Effects of the Sudakov form factor and exact kinematics on the Golec-Biernat–Wüsthoff and Bartels–Golec-Biernat–Kowalski models

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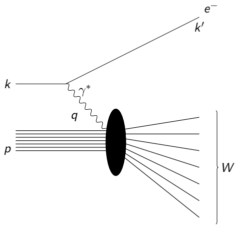
outline

Introduction

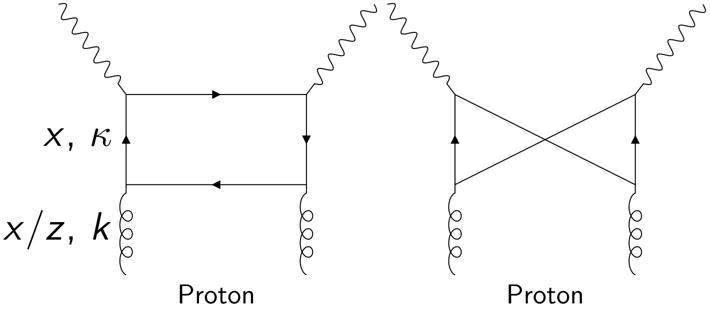
General introduction

- DIS has been central to the study of proton structure.
- The knowledge of the structure of protons is vital for measurements of processes such as the p-p collision.
- With Electron Ion Collider (EIC) on horizon, DIS remains to be of great interest.
- Factorization allows long-distance effects to be encoded inside process-independent parton distribution functions (PDFs).



k_t-factorization

The inclusive DIS process $ep \to \gamma^*(q)p(p) \to X$.



with $q' \equiv q + xp$,

$$\kappa = \alpha p - \beta q' + \kappa_t \qquad \text{and} \qquad k = ap - bq' + k_t.$$

The structure function can be expressed in terms of the dipole gluon density $\mathcal{F}$ [Kimber, 2001, Kwiecinski et al., 1997]:

$$F_{2T}(x,Q^2) = \sum e_f^2 \frac{Q^2}{4\pi} \int d\tilde{\Pi} \;\; \alpha_s(\mu^2)\mathcal{F}(x/z, k_t^2)$$
$$\times \left[\left[\beta^2 + (1-\beta)^2\right] \left(\frac{\boldsymbol{\kappa}_t}{D_1} - \frac{\boldsymbol{\kappa}_t - \mathbf{k}_t}{D_2} \right)^2 + m_f^2 \left(\frac{1}{D_1} - \frac{1}{D_2} \right)^2 \right],$$

$$F_{2L}(x,Q^2) = \sum e_f^2 \frac{Q^2}{4\pi} \int d\tilde{\Pi} \;\; \alpha_s(\mu^2)\mathcal{F}(x/z, k_t^2)$$
$$\times \left[4Q^2\beta^2(1-\beta)^2 \left(\frac{1}{D_1} - \frac{1}{D_2} \right)^2 \right],$$

where

$$\int d\tilde{\Pi} = \int \frac{dk_t^2}{k_t^2} \int_0^1 d\beta \int d\kappa_t^2 \frac{d\phi}{2\pi} \Theta(1 - x/z)$$
$$D_1 = \kappa_t^2 + \beta(1-\beta)Q^2 + m_f^2, \qquad D_2 = (\boldsymbol{\kappa}_t - \mathbf{k}_t)^2 + \beta(1-\beta)Q^2 + m_f^2$$
$$\frac{1}{z} = 1 + \frac{\kappa'^2_t + m_f^2}{\beta(1-\beta)Q^2} + \frac{k_t^2}{Q^2}.$$

