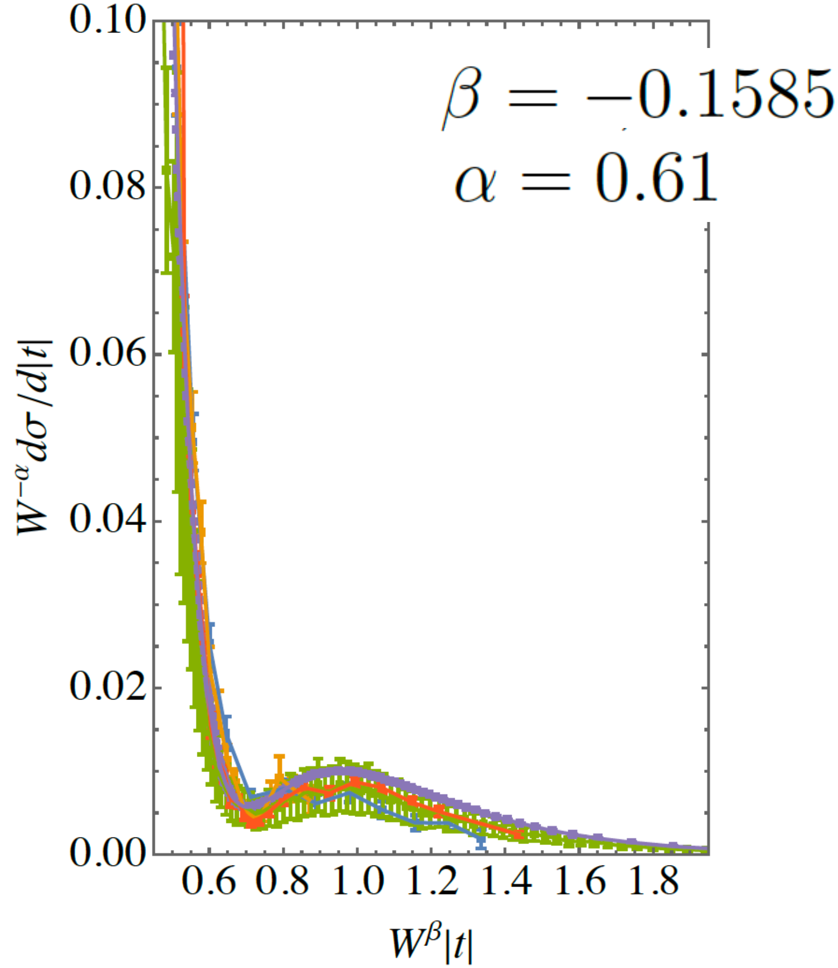
Scaling at the LHC – first step



Bump and dip positions are superimposed. Now we have to superimpose bump and dip values.

