Relativistic Spin-hydrodynamics

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Based On : PLB 814 (2021) 136096; PRD 103, 014030 (2021); PRL 129, 192301 (2022)

Heavy-ion Collisions :

- Signatures of Quark-Gluon Plasma phase found in collider experiments.

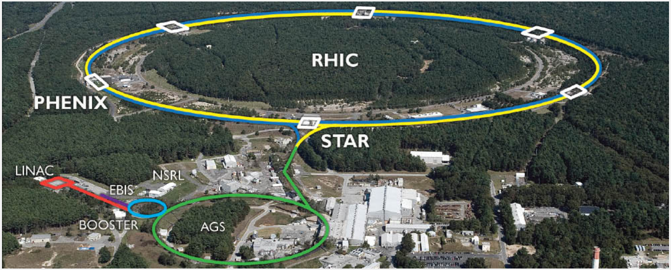


Figure 1: Relativistic Heavy-Ion Collider, BNL. [U.S. Department of Science.]

Figure 2: Large Hadron Collider, CERN. [FORBES, 2016.]

Features of Non-central Collisions :

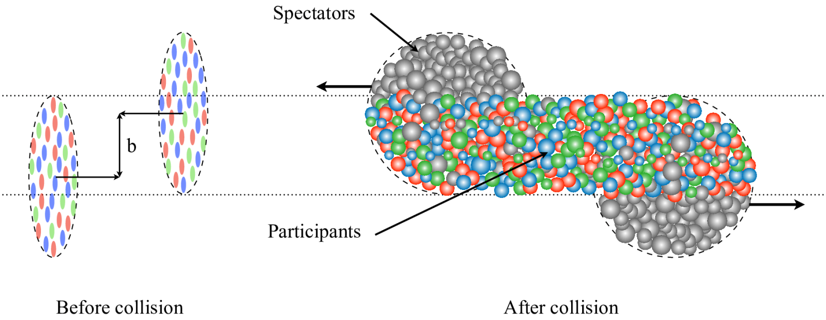


Figure 3: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- General properties of the matter produced :
  - Behaves like a fluid (Hydrodynamics applicable).
  - The viscosity (η/s) is lowest (Dissipative hydrodynamics required).
  - The vorticity is highest (for non-central collisions).

  [P. Kovtun, D. T. Son and, A. Starients, PRL 94, 111601 (2005); STAR Collaboration, Nature 548, 62 (2017)]

Features of Non-central Collisions :

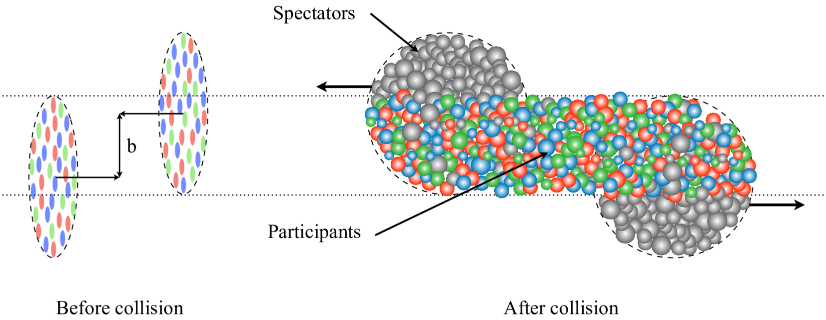


Figure 3: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Non-central collisions ($b \neq 0$) are more common.

- Special feature of Non-Central Collisions :
  - – Large Angular Momentum.
  - – Large Magnetic Field. [A. Bzdak and, V. Skokov, PLB 710 (2012) 171–174]
  - – Finite particle polarization at small energies.

Generation of Angular Momentum :

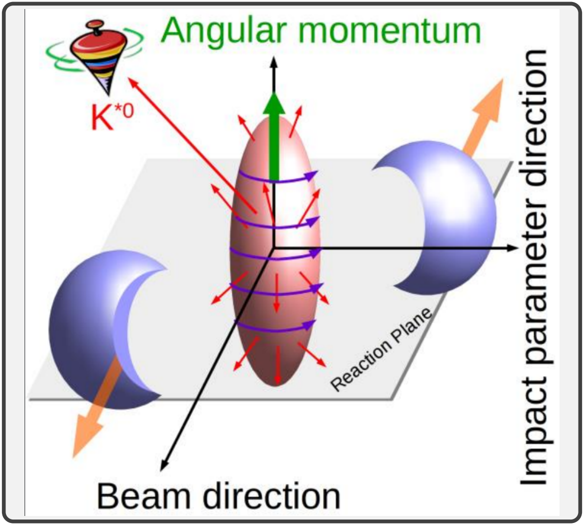


Figure 4: Generation of angular momentum in non-central collisions. [B. Mohanty, ICHEP 2020]

Generation of Angular Momentum :

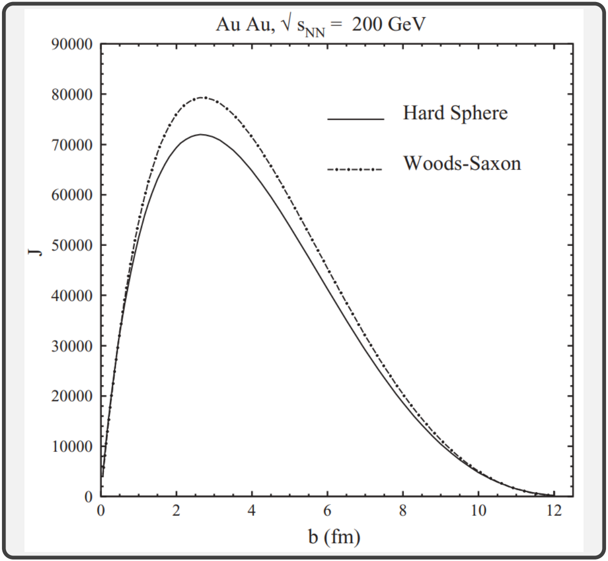


Figure 5: Angular momentum vs impact parameter. [Becattini, Piccinini and, Rizzo, PRC 77 (2008) 204906]