Dipole factorization

The formula can also be written in the impact parameter space [Nikolaev & Zakharov, 1991]:

$$F_{2T}(x,Q^2) = 6\sum e_f^2 \frac{Q^2}{4\pi^2}\int d\Pi \;\; \sigma_{\text{dipole}}(x/z,r) \\ \times \left[\left[\beta^2+(1-\beta)^2\right] K_0^2(\epsilon r) + m_f^2 K_1^2(\epsilon r)\right],$$

$$F_{2L}(x,Q^2) = 6\sum e_f^2 \frac{Q^2}{4\pi^2}\int d\Pi \;\; \sigma_{\text{dipole}}(x/z,r) \\ \times \left[4Q^2\beta^2(1-\beta)^2 K_1^2(\epsilon r)\right],$$

where

$$\int d\Pi = \int_0^1 d\beta \int \frac{d^2\mathbf{r}}{(2\pi)^2}\Theta(1-x/z)$$
$$\epsilon^2 = \beta(1-\beta)Q^2 + m_f^2$$

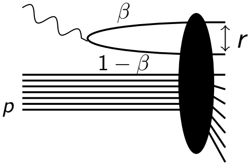
Dipole factorization

The above formula is factorized in the form [Nikolaev & Zakharov, 1991]

$$F_2\left(x, Q^2\right) \sim \int_0^1 d\beta \int d^2\mathbf{r} \left|\Psi\left(\beta, r, Q^2\right)\right|^2 \sigma_{\rm dipole}\left(x, r\right).$$

This form has a nice interpretation:

- Ψ... describes fluctuation of γ^* into $q\bar{q}$ pair with momentum fractions β and $1-\beta$,
- $\sigma_{\rm dipole}$... describes interaction of the $q\bar{q}$ pair of a size r with the target proton.



Dipole cross section

The dipole cross section is related to the gluon density by the relation

$$\sigma_{\rm dipole}(x,r) = \frac{4\pi}{C_A}\int \frac{d^2\mathbf{k_t}}{k_t^2}\left(1 - e^{-i\mathbf{r}\cdot\mathbf{k}_t}\right)\alpha_s \mathcal{F}(x,k_t^2).$$

When the dipole size r is small

$$\sigma_{\rm dipole}(x,r) \approx r^2 \frac{4\pi^2\alpha_s x g(x,1/r)}{C_A}.$$

$\rightarrow$ Colour transparency.
(When the dipole is small, it looks colourless.)

But what happens when r increases? Do we have unlimited growth?

Saturation!

Saturation and non-linear evolution

- Balitsky–Fadin–Kuraev–Lipatov (BFKL) [Balitsky & Lipatov, 1978, Kuraev et al., 1977] which resums large logarithms, $\log(1/x)$, predicts sharp rise of cross section $\sim x^{-\lambda}$.
- Froissart bound limits the growth to be $\lesssim \log^2(1/x)$ [Froissart, 1961].
- Saturation is described by nonlinear equations.
  e.g.
  Balitsky–Kovchegov (BK) [Balitsky, 1996, Kovchegov, 1999],
  Gribov–Levin–Ryskin (GLR) [Gribov et al., 1983]
  etc.

But it is often useful to have a simple model to describe the phenomenon!

GBW and BGK models

GBW model is in the form [Golec-Biernat & Wusthoff, 1998]

$$\sigma_{\mathrm{GBW}}(x,r) = \sigma_0 \left(1 - e^{-\frac{r^2 Q_s^2}{4}}\right),$$

where

$$Q_s^2(x) = \frac{1}{4}\left(\frac{x_0}{x}\right)^{\lambda}$$

is the *saturation scale*, which separates the scaling region ($\sim r^2$), and the saturated region ($\sim \sigma_0$).

The DGLAP-improved BGK [Bartels et al., 2002] model reads:

$$\sigma_{\mathrm{BGK}}(x,r) = \sigma_0 \left(1 - \exp\left[-\frac{r^2 4\pi^2 \alpha_s(\mu^2) x g(x,\mu^2)}{3\sigma_0}\right]\right),$$

where

$$\mu^2 = C/r^2 + \mu_0^2 \qquad \text{and} \qquad xg(x,Q_0^2) = A_g x^{-\lambda_g}(1-x)^{5.6}$$

Sudakov Form Facor

arXiv:2210.16084

Sudakov form factor

- Sudakov form factor resumms logarithms of two vastly different scales, such as $\log(q_t/Q)$ [Collins et al., 1985].
- It was shown by Xiao et al. [Xiao et al., 2017], that large logarithms of type $\log(1/x)$ and the Sudakov logarithms can be resummed consistently and simultaneously.
- Hard scale dependence is subleading in the leading $\log(1/x)$ approximation [Kimber et al., 2000].

