Kinetic equation for the early stage

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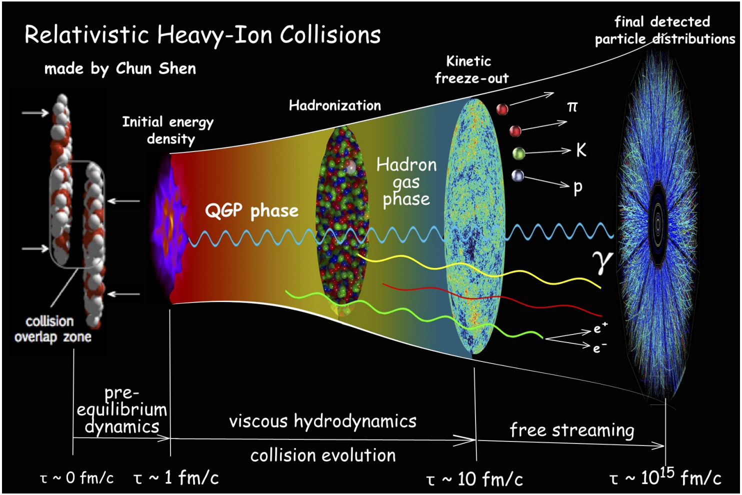
Collision dynamics



Chun Shen, OSU

Hydrodynamic regime

- energy-momentum tensor

$$T_{id}^{\mu\nu} = (\epsilon + p)g^{\mu\nu} - Pu^{\mu}u^{\nu} + \pi^{\mu\nu}$$

- close to local equilibrium
- local energy density ϵ, pressure p, flow u^{μ}, stress (viscous) corrections $\pi^{\mu\nu}$
- local momentum distribution close to equilibrium

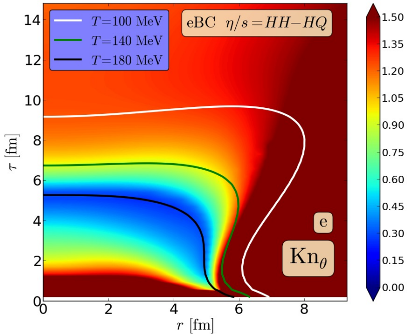
$$f(p) = f_{eq}(p) + \delta f(p)$$

$f(p)$ not explicitly given

Early times

- large gradients
- viscous corrections dominant

Knudsen number $K = l_{micro}/L_{macro}$



H. Niemi, G. Denicol, , arXiv: 1404.7327

Kinetic equation

equation for the phase-space distribution $f(x,p)$

$$p^\mu \partial_\mu f(x,p) = C\left[f(x,p)\right]$$

$C\left[f(x,p)\right]$ collisions integral ($2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ processes)

- for on-shell particles $f(t, \mathrm{x}, \mathrm{p})$ 7-dimensional function

- more general than viscous hydrodynamics

- most solutions for boost-invariant geometry

boost-invariant, relaxation-time approximation

- relaxation time approximation

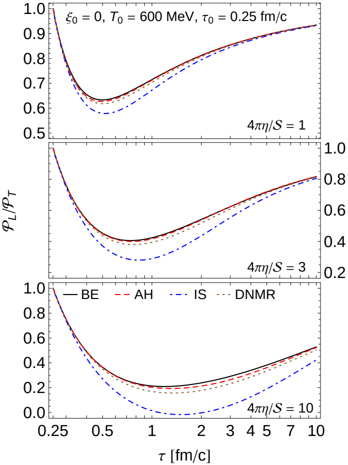
$$p^\mu \partial_\mu f(x,p) = \frac{1}{\tau_{relax}} \left(f(x,p) - f_{eq}(x,p) \right)$$

- boost invariant solution

$$f(x,p) = f(t, p_\perp, x_\perp, w)$$

$w = tp_{||} - zE_p$

testing hydrodynamics



Nonequilibrium
$P_L \neq P_T$

larger viscosity η
$\updownarrow$
larger relaxation time τ_{relax}
(less collisions)

