Phenomenology of light mesons with $J = 2, 3$

Shahriyar Jafarzade

Jan Kochanowski University of Kielce, Poland

joint works with F.Giacosa, R.D.Pisarski, A.Koenigstein,
M.Piotrowska , A.Vereijken

PRD 103(2021)9, 096027 && PRD 106(2022)3, 036008 && arXiv:(2023) 2304.05225

Białasówka seminar, AGH, Kraków, 14.04.2023

Introduction

- The low-energy sector of QCD is rich in experimental data
- Some experiments fo hadron spectroscopy: CLAS12(JLab), GlueX (JLab), BESIII (CHINA), BaBar(SLAC), Belle (JAPAN), CLEO (Syracuse), COMPASS (CERN), LHCb (CERN), PANDA (FAIR) etc.
- Experiments to study light mesons: a)BESIII (charmonium decay) $M \leq 2.5$ GeV; b)GlueX (photoproduction) $M \leq 2.8$ GeV
- Hadrons: i) Mesons (integer spin) and ii) Baryons (half-integer spin)
- Conventional mesons: quark-anti-quark pairs
- Other mesons: glueballs, exotic states, hybrids etc.
- Theoretical methods to study hadrons: LQCD, Effective models of hadrons, QCD sum rules, Functional methods

Quark Model

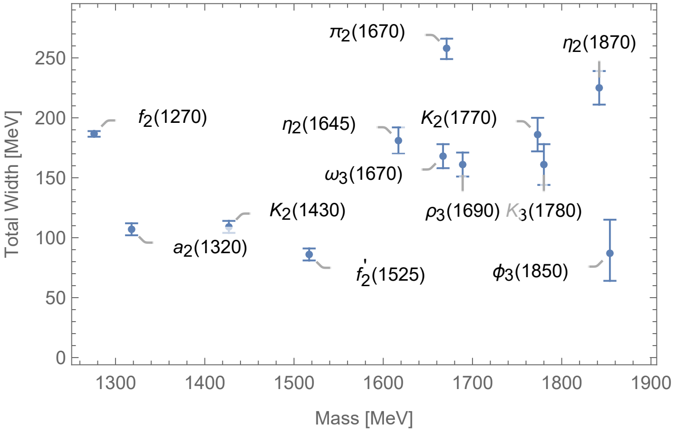
- "A Schematic Model of Baryons and Mesons" [M. Gell-Mann, Physics Letters.8(3) 214-215 1964]
- "An SU(3) Model for Strong Interaction Symmetry and its Breaking" [G.Zweig, CERN Report 1964]
- Three light quarks $\{u, d, s\}$ within mesons leads their classification as octets and a singlet because of an $SU(3)$ representation: $3 \otimes \bar{3} = 8 \oplus 1$
- Lightest mesons are so-called the pseudoscalars $\{\pi^+, \pi^0, \pi^-\}$
- Other pseudoscalars: four Kaons (K), η and η'
- Isoscalar states $\eta \,\&\, \eta'$ are mixed
- The quark model fails to explain the larger mass of $\eta'(958)$ than of η

- Conventional mesons are classified according to their quantum numbers such as angular momentum-L, spin-S, total spin-J
- List of light mesons up to $J = 3$ [R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 083C01 (2022)]

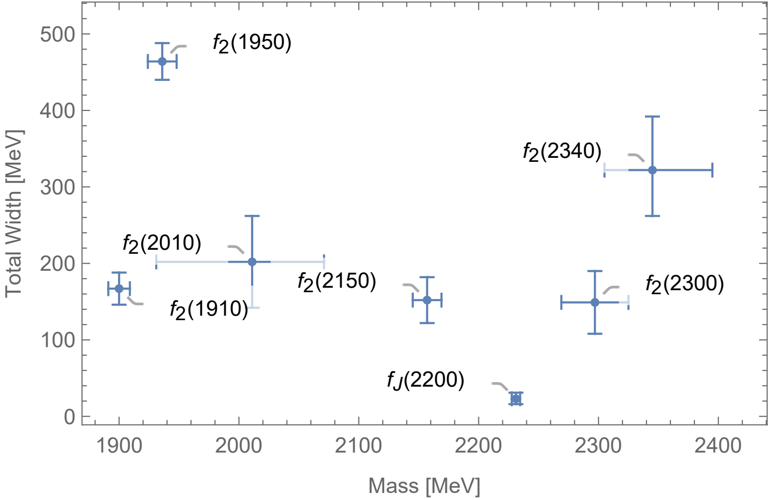
$n^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = \frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	$I = 0$ f'	$I = 0$ f	
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$	
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$	
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$	
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	???
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$	???
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$	
1^3D_2	2^{--}	?	$K_2(1820)$[a]	?	?	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	???