We use Xiao et al.'s formula for the dipole gluon density and introduce a new, hard-scale-dependent dipole cross section $\Sigma_{\rm dipole}$:

$$\Sigma_{\rm dipole}(x, r, Q^2) = \int \frac{d^2\mathbf{k}_t}{(2\pi)^2 k_t^2}\left(1 - e^{i\mathbf{k}_t\cdot\mathbf{r}}\right) \times \int d^2\mathbf{r}' e^{i\mathbf{k}_t\cdot\mathbf{r}'} e^{-S(r',Q^2)} \nabla^2_{\mathbf{r}'}\sigma_{\rm dipole}(x, r')$$

$$= \int_0^r dr' r' \log\left(\frac{r}{r'}\right) e^{-S(r',Q^2)} \nabla^2_{r'}\sigma_{\rm dipole}(x, r').$$

This new object $\Sigma_{\rm dipole}$ is hard scale, Q^2, dependent, and does not converge to a constant at large r.

In the present study, we focus on the leading-order, perturbative Sudakov factor [Xiao et al., 2017],

$$S^{(1)}_{\rm pert}(r, Q^2) = \frac{C_A}{2\pi} \int_{\mu_b^2}^{Q^2} \alpha(\mu^2) \frac{d\mu^2}{\mu^2} \log\left(\frac{Q^2}{\mu^2}\right).$$

For the case of the running coupling $\alpha_s(\mu^2) = 1/(b_0 \log \frac{\mu^2}{\Lambda^2_{\rm QCD}})$,

$$S^{(1)}_{\rm pert}(r, Q^2) = \frac{C_A}{2\pi b_0}\Bigg[- \log\left(\frac{Q^2}{\mu_b^2}\right) + \left(\frac{1 + \alpha(\mu_b^2) b_0 \log\left(\frac{Q^2}{\mu_b^2}\right)}{\alpha(\mu_b^2) b_0}\right) \log\left(1 + \alpha(\mu_b^2) b_0 \log\left(\frac{Q^2}{\mu_b^2}\right)\right)\Bigg],$$

where

$$b_0 = \frac{11 C_A - 2 n_f}{12} \qquad \mu_b = C_S/r \qquad C_S = 2e^{-\gamma_{\rm E}}$$

and $\gamma_{\rm E} \approx 0.577$ is the Euler-Mascheroni constant.

- We consider $S(r, Q^2) = 0$ for $\mu_b^2 > Q^2$.
- For the large-r region, te Sudakov factor is frozen with modified b_*-prescription of [Golec-Biernat & Sapeta, 2006]:

$$\mu_b^2 = \frac{\mu_0^2}{1 - e^{-r^2 \frac{\mu_0^2}{C}}},$$

where $\mu_0^2 = C/r_{\mathrm{max}}^2$.

$\rightarrow$ No modification for $Q^2 r^2 \lesssim 1$

Set-up

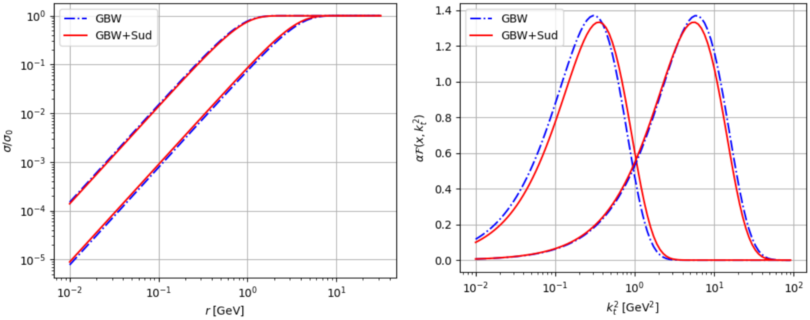
- The models are fitted to Inclusive DIS data from HERA [Abt et al., 2017].
- The data are selected to be in the range