Generation of Magnetic Field :

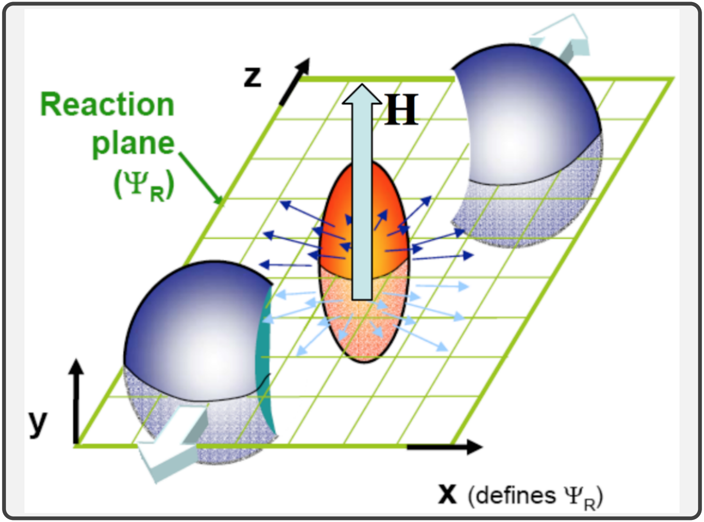


Figure 6: Generation of magnetic field in non-central collisions. [D. E. Kharzeev, PPNP 75 (2014) 133–151]

Generation of Magnetic Field :

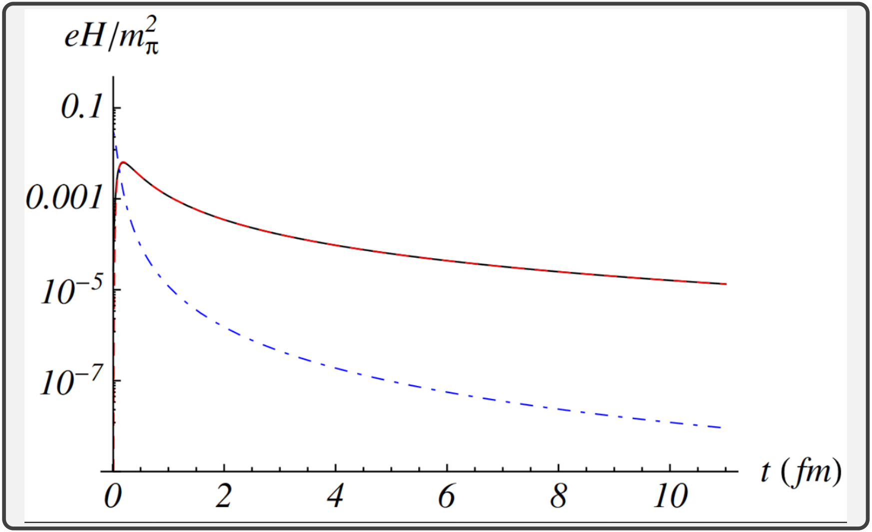


Figure 7: Time evolution of magnetic field. [K. Tuchin, IJMPE 23, No. 1 (2014) 1430001]

Particle Polarization :

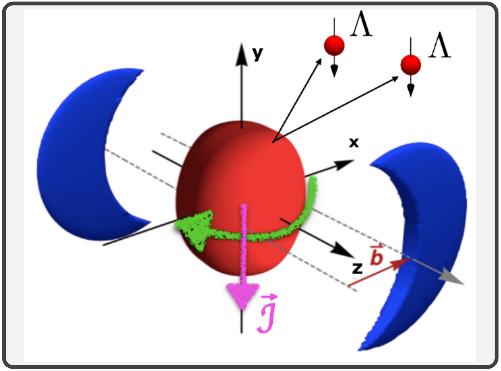


Figure 8: Origin of particle polarization. [W. Florkowski *et al*, PPNP 108 (2019) 103709]

- Large angular momentum → Local vorticities → spin alignment.

  [Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005); Phys. Lett. B 629, 20 (2005)]

Particle Polarization :

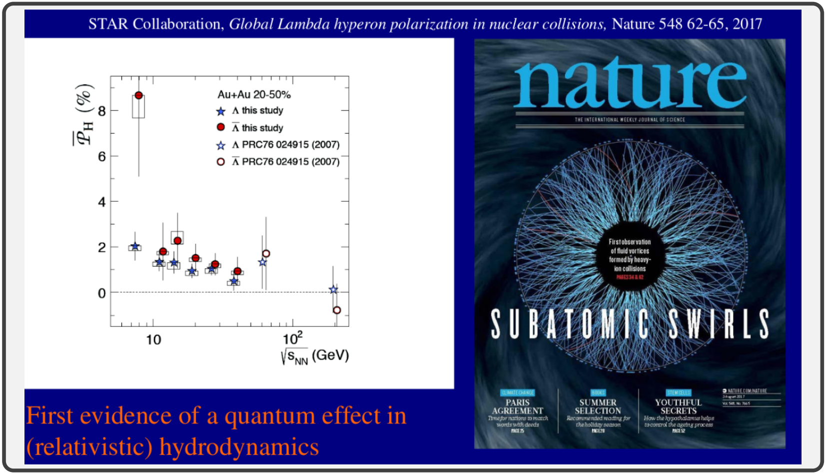


Figure 9: First observation of Λ-hyperon polarization. [F. Becattini - 'Subatomic Vortices'.]

- STAR collaboration of RHIC provided the first experimental evidence.

  [STAR Collaboration, Nature 548, 62 (2017), Phys. Rev. Lett. 123, 132301 (2019), Phys. Rev. Lett. 126, 162301 (2021)]

- Theoretical models assuming equilibration of spin d.o.f. explains the data.

