An evolution equation for elastic scattering of hadrons

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NCN GRANT 2020/37/K/ST2/02665

Białasowka
April 28, 2023

Iceland Liechtenstein Norway grants

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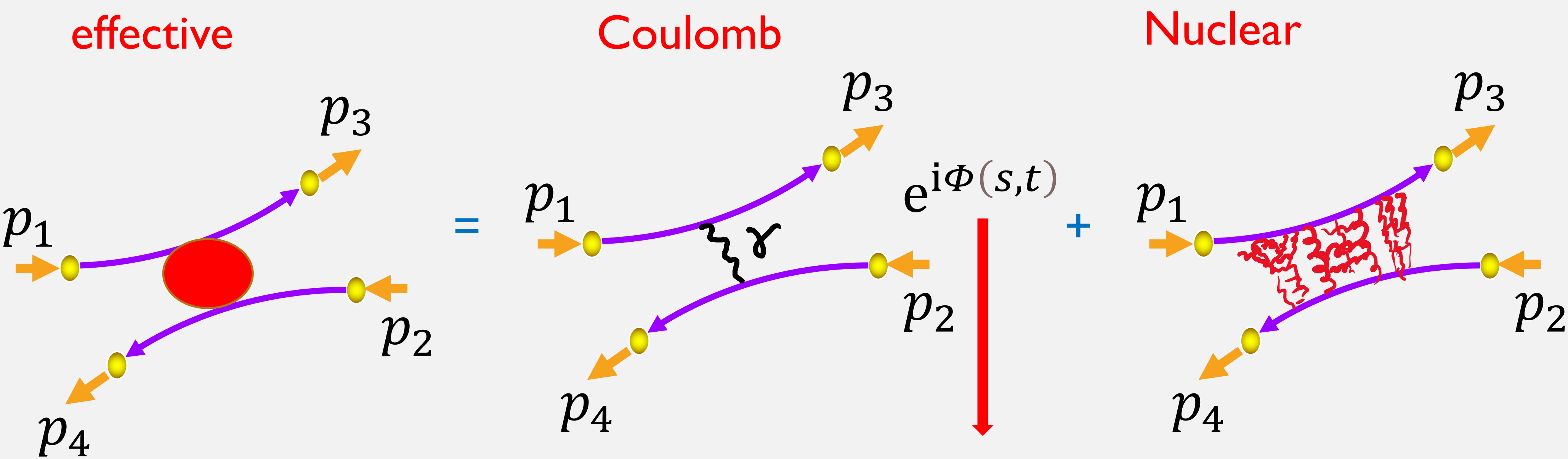
Some challenges:

- Seemingly simple kinematics but complicated dynamics
- Non-perturbative phenomenon
- Long range force interplays with short ranged
- Experimental gap among energies
- Differential cross sections with fluctuations (and/or) large uncertanties
- Lack of cross-symmetric experiments at the same energies

Interesting problems:

- It is the most basic diffractive phenomenon at high energies
- It is vaccum interaction
- One Pomeron, two Pomerons, how many?
- Where is the odderon?
- Several different models, but is there an equation to describe it?

Relativistic Elastic Scattering



Coulomb phase

L.D. Solov'ev, *JETP* **22**, 205 (1966) 26;
H. Bethe, *Ann. Phys. (N.Y.)* **3**, 190 (1958) 27;
G.B. West, D.R. Yennie, *Phys. Rev.* **172**(5), 1413 (1968);
V. Kundrat and M. Lokajcek, *Phys. Lett. B* **611** (2005) 102 ;
R. Cahn, *Z. Phys. C* **15** (1982) 253.

Mandelstam variables

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Basic physical quantities we are interested in:

Forward quantities

Optical theorem $\sigma_{tot} = 4\pi(\hbar c)^2 T_I^N(s,0)$

Ratio of real and imaginary amplitudes $\rho = \frac{T_R^N(s,0)}{T_I^N(s,0)}$

And the slopes

$$B_I = \frac{2}{T_I^N(s,t)} \frac{\mathrm{d}}{\mathrm{d}t} T_I^N(s,t)\Big|_{t=0}$$

$$B_R = \frac{2}{T_R^N(s,t)} \frac{\mathrm{d}}{\mathrm{d}t} T_R^N(s,t)\Big|_{t=0}$$

$$B = \frac{\rho^2 B_R + B_I}{\rho^2 + 1}$$

Differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = |T(s,t)|^2$$

Some models of elastic scattering amplitudes

- Bourrely Soffer Wu (eikonalized model based on crossing symmetry properties)
- Kohara Ferreira Kodama (based on stochastic vaccum model)
- Donnachie Landshoff (double Pomeron exchange + “perturbative tri-gluon exchange”)
- DGM (dinamical gluon mass)
- Jenkovszky model (regge based)
- Białas Bzdak (glauber based)
- HEGS ()

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If you want to, you can increase this list ....

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BSW model

C. Bourrely, J. Soffer, and T. T. Wu, Phys. Rev. D 19, 3249 (1979)

$$M(s,t) = \frac{is}{2\pi}\int \mathrm{d}^2 b\, e^{-iq\cdot b}\left(1 - \mathrm{e}^{-\Omega(s,\,b)}\right)$$

eikonalized model

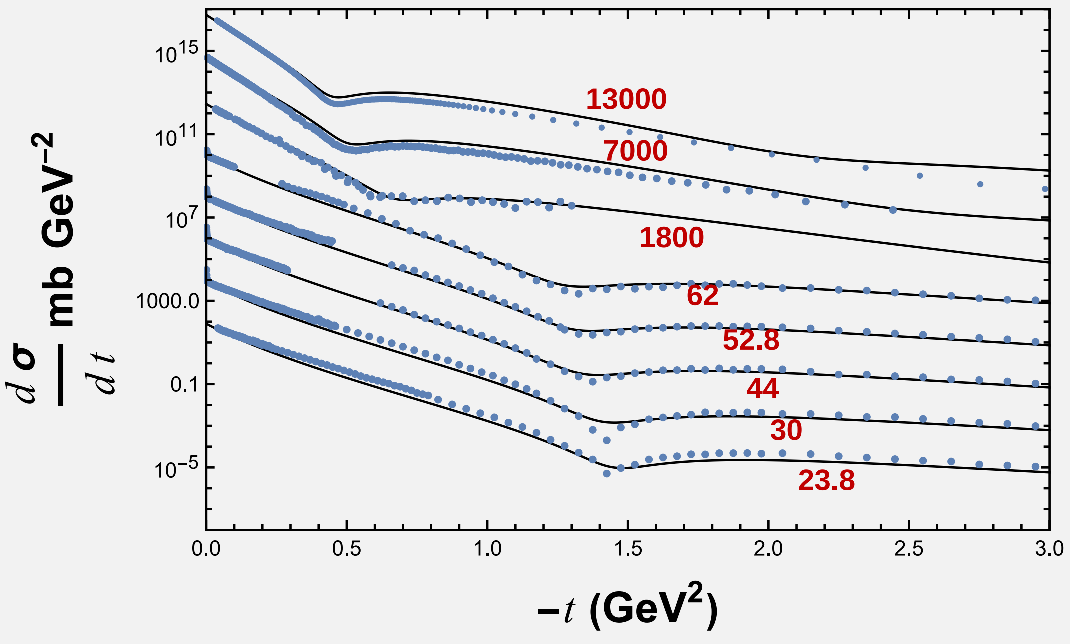
$$\Omega(s,b) = S(s)F(b) \qquad \text{opacity}$$

Symmetryzed energy dependece $S(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}$ $\qquad F(b) = \frac{1}{2\pi}\int d^2 q e^{iq\cdot b}\,\tilde{F}(-q^2)$

with $\ln u = \ln s - \mathrm{i}\pi$

Form factor

$$G(t) = \frac{1}{\left(1 - \frac{t}{m_1^2}\right)\left(1 - \frac{t}{m_2^2}\right)} \qquad \tilde{F}(t) = f G^2(t)\frac{a^2+t}{a^2-t}$$



c = 0.167	m_1 = 0.577 GeV	a = 1.858 GeV
c' = 0.748	m_1 =1.719 GeV	f = 6.971 GeV^{-2}

Parameters of the model

KFK model

A. K. K., E. Ferreira, and T. Kodama, Eur. Phys. J. C 74, 3175 (2014)

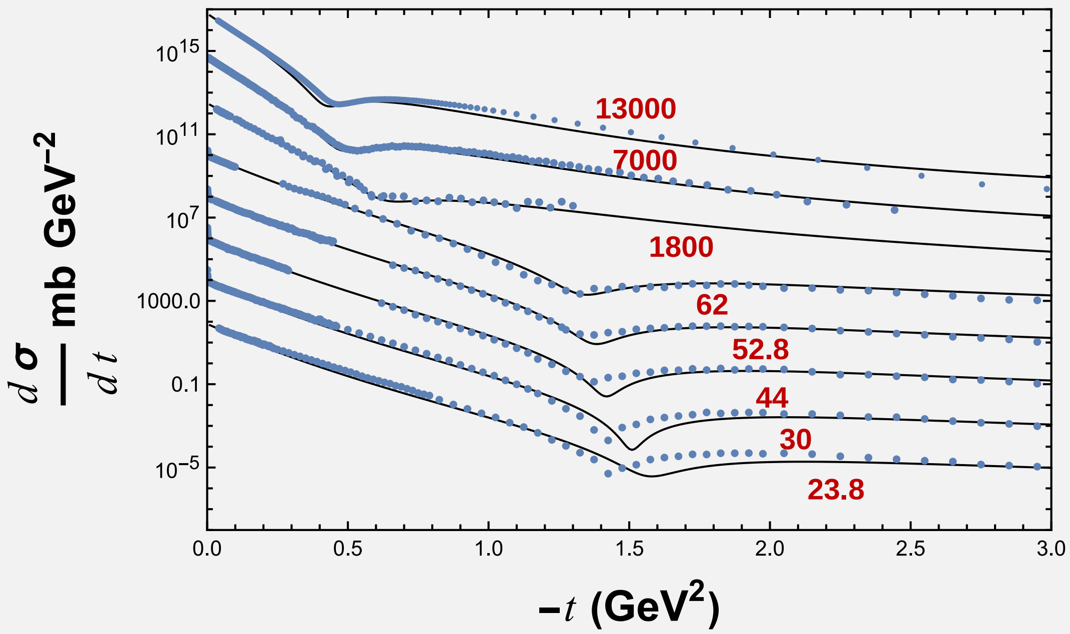
$$\widetilde{T_\mathrm{k}}(s,b) = \frac{\alpha_k(s)}{2\beta_k(s)} \mathrm{e}^{-\frac{b^2}{4\beta_k(s)}} + \lambda_k(s)\widetilde{\Psi}(s,b)$$

Based on Stochastic vaccum model

$$T_k(s,t) = \frac{1}{2\pi}\int \mathrm{d}^2 b\, \mathrm{e}^{-\mathrm{i}q\cdot b}\tilde{T}_k(s,b)$$

$$\widetilde{\Psi}_k(s,b) = \frac{2\mathrm{e}^{\gamma_k(s)-\sqrt{\gamma_k(s)+\frac{b^2}{a_0}}}}{a_0\sqrt{\gamma_k(s)+\frac{b^2}{a_0}}}\left[1-\mathrm{e}^{\gamma_k(s)-\sqrt{\gamma_k(s)+b^2/a_0}}\right]$$

Fourier transformation in closed forms in both *t* and *b*



Donnachie Landshoff model   Pomeron exchange (Regge-like)

A. Donnachie and P.V. Landshoff , Physics Letters B 727 (2013)

$$\alpha_i(E) = 1 + \varepsilon_i + \alpha_i' t \qquad \mathrm{i} = P\,,\pm$$ Regge trajectories

One Pomeron, even and odd regge contributions

$$A(s,t) = -\frac{X_P F_P(t)}{2\nu} \mathrm{e}^{-\frac{1}{2}\mathrm{i}\pi\alpha_P(t)} (2\nu\alpha_P')^{\alpha_P(t)} - \frac{X_+ F_+(t)}{2\nu} \mathrm{e}^{-\frac{1}{2}\mathrm{i}\pi\alpha_+(t)} (2\nu\alpha_+')^{\alpha_+(t)} - i\,\frac{X_- F_-(t)}{2\nu} \mathrm{e}^{-\frac{1}{2}\mathrm{i}\pi\alpha_-(t)} (2\nu\alpha_-')^{\alpha_-(t)}$$

Two Pomeron exchange (negative contribution)

$$\frac{X_P^2}{64\pi\nu} \mathrm{e}^{-\frac{1}{2}\mathrm{i}\pi\alpha_{PP}(t)} (2\nu\alpha_P')^{\alpha_{PP}(t)} \left[\frac{A^2}{{a}/{\alpha_p'} + L} \mathrm{e}^{\frac{1}{2}at} + \frac{(1-A)^2}{{b}/{\alpha_p'} + L} \mathrm{e}^{\frac{1}{2}bt} \right]$$

Form factor

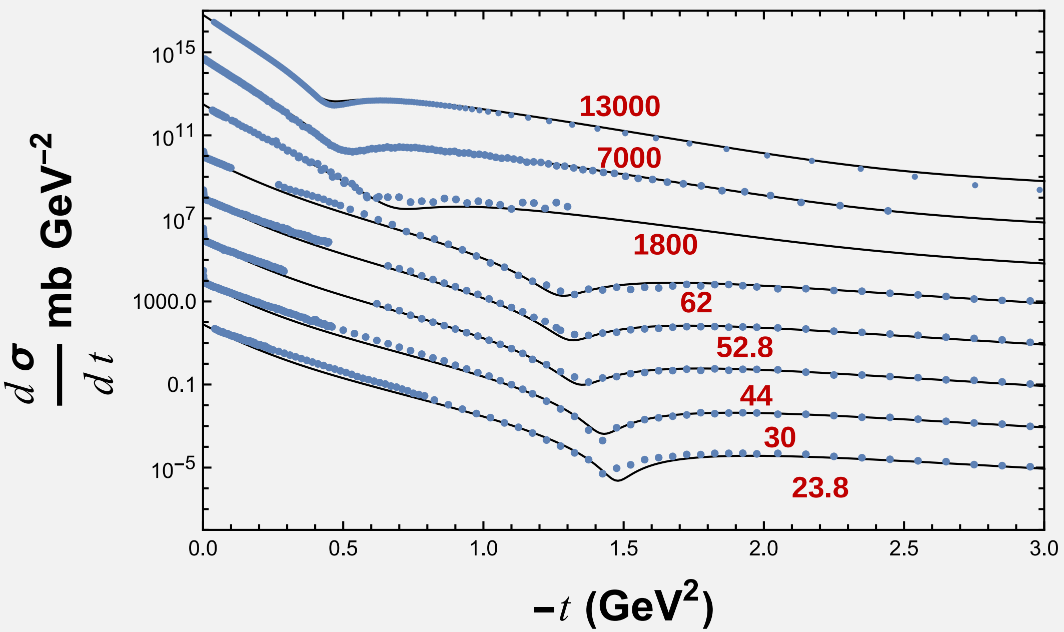
$$F(t) = A\mathrm{e}^{at} + (1-A)\mathrm{e}^{bt}$$

$$L = \ln(2\nu\alpha_P') - \frac{1}{2}\mathrm{i}\pi$$

"Perturbative" tri-gluon exchange

A. Donnachie and P. V. Landshoff, Phys.Lett. B387 (1996) 637-641

$$\frac{C}{t_0} \mathrm{e}^{2\left(1 - \frac{t^2}{t_0^2}\right)}$$



$\varepsilon_P = 0.110$	$\varepsilon_+ = -0.327$	$\varepsilon_- = -0.505$
X_P=339	$X_+ = 212$	$X_- = 104$
$\alpha_P' = 0.165\ \mathrm{GeV}^{-2}$	a=7.854 GeV^{-2}	b=2.470 GeV^{-2}
$A = 0.682$	$C = 0.0406$	t_0 =4.230 GeV^{-2}

Parameters of the model

Dynamical Gluon Mass model (QCD inspired)

E. G. S. Luna, A. F. Martini, M. J. Menon, A. Mihara, and A. A. Natale, Phys. Rev. D 72 , 034019 (2005)

Amplitude $$A(s,q^2) = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}^2 b\, e^{-iq\cdot b}\left(1 - \mathrm{e}^{\mathrm{i}\chi_{p\bar p}^{pp}(s,b)}\right)$$

Eikonal $$\chi_{p\bar p}^{pp}(s,b) = \frac{\mathrm{i}}{2}\left[\sigma_{qq}(s)W(b;\mu_{\mathrm{qq}}) + \sigma_{qg}(s)W(b;\mu_{\mathrm{qg}}) + \sigma_{gg}(s)W(b;\mu_{\mathrm{gg}}) \pm kC_{odd}\frac{m_g}{\sqrt{s}}\mathrm{e}^{\frac{\mathrm{i}\pi}{4}}W(b;\mu_{\mathrm{odd}})\right]$$

Elementary cross sections

$$\sigma_{qq}(s) = kC_{qq}\frac{m_g}{\sqrt{s}}\mathrm{e}^{\frac{\mathrm{i}\pi}{4}}$$

$$\hat\sigma_{gg}(\hat s) = \left(\frac{3\pi_{\alpha_s}^{-2}}{\hat s}\right)\left[\frac{12\hat s - 55M_g^2\hat s^3 + 12M_g^4\hat s^2 + 66M_g^6\hat s \quad - 8M_g^8}{4M_q^2\hat s\left[\hat s - M_g^2\right]^2} - 3\ln\left(\frac{\hat s - 3M_g^2}{M_\delta^2}\right)\right]$$

$$\sigma_{qg}(s) = \left[C_{qg} + C_{qg}{}'\left[\ln\left(\frac{s}{m_g^2}\right) - \mathrm{i}\frac{\pi}{2}\right]\right]$$

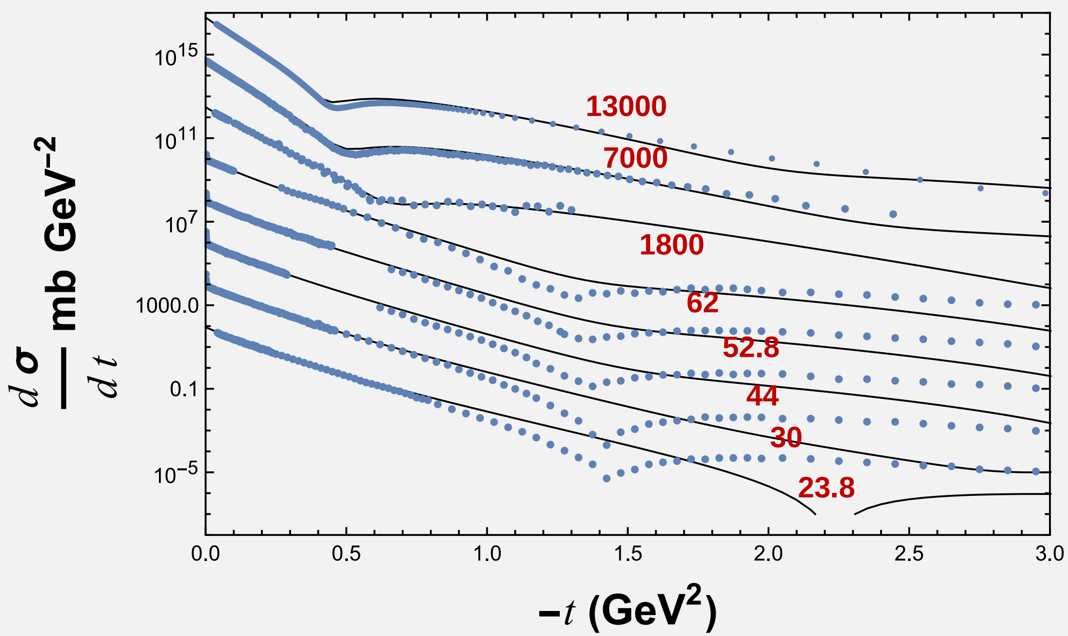
$$\sigma_{gg}(s) = C_{gg}\int_{4m_g^2/s}^{1}\mathrm{d}\tau\, F_{gg}(\tau)\hat\sigma_{gg}(\hat s)$$

$$F_{\mathrm{gg}}(\tau) = \int_\tau^1 \frac{\mathrm{d}x}{x}g(x)g({}^{\tau}\!/_{x}) \qquad g(x) = N_g\frac{(1-x)^5}{x^{1+\epsilon}}$$

b-profile $$W(b;\mu) = \frac{\mu^2}{96\pi}(\mu b)^3 k_3(\mu b)$$

running coupling $$\bar\alpha_s(\hat s) = \frac{4\pi}{\beta_0\ln\left[\left(\hat s + 4M_g^2(\hat s)\right)/\Delta^2\right]}$$

dynamic mass $$M_g^2(\hat s) = m_g^2\left[\frac{\ln\left[\left(\hat s + 4m_g^2\right)/\Delta^2\right]}{\ln\left(4m_g{}^2/\Delta^2\right)}\right]^{-12/11}$$



m_g	400 MeV
C_{odd}	3.03
C_{qq}	10.07
C_{qg}	0.874
C'_{qg}	0.0451
C_{gg}	0.00379
μ_{odd}	0.41
μ_{gg}	0.651
μ_{qq}	1.32
μ_{qg}	0.838