$$x \leq 0.01 \qquad 0.045\ \mathrm{GeV}^2 \leq Q^2 \leq 650\ \mathrm{GeV}^2.$$

- Light quarks are taken massless. c and b quark masses are $m_c = 1.3\ \mathrm{GeV}$ and $m_b = 4.6\ \mathrm{GeV}$.
- Minuit package [James & Roos, 1975] was used to fit the model.

GBW + Sudakov

	σ_0 [mb]	$x_0(10^{-4})$	λ	χ^2/dof
GBW	19.1	2.58	0.322	4.44
GBW + Sud	18.6	3.11	0.299	2.66

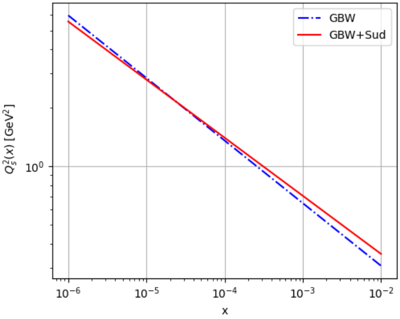


- Considerable improvement in the fit quality for moderate change in the parameters.
- Milder dependence on x.

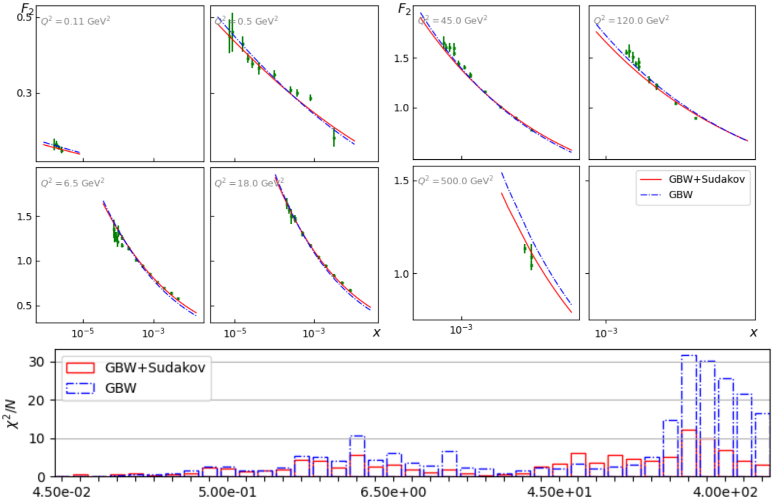
Saturation scale

In terms of the gluon density, the saturation scale can ge thought of as a typical k_t^2 of gluons where $\mathcal{F}$ peaks. For the GBW model,

$$\frac{\partial \mathcal{F}(x, k_t^2)}{\partial k_t^2} = Q_s^{-4}(Q_s^2 - k_t^2)e^{-k_t^2/Q_s^2} = 0$$

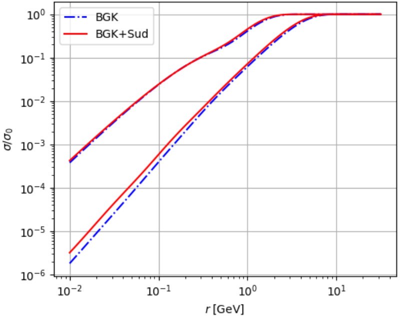


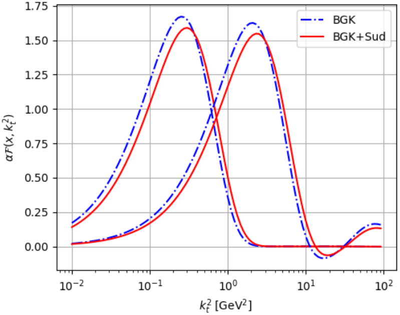
Comparison with data at selected Q^2.