Scaling at the LHC – second step

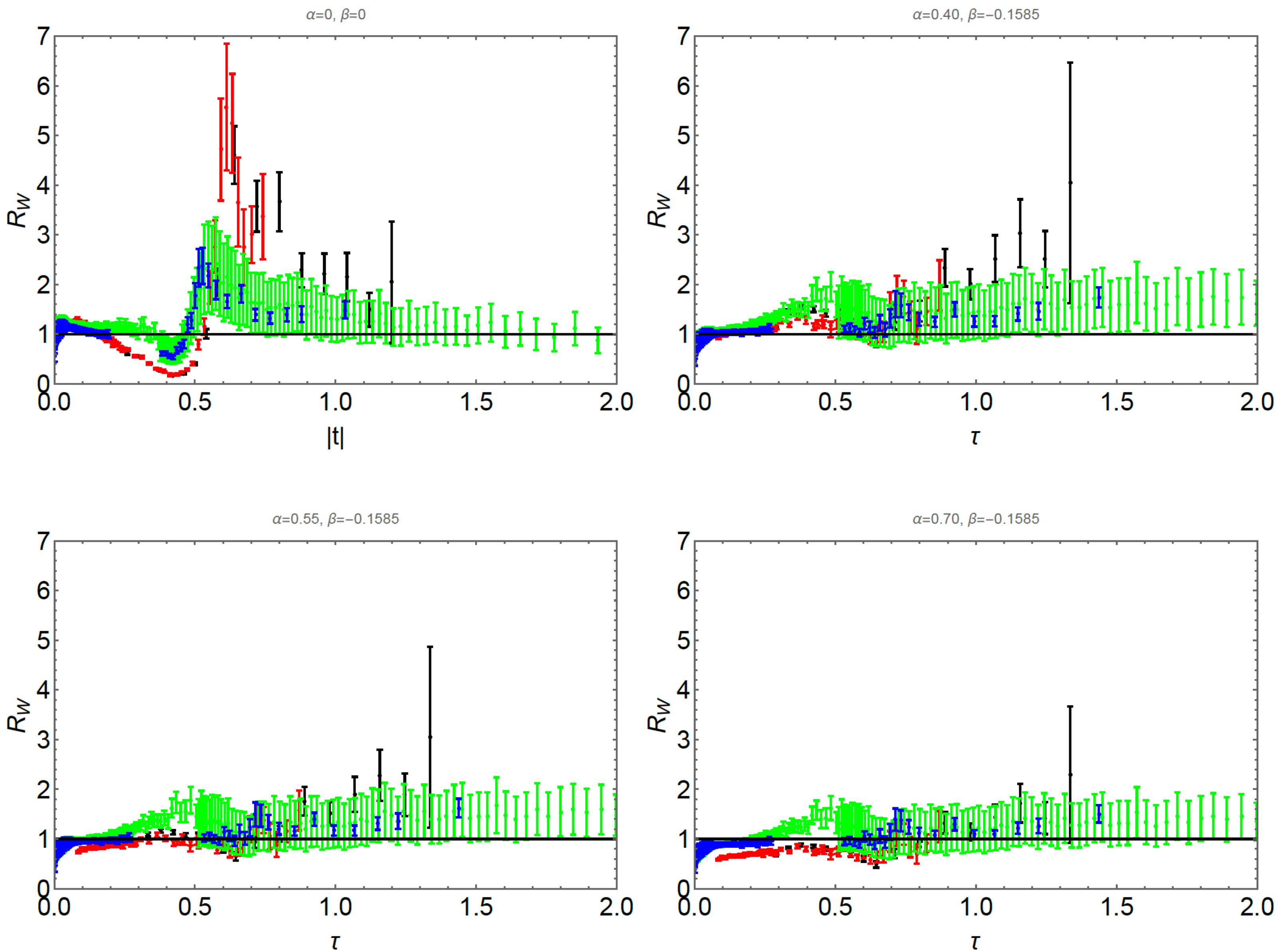




Scaling at the LHC – second step ratio method

$$W_{\mathrm{ref}} = 13\ \mathrm{TeV}$$

$$R_W(\tau_i) = \frac{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W_{\mathrm{ref}}, \tau_i)}{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W, \tau_i)}$$



α=0, β=0
α=0.40, β=−0.1585
α=0.55, β=−0.1585
α=0.70, β=−0.1585
R_W
|t|
τ
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5
6
7