Jenkovszky model (reggeons +Pomeron+Odderon)

L.L. Jenkovszky, A.I. Lengyel, D.I. Lontkovsky, Int. J. Mod. Phys. A, 26, 4755 (2011)

Amplitudes $A(s,t)_{pp}^{p\bar{p}} = A_P(s,t) + A_f(s,t) \pm [A_\omega(s,t) + A_O(s,t)]$

trajectories

$$\alpha_k(t) = 1 + \delta_k + \alpha'_k t$$

$$A_R(s,t) = a_R \mathrm{e}^{-\frac{i\pi}{2}\alpha_R(t)} \mathrm{e}^{b_R t} \left({}^{s}/_{s_0}\right)^{\alpha_R(t)}$$ reggeons

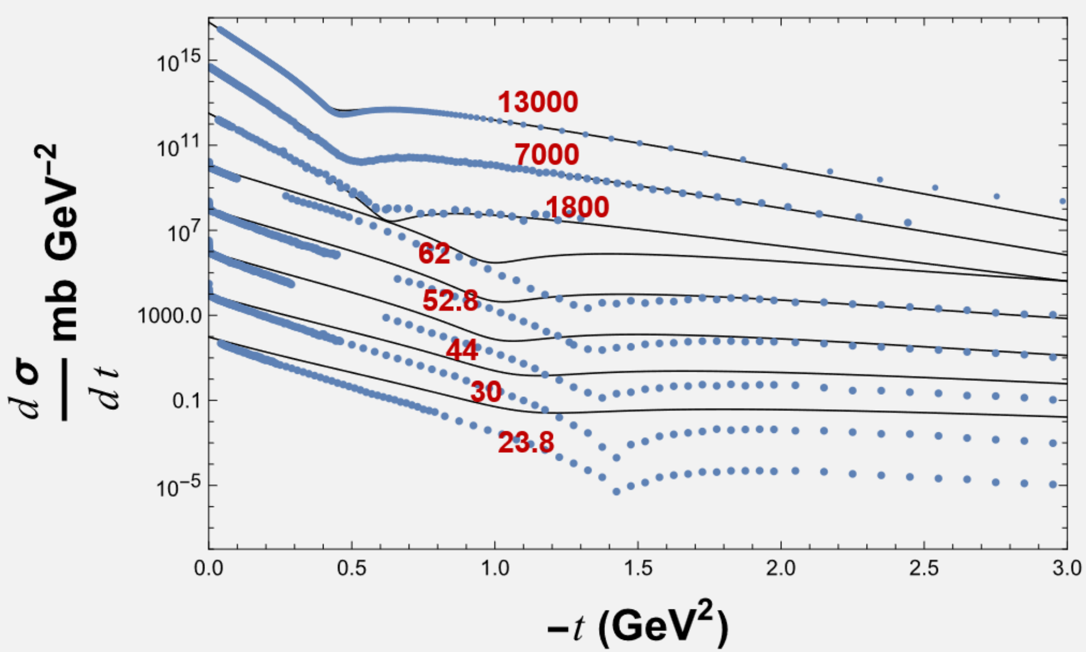
Energy dependence

$$r_1^2(s) = b_P + \ln(s/s_0) - \frac{i\pi}{2}$$

$$A_P(s,t) = i\frac{a_P s}{b_P s_0}\left[r_1^2(s) e^{r_1^2(s)[\alpha_P(t)-1]} - \varepsilon_P r_2^2(s) e^{r_2^2(s)[\alpha_P(t)-1]}\right]$$ Pomeron

$$r_2^2(s) = \ln(s/s_0) - \frac{i\pi}{2}$$

$$A_O(s,t) = \frac{a_O s}{b_O s_0}\left[r_{1O}^2(s) e^{r_{1O}^2(s)[\alpha_O(t)-1]}\right]$$ odderon



These curves are done with fixed parameters

b-space (non-observable)

Physical cross sections are written

$$\sigma_{el}(s) = \int d^2\vec{b}\,\left|\tilde{T}(s,\vec{b})\right|^2 = \int d^2\vec{b}\,\frac{d\tilde{\sigma}_{el}}{d^2\vec{b}}(s,\vec{b})$$

$$\sigma_{tot}(s) = 2\int d^2\vec{b}\,\tilde{T}_I(s,\vec{b}) = \int d^2\vec{b}\,\frac{d\tilde{\sigma}_{tot}}{d^2\vec{b}}(s,\vec{b})$$

$$\sigma_{inel}(s) = \int d^2\vec{b}\,G_{inel}(s,\vec{b}) = \int d^2\vec{b}\,\frac{d\tilde{\sigma}_{inel}}{d^2\vec{b}}(s,\vec{b})$$

Unitarity constraint in b space

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$$

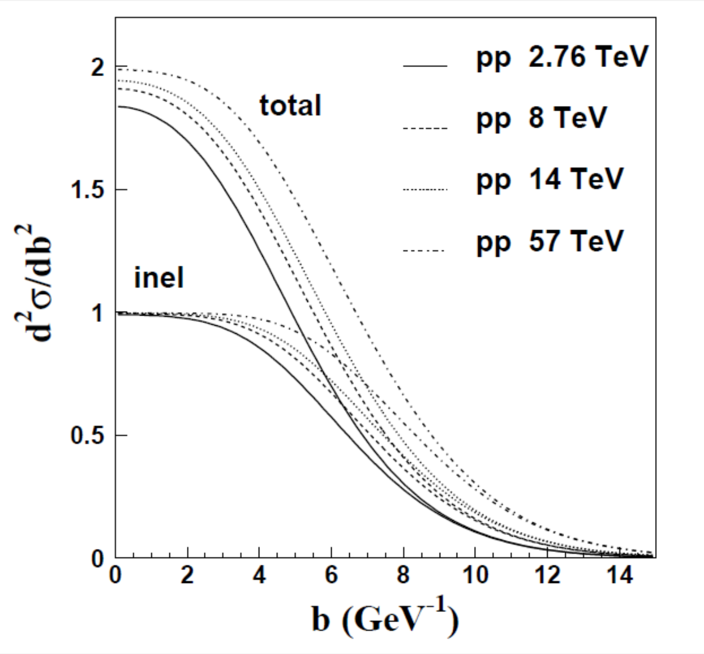
Assuming independent $d^2\vec{b}$ increments

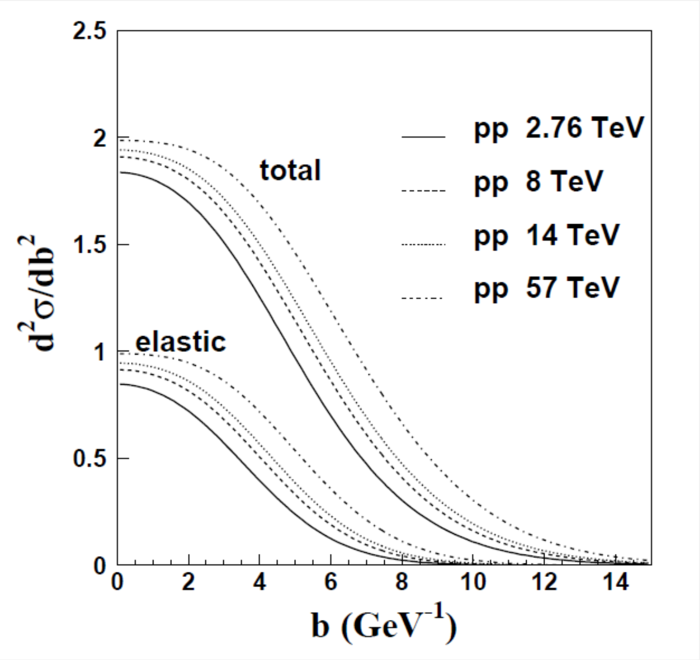
$$2\,\tilde{T}_I(s,b) = \left|\tilde{T}(s,b)\right|^2 + G_{inel}(s,b)$$

$$G_{inel}(s,b) \approx \tilde{T}_I(s,b)[2 - \tilde{T}_I(s,b)] - \cancelto{0}{\tilde{T}_R(s,b)^2}$$

$$G_{inel}(s,b) \leq 1$$

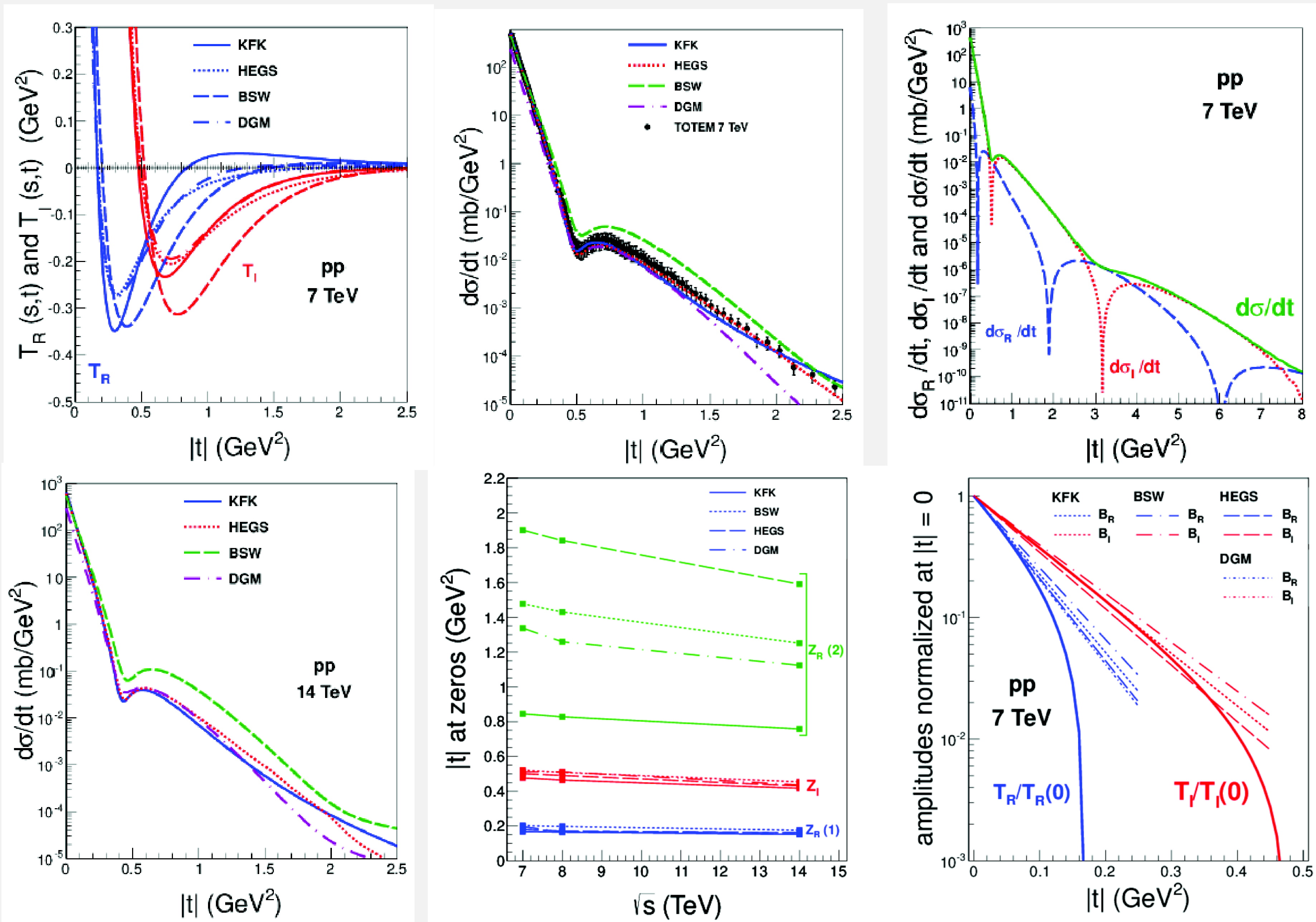
Monotonic results for elastic differential 'cross sections' profile functions





- Interesting diffusive behaviour with increasing energy
- Unitarity bound saturation

Different models: amplitudes, differential cross sections, zeros, slopes



E. Ferreira, T. Kodama, A. K. K., D. Szilard, Acta Phys.Polon.Supp. ***8*** *(2015) 1017*

Fitting an Elephant

Drawing an elephant with four complex parameters

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(Received 20 August 2008; accepted 5 October 2009)

We define four complex numbers representing the parameters needed to specify an elephantine shape. The real and imaginary parts of these complex numbers are the coefficients of a Fourier coordinate expansion, a powerful tool for reducing the data required to define shapes.

[DOI: 10.1119/1.3254017]

A turning point in Freeman Dyson's life occurred during a meeting in the Spring of 1953 when Enrico Fermi criticized the complexity of Dyson's model by quoting Johnny von Neumann:[1] "With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." Since then it has become a well-known saying among physicists, but nobody has successfully implemented it.

To parametrize an elephant, we note that its perimeter can be described as a set of points $(x(t), y(t))$, where t is a parameter that can be interpreted as the elapsed time while going along the path of the contour. If the speed is uniform, t becomes the arc length. We expand x and y separately[2] as a Fourier series

$$x(t) = \sum_{k=0}^{\infty} (A_k^x \cos(kt) + B_k^x \sin(kt)), \tag{1}$$

$$y(t) = \sum_{k=0}^{\infty} (A_k^y \cos(kt) + B_k^y \sin(kt)), \tag{2}$$

where A_k^x, B_k^x, A_k^y, and B_k^y are the expansion coefficients. The lower indices k apply to the kth term in the expansion, and the upper indices denote the x or y expansion, respectively.

Using this expansion of the x and y coordinates, we can analyze shapes by tracing the boundary and calculating the coefficients in the expansions (using standard methods from Fourier analysis). By truncating the expansion, the shape is smoothed. Truncation leads to a huge reduction in the information necessary to express a certain shape compared to a pixelated image, for example. Székely *et al.*[3] used this approach to segment magnetic resonance imaging data. A similar approach was used to analyze the shapes of red blood cells,[4] with a spherical harmonics expansion serving as a 3D generalization of the Fourier coordinate expansion.

The coefficients represent the best fit to the given shape in the following sense. The $k=0$ component corresponds to the center of mass of the perimeter. The $k=1$ component corresponds to the best fit ellipse. The higher order components trace out elliptical corrections analogous to Ptolemy's epicycles.[5] Visualization of the corresponding ellipses can be found at Ref. 6.

We now use this tool to fit an elephant with four parameters. Wei[7] tried this task in 1975 using a least-squares Fourier sine series but required about 30 terms. By analyzing the picture in Fig. 1(a) and eliminating components with amplitudes less than 10% of the maximum amplitude, we obtained an approximate spectrum. The remaining amplitudes were slightly modified to improve the aesthetics of the final image. By incorporating these coefficients into complex numbers, we have the equivalent of an elephant contour coded in a set of four complex parameters (see Fig. 1(b)).

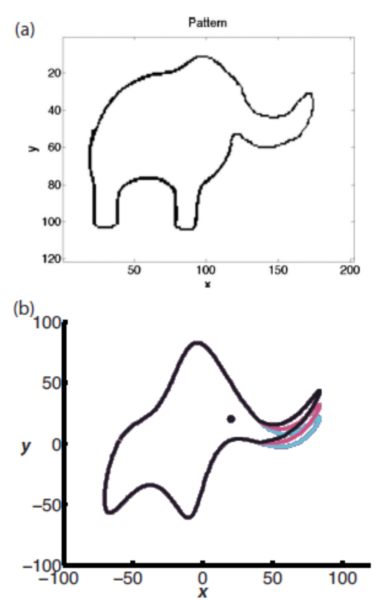


Fig. 1. (a) Outline of an elephant. (b) Three snapshots of the wiggling trunk.

Table I. The five complex parameters $p_1, \ldots, p_5$ that encode the elephant including its wiggling trunk.

Parameter	Real part	Imaginary part
$p_1 = 50 - 30i$	$B_1^x = 50$	$B_1^y = -30$
$p_2 = 18 + 8i$	$B_2^x = 18$	$B_2^y = 8$
$p_3 = 12 - 10i$	$A_3^x = 12$	$B_3^y = -10$
$p_4 = -14 - 60i$	$A_5^x = -14$	$A_1^y = -60$
$p_5 = 40 + 20i$	Wiggle coeff. $= 40$	$x_{eye} = y_{eye} = 20$

The real part of the fifth parameter is the "wiggle parameter," which determines the x-value where the trunk is attached to the body (see the video in Ref. 8). Its imaginary part is used to make the shape more animal-like by fixing the coordinates for the elephant's eye. All the parameters are specified in Table I.

The resulting shape is schematic and cartoonlike but is still recognizable as an elephant. Although the use of the Fourier coordinate expansion is not new,[2,3] our approach clearly demonstrates its usefulness in reducing the number of parameters needed to describe a two-dimensional contour. In the special case of fitting an elephant, it is even possible to lower it to four complex parameters and therein implement a well-known saying.

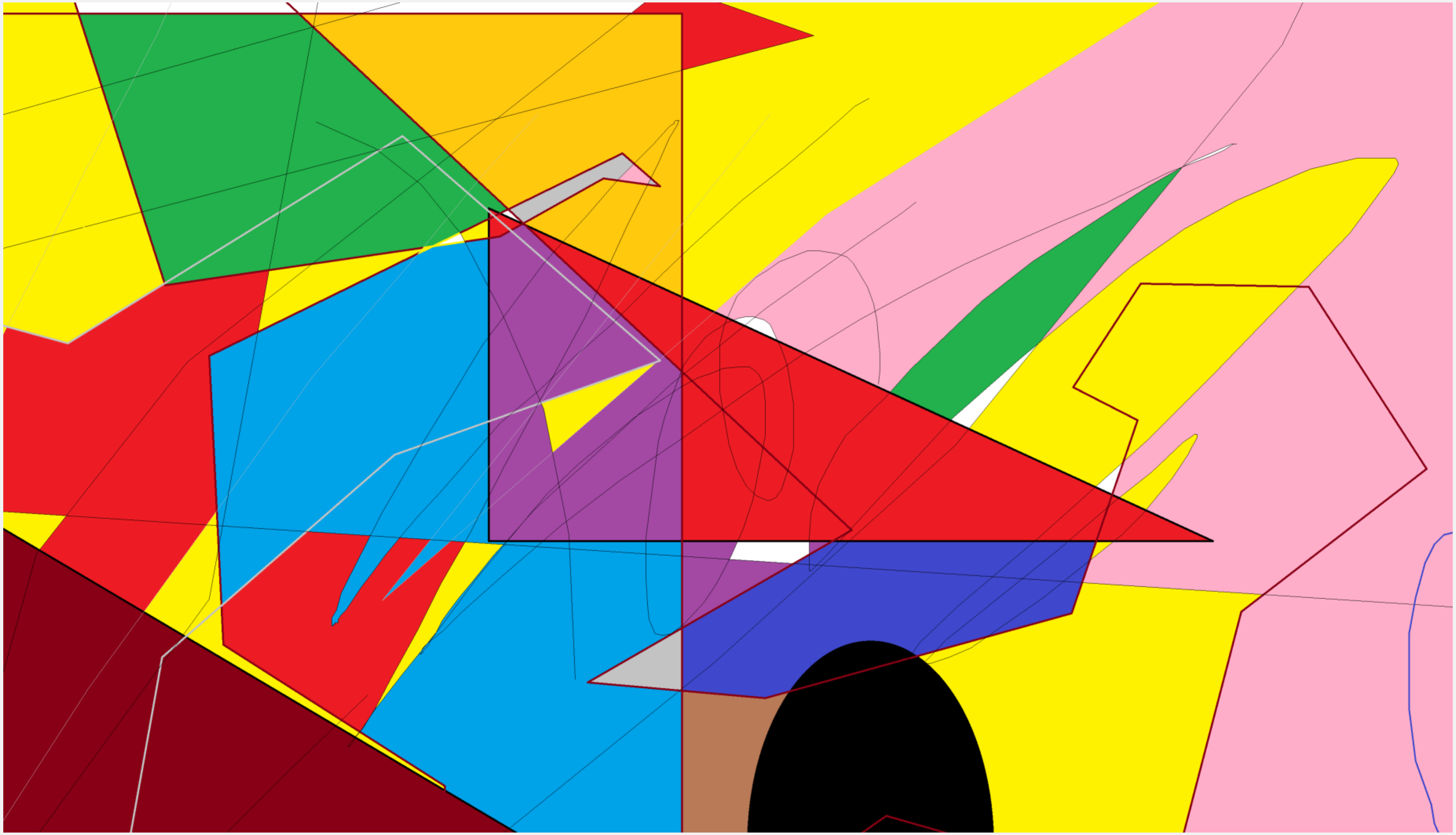
ACKNOWLEDGMENTS

Many thanks to Jean-Yves Tinevez and Marija Žanić, as well as the anonymous reviewers, for revising and improving this article.