BGK + Sudakov

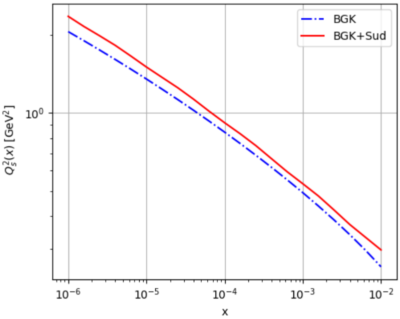
	σ_0 [mb]	A_g	λ_g	C	μ_0^2 [GeV^2]	χ^2/dof
BGK	23.3	1.18	0.0832	0.329	1.87	1.56
BGK + Sud	22.2	8.67	-0.500	0.670	3.83	1.21



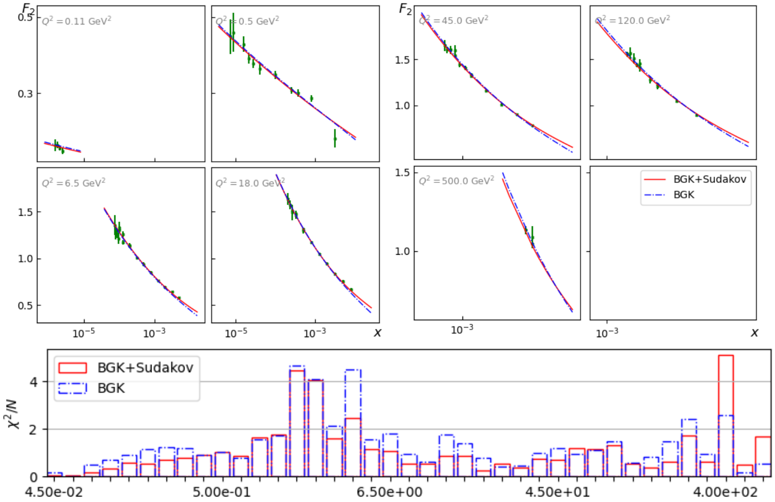


- $\lambda_g < 0$ $\rightarrow$ small-x rise comes from DGLAP evolution.
- The gluon density shifts to the higher k_t region.

Saturation scale



Comparison with data at selected Q^2.



Q^2 dependence

For data in the range $0.045 \leq Q^2 \leq Q_{\mathrm{up}}$:

Q^2_{up} $[\mathrm{GeV}^2]$	GBW	GBW+Sud
5	1.55	1.55
25	1.46	1.41
50	1.97	1.83
100	2.36	2.15
650	4.44	2.66

Q^2_{up} $[\mathrm{GeV}^2]$	BGK	BGK+Sud
5	1.63	1.59
25	1.42	1.30
50	1.52	1.23
100	1.55	1.25
650	1.56	1.21

- Introduction of the hard scale by the Sudakov factor accounts for the Q^2 dependence better.
- No more deterioration with increased range of Q^2 for the BGK model.

Non-perturbaive Sudakov factor

Non-perturbative Sudakov factor
[Collins et al., 1985, Prokudin et al., 2015]:

$$S(r, Q^2) = S_{\mathrm{pert}}(r, Q^2) + S_{\mathrm{np}}(r, Q^2),$$

where [Prokudin et al., 2015]

$$S_{\mathrm{np}}(r, Q^2) = g_1 r^2 + g_2 \log\left(\frac{r}{r_*}\right) \log\left(\frac{Q}{Q_0}\right).$$

$\mu_{0S}^2[\mathrm{GeV}^2]$	$\mathrm{GBW} + \mathrm{Sud}_{\mathrm{pert}}$	$\mathrm{GBW} + \mathrm{Sud}_{\mathrm{pert+np}}$
1	2.71	2.72
2	2.66	2.67
3	2.64	2.65
4	2.64	2.64
5	2.64	2.65

