Fluctuation of conserved charges: Exploring the phase diagram and other applications

- Why fluctuations
- Making the connection between experiment and theory
- Constraining proton annihilation and local charge conservation

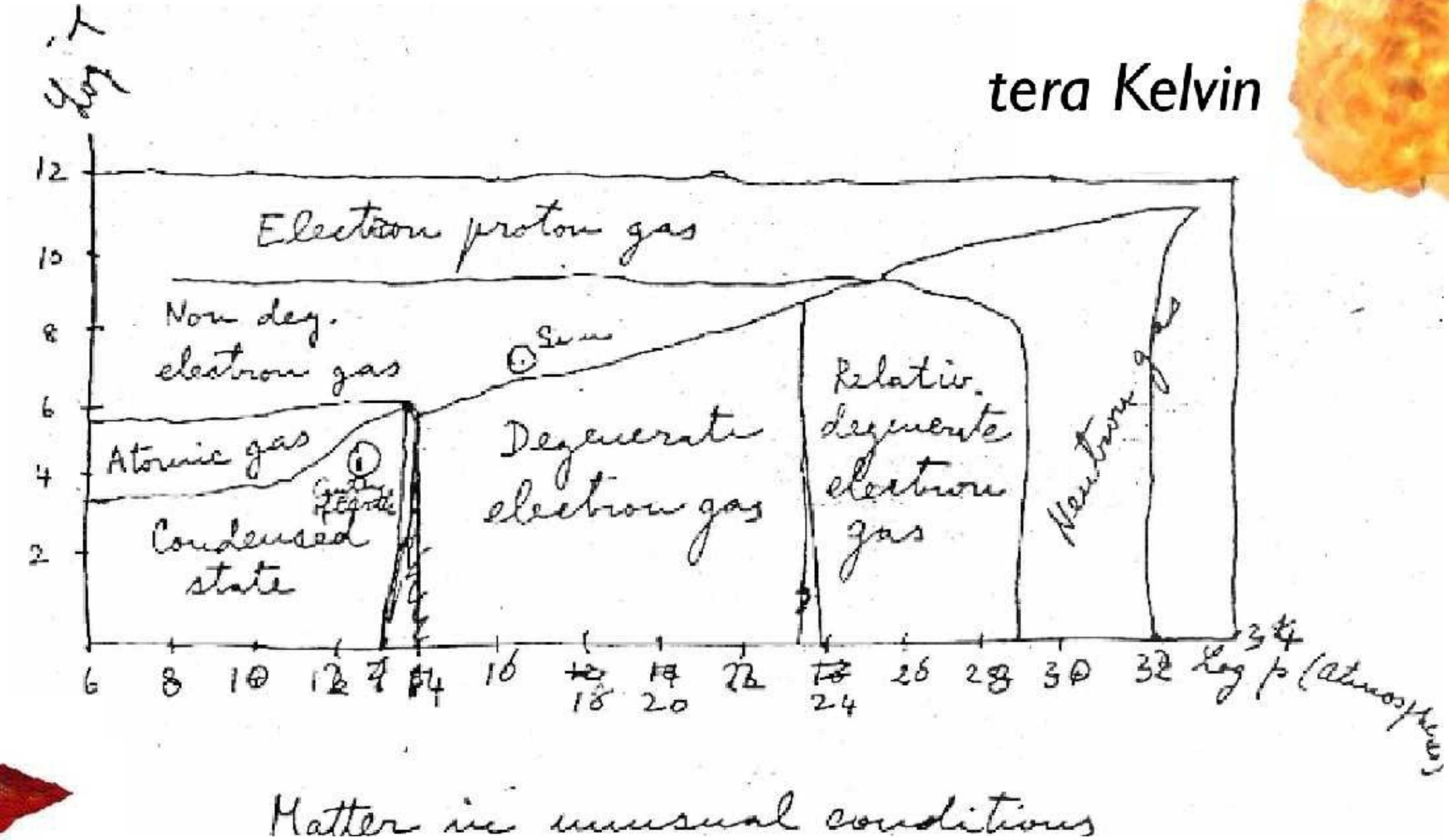
Thanks to:
D. Oliinychenko, A. Sorensen, J. Steinheimer, V. Vovchenko

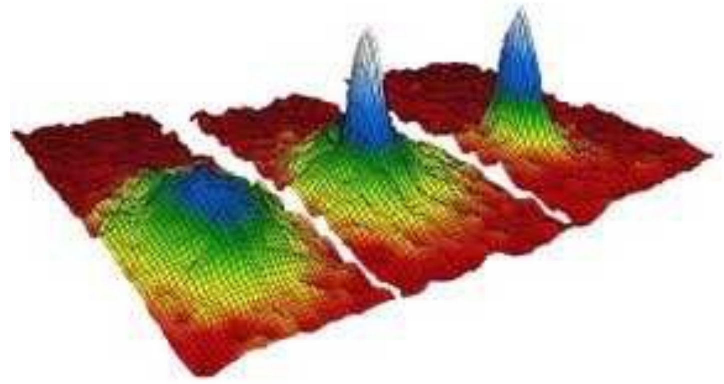
EMMI

An old question

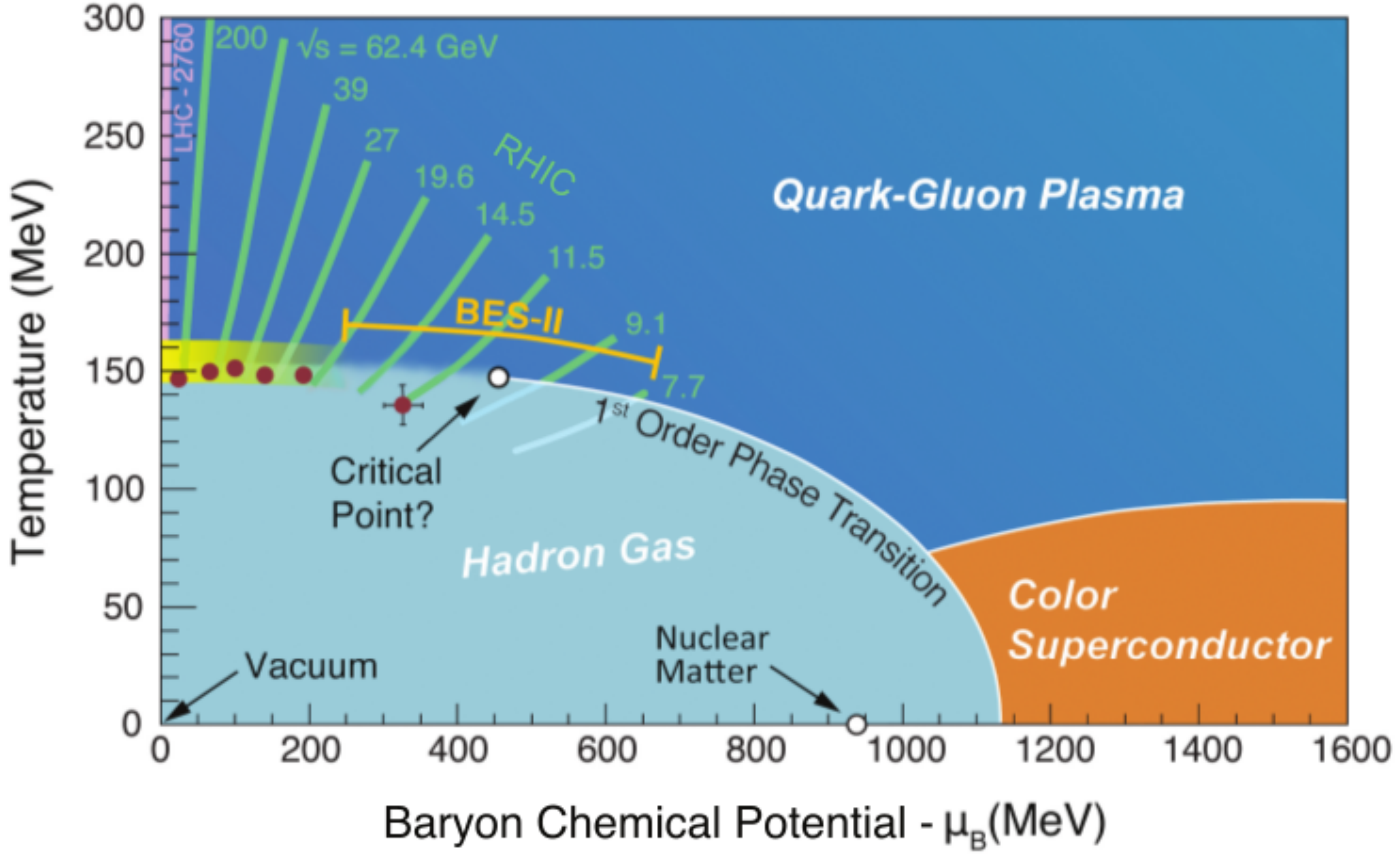
Fermi 1953

tera Kelvin





The phase diagram



Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

What we know about the Phase Diagram

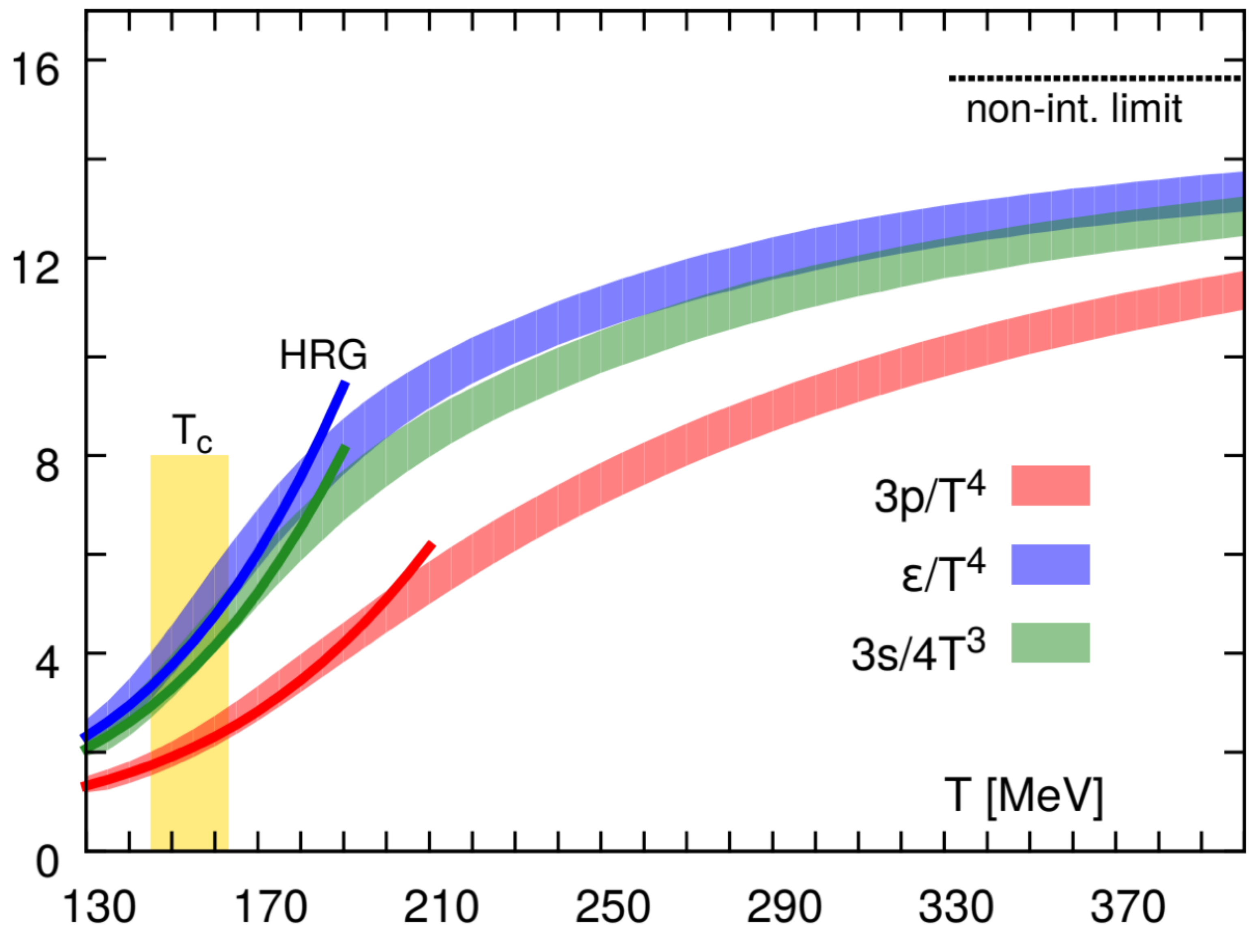
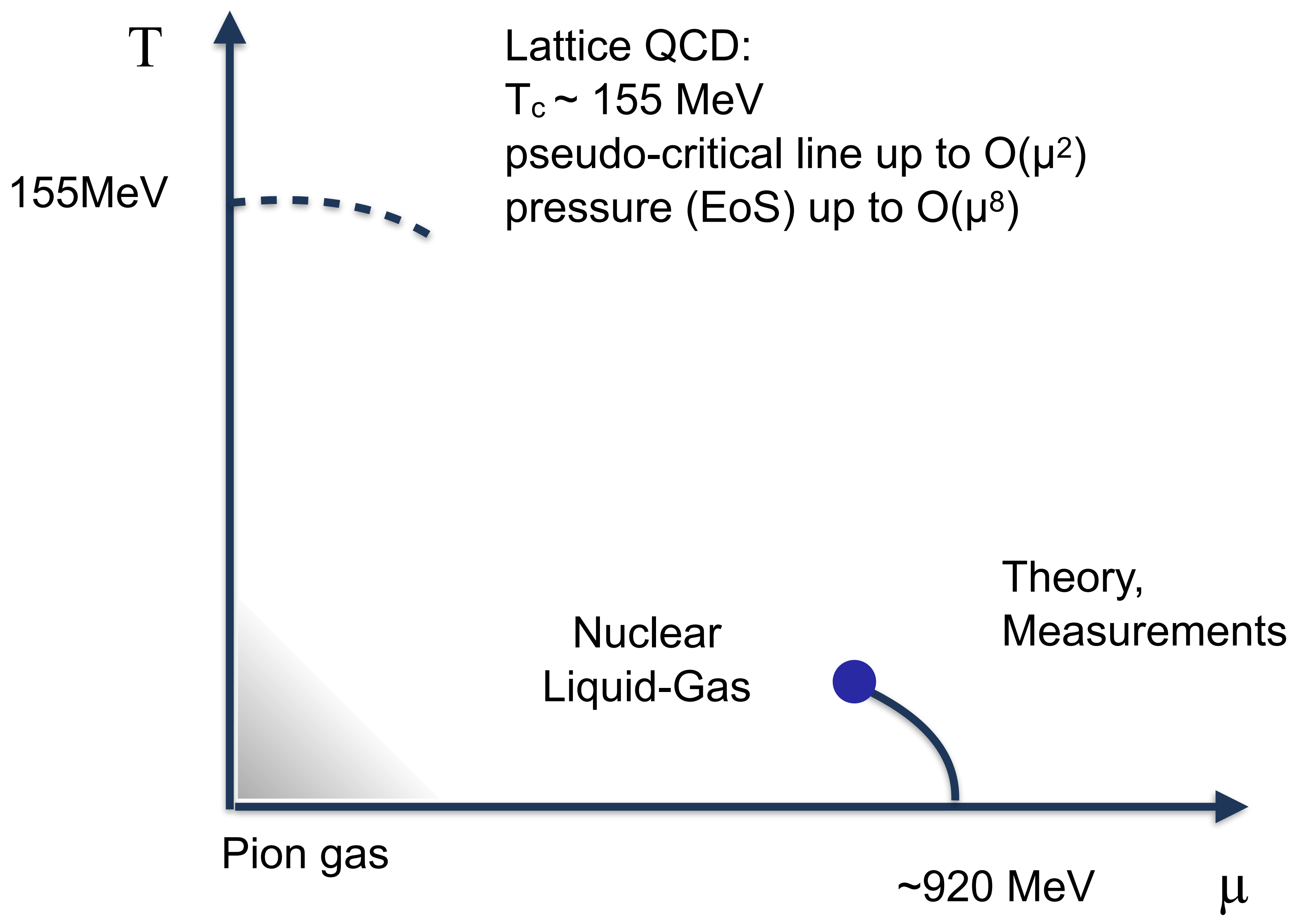
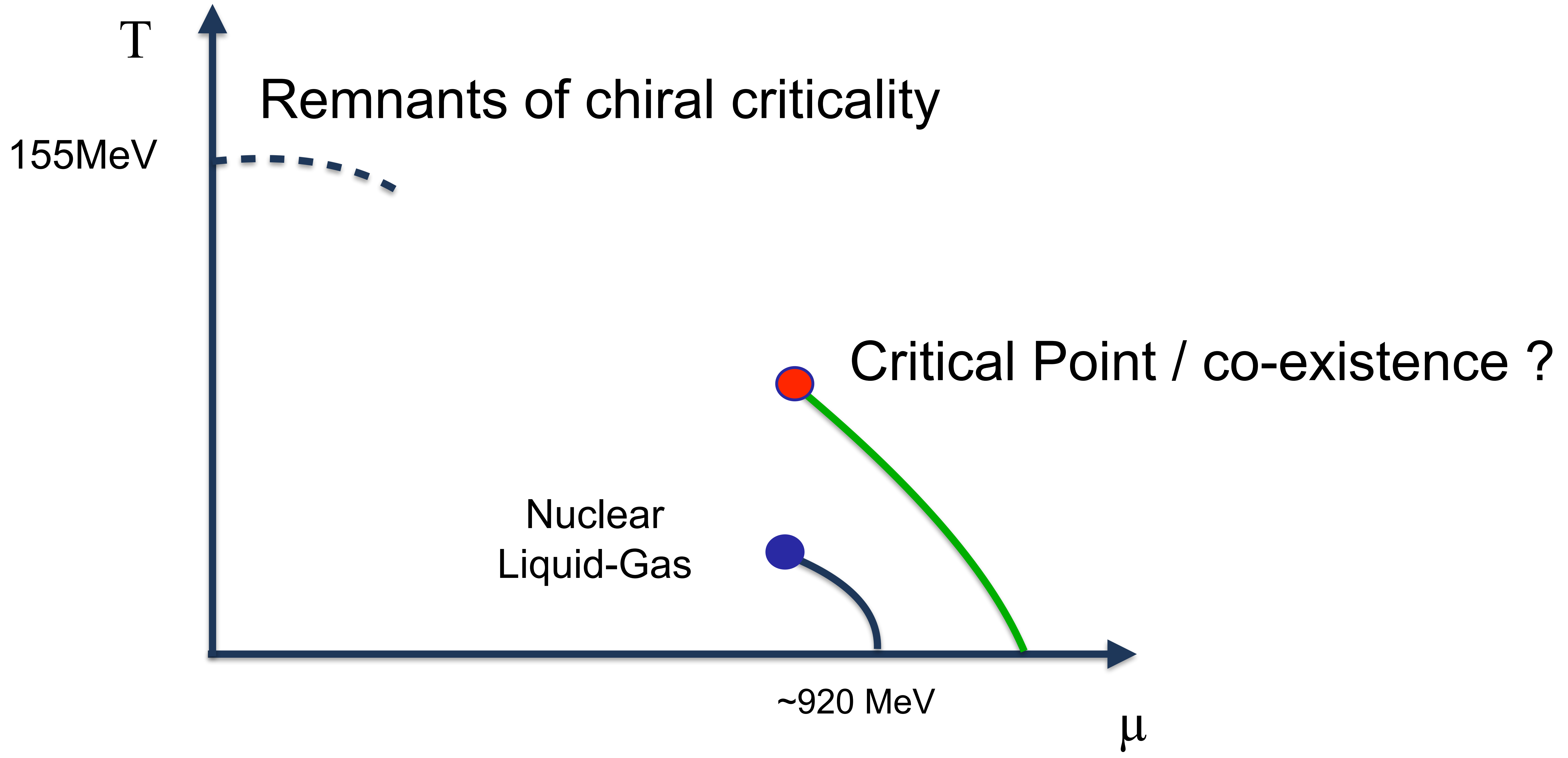