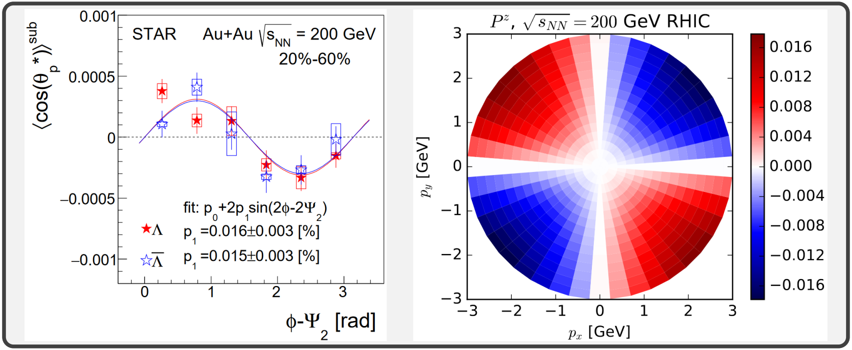


Figure 10: Observation (L) and prediction (R) of longitudinal polarization.
[Left: PRL **123** 132301 (2019); Right: PRL **120** 012302 (2018)]

- Theoretical models assuming equilibration of spin d.o.f. predict the opposite sign.
- Not enough time to thermalize. Dissipative forces at play?

Spin-polarization in Heavy-ion Collisions :

- Spin-polarization of two kinds have been studied -
  1. The Global Spin-Polarization (GSP). [STAR Collaboration, Nature 548, 62 (2017)]
  2. The Longitudinal Spin-Polarization (LSP). [STAR Collaboration, PRL 123, 132301 (2019)]

- **Explanation of these two effects?**
  — Theoretical models developed assuming equilibration of spin d.o.f.
  $\rightarrow$ Explains the GSP aptly.
  $\rightarrow$ Doesn't explain the LSP. (Both quantitative and qualitative mismatch)

  [I. Karpenko and F. Becattini, EPJC 77 (2017) 4, 213, PRL 120, 012302 (2018)]

- **The two processes differ.**

- Let's look at the plausible origin of these processes.
  — GSP is due to the large vorticities about $\hat{\mathcal{J}}$ leading to spin-alignment.
  — LSP is due to the elliptic flow in the transverse plane.

- Global angular momentum produced early, giving time to thermalize $\rightarrow$ GSP.
- Elliptic flow requires some time to generate, no time to thermalize $\rightarrow$ LSP.

- Probable resolution $\rightarrow$ Let go of the notion of spin equilibration.

Einstein-de Haas effect :

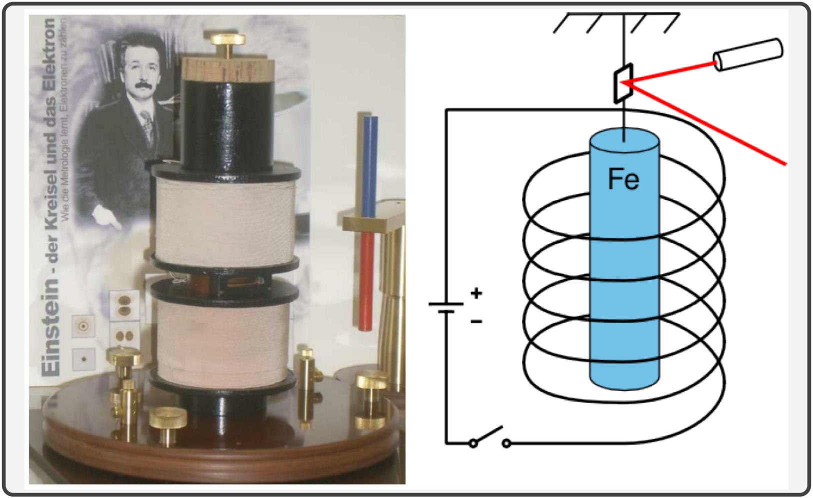
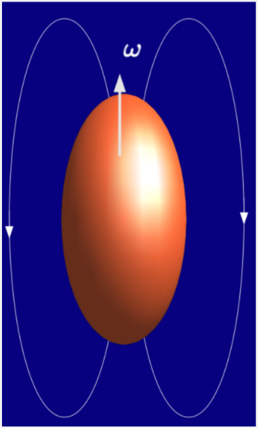
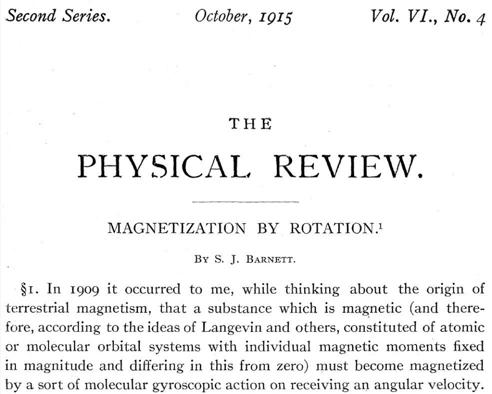


Figure 11: Einstein-de Haas effect. [Amaresh Jaiswal - Excited QCD 2022]

Magnetic field aligns electron spins → Matter rotates to conserve angular momentum.

Barnett effect :





Second Series. *October, 1915* *Vol. VI., No. 4*

THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

BY S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

Figure 12: Barnett effect. [Amaresh Jaiswal - Excited QCD 2022]

Non-zero angular momentum $\rightarrow$ Generation of magnetic field.

Gist of the Problems :

The two problems we wish to address are :

- To search for a resolution of the ‘spin sign puzzle’ in longitudinal polarization.
  - $\rightarrow$ Dissipative Spin-hydrodynamics.
- To understand the origin of EdH and Barnett effects in relativistic fluids.
  - $\rightarrow$ Dissipative Spin-magnetohydrodynamics.

Section Outline :

Relativistic Hydrodynamics :

- Ideal Hydrodynamics
- Dissipative Hydrodynamics
- Relativistic Kinetic Theory

Relativistic Spin-hydrodynamics :

Relativistic Spin-Magnetoydrodynamics

Summary and Outlook :

Relativistic Hydrodynamics :

- Recall fluid-like properties of the matter produced in heavy-ion collisions.

- First formulation of relativistic hydrodynamics was for an ideal fluid.