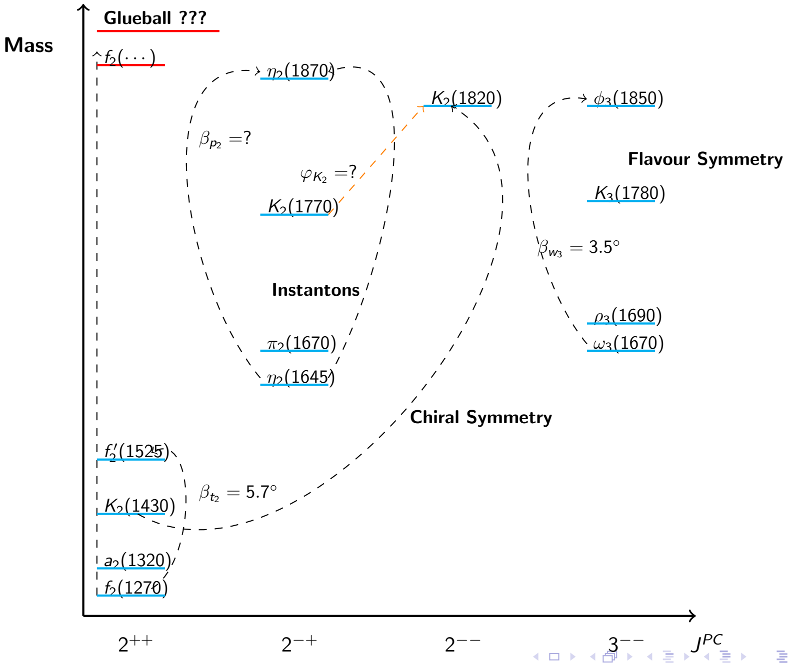
Light mesons with $2 \leq J \leq 3$

- i) Are 3^{--} mesons quark-anti-quark objects? ii) What is the mixing angle in 2^{-+} sector? iii) Where are 2^{--} mesons?



- iv) Where is the 2^{++} glueball (meson made of gluons only)?




Glueball ???
Mass
$f_2(\cdots)$
$\eta_2(1870)$
$K_2(1820)$
$\phi_3(1850)$
$\beta_{p_2} = ?$
$\varphi_{K_2} = ?$
Flavour Symmetry
$K_2(1770)$
$K_3(1780)$
$\beta_{w_3} = 3.5°$
Instantons
$\rho_3(1690)$
$\pi_2(1670)$
$\omega_3(1670)$
$\eta_2(1645)$
Chiral Symmetry
$f_2'(1525)$
$\beta_{t_2} = 5.7°$
$K_2(1430)$
$a_2(1320)$
$f_2(1270)$
2^{++}
2^{-+}
2^{--}
3^{--}
J^{PC}

Symmetries of QCD

- QCD Lagrangian: G_μ-gluon, q_i-quark, $\overline{q}_i$-anti-quark fields

$$\mathcal{L}_{QCD} = \mathrm{tr}\Big(\bar{q}_i(i\gamma_\mu D^\mu - m_i)q_i - \frac{1}{2}G_{\mu\nu}G^{\mu\nu}\Big),\ G_{\mu\nu:} = D_\mu G_\nu - D_\nu G_\mu - ig[G_\mu, G_\nu]$$

$$D_\mu := \partial_\mu - igG_\mu,\ G_\mu := G^a_\mu t^a,\ [t^a, t^b] = if^{abc}t^c$$

- Color symmetry: $SU(3)_c \rightarrow$ Confinement (we can see hadrons e.g. mesons and glueballs not quark and gluons separately)
- Chiral symmetry ($m_i \rightarrow 0$) $N_f = 3$: $U(3)_R \times U(3)_L \equiv U(1)_{V=R+L} \times SU(3)_V \times SU(3)_A \times U(1)_{A=R-L}$ (Hadronic model for 2^{++} and 2^{--} mesons)
- Broken: 1) explicitly by $m_i \neq 0$ and 2) spontaneously breaking to $SU(3)_V \times U(1)_V$ (Hadronic model for 3^{--} mesons)
- Dilitation invariance: $x^\mu \rightarrow \lambda^{-1}x^\mu$ works in chiral limit and classically
- Quantum level $\rightarrow$ Trace anomaly
- $U(1)_A$: Classical symmetry, broken by quantum effects $\rightarrow$ Axial anomaly (Hadronic model for 2^{-+} mesons)

Mesons within nonets

- Mesons can be grouped into the nonets which transform under the adjoint transformation of the flavour symmetry $U_V(3)$

$$P=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad V^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega^\mu_{1,N}+\rho^{0\mu}_1}{\sqrt{2}} & \rho^{+\mu}_1 & K^{*+\mu}_1 \\ \rho^{-\mu} & \frac{\omega^\mu_N-\rho^{0\mu}}{\sqrt{2}} & K^{*0\mu} \\ K^{*-\mu} & \bar{K}^{*0\mu} & \omega^\mu_S \end{pmatrix}$$

- Nonets can be extended to the chiral ones based on the chiral symmetry $U_L(3)\times U_R(3)$

$n^{2S+1}L_J$	J^{PC}	$I=1$	$I=\frac{1}{2}$	$I=0$	$I=0$	Chiral nonet
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	$\frac{\Phi-\Phi^\dagger}{2i}$
1^3D_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	$\frac{L_\mu+R_\mu}{2}$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	$\frac{L_\mu-R_\mu}{2}$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	$\frac{\mathbf{L}_{\mu\nu}+\mathbf{R}_{\mu\nu}}{2}$
1^3D_2	2^{--}	$\rho_2(?)$	$K_2(1820)$	$\omega_2(?)$	$\phi_2(?)$	$\frac{\mathbf{L}_{\mu\nu}-\mathbf{R}_{\mu\nu}}{2}$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	$\frac{\mathbf{\Phi}_{\mu\nu}-\mathbf{\Phi}^\dagger_{\mu\nu}}{2i}$
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$\frac{\mathbf{L}_{\mu\nu\rho}+\mathbf{R}_{\mu\nu\rho}}{2}$