$\mu_{0S}^2[\mathrm{GeV}^2]$	$\mathrm{BGK} + \mathrm{Sud}_{\mathrm{pert}}$	$\mathrm{BGK} + \mathrm{Sud}_{\mathrm{pert+np}}$
1	1.18	1.17
2	1.21	1.17
3	1.25	1.21
4	1.29	1.21
5	1.32	1.22

- The non-perturbative Sudakov factor is relevant in the large-r region, where the photon wave function strongly suppresses.
- For a small value ($\sim 2\ \mathrm{GeV}^2$), effects of the non-perturbative Sudakov factor is negligible.

Summary so far

- The Sudakov form factor can be incorporated in the dipole cross section.
- The Sudakov form factor introduces the hard scale dependence.
- Better description of data over a wide range of Q^2, for both the GBW and BGK models.
- Improvement at the moderate-x region ~ 0.01.
- For the process studied, effects of non-perturbative Sudakov factor is negligible.

Exact Gluon Kinematics

(in progress)

x/z vs x

As we saw earlier, the derivation of the dipole factorization involves with the dipole cross section $\sigma_{\rm dipole}(x/z, r)$, while the GBW formula uses $\sigma_{\rm dipole}(x, r)$.

$1/z$ enters implicitly as the modified variable

$$\tilde{x} \equiv x \left(1 + 4\frac{m_f^2}{Q^2}\right).$$

The derivation of dipole factorization requires that $1/z$ do not depend on k_t nor κ'_t. However in the k_t factorization,

$$\frac{1}{z} = 1 + \frac{\kappa'^2_t + m_f^2}{\beta(1-\beta)Q^2} + \frac{k_t^2}{Q^2} \qquad \geq \left(1 + 4\frac{m_f^2}{Q^2}\right).$$

- Massive light quarks partially simulate non-zero k_t and κ_t.
- Larger value of $1/z$ suppresses the cross section.
- Dipole factorization assumes that the dipole does not change its size throughout the interaction.
- Argument of the running coupling can depend of k_t and κ_t.

What are the effects of these points?

Factorization Formula

Using $\boldsymbol{\kappa}'_t \equiv \boldsymbol{\kappa}_t - (1-\beta)\mathbf{k}_t$, angle ϕ can be integrated [Kimber, 2001, Kwiecinski et al., 1997]:

$$F_2(x, Q^2) = \sum_f e_f^2 \frac{Q^2}{2\pi} \int \tilde{\Pi}' \, \alpha_s \mathcal{F}(x/z, k_t^2) \Theta(1 - x/z) \times \Big[\left(\beta^2 + (1-\beta)^2\right) \left(\frac{I_1}{2\pi} - \frac{I_2}{2\pi}\right) + \left(m_f^2 + 4Q^2\beta^2(1-\beta)^2\right) \left(\frac{I_3}{2\pi} - \frac{I_4}{2\pi}\right) \Big],$$

where

$$\frac{I_1}{2\pi} = \frac{N_1 N_2 + N_3^2}{\left(N_1^2 + 2N_1N_2 + N_3^2\right)^{3/2}} \qquad \frac{I_2}{2\pi} = \frac{N_3 - (1-2\beta)N_1}{(N_1 + N_4)\sqrt{N_1^2 + 2N_1N_2 + N_3^2}}$$

$$\frac{I_3}{2\pi} = \frac{N_1 + N_2}{\left(N_1^2 + 2N_1N_2 + N_3^2\right)^{3/2}} \qquad \frac{I_4}{2\pi} = \frac{(1-\beta)}{(N_1 + N_4)\sqrt{N_1^2 + 2N_1N_2 + N_3^2}}$$

for

$$N_1 \equiv \beta(1-\beta)Q^2 + m_f^2 \qquad N_2 \equiv {\kappa'}_t^2 + (1-\beta)^2 k_t^2$$

$$N_3 \equiv {\kappa'}_t^2 - (1-\beta)^2 k_t^2 \qquad N_4 \equiv {\kappa'}_t^2 + \beta(1-\beta)k_t^2.$$

Set-up

The same configuration as the previous fit.