Figure from HotQCD coll., PRD ‘14



What we are looking for

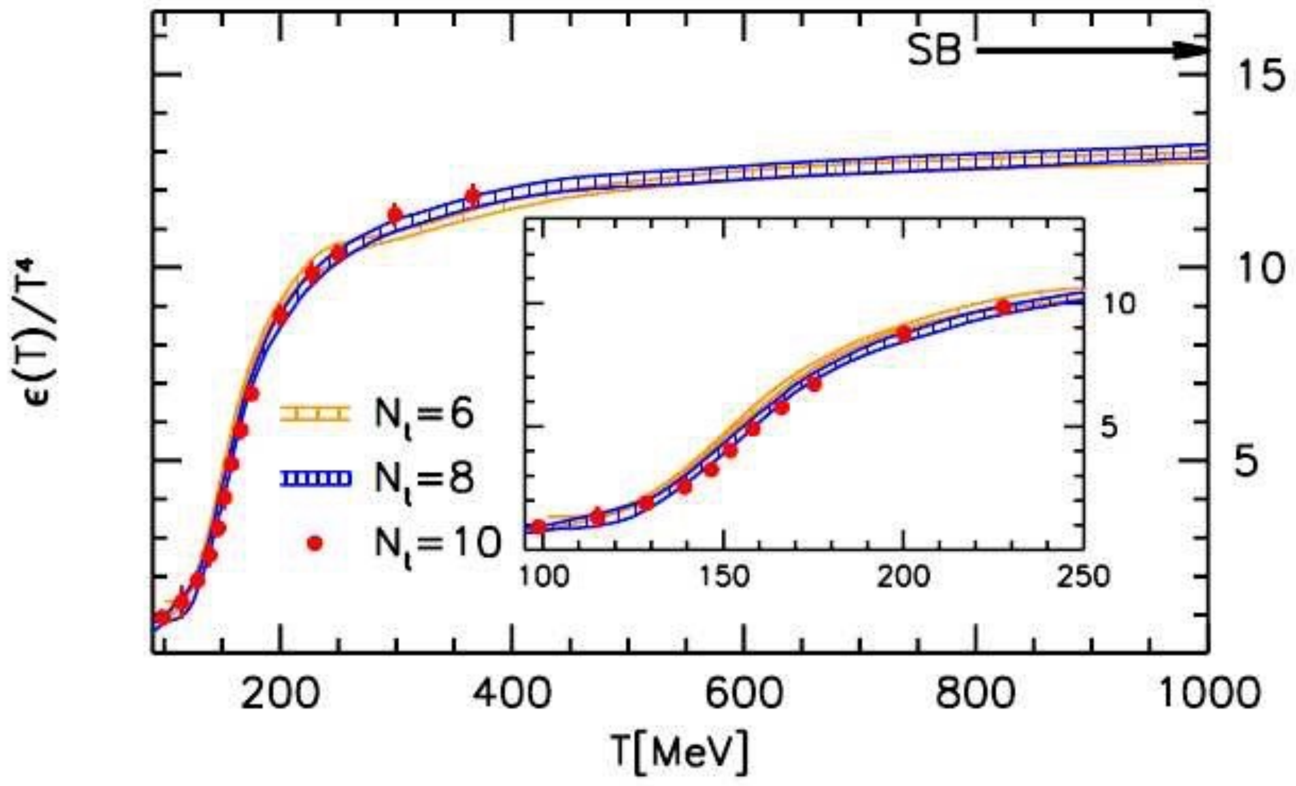


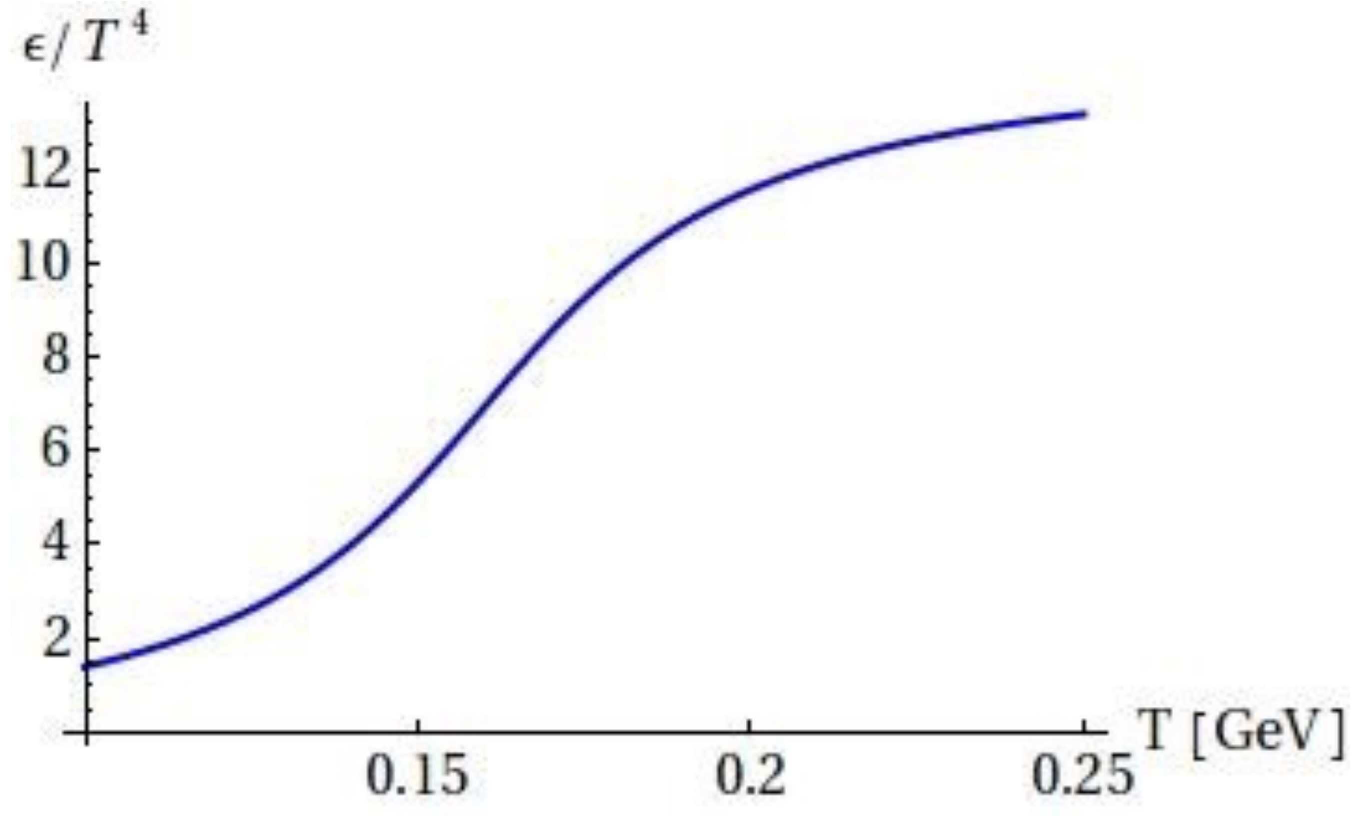
We are dealing with small system of finite lifetime

NO real singularities!

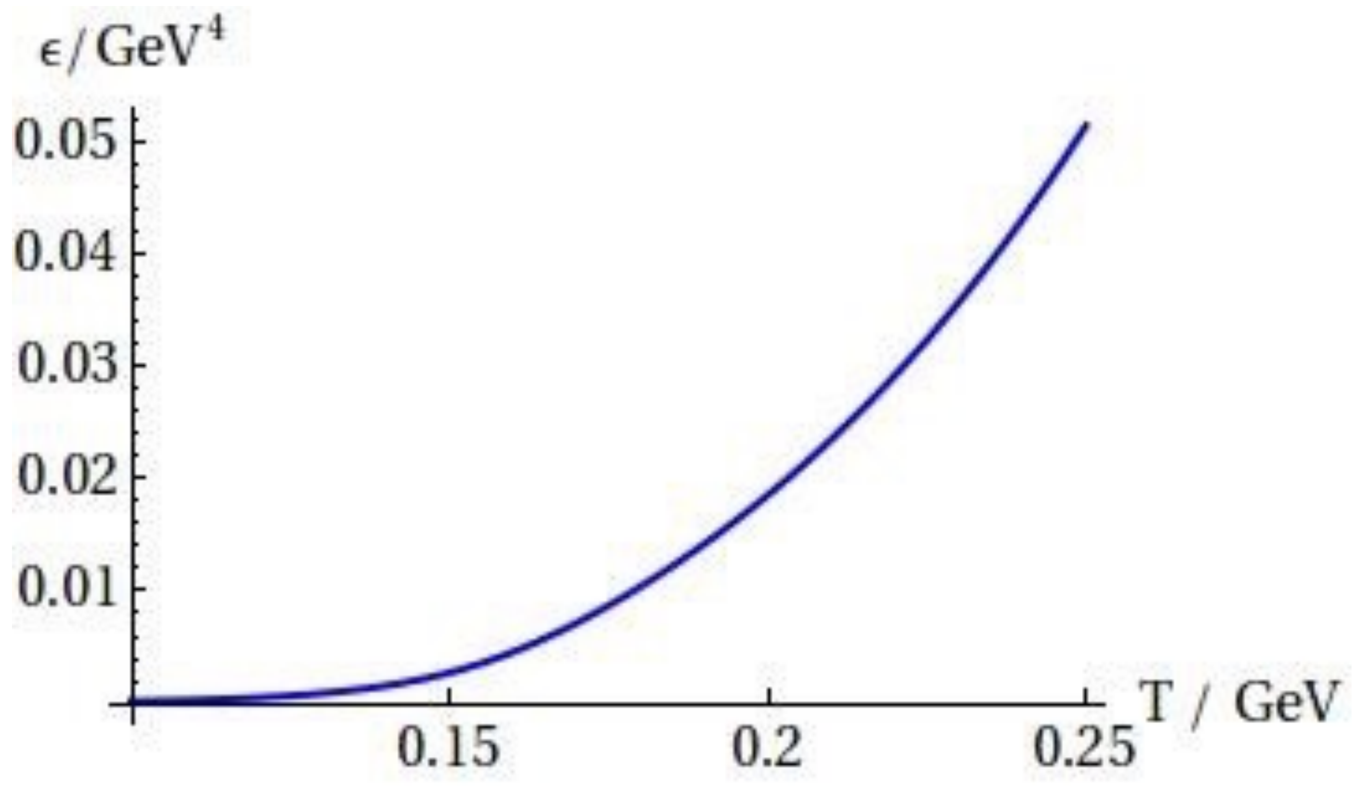
Cumulants and Phase structure

S. Borsanyi et al, JHEP 1011 (2010) 077





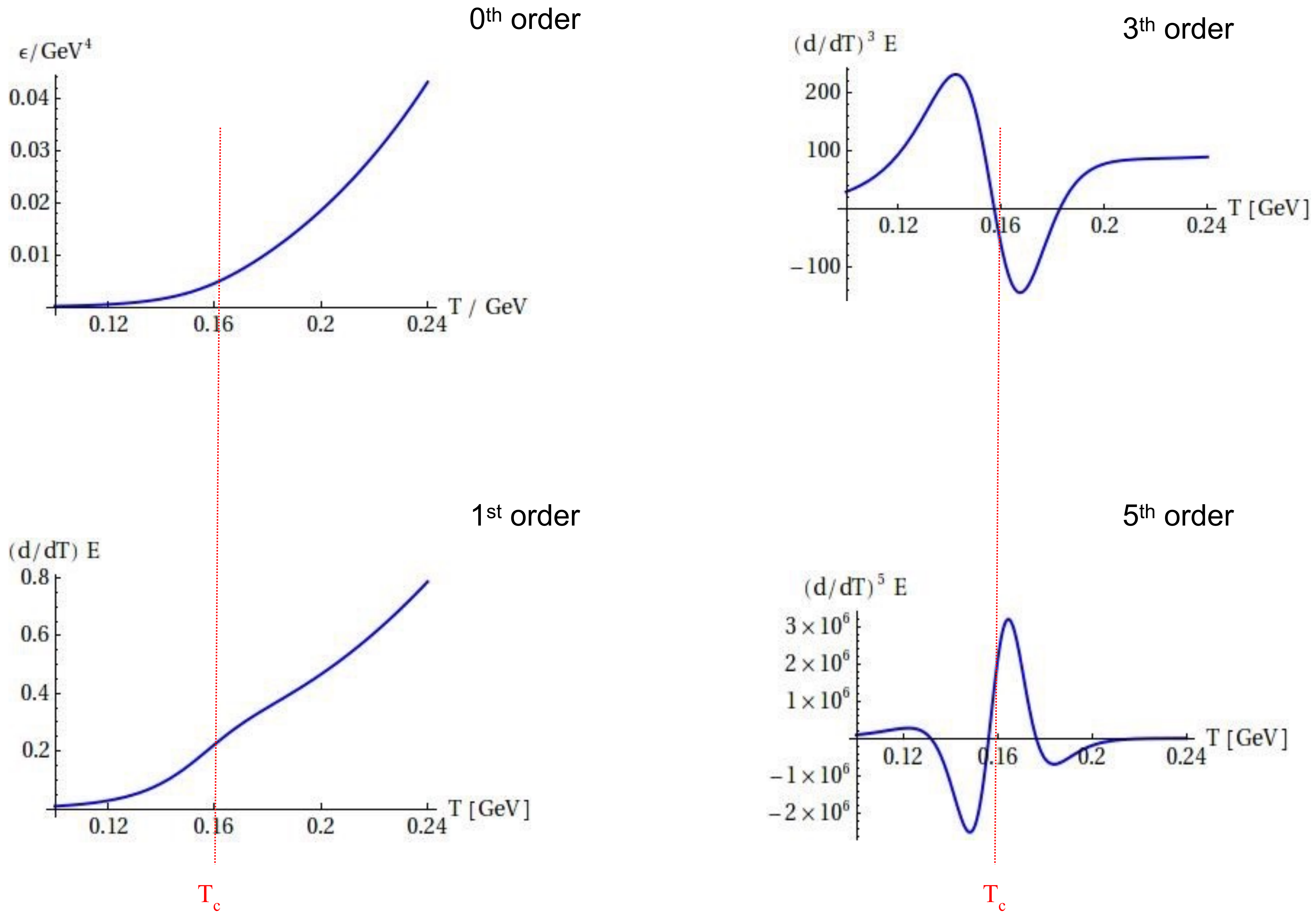
What we always see....



What it really means....

“T_c” ~ 155 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Cumulants of (Baryon) Number

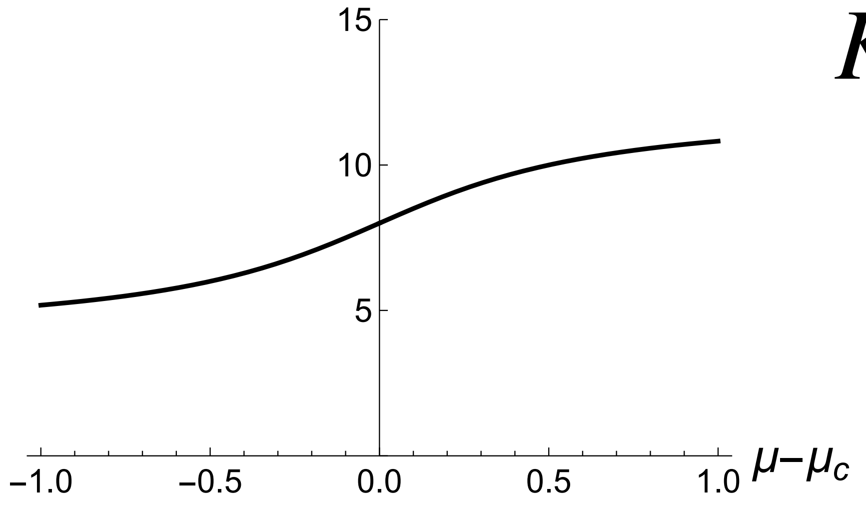
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \;\; K_2 = \langle N - \langle N \rangle \rangle^2\,, \;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

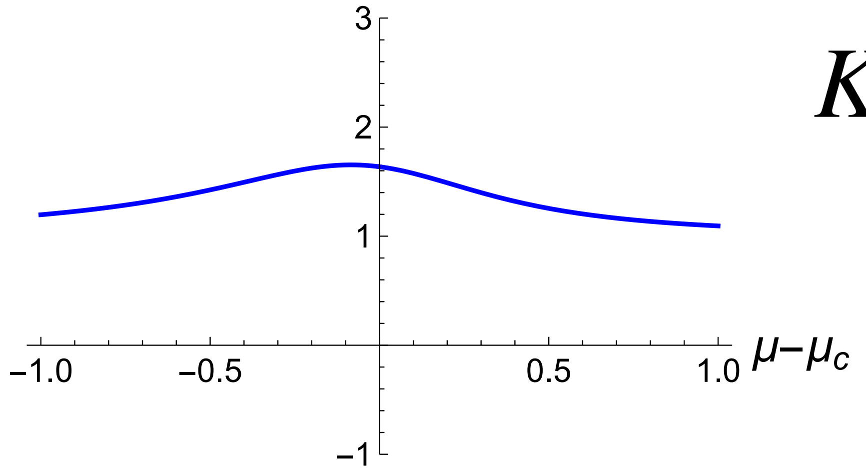
Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

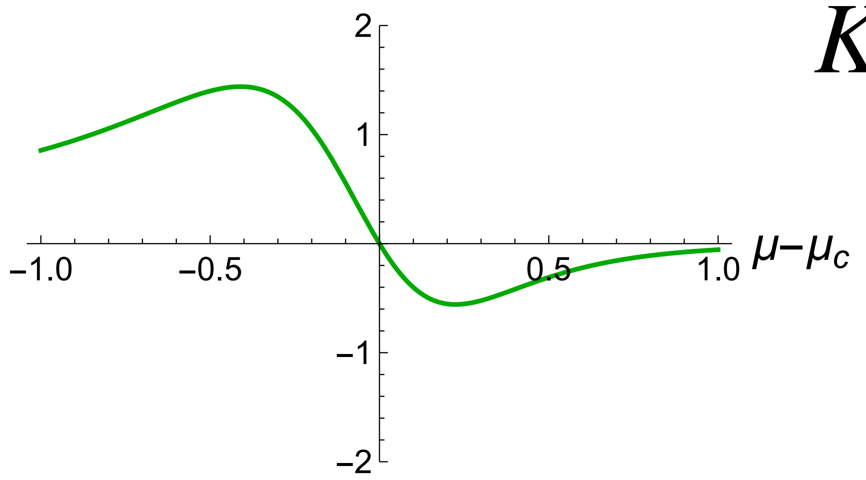
Cumulant Ratios: $\frac{K_2}{\langle N \rangle},\ \frac{K_3}{K_2},\ \frac{K_4}{K_2}$



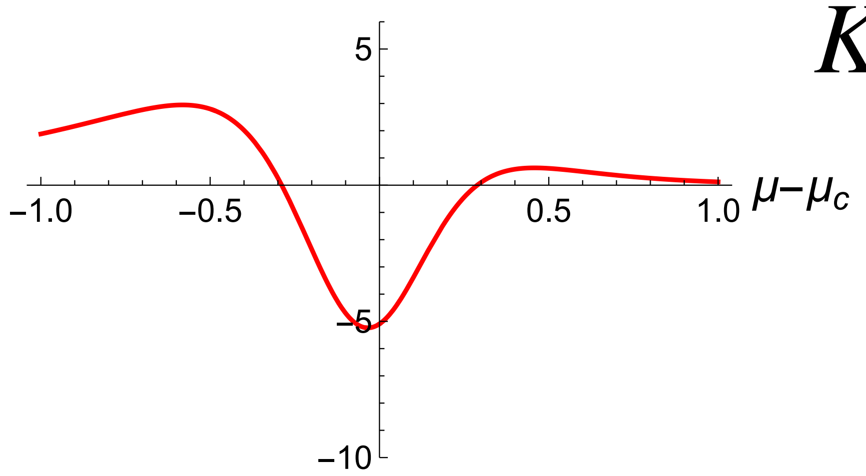
$K_1 = \langle N \rangle$



$K_2 = \frac{\partial}{\partial(\mu/T)} K_1$

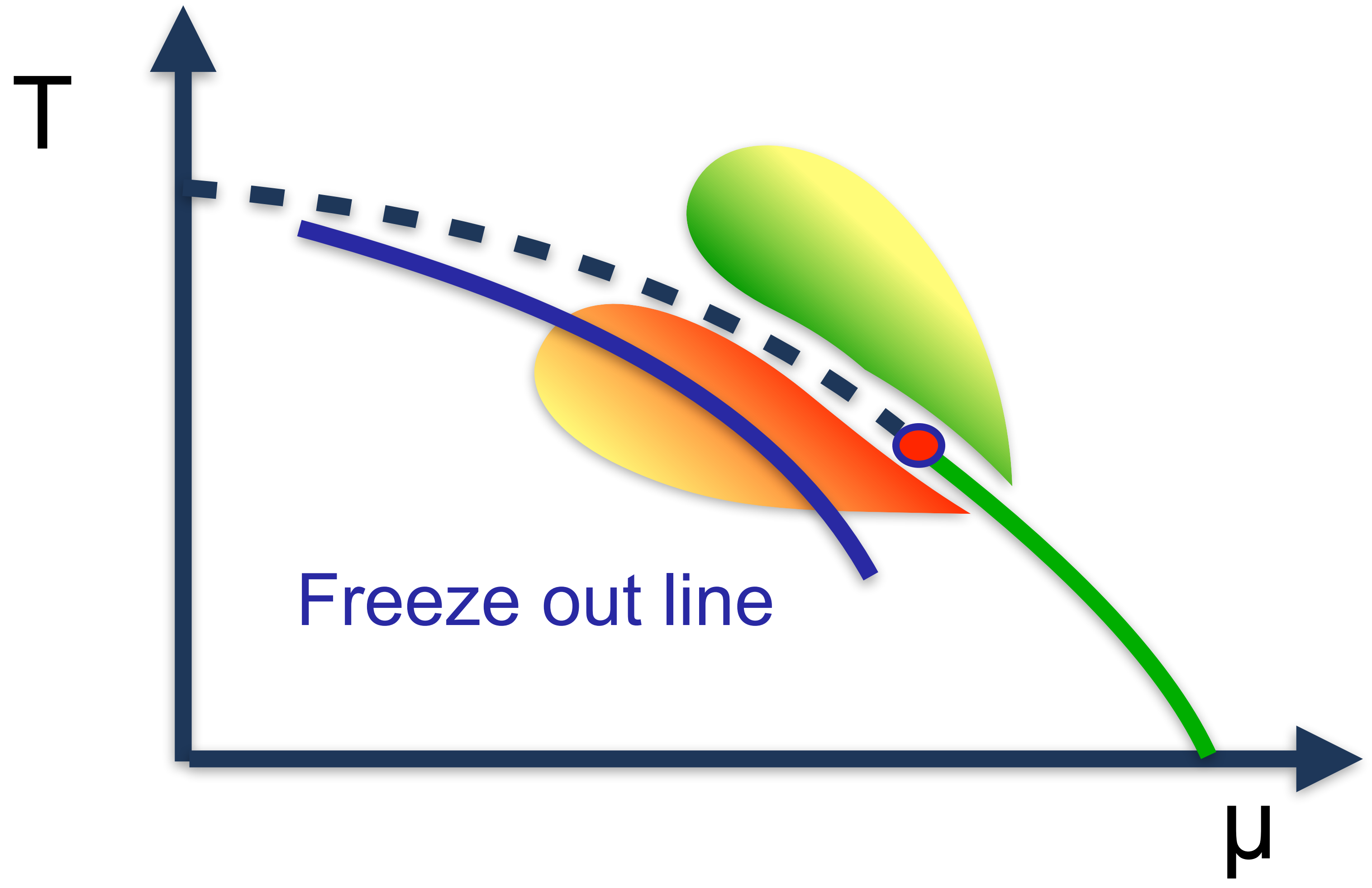


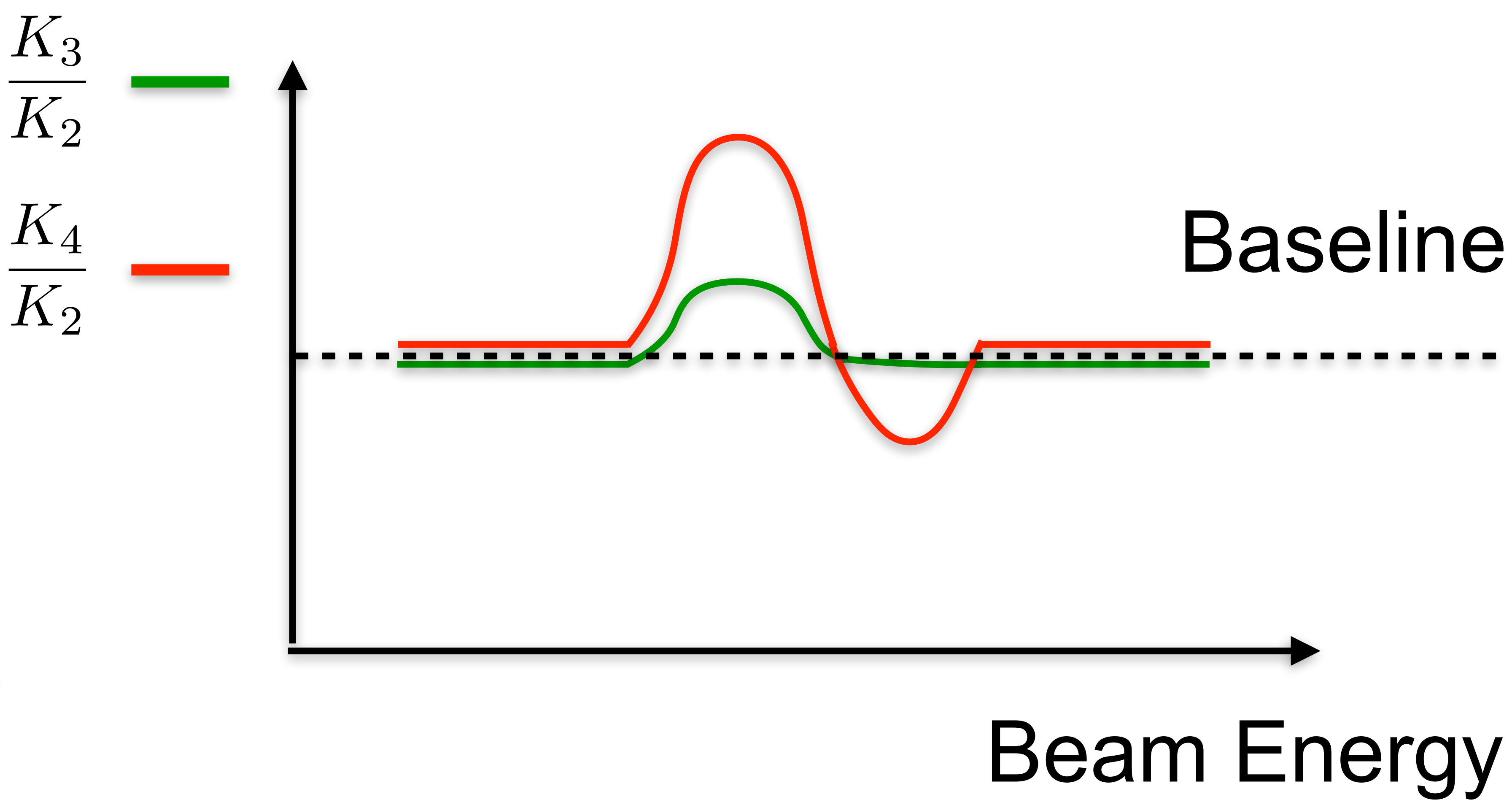
$K_3 = \frac{\partial}{\partial(\mu/T)} K_2$