W. Florkowski, R. Ryblewski, M. Strickland,

arXiv 1304.0665

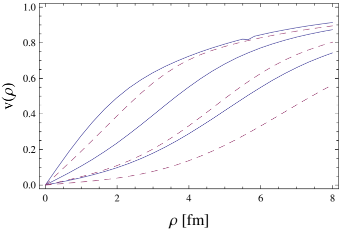
free-streaming + hydrodynamics

- free-streaming until τ_{hydro}
- matching to fluid $T^{\mu\nu}$ at τ_{hydro}

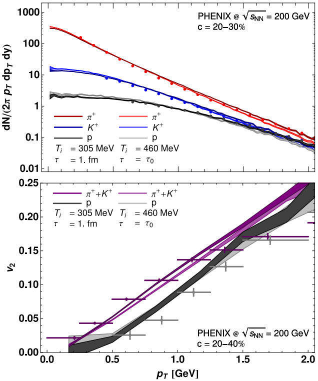
$\longrightarrow$ pre-equilibrium flow

$$v_T \simeq -\frac{\tau_{hydro}-\tau_0}{3}\frac{\nabla_T n(x,y)}{n(x,y)}$$

universal flow J. Vredevoogt, S. Pratt, arXiv: 0810.4325

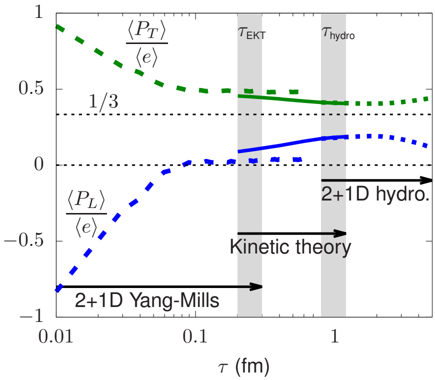


W. Broniowski, W. Florkowski, M. Chojnacki, A. Kisiel, arXiv: 0812.3393



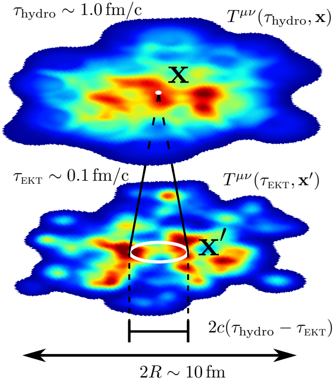
hybrid model

kinetic equation + hydrodynamics



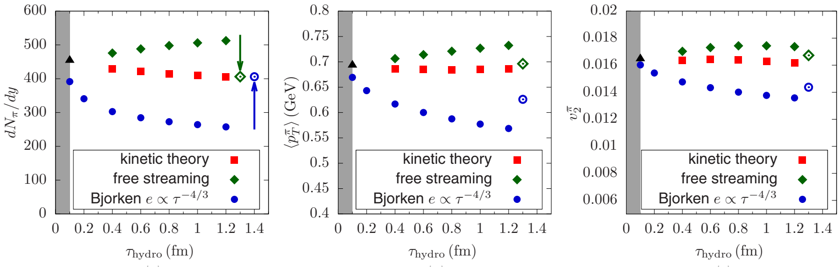
A. Kurkela, A. Mazeliauskas, J. F. Paquet, S. Schlichting, D. Teaney arXiv: 1805.01604

$$\delta T_{x}^{\mu\nu}(\tau_{hydro}, x) = \int d^2x' \; G_{\alpha\beta}^{\mu\nu}\left(x, x', \tau_{hydro}, \tau_{ekt}\right) \times \delta T_{x}^{\alpha\beta}(\tau_{ekt}, x') \frac{\overline{T}_{x}^{\tau\tau}(\tau_{hydro})}{\overline{T}_{x}^{\tau\tau}(\tau_{ekt})}$$