- massless light quarks
- $m_c = 1.3\ \mathrm{GeV}$ & $m_b = 4.6\ \mathrm{GeV}$
- HERA F_2 data.
- $0.045 \leq Q^2 \leq 650\ \mathrm{GeV}^2$ & $x < 0.01$

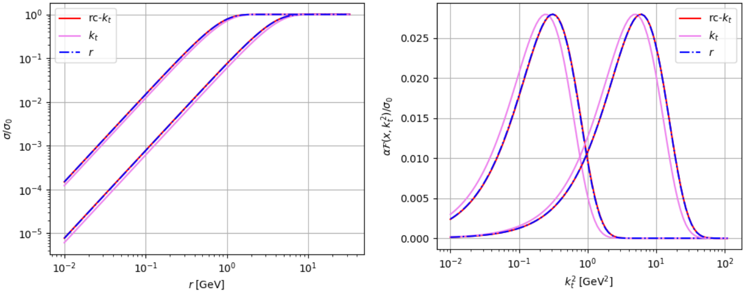
For the GBW model we additionally investigate the k_t-factorization with running coupling, assuming

$$\alpha_s(\mu^2)\mathcal{F}(x, k_t^2) = \frac{\alpha_s(\mu^2)}{0.2}\frac{N_c}{4\pi}\int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{i\mathbf{k}_t\cdot\mathbf{r}}\nabla_{\mathbf{r}}^2\sigma_{\mathrm{dipole}}(x, r),$$

where $\mu^2 = k_t^2 + \kappa'^2_t + m_f^2$ [Kwiecinski et al., 1997].

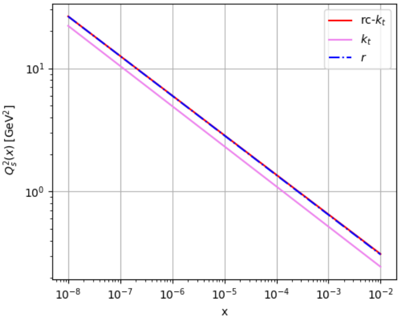
GBW

-	σ_0 [mb]	x_0 (10^{-4})	λ	χ^2/dof
r-GBW	1.907e+01	2.582e+00	3.219e-01	4.438e+00
r-GBW-massive	2.384e+01	1.117e+00	3.082e-01	5.274e+00
k_t-GBW	3.344e+01	1.333e+00	3.258e-01	4.396e+00
rc-k_t-GBW	5.613e+01	2.639e+00	3.213e-01	2.448e+00

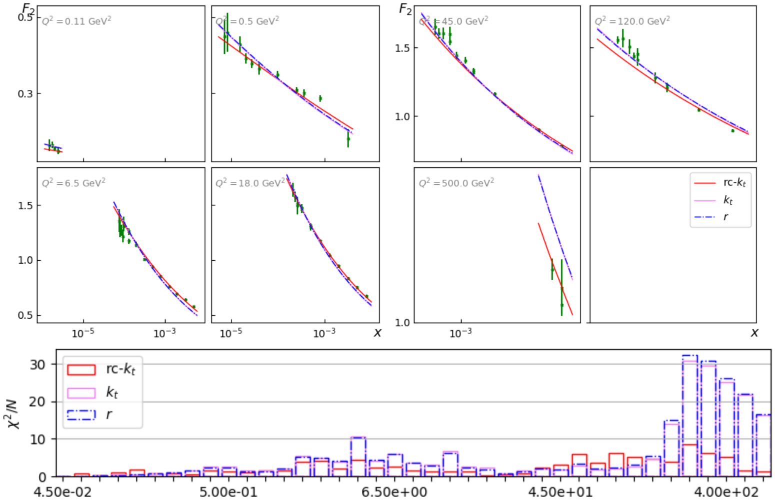


- The fit qualities of dipole factorization and k_t-factorization are comparable.
- Fit parameters are similar too, except for σ_0.
- Considerable improvement from the running coupling.
- x_0 of k_t-GBW and r-GBW-massive are similar.