Symmetries of chiral nonets

- Chiral nonets under
  1. Parity: $P = (-1)^{L+1}$,
  2. Charge conjugation: $C = (-1)^{L+S}$
  3. Chiral symmetry: $U_L(3) \times U_R(3)$

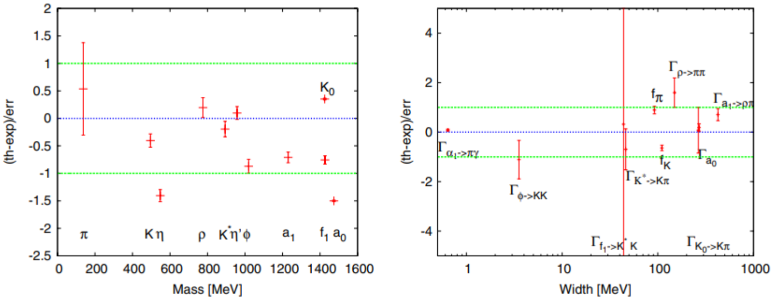
Nonet	Parity (P)	Charge conjugation (C)	$U_L(3) \times U_R(3)$
$\Phi(t, \vec{x})$	$\Phi^\dagger(t, -\vec{x})$	$\Phi^t(t, \vec{x})$	$U_L \Phi U_R^\dagger$
$R^\mu(t, \vec{x})$	$L_\mu(t, -\vec{x})$	$-(L^\mu(t, \vec{x}))^t$	$U_R R^\mu U_R^\dagger$
$L^\mu(t, \vec{x})$	$R_\mu(t, -\vec{x})$	$-(R^\mu(t, \vec{x}))^t$	$U_L L^\mu U_L^\dagger$
$\mathbf{R}^{\mu\nu}(t, \vec{x})$	$\mathbf{L}_{\mu\nu}(t, -\vec{x})$	$(\mathbf{L}^{\mu\nu}(t, \vec{x}))^t$	$U_R \mathbf{R}^{\mu\nu} U_R^\dagger$
$\mathbf{L}^{\mu\nu}(t, \vec{x})$	$\mathbf{R}_{\mu\nu}(t, -\vec{x})$	$(\mathbf{R}^{\mu\nu}(t, \vec{x}))^t$	$U_L \mathbf{L}^{\mu\nu} U_L^\dagger$
$\mathbf{\Phi}^{\mu\nu}(t, \vec{x})$	$\mathbf{\Phi}^\dagger_{\mu\nu}(t, -\vec{x})$	$(\mathbf{\Phi}^{\mu\nu}(t, \vec{x}))^t$	$U_L \mathbf{\Phi}^{\mu\nu} U_R^\dagger$
$\mathbf{R}^{\mu\nu\rho}(t, \vec{x})$	$\mathbf{L}_{\mu\nu\rho}(t, -\vec{x})$	$-(\mathbf{L}^{\mu\nu\rho}(t, \vec{x}))^t$	$U_R \mathbf{R}^{\mu\nu\rho} U_R^\dagger$

Sigma Models

- Sigma models used to study the interaction between mesons
- The mathematical structure developed [F. Gürsey Nuovo Cimento 16, 230-240 (1960)]
- A non-linear sigma model: scalar states are integrated out, leaving the pseudoscalar states [J. S. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957); M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960); S.Weinberg, Phys. Rev. Lett. 18, 188 (1967).]
- A Linear Sigma Model (LSM): keep the scalar and pseudoscalar degrees of freedom [J. S. Schwinger, Phys. Lett. B 24, 473 (1967); S. Weinberg, Phys. Rev. 166, 1568 (1968).]
- Spin-1 eLSM (Frankfurt model) [D.Parganlija, F.Giacosa, and D.H.Rischke PRD 82, 054024 (2010)]

eLSM results

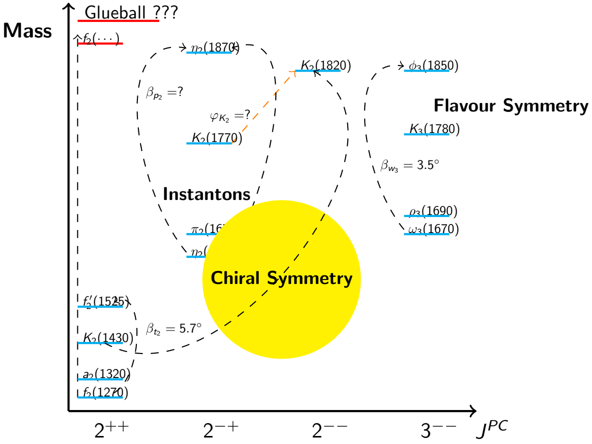
- (Axial)-vector mesons [D. Parganlija, F. Giacosa, et al. PRD 87 (2013) 014011]



- The pseudoscalar glueball [S.Janowski, F. Giacosa, D.H.Rischke PRD 90 (2014) 11, 114005]
- Charmed Mesons [W.I. Eshraim, F. Giacosa, D.H. Rischke EPJA 51 (2015) 9, 112]
- Baryons [L. Olbrich, F. Giacosa, et.al. PRD 93 (2016) 3, 034021]
- Other hadrons, finite temperature effects, exotic mesons · · ·

Spin-2 eLSM

- "From well-known tensor mesons to yet unknown axial-tensor mesons" [S.Jafarzade, A.Vereijken, M.Piotrowska and F.Giacosa, PRD 106(2022)3, 036008]