[1] F. Dyson, "A meeting with Enrico Fermi," Nature (London) **427**(6972), 297 (2004).
[2] F. P. Kuhl and C. R. Giardina, "Elliptic Fourier features of a closed contour," Comput. Graph. Image Process. **18**, 236–258 (1982).
[3] G. Székely, A. Kelemen, C. Brechbühler, and G. Gerig, "Segmentation of 2D and 3D objects from MRI volume data using constrained elastic deformations of flexible Fourier contour and surface models," Med. Image Anal. **1**(1), 19–34 (1996).
[4] K. Khairy and J. Howard, "Spherical harmonics-based parametric deconvolution of 3D surface images using bending energy minimization," Med. Image Anal. **12**(2), 217–227 (2008).
[5] The interactive Java applet written by Rosemary Kennett, ⟨physics.syr.edu/courses/java/demos/kennett/Epicycle/Epicycle.html⟩.
[6] Interactive Java applet of elliptic descriptors by F. Puente León, ⟨www.vms.ei.tum.de/lehre/vms/fourier/⟩.
[7] J. Wei, "Least square fitting of an elephant," CHEMTECH **5**, 128–129 (1975).
[8] See supplementary material at http://dx.doi.org/10.1119/1.3254017 for the movie showing the wiggling trunk.

J. Mayer, K. Khairy and J. Howard, American Journal of Physics 78, 648–649 (2010)

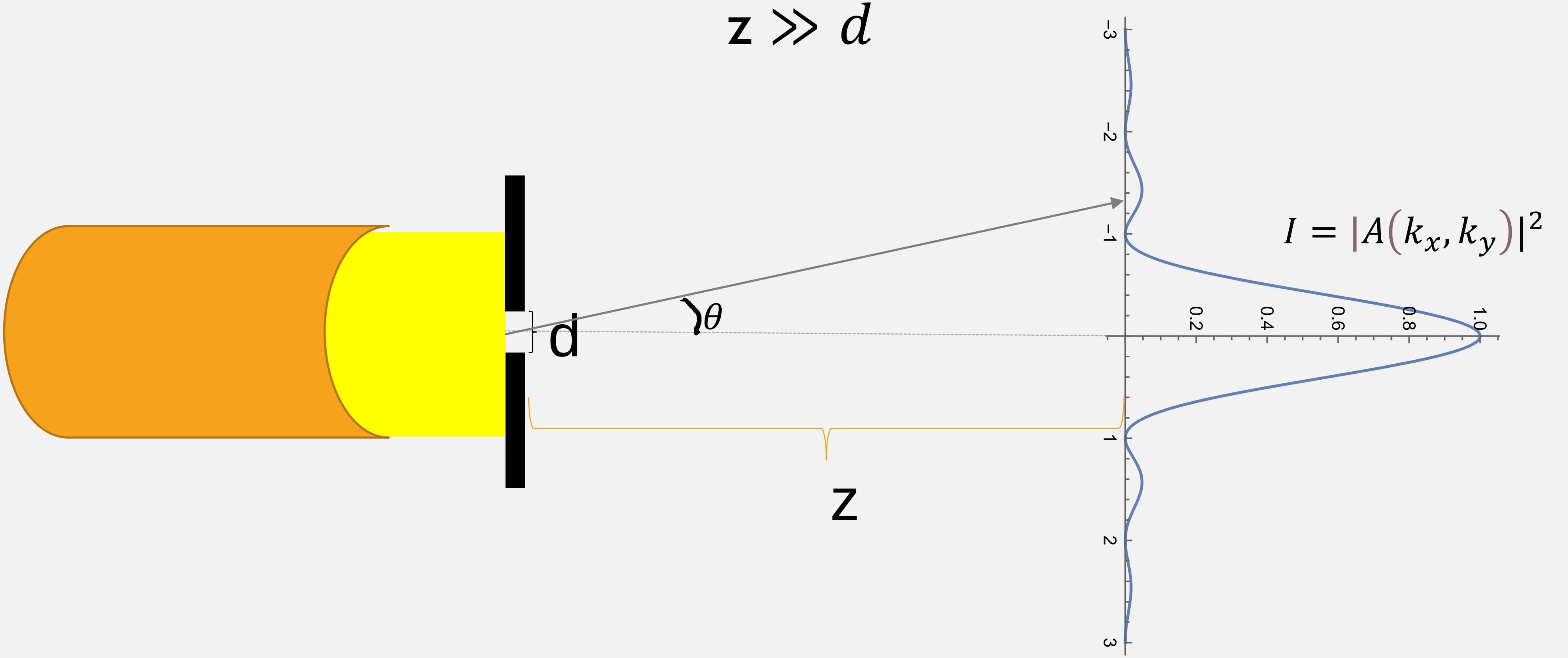
What if we could study this problem through a different point of view?



Fraunhofer diffraction

$$A(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_i(x,y)t_M(x,y)e^{-i(k_x x+k_y y)}\,dx\,dy$$

$$z \gg d$$



Our initial motivation

A Pomeron-like amplitude is $T(s,t) \sim i g_a(t) g_b(t) e^{[\alpha_P(t)-1]\ln s}$

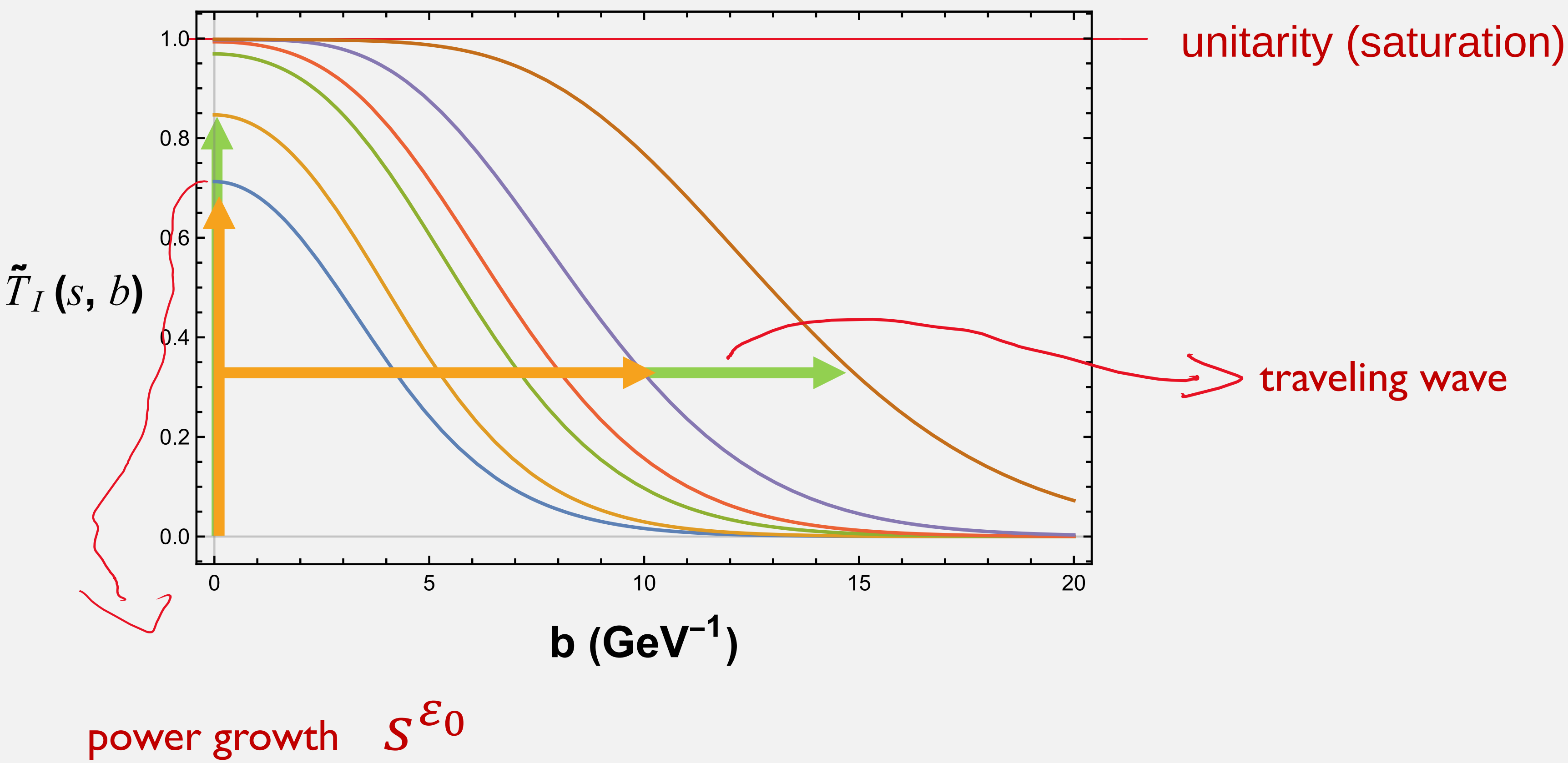
Fourier transforming it to b-space $\tilde{T}(s,b) = \mathrm{i}\int \mathrm{d}^2 q\, \mathrm{e}^{iq\cdot b} T(s,q^2)$

If $g_a(t)$ and $g_b(t)$ do not vary much on t $\tilde{T}(s,b) \sim \tilde{g}_a\, \tilde{g}_b\, s^{\varepsilon_0} \dfrac{\mathrm{e}^{-\frac{b^2}{4\alpha' \ln s}}}{2\alpha' \ln s}$

It is trivial to show the identity $\left(\dfrac{\partial}{\partial\tau} - \nabla_b^2\right)\tilde{T}(s,b) = 0$ where $\nabla_b^2 = \partial_b^2 + \dfrac{1}{b}\partial_b$

But diffusion alone would not take into account the unitarity saturation and asymptoticaly the Froissart bound would be violated.

We need non-linear terms.



But diffusion alone would not take into account the unitarity saturation and asymptoticaly the Froissart bound would be violated.

We need non-linear terms.

Which framework contained already these tools?

Regge Field Theory

Reggeons propagate in two space dimensions $\vec{x}$ and imaginary time τ

The action for a free Pomeron field is written

$$A_0 = \int d^2\vec{x}\ \mathrm{d}\tau\, \mathcal{L}_0(\vec{x},\tau)$$

with the free Lagrangian

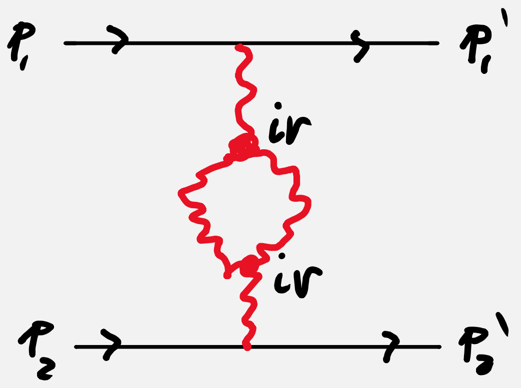
$$\mathcal{L}_0(\vec{x},\tau) = \frac{1}{2}\varphi^+(\vec{x},\tau)\frac{\overleftrightarrow{\partial}}{\partial\tau}\varphi(\vec{x},\tau) - \alpha_0'\nabla\varphi^+(\vec{x},\tau)\cdot\nabla\varphi(\vec{x},\tau) - \varepsilon_0\varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau)$$

The interecting part is written in terms of triple Pomeron coupling

$$\mathcal{L}_I = -i\lambda[\varphi^+(\vec{x},\tau)\varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau) + \varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau)\varphi(\vec{x},\tau)]$$

H. D. Abarbanel, J. D. Bronzan, R. L. Sugar, and A. R. White, Physics Reports 21, 119 (1975).

In RFT the imaginary time τ is the rapidity $\tau = \ln(s)$



Typical graph contributing to the amplitude

To avoid the imaginary term one can transform the Gribov fields such as $q = \mathrm{i}\varphi^{+}$ and $p = \mathrm{i}\varphi$

$$\mathcal{L} = \frac{1}{2} q \overleftrightarrow{\partial_\tau} p + \alpha' \nabla_b q \cdot \nabla_b p - \varepsilon_0 q p + \lambda q (p + q) p$$

The Hamiltonian is given

$$H = \int d^2 b \, [-\alpha' \nabla_b q(b) \cdot \nabla_b p(b) + \varepsilon_0 q(b) p(b) - \lambda q(b)[p(b) + q(b)] p(b)]$$

Where *q* and *p* are creation and anihillation operators respectively satisfiying the comutation relation $[p(b,\tau), q(b',\tau)] = -\delta^{(2)}(b - b')$

In discretized two-dimensional *b*-space lattice it was shown that for $\varepsilon_0 > 0$ the zero-energy ground state $|\phi_O >$ acquires a non-zero energy state $|\phi_1 >$ which approaches a coherent state

$$|\phi_1 >= e^{-\frac{\varepsilon_0}{\lambda} \int q(b,0)\, \mathrm{d}b} |\phi_0 > \qquad \text{such that} \qquad p(b,0)|\phi_1 >= \frac{\varepsilon_0}{\lambda} |\phi_1 >$$

A generalized state is written as

$$|\psi(\tau) >= e^{-\hat{A}(\tau)} |\phi_O >$$

With the operator

n+1 correlation functions

$$\hat{A}(\tau) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \mathrm{d}^2 \overrightarrow{b_1} \ldots d^2 \overrightarrow{b_n} \; q(\overrightarrow{b_1}) \cdots q(\overrightarrow{b_n}) \; G_n(\tau, \bar{b}_1, \ldots, \vec{b}_n)$$

V. Alessandrini, D. Amati, and M. Ciafaloni, Nucl. Phys. B 130, 429 (1977)

Expanding $|\psi> = e^{-\hat{A}(\tau)}|\phi_0> = (1 - \hat{A} + \frac{1}{2!}\hat{A}^2 + \cdots)\,|\phi_0>$

And solving the Schrodinger equation $\partial_\tau|\psi>= -H|\psi>$

By colecting powers of q such as $\int \mathrm{d}^2b\, q(b), \int \mathrm{d}^2b\, q(b)q(b)$ ...

One arrive to as set of coupled differential equations

$$\partial_\tau G_1(b,\tau) = \left(\varepsilon_0 + \alpha'\nabla_b^2\right)G_1(b,\tau) - \lambda G_1^2(b,\tau) - \lambda G_2(b,b,\tau)$$

$$\partial_\tau G_2(b,b',\tau) = \left(\varepsilon_0 + \alpha'\nabla_b^2\right)G_2(b,b',\tau) + \left(\varepsilon_0 + \alpha'\nabla_{b'}^2\right)G_2(b,b',\tau) - 2\lambda\,\delta^2(b-b')G_1(b,\tau)$$
$$-2\lambda[G_1(b,\tau) + G_1(b',\tau)]G_2(b,b',\tau) - 2\lambda G_3(b,b,b',\tau)$$

.

.

In the semiclassical approximation $\partial_\tau G_1(b,\tau) = \left(\varepsilon_0 + \alpha'\nabla_b^2\right)G_1(b,\tau) - \lambda G_1^2(b,\tau)$

Assumption of our approach:

2-POINT CORRELATION FUNCTION $\propto$ ELASTIC SCATTERING AMPLITUDE

$$G_1(b,\tau) \propto \mathrm{i}\tilde{T}(b,\tau)$$

We found similar ideias *R. Peschanski, Phys. Rev. D 79, 105014 (2009).*
The author explore the analytical properties of the imaginary amplitude with a noise term

We are focused in the complex equation $T(s,t) = T_R(s,t) + \mathrm{i}T_I(s,t)$

In b-space $\tilde{T}(s,\vec{b}) = \int \mathrm{d}^2\vec{q}\, \mathrm{e}^{-\mathrm{i}\vec{b}\cdot\vec{q}}\, T(s,-q^2)$

$$\partial_\tau(i\tilde{T}(b,\tau)) = \left(\varepsilon_0 + \alpha'\nabla_b^2\right)(i\tilde{T}(b,\tau)) - \lambda\left(i\tilde{T}_I(b,\tau)\right)^2$$

The imaginary and real parts respectively are *H. Kakkad, A. K. K. and P. Kotko, Eur. Phys. J. C* 82 (2022) 9, 830

$$\left.\begin{aligned}\frac{\partial\tilde{T}_I}{\partial\tau} &= \alpha'\frac{\partial^2\tilde{T}_I}{\partial b^2} + \varepsilon_0\left[\tilde{T}_I\left(1-\frac{\lambda}{\varepsilon_0}\tilde{T}_I\right) + \frac{\lambda}{\varepsilon_0}\tilde{T}_R{}^2\right]\\ \frac{\partial\tilde{T}_R}{\partial\tau} &= \alpha'\frac{\partial^2\tilde{T}_R}{\partial b^2} + \varepsilon_0\tilde{T}_R\left(1-2\frac{\lambda}{\varepsilon_0}\tilde{T}_I\right)\end{aligned}\right\}$$

EVOLUTION EQUATION FOR COMPLEX AMPLITUDES

They look like BK type equations:

S. Munier and R. B. Peschanski, Phys. Rev. Lett. 91, 232001 (2003)

Y. V. Kovchegov, L. Szymanowski, and S. Wallon, Phys. Lett. B 586, 267 (2004),

To evolve the evolution equation for amplitudes we use as initial conditions two different models with analytical forms in b-space: KFK and BSW

A. K. K., E. Ferreira, and T. Kodama, Eur. Phys. J. C 74, 3175 (2014)

C. Bourrely, J. Soffer, and T. T. Wu, Phys. Rev. D 19, 3249 (1979)

At first our equation was aimed for high energies, so we start at $\tau = \ln\frac{500GeV}{1GeV}$

Birth and death statistical processes

L. Peliti, J. Phys. (Paris) 46, 1469 (1985)

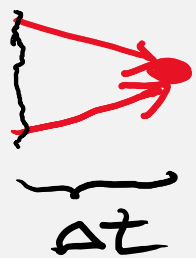
L. Pechenik, H. Levine, Phys. Rev. E 59, 3893 (1999)

Consider a statistical model with the following possibilitites:

Particle spliting

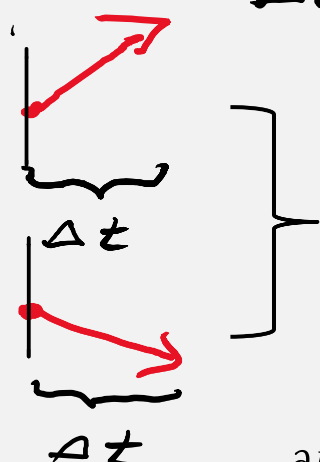
$$\left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\varepsilon_0} = \varepsilon_0\left[(n_i-1)P(\dots,n_i-1,\dots;t) - n_i P(\dots,n_i;t)\right]$$

Two particles coalescing



$$\left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\lambda} = \lambda\left[(n_i+1)n_i P(\dots,n_i+1,\dots;t) - n_i(n_i-1)P(\dots,n_i,..;t)\right]$$

Particle diffusing or



$$\left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\alpha'} = \alpha'\sum_{\mathrm{e}}\left[(n_e+1)P(\dots,n_i-1,n_e+1,\dots;t) - n_i P(\dots,n_i,..;t)\right]$$

$$\left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{tot} = \sum_{\mathrm{i}} \left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\varepsilon_0} + \left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\lambda} + \left.\frac{\partial P(\{n_i\};t)}{\partial t}\right|_{\alpha'}$$

creation and annihilation operators $\hat{a}|\ldots, n_i \ \ldots\rangle = n_i|\ldots, n_i - 1, \ldots\rangle$ with $[\hat{a}_i, \hat{a}_j^+] = \delta_{ij}$

$$\hat{a}^+|\ldots, n_i \ \ldots\rangle = |\ldots, n_i + 1, \ldots\rangle$$

The initial condition for the master equation at t=0 is Poissonian $P(\{n_i\}; t=0) = \mathrm{e}^{-N_A(0)} \prod_i \frac{n_{0i}^{n_i}}{n_i!}$

with $N_A(0) = \sum_i n_{0i}$ and a multiparticle state is $|\phi(t)\rangle = \sum_{\{n_i\}} P(\{n_i\}; t)\, |\{n_i\}\rangle$

The master equation follows a Schrodinger type $\frac{\partial}{\partial t}|\phi(t)\rangle = -\hat{H}|\phi(t)\rangle$ where $\hat{H} = \sum_i \hat{H}_i$

with $|\phi(t)\rangle = \mathrm{e}^{-\hat{H}t}|\phi(0)\rangle = \mathrm{e}^{-N_A(0)}\mathrm{e}^{-\hat{H}t}e^{\sum_i \hat{a}_i^+ n_{0i}}|0\rangle$ (coherent states)

and $\hat{H} = -\alpha' \sum_e \hat{a}_i^+ (\hat{a}_e - \hat{a}_i) + \varepsilon_0[1 - \hat{a}_i^+]\hat{a}_i^+ a_i - \lambda[1 - \hat{a}_i^+]\hat{a}_i^+ \hat{a}_i^2$