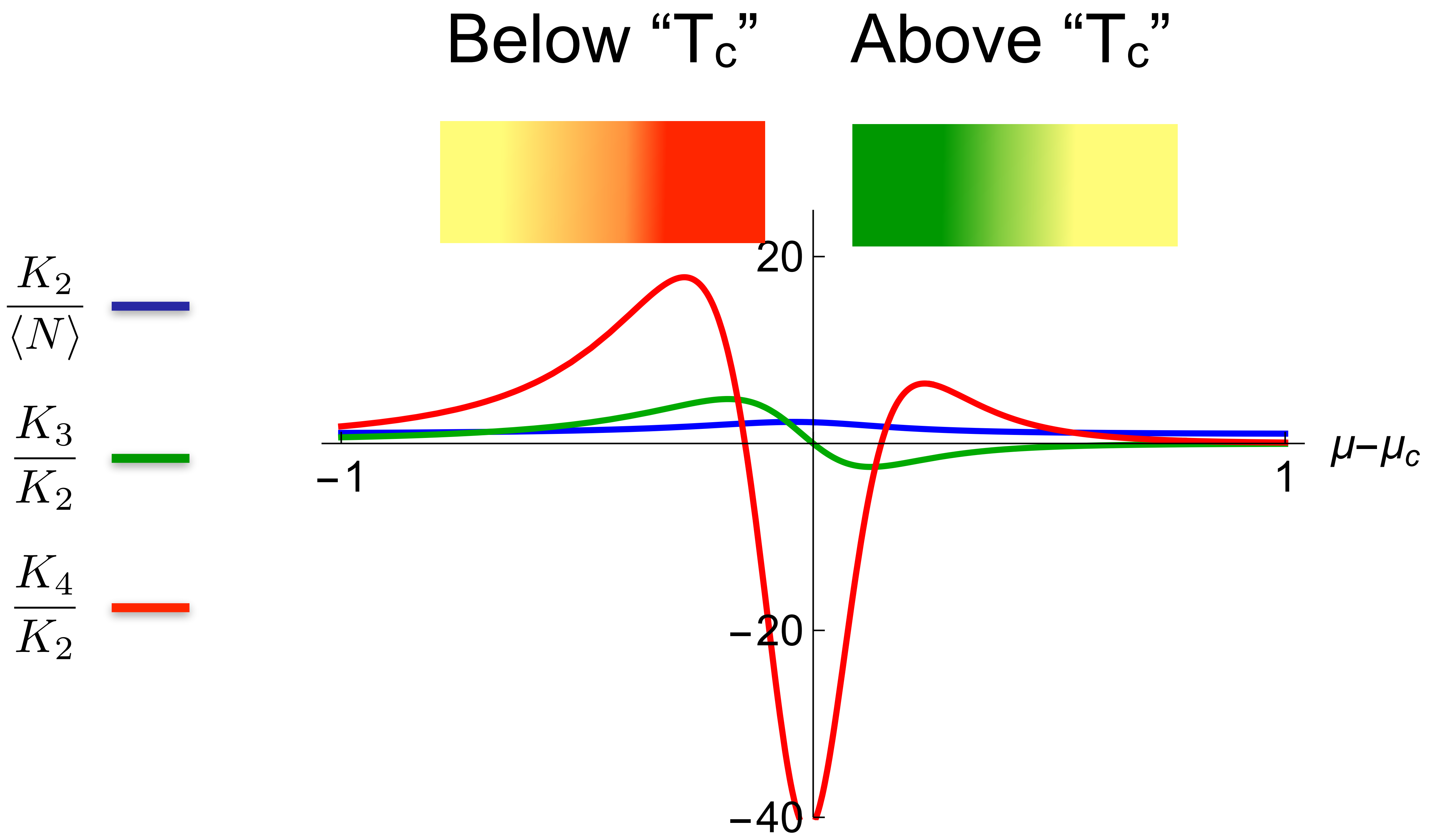


$K_4 = \frac{\partial}{\partial(\mu/T)} K_3$

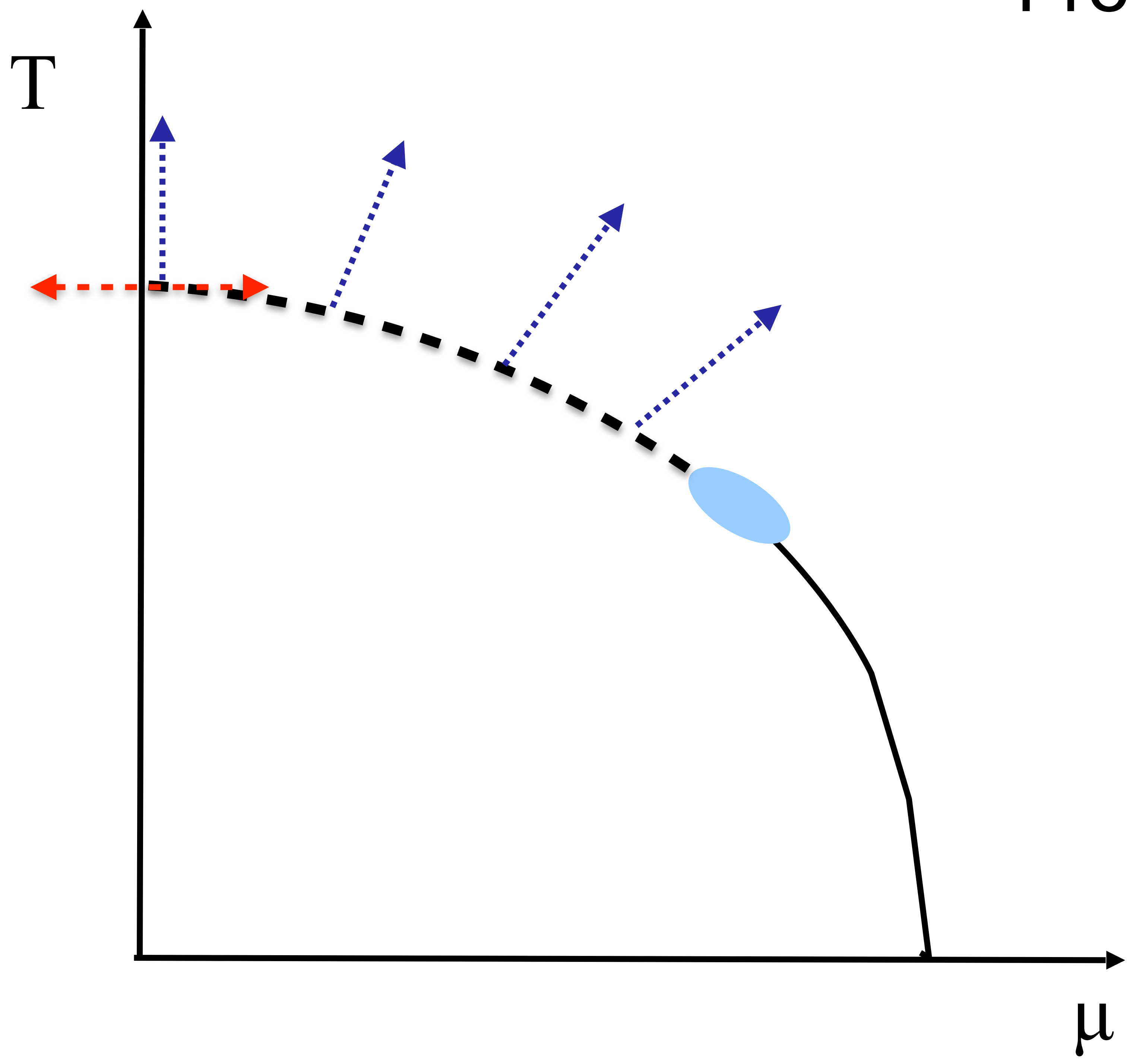
What to expect?







Close to μ=0



Free energy: $F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$

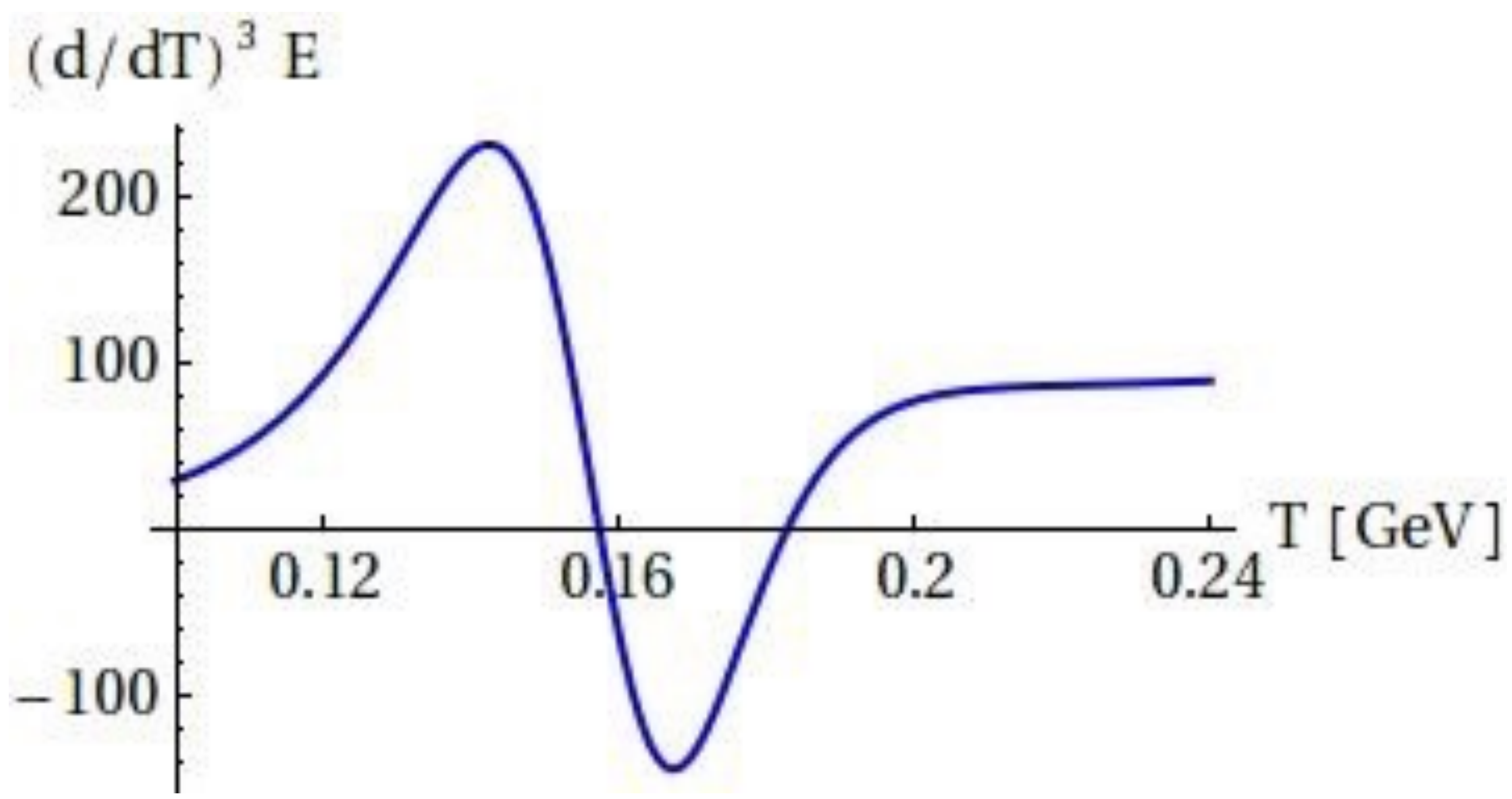
a ~ curvature of critical line

$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

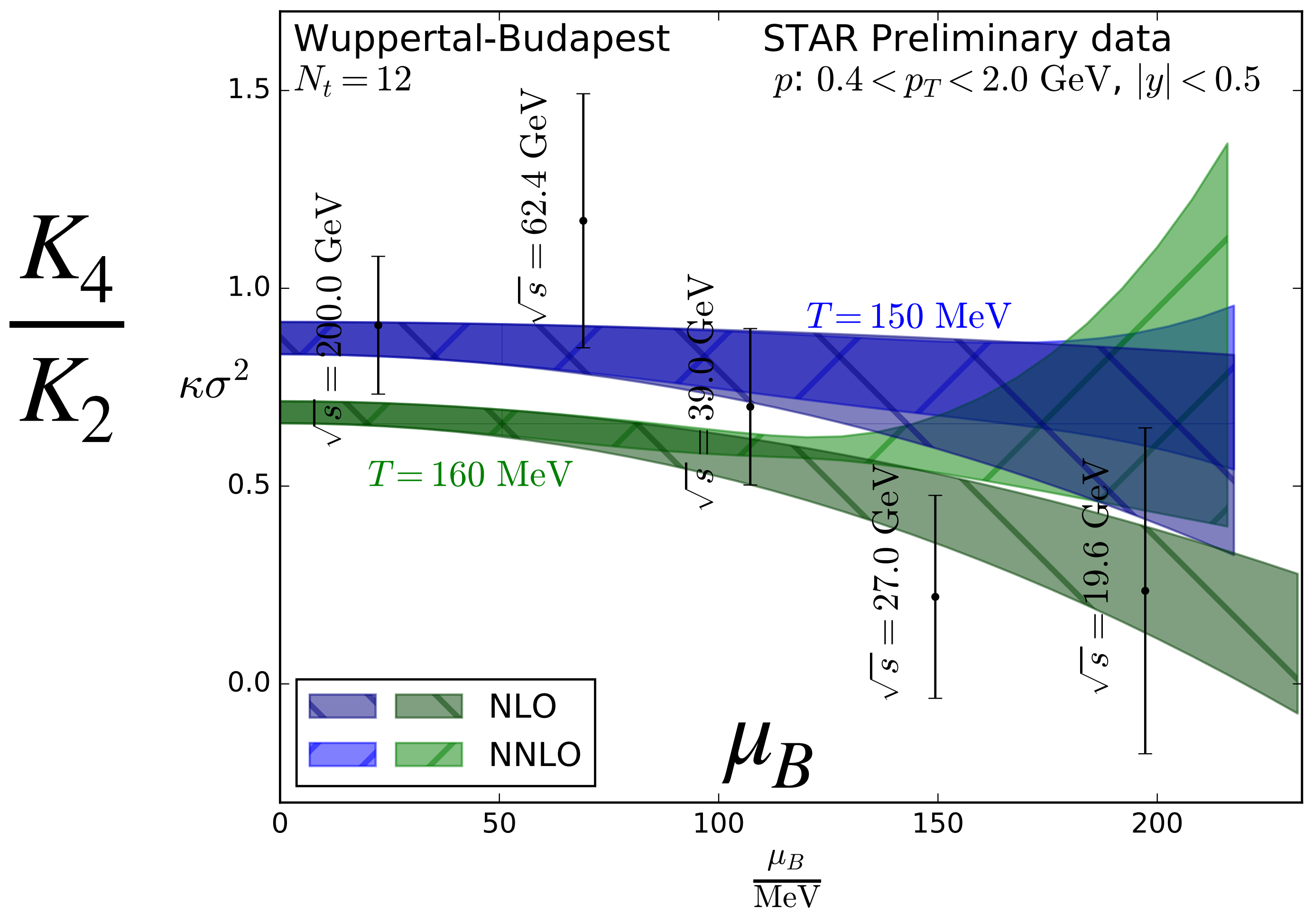
Needs higher order cumulants (derivatives) at μ ~ 0

Cumulants at small μ

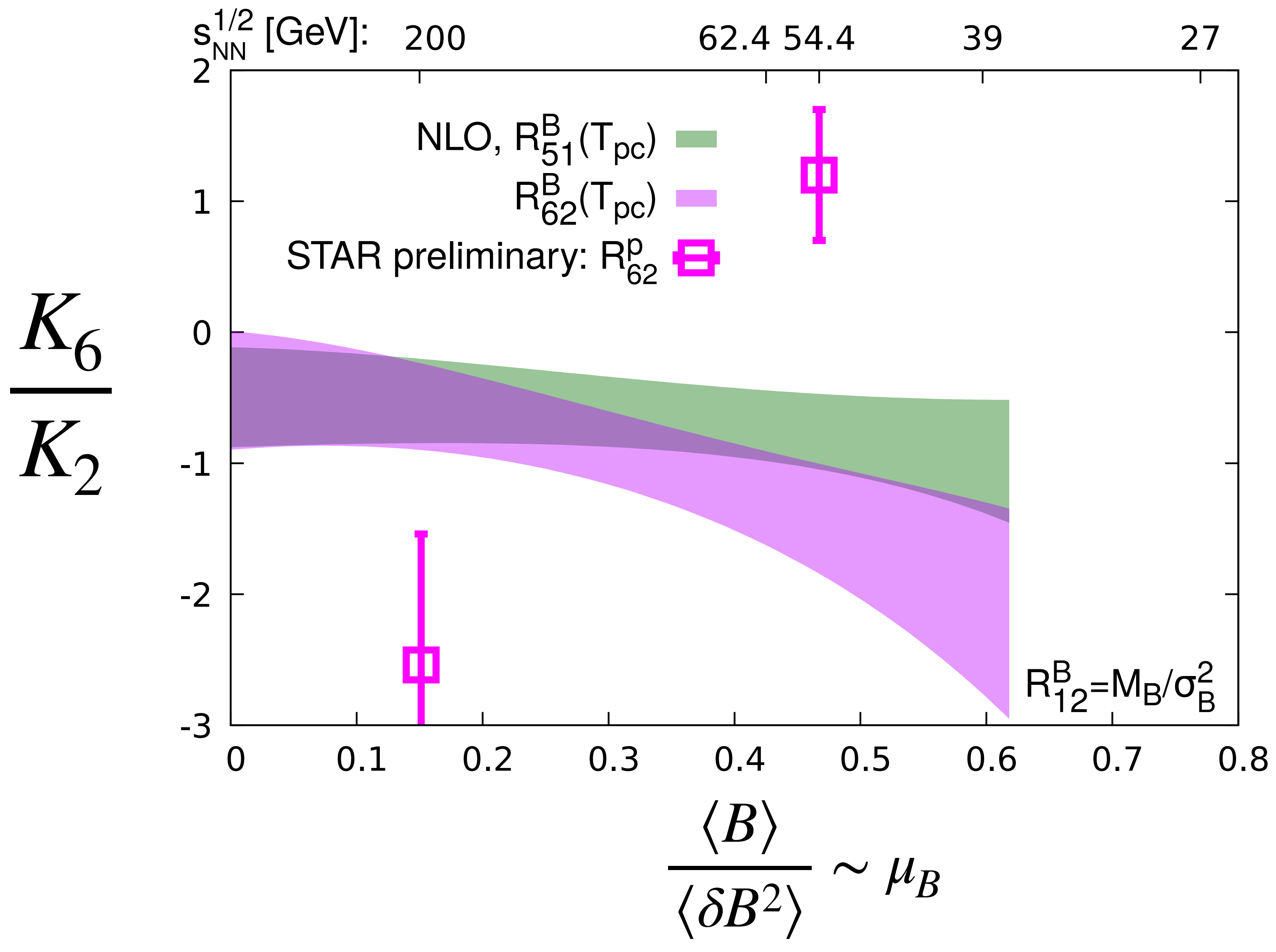
- Baryon number cumulants can be calculated in Lattice QCD
  - possible test of chiral criticality ? Friman et al, ’11



