Revisiting "target mass corrections" in lepton-nucleus deeply inelastic scattering

AGH, Kraków

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[1] w/ Olness, Muzakka, Leger, Schienbein, et al (nCTEQ Collaboration) [2301.07715]

Thank you for the invitation!

Today's talk

A few highlights from a "short" ☺ study on **T**arget **M**ass **C**orrections (TMCs) (more in a bit!) in deeply inelastic scattering (DIS) off **nuclear targets**

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Target mass corrections in lepton-nucleus DIS: theory and applications to nuclear PDFs

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Abstract: Motivated by the wide range of kinematics covered by current and planned deep-inelastic scattering (DIS) facilities, we revisit the formalism, practical implementation, and numerical impact of target mass corrections (TMCs) for DIS on unpolarized nuclear targets. An important aspect is that we only use nuclear and later partonic degrees of freedom, carefully avoiding a picture of the nucleus in terms of nucleons. After establishing that formulae used for individual nucleon targets (p, n), derived in the Operator Product Expansion (OPE) formalism, are indeed applicable to nuclear targets, we rewrite expressions for nuclear TMCs in terms of re-scaled (or averaged) kinematic variables. As a consequence, we find a representation for nuclear TMCs that is approximately independent of the nuclear target. We go on to construct a single-parameter fit for all nuclear targets that is in good numerical agreement with full computations of TMCs. We discuss in detail qualitative and quantitative differences between nuclear TMCs built in the OPE and the parton model formalisms, as well as give numerical predictions for current and future facilities.

Keywords: DIS, Structure Functions, Target Mass Corrections, OPE, nuclear PDFs

Contents

w/ Olness, Muzakka, Leger, Schienbein, et al (nCTEQ Collaboration) [2301.07715]

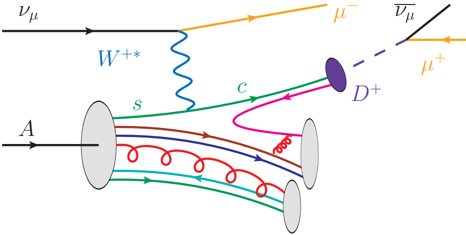
why?

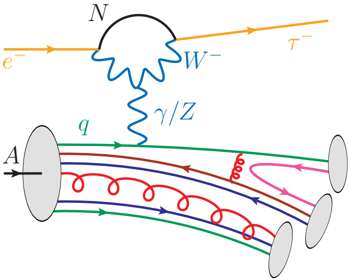
the big picture

Several νDIS and $e^{\pm}$DIS programs collecting data now:
– Fermilab
– JLab
– CERN

with more planned for $\gtrsim$'20s:
– BNL
– LBNF
– CERN

and with various agendas:
– precision hadronic structure
– QCD at the extremes
– precision ν oscillations
– search for more new physics





but why nTMCs?

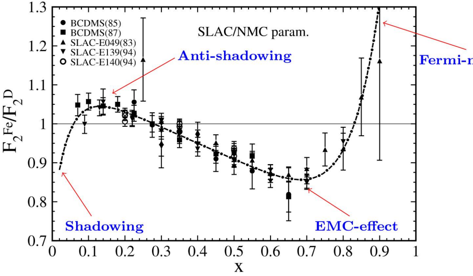
nuclei (A) are not protons $(\mathcal{P})$ ☹

Consider need for A in νDIS

· ν only interact through weak force: targets must be bigger ($\mathcal{O}(10)$tons), denser (Pb,Fe) $\implies$ more nuclear

· fact of life: dynamic nuclear structure impacts sensitivity to hadronic structure $\implies$

Plotted: $\frac{F_2^{\rm iron}}{F_2^{\rm deuteron}}$ for ν-DIS



Schienbein, et al [0710.4897]

For non-expert, QED (γ) contribution to F_2:

$F_2(\xi) \approx \sum_{i\in\{q,\overline{q},g\}} Q_i^2\ \xi\ f_i^A(\xi)$, $\quad Q_i$=electric charge of i

Consider need for A in νDIS

Plotted: $\frac{F_2^{\rm deuteron}}{F_2^{\rm proton}+F_2^{\rm neutron}}$ for ℓ-DIS

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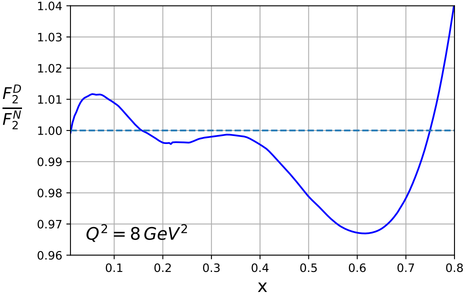

Muzakka, et al [2204.13157]