The physical quantites are

$$\langle A(t)\rangle = \sum_{\{n_i\}} A(\{n_i\})P(\{n_i\}; t)$$

$$\langle A(t_0)\rangle = \frac{\int \prod_i D\bar{z}_i D z_i A(\{z_i(t_0)\})\mathrm{e}^{-S[\{\bar{z}_i(t)\},\{z_i(t)\};t_0]}}{\int \prod_i D\bar{z}_i D z_i \mathrm{e}^{-S[\{\bar{z}_i(t)\},\{z_i(t)\};t_0]}}$$

$$S[\{\bar{z}_i(t)\},\{z_i(t)\}; t_0] = \sum_i \int_0^{t_0} \left[\bar{z}_i(t)\left(\frac{d}{dt} - \alpha'\nabla^2\right) z_i(t) - \varepsilon_0\big(1 + \bar{z}_i(t)\big)\bar{z}_i(t) z_i(t) + \lambda(1 + \bar{z}_i(t))\bar{z}_i(t) z_i^2(t)\right]$$

After performing a Stratonovich transformation

$$\Longrightarrow \quad dz_i(t) = [\alpha'\nabla^2 + \varepsilon_0 - \lambda z_i(t)]z_i(t)\,\mathrm{d}t + \sqrt{2[\varepsilon_0 - \lambda z_i(\mathrm{t})]z_i(t)}\,\mathrm{d}W_i$$

$\frac{\mathrm{d}W_i}{\mathrm{d}\tau} = \eta_i$ White noise

$z_i(t) \to \tilde{T}_i(\tau, b)$

$$\frac{\mathrm{d}\tilde{T}_i}{\mathrm{d}\tau} = [\alpha'\nabla^2 + \varepsilon_0 - \lambda\tilde{T}_i]\tilde{T}_i + \sqrt{2[\varepsilon_0 - \lambda\tilde{T}_i]\tilde{T}_i}\,\eta_i$$

creation and annihilation operators $\hat{a}|\dots,n_i \dots\rangle = n_i|\dots,n_i - 1,\dots\rangle$ with $[\hat{a}_i,\hat{a}_j^+] = \delta_{ij}$

$\hat{a}^+|\dots,n_i \dots\rangle = |\dots,n_i + 1,\dots\rangle$

The initial condition for the master equation at t=0 is Poissonian $P(\{n_i\};t=0) = \mathrm{e}^{-N_A(0)}\prod_i \frac{n_{0i}^{n_i}}{n_i!}$

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and $\hat{H} = -\alpha'\sum_e \hat{a}_i^+(\hat{a}_e - \hat{a}_i) + \varepsilon_0[1-\hat{a}_i^+]\hat{a}_i^+ a_i - \lambda[1-\hat{a}_i^+]\hat{a}_i^+\hat{a}_i^2$

The physical quantites are

$$\langle A(t)\rangle = \sum_{\{n_i\}} A(\{n_i\})P(\{n_i\};t)$$

$$\langle A(t_0)\rangle = \frac{\int\prod_i D\bar{z}_i Dz_i A(\{z_i(t_0)\})\mathrm{e}^{-S[\{\bar{z}_i(t)\},\{z_i(t)\};t_0]}}{\int\prod_i D\bar{z}_i Dz_i \mathrm{e}^{-S[\{\bar{z}_i(t)\},\{z_i(t)\};t_0]}}$$

$$S[\{\bar{z}_i(t)\},\{z_i(t)\};t_0] = \sum_i \int_0^{t_0}\left[\bar{z}_i(t)\left(\frac{d}{dt} - \alpha'\nabla^2\right)z_i(t) - \varepsilon_0\big(1+\bar{z}_i(t)\big)\bar{z}_i(t)z_i(t) + \lambda(1+\bar{z}_i(t))\bar{z}_i(t)z_i^2(t)\right]$$

After performing a Stratonovich transformation

$$\Longrightarrow \quad dz_i(t) = [\alpha'\nabla^2 + \varepsilon_0 - \lambda z_i(t)]z_i(t)\,\mathrm{d}t + \sqrt{2[\varepsilon_0 - \lambda z_i(\mathrm{t})]z_i(t)}\,\mathrm{d}W_i$$

$\frac{\mathrm{d}W_i}{\mathrm{d}\tau} = \eta_i$ White noise

$z_i(t) \to \tilde{T}_i(\tau,b)$

$$\frac{\mathrm{d}\tilde{T}_i}{\mathrm{d}\tau} = [\alpha'\nabla^2 + \varepsilon_0 - \lambda\tilde{T}_i]\tilde{T}_i + \boxed{\sqrt{2[\varepsilon_0 - \lambda\tilde{T}_i]\tilde{T}_i}\,\eta_i}$$

Our first results

Initial conditions starting at $\sqrt{s_{fix}} = 500$ GeV

$$\tilde{T}_R(s_{fix}, b) = \tilde{T}_R^{KFK}(s_{fix}, b) \qquad \tilde{T}_R(s_{fix}, b) = \tilde{T}_R^{BSW}(s_{fix}, b)$$

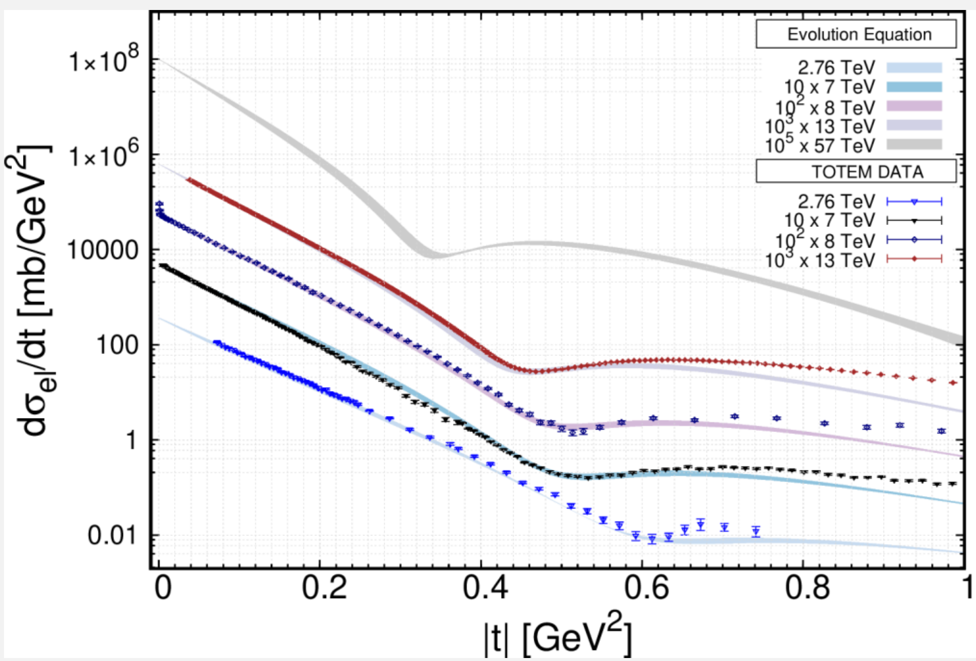
$$\tilde{T}_I(s_{fix}, b) = \tilde{T}_I^{KFK}(s_{fix}, b) \qquad \tilde{T}_I(s_{fix}, b) = \tilde{T}_I^{BSW}(s_{fix}, b)$$

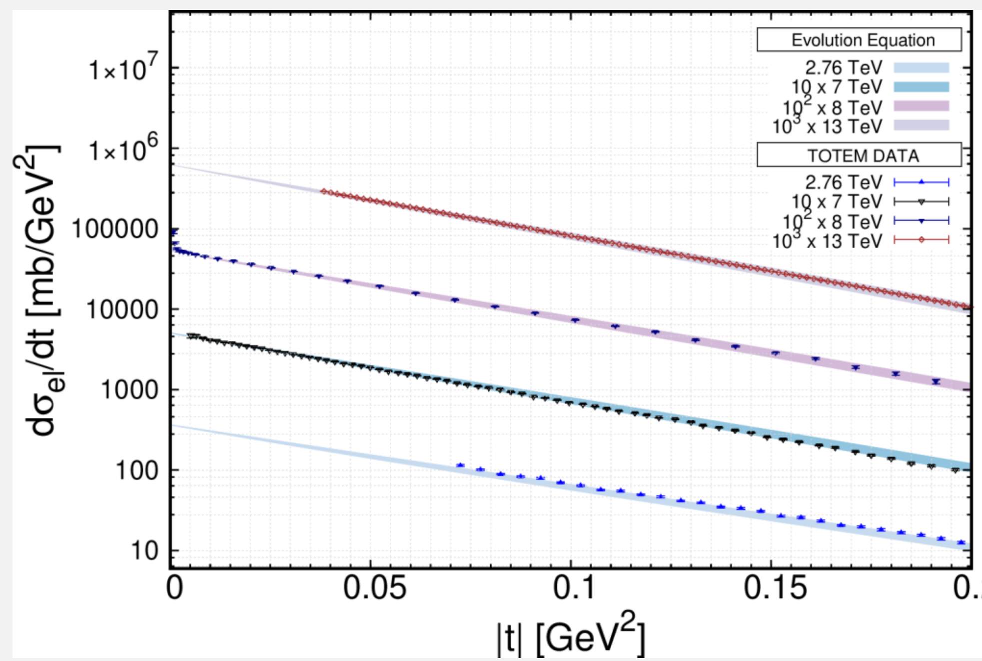
Boundary conditions set $\tilde{T}_I(s, b \to \infty) = \tilde{T}_R(s, b \to \infty) = 0$

The obtained parameters are

	α' (GeV^{-2})	ε_0	λ/ε_0
KFK	0.105	0.129	0.712
BSW	0.090	0.140	0.820

Our predictions for differential cross section



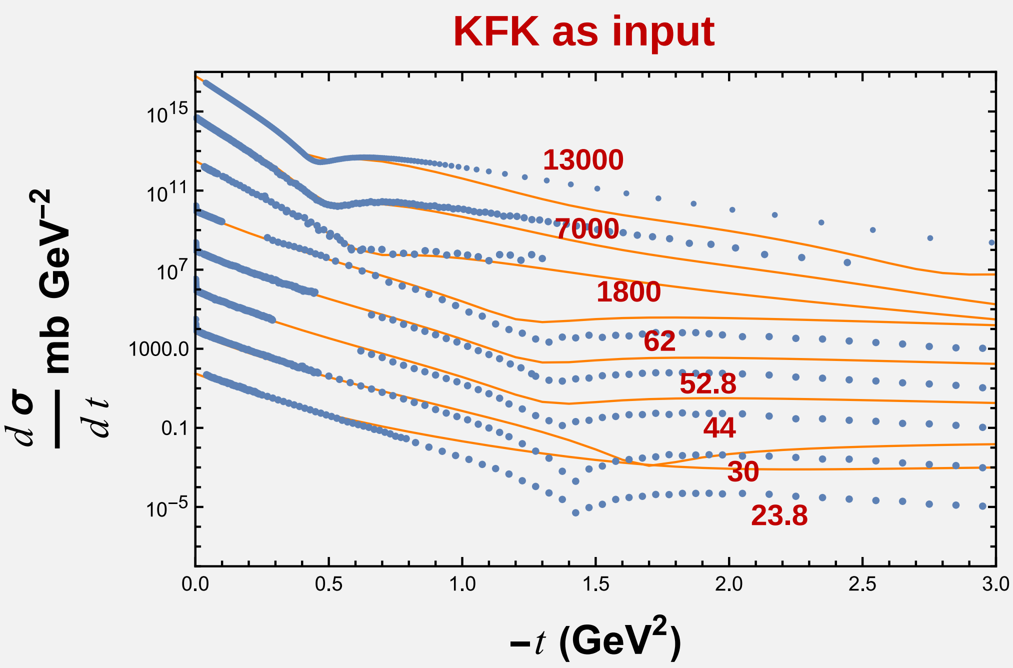


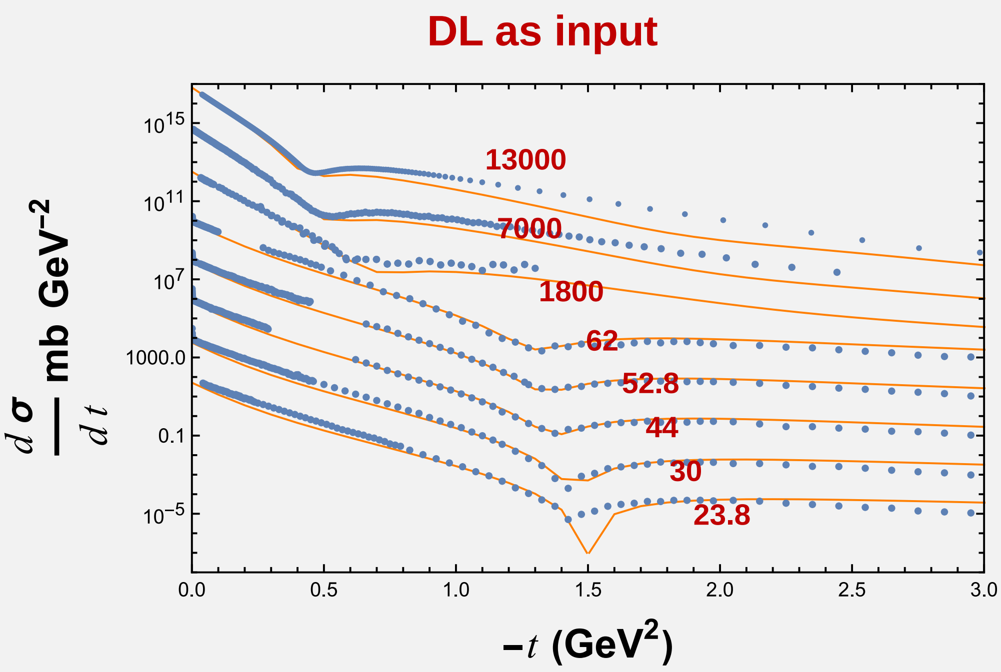
The integrated quantities are

	$\sqrt{s}$ [TeV]	σ_{tot} [mb]	ρ	B [GeV^{-2}]
KFK initial condition	2.76	84.31	0.123	17.28
	7	99.07	0.117	18.47
	8	101.32	0.116	18.65
	13	109.78	0.113	19.32
	57	138.32	0.105	21.55
BSW initial condition	2.76	83.14	0.143	19.69
	7	98.40	0.134	21.12
	8	100.73	0.132	21.34
	13	109.52	0.127	22.15
	57	142.30	0.115	24.87
TOTEM	2.76	84.7 ± 3.3	–	17.1 ± 0.30
	7	98.0 ± 2.5	0.145 ± 0.091	19.73 ± 0.40
	8	101.7 ± 2.9	0.12 ± 0.03	19.74 ± 0.28
	13	110.6 ± 3.4	0.10 ± 0.01	20.40 ± 0.01
ATLAS	7	95.35 ± 0.38	0.14 (fix)	19.73 ± 0.14
	8	96.07 ± 0.18	0.136 (fix)	19.74 ± 0.05
AUGER	57	133 ± 29	–	–

G. Antchev et al.(TOTEM Collaboration), Eur. Phys.J.C 80, 91(2020); EPL 101, 21002(2013); Eur. Phys.J.C 76, 661(2016); Eur. Phys.J.C 79, 103(2019)
G. Aad et al.(ATLAS Collaboration), Nucl. Phys.B 889, 486(2014); Phys. Lett. B 761, 158 (2016)
P. Abreu et al. (Pierre Auger Collaboration) Phys. Rev. Lett. 109, 062002 (2012)

Some problems at ISR energies





Could it be like a broken clock?

Can we reproduce ISR energies?

$\alpha' \to 0$

We have just a **complex differential logistic equation**

$$\frac{\partial \tilde{T}}{\partial \tau} = \varepsilon_0 \tilde{T}\left(1 - \frac{\lambda}{\varepsilon_0}\tilde{T}\right)$$

Where $\dfrac{\varepsilon_0}{\lambda}$ is the saturation bound and ε_0 is the growth rate

and the real and imaginary parts are

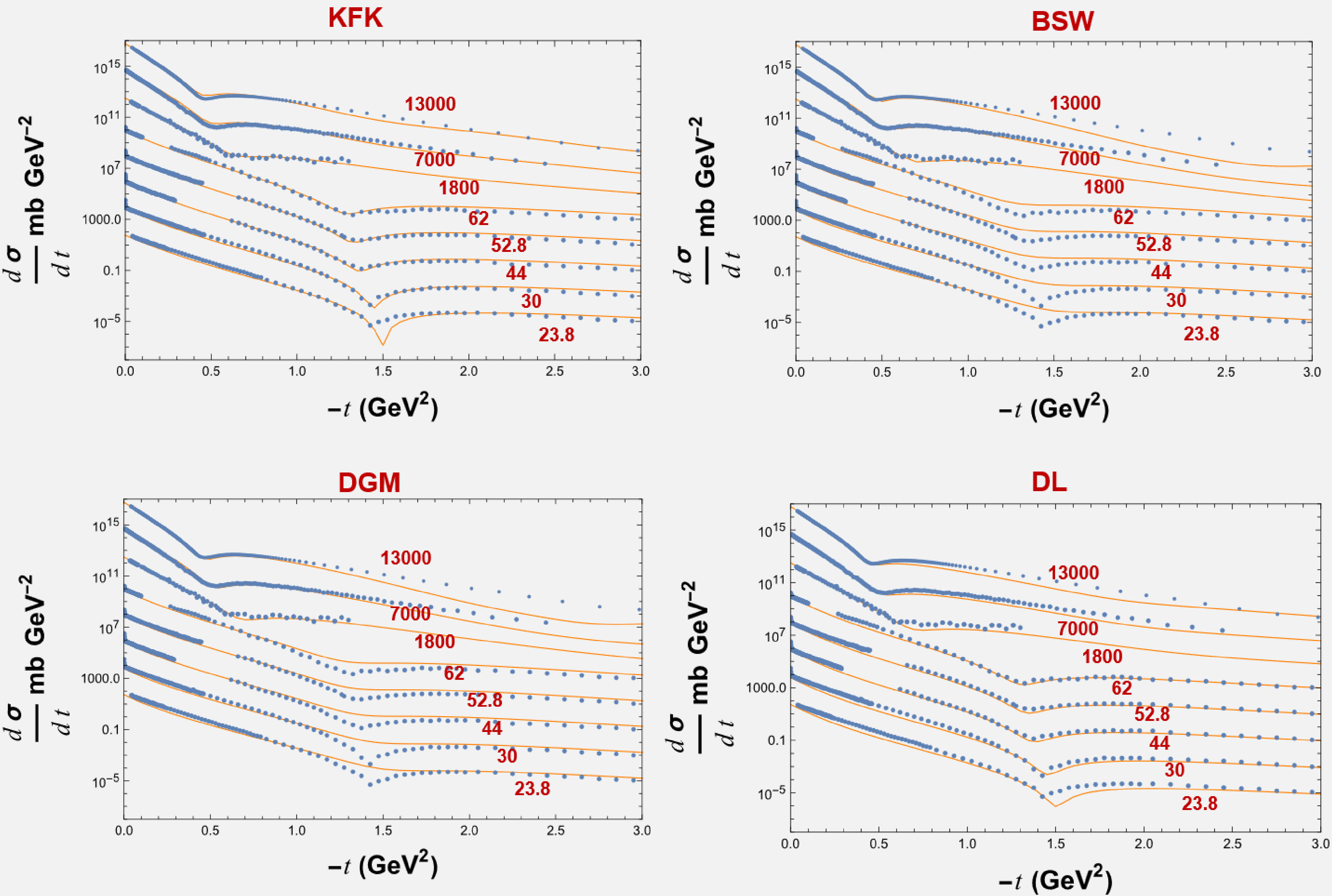
$$\frac{\partial \tilde{T}_I}{\partial \tau} = \varepsilon_0 \left[\tilde{T}_I\left(1 - \frac{\lambda}{\varepsilon_0}\tilde{T}_I\right) + \frac{\lambda}{\varepsilon_0}\tilde{T}_R{}^2\right]$$