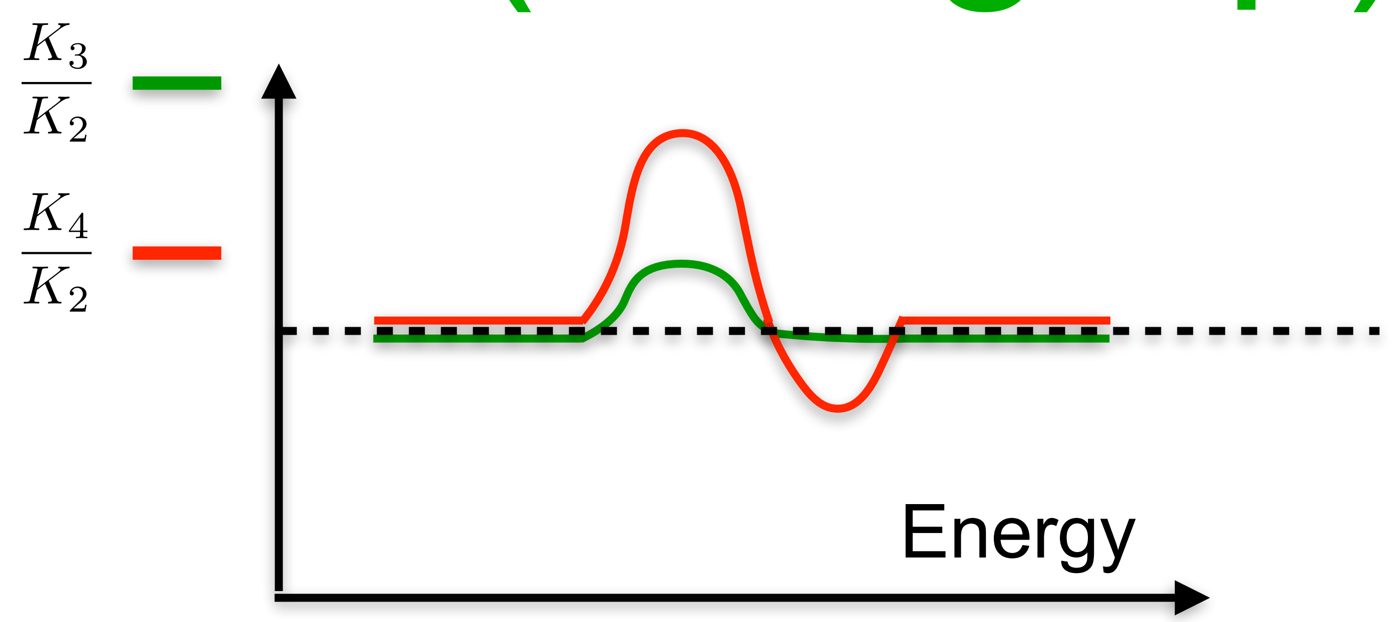
Wuppertal-Budapest, arXiv:1805.04445



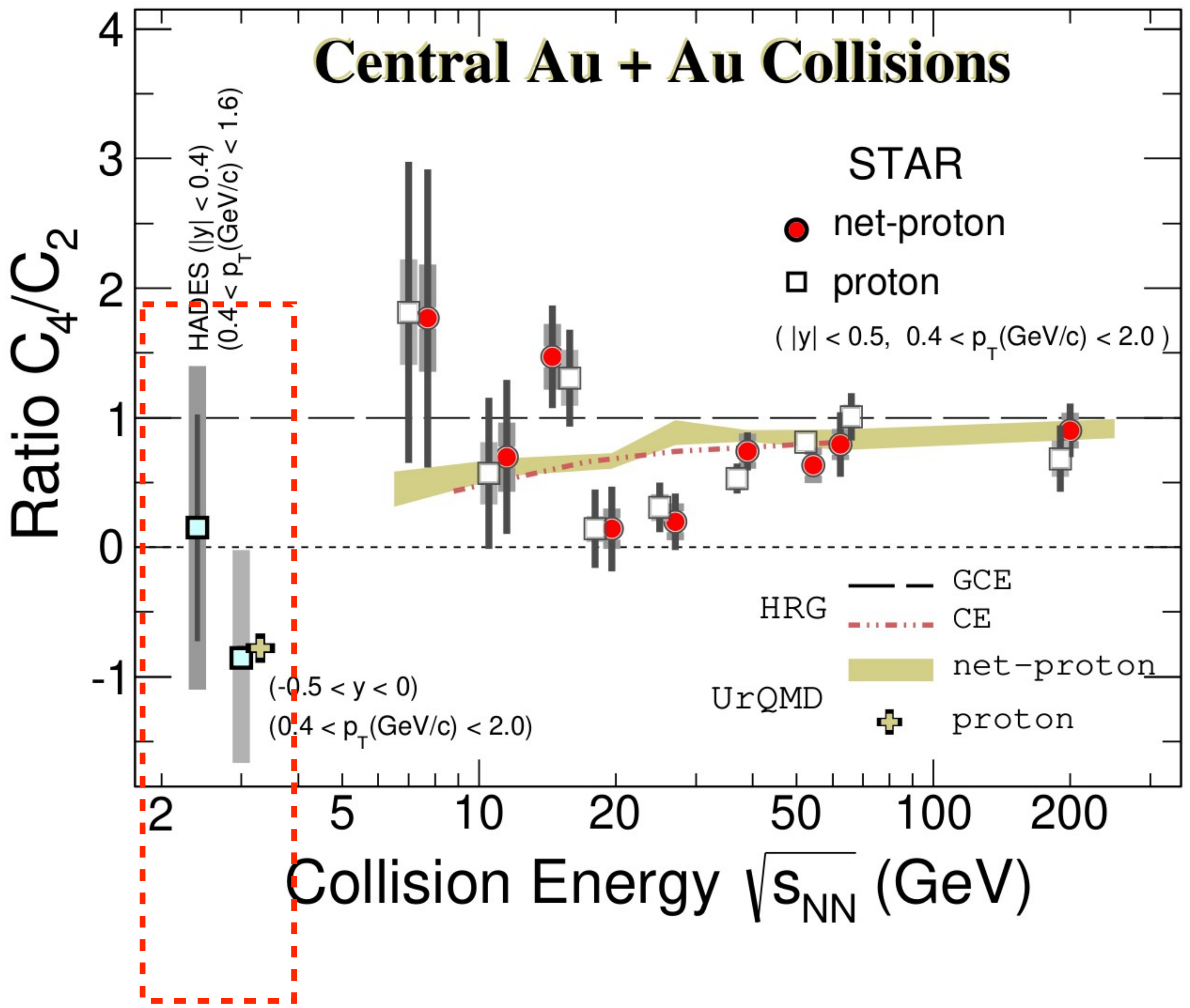
HotQCD, arXiv:2001.08530



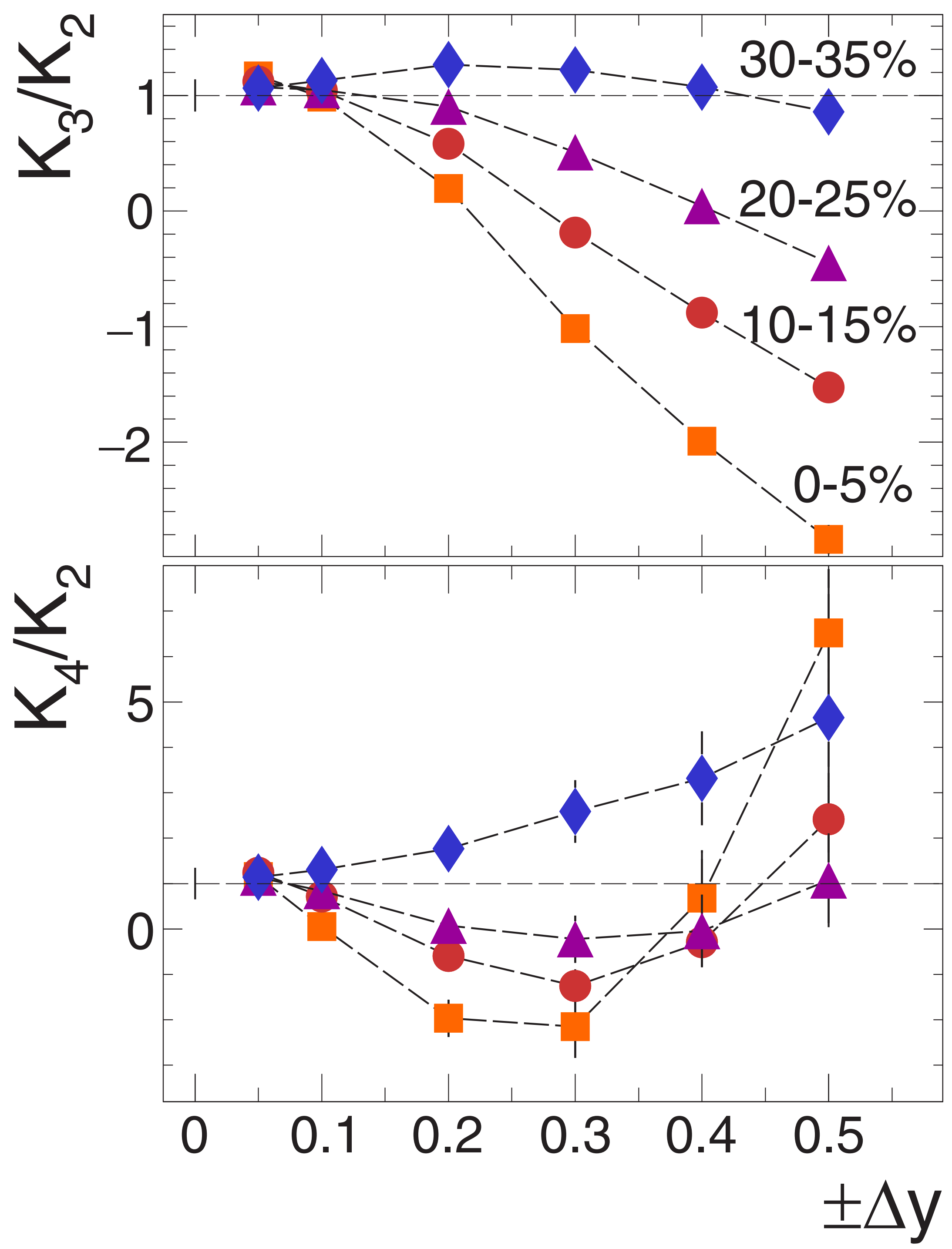
Data (at high μ)



STAR: arXiv:2112.00240



Different acceptance!



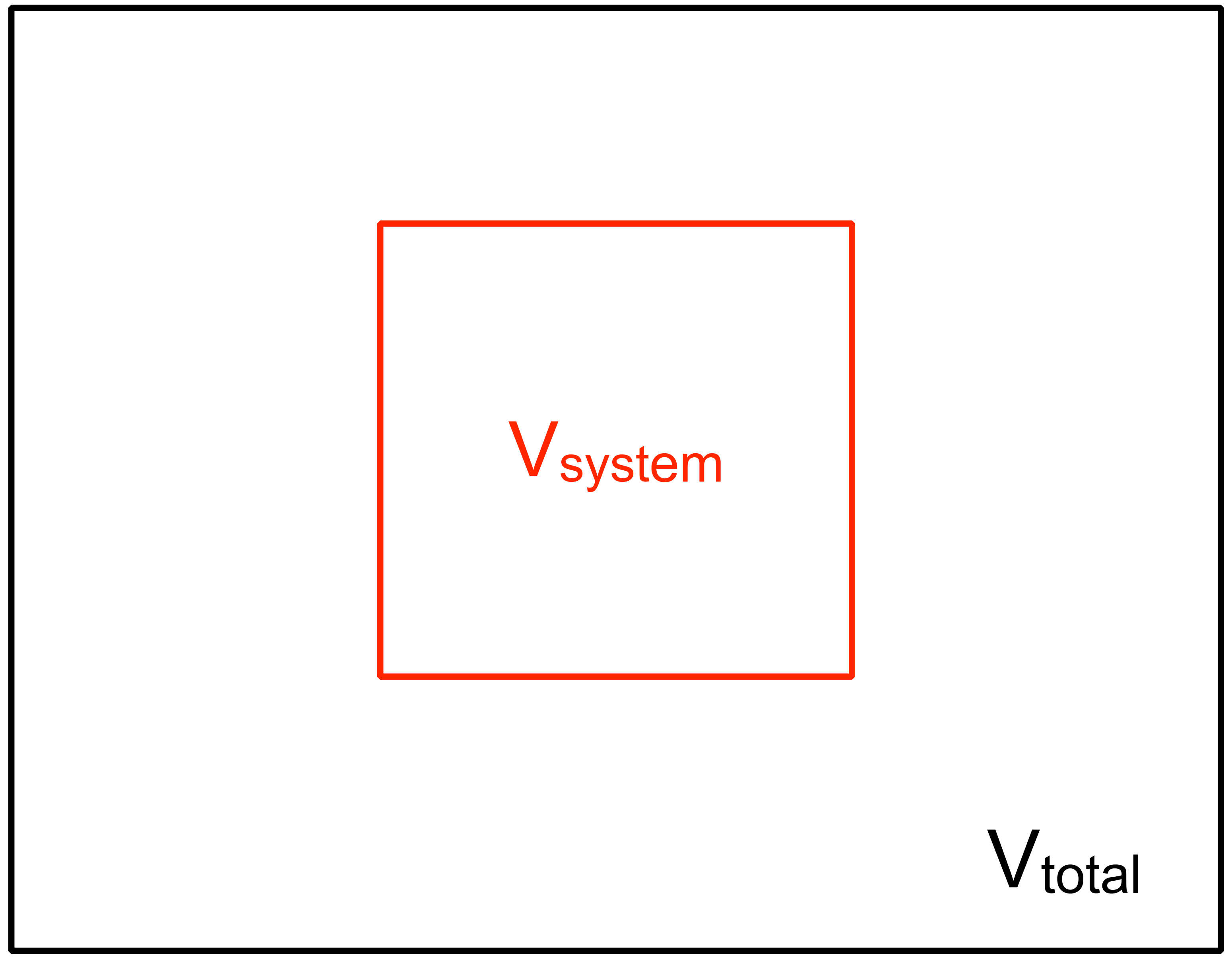
HADES: arXiv:2002.08701

$\sqrt{s} = 2.4$ GeV

Compare Data with Lattice QCD and other field theoretical models

- Lattice cannot calculate hadron abundances
- Cumulants are well defined quantities
- Compare cumulants !?
  - Detector fluctuates (efficiency etc…)
  - Volume is not fixed in experiment
    - Possible solution (Rustamov et al, 2211.14849)
  - Baryon number conservation
    - Lattice uses grand canonical ensemble
  - Experiment measures protons not all baryons

Grand canonical ensemble

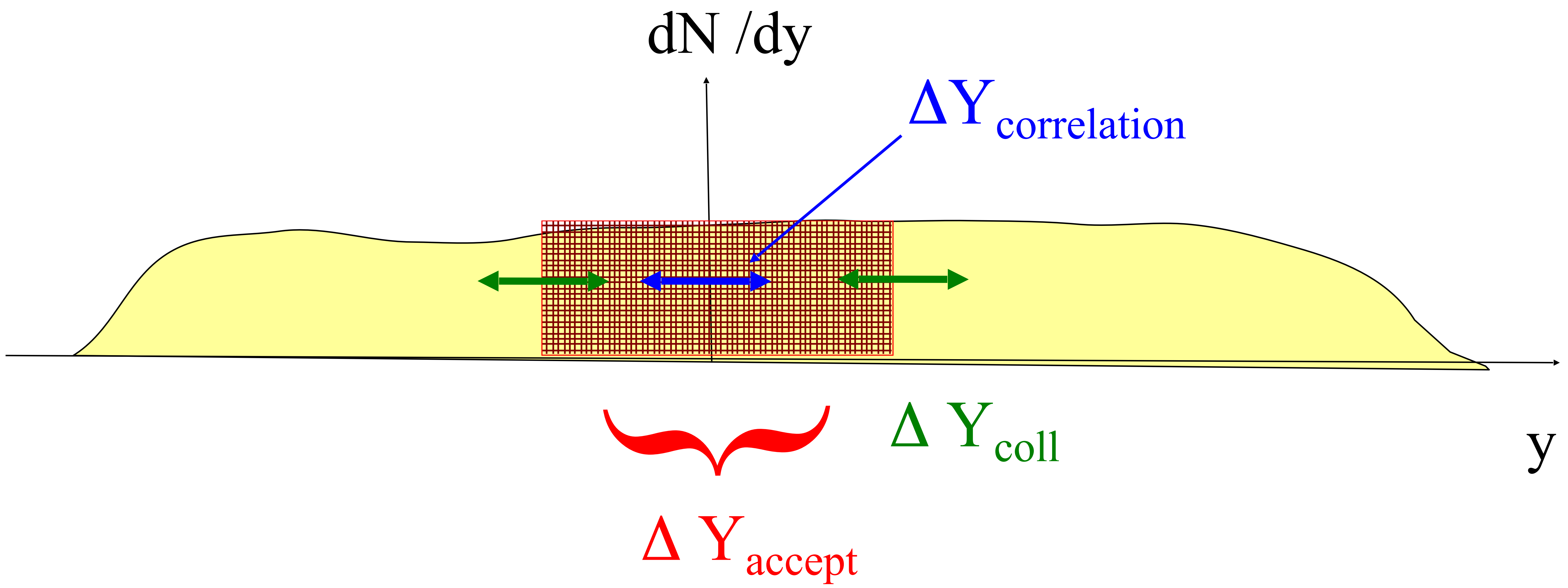


$$V_{total} \to \infty$$
$$V_{system} \to \infty$$

$$\frac{V_{system}}{V_{total}} \to 0$$

In coordinate space!!!!

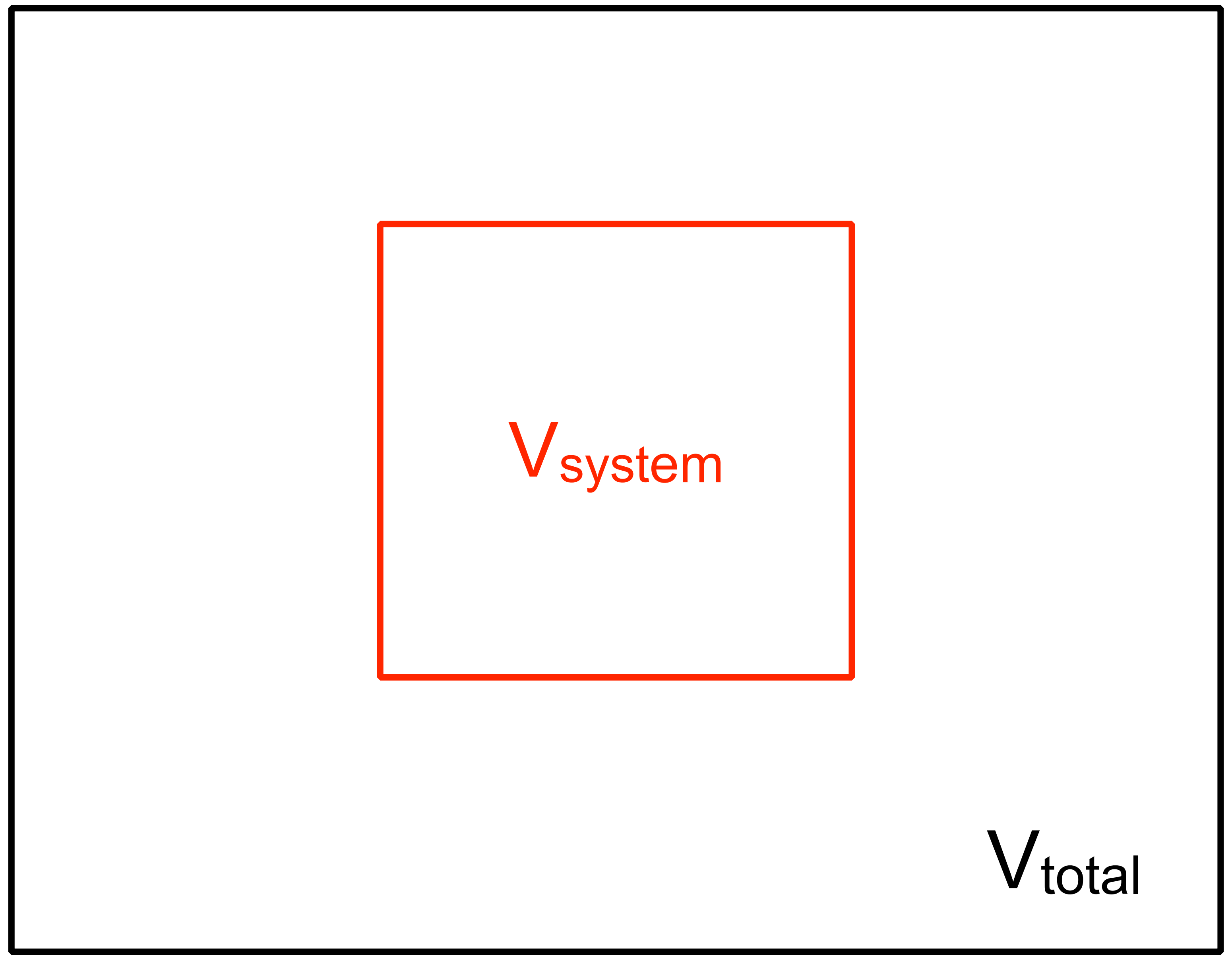
How to make a grand-canonical ensemble in experiment



Conditions for "charge" fluctuations:

- $\Delta Y_{corrrelation} << \Delta Y_{accept}$ **(catch the physics)**
- $\Delta Y_{total} >> \Delta Y_{accept} >> \Delta Y_{coll}$ **(keep the physics and minimize charge conservation effect)**

Grand canonical ensemble



$$V_{total} \to \infty$$

$$V_{system} \to \infty$$

$$\frac{V_{system}}{V_{total}} \to 0$$

In coordinate space!!!!

Lattice:

$V_{total} \to \infty$

grand-canonical ensemble

Coordinate space

Experiment:

V_{total} finite!

$V_{system} \ll V_{total}$ (hopefully)

effect of global charge conservation

Momentum Space

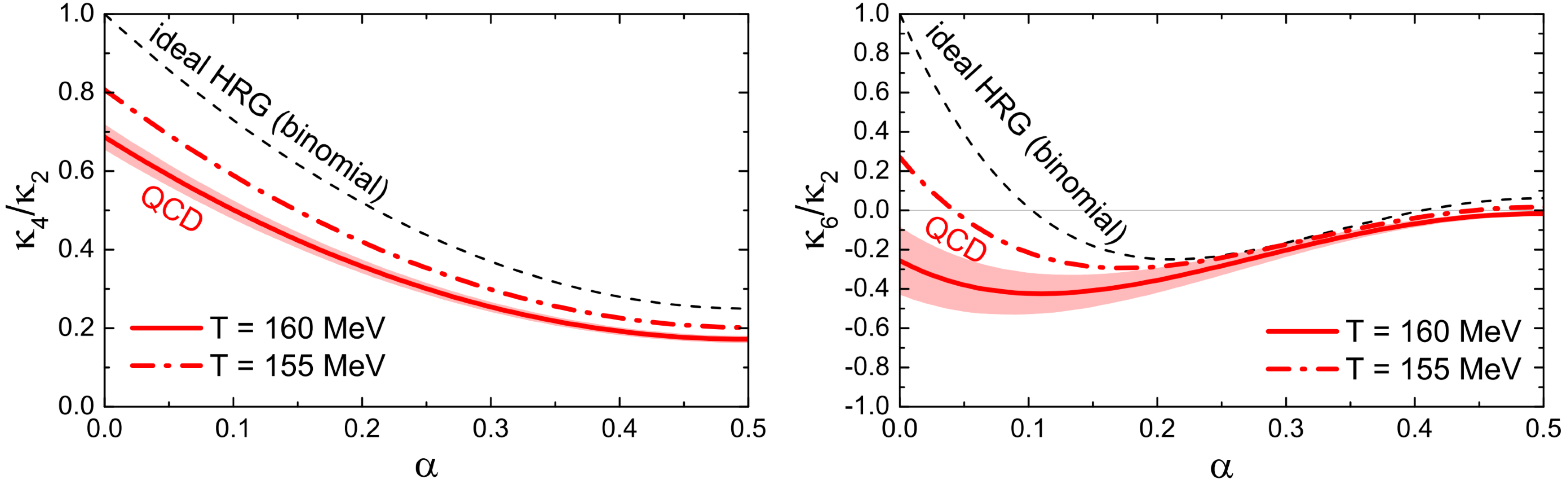
Global charge conservation

Solved for ANY equation of state (including QCD)