Saturation scale

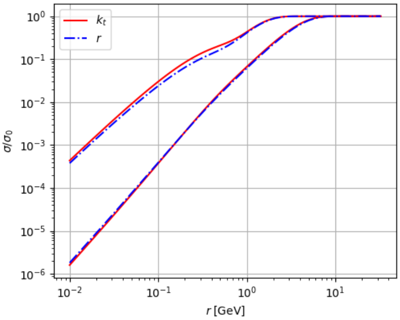


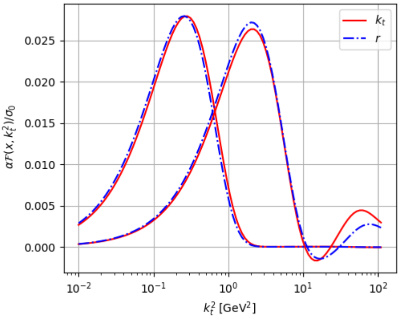
Comparison with data at selected Q^2.



BGK

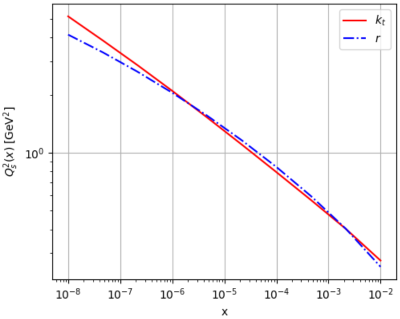
-	σ_0 [mb]	A_g	λ_g	C	μ_0^2 [GeV^2]	χ^2/dof
r-BGK	2.328e+01	1.229e+00	7.311e-02	3.420e-01	1.928e+00	1.564e+00
k_t-BGK	3.470e+01	1.048e+00	2.205e-01	2.391e-01	9.954e-01	1.527e+00



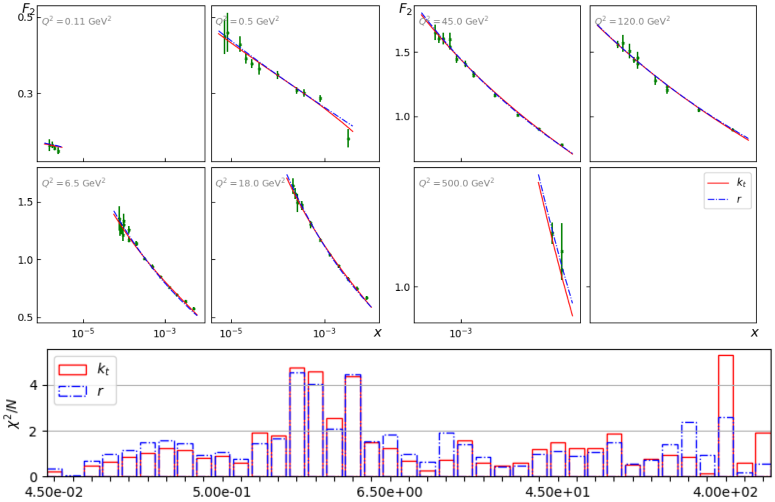


- The fit qualities of dipole factorization and k_t-factorization are comparable.
- Some difference in the second peak of the gluon density.
- Difference is more visible at the small-x region.

Saturation scale



Comparison with data at selected Q^2.



Summary & Next step

- The exact gluon kinematics suppresses the cross section.
- For inclusive DIS, the fit qualities are comparable for the dipole factorization and k_t-factorization.
- The effect shows predominantly in the normalization σ_0.
- Running coupling is important in the description of the large-Q^2 region.

As a next step we will investigate less inclusive processes:

- Dijets in DIS relevant for EIC
- Single Inclusive jet production in p-p collision.

Acknowledgement

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