- Later extended to dissipative cases.

  [P. Romatschke, IJMPE 19 (2010) 1-53, J. Y. Ollitrault EJP 29 (2008) 275-302, Jaiswal and Roy AHEP 2016 (2016) 9623034]

Relativistic Hydrodynamics :

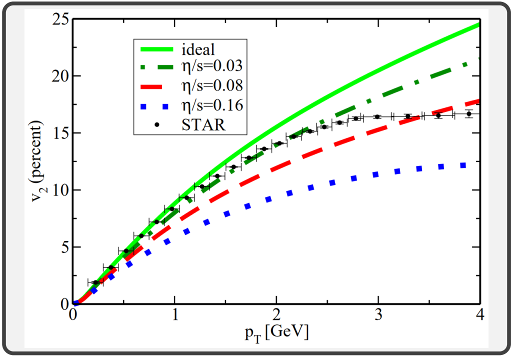


Figure 13: Elliptic flow. [P. Romatschke, PRL **99** (2007) 172301]

- Initial formulations of dissipative theories - acausal and unstable.
- Several of causal theories exist - MIS, BRSSS, DNMR, BDNK etc.

Ideal Relativistic Hydrodynamics :

- The framework of hydrodynamics is built on the conservation laws.

- For a non-polarizable relativistic charged fluid, the conservation laws are:

$$\partial_\mu N^\mu_{(0)} = 0, \qquad \partial_\mu T^{\mu\nu}_{(0)} = 0.$$

- These two conserved currents can be tensor decomposed in terms of available hydrodynamic variables $\rightarrow u^\mu$ and the metric tensor $\rightarrow g^{\mu\nu}$ as,

$$N^\mu_{(0)} = n_0 u^\mu, \qquad T^{\mu\nu}_{(0)} = \mathcal{E}_0 u^\mu u^\nu - \mathcal{P}\Delta^{\mu\nu}$$

where, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is a projection operator, $u^\mu u_\mu = 1$ and, $u_\mu \Delta^{\mu\nu} = 0$.

- Available equations $\rightarrow 1 + 4 = 5$.
The number of unknown variables $\rightarrow$ 1+2+3=6.
The system of equations can be closed if we provide an Equation of State (EoS) i.e. $\mathcal{P} = \mathcal{P}(\mathcal{E}_0)$

Ideal Relativistic Hydrodynamics :

- The conservation laws lead to the ideal hydro equations as,

$$\begin{aligned} \dot{\mathcal{E}}_0 + (\mathcal{E}_0 + \mathcal{P})\,\theta &= 0 \\ (\mathcal{E}_0 + \mathcal{P})\,\dot{u}^\alpha - \nabla^\alpha \mathcal{P} &= 0 \\ \dot{n}_0 + n_0\theta &= 0 \end{aligned}$$

  where, $\theta = \partial_\mu u^\mu$ is the scalar expansion.
  The differential operator is decomposed as, $\partial_\mu \equiv u^\mu D + \nabla^\mu$.
  For any quantity, A, we define, $\dot{A} = DA = (u \cdot \partial)\, A$.

- These equations of ideal hydrodynamics describe a fluid at local equilibrium.

Dissipative Relativistic Hydrodynamics :

- Inclusion of dissipation is carried out through modification of N^μ and $T^{\mu\nu}$ as,

$$N^\mu = n_0 u^\mu + n^\mu, \qquad T^{\mu\nu} = \mathcal{E}_0 u^\mu u^\nu - (\mathcal{P} + \mathit{\Pi})\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

  where, $u_\mu n^\mu = 0$, $u_\mu \pi^{\mu\nu} = 0$ and, $\pi^\mu_\mu = 0$.

- We have chosen the Landau frame $u_\mu T^{\mu\nu} = \mathcal{E} u^\nu$ and Landau matching conditions $\mathcal{E} = \mathcal{E}_0, n = n_0$.

- Thus the number of unknown variables are increased as, 4 + 8 + 3 = 15.

- However, the number of conservation laws remain the same i.e. 5.

- So, apart from EoS, 9 more equations needed to close the system of equations.

Dissipative Relativistic Hydrodynamics :

- The conservation laws lead to the following dissipative hydro equations,

$$\begin{aligned}\dot{\mathcal{E}}_0 + (\mathcal{E}_0 + \mathcal{P} + \Pi)\,\theta - \pi^{\mu\nu}\sigma_{\mu\nu} &= 0\\ (\mathcal{E}_0 + \mathcal{P} + \Pi)\,\dot{u}^\alpha - \nabla^\alpha(\mathcal{P} + \Pi) + \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} &= 0\\ \dot{n}_0 + n_0\theta + \partial_\mu n^\mu &= 0\end{aligned}$$

  where, $\sigma^{\mu\nu} = (\nabla^\mu u^\nu + \nabla^\nu u^\mu)/2 - \Delta^{\mu\nu}\theta/3$ is the shear stress tensor.

- These equations are exact up to all order in gradients.

- Next we incorporate the order-by-order gradient corrections :

$$\begin{aligned}N^\mu &= N^\mu_{(0)} + N^\mu_{(1)} + N^\mu_{(2)} + \cdots\\ T^{\mu\nu} &= T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots\end{aligned}$$

Dissipative Relativistic Hydrodynamics up to $\mathcal{O}(\partial)$:

- Truncating terms up to first order in spacetime gradients, we get the Navier-Stokes equations within Landau-Lifshitz frame and matching conditions as,

$$\pi^{\mu\nu} = 2\,\eta\,\sigma_{\mu\nu}\,,$$
$$\Pi = -\zeta\,\theta,$$
$$n^{\mu} = \kappa\,(\nabla^{\mu}\xi)\,.$$

where, $\xi = \mu/T$, η, ζ and κ are the $\mathcal{O}(\partial)$ transport coefficients.

- The details of the transport coefficients can only be obtained from a microscopic theory.