Spin-2 eLSM

- Chiral invariant Lagrangian with $\Delta := \mathrm{diag}\{\delta_N, \delta_N, \delta_S\}$

$$\mathcal{L} = \mathrm{Tr}\Big\{\big(\frac{m^2}{2} + \Delta\big)(\mathbf{L}_{\mu\nu}^2 + \mathbf{R}_{\mu\nu}^2)\Big\} + \frac{h_1^{\mathrm{ten}}}{2}\mathrm{Tr}\{\Phi^\dagger\Phi\}\mathrm{Tr}\{\mathbf{L}^{\mu\nu}\mathbf{L}_{\mu\nu} + \mathbf{R}^{\mu\nu}\mathbf{R}_{\mu\nu}\} +$$
$$+ h_2^{\mathrm{ten}}\mathrm{Tr}\{\Phi^\dagger\mathbf{L}^{\mu\nu}\mathbf{L}_{\mu\nu}\Phi + \Phi\mathbf{R}^{\mu\nu}\mathbf{R}_{\mu\nu}\Phi^\dagger\} + 2h_3^{\mathrm{ten}}\mathrm{Tr}\{\Phi\mathbf{R}^{\mu\nu}\Phi^\dagger\mathbf{L}_{\mu\nu}\},$$

- Masses of spin-2 mesons in terms of $\phi_N \approx 0.158\,\mathrm{GeV}$ and $\phi_S \approx 0.138\,\mathrm{GeV}$

$$m_{\rho_2}^2 - m_{a_2}^2 = -h_3^{\mathrm{ten}}\phi_N^2, \quad m_{K_{2A}}^2 - m_{K_2}^2 = -\sqrt{2}h_3^{\mathrm{ten}}\phi_N\phi_S, \quad m_{f_{2s}}^2 - m_{\omega_{2,S}}^2 = 2h_3^{\mathrm{ten}}\phi_S^2$$
$$m_{\rho_2}^2 = m_{\omega_{2,N}}^2\,, \quad m_{a_2}^2 = m_{f_{2n}}^2$$

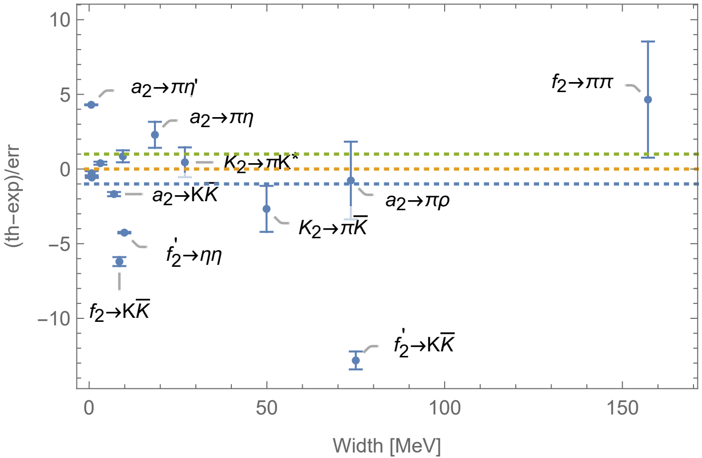
Resonance	Mass (in MeV)	Resonance	Mass (in MeV)
$a_2(1320)$	**1317**	$\rho_2(?)$	1661
$K_2^*(1430)$	**1427**	$K_2^*(1820)$	**1819**
$f_2(1270)$	1315	$\omega_{2,N}(?)$	1663
$f_2'(1525)$	1522	$\omega_{2,S}(?)$	1966

- Mass prediction for missing $\rho_2(?)$ is near to [S. Godfrey and N. Isgur PRD (1985) 32, 189]

- Test of chiral model with three parameters for spin-2 mesons

Decay process	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \bar{K}\,K$	4.06 ± 0.14	$7.0^{+2.0}_{-1.5} \leftrightarrow (4.9 \pm 0.8)\%$
$a_2(1320) \longrightarrow \pi\,\eta$	25.37 ± 0.87	$18.5 \pm 3.0 \leftrightarrow (14.5 \pm 1.2)\%$
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	1.01 ± 0.03	$0.58 \pm 0.10 \leftrightarrow (0.55 \pm 0.09)\%$
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	44.82 ± 1.54	$49.9 \pm 1.9 \leftrightarrow (49.9 \pm 0.6)\%$
$f_2(1270) \longrightarrow \bar{K}\,K$	3.54 ± 0.29	$8.5 \pm 0.8 \leftrightarrow (4.6^{+0.5}_{-0.4})\%$
$f_2(1270) \longrightarrow \pi\,\pi$	168.82 ± 3.89	$157.2^{+4.0}_{-1.1} \leftrightarrow (84.2^{+2.9}_{-0.9})\%$
$f_2(1270) \longrightarrow \eta\,\eta$	0.67 ± 0.03	$0.75 \pm 0.14 \leftrightarrow (0.4 \pm 0.08)\%$
$f_2'(1525) \longrightarrow \bar{K}\,K$	23.72 ± 0.60	$75 \pm 4 \leftrightarrow (87.6 \pm 2.2)\%$
$f_2'(1525) \longrightarrow \pi\,\pi$	0.67 ± 0.14	$0.71 \pm 0.14 \leftrightarrow (0.83 \pm 0.16)\%$
$f_2'(1525) \longrightarrow \eta\,\eta$	1.81 ± 0.05	$9.9 \pm 1.9 \leftrightarrow (11.6 \pm 2.2)\%$
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$