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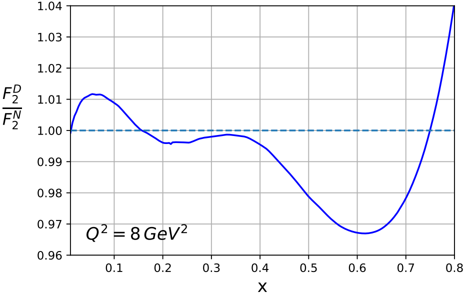
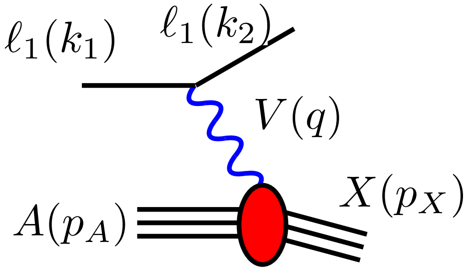


Muzakka, et al [2204.13157]

Take away: nuclear dynamics evident at $x \gtrsim 0.1$ and smaller Q^2 $\implies$ good control over pQCD needed in this double limit

target mass corrections

Formally, inclusive DIS of $\ell \in \{\ell^\pm, \nu, \overline{\nu}\}$ off protons can be described by the collinear factorization theorem (**CFT**) Collins, Soper ('87); Collins ('11)



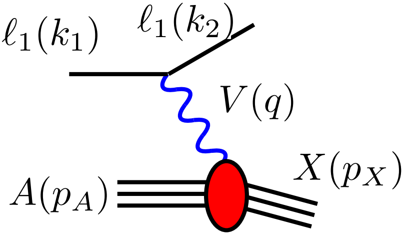
$$d\sigma(\ell_1 p \to \ell_2 X) = \underbrace{\sum_{i,X_n}}_{inclusive} \underbrace{\Delta_{ii'}}_{\text{shower/RGE}} \otimes \underbrace{f_i}_{\text{PDF}} \otimes \underbrace{d\hat{\sigma}_{i'\ell\to X_n}}_{\text{hard scattering}} + \mathcal{O}\left(\frac{\Lambda_{\rm NP}^{2+k}}{Q^{2+k}}\right)$$

TMCs are a class of $\mathcal{O}(x_A^2 M_A^2/Q^2)$ "power corrections" to the **CFT** for DIS that can be incorporated in structure functions

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more; Kretzer, Reno ('02,'03); Schienbein, et al [0709.1775]

Fun facts about TMCs:

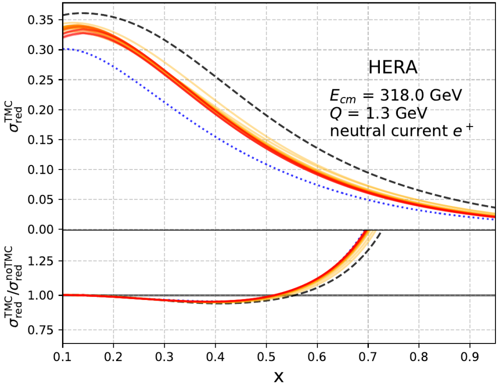
- Appear at "leading twist" in operator product expansion (OPE)
- "kinematical" in origin not "dynamical" Ellis, Furmanski, Petronzio ('82,'82)
- Standard for PDF groups to incorporate TMCs to some degree
- Has correspondence to ACOT formalism RR, et al [2301.07715]

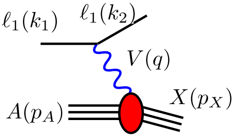


TMCs in practice: (reduced) cross sections

Example: Even for $\mathcal{P}$, **TMCs** can be sizable in certain kinematical limits

(ignore the many curves and consider the general trend in lower plot!)





$$\frac{d^2\sigma^{\rm NC}}{dx\,dy} = x(s-M^2)\,\frac{d^2\sigma^{\rm NC}}{dxdQ^2} = \frac{4\pi\alpha^2}{xyQ^2}\left[\frac{Y_+}{2}\ \sigma^{NC}_{\rm Red.}\right],$$

$$\sigma^{NC}_{\rm Red.} = \left(1+\frac{2y^2\varepsilon^2}{Y_+}\right)F_2^{\rm NC} \mp \frac{Y_-}{Y_+}\,xF_3^{\rm NC} - \frac{y^2}{Y_+}F_L^{\rm NC},$$

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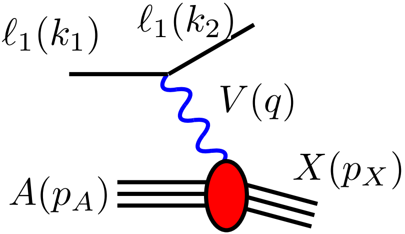

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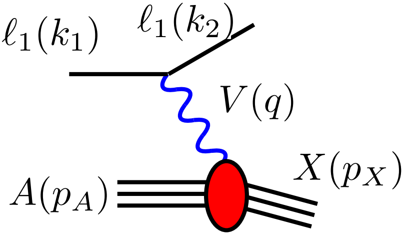

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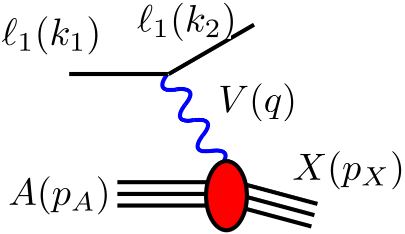