Dissipative Relativistic Hydrodynamics up to $\mathcal{O}(\partial^2)$:

- Truncating terms up to second order in spacetime gradients, we get the evolution equations of the dissipative currents as,

$$\begin{aligned}
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\,\beta_\pi\,\sigma_{\mu\nu} + \lambda_1 \pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_2 \pi^{\mu\nu}\theta + \lambda_3 \pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} + \lambda_4 \Pi\sigma^{\mu\nu}, \\
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= \beta_\Pi\,\sigma_{\mu\nu} + \delta_1 \Pi\theta + \delta_2 \pi^{\mu\nu}\sigma_{\mu\nu}, \\
\dot{n}^\mu + \frac{n^\mu}{\tau_n} &= \beta_n\,(\nabla^\mu \xi) + \psi_1 n_\nu \omega^{\nu\mu} + \psi_2 n^\mu \theta + \psi_3 n_\nu \sigma_{\nu\mu} + \psi_4 \pi^{\mu\nu}\,(\nabla_\nu \xi).
\end{aligned}$$

- The dissipative currents can no longer be completely determined from other hydrodynamic variables and have to be promoted to independent variables.

- Higher order evolution equations can also be obtained. However, to completely specify the theory a microscopic theory is required $\rightarrow$ Kinetic Theory.

Kinetic Theory

- The central object withing relativistic kinetic theory is the phase-space distribution function, $f(x,p)$.

- Different moments of $f(x,p)$ correspond to different hydrodynamical variables,

$$N^\mu = \int \mathrm{dp}\, p^\mu \left(f - \bar{f}\right) \qquad\qquad T^{\mu\nu} = \int \mathrm{dp}\, p^\mu p^\nu \left(f + \bar{f}\right)$$

  where, $\mathrm{dp} = \mathrm{d}^3|\vec{\mathrm{p}}|/(2\pi)^3 p^0$ and p^0 is the particle energy.

- Equilibrium quantities are given by,

$$n_0 = u_\mu N^\mu = \int \mathrm{dp}\, (u\cdot p)\left(f_{\mathrm{eq}} - \bar{f}_{\mathrm{eq}}\right),$$

$$\mathcal{E}_0 = u_\mu u_\nu T^{\mu\nu} = \int \mathrm{dp}\, (u\cdot p)^2\left(f_{\mathrm{eq}} + \bar{f}_{\mathrm{eq}}\right),$$

$$\mathcal{P} = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu} = -\frac{1}{3}\int \mathrm{dp}\, (p\cdot\Delta\cdot p)\left(f_{\mathrm{eq}} + \bar{f}_{\mathrm{eq}}\right)$$

- The equilibrium distribution function can be found to be given by,

$$f_{\text{eq}} = \frac{1}{e^{\beta(u\cdot p)-\xi} + a}$$

where, $\beta = 1/T$, $\xi = \mu/T$ and, a can take the values $0, +1$, and -1.

- Dissipative hydrodynamics describes a fluid that is out-of-equilibrium.

- From Chapman-Enskog like expansion we have, $f(x,p) = f_{\text{eq}}(x,p) + \delta f(x,p)$.

- Thus, the dissipative quantities are expressed as,

$$n^\mu = \Delta^\mu_\alpha N^\alpha = \Delta^\mu_\alpha \int \mathrm{dp}\, p^\alpha \left(\delta f - \delta \bar{f}\,\right),$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} T^{\alpha\beta} = \Delta^{\mu\nu}_{\alpha\beta} \int \mathrm{dp}\, p^\alpha p^\beta \left(\delta f + \delta \bar{f}\,\right),$$

$$\Pi = -\frac{1}{3}\Delta_{\alpha\beta} T^{\alpha\beta} = -\frac{1}{3}\int \mathrm{dp}\, \left(p\cdot\Delta\cdot p\right)\left(\delta f + \delta \bar{f}\,\right)$$

Boltzmann Equation and Relaxation Time Approximation

- Expression of $\delta f(x,p)$ is required and can be obtained from the Boltzmann equation,

$$p^\mu \partial_\mu f = C[f]$$

where, $C[f]$ is the collision kernel, that controls the interaction process.

- Under relaxation time approximation (RTA) we get,

$$p^\mu \partial_\mu f = -\frac{(u \cdot p)}{\tau_{\mathrm{R}}} \delta f$$

where, τ_{R} is the relaxation time.

- We can solve for δf in an iterative manner to get the correction up to the required order and obtain the expressions for the transport coefficients.

Relativistic Spin-hydrodynamics :

- Inspired by the success of Relativistic Hydrodynamics (RH) in explaining the multitude of properties of QGP evolution, development of a framework of RH with spin was started.

  [F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

  [W. Florkowski *et al*, PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017]

  [D. Montenegro *et al*, PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016]

How to include internal degrees of freedom in a macroscopic theory?

[J. Weyssenhoff, A. Raabe, Acta Phys. Pool. 9 (1947) 7]

Relativistic Spin-hydrodynamics :

- Origin of spin is purely quantum mechanical.

- Any theory with spin should be built up from Quantum Field Theory (QFT).

- To derive a hydrodynamical description of a spin-polarized fluid starting from QFT, it was proved that a spin-polarization tensor ($\omega^{\mu\nu}$) must be introduced.

  [F. Becattini *et al*, PLB 789 (2019) 419-425]

- It has been argued that, at global equilibrium, the spin-polarization tensor should be same as the thermal vorticity.

  [F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

  [N. Weickgenannt *et al*, PRL 127 (2021) 5, 052301]

$$\boxed{\omega^{\mu\nu}|_{\rm geq} \propto \varpi^{\mu\nu} = \left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)/2}$$

$\beta^\mu = u^\mu/T$ is the inverse temperature four-vector.

Relativistic Spin-hydrodynamics :

- A theory of ideal spin-hydrodynamics was formulated for fluids in equilibrium.
  [W. Florkowski *et al*, PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017]
  [D. Montenegro *et al*, PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016]
- But, we want description of fluid with non-thermalized spin, where the relation, $\omega^{\mu\nu}|_{\rm geq} \propto \varpi^{\mu\nu}$ may not hold.
- Thus, we need to understand, how the out-of-equilibrium system and hence $\omega^{\mu\nu}$ evolves.
- We develop a theory of dissipative spin-hydrodynamics.