- Chiral Lagrangian $\mathcal{L} = g_2\left(\mathrm{Tr}\left\{\mathbf{L}_{\mu\nu}L^{\mu}L^{\nu}\right\} + \mathrm{Tr}\left\{\mathbf{R}_{\mu\nu}R^{\mu}R^{\nu}\right\}\right)$



- There is an overall agreement between theory and PDG results
- The fit result for the mixing angle agrees well with PDG
- Results can be improved by including further subdominant chiral invariant terms

- Decay rates for missing 2^{--} imply broad resonance even for the lightest member of it $\rho_2(?)$
- Using the coupling of chiral model determined from 2^{++} decays leads larger decay rates compared to the fit of LQCD results

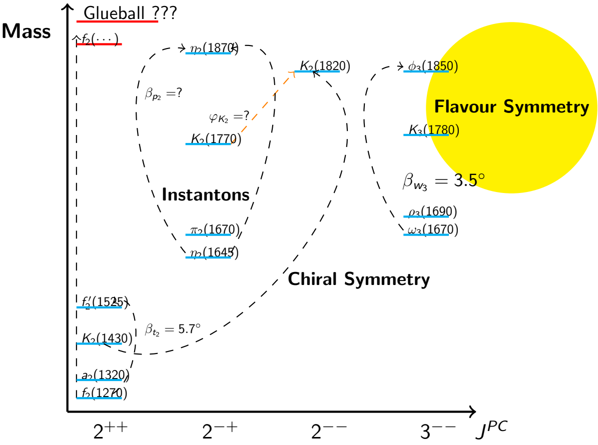
Decay process ($2^{--} \to 1^{--} + 0^{-+}$)	eLSM (PDG)	eLSM (LQCD)	LQCD
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	99 ± 50	30	–
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	85 ± 43	27	36
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	419 ± 210	122	125
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	0.8	0.3	–

- Compared to the well-established 3^{--} meson

Decay process ($3^{--} \to 1^{--} + 0^{-+}$)	PDG	eLSM (PDG)	LQCD
$\rho_3(1690) \longrightarrow \rho(770)\,\eta$	–	3.8 ± 0.8	–
$\rho_3(1690) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	–	3.4 ± 0.7	2
$\rho_3(1690) \longrightarrow \omega(782)\,\pi$	25.8 ± 9.8	35.8 ± 7.4	22
$\rho_3(1690) \longrightarrow \phi(1020)\,\pi$	–	0.036 ± 0.007	–

Spin-3 eLSM

- "Phenomenology of $J^{PC} = 3^{--}$ tensor mesons" [S.Jafarzade, A.Koenigstein and F.Giacosa, PRD 103(2021)9, 096027]



Spin-3 eLSM

- Chiral Lagrangian $\mathcal{L} = g_3\Big(\mathbf{tr}\big\{\mathbf{L}^{\mu\nu\rho}(\partial_\mu L_\nu)L_\rho + \mathbf{R}^{\mu\nu\rho}R_\nu(\partial_\mu R_\rho) + \cdots\big\}\Big)$

- Following terms are reduced from the chiral Lagrangians

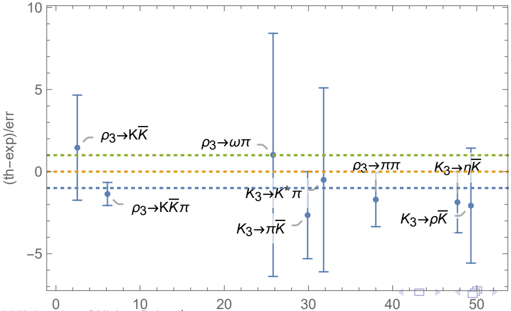
Decay Mode	Interaction Lagrangians	$\frac{1}{7} \times \lvert -i\mathcal{M}\rvert^2$
$3^{--} \to 0^{-+} + 0^{-+}$	$\mathcal{L}_{WPP} = g_{WPP}\,\mathbf{tr}\big[W^{\mu\nu\rho}\big[P\,,(\partial_\mu\partial_\nu\partial_\rho P)\big]_-\big]$	$g_{WPP}^2 \times \frac{2\lvert\vec{k}_{P_1,P_2}\rvert^6}{35}$
$3^{--} \to 0^{-+} + 1^{--}$	$\mathcal{L}_{WVP} = g_{WVP}\,\varepsilon^{\mu\nu\rho\sigma}\,\mathbf{tr}\big[W_{\mu\alpha\beta}\big\{(V_{\nu\rho})\,,(\partial^\alpha\partial^\beta\partial_\sigma P)\big\}_+\big]$	$g_{WVP}^2 \times \frac{8\lvert\vec{k}_{V,P}\rvert^6\, m_W^2}{105}$
Subdominant channels	$\cdots$	$\cdots$

- Decay rate with momentum $\lvert\vec{k}_{A,B}\rvert = \frac{1}{2m_W}\sqrt{(m_W^2 - m_A^2 - m_B^2)^2 - 4m_A^2 m_B^2}$

$$\Gamma(W \to A + B) = \frac{\lvert\vec{k}_{A,B}\rvert}{8\pi m_W^2} \times \lvert -i\mathcal{M}\rvert^2 \times \kappa_i \times \Theta(m_W - m_A - m_B)$$