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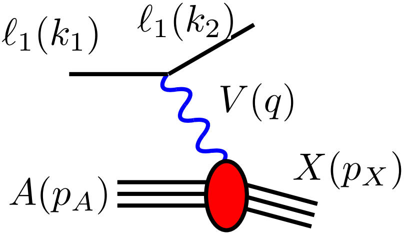

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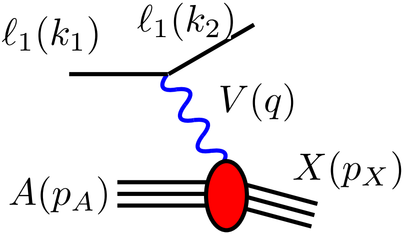

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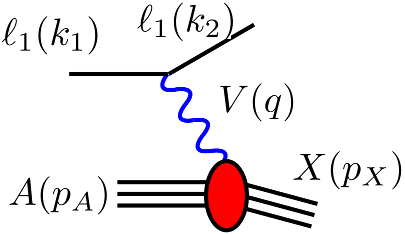

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  ("right answer for wrong reasons")
- Green's function approach by EFP "only" next-to-leading twist

  ("correct but hard...")

what did we do?

many things

What did we do?

- Starting from OPE, derived TMCs for arbitrary A for $F_1, \ldots, F_6$ ☺
  (lenthy appendix to avoid ambiguities in conventions!) sketched in next few slides!
- found a way of writing nTMCs formulae that are A-independent ☺
  (major sanity check!)
- ran numbers for JLAB, EIC, LBNF ☺
- found A-independent, 2-parameter fit for leading nTMCs ☹ – no time!
  (easy to implement in numerical codes!)
- Established correspondence between TMCs in OPE and TMCs in ACOT/parton model ☹ – no time! (nice intuition!)
- discovered some interesting pheno. along the way ☺

build nTMCs

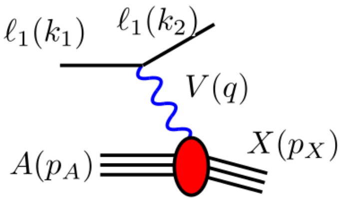
building TMCs for nuclear targets

To build **TMCs** for nuclear targets, follow Georgi & Politzer ('76,'76) with guidance from "modern" literature:

1. Get coffee ☺
2. Build inclusive, had. tensor in DIS: $W_A^{\mu\nu}$
3. Define forward-scatt. tensor $T_A^{\mu\nu}$, which has nice analytic properties
4. Decompose according to Lorentz structures $(F_i, \Delta T_i)$, relate the two
5. Get more coffee (magic!)
6. Take the OPE of $T_A^{\mu\nu}$, simplify (soooo tedious)
7. Take $(M_A^2/Q^2) \to 0$ to fix OPE Wilson coefficients (major sanity check!)
8. Repeat Steps 5-6, simplify (lots of identities and magic)

start from the beginning

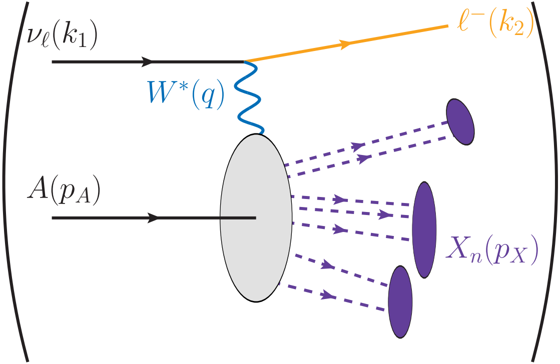
definitions



Kinematic variable	Description
$\nu_A = \frac{q \cdot p_A}{M_A} \overset{lab}{=} E_{\ell_1} - E_{\ell_2}$	Lepton energy loss in the laboratory (nucleon rest) frame
$y_A = \frac{q \cdot p_A}{k_1 \cdot p_A} \overset{lab}{=} \frac{\nu_A}{E_{\ell_1}}$	Inelasticity $y_A \in [0,1]$
$Q^2 = -q^2 > 0$	Boson squared momentum transfer
$x_A = \frac{Q^2}{2 p_A \cdot q} = \frac{Q^2}{2 M_A \nu_A}$	Bjorken x_A with $x_A \in [0,1]$
$W^2 = (p_A + q)^2 = M_A^2 + Q^2 \frac{1 - x_A}{x_A}$	Mass squared of the recoil system
$s = (k_1 + p_A)^2 = \frac{Q^2}{x_A y_A} + M_A^2 + m_\ell^2$	Center of Mass System (CMS) energy squared

draw diagrams, currents, and build the matrix element

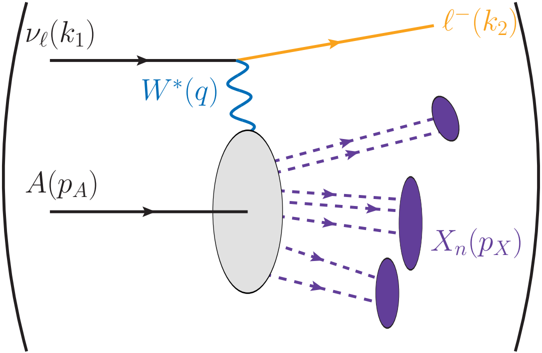
$-i\mathcal{M}$



$= \; J^{\alpha}_{\ell_f \ell_i} \;\cdot\; \Delta_{\alpha\mu} \;\cdot\; J^{\nu}_{X_n A}$

squaring and summing over spins gives us $H_n^{\mu\nu}$ (exclusive, n-body)