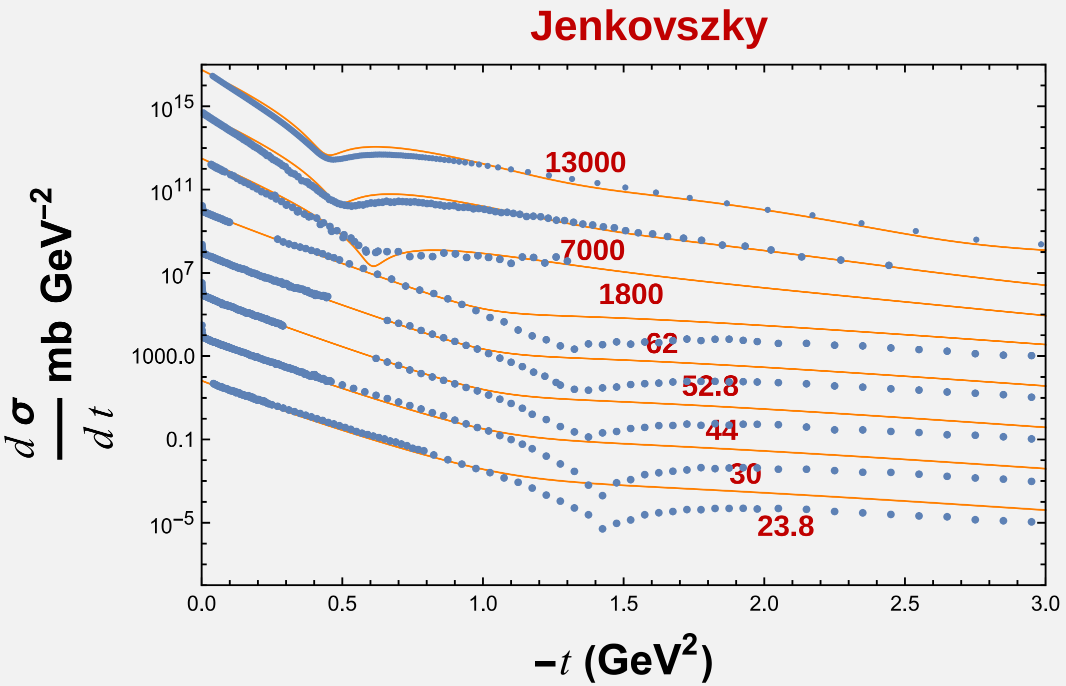
$$\frac{\partial \tilde{T}_R}{\partial \tau} = \varepsilon_0 \tilde{T}_R\left(1 - 2\frac{\lambda}{\varepsilon_0}\tilde{T}_I\right)$$

pp differential cross sections at 23, 30, 44, 52 and 62 GeV

U. Amaldi et al., Nucl.Phys.B 166 (1980) 301-320, 1980.
https://doi.org/10.17182/hepdata.7940

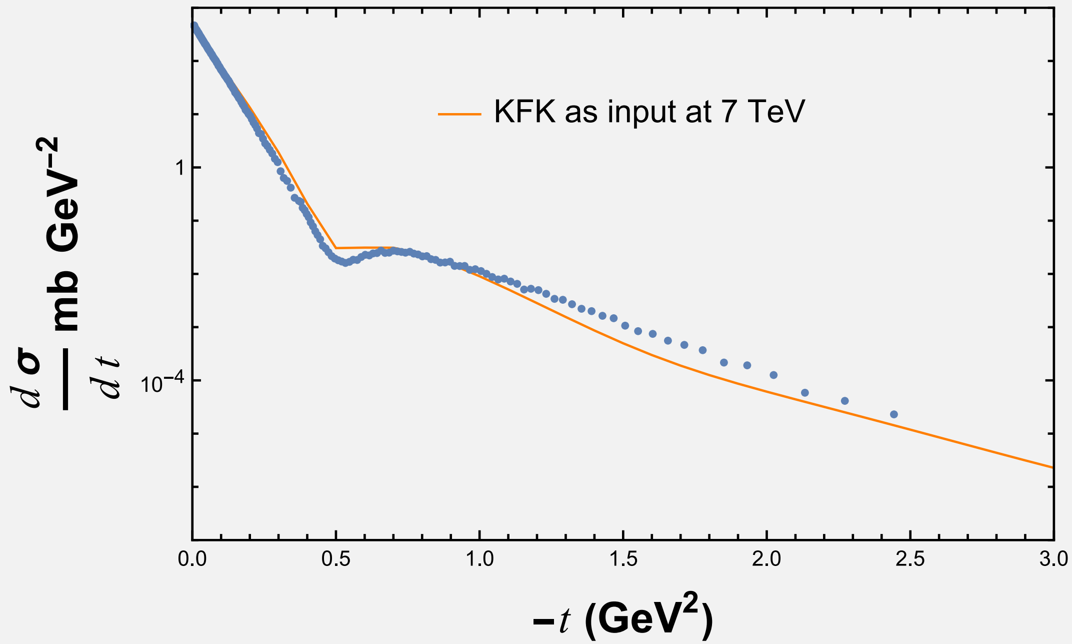
Results for $\frac{\partial \tilde{T}}{\partial \tau} = \varepsilon_0 \tilde{T} \left(1 - \frac{\lambda}{\varepsilon_0} \tilde{T} \right)$



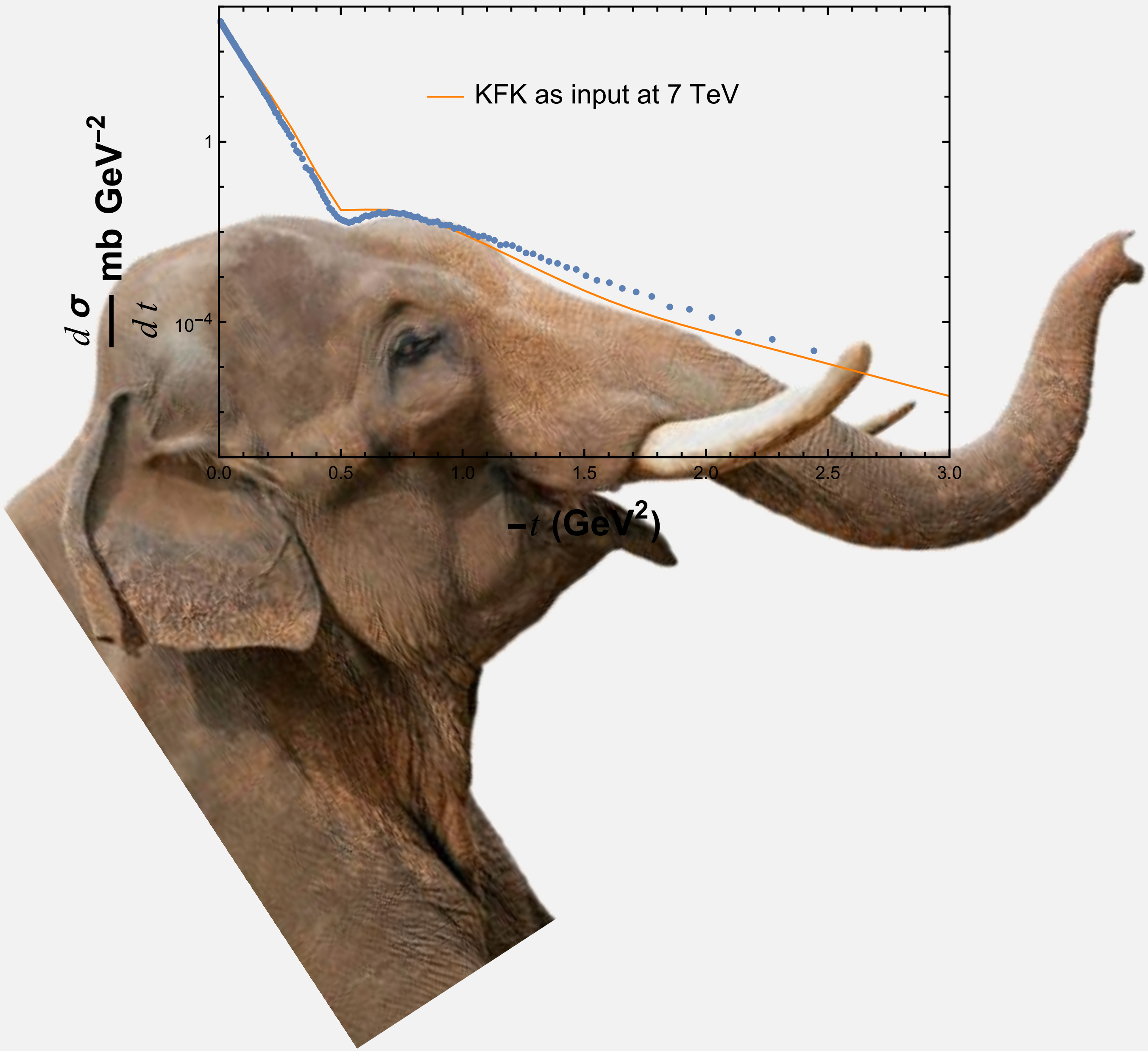


Results for $\alpha' \to 0$

I.C.	ε_0	λ
KFK	0.14	0.101
DL	0.14	0.105
BSW	0.14	0.112
DGM	0.143	0.107
Jenkovszky	0.128	0.090



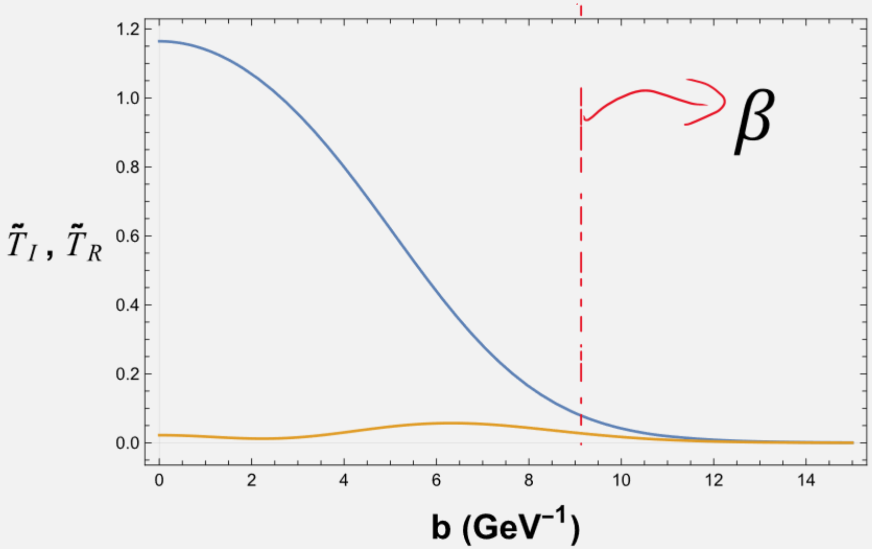
KFK as input at 7 TeV
1
10^-4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
-t (GeV^2)
dσ/dt mb GeV^-2



KFK as input at 7 TeV
1
10^-4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−t (GeV^2)
dσ/dt mb GeV^-2

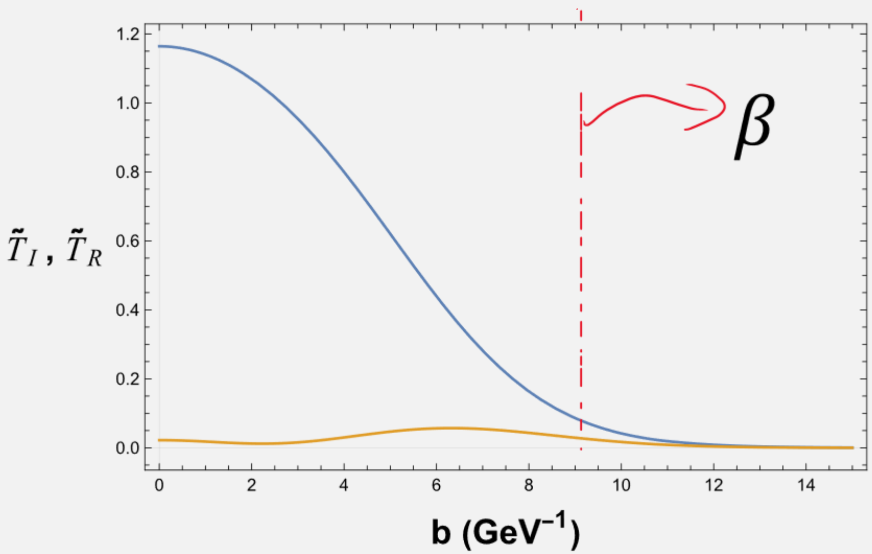
Turning on diffusion (phenomenologically)

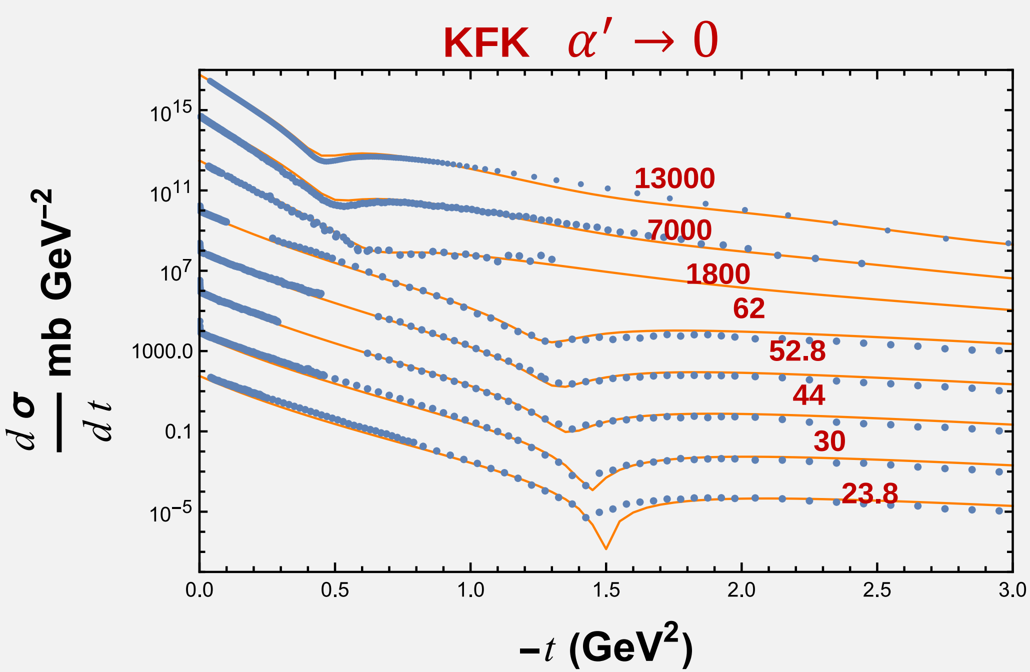
$$\frac{\partial \tilde{T}}{\partial \tau} = \alpha' \theta(b-\beta) \nabla_b^2 \tilde{T} + \varepsilon_0 \tilde{T}\left(1 - \frac{\lambda}{\varepsilon_0}\tilde{T}\right) \quad \text{with} \quad \nabla_b^2 = \partial_b^2 + \frac{1}{b}\partial_b$$

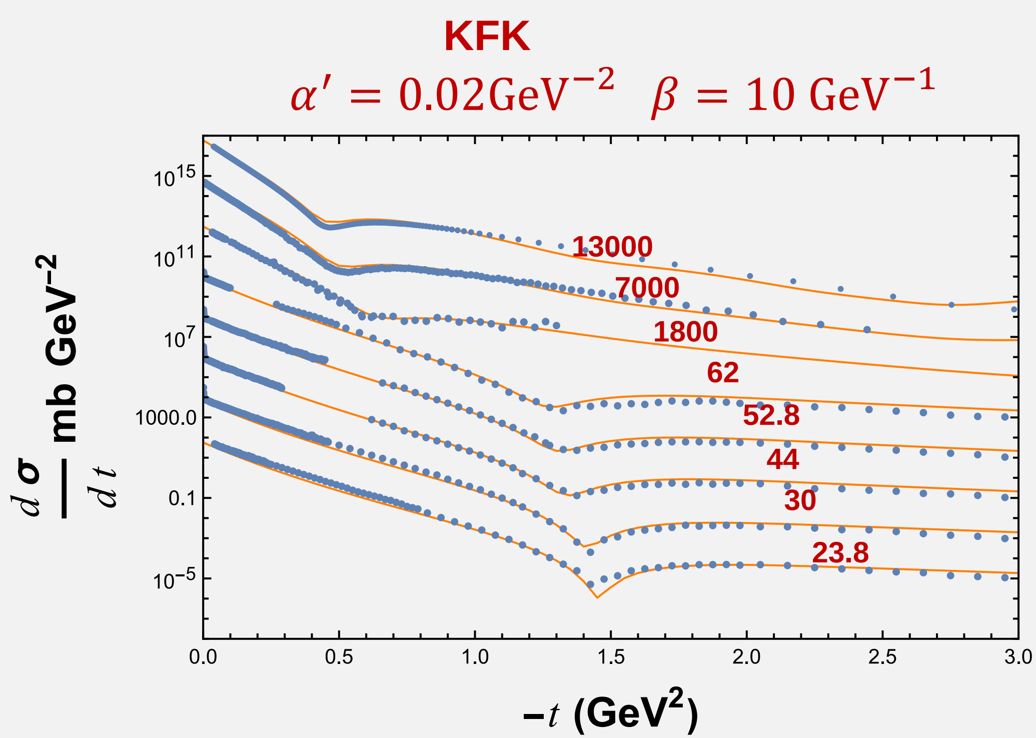


Turning on diffusion (phenomenologically)

$$\frac{\partial \tilde{T}}{\partial \tau} = \alpha' \theta(b-\beta) \nabla_b^2 \tilde{T} + \varepsilon_0 \tilde{T} \left(1 - \frac{\lambda}{\varepsilon_0} \tilde{T}\right) \quad \text{with} \quad \nabla_b^2 = \partial_b^2 + \frac{1}{b} \partial_b$$

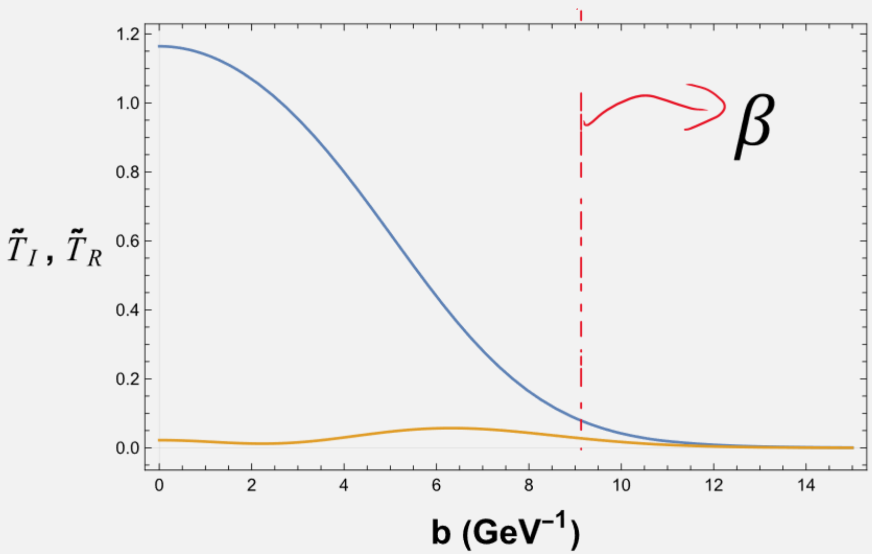




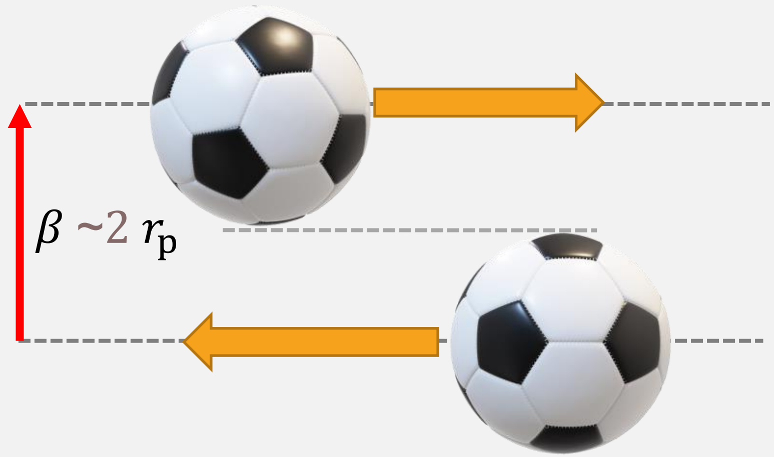


Turning on diffusion (phenomenologically)

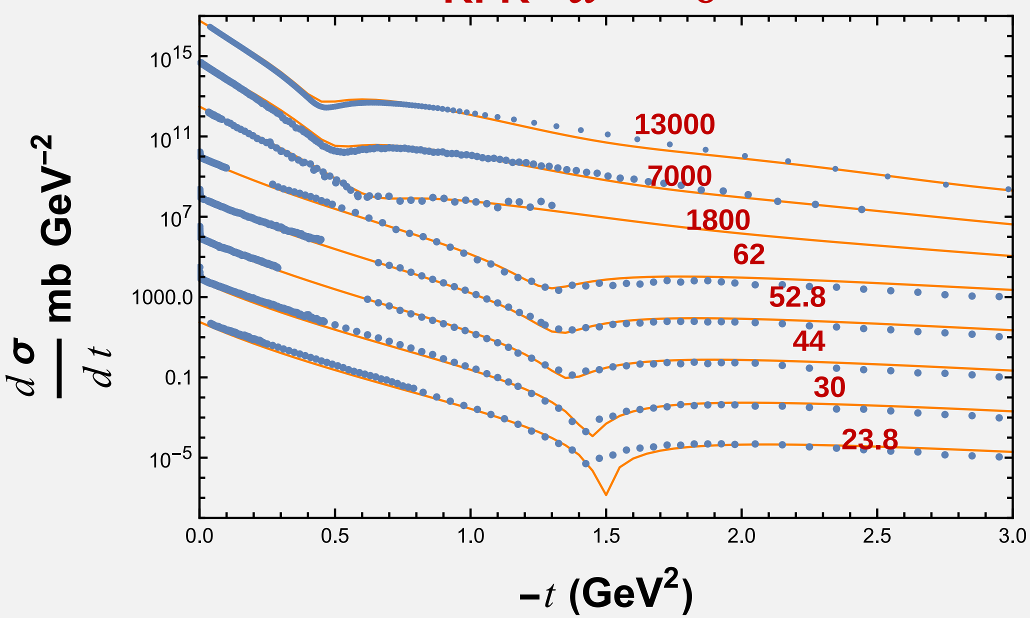
$$\frac{\partial \tilde{T}}{\partial \tau} = \alpha' \theta(b-\beta)\nabla_b^2 \tilde{T} + \varepsilon_0 \tilde{T}\left(1 - \frac{\lambda}{\varepsilon_0}\tilde{T}\right) \quad \text{with} \quad \nabla_b^2 = \partial_b^2 + \frac{1}{b}\partial_b$$



Apparently the data wants ...