V. Vovchenko et al, arXiv 2003.13905, arXiv:2007.03850

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B}$$

$$\left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$



α = fraction of measured baryons

$\beta = 1 - \alpha$

Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

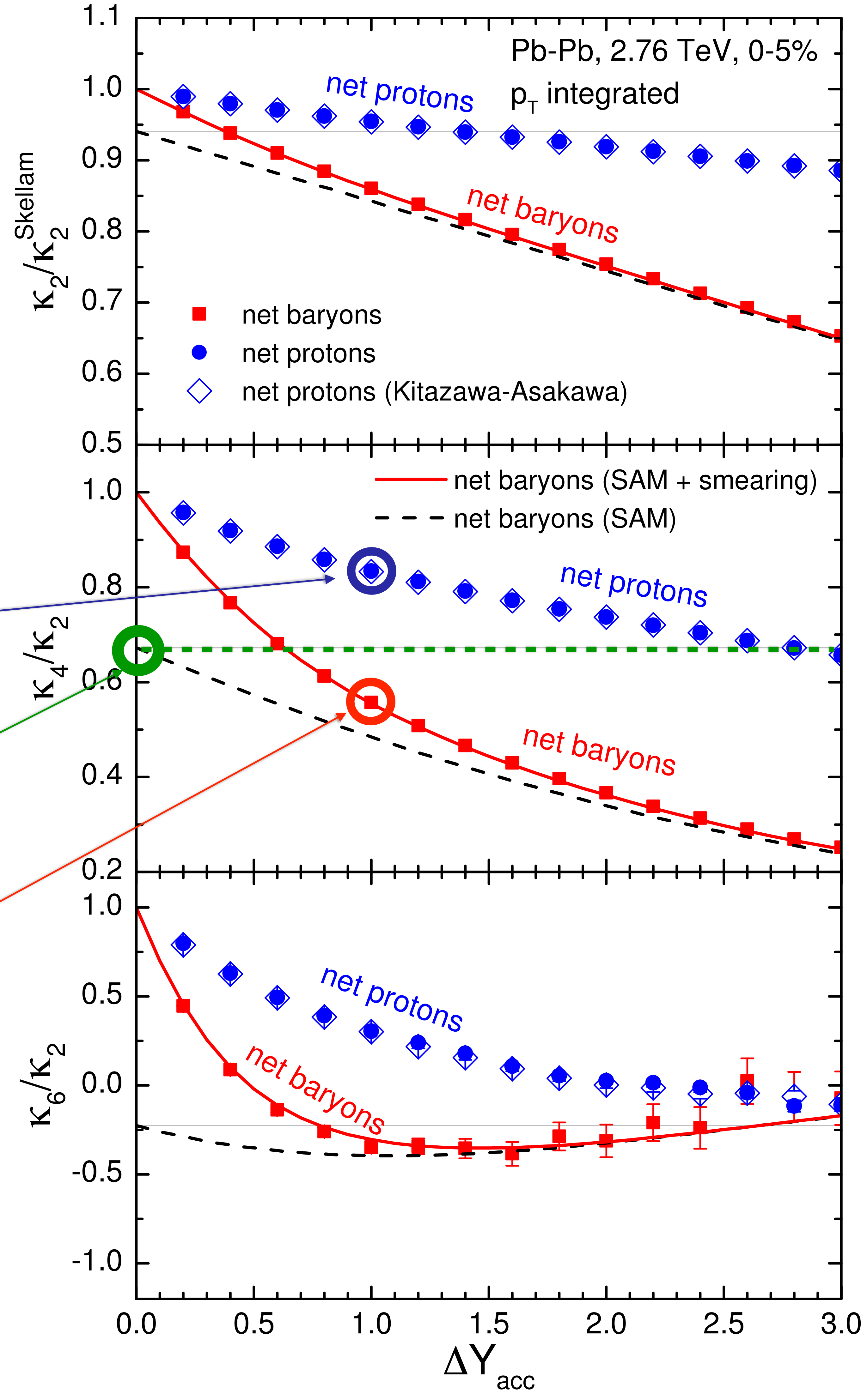
Alternative derivation:
M. Barey, and A. Bzdak 2205.05497, 2210.15394

For ideal gas:
Bleicher et al: hep-ph/0006201
Bzdak et al: 1203.4529
Braun-Munzinger et al, 1807.08927

Protons vs Baryons

- Proton are subset of all baryons
  - dilutes the signal
  - need to do binomial unfolding
    - Kitazawa, Asakawa PRC ‘12
  - Otherwise Apples vs. Oranges

V. Vovchenko et al 2012.09954



Measure only protons

Lattice QCD

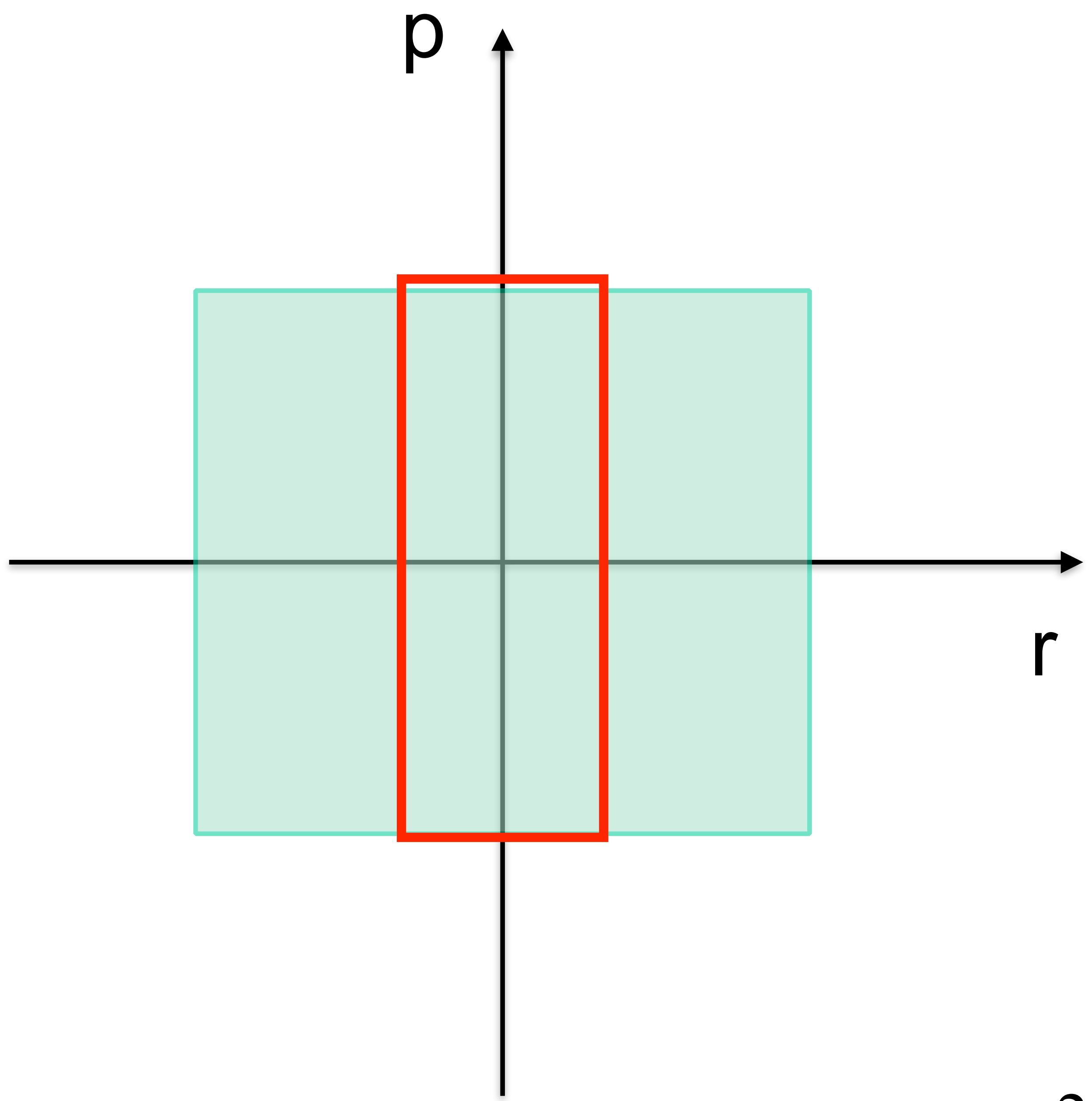
Measure all Baryons

Compare Data with Lattice QCD and other field theoretical models

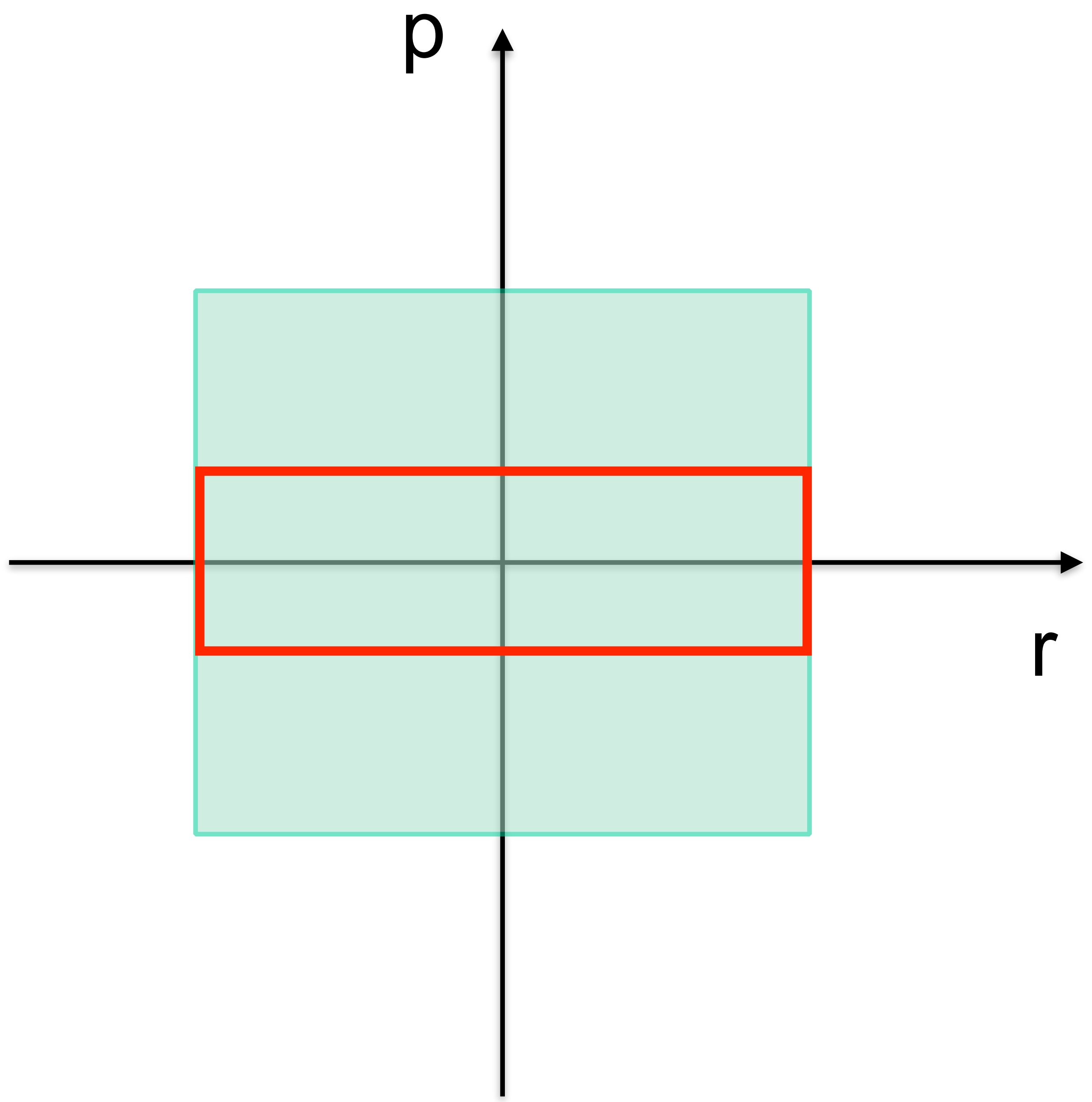
- Lattice cannot calculate hadron abundances
- Cumulants are well defined quantities
- Compare cumulants !?
  - Detector fluctuates (efficiency etc…)
  - Experiment measures protons not all baryons
  - Volume is not fixed in experiment
    - Possible solution (Rustamov et al, 2211.14849)
  - Baryon number conservation
    - Lattice uses grand canonical ensemble
  - Experiment cuts **momentum** space, Theory cuts **configuration** space

Coordinate vs momentum space cuts

Small Spatial Volume



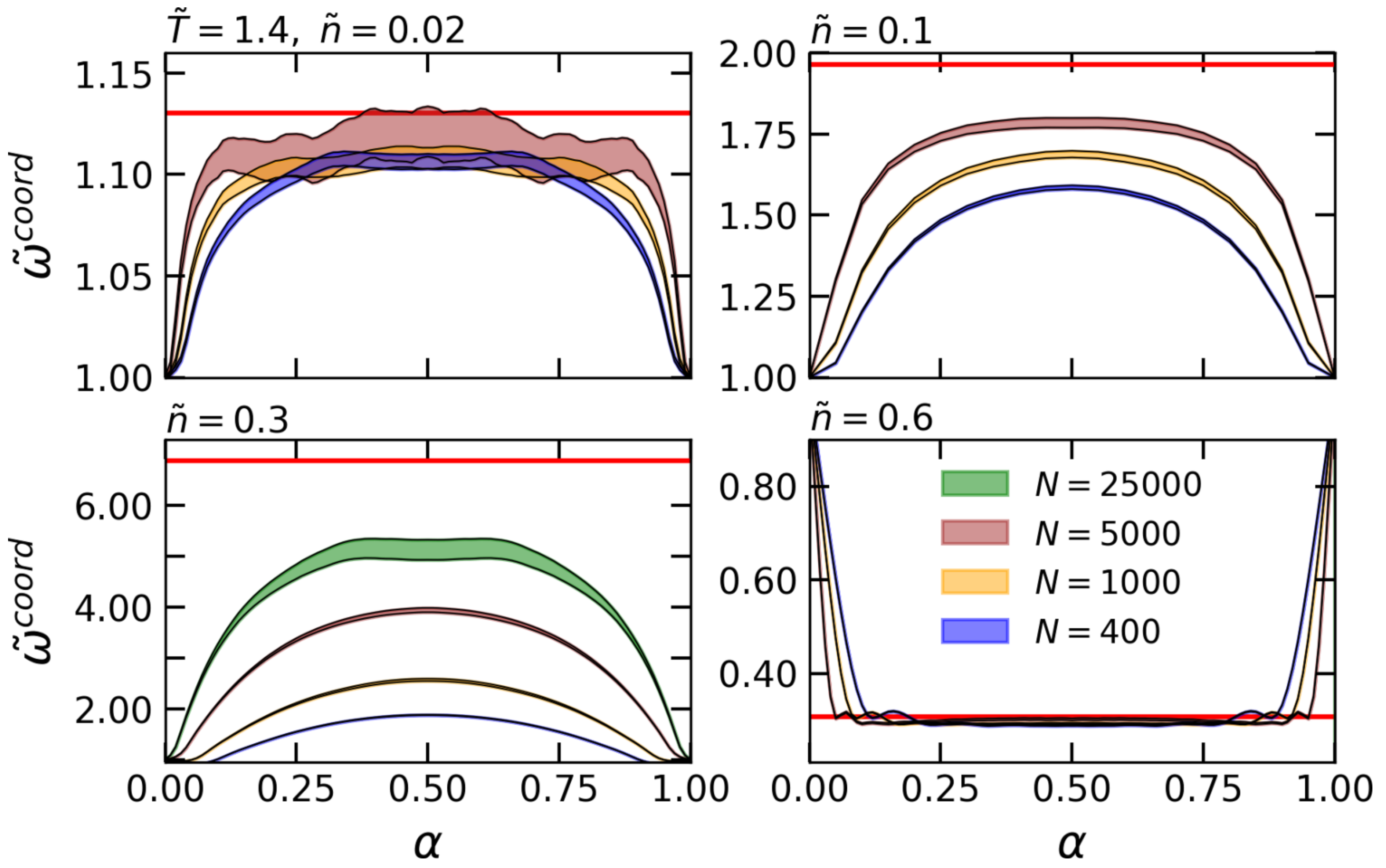
Limited Acceptance



classical Molecular dynamics: $H = \frac{p^2}{2m} + V(x_1 - x_j)$

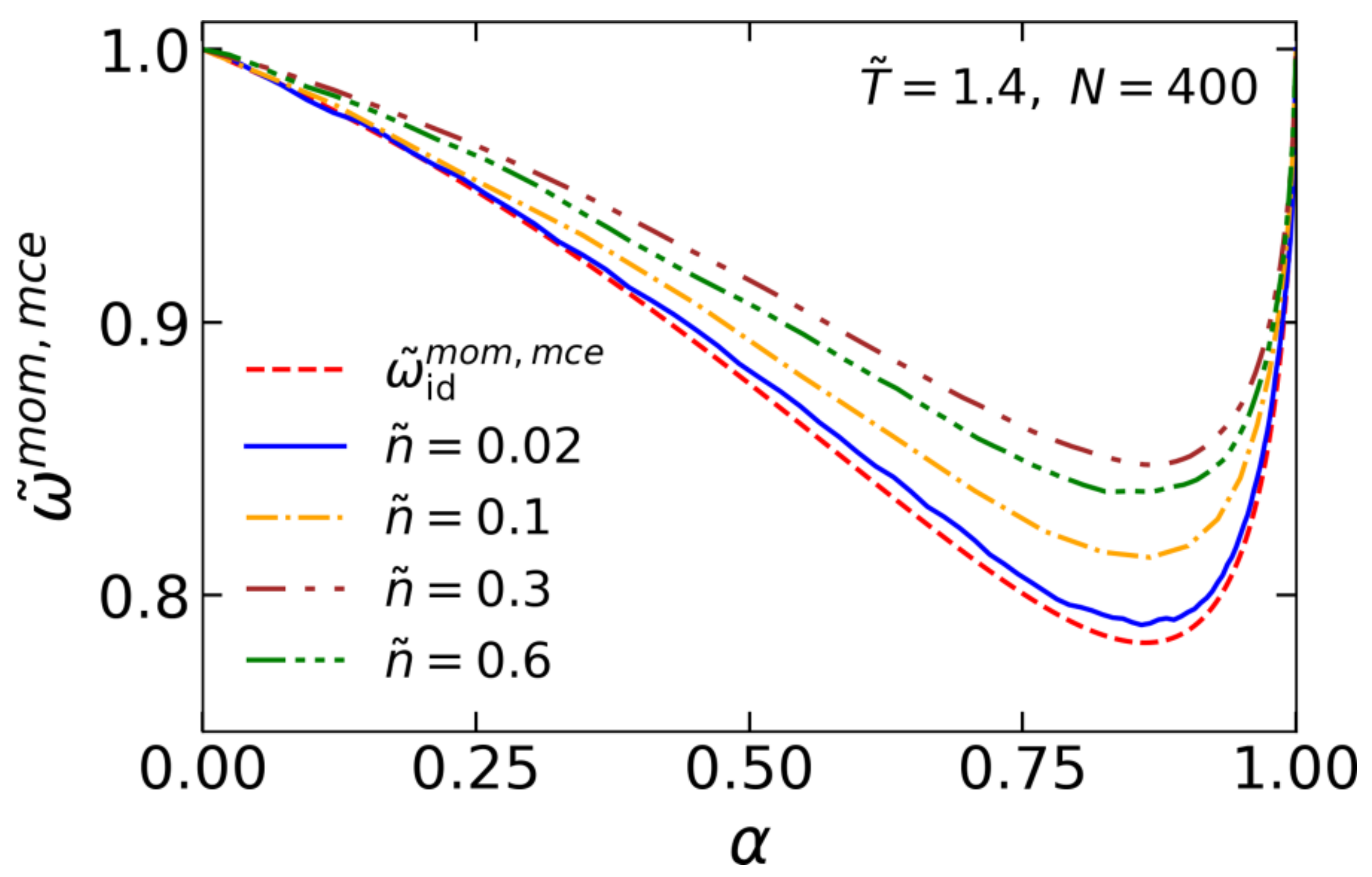
partition function $Z = \int \prod_i (dp_i dx_i) e^{[} = \beta H = \int \prod_i dp_i e^{\frac{p^2}{2m}} \int \prod_i dx_i e^{V(x_i - x_j)} = Z_p \times Z_x$ factorizes!

Correlations live in coordinate space



Cut in coordinate space
Integrate over all Momenta

Fluctuations close to expectation from grand canonical
Correlations clearly visible



Cut in momentum space
Integrate over all Space

Fluctuations close to non-interacting gas
NO correlations or criticality visible

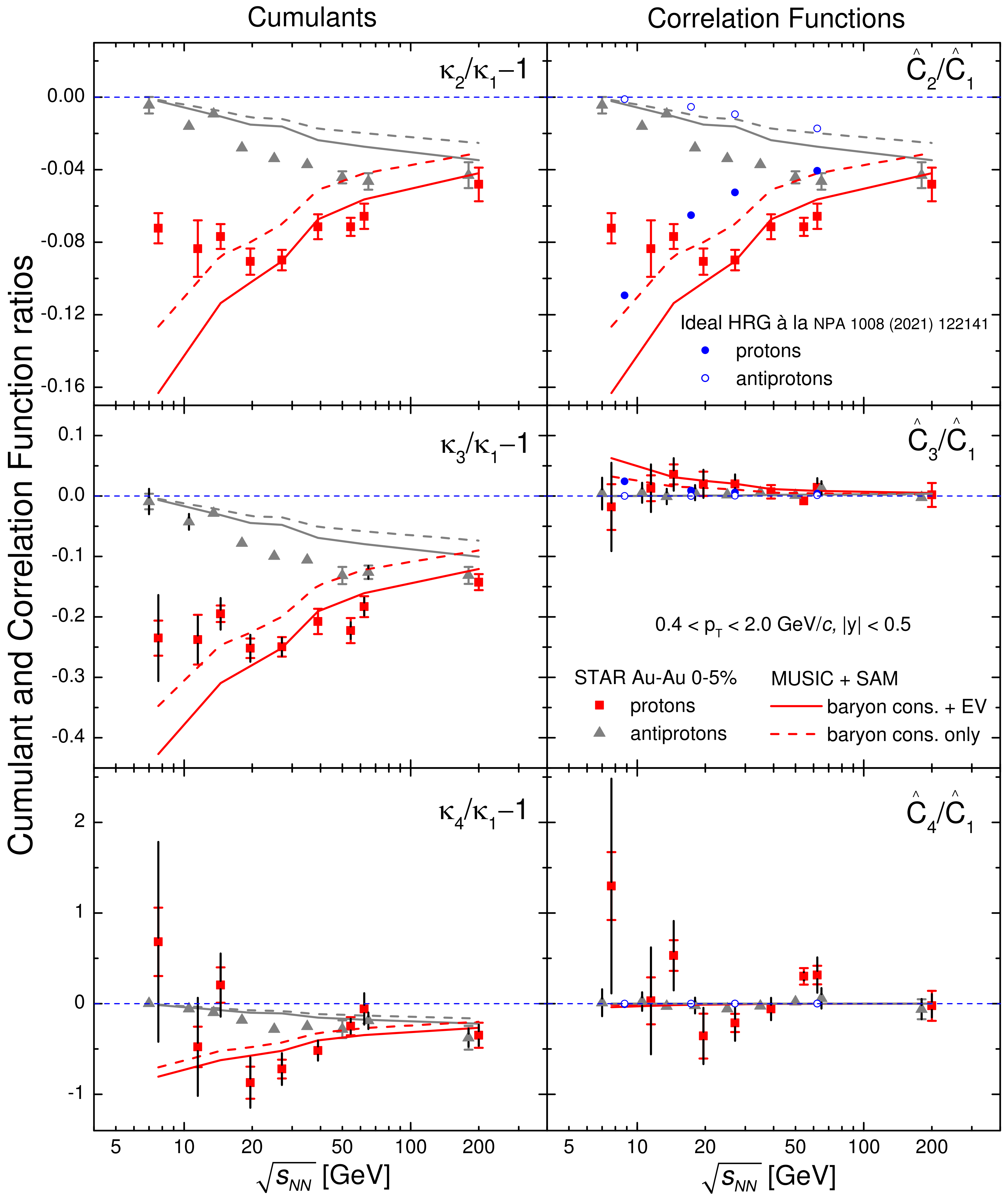
Need Space-Momentum correlations → Flow!