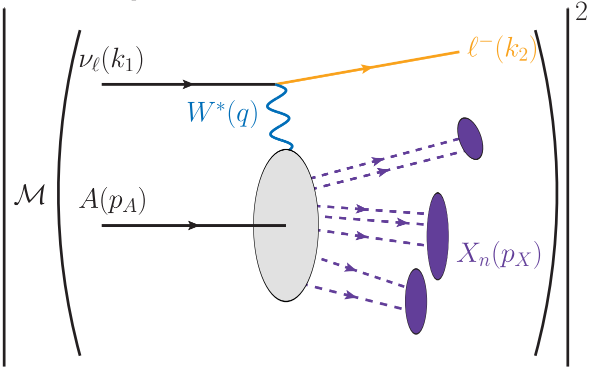
$\Sigma_{\text{spins}} \left| \mathcal{M} \right.$



$\left. \right|^2 \sim L_{\mu\nu} \cdot H_n^{\mu\nu}$

n-body phase space integral *and* summing over n gives us $W_A^{\mu\nu}$

$$\frac{d^3\sigma}{dk_2^3} \sim \int dPS_n \, \Sigma_n \, \Sigma_{\rm spins} \left| \mathcal{M} \left(\text{diagram} \right) \right|^2 \sim L_{\mu\nu} \cdot W_A^{\mu\nu}$$



this step sometimes omitted in textbooks, e.g., Halzen & Martin

in the paper, we use exact expressions for $d\sigma$, etc., so $\sim \to =$

Summing over X_n ensures "inclusivity" and closure, $1 = \sum_n |X_n\rangle\langle X_n|$

$$W^A_{\mu\nu} = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\langle A|J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$$

[2] parton model says $F_i = \sum f_{j/p}$; see Collins ('11) for nice discussion on this!

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$$= -g_{\mu\nu}\ F^A_1\ + \frac{p_{A\mu}p_{A\nu}}{Q^2}\ 2x_A\ F^A_2\ - i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho_A q^\sigma}{Q^2}\ x_A F^A_3$$

$$+ \frac{q_\mu q_\nu}{Q^2}\ 2F^A_4\ + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_5\ + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_6$$

– **point #1:** $F_i(x, Q^2)$ are structure functions and can be measured[2]

– **point #2:** $W^A_{\mu\nu}$ is defined in the "DIS" limit:

$$x_A = \frac{Q^2}{2p_A\cdot q}\ \text{is fixed and}\ (Q^2/M^2_A) \to \infty$$

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Define the time-ordered ME for (virtual) $AV^* \to AV^*$ scattering

$$T^A_{\mu\nu} = \int \ d^4z \ e^{iq\cdot z} \langle A| \ \mathcal{T} J^\dagger_{had.\mu}(z) \ J_{had.\nu}(0)|A\rangle$$

$$= -g_{\mu\nu} \ \Delta T^A_1 \ + \frac{p_{A\mu} p_{A\nu}}{M_A^2} \ \Delta T^A_1 \ - i\epsilon_{\mu\nu\rho\sigma} \frac{p_A^\rho q^\sigma}{M_A^2} \ \Delta T^A_3$$

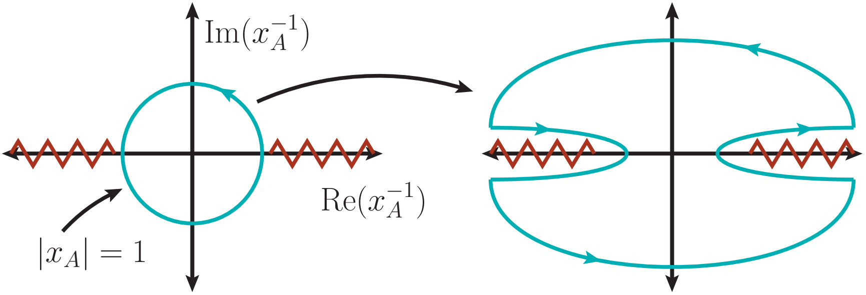
$$+ \frac{q_\mu q_\nu}{M_A^2} \ \Delta T^A_4 \ + \frac{p_{A\mu} q_\nu + p_{A\nu} q_\mu}{M_A^2} \ \Delta T^A_5 \ + \frac{p_{A\mu} q_\nu - p_{A\nu} q_\mu}{M_A^2} \ \Delta T^A_6$$

point #1: related to $W^A_{\mu\nu}$ by Fourier transformations

point #2: $T^A_{\mu\nu}$ is defined in the "short-distance" limit:

$$\frac{x_A}{Q} \text{ is fixed and } (Q^2/M_A^2) \to \infty$$

$W^A_{\mu\nu}$ and $T^A_{\mu\nu}$ are different (DIS vs short-distance limit) but related by a contour



$$\Delta T_i^A(x_A^{-1} + i\varepsilon) - \Delta T_i^A(x_A^{-1} - i\varepsilon) = (\text{some factor}) \times F_i^A(x_A, Q^2)$$