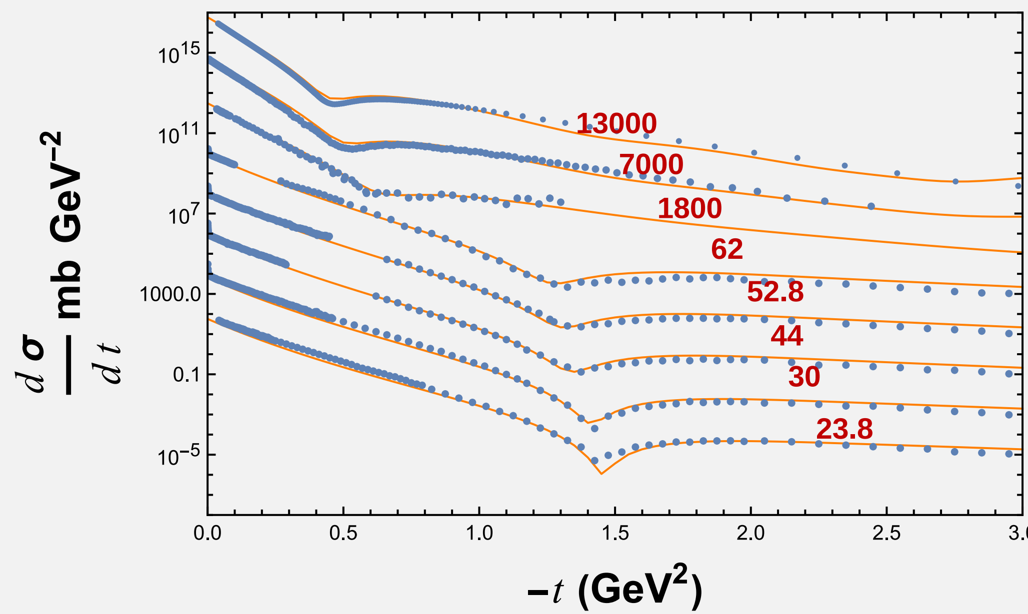


KFK $\alpha' \to 0$

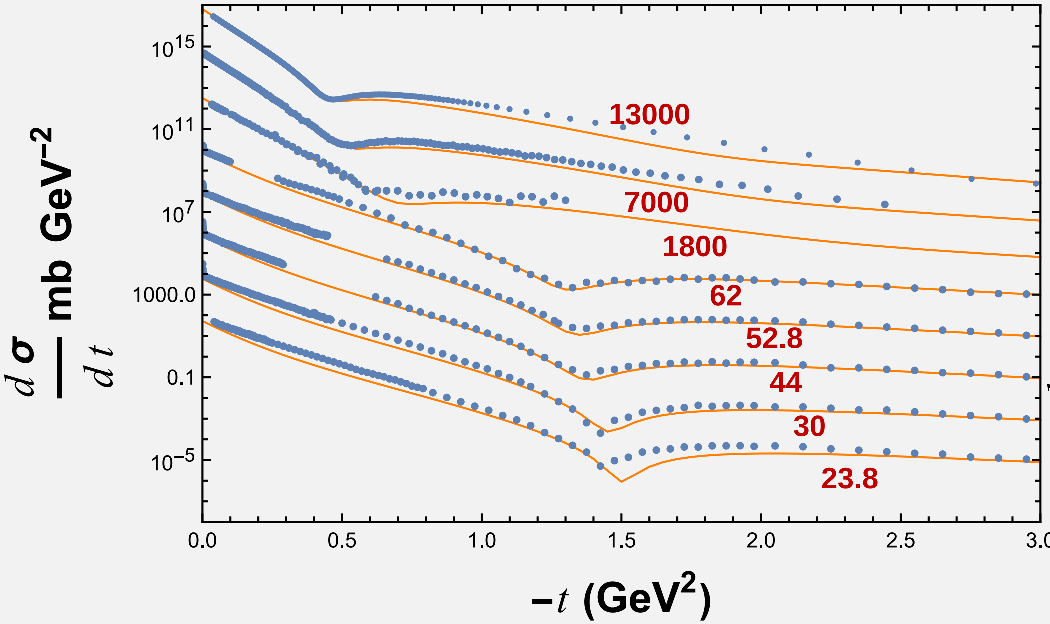


KFK

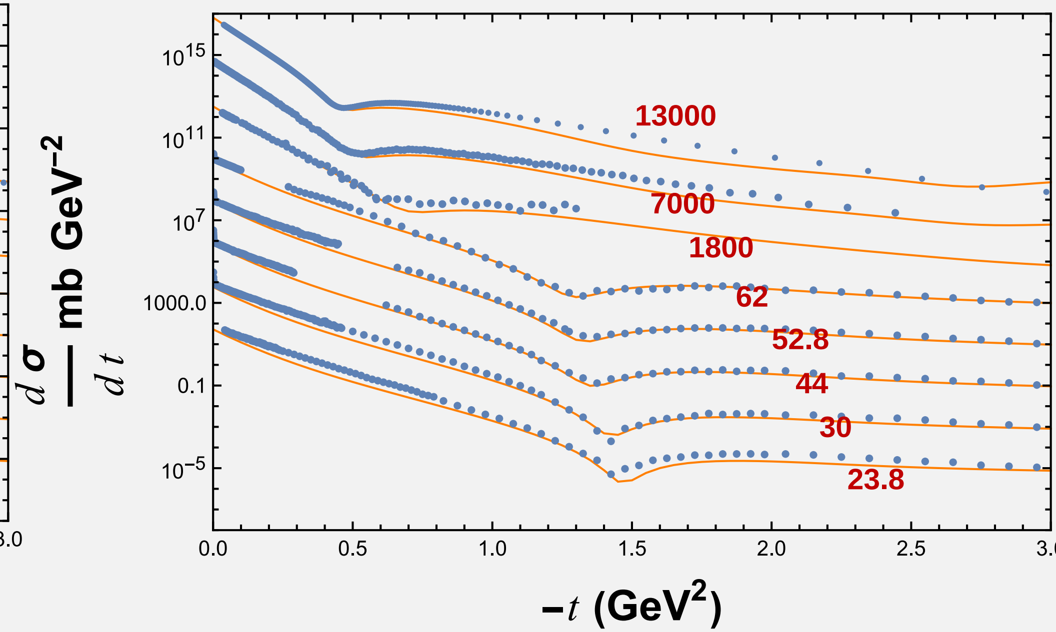
$\alpha' = 0.02\text{GeV}^{-2} \quad \beta = 10\ \text{GeV}^{-1}$



DL



DL
$\alpha' = 0.02 \quad \beta = 10$



Dip difference in crossed channel (Odderon)?

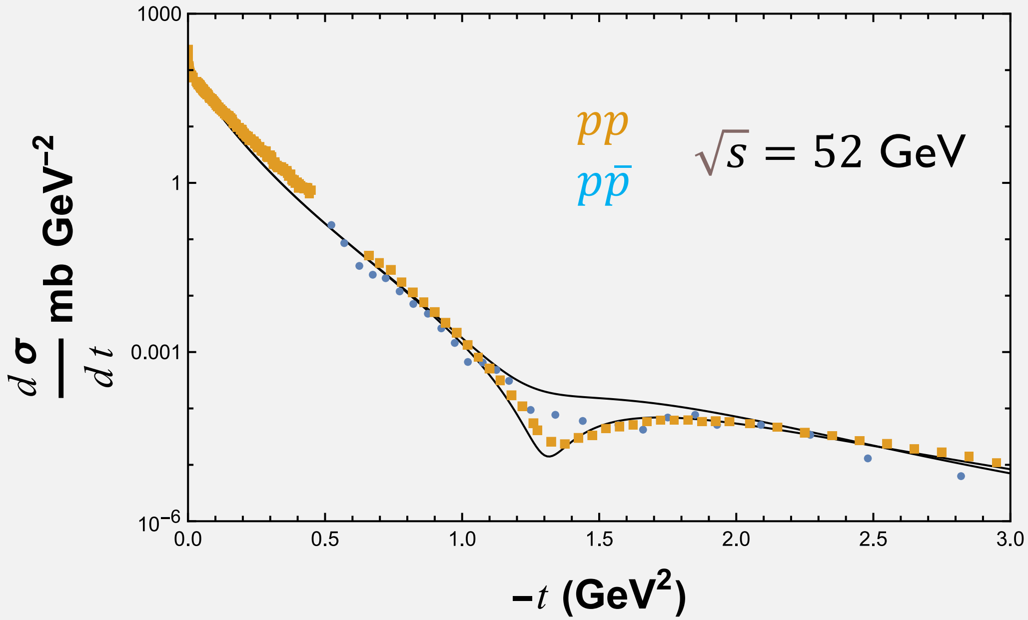
Odderon: an odd exchange in elastic scattering (changes sing for pp and $p\bar{p}$)

L. Lukaszuk, B. Nicolescu, Lett. Nuovo Cim. 8, 405 (1973)

Few energies to directly compare pp with $p\bar{p}$

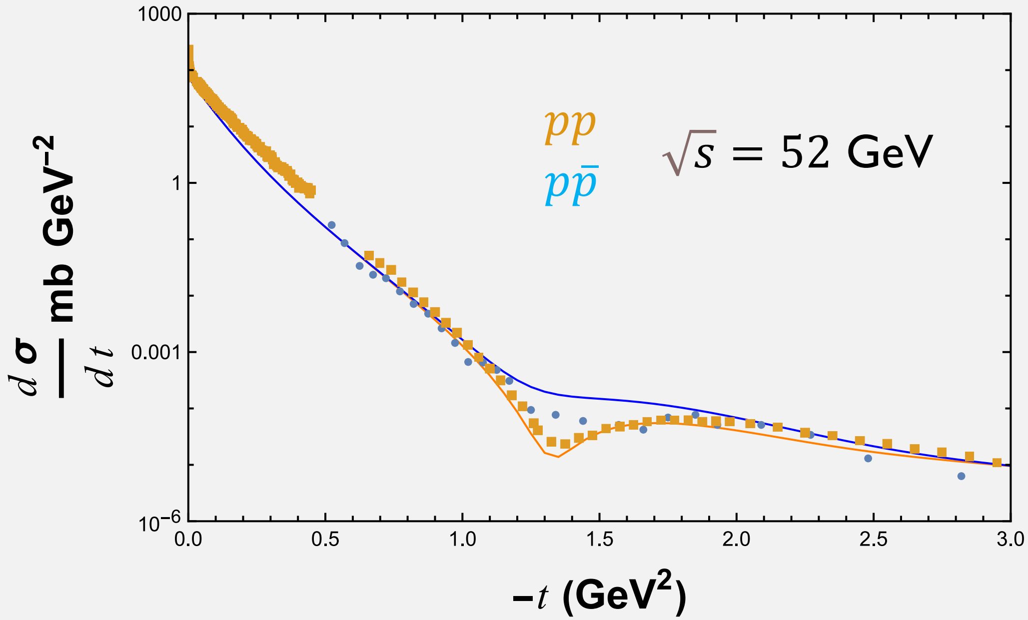
A. Breakstone et al, Phys. Rev. Lett. 54, 2180 (1985)
U. Amaldi et al, Nucl.Phys.B 166 (1980) 301-320, (1980)

DL model predictions



The dip difference due to the tri-gluon exchange

Evolution Equation with DL input

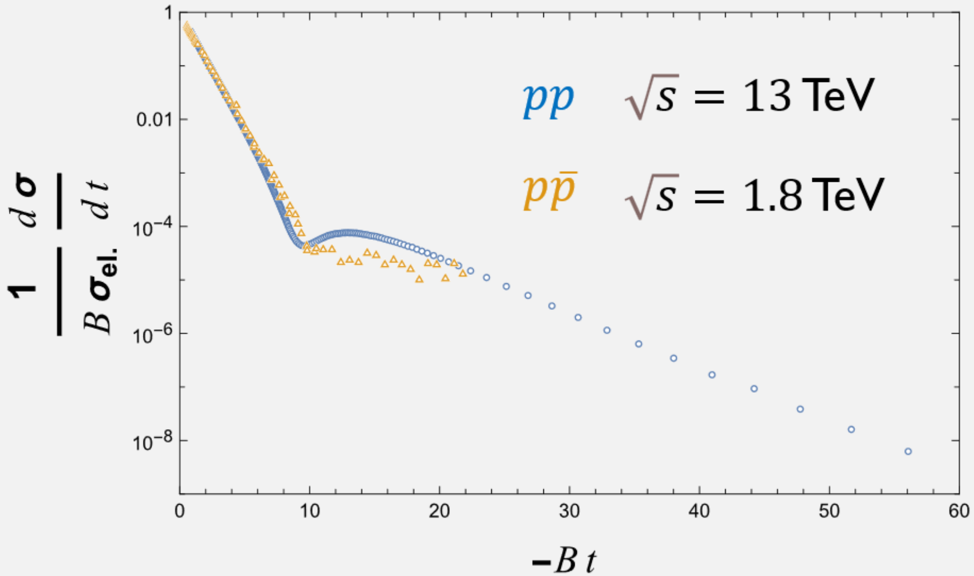


Same evolution equation, but different I.C.

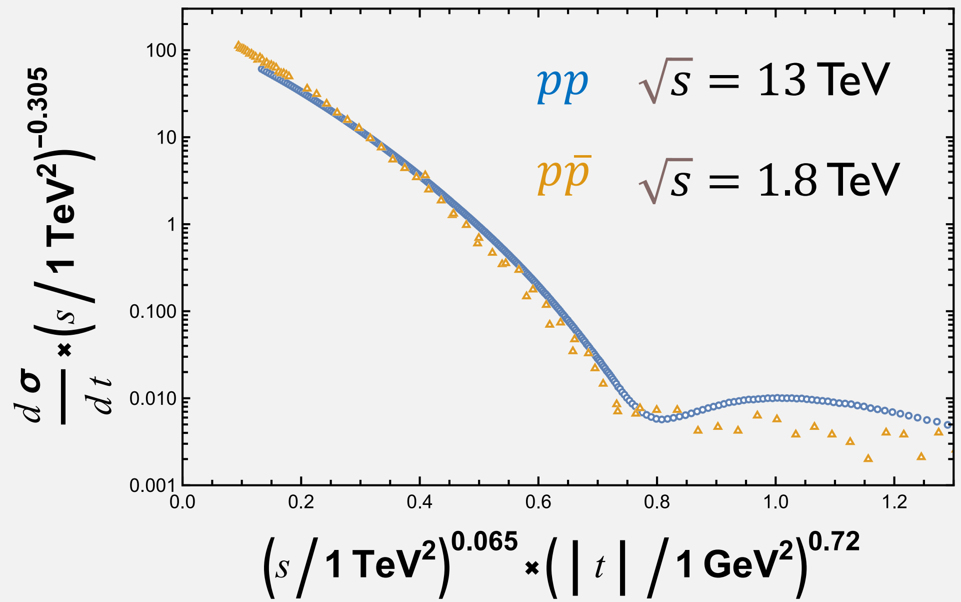
Dip difference in crossed channel (Odderon)?

Scaling at TeV energies

T. Csorgo, I. Szanyi, Eur. Phys. J. C (2021) 81:611



C. Baldenegro, C. Royon and A. M. Stasto, Phys. Lett. B 830, 137141 *(2022)*



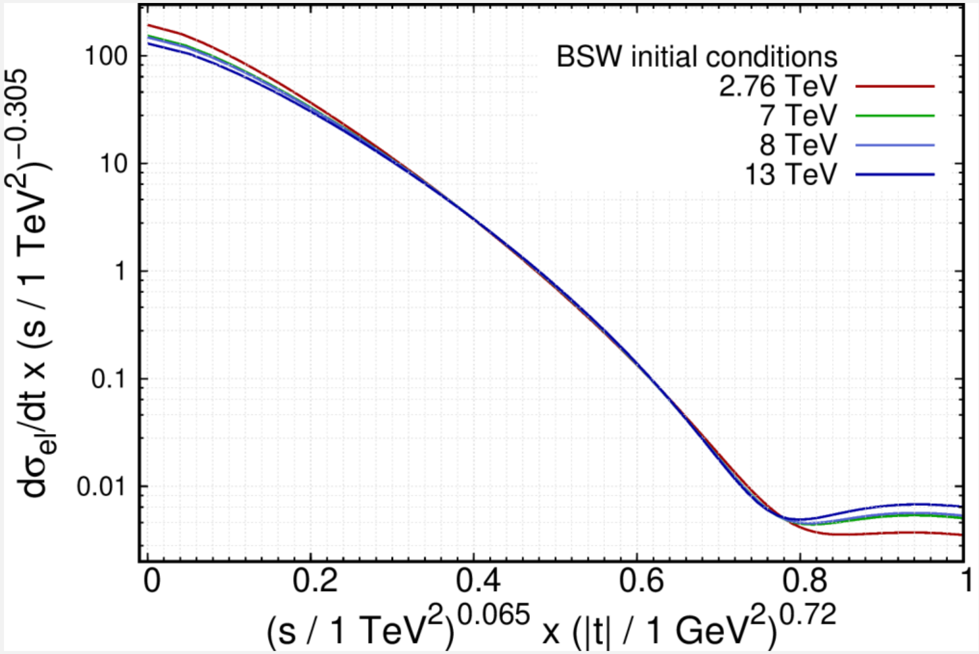
Comparing the shape of the differential cross section between crossed channel...

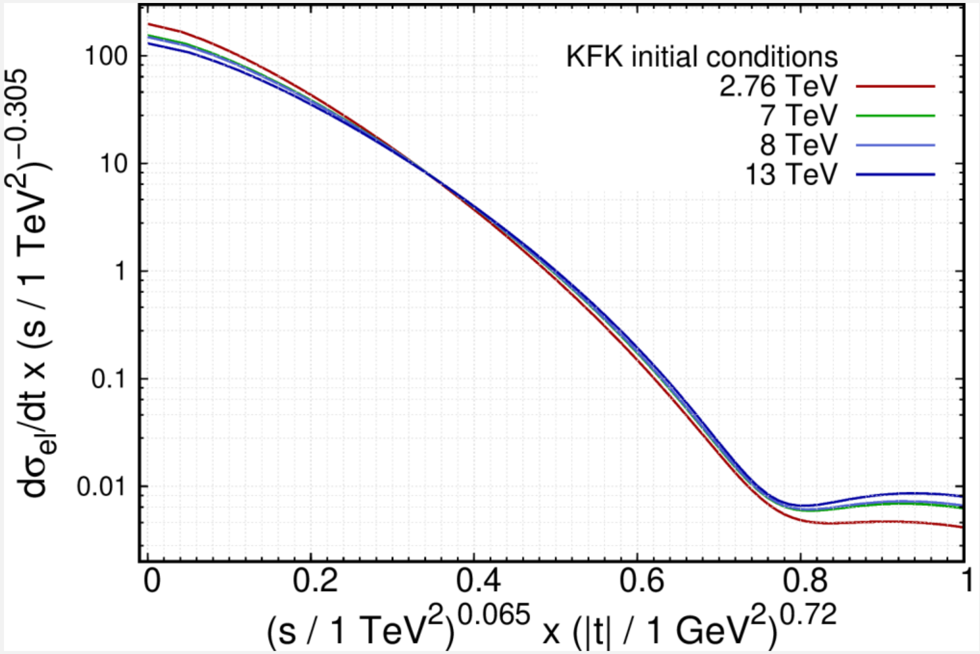
V. M. Abazov et al. (D0 and TOTEM Collaboration) Phys. Rev. Lett. 127, 062003 (2021)

Does the dip-bump difference indicate the existence of the odderon?