Relativistic Spin-hydrodynamics :

- We first note that, spin-polarization originates from rotation of fluid.
- Thus, apart from the number current and stress-energy tensor conservation, we need to allow conservation of angular momentum. Thus the conservation laws are :

$$\partial_\mu N^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda J^{\lambda,\mu\nu} = 0$$

where, $J = L + S$. Also, $L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$.

- For symmetric $T^{\mu\nu}$ we have, $\boxed{\partial_\lambda S^{\lambda,\mu\nu} = 0}$
- Introduction of $S^{\lambda,\mu\nu}$ increases the number of unknown variables. More equations are required. Hence a relativistic kinetic theory with spin is necessary.

Kinetic Theory with Spin :

- To import spin in kinetic theory (KT), we start from the Wigner function ($\mathcal{W}_{\alpha\beta}$), that bridges the gap between QFT and KT.

- For spin-1/2 particles we set up kinetic equation of $\mathcal{W}_{\alpha\beta}$ using Dirac equation,

$$\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right] \mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

[Xin-Li Sheng, PhD Thesis (2019), N. Weickgenannt *et al*, PRL 127 (2021) 5, 052301, PRD 100, 056018 (2019).]

- The Wigner function can be decomposed as,

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta}$$

$\mathcal{F} \rightarrow$ scalar component,

$\mathcal{P} \rightarrow$ pseudoscalar component,

$\mathcal{V}_\mu \rightarrow$ vector component,

$\mathcal{A}_\mu \rightarrow$ axial vector component,

$\mathcal{S}_{\mu\nu} \rightarrow$ tensor component.

where, the γ-matrices are the 4×4 Dirac γ-matrices and, $\Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$.

Kinetic Theory with Spin :

- For spin-hydrodynamics it suffices to consider only $\mathcal{F}$ and $\mathcal{A}_\mu$ components.

  [Xin-Li Sheng, PhD Thesis (2019)]

	Scalar Component	***Axial Component***
Kin. Eq.	$k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$
RTA	$C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\Big]$	$C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\Big]$
Dist. fn.	$\mathcal{F}^\pm(x,k) = 2m\int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$	$\mathcal{A}^\mu_\pm(x,k) = 2m\int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103 (2021) 1, 014030]

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int \mathrm{dP}(\cdots), \quad \int \mathrm{dP} = d^3p/\,(2\pi)^3\,p^0.$

Spin measure $\rightarrow \quad \int \mathrm{dS} = (m/\pi\mathfrak{s})\int d^4s\,\delta(s\cdot s + \mathfrak{s}^2).$

Relativistic Kinetic Equation :

- We take the equilibrium phase-space distribution function to be :

$$f^{\pm}_{\text{eq}}(x,p,s) = e^{-\beta(u\cdot p)\pm\xi}\left(1+\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right)+\mathcal{O}(\omega^2)$$

[F. Becatinni et al., Annals Phys. 338 (2013) 32-49, W. Florkowski et al., PRD 97 (2018) 11, 116017]

- Near local equilibrium $f(x,p,s)$ can be expanded in Chapman-Enskog like expansion as,

$$f^{\pm}(x,p,s) = f^{\pm}_{\text{eq}}(x,p,s) + \delta f^{\pm}(x,p,s).$$

- δf is the non-equilibrium correction and is obtained solving the kinetic equation of 'f' i,e, the Boltzmann equation,

$$p^{\mu}\partial_{\mu}f^{\pm}(x,p,s) = -\frac{(u\cdot p)}{\tau_{\text{eq}}}\delta f^{\pm}(x,p,s)$$

Conserved Currents (In and Out-of-Equilibrium) :

- The conserved currents are expressed in kinetic theory as,

$$N^{\mu} = \int \mathrm{dPdS}\, p^{\mu} \left(f^{+} - f^{-}\right),$$

$$T^{\mu\nu} = \int \mathrm{dPdS}\, p^{\mu} p^{\nu} \left(f^{+} + f^{-}\right),$$

$$S^{\lambda,\mu\nu} = \int \mathrm{dPdS}\, p^{\lambda} s^{\mu\nu} \left(f^{+} + f^{-}\right)$$

- The non-equilibrium parts give the transport coefficients:

$$\delta N^{\mu} = \tau_{\rm eq} \beta_n (\nabla^{\mu}\xi),$$

$$\delta T^{\mu\nu} = \tau_{\rm eq} \left[-\beta_{\Pi}\, \Delta^{\mu\nu}\, \theta + 2\, \beta_{\pi}\, \sigma^{\mu\nu} \right],$$

$$\delta S^{\lambda,\mu\nu} = \tau_{\rm eq} \Big[B_{\Pi}^{\lambda,\mu\nu} \theta + B_{n}^{\phi\lambda,\mu\nu} (\nabla_{\phi}\xi) + B_{\pi}^{\alpha\beta\lambda,\mu\nu} \sigma_{\alpha\beta} + B_{\Sigma}^{\rho\gamma\phi\lambda,\mu\nu} (\nabla_{\rho}\omega_{\gamma\phi}) \Big]$$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103, 014030 (2021)]

Time Evolution of Thermodynamic Variables :

- By choosing the Landau frame and matching conditions we found the following relations:

$$\dot{\xi} = \xi_\theta\,\theta, \qquad \dot{\beta} = \beta_\theta\,\theta, \qquad \beta\dot{u}_\mu = -\nabla_\mu\beta + \frac{n_0\ \tanh\xi}{(\mathcal{E}+\mathcal{P})}\left(\nabla_\mu\xi\right)$$

$$\dot{\omega}^{\mu\nu} = \mathcal{D}^{\mu\nu}_{\Pi}\theta + \mathcal{D}^{\mu\nu\alpha}_{\mathrm{n}}\left(\nabla_\alpha\xi\right) + \mathcal{D}^{\mu\nu\alpha\beta}_{\pi}\sigma_{\alpha\beta} + \mathcal{D}^{\lambda\mu\nu\alpha\beta\gamma}_{\Sigma}\left(\nabla_\alpha\omega_{\beta\gamma}\right),$$

- Thus, the non-equilibrium parts of N^μ and $T^{\mu\nu}$ remain same as in the case of un-polarized fluid.