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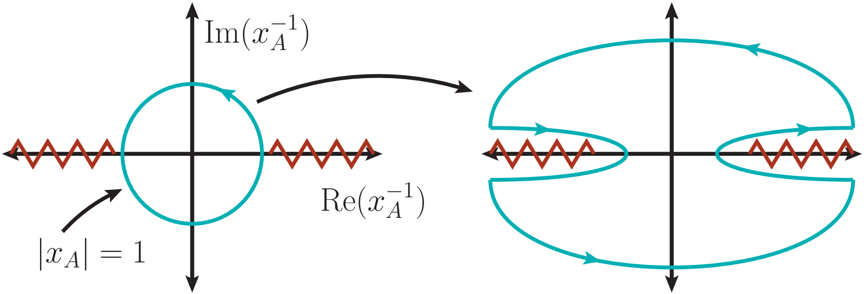

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Taylor-expand ΔT_i around $x_A^{-1} \to 0$ + Cauchy's Thm see also Collins ('84)!

$$\Delta T_i^A = (\text{some other factor}) \times \sum_N^\infty \underbrace{F_i^{AN}(Q^2)}_{N^{th}\ \text{Mellin moment} = \int_0^1 dy\ y^{(N-1)} F_i(y)} x_A^{-N}$$

what is next?

building TMCs for nuclear targets

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4. Relate F_i^A and ΔT_i^A ✓
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the operator product expansion (in a nutshell)

The OPE is a formalism for decomposing products of operators

Wilson ('69); Brandt, Preparata ('71); Christ, et al ('72)

$$\langle \text{some number of operators } \hat{\mathcal{O}} \rangle = \sum_k \underbrace{\mathcal{C}_k}_{\text{Wilson coeff.}} \times \langle \text{fewer operators } \hat{\mathcal{O}} \rangle$$

Assume $T^A_{\mu\nu}$ has an OPE in the short-distance limit:[3]

$$\lim_{z\to 0} \ T^A_{\mu\nu} \overset{\text{OPE}}{=} -2i \sum_{\iota,n} \underbrace{c_{\mu\nu\mu_1\ldots\mu_n}(q)}_{\text{Wilson coeff.}} \underbrace{\langle A|\mathcal{O}^{\mu_1\ldots\mu_n}_{\iota,\tau=2}|A\rangle}_{\text{hadronic ME}}$$

$$+ \ \text{power corrections at } \mathcal{O}(\tau > 2)$$

[3] Power counting is ordered by "twist", τ = (dim. of EFT operator) − (# of Lorentz indices); see also Sterman (TASI'95)

applying the OPE (massless target)

After combinatorics and collecting like-terms, get things like this:

$$\Delta\,\tilde{T}_1^A = (-4i)\ \sum_{k=1}^{\infty}\ \left[C_1^{2k} A_{\tau=2}^{2k}\right]\ \sum_{j=0}^{k}\ \frac{(2k-j)!(2k)!}{(2k)!j!(2k-2j)!}\ \left(\frac{M_A^2}{Q^2}\right)^j\ x_A^{-(2k-2j)}$$
$$+ (-4i)\ \sum_{k=1}^{\infty}\left[C_2^{2k} A_{\tau=2}^{2k}\right]\ \sum_{j=1}^{k}\ \frac{(2k-j)!(2k-2)!}{(2k)!(j-1)!(2k-2j)!}\ \left(\frac{M_A^2}{Q^2}\right)^j\ x_A^{-(2k-2j)}$$
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$$+ \mathcal{O}(\tau > 2)$$

After taking the $(M_A^2/Q^2) \to 0$ limit, recover something remarkable:

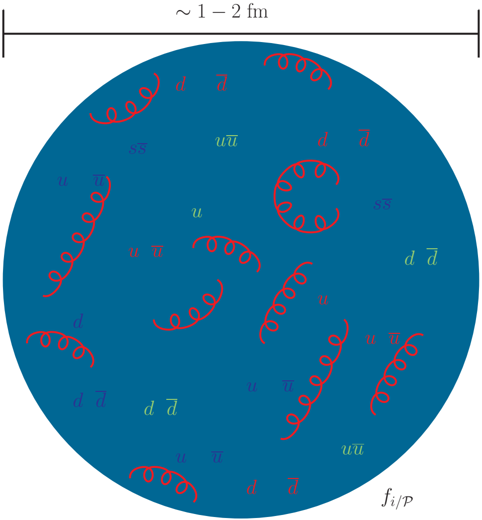
$$\tilde{F}_i^{AN}\Big|_{\text{No TMC}} = C_i^N A_{\tau=2}^N + \mathcal{O}(\tau > 2) \quad \text{for} \quad i = 1, 3-6 \,,$$
$$\tilde{F}_2^{A(N-1)}\Big|_{\text{No TMC}} = C_2^N A_{\tau=2}^N + \mathcal{O}(\tau > 2)$$

nuclear str. fns. = (short-dist. phys.) $\times$ (long-dist./hadronic phys.)