Differential cross section ‘acidental’ scaling behaviour among LHC energies

Our results with different initial conditions





Differential cross section ‘acidental’ scaling behaviour among LHC energies

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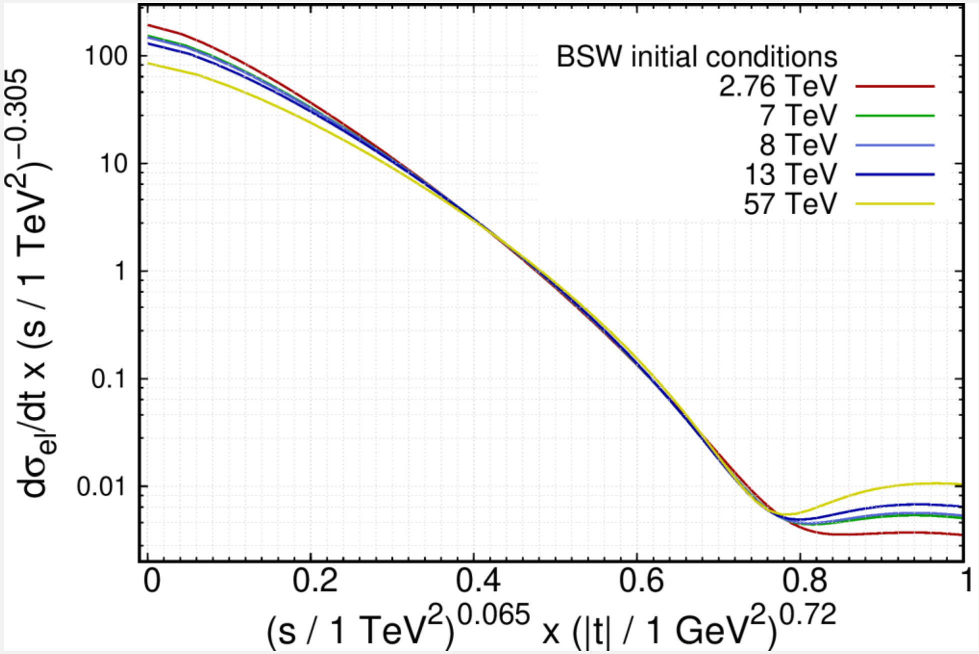
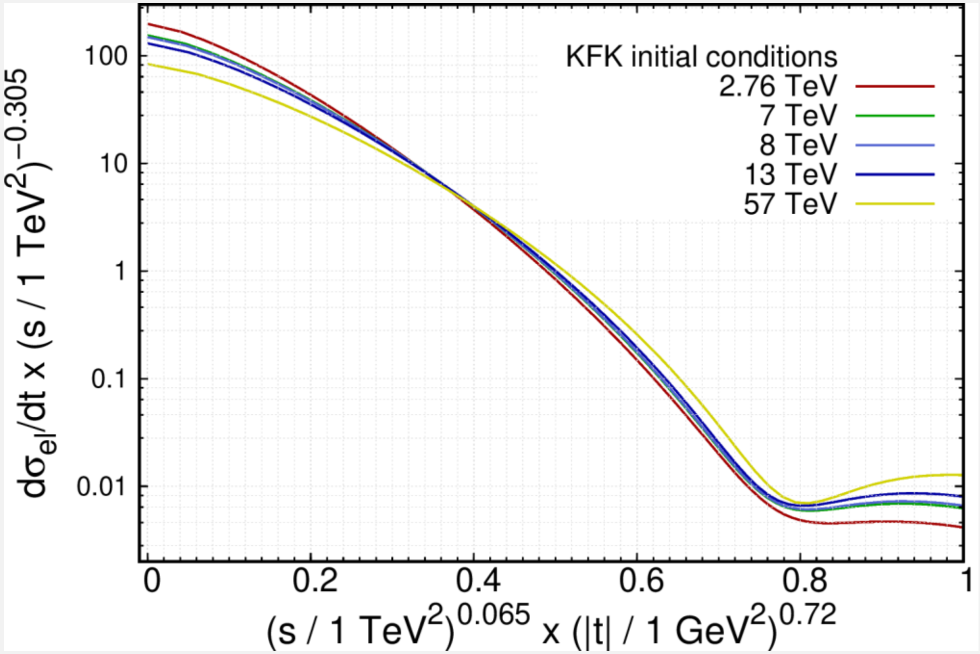
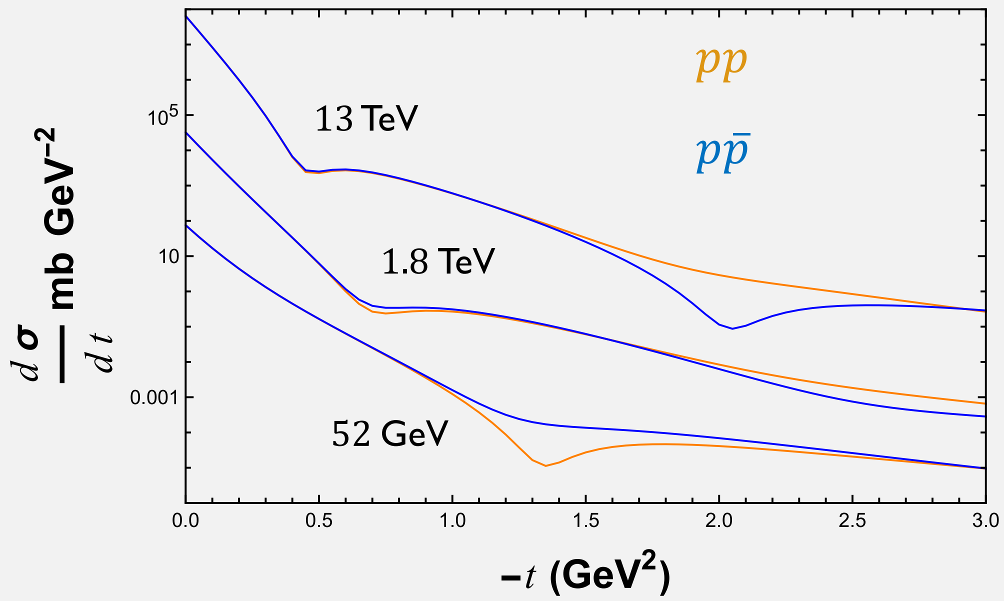




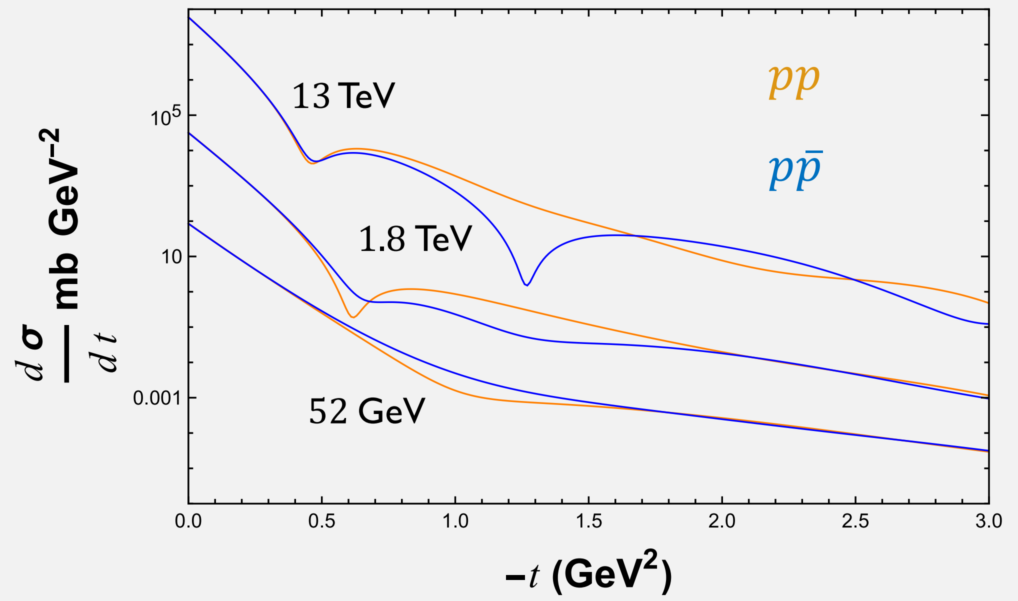
Dip evolution for different initial conditions

Note that at high energies the “first” dip structure for pp and $p\bar{p}$ seems to converge!

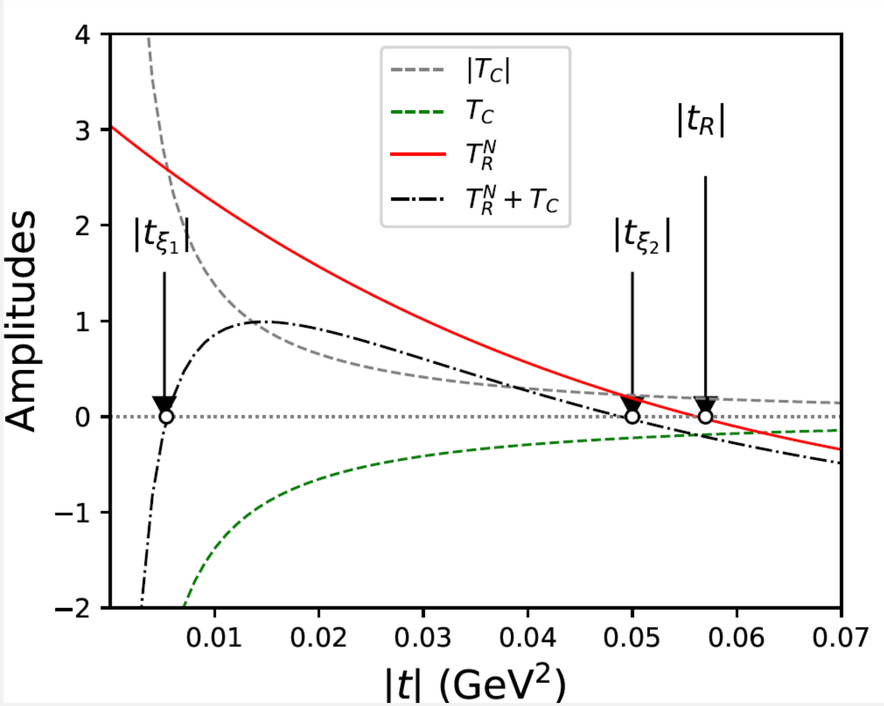
Evolution Equation with DL input



Evolution Equation with Jenkovszky input



The real-Coulomb interference
(very forward scattering)



Let $T_R(s,t)$ be the real part of the sum of the nuclear and Coulomb pp amplitudes,

$$T_R(s,t) \equiv T_R^N(s,t) + T_C(s) \ , \tag{8}$$

then, for s large, if $T_R^N(s,t) > |T_C(t)|$ in a region $0 < |t| < |t_R|$ then $T_R(s,t)$ has two zeros,

$$T_R(s,t_{\xi_1}) = T_R(s,t_{\xi_2}) = 0, \quad 0 < |t_{\xi_1}| < |t_{\xi_2}| < |t_R| \tag{9}$$

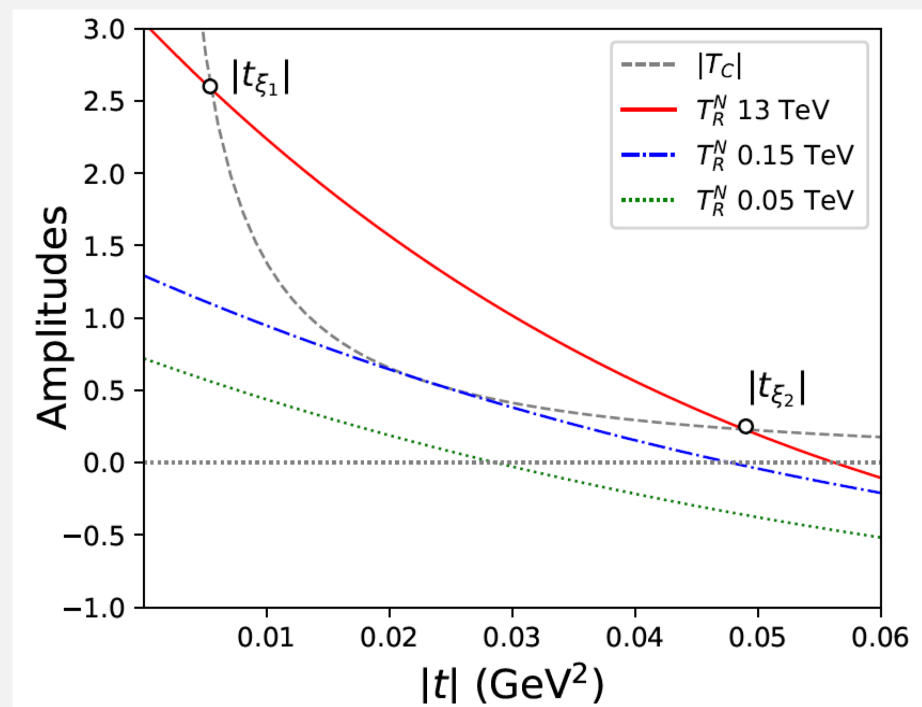
A.K.K, Eur. Phys. J. C 83 (2023) 2, 126

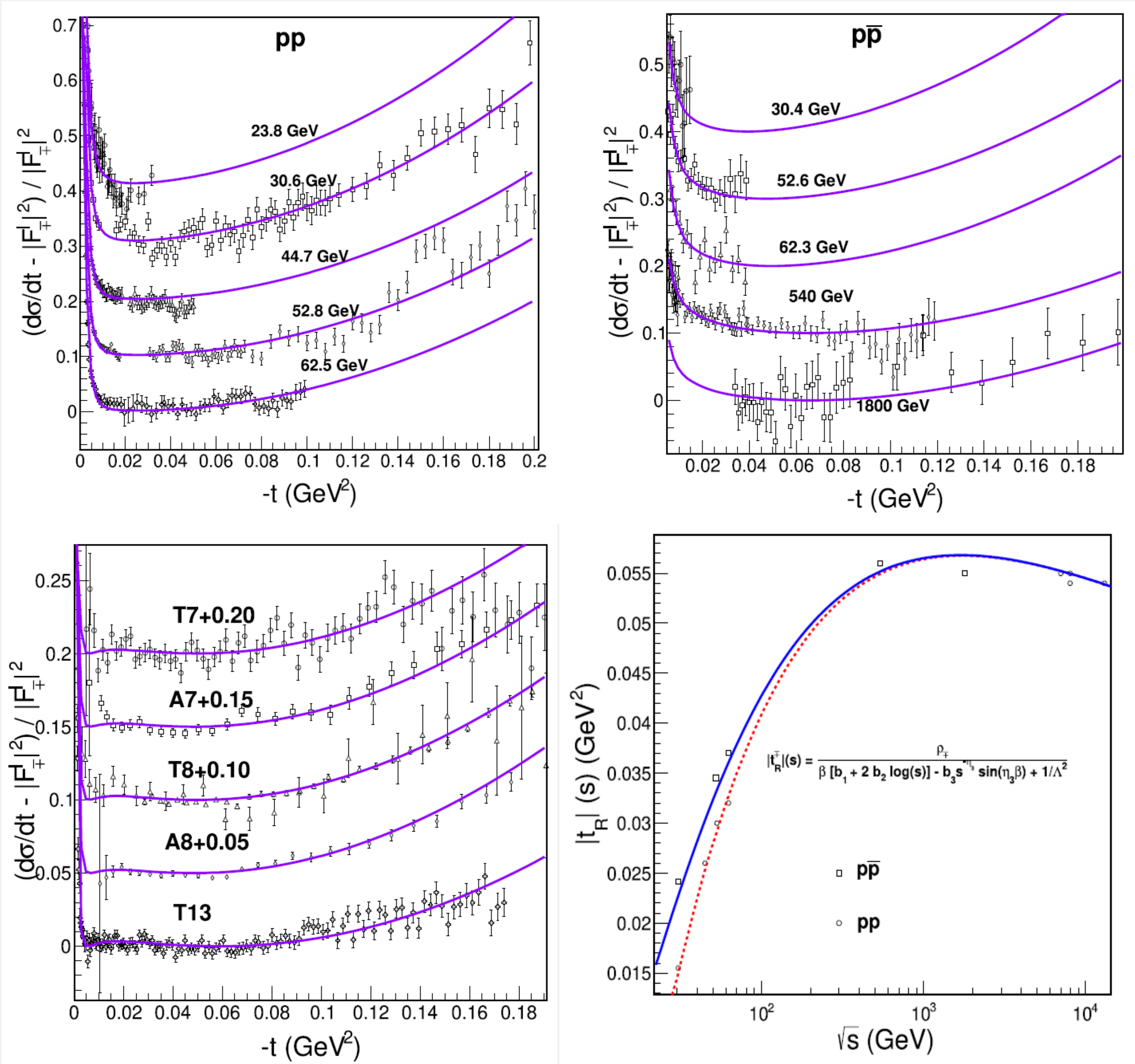
For pp the Coulomb amplitude is negative

The real nuclear amplitude is positive in the forward range (Martin's theorem)

A. Martin, Phys. Lett. B 404, 137 (1997).

As the energy increases the real nuclear amplitude also increases



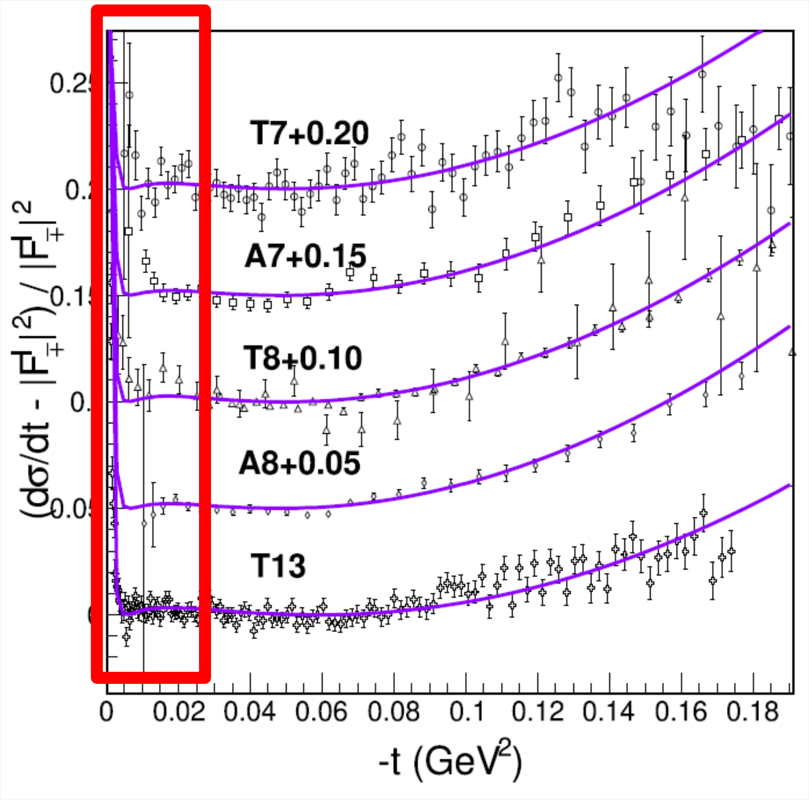


Analytic model for the very forward amplitudes

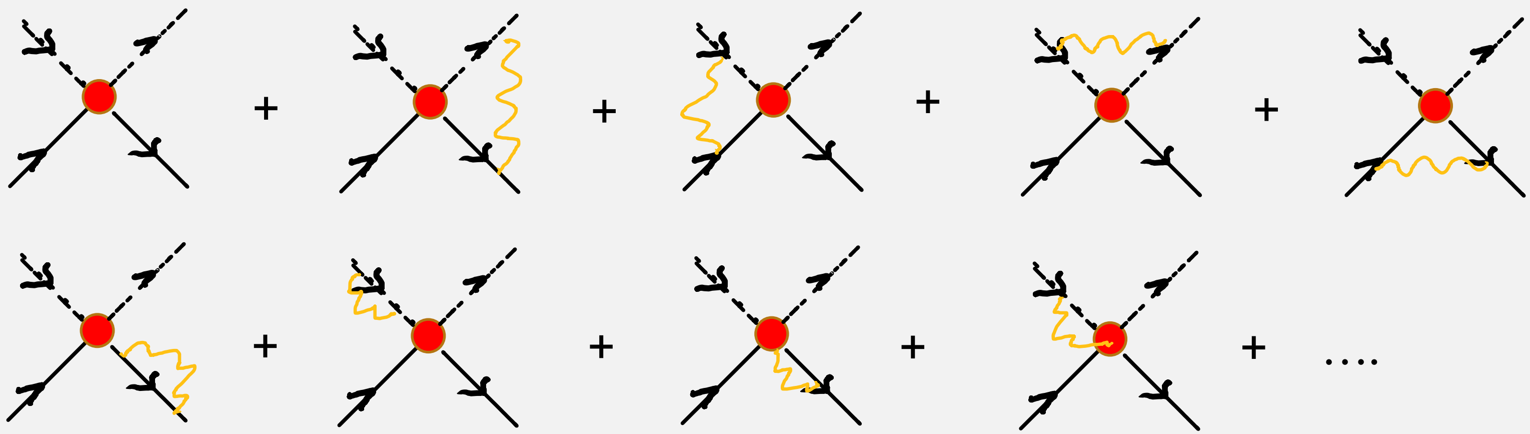
A.K.K., J.Phys.G 46 (2019) 12, 125001

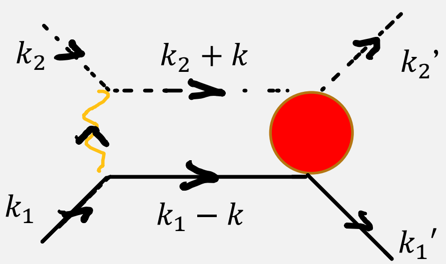
$$T_R(s,t) + T_C(t) = 0$$

Is it possible to observe any dip due to the interplay between real and Coulomb amplitude?