A. Kurkela, A. Mazeliauskas, J. F. Paquet, S. Schlichting, D. Teaney arXiv: 1805.01604

Early dynamics gives a transverse push

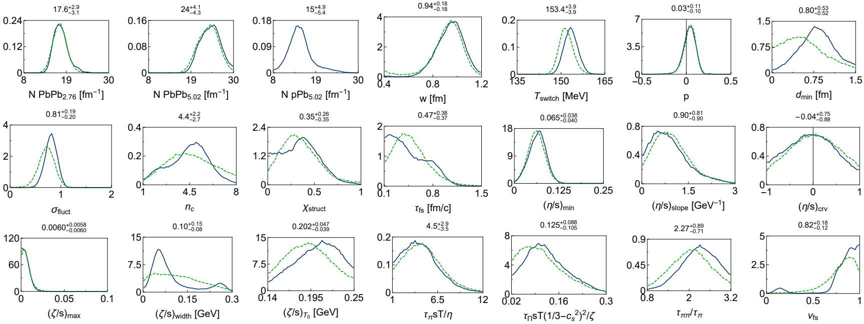


A. Kurkela, A. Mazeliauskas, J. F. Paquet, S. Schlichting, D. Teaney arXiv: 1805.00961

hydrodynamics - free streaming - kinetic equations : SMALL differences

free-streaming+hydro

Bayesian analysis - includes identified spectra



G. Nijs, W. van der Schee, U. Gürsoy, R. Snellings arXiv: 2010.15130

- free-streaming with $v_s = 0.82 < 1$
- short free-streaming evolution $\tau_{fs} = 0.47\text{fm/c}$

Kinetic equations in RTA

- boost-invariant
- relaxation time approximation
- massless particles $\longrightarrow$ evolution of an integral (simplification)

$f(\tau, \vec{x}_T, \vec{p}_T, p_{\|}, w) \longrightarrow F(\tau, \vec{x}_T, \phi, v_z) = \int p^3 dp f(\tau, x_T, p_T, p_{\|}, w)$
at $z = 0$

$$v = (1, \vec{v}_T, v_z) = (1, \sqrt{1 - v_z^2} cos\phi, \sqrt{1 - v_z^2} sin\phi, v_z)$$

$$\partial_\tau F + \vec{v}_T \partial_{\vec{x}_T} - \frac{v_z}{\tau}(1 - v_z^2)\partial_{v_z} F + \frac{4v_z^2}{\tau} = \frac{1}{\tau_{relax}} (F - F_{iso})$$

Energy-momentum tensor can be reconstructed from F

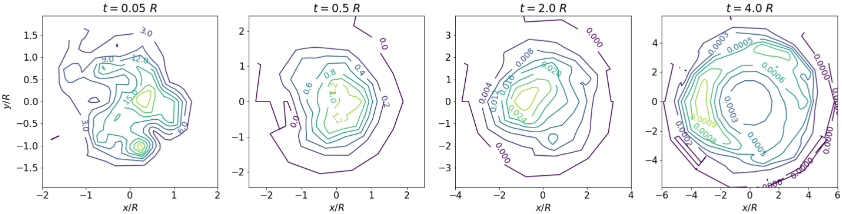
$$T^{\mu\nu}(\tau, \vec{x}_T) = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-1}^{1} \frac{dv_z}{2} F(\tau, \vec{x}_T, \phi, v_z)$$

A. Kurkela, U. Wiedemann, B. Wu : arXiv: 1803.02072

Kinetic evolution in 2+1 dim

boost-invariant, small longitudinal pressure
$F = \delta(v_z)\tilde{F}$

$$\partial_\tau \tilde{F} + v_T \partial_T \tilde{F} + \frac{1}{\tau}\tilde{F} = \frac{1}{\tau_{relax}}\left(\tilde{F} - \tilde{F}_{iso}\right)$$



A. Kurkela, S.F. Taghavi, U. Wiedemann, B. Wu, arXiv: 2007.06851

kinetic equations in 1+1+1 dim

- evolution in time t, longitudinal z, and one transverse direction x

$$\partial_t F + v_x \partial_x F + v_z F = \frac{1}{\tau_{relax}} (F - F_{iso})$$

- variables τ, x, η, ϕ, v_z : $F(\tau, x, \eta, \phi, v_z)$

- expansion in azimuthal angle:

$$F = F_0 + 2cos(\phi)F_1 + \dots$$

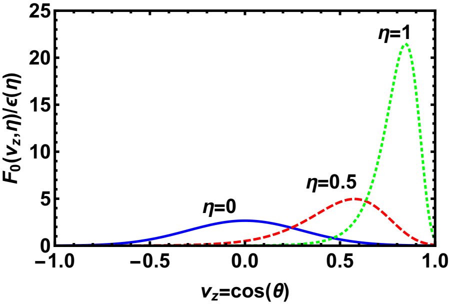
coupled equations for F_0 and F_1
solved in the global frame