- Non-equilibrium part of spin tensor depends on gradients of multiple hydrodynamics variables.

- The evolution of spin-polarization tensor depend on scalar expansion, shear stress, particle and spin diffusion but not on vorticity.

Section Outline :

Relativistic Hydrodynamics :

- Ideal Hydrodynamics
- Dissipative Hydrodynamics
- Relativistic Kinetic Theory

Relativistic Spin-hydrodynamics :

Relativistic Spin-Magnetoydrodynamics

Summary and Outlook :

Covariant Theory of Non-resistive Electrodynamics :

- In the limit of infinite conductivity, field strength tensor is,

$$F^{\mu\nu} \to B^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\, u_\alpha\, B_\beta$$

- B^μ is orthogonal to u^μ and spacelike i.e. $u_\mu B^\mu = 0$ and, $B_\mu B^\mu \leq 0$.

[G. S. Denicol et al. Phys.Rev.D 98 (2018) 7, 076009; A. K. Panda et al., JHEP 03 (2021) 216]

- If the medium if magnetizable, then the Maxwell's equations are given by,

$$\partial_\mu H^{\mu\nu} = J^\nu, \qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0,$$
$$\left(\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\, F_{\alpha\beta}\right)$$

where, $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$ is the induction tensor and $M^{\mu\nu}$ is the magnetization tensor, J^ν is the charged four-current.

[Balakin, Grav.Cosmol. 13 (2007) 163-177; Hehl and, Obukhov, Phys. Lett. A 311, 277 (2003)]

Current Sources :

- The charged four-current may have two different origins -

$$J^\mu = J^\mu_{\rm f} + J^\mu_{\rm ext}$$

- In case of HICs, one may identify $J^\mu_{\rm f}$ with the charged 4-current within the fluid and $J^\mu_{\rm ext}$ with the charged 4-current of the spectators.

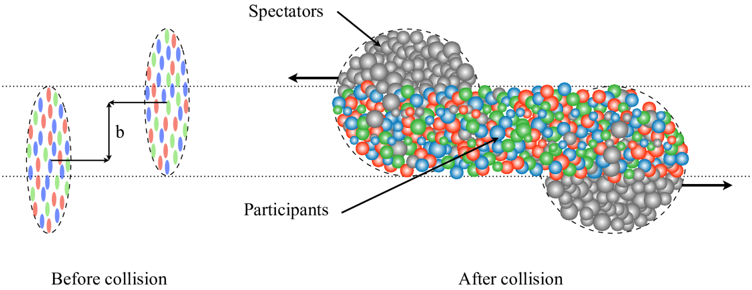


Figure 14: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

Current Sources :

- The charged four-current may have two different origins -

$$J^{\mu} = J^{\mu}_{\mathrm{f}} + J^{\mu}_{\mathrm{ext}}$$

- In case of HICs, one may identify J^{μ}_{f} with the charged 4-current within the fluid and J^{μ}_{ext} with the charged 4-current of the spectators.

- Consequently, the field strength will have two parts,

$$F^{\mu\nu} = F^{\mu\nu}_{\mathrm{f}} + F^{\mu\nu}_{\mathrm{ext}}$$

- J^{μ} can be related to particle four-current as, $J^{\mu} = \mathfrak{q} N^{\mu}$.

Conserved Currents (Particle Current and Stress-Energy Tensor) :

- The net particle current within the system remains conserved. Hence we have,

$$\partial_\mu N^\mu_{\mathrm{f}} = 0$$

- Total stress-energy tensor is, $T^{\mu\nu} = T^{\mu\nu}_{\mathrm{f}} + T^{\mu\nu}_{\mathrm{int}} + T^{\mu\nu}_{\mathrm{B}} + T^{\mu\nu}_{\mathrm{ext}}$

- The first three stress-energy tensors are given by,

$$\begin{aligned} T^{\mu\nu}_{\mathrm{f}} &= \mathcal{E}\, u^\mu u^\nu - (\mathcal{P} + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu} \\ T^{\mu\nu}_{\mathrm{int}} &= -F^\mu{}_\alpha M^{\nu\alpha} \\ T^{\mu\nu}_{\mathrm{B}} &= -F^{\mu\alpha} F^\nu{}_\alpha + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \end{aligned}$$

- Due to the external field, stress-energy tensor is not conserved

$$\partial_\nu T^{\mu\nu} = -f^\mu_{\mathrm{ext}}\,, \qquad \partial_\nu T^{\mu\nu}_{\mathrm{f}} = F^\mu{}_\alpha J^\alpha_{\mathrm{f}} + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}\,, \qquad f^\mu_{\mathrm{ext}} = F^\mu{}_\alpha J^\alpha_{\mathrm{ext}}$$

Conserved Currents (Angular Momentum Tensor) :

- Similar to stress-energy tensor, the total angular momentum is not conserved in presence of external field and we have,

  $$\partial_\lambda J^{\lambda,\mu\nu} = \partial_\lambda L^{\lambda,\mu\nu} + \partial_\lambda S^{\lambda,\mu\nu} = -\tau^{\mu\nu}_{\text{ext}},$$

  where, $\tau^{\mu\nu}_{\text{ext}} = x^\mu f^\nu_{\text{ext}} - x^\nu f^\mu_{\text{ext}}$ is the torque exerted by J_{ext} on the system.