Comparison with data from Beam energy scan

Vovchenko et al, 2107.00163

- Viscous hydro
- EOS tuned to LQCD
- Correct for global charge conservation
- Protons NOT baryons

- Baseline!
  No critical point or phase transition



Comparison with data from Beam energy scan

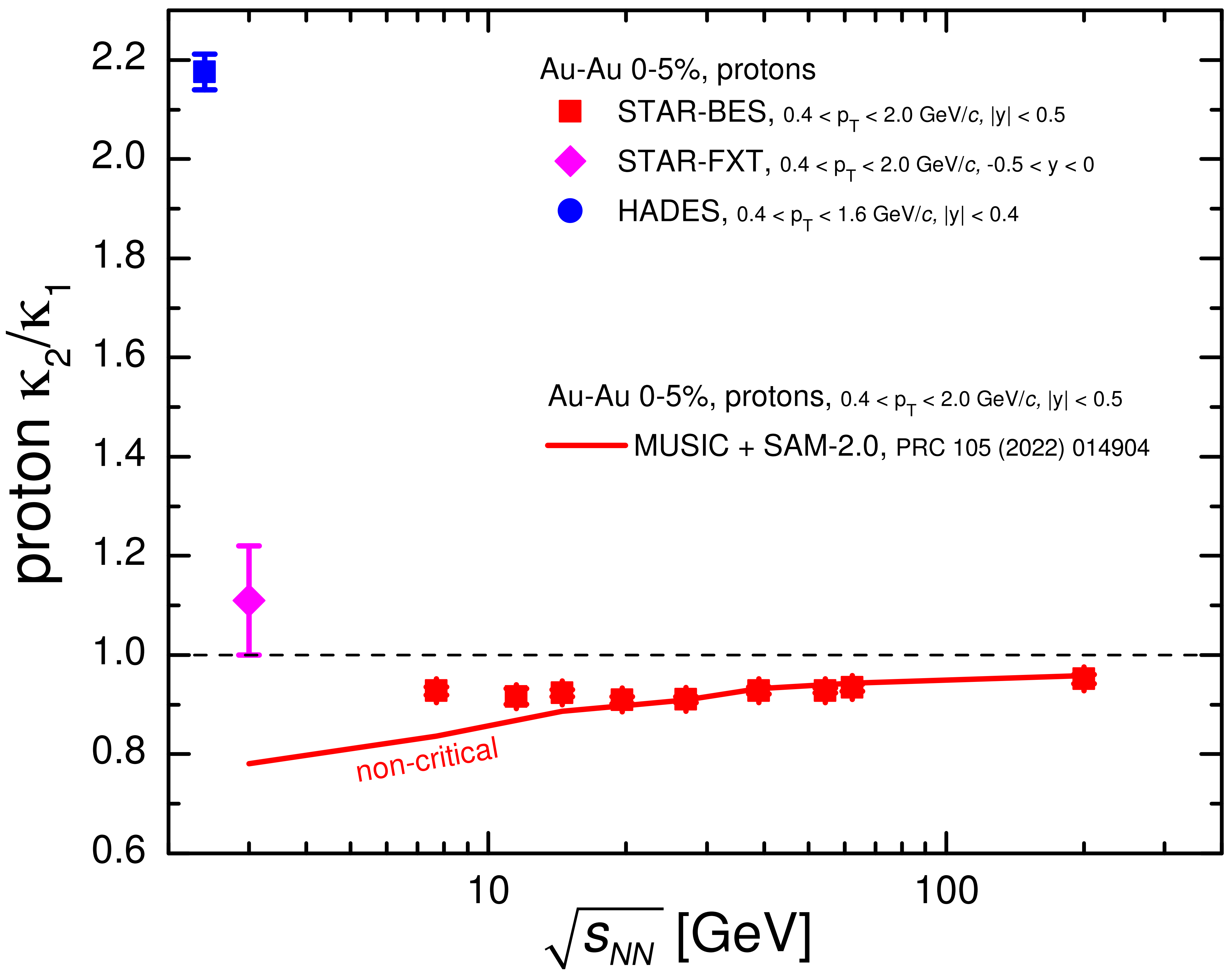
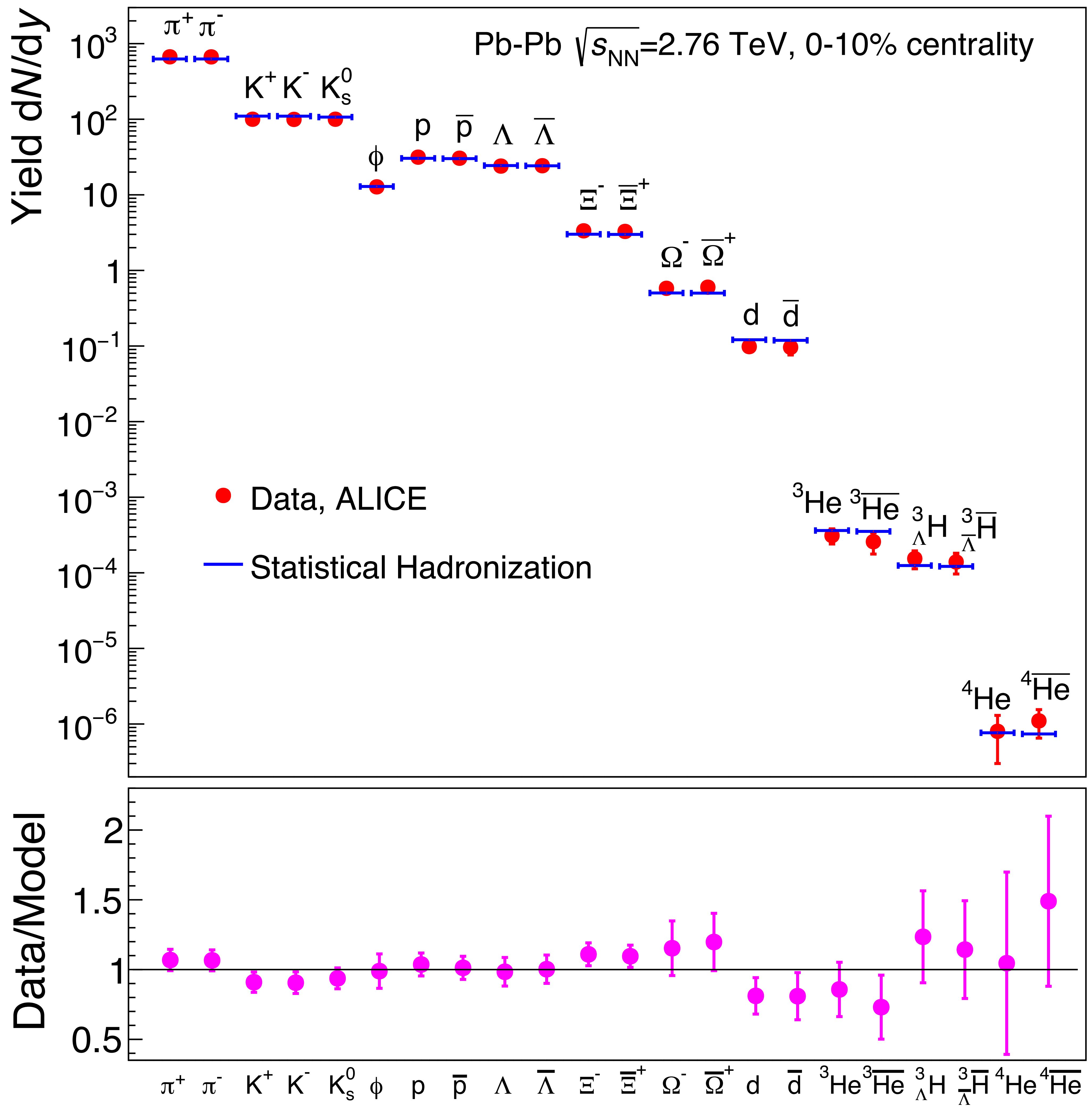


Figure courtesy of V. Vovchenko

Let's understand the second order cumulants first!

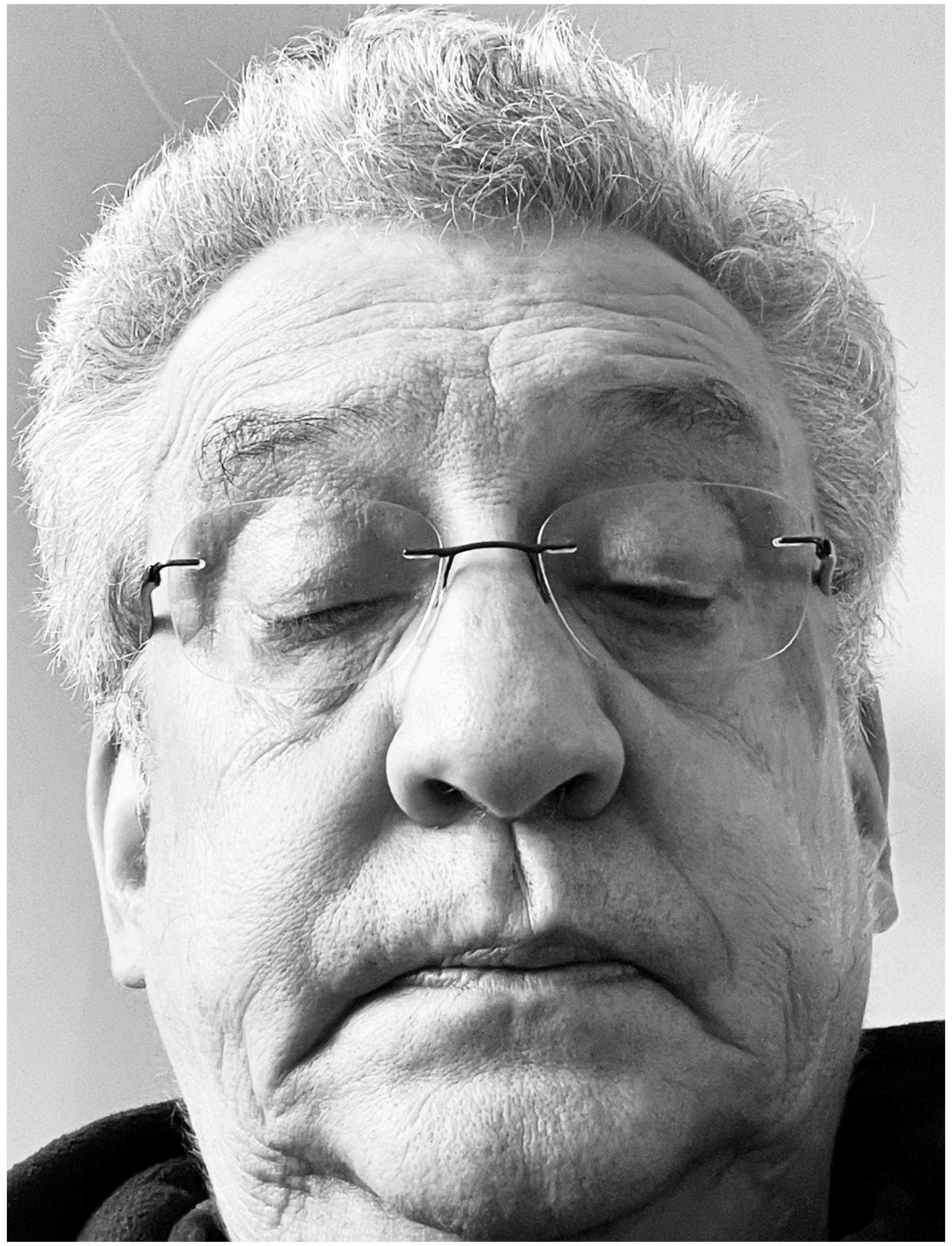
Proton annihilation in the hadronic phase ?



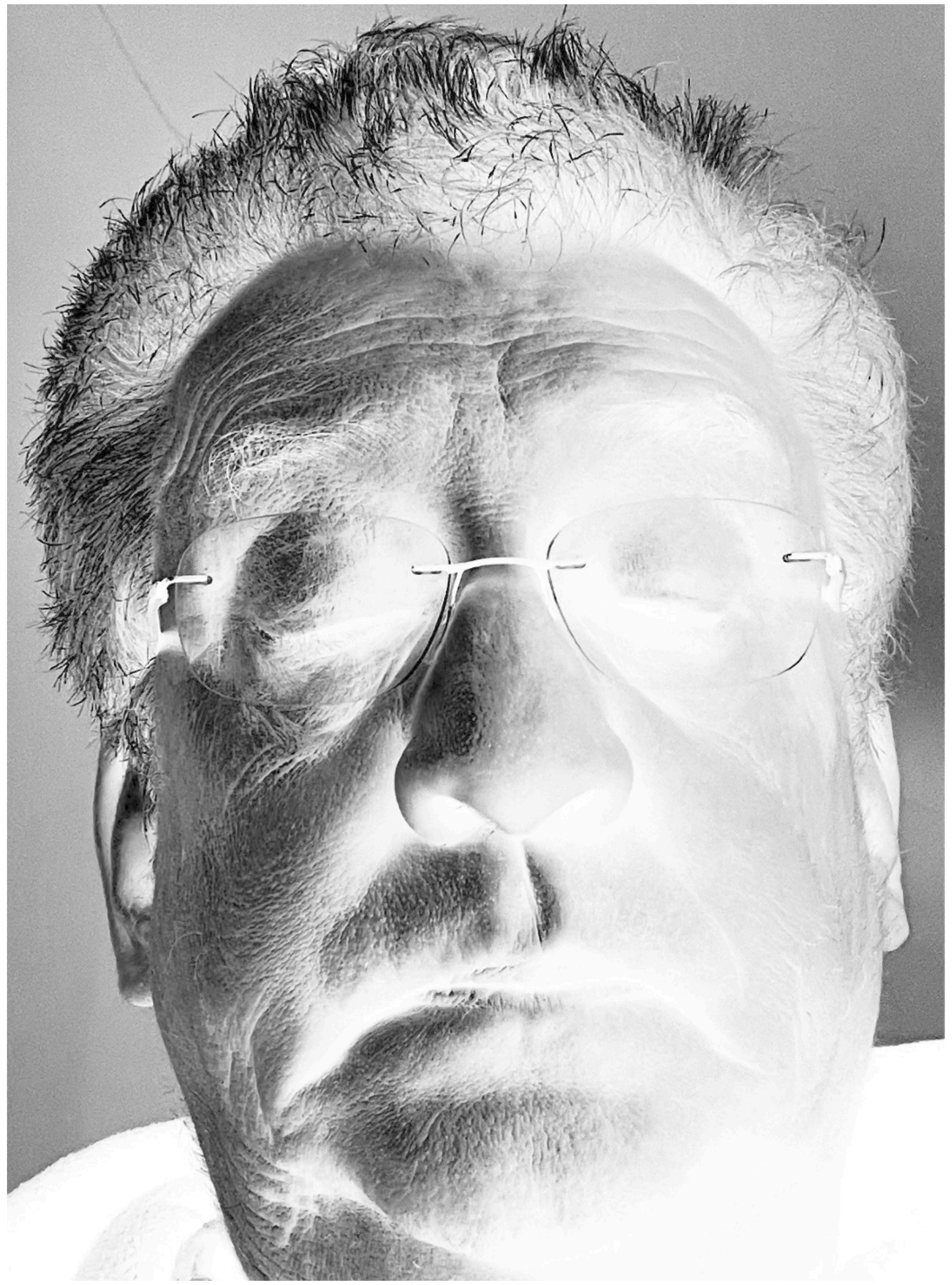
- Thermal Model with phase shift corrections:
  - No room for annihilation in hadronic phase

Andronic et al, 2101.05747

Me



$\overline{\text{Me}}$



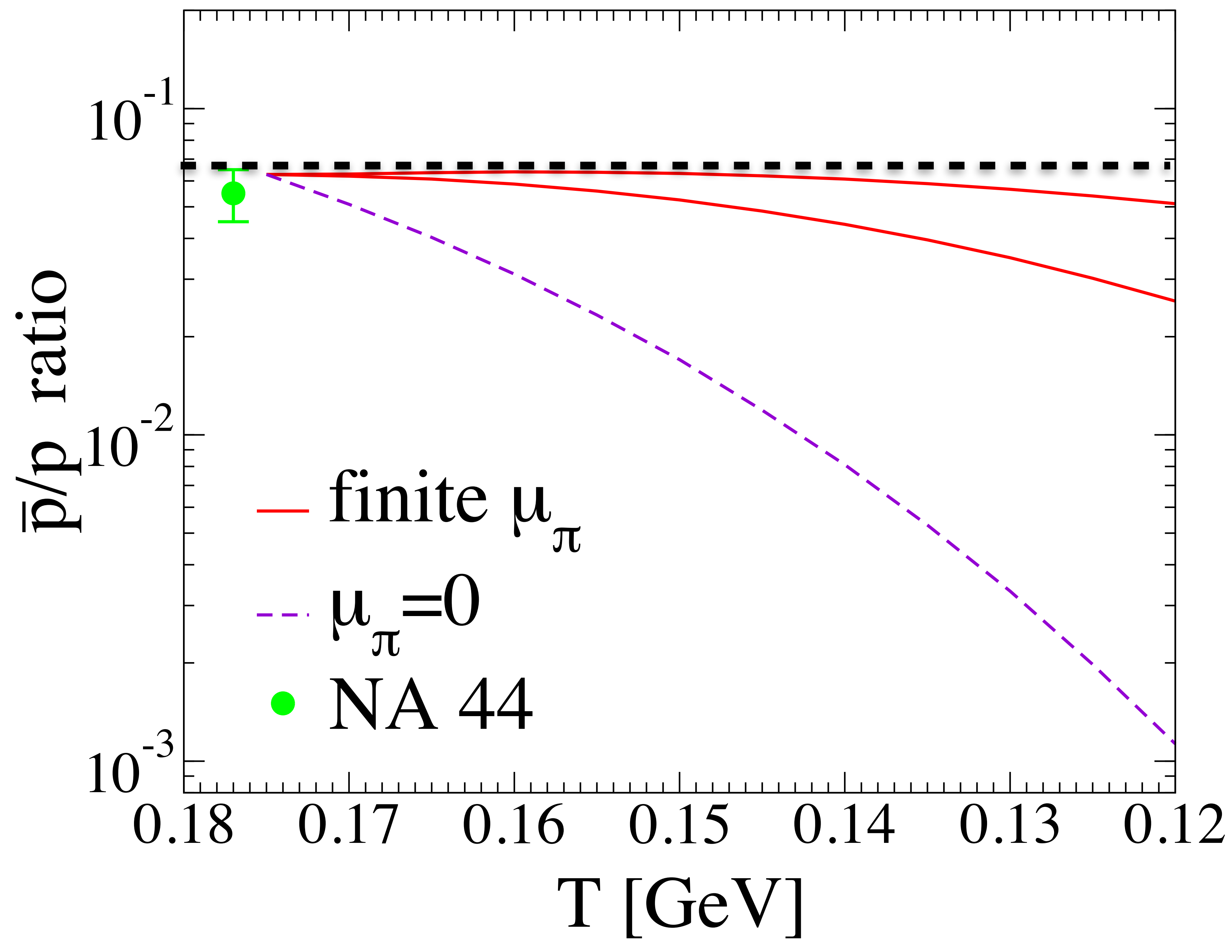
Why the discussion?

- Lifetime of hadronic phase is short
- pion number effectively conserved

  - $4\pi \Leftrightarrow 2\pi$ suppressed (chiral symmetry)
- $\Rightarrow$ finite μ_π
- increased re-generation of anti-protons

  - $5\pi \Leftrightarrow p + \bar{p}$
- Most transport calculations violate
  - detailed balance
  - exceptions:
    - E. Seifert, W. Cassing, PRC 97 (2018) 024913,
    - O. Garcia-Montero et al, Phys. Rev. C 105 (2022) 064906

Rapp, Shuryak, PRL 86 (2001) 2980;



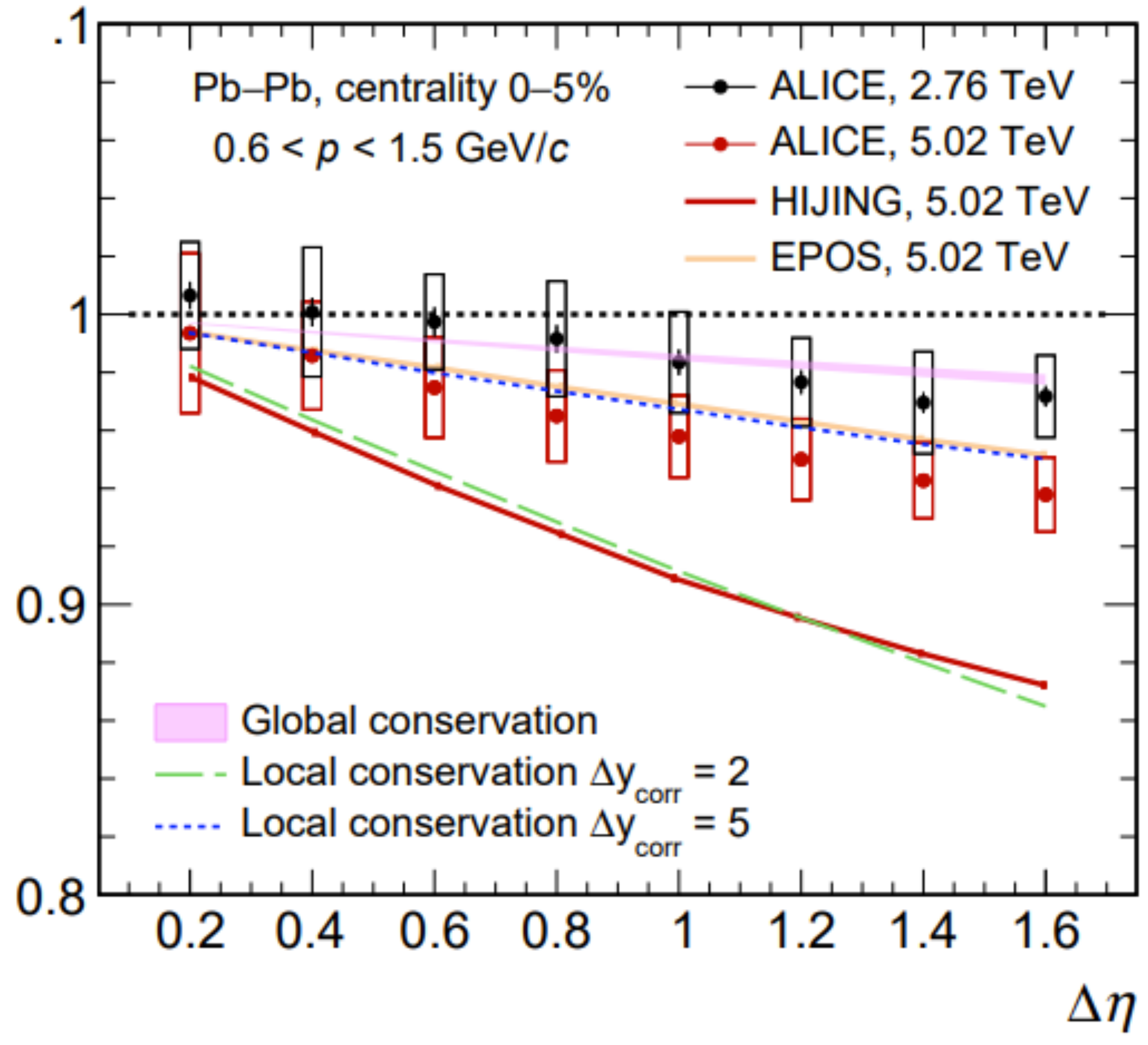
.