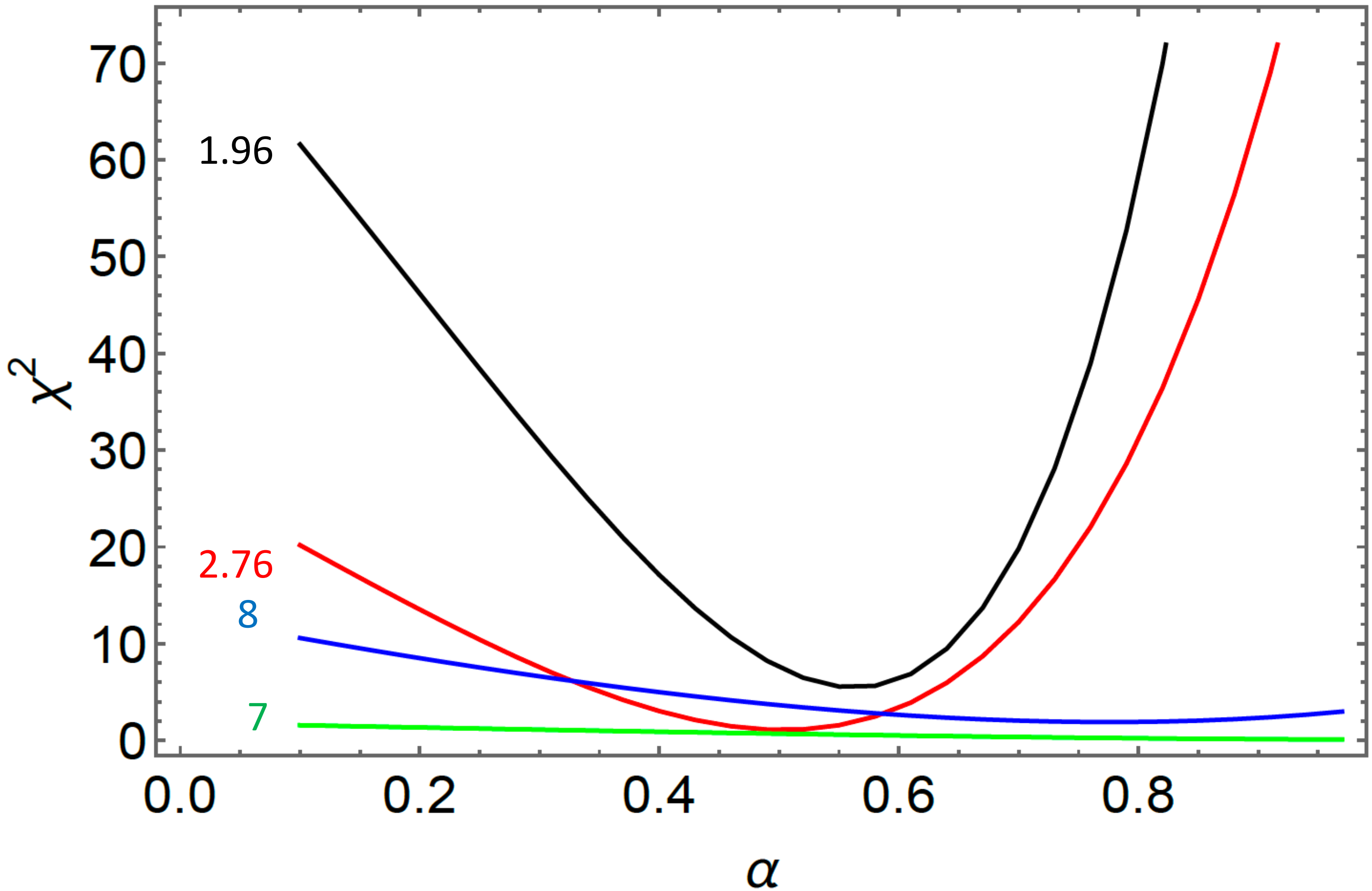
A few observations

- poor quality of lower energy data
- hard to find the best value of *α* "by an eye"
- try χ^2

$$\chi^2(W) = \frac{1}{N_W - n_{\rm par}} \sum_{i=1}^{N_W} \left(\frac{R_W(\tau_i) - 1}{\delta R_W(\tau_i)} \right)^2$$

$$0.35 \ \mathrm{GeV}^2 < |t| < 1.5 \ \mathrm{GeV}^2$$

Fixing α



A few observations

- poor quality of lower energy data
- hard to find the best value of α “by eye”
- try χ^2
- best value of α is determined by the lowest energy data
- 7 and 8 TeV data have large errors, χ^2 is flat
- small $|t|$ and large $|t|$ points do not scale as well as at the ISR
- no universal power for τ and normalization
- no problem with scaling in the dip region

Other scaling laws

Physics Letters B 830 (2022) 137141

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Scaling properties of elastic proton-proton scattering at LHC energies

C. Baldenegro [a], C. Royon [b,*], A.M. Stasto [c]

[a] École Polytechnique, Laboratoire Leprince-Ringuet, Av. Chasles, 91120 Palaiseau, France
[b] Department of Physics and Astronomy, The University of Kansas, Lawrence, KS 66045, USA
[c] Department of Physics, Penn State University, University Park, PA 16802, USA

$$\frac{1}{s^{\alpha/2}} \frac{d\sigma_{\text{el}}}{d|t|}(s,t) = \Phi(\tau) \qquad \tau = s^a t^b$$

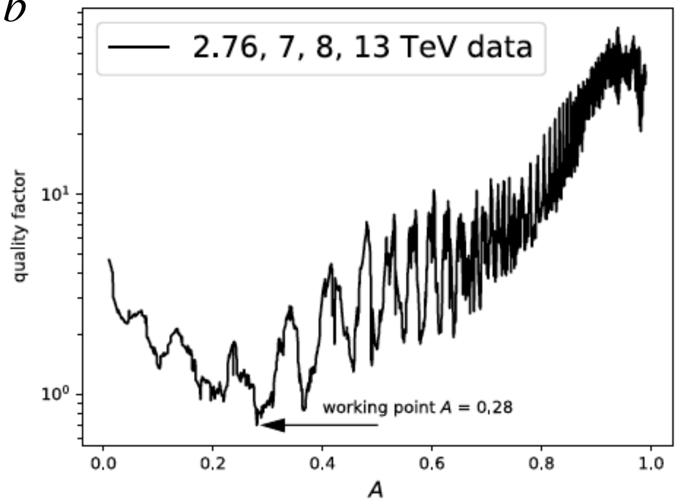
Baldenegro, Royon, Staśto

$$\frac{1}{s^{\alpha/2}} \frac{d\sigma_{\mathrm{el}}}{d|t|}(s,t) = \Phi(\tau) \qquad \tau = s^a t^b$$