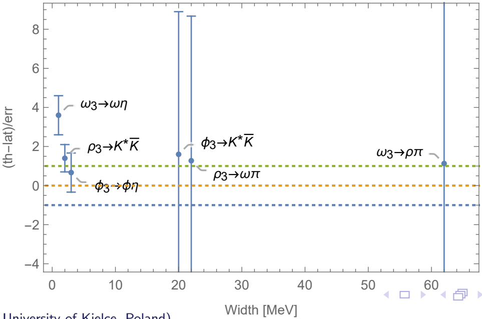
- Model predictions using two parameters show qualitative agreement with PDG data

Decay process	eLSM (MeV)	PDG (MeV)
$\rho_3(1690) \longrightarrow \pi\,\pi$	32.7 ± 2.3	38.0 ± 3.2
$\rho_3(1690) \longrightarrow \bar{K}\,K$	4.0 ± 0.3	2.54 ± 0.45
$\rho_3(1690) \to \omega(782)\,\pi$	35.8 ± 7.4	25.8 ± 9.8
$K_3^*(1780) \to \rho(770)\,K$	16.8 ± 3.5	49.3 ± 15.7
$K_3^*(1780) \longrightarrow \pi\,\bar{K}$	18.5 ± 1.3	29.9 ± 4.3
$K_3^*(1780) \longrightarrow \bar{K}\,\eta$	7.4 ± 0.6	47.7 ± 21.6
$K_3^*(1780) \to \rho(770)\,K$	16.8 ± 3.5	49.3 ± 15.7



- Comparison to the LQCD data [C.Johnson and J.Dudek PRD 103, 074502 (2021)]

Decay process ($3^{--} \to 1^{--} + 0^{-+}$)	eLSM (MeV)	LQCD (MeV)
$\rho_3(1690) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	3	2
$\rho_3(1690) \longrightarrow \omega(782)\,\pi$	36	22
$\omega_3(1670) \longrightarrow \rho(770)\,\pi$	97	62
$\omega_3(1670) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	2.9	2
$\omega_3(1670) \longrightarrow \omega(782)\,\eta$	2.8	1
$\phi_3(1850) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	36	20
$\phi_3(1850) \longrightarrow \phi(1020)\,\eta$	4	3

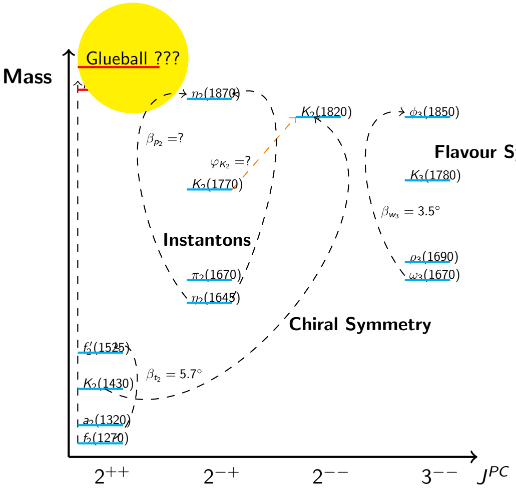


- Model results are internally consistent, namely, the sum of the various decay channels do not overshoot the PDG total decay widths
- Chiral symmetry is the guiding principle even for high-spin (3^{--}) mesons
- Some predictions can be tested in *GlueX* and *CLAS12* experiments at Jefferson Lab

decay process	eLSM (keV)
$\rho_3^{\pm/0}(1690) \to \gamma\,\pi^{\pm/0}$	69 ± 14
$\rho_3^{0}(1690) \to \gamma\,\eta$	157 ± 32
$\rho_3^{0}(1690) \to \gamma\,\eta'(958)$	20 ± 4
$K_3^{\pm}(1780) \to \gamma\,K^{\pm}$	58 ± 12
$K_3^{0}(1780) \to \gamma\,K^{0}$	231 ± 48
$\omega_3(1670) \to \gamma\,\pi^{0}$	560 ± 120
$\omega_3(1670) \to \gamma\,\eta$	19 ± 4
$\omega_3(1670) \to \gamma\,\eta'(958)$	1.4 ± 0.3
$\phi_3(1850) \to \gamma\,\pi^{0}$	4 ± 1
$\phi_3(1850) \to \gamma\,\eta$	129 ± 26
$\phi_3(1850) \to \gamma\,\eta'(958)$	35 ± 7

Spin-2 glueball eLSM

- "Is $f_2(1950)$ the tensor glueball?" [A.Vereijken, S.Jafarzade, M.Piotrowska and F.Giacosa, (arXiv:2304.05225)]



Spin-2 glueball

- Glueballs are not experimentally observed yet
- In radiative J/ψ decays, the primary $c\bar{c}$ pair converts into two gluons and a photon. Scalar and tensor glueballs can be produced
- Spin-2 glueball with the quantum number $J^{PC} = 2^{++}$ is the second lightest state after scalar glueball according to all LQCD simulations
- Analyses of the BESIII data showed an enhancement in the mass distribution around 2210 MeV.[E. Klempt et.al. Phys.Lett.B 830 (2022) 137171]
- Glueball extended chiral Lagrangian

$$\mathcal{L}_\lambda = \lambda G_{2\,,\mu\nu}\Big(\mathrm{Tr}\Big[\{L^\mu, L^\nu\}\Big] + \mathrm{Tr}\Big[\{R^\mu, R^\nu\}\Big]\Big)$$