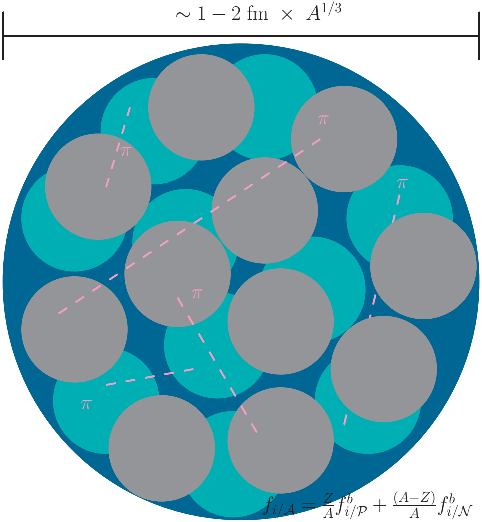
what does this mean?

for proton, $F_i^N = C_i^N \times A^N$ + power corrections

$\implies$ "PDFs = had. ME"

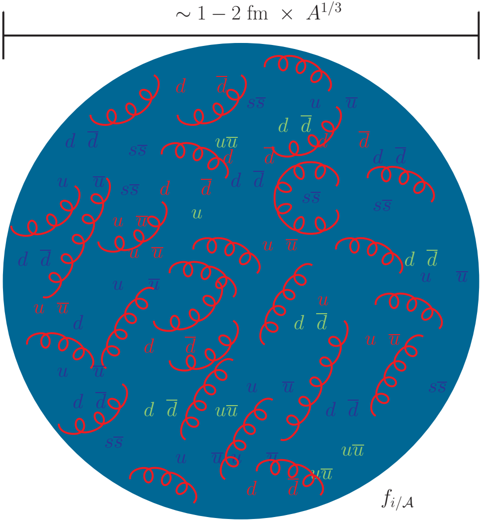


for A, common to parameterize PDF as combination of "bound" $\mathcal{P}$ and $\mathcal{N}$ PDFs



$$f_{i/A} = \frac{Z}{A} f^b_{i/\mathcal{P}} + \frac{(A-Z)}{A} f^b_{i/\mathcal{N}}$$

for A, $F_i^{AN} = C_i^N \times A^N$ + power corrections
$\Longrightarrow$ "PDFs = had. ME" (no need for intermediate picture!)



After combinatorics and collecting like-terms, get things like this:

$$\underbrace{\Delta\,\tilde{T}_1^A}_{\sim\sum F_1^{AN}} = (-4i)\ \sum_{k=1}^{\infty}\ \underbrace{\left[C_1^{2k} A_{\tau=2}^{2k}\right]}_{F_1^{A(2k)}|_{\text{no TMC}}}\ \sum_{j=0}^{k}\ \frac{(2k-j)!(2k)!}{(2k)!j!(2k-2j)!}\ \left(\frac{M_A^2}{Q^2}\right)^j\ x_A^{-(2k-2j)}$$

$$+(-4i)\ \sum_{k=1}^{\infty}\ \underbrace{\left[C_2^{2k} A_{\tau=2}^{2k}\right]}_{F_2^{A(2k-1)}|_{\text{no TMC}}}\ \sum_{j=1}^{k}\ \frac{(2k-j)!(2k-2)!}{(2k)!(j-1)!(2k-2j)!}\ \left(\frac{M_A^2}{Q^2}\right)^j\ x_A^{-(2k-2j)}$$

$$+\ \mathcal{O}(\tau > 2)$$

In massive limit, "massless" structure functions mix

after applying several integral identities[4]

[4] see also Detmold [hep-ph/0509011]

Nuclear structure functions with TMCs

Generically, str. fn. with TMCs have the form:

$$F_j^{A,\mathrm{TMC}}(x_A, Q^2) \;=\; \sum_{i=1}^{6} A_j^i \underbrace{F_i^{A,(0)}(\xi_A, Q^2)}_{\text{no TMCs}} + B_j^i h_i^A(\xi_A, Q^2) + C_j g_2^A(\xi_A, Q^2)$$

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Lots to unpack!