F in boosted frame

Initial conditions $F = \epsilon g(v_z)$ (note $v_z = cos(\theta)$)

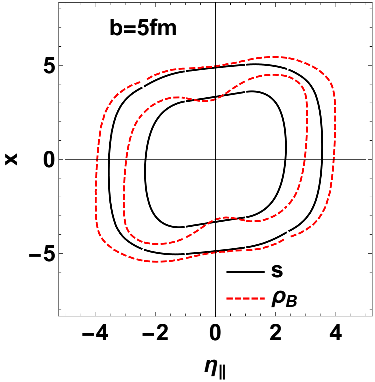
after boost by rapidity $y = \eta$ (Bjorken flow)

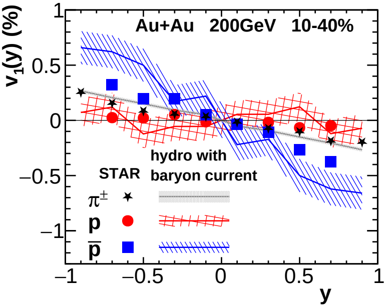
$$F = \epsilon \frac{g((v_z cosh(y) - sinh(y))/(cosh(y) - v_z sinh(y)))}{(cosh(y) - v_z sinh(y))^4}$$



- difficult numerics at large rapidities

tilted fireball initial conditions

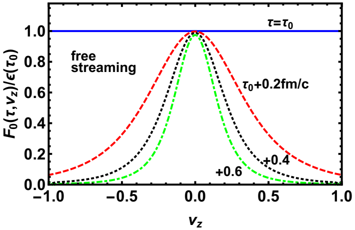


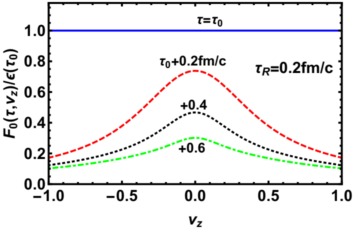


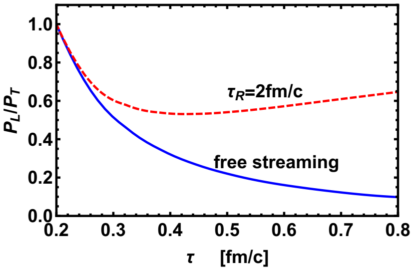
can describe the directed flow with early start of hydrodynamics

τ_{hydro}=0.2fm/c

longitudinal evolution

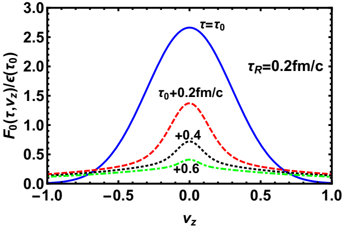


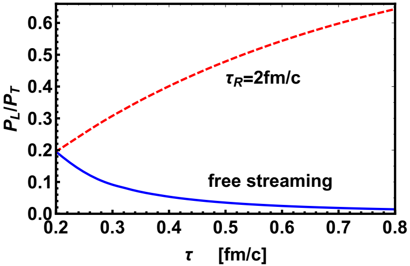




- redshift $\longrightarrow$ reduction of longitudinal pressure
- collisions $\longrightarrow$ isotropization