Need additional data to settle this issue

"Local" or "Global" Charge conservations

ALICE Coll., arXiv:2206.03343

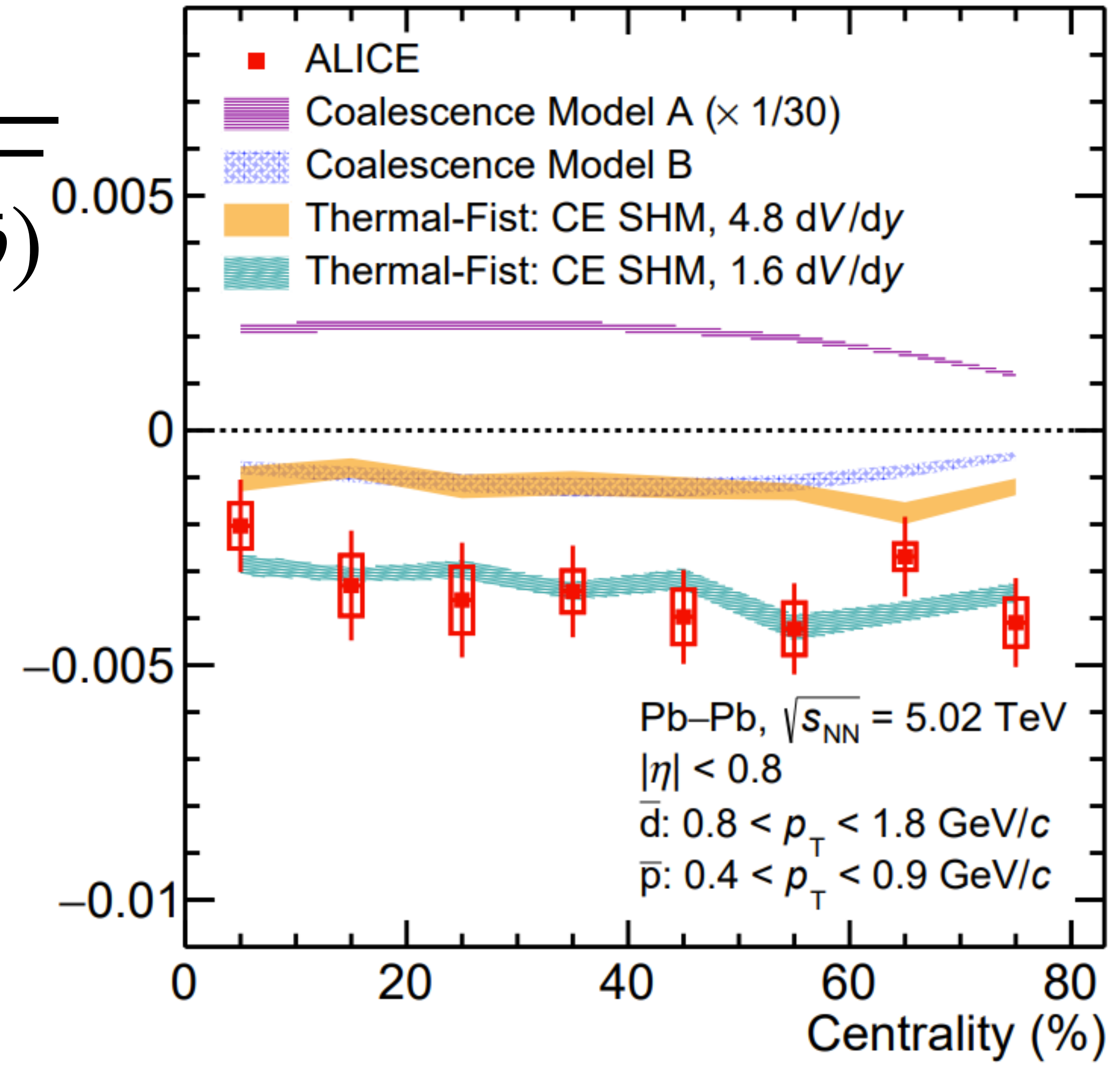
$$\frac{\kappa_2(p-\bar{p})}{\langle p+\bar{p}\rangle}$$



No annihilation

ALICE Coll., arXiv:2204.10166

$$\frac{\mathrm{cov}(\bar{\mathrm{d}},\bar{\mathrm{p}})}{\sqrt{\kappa_2(\bar{d})\kappa_2(\bar{p})}}$$



No annihilation

"wants" **long** range charge correlation

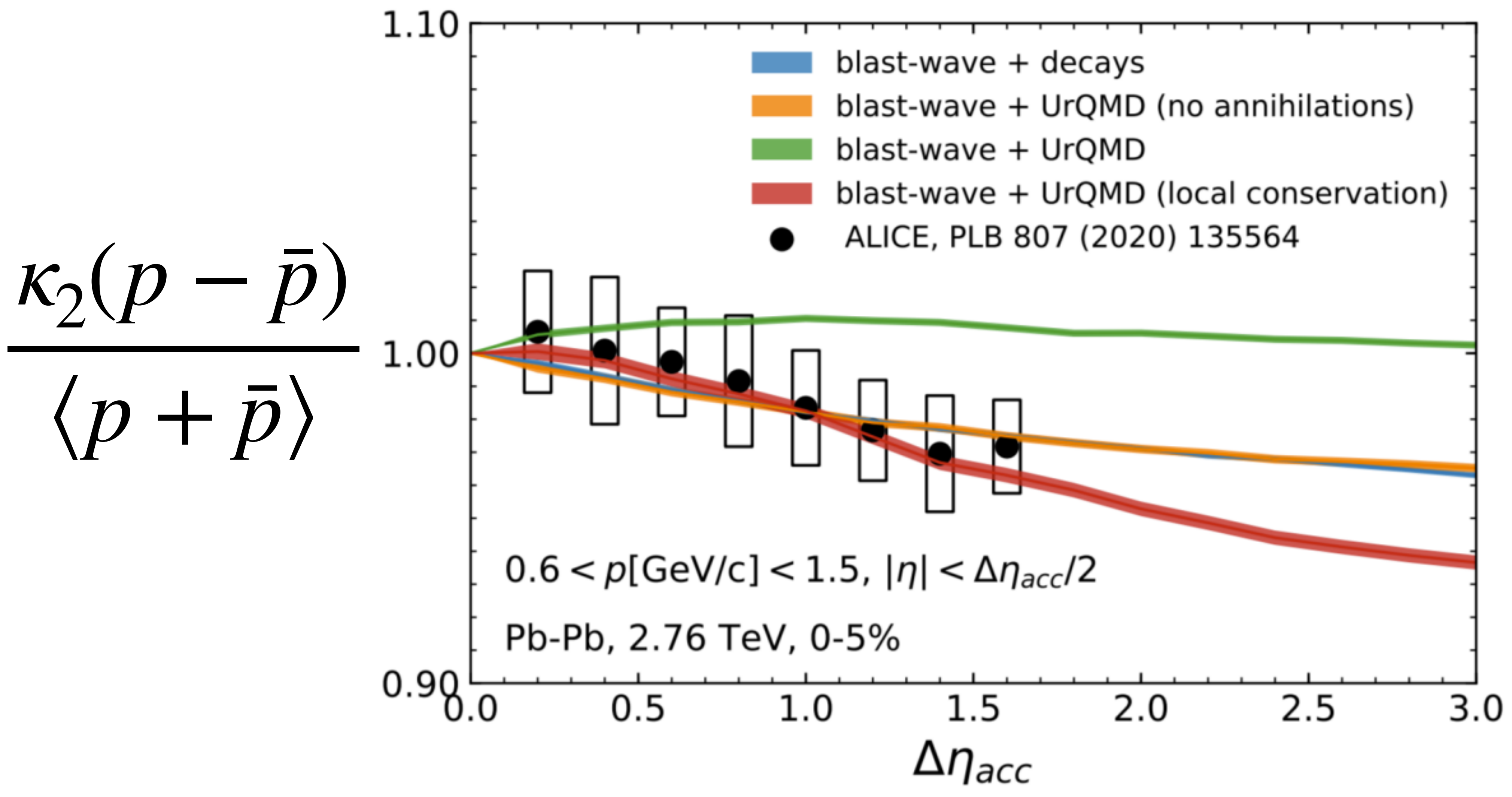
"wants" **short** range charge correlations

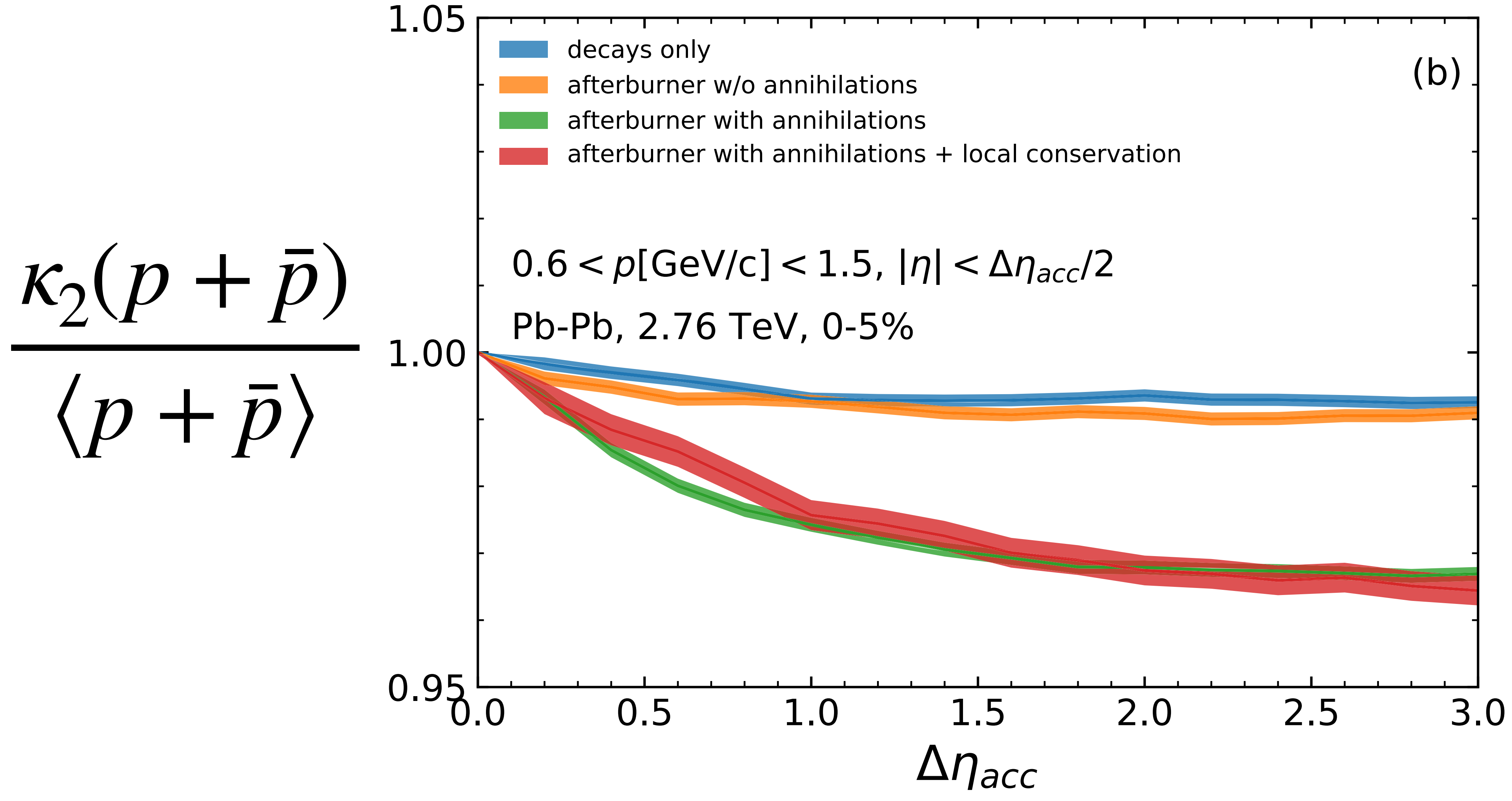
May resolve the tension between proton fluctuations that seem to prefer "global" baryon conservation vs light $\bar{d}$ - $\bar{p}$ correlations that prefer more "local" baryon conservation

Baryon annihilation and fluctuations

Savchuk et al., PLB 827, 136983 (2022)

- $\kappa_2(p - \bar{p})$:
  - **Not (really)** affected by annihilation
  - affected by baryon number conservation
- $\kappa_2(p + \bar{p})$:
  - affected by annihilation
  - **NOT** affected by baryon number conservation





N.B.:
In UrQMD annihilation has NO detailed balance
$\rightarrow$*No reaction* $5\pi \rightarrow p + \bar{p}$
$\rightarrow$ *maximum effect*

Measure $\kappa_2(p - \bar{p})$ AND $\kappa_2(p + \bar{p})$ to constrain both amount of annihilation AND baryon correlation length

New data @ 5.02 TeV

ALICE Collaboration, Phys. Rev. C 101 (2020) 044907

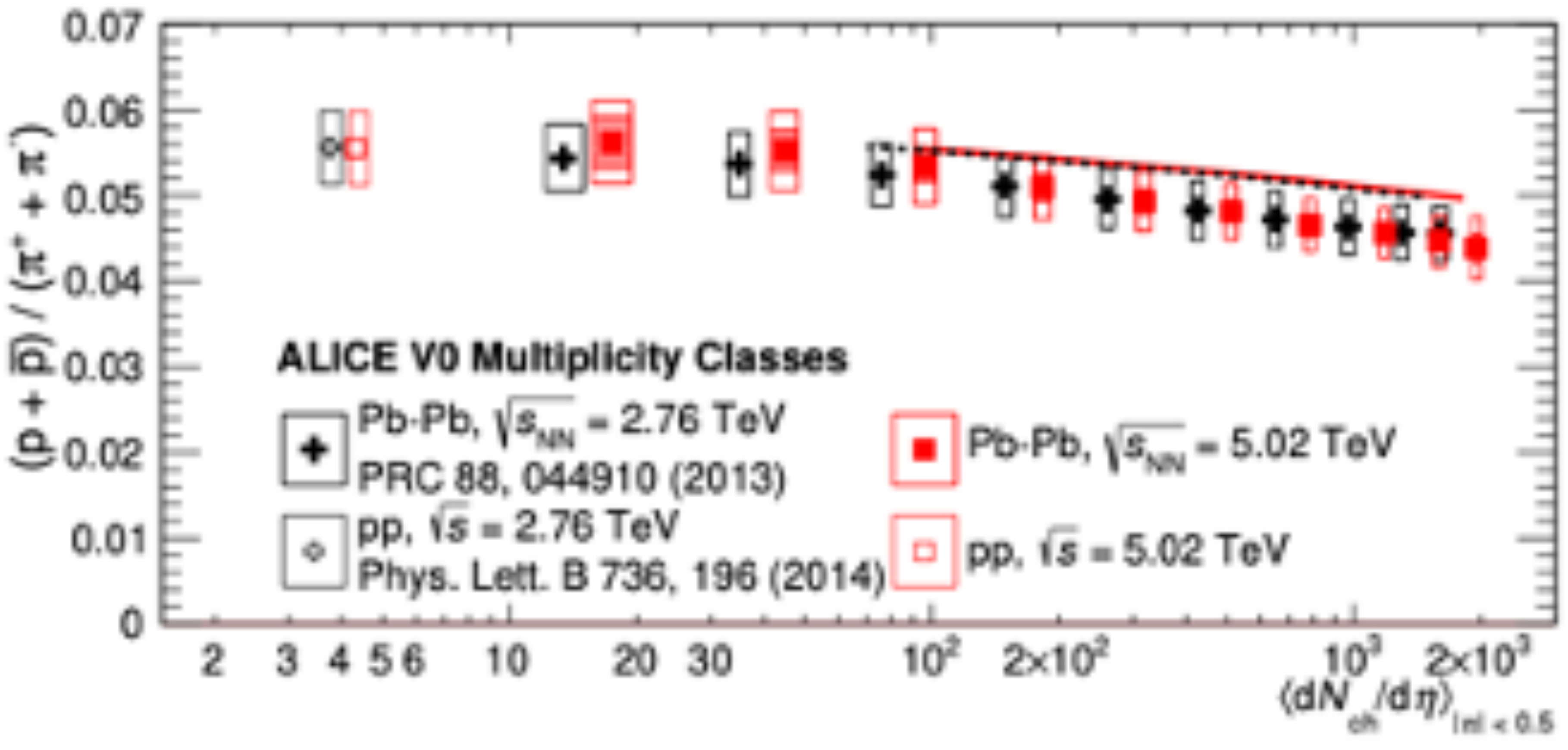
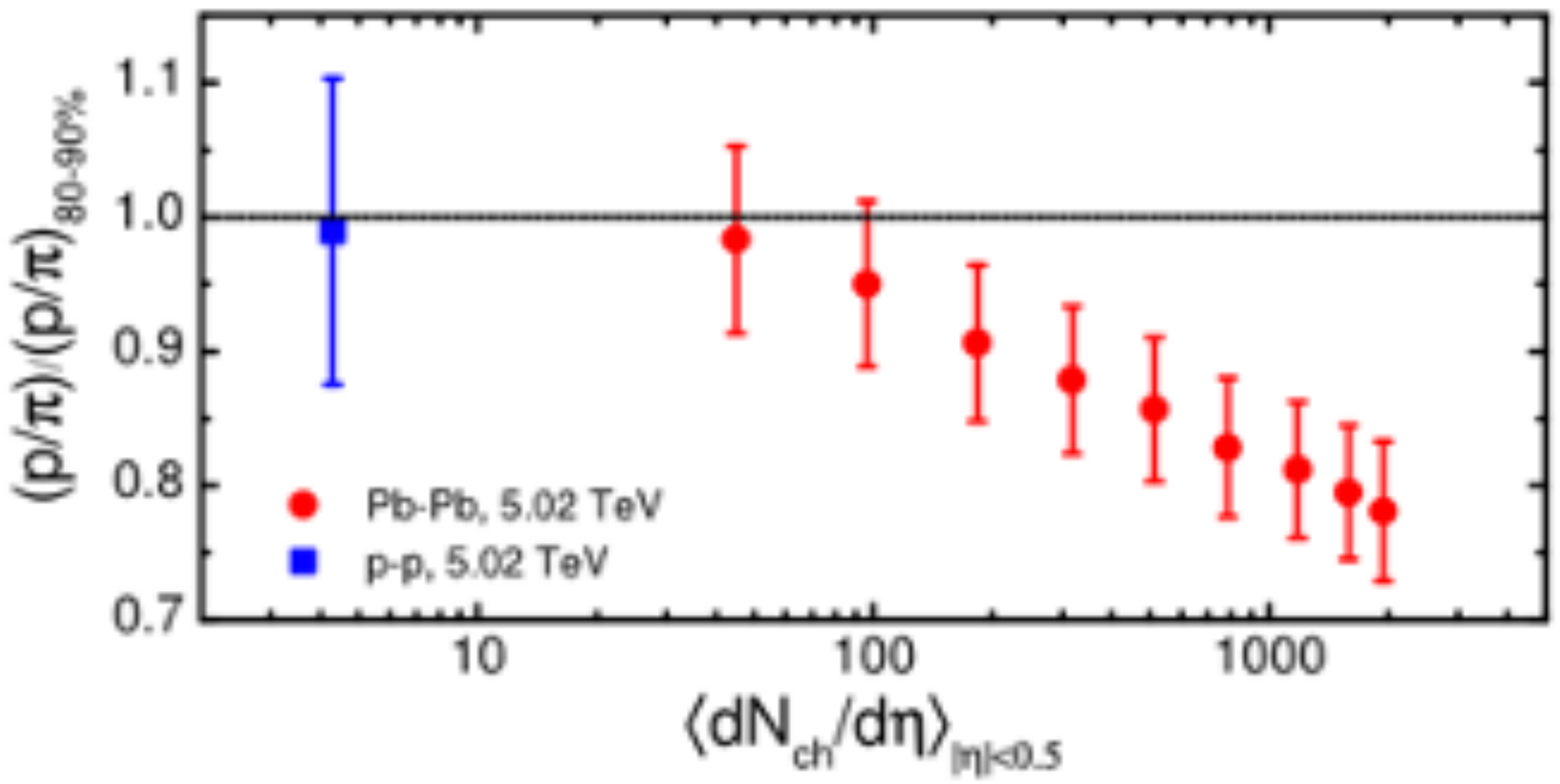


Figure 7: Transverse momentum integrated K/π (top) and p/π (bottom) ratios as a function of $\langle dN_{ch}/d\eta \rangle$ in Pb – Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, compared to Pb – Pb at 2.76 TeV [14]. The values in pp collisions at $\sqrt{s} = 5.02$ and 2.76 TeV are also shown. The empty boxes show the total systematic uncertainty; the shaded boxes indicate the contribution uncorrelated across centrality bins (not estimated in Pb – Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV).

Short hadronic phase

Long hadronic phase



- Evidence for suppression of p/π ration in central collisions (~20%, >4σ level)
- Due to hadronic phase?