- x_A = Bjorken x
- $\xi_A(x_A)$ = Natchmann $x = 2x_A/(1+\sqrt{1+4x_A^2M_A^2/Q^2})$
- A_j^i, B_j^i, C_j are closed-form coefficient fns. $\sim f\left([x_A, (x_A^2M_A^2/Q^2)\right]$
- for $i \neq j$, structure function mixing!
- $A_i^i \sim \mathcal{O}(1)$; all other coeff. $\sim \mathcal{O}(x_A M_A^2/Q^2)$.
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Example:

$$F_1^{A,\mathrm{TMC}}(x_A) \;=\; \left(\frac{x_A}{\xi_A r_A}\right) F_1^{A,(0)}(\xi_A) \;+\; \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right) h_2^A(\xi_A) \;+\; \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right) g_2^A(\xi_A)$$

Nuclear structure functions with TMCs

Generically, str. fn. with TMCs have the form (at leading power):

$$F_j^{A,\mathrm{TMC}}(x_A, Q^2) \;=\; \sum_{i=1}^{6} A_j^i \underbrace{F_i^{A,(0)}(\xi_A, Q^2)}_{\text{no TMCs}} + B_j^i h_i^A(\xi_A, Q^2) + C_j g_2^A(\xi_A, Q^2)$$

$$\tilde{F}_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right) \tilde{F}_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right) \tilde{h}_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right) \tilde{g}_2^A(\xi_A),$$

$$\tilde{F}_2^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A^2}{\xi_A^2 r_A^3}\right) \tilde{F}_2^{A,(0)}(\xi_A) + \left(\frac{6M_A^2 x_A^3}{Q^2 r_A^4}\right) \tilde{h}_2^A(\xi_A) + \left(\frac{12M_A^4 x_A^4}{Q^4 r_A^5}\right) \tilde{g}_2^A(\xi_A),$$

$$\tilde{F}_3^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right) \tilde{F}_3^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right) \tilde{h}_3^A(\xi_A),$$

$$\begin{aligned}\tilde{F}_4^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A}\right) \tilde{F}_4^{A,(0)}(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^2}\right) \tilde{F}_5^{A,(0)}(\xi_A) + \left(\frac{M_A^4 x_A^3}{Q^4 r_A^3}\right) \tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right) \tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^4 x_A^4}{Q^4 r_A^4}\right) \left(2 - \xi_A^2 M_A^2/Q^2\right) \tilde{h}_2^A(\xi_A) \\ & + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^5}\right) \left(1 - 2x_A^2 M_A^2/Q^2\right) \tilde{g}_2^A(\xi_A),\end{aligned}$$

$$\begin{aligned}\tilde{F}_5^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A^2}\right) \tilde{F}_5^{A,(0)}(\xi_A) - \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3 \xi_A}\right) \tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right) \tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^4}\right) \left(1 - x_A \xi_A M_A^2/Q^2\right) \tilde{h}_2^A(\xi_A) \\ & + \left(\frac{6M_A^4 x_A^3}{Q^4 r_A^5}\right) \tilde{g}_2^A(\xi_A),\end{aligned}$$

$$\tilde{F}_6^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right) \tilde{F}_6^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right) \tilde{h}_6(\xi_A).$$

Rescaling

Interestingly, TMCs have particular kinematical dependence:

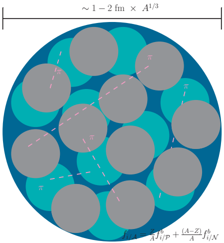
$$\frac{x_A}{\xi_A} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right)$$

Define "average (nucleon) kinematics": $M_N \equiv M_A/A$ **and** $x_N \equiv A x_A$

$$\frac{x_A}{\xi_A} = \frac{x_N}{\xi_N} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right) = \left(\frac{x_N^2 M_N^2}{Q^2}\right)^j$$

Consequence: TMCs for A-independent, "nucleon" str. fns. that matches intuitive picture of nuclei →

– same expressions as for A but replace "$_A$" with "$_N$"



some numbers

running the numbers

we use NLO PDFs (`nCTEQ15`) to build str. fns. At LO, these are

$$F_1^{\nu A} = \quad (d + s + \bar{u} + \bar{c}), \qquad F_1^{\bar{\nu} A} = \quad (u + c + \bar{d} + \bar{s})$$

$$F_2^{\nu A} = 2x\,(d + s + \bar{u} + \bar{c}), \qquad F_2^{\bar{\nu} A} = 2x\left(u + c + \bar{d} + \bar{s}\right)$$

$$F_3^{\nu A} = +2\,(d + s - \bar{u} - \bar{c}), \qquad F_3^{\bar{\nu} A} = -2\left(u + c - \bar{d} - \bar{s}\right)$$