Using quality factor method they find

$$\alpha \simeq 0.61 \qquad a \simeq 0.065 \qquad b \simeq 0.72$$

$A = 1 - b$



W TeV	χ^2 this paper	[9]
1.96	5.56	9.71
2.76	1.57	1.39
7	0.56	0.66
8	3.09	3.27

Other scaling laws

$$\frac{1}{s^{\alpha/2}}\frac{d\sigma_{\rm el}}{d|t|}(s,t)=\Phi(\tau) \qquad\qquad \tau=s^a t^b$$

In terms of variable τ positions of dips (and bumps) should be the same at *all* energies. We know from $t_{\rm bump}/\,t_{\rm dip}$=const. that $t_d=s^{\beta/2}B_{\rm dip}$

Hence,

$$\tau_{\rm d}=s^a t_{\rm d}^b=s^{a+b\,\beta/2}B_{\rm dip}^b$$

is energy independent. Therefore

$$a+b\,\beta/2=0$$

Other scaling laws

$$a + b\,\beta/2 = 0 \qquad (*)$$

Experimental fact at the LHC energies

$$\beta \ = \ -0.1585$$

Baldenegro, Royon, Staśto fit:

$$a \simeq 0.065 \qquad b \simeq 0.72$$

Substituting their b to the constraint (*)

$$a \ = \ 0.057$$

9% off

Other scaling laws

BRS method prefers to misalign dip and bump positions to have better overlap in off-dip and off-bump regions.

If we want to have dips and bumps positions aligned at all LHC energies with scaling variable $\tau = s^a t^b$ condition

$$a + b\,\beta/2 = 0$$

with $\beta = -0.1585$ must be satisfied. Best value of a has to be found by fitting.

Amplitude parametrizations

One commonly uses two exponent parametrizations of elastic amplitude

$$\mathcal{A}(s,t) = i\left(\mathcal{A}_1(s,t) + \mathcal{A}_2(s,t)e^{i\phi}\right)$$

with

$$\mathcal{A}_i(s,t) = N_i(s)\, e^{-B_i(s)|t|}$$

Solving $t_{\text{bump}}/\, t_{\text{dip}}$=const. condition gives

$$N_i(s) = n_i N(s) \quad \text{and} \quad B_i(s) = b_i B(s)$$

Hard disk diffraction – Airy pattern

$$\frac{|t|_{\mathrm{bump}}}{|t|_{\mathrm{dip}}} = 1.34$$

$$A(s,t) = 2is \int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left[1 - e^{-\Omega(s,b)}\right]$$

Hard disk diffraction – Airy pattern

$$\frac{|t|_{\rm bump}}{|t|_{\rm dip}} = 1.34$$

$$A(s,t) = 2is\int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \overbrace{\left[1 - e^{-\Omega(s,b)}\right]}^{\Theta(b - R(s))}$$

Hard disk diffraction – Airy pattern

$$\frac{|t|_{\rm bump}}{|t|_{\rm dip}} = 1.34$$

$$A(s,t) = 2is\int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\overbrace{\left[1 - e^{-\Omega(s,b)}\right]}^{\Theta(b-R(s))} = 4\pi isR^2\frac{1}{R\sqrt{|t|}}J_1\left(R\sqrt{|t|}\right)$$