For analysis and discussion: See V.Vovlchenko and V.K 2210.15641

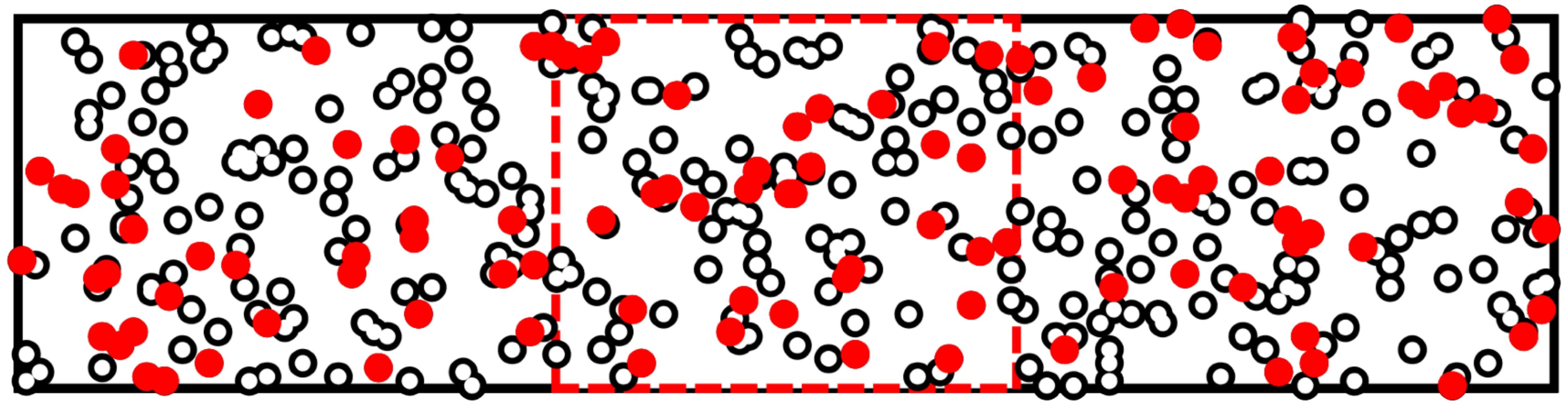
Summary

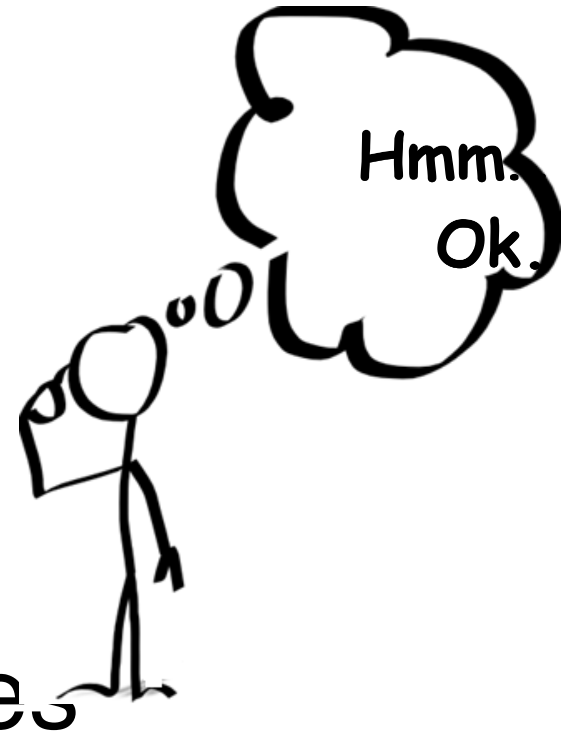
- Fluctuations measure derivatives of the Free Energy
  - They are a powerful tool to explore QCD phase diagram and other stuff
    - critical point
    - nuclear liquid gas transition
    - remnants of chiral criticality at $\mu \sim 0$
- Quantitative interpretation of measurements requires care:
  - Global (local) charge conservation
  - Protons vs baryons
  - momentum vs coordinate space
- Fluctuations may constrain proton annihilation together with locality of baryon number conservation

Thank You

Binomial acceptance vs actual acceptance

12

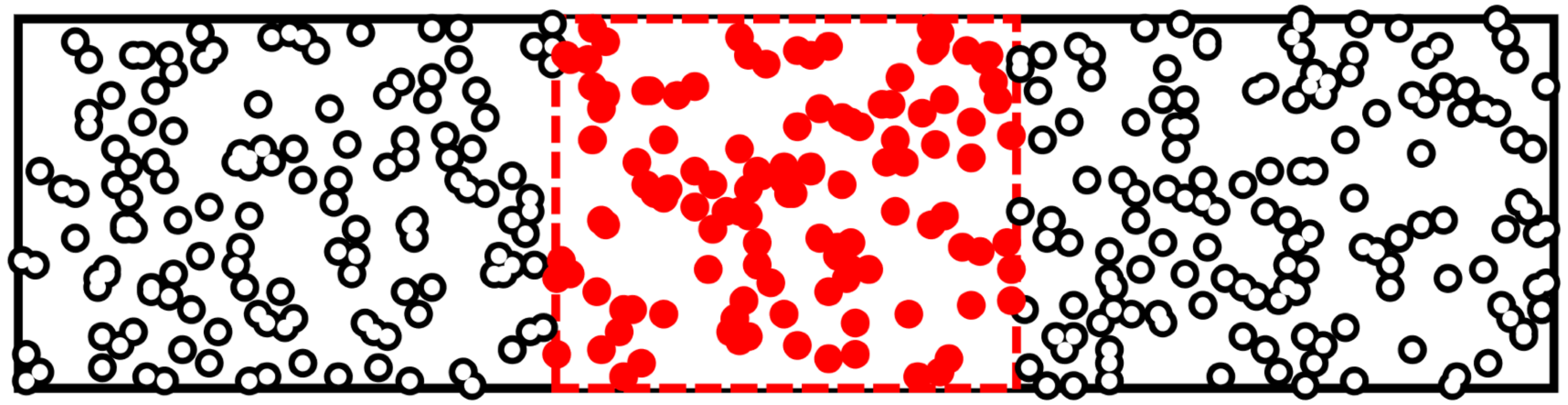




Binomial acceptance: accept each particle (charge) with probability α independently from all other particles

The binomial acceptance will not provide the correct result (except for a gas of uncorrelated particles)

What we really need is



Cumulants of (baryon) number distribution

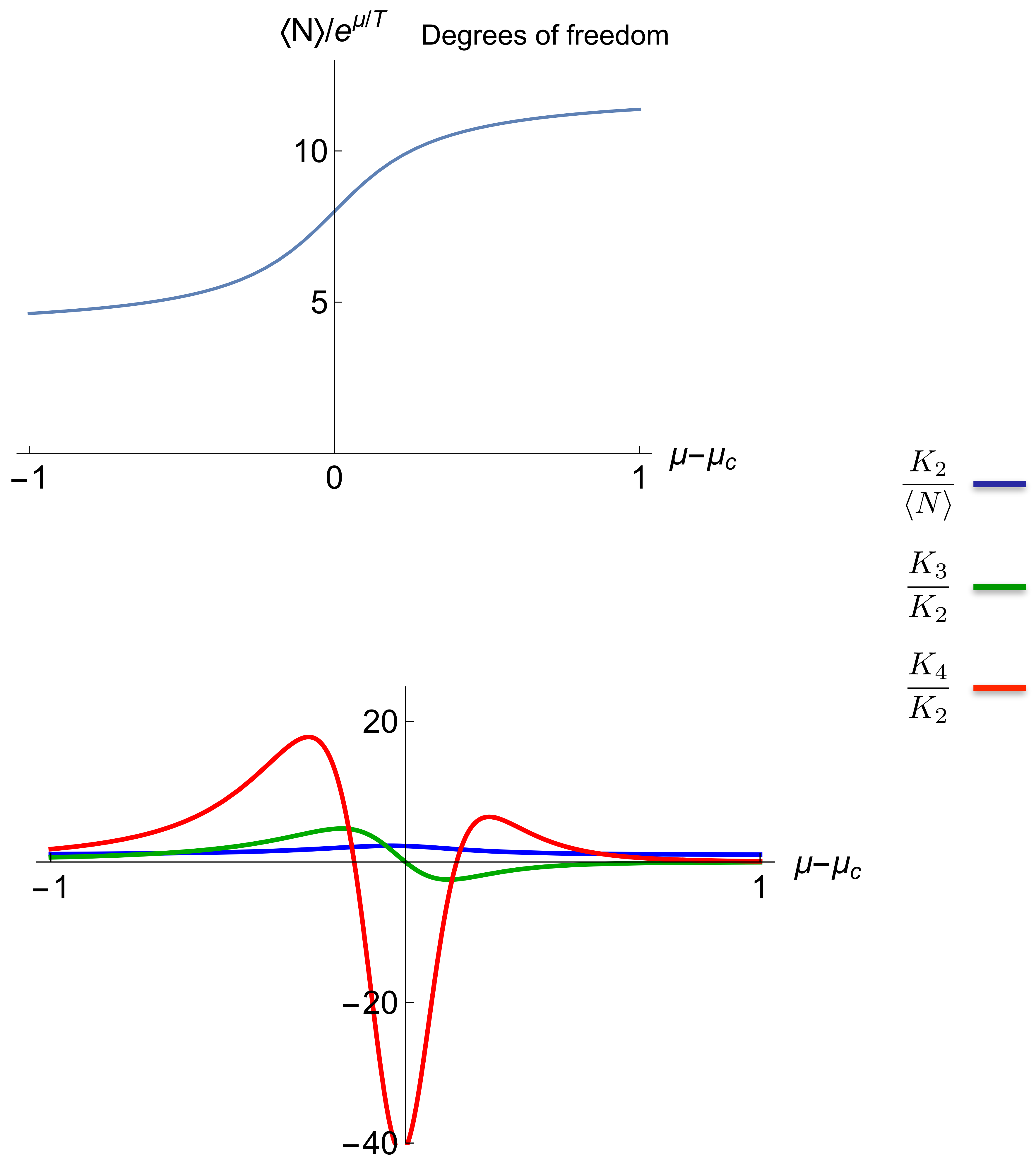
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle \,, \;\; K_2 = \langle N - \langle N \rangle \rangle^2 \,, \;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \; \frac{K_3}{K_2}, \; \frac{K_4}{K_2}$

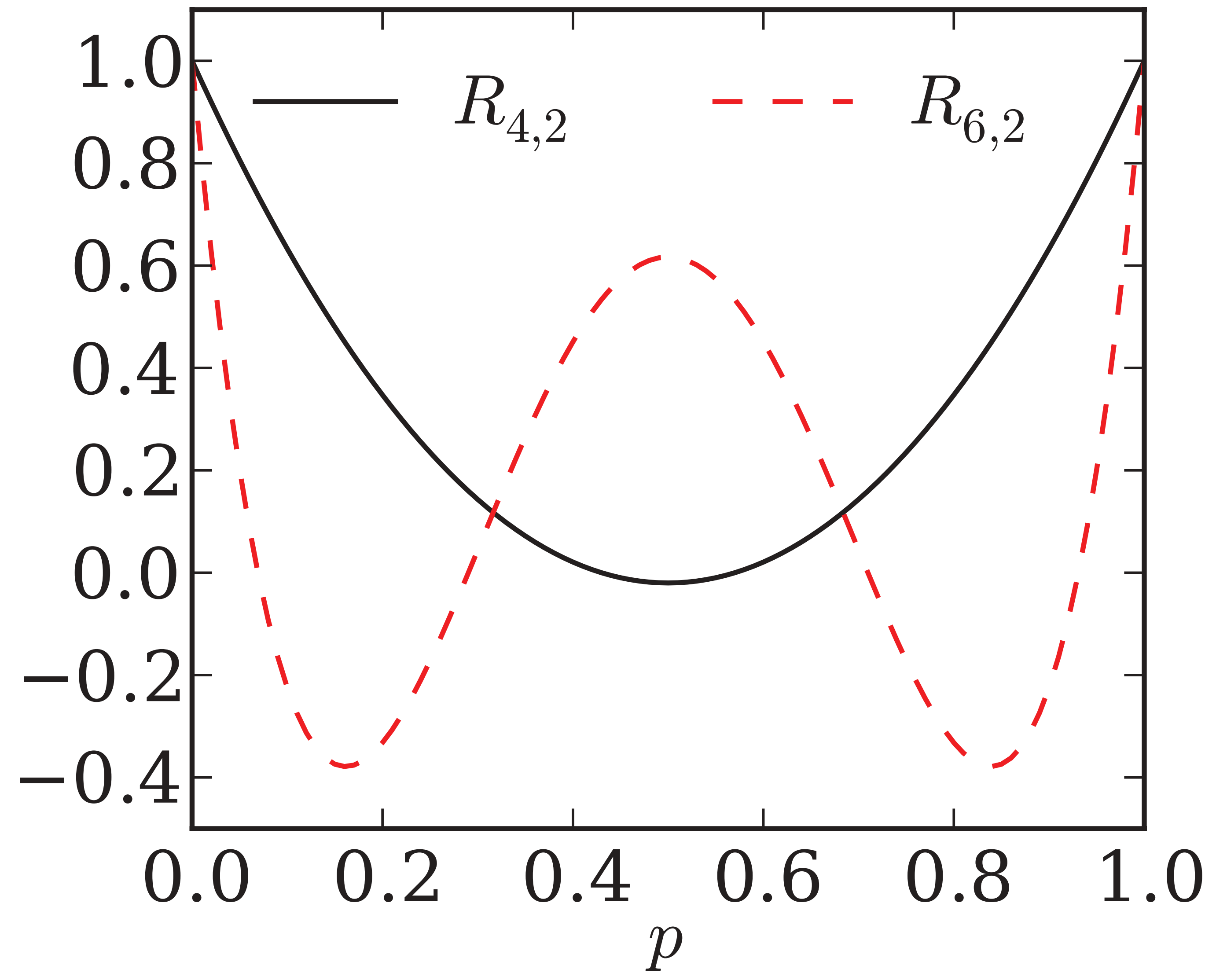


Baryon number cumulants measure derivatives of the EOS w.r.t chemical potential

Baryon number conservation and lattice susceptibilities

- Charges (baryon number, strangeness, electric charge) are conserved globally in HI collisions
- Lattice (and most other calculations) work in the grand canonical ensemble: charges may fluctuate
- Effect of charge conservation have been calculated in the **ideal gas/HRG** limit. NON-neglibile corrections especially for higher order cumulants (Bzdak et al 2013, Rustamov et al. 2017,…)
- Wouldn't it be nice to know what the effect of charge conservation on **real QCD** (aka lattice) susceptibilities is?

This can actually be done!



Bzdak et al, 2013

V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V.K., arXiv 2003.13905,

V. Vovchenko, R. Poberezhnyuk, V.K., arXiv:2007.03850

Subensemble acceptance method
(SAM)

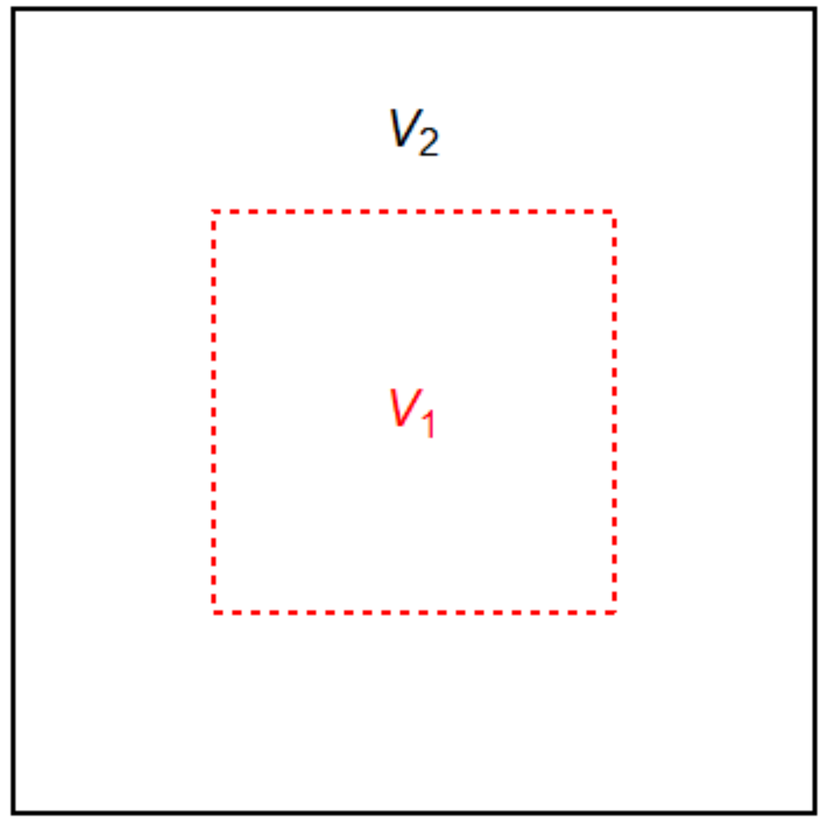
Partition a thermal system with a globally conserved charge *B (canonical ensemble)* into two subsystems which can exchange the charge

$$V = V_1 + V_2$$

Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

$$V_1, V_2 \gg \xi^3 \qquad \xi = correlation\, length$$



The canonical partition function then reads:

$$Z^{ce}(T, V, B) = \sum_{B_1} Z^{ce}(T, V_1, B_1) Z^{ce}(T, V - V_1, B - B_1)$$

The probability to have charge B_1 *in* V_1 is:

$$P(B_1) \sim Z^{ce}(T, \alpha V, B_1) Z^{ce}(T, (1-\alpha)V, B - B_1), \qquad \alpha \equiv V_1/V$$

Subensemble acceptance method
(SAM)

In the thermodynamic limit, $V \to \infty$, Z^{ce} expressed through free energy *density*

$$Z^{\text{ce}}(T,V,B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T} f(T,\rho_B)\right]$$

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1} \exp\left[-\frac{\alpha V}{T} f(T,\rho_{B_1})\right] \exp\left[-\frac{\beta V}{T} f(T,\rho_{B_2})\right]\right\} + \tilde{C}$$

Cumulants of B_1:

$$\kappa_n[B_1] = \left.\frac{\partial^n G_{B_1}(t)}{\partial t^n}\right|_{t=0} \equiv \tilde{\kappa}_n[B_1(t)]|_{t=0}$$

or

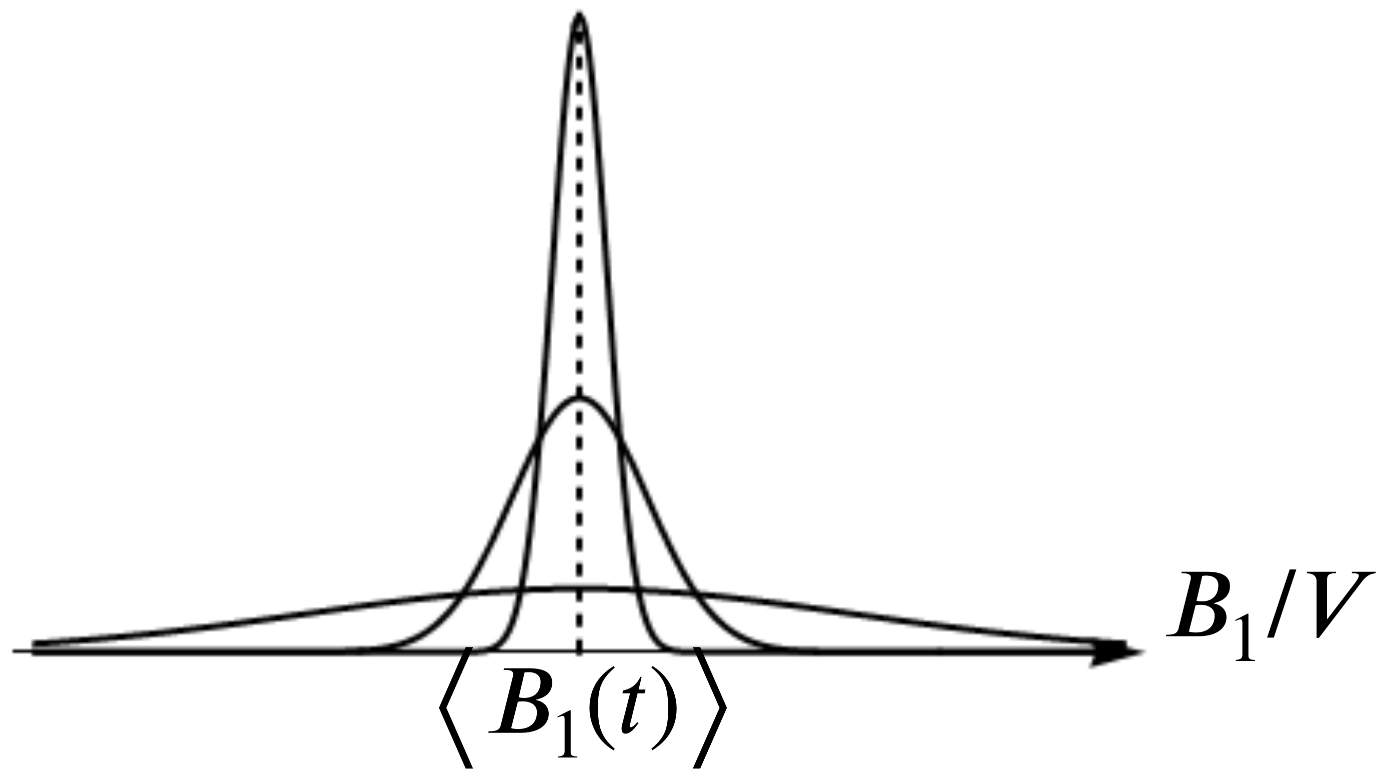
$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0}$$

All κ_n can be calculated by determining the *t*-dependent first cumulant $\tilde{\kappa}_1[B_1(t)]$

Making the connection…

$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1 \tilde{P}(B_1;t)}{\sum_{B_1} \tilde{P}(B_1;t)} \equiv \langle B_1(t) \rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{ tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T} \right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t) \rangle$



$\langle B_1(t) \rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)] \qquad \text{with} \qquad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial \rho_B$$

t = 0:

$$\rho_{B_1} = \rho_{B_2} = B/V,\ B_1 = \alpha B,$$

i.e. conserved charge uniformly distributed between the two subsystems

Second order cumulant

Differentiate condition for maximum of $\widetilde{P}(B_1;t)$,

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)] \qquad (*)$$

$$\frac{\partial(*)}{\partial t}: \qquad 1 = \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1}}\right)_T \left(\frac{\partial \rho_{B1}}{\partial \langle B_1 \rangle}\right)_V \frac{\partial \langle B_1 \rangle}{\partial t} - \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B2}}\right)_T \left(\frac{\partial \rho_{B2}}{\partial \langle B_2 \rangle}\right)_V \frac{\partial \langle B_2 \rangle}{\partial \langle B_1 \rangle} \frac{\partial \langle B_1 \rangle}{\partial t}$$

$$\left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1,2}}\right)_T \equiv \left[\chi_2^B\left(T, \rho_{B_{1,2}}\right) T^3\right]^{-1}, \quad \rho_{B_1} \equiv \frac{\langle B_1 \rangle}{\alpha V}, \quad \rho_{B_2} \equiv \frac{\langle B_2 \rangle}{(1-\alpha)V}, \quad \langle B_2 \rangle = B - \langle B_1 \rangle, \quad \frac{\partial \langle B_1 \rangle}{\partial t} \equiv \tilde{\kappa}_2[B_1(t)]$$

Solve the equation for $\widetilde{\kappa}_2$:

$$\tilde{\kappa}_2[B_1(t)] = \frac{V\,T^3}{[\alpha\,\chi_2^B(T, \rho_{B_1})]^{-1} + [(1-\alpha)\,\chi_2^B(T, \rho_{B_2})]^{-1}}$$

t = 0: $$\kappa_2[B_1] = \alpha\,(1-\alpha)\,V\,T^3\,\chi_2^B$$

Higher-order cumulants: iteratively differentiate $\widetilde{\kappa}_2$ w.r.t. t

Full result up to sixth order

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B \; - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\, \chi_4^B}{\chi_2^B}\right]$$

$$\kappa_5[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha) \left\{[1-2\beta\alpha]\chi_5^B \; - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B}\right\}$$

$$\kappa_6[B_1] = \alpha\, VT^3\, \beta\, [1-5\alpha\beta(1-\alpha\beta)]\, \chi_6^B + 5\, VT^3\, \alpha^2\, \beta^2 \left\{9\alpha\beta \frac{(\chi_3^B)^2\, \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} - 2(1-2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1-3\beta\alpha] \frac{\chi_3^B\, \chi_5^B}{\chi_2^B}\right\}$$

$$\beta = 1 - \alpha$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$

– grand-canonical susceptibilities e.g from Lattice QCD!!

Details: Vovchenko, et al. arXiv:2003.13905

Cumulant ratios

Some common cumulant ratios:

scaled variance $$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness $$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis $$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$: Grand canonical limit
- $\alpha \to 1$: canonical limit
- $\chi_{2n} = <N> + <\bar{N}>$; $\chi_{2n+1} = <N> - <\bar{N}>$: recover known results for ideal gas