G.B. West, D.R. Yennie, Phys. Rev. ***172***(5), *1413 (1968)*





$$I_N = -4\pi \mathrm{i}\alpha(4k_1\cdot k_2)\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{f_N(2k_2\cdot k + k^2, -2k_1\cdot k + k^2, s, t')}{(k^2 + i_\epsilon)[(k_2+k)^2 - m^2 + \mathrm{i}\epsilon][(k_1-k)^2 - M^2 + \mathrm{i}\epsilon]}$$

Similar results from adding Coulomb and Strong force eikonals *R. Cahn, Z. Phys. C* ***15*** *(1982) 253.*

$$T_{C+N}(s,t) = \frac{s}{4\pi\mathrm{i}}\int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}(\chi_C+\chi_N)} - 1\right]$$

$$T_C(t) = \frac{s}{4\pi\mathrm{i}}\int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_C} - 1\right]$$

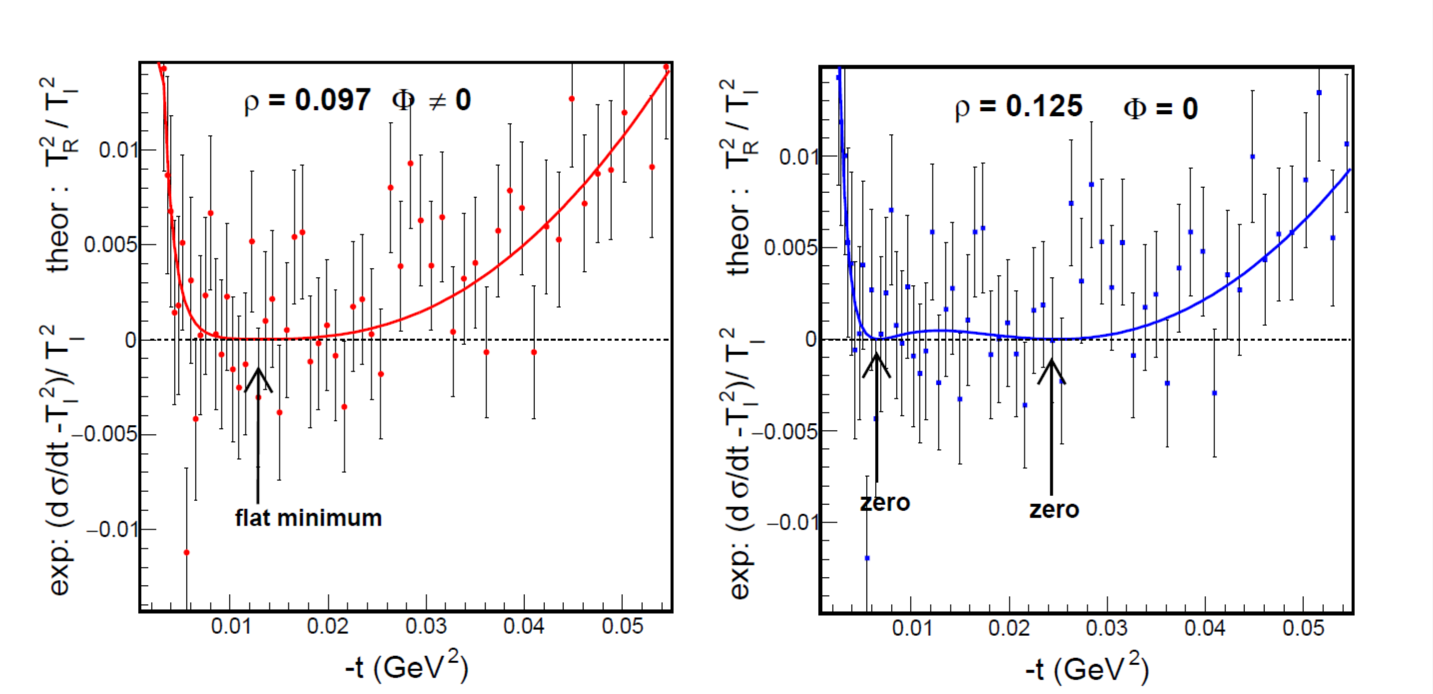
$$T_N(s,t) = \frac{s}{4\pi\mathrm{i}}\int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_N} - 1\right]$$

$$T_{C+N}(s,t) = T_C(t) + T_N(s,t) + \frac{s}{4\pi\mathrm{i}}\int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_C} - 1\right]\left[\mathrm{e}^{2\mathrm{i}\chi_N} - 1\right]$$

Relative Coulomb phase

$$\phi(s,t) = -\ln\left(\frac{-t}{s}\right) + \int_0^s \frac{\mathrm{d}t'}{|t'-t|}\left[1 - \frac{F^{\mathrm{N}}(s,t')}{F^{\mathrm{N}}(s,t)}\right]$$

Explicity $$\phi(s,t) \sim -\left[\gamma + \ln\left(-\frac{Bt}{2}\right) + O(-Bt)\right]$$



A.K.K., E. Ferreira and M. Rangel, Phys.Lett.B 789 (2019) 1-6
A.K.K., E. Ferreira, T. Kodama and M. Rangel, Eur.Phys.J.C 77 (2017) 12, 877

The presence of the Coulomb phase reduces the magnitude of ρ at LHC!

Questions

- Is the odderon an initial conditions problem?
- Is there diffusion in b space?
- How does the random noise will affect the real part of the amplitude?
- Is it possible to foreseen any asymptotic behaviour?
- Is the Coulomb phase really fundamental in charged hadrons?

For the future

- Use other models as initial conditions
- Implement the white noise in the evolution equation
- Include odderon fields and higher order Pomeron couplings

"**Entities must not be multiplied beyond necessity**"

Illustration of William of Ockham (from Wikipedia)

Thank you!

Hadronic Collider Experiments

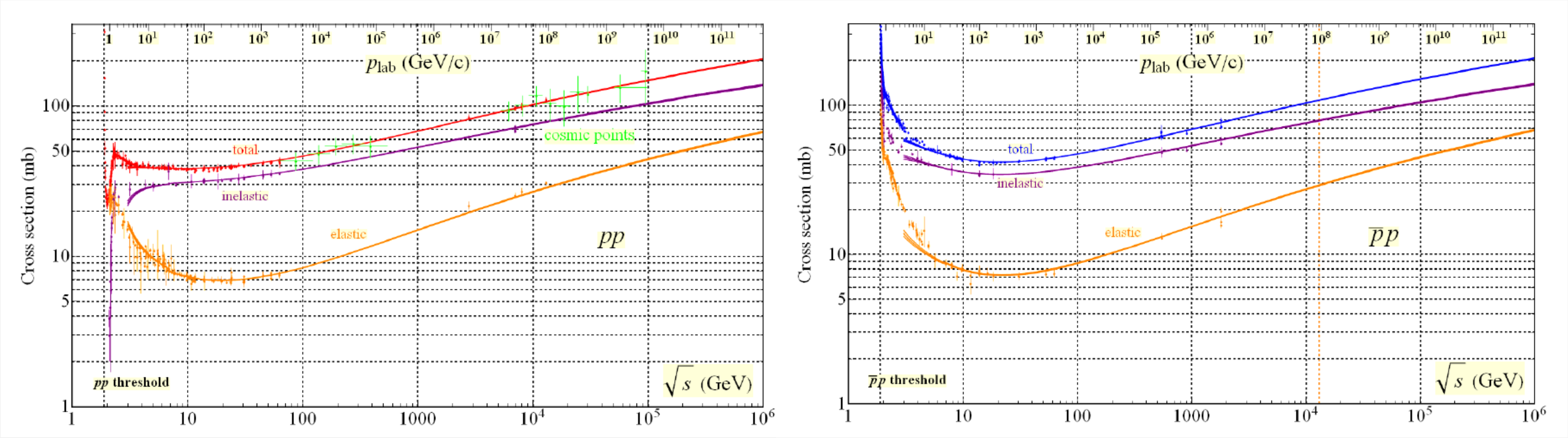
Intersecting Storage Rings-CERN, 1971–1984

Proton-Antiproton Collider(SPS)-CERN, 1981–1991

Tevatron-Fermilab, 1987–2011

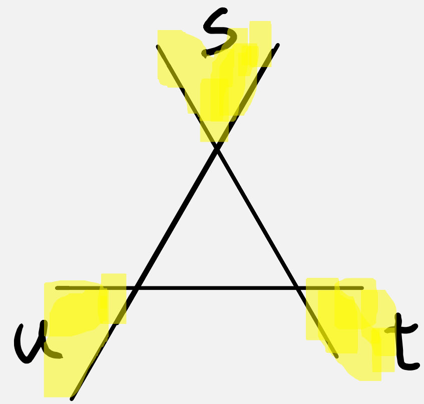
Relativistic Heavy Ion Collider-BNL, 2000–...

Large Hadron Collider-CERN, 2009–...



P.A. Zyla et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020)

Assumptions



Mandelstam plane

Analytic nuclear amplitude $A(s,t,u)$

Singularities have a physical meaning

Crossing symmetric amplitudes $A_{pp}(s,t,u) = A_{p\bar{p}}(u,t,s)$

Unitarity of S matrix $ss^{\dagger} = 1$

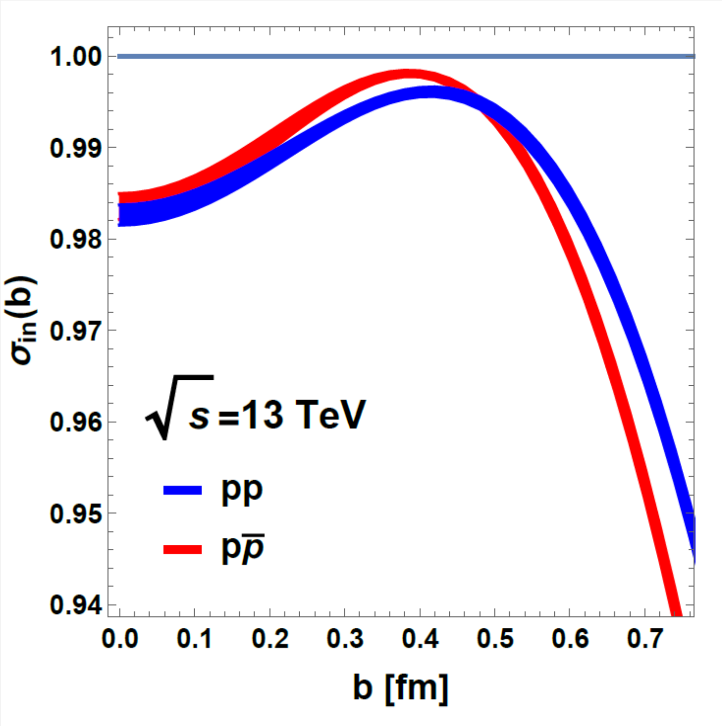
Theorems

Optical theorem $\sigma_T = \dfrac{1}{2|p|\sqrt{s}} \operatorname{Im} A(s,t)$

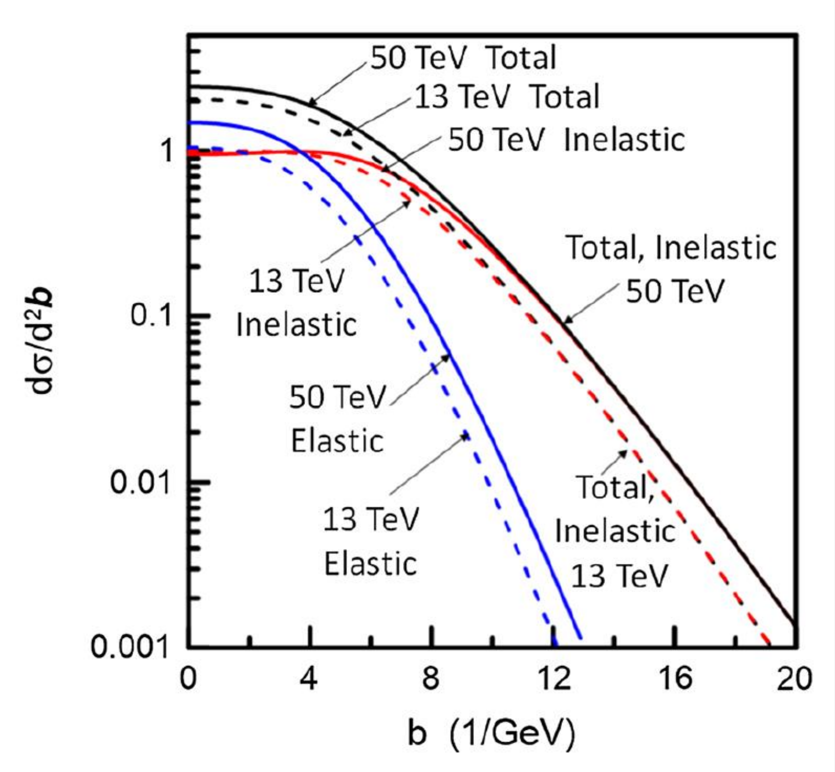
Froissart theorem/bound $\sigma_T(s) \leq C \log^2\left(\dfrac{s}{s_0}\right)$ $\quad s \to \infty$

Pomeranchuck theorem $\dfrac{\sigma_T^{pp}(s)}{\sigma_T^{p\bar{p}}(s)} \to 1$ $\quad s \to \infty$

Something is going on at 13 TeV

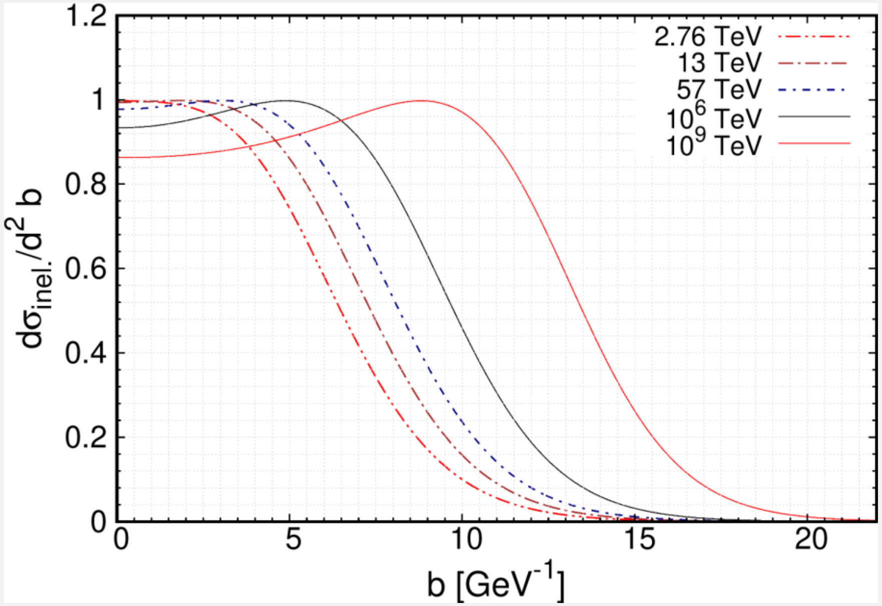


W. Broniowski, L. Jenkovszky, E. R. Arriola, I. Szanyi, *Phys. Rev. D* **98**, 074012 (2018); E. R. Arriola, W. Broniowski, *Few Body Syst.* **57** (2016) 7, 485-490; *Phys. Rev. D* **95** (2017) 7, 074030

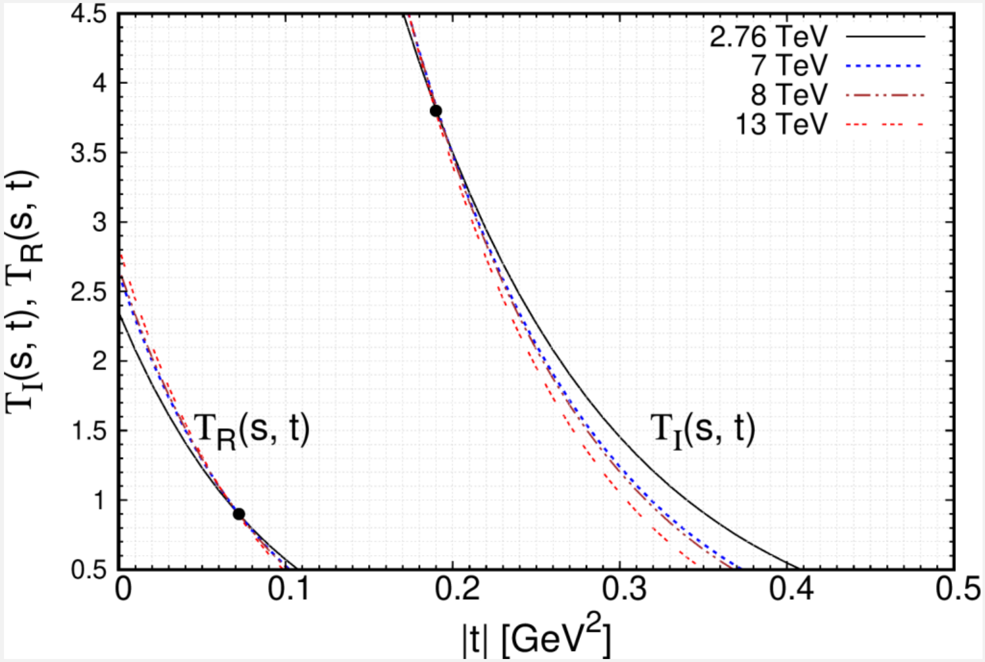


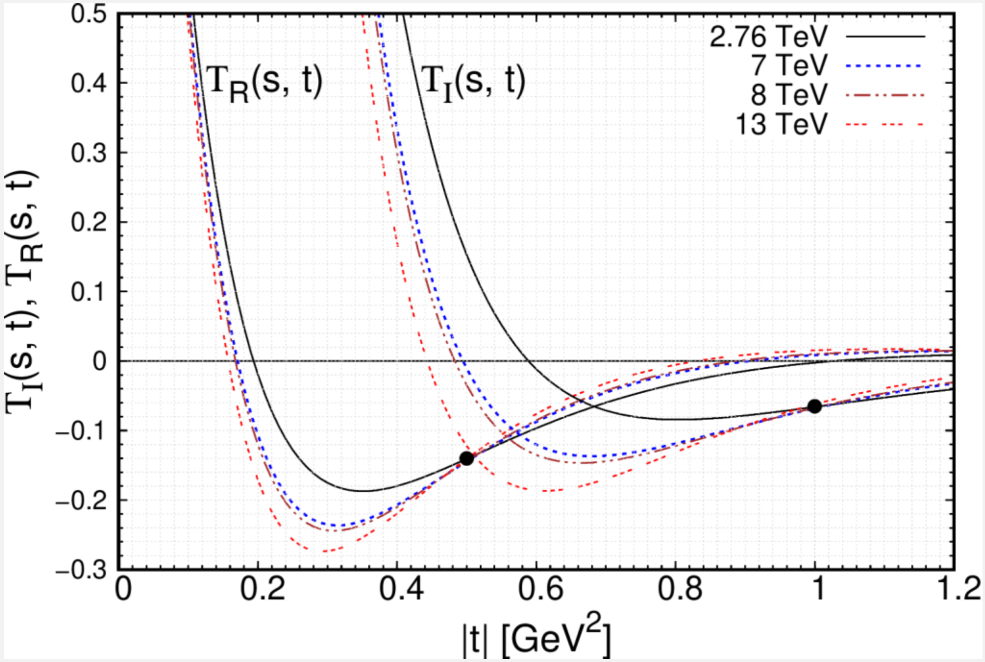
E. Ferreira, A. K. K. and T. Kodama; *Eur. Phys. J. C* **81** (2021) 4, 290

For LHC energies we also observe the ‘halo’ effect



The complex amplitudes in t-space are





We also observe estationary points in t-space. This was observed by Csorgo et. al

T. Csorgo and I. Szanyi, in International Scientific Days - Femtoscopy Session (2022) https://indico.cern.ch/event/1152630/.