Branching Ratio	eLSM	Branching Ratio	eLSM
$\frac{G_2(2210)\longrightarrow \bar{K}\ K}{G_2(2210)\longrightarrow \pi\ \pi}$	0.4	$\frac{G_2(2210)\longrightarrow \rho(770)\ \rho(770)}{G_2(2210)\longrightarrow \pi\ \pi}$	55
$\frac{G_2(2210)\longrightarrow \eta\ \eta}{G_2(2210)\longrightarrow \pi\ \pi}$	0.1	$\frac{G_2(2210)\longrightarrow \bar{K}^*(892)\ \bar{K}^*(892)}{G_2(2210)\longrightarrow \pi\ \pi}$	46
$\frac{G_2(2210)\longrightarrow \eta\ \eta'}{G_2(2210)\longrightarrow \pi\ \pi}$	0.004	$\frac{G_2(2210)\longrightarrow \omega(782)\ \omega(782)}{G_2(2210)\longrightarrow \pi\ \pi}$	18
$\frac{G_2(2210)\longrightarrow \eta'\ \eta'}{G_2(2210)\longrightarrow \pi\ \pi}$	0.006	$\frac{G_2(2210)\longrightarrow \phi(1020)\ \phi(1020)}{G_2(2210)\longrightarrow \pi\ \pi}$	6

- BR for the various spin-2 PDG resonances

Resonances	Branching Ratios	PDG	eLSM
$f_2(1910)$	$\rho(770)\rho(770)/\omega(782)\omega(782)$	2.6 ± 0.4	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	0.09 ± 0.05	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'(958)$	> 0.05	~ 8
$f_2(1910)$	$\omega(782)\omega(782)/\eta\eta\prime(958)$	2.6 ± 0.6	~ 200
$f_2(1950)$	$\eta\eta/\pi\pi$	0.14 ± 0.05	0.081
$f_2(1950)$	$K\overline{K}/\pi\pi$	~ 0.8	0.32
$f_2(1950)$	$4\pi/\eta\eta$	> 200	> 700
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	0.79 ± 0.11	0.1
$f_2(2150)$	$K\overline{K}/\eta\eta$	1.28 ± 0.23	~ 4
$f_2(2150)$	$\pi\pi/\eta\eta$	< 0.33	~ 10
$f_J(2220)$	$\pi\pi/K\overline{K}$	1.0 ± 0.5	~ 2.5

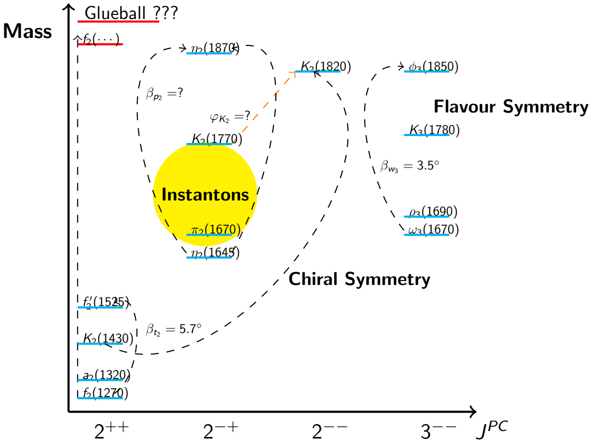
- Our results for $f_2(1950)$ does not contradict with PDG data
- PDG result for the total decay width $\Gamma_{f_2(1950)} = 464 \pm 24$ MeV

- Test of spin-2 resonances in PDG as a glueball

Resonances	Interpretation status
$f_2(1910)$	Agreement with some data, but excluded by $\eta\eta/\eta\eta'$ and $\omega\omega/\eta\eta\prime$ ratios
$f_2(1950)$	$\eta\eta/\pi\pi$ agrees with data, no contradictions with data, but implies broad tensor glueball Best fit as predominantly glueball of considered resonances
$f_2(2010)$	Likely primarily strange-antistrange content
$f_2(2150)$	All available data contradicts theoretical prediction
$f_J(2220)$	Data on $\pi\pi/K\bar{K}$ disagrees with theory, largest predicted decay channels are not seen
$f_2(2300)$	Likely primarily strange-antistrange content
$f_2(2340)$	Likely primarily strange-antistrange content, would also imply a broad glueball

$U(1)_A$ breaking effect on spin-2 mesons

- [R.D.Piasrski, F.Giacosa, and S.Jafarzade (to appear soon)]



Mixing angles

- Singlet-octet mixing angles of mesons [R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 083C01 (2022)]

$n^{2s+1}\ell_J$	J^{PC}	θ_{quad} [°]	θ_{lin} [°]
1^1S_0	0^{-+}	-11.3	-24.5
1^3S_1	1^{--}	39.2	36.5
1^3P_2	2^{++}	29.6	28.0
1^3D_3	3^{--}	31.8	30.8

- In case of 2^{++}, mixing angle from decay widths within eLSM (32.5°) agrees with the PDG result obtained from

$$\theta_{\text{quad}} = \arctan\left(\sqrt{\frac{4m_{K_2}^2 - m_{a_2}^2 - 3m_{f_2'}^2}{-4m_{K_2}^2 + m_{a_2}^2 + 3m_{f_2}^2}}\right)$$

- Mixing relation works for 1^{--} and 3^{--} mesons
- The same mixing formula does not explain the decay results for 2^{-+} mesons. What is the sign and numerical value of the mixing angle in this sector?