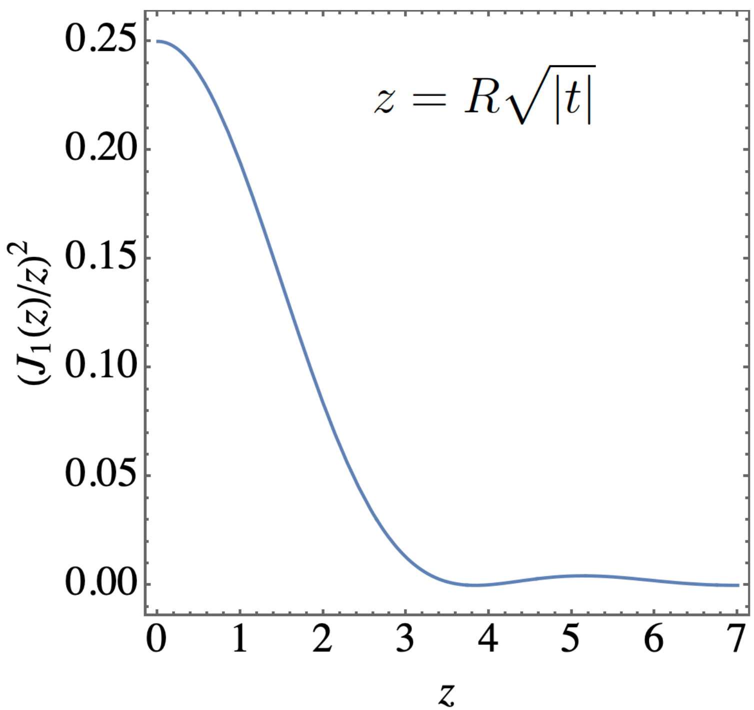
Hard disk diffraction – Airy pattern

$$\frac{|t|_{\text{bump}}}{|t|_{\text{dip}}} = 1.34$$

$$A(s,t) = 2is \int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \overbrace{\left[1 - e^{-\Omega(s,b)}\right]}^{\Theta(b-R(s))} = 4\pi i s R^2 \frac{1}{R\sqrt{|t|}} J_1\left(R\sqrt{|t|}\right)$$

$$\begin{aligned} z_{\text{dip}} &= 3.83 \\ z_{\text{bump}} &= 5.13 \end{aligned}$$

$$\frac{z_{\text{bump}}}{z_{\text{dip}}} = 1.34$$



Hard disk diffraction – Airy pattern

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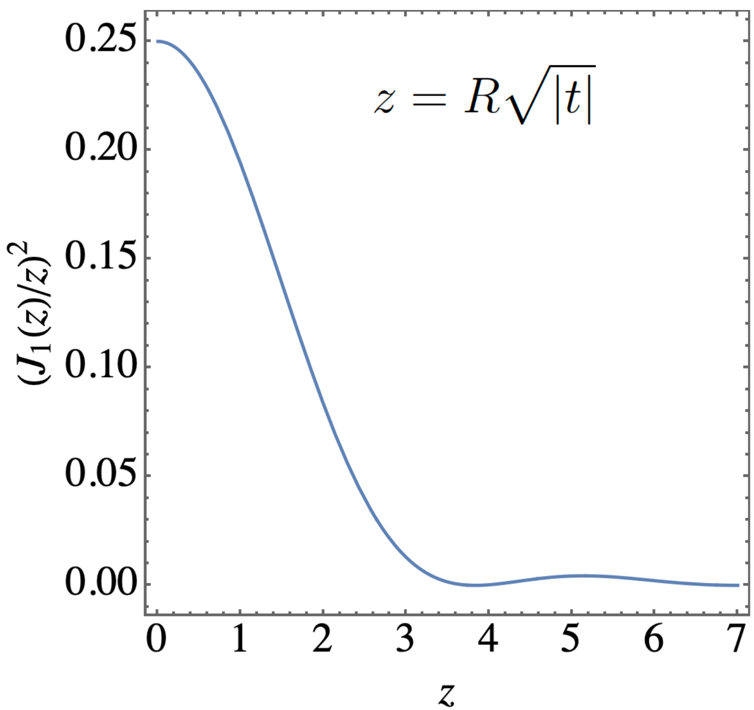
$$\frac{\sqrt{|t|_{\rm bump}}}{\sqrt{|t|_{\rm dip}}} = 1.34$$

$$\frac{z_{\rm bump}}{z_{\rm dip}} = 1.34$$



Summary and Conclusions

- The fact that ratio of the bump to dip positions ($t_{\rm b}/t_{\rm d}$) of $d\sigma_{\rm el}/d|t|$ is constant over the wide energy range (ISR to LHC) suggests the scaling variable $\tau = f(s)|t|$;
- Despite lower energy the ISR data are better prone to geometrical scaling because elastic, inelastic and total cross-sections have the same (within uncertainties) energy dependence;
- On theoretical side this is due to the fact that at the ISR geometrical scaling is present at the lowest values of $|t|$, which is not the case at the LHC where the integrated cross-sections have large contributions from the non-scaling region at small $|t|$;
- Violation of geometrical scaling of the ISR data at the dip has been attributed to the vanishing of the imaginary part of the elastic amplitude in the vicinity of $t = t_{\rm d}$, such violation does not happen at the LHC;
- Both at the ISR and LHC energies it seems that $f(s) \sim \sigma_{\rm inel}(s)$;
- Simple scaling law $\tau = f(s)|t|$ is not the only one that makes dip and bump positions energy independent, there exists in fact a family of such transformations, that may lead to equally good scaling laws.
- The fact that $t_{\rm b}/t_{\rm d}$ is energy independent constrains possible parametrizations of the elastic scattering amplitude.