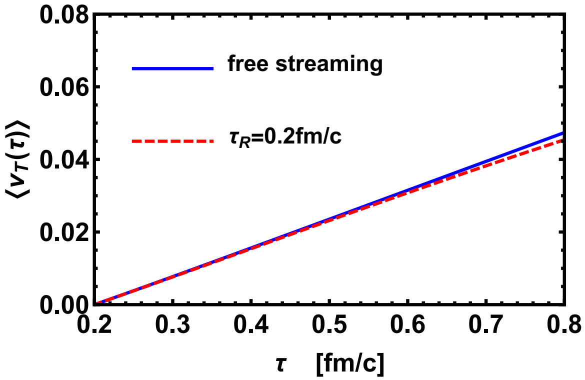
pressure asymmetry in initial conditions





nonisotropic (non-equilibrium) initial conditions can be implemented

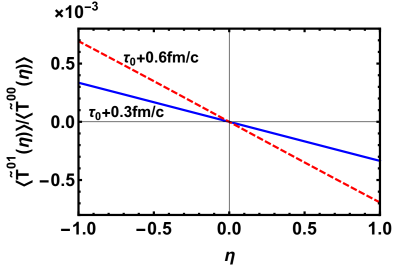
transverse flow

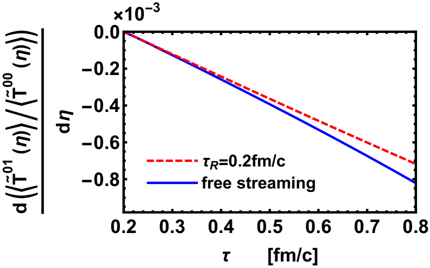


universal transverse flow

directed flow

$\tau_{relax} = 0.2\text{fm/c}$

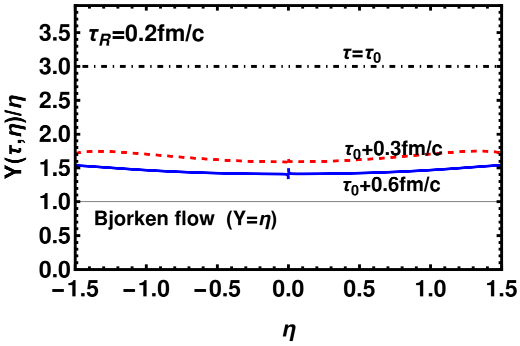




directed flow builds up in the early stage

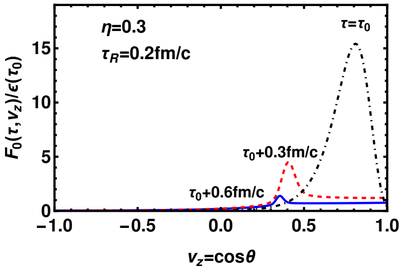
pre-Bjorken flow in 1+1 dim

- non Bjorken flow in the initial conditions $Y \simeq 3\eta$

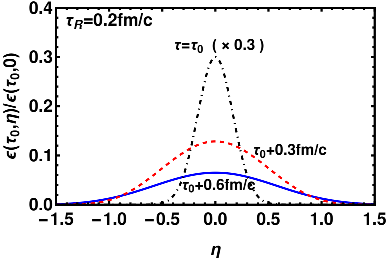


- onset of the Bjorken flow

local distribution at $Y \simeq \eta$ forms



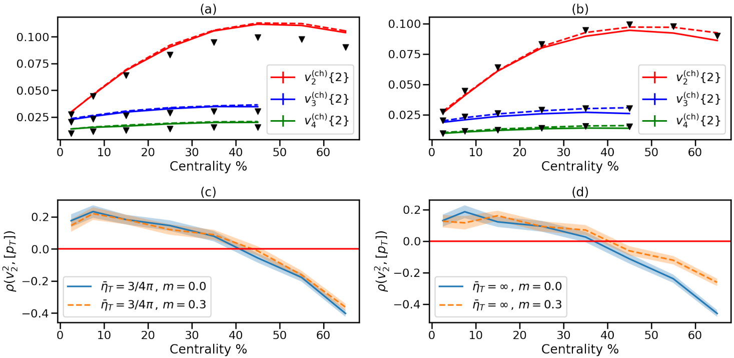
broadening ($\times 3$)
reflects the formation of Bjorken flow



- Kinetic equation for early dynamics
- Non-boost invariant solutions
- Early formation of directed flow
- Effects of non-Bjorken flow ?

RTA pre-equilibrium + hydrodynamics

relaxation time isotropization	free-streaming



D. Liyanage, D. Everett, C. Chattopadhyay, U. Heinz arXiv: 2205.00964