- For the parameter α of $U_A(1)$, quark fields under $SU_L(3) \times SU_R(3) \times U_A(1)$

$$q_{L,R} := \mathbb{P}_{L,R}\, q = \frac{1}{2}(1 \mp \gamma_5) q\,, \qquad q_{L,R} \to e^{\mp i \frac{\alpha}{2}} U_{L,R}\, q_{L,R}$$

- Mesonic fields with $J = 0, 2$

Mesons	Chiral Nonet
$\{\pi, K, \eta, \eta'(958)\}$ Scalar mesons	$\Phi := \overline{q}_R q_L$
$\{\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1875)\}$ Orbitally excited tensor mesons	$\Phi_{\mu\nu} := -\overline{q}_R(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu - \frac{1}{4}\delta_{\mu\nu} \overleftrightarrow{D}^2) q_L$

- Following interaction terms are violating $U(1)_A$

$$\frac{\varepsilon^{ijk}\varepsilon^{i'j'k'}}{3!}\left(\Phi^{ii'}\Phi^{jj'}\Phi^{kk'}\,,\ \Phi^{ii'}\Phi^{jj'}_{\mu\nu}\Phi^{\mu\nu\,kk'}\,,\cdots\right)$$

- Why $\eta'(958)$ is much larger than η? Solved by using the instantons ['t Hooft, Phys. Rev. Lett. 37, 8 (1976)]
- Expansion of Euclidean action assuming the zero modes dominance leads to effective quark interaction ['t Hooft, Phys. Rev. D 14, 3432 (1976)]

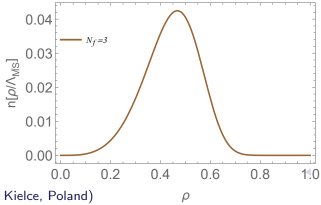
- Instantons are the self dual solutions of the Yang-Mill equations in Euclidean space-time [Belavin et.al., Phys. Lett. B 59, 85 (1975)]
- Dirac operator has a zero mode in the presence of instanton
- Unknown parameter M relates mesonic and quark fields

$$\Phi = \frac{1}{2M^2}\left(\frac{1}{3}(\overline{\psi}\lambda^0\psi + \overline{\psi}\gamma_5\lambda^0\psi)\lambda^0 + \sum_{a=1}^{8}(\overline{\psi}\lambda^a\psi + \overline{\psi}\gamma_5\lambda^a\psi)\lambda^a\right)$$

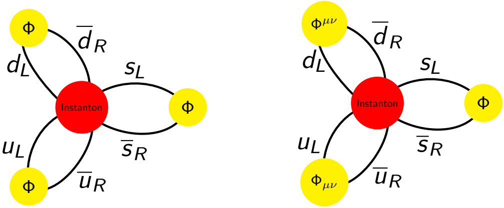
- $U_A(1)$ violating term for $J=0$

$$\mathcal{L}_{\text{eff}}^{J=0} = -a\left(\det\Phi + \det\Phi^\dagger\right) = -\frac{g_0}{3!}\left(\det(\overline{\psi}\mathbb{P}_R\psi) + \det(\overline{\psi}\mathbb{P}_L\psi)\right), a = \frac{g_0 M^6}{8\cdot 3!}$$

- Fix a from $\eta-\eta'$ mixing and $g_0 = \int d\rho\, n(\rho)(8\pi^2)^3\rho^9$ within dilute instanton gas model [R.D.Pisarski & F.Rennecke , PRD 101, 114019 (2020)] leads $M = 165$ MeV



- Instanton induced interactions for $J = 0$ and $J = 2$



- Lagrangian for spin-2 case [F.Giacosa, A.Koenigstein & R.D.Pisarski PRD 97 (2018) 9, 091901]

$$\mathcal{L}_{\text{eff}}^{J=2} = -\frac{c}{3!}\left[\left(\epsilon^{ijk}\epsilon^{i'j'k'}\cdot\Phi_{jj'}\cdot\Phi_{\mu\nu\;kk'}\cdot\Phi^{\mu\nu}_{ii'}\right)+\text{c.c.}\right], \quad c = \frac{g_2 M_2^8 M^2}{8\cdot 3!}$$

- Coupling $g_2 = \int d\rho\, n(\rho)(8\pi^2)^3\rho^{9+4}$ within DGI assuming $\Lambda_{\overline{MS}} = 0.3$ GeV
- Compared to $\mathcal{L}_{\text{eff}}^{J=0}$, we obtain the ratio $\frac{c}{a} = 0.01$ if $M_2 \approx M$ and
- Mixing has the same sign as 0^{-+} and strongly dependent on M_2
- Large mixing from decay channels [A.Koenigstein & F.Giacosa EPJA 52 (2016) 12, 356] is expected for $M_2 > M$

Conclusion

- We have extended the LSM
  1. Spin-2 mesons: $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, $f_2'(1525)$
  2. Spin-3 mesons: $\rho_3(1690)$, $K_3^*(1780)$, $\phi_3(1850)$, $\omega_3(1670)$
  3. Spin-2 glueball: $f_2(1950)$

- Qualitative agreement with PDG and LQCD data for well-established $J = 2, 3$ mesons

- Various predictions including the radiative decays can be the subject of future experimental studies (GlueX, CLAS12, BESIII)

- Model results can be useful for the assignment of missing 2^{--} mesons and the 2^{++} glueball

- Instanton induced interactions should be considered in the mixing between 2^{-+} mesons: $\eta_2(1645), \eta_2(1875)$

Thank you for the attention!