- However, since $\partial_\lambda L^{\lambda,\mu\nu} = -\tau^{\mu\nu}_{\text{ext}}$, we get a conserved spin angular momentum tensor i.e.

  $$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

Boltzmann Equation :

- In presence of electromagnetic fields, the Boltzmann equation under RTA is,

$$p^\mu \partial_\mu^{(x)} f^\pm + \mathcal{F}^\mu \partial_\mu^{(p)} f^\pm + \mathcal{S}^{\mu\nu} \partial_{\mu\nu}^{(s)} f^\pm = -\frac{(u \cdot p)}{\tau_{\mathrm{R}}} \delta f^\pm$$

where, [Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970)]

$$\partial_\mu^{(x)} \equiv \frac{\partial}{\partial x^\mu}, \qquad \partial_\mu^{(p)} \equiv \frac{\partial}{\partial p^\mu}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}},$$

- We can obtain a simplified expression for the equation of motion as,

[Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970), Nora W. et al. PRD 100 (2019) 5, 056018]

$$\mathcal{F}^\alpha = \mathfrak{q}\, F^{\alpha\beta} p_\beta + \frac{m}{2}\left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}$$

where, $m^{\alpha\beta} = \chi\, s^{\alpha\beta}$ is dipole moment tensor, m is the mass of the particle.

- We can use the dipole moment tensor to provide a definition of $M^{\alpha\beta}$ as,

$$M^{\alpha\beta} = m \int \mathrm{dPdS}\, m^{\alpha\beta} \left(f^+ - f^-\right)$$

Solving Boltzmann Equation :

- Using RTA in Boltzmann equation we can write the 1st order gradient correction as,

$$\delta f^{\pm}_{(1)} = -\mathcal{D} f^{\pm}_{\text{eq}},$$

where,

$$\mathcal{D} = \frac{\tau_R}{(u \cdot p)} \left(p^{\alpha} \frac{\partial}{\partial x^{\alpha}} + \mathcal{F}^{\alpha} \frac{\partial}{\partial p^{\alpha}} \right)$$

[A. K. Panda et al., JHEP 03 (2021) 216]

- For equilibrium distribution function, we use,

$$f^{\pm}_{\text{eq}}(x, p, s) = \left(1 + \frac{1}{2} \omega_{\alpha\beta}\, s^{\alpha\beta}\, \tilde{f}^{\pm}_{0} \right) f^{\pm}_{0}$$

$$\text{with,} \quad f^{\pm}_{0} = \left[e^{\beta \cdot p \mp \xi} + 1 \right]^{-1} \qquad \text{and,} \qquad \tilde{f}^{\pm}_{0} = 1 - f^{\pm}_{0}$$

[F. Becattini et. al., Annals Phys. 338 (2013); W. Florkowski et. al., Phys.Rev.D 97 (2018)]

Dissipative Currents in Spin-magnetohydrodynamics:

- The dissipative quantities are defined as,

$$
\begin{aligned}
\Pi &= -\frac{\Delta_{\alpha\beta}}{3}\int dP \int dS\, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\
\pi^{\mu\nu} &= \Delta^{\mu\nu}_{\alpha\beta}\int dP \int dS\, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\
n^{\mu} &= \Delta^{\mu}_{\alpha}\int dP \int dS\, p^{\alpha} \left(\delta f^{+} - \delta f^{-}\right) \\
\delta S^{\lambda,\mu\nu} &= \int dP \int dS\, p^{\lambda} s^{\mu\nu} \left(\delta f^{+} + \delta f^{-}\right)
\end{aligned}
$$

where, $\Delta^{\mu\nu}_{\alpha\beta} = (1/2)(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\beta}\Delta^{\mu}_{\alpha}) - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$ is a traceless symmetric projection operator.

Dissipative Currents in Spin-magnetohydrodynamics:

- So, the dissipative currents are :

$$X = \tau_{\text{eq}}\Big[\beta_{X\Pi}\,\theta + \beta^{\alpha}_{Xn}\left(\nabla_\alpha \xi\right) + \beta^{\alpha}_{Xa}\dot{u}_\alpha + \beta^{\alpha\beta}_{X\pi}\sigma_{\alpha\beta} + \beta^{\alpha\beta}_{X\Omega}\Omega_{\alpha\beta} + \beta^{\alpha\beta}_{XF}\left(\nabla_\alpha B_\beta\right) + \beta^{\alpha\beta\gamma}_{X\Sigma}\left(\nabla_\alpha \omega_{\beta\gamma}\right)\Big],$$

where, $X \equiv n^\mu,\ \Pi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}$

- Evolution of spin-polarization tensor is given by,

$$\dot{\omega}^{\mu\nu} = \mathcal{D}^{\mu\nu}_{\Pi}\,\theta + \mathcal{D}^{\mu\nu\gamma}_{\mathrm{n}}\left(\nabla_\gamma \xi\right) + \mathcal{D}^{\mu\nu\gamma}_{\mathrm{a}}\dot{u}_\gamma + \mathcal{D}^{\mu\nu\rho\kappa}_{\pi}\sigma_{\rho\kappa} + \mathcal{D}^{\mu\nu\rho\kappa}_{\Omega}\Omega_{\rho\kappa} + \mathcal{D}^{\mu\nu\phi\rho\kappa}_{\Sigma}\left(\nabla_\phi \omega_{\rho\kappa}\right)$$

- Equilibrium magnetization tensor is given by,

$$M^{\mu\nu}_{\text{eq}} = a_1(T,\mu)\,\omega^{\mu\nu} + a_2(T,\mu)\,u^{[\mu}u_\gamma\omega^{\nu]\gamma}$$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PRL 129, 192301 (2022)]

Section Outline :

Relativistic Hydrodynamics :

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Relativistic Spin-Magnetoydrodynamics

Summary and Outlook :

Summary and Outlook :

- **Summary :**
  1. Viscous effects may be necessary for explanation of LSP.
  2. We found the dissipative currents depend on multiple hydrodynamic variables.
  3. Vorticity may affect the evolution of spin-polarization tensor.
  4. Magnetomechanical effects exists in a spin-polarizable and magnetizable fluid.

- **Outlook :**
  1. Formulation of a causal spin-hydrodynamics is required.
  2. A spin-hydrodynamics with non-local collisions is necessary.
  3. A spin-hydrodynamics for spin-1 particles needs to be formulated.
  4. Need to study phenomenological consequences of the theory.

Thank you.