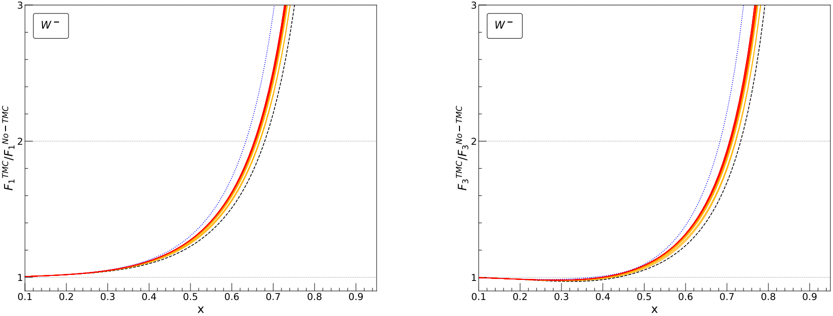
$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u+\bar{u}) + (d+\bar{d}) + 4(c+\bar{c}) + (s+\bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
H	1	1	Be	9	4	Ca	40	20	Xe	131	54
D	2	1	C	12	6	Fe	56	26	W	184	74
^{3}He	3	2	N	14	7	Cu $_{\rm iso}$	64	32	Au	197	79
He	4	2	Ne	20	10	Kr $_{\rm iso}$	84	42	Au $_{\rm iso}$	197	98.5
Li	6	3	Al	27	13	Ag $_{\rm iso}$	108	54	Pb $_{\rm iso}$	207	103.5
Li	7	3	Ar	40	18	Sn $_{\rm iso}$	119	59.5	Pb	208	82

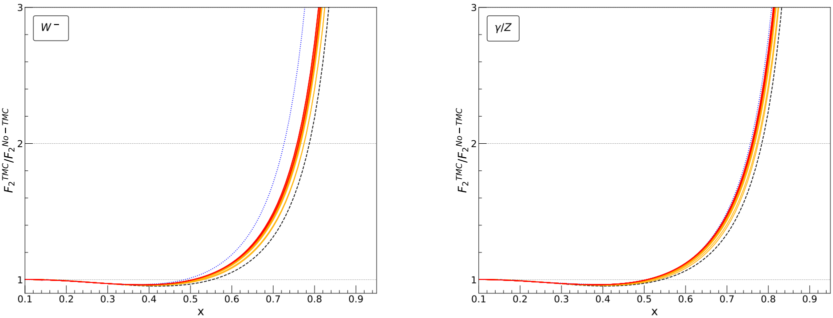
ratio of F_i^{TMC} / $F_i^{\mathrm{no\ TMC}}$

Plotted: ratio for (L) $F_1^{W^-}$ and (R) $F_3^{W^-}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

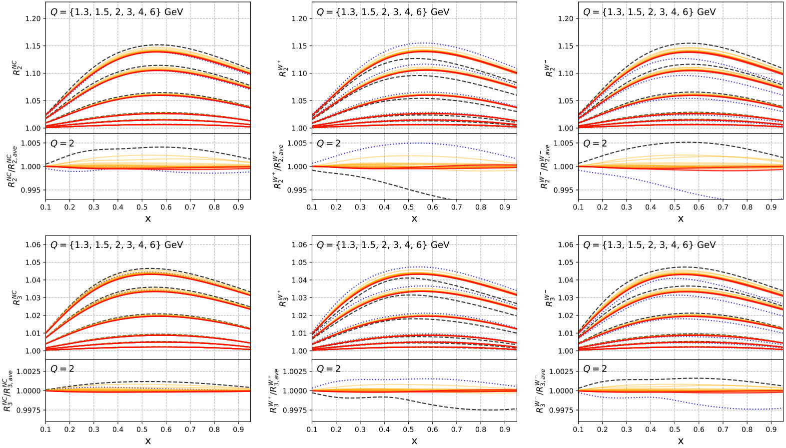
Plotted: ratio for (L) $F_2^{W^-}$ and (R) $F_2^{\gamma/Z}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

ratio of F_i^{TMC} / $F_i^{\mathrm{leading\ TMC}}$

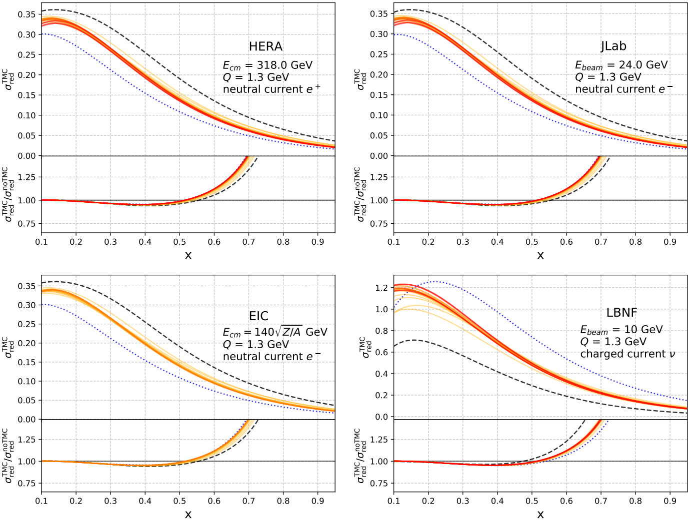
Plotted: ratio for (L) $F_i^{Z/\gamma}$, (C) $F_i^{W^+}$, (R) $F_i^{W^-}$ for $i = 2$ (upper) and $i = 3$ (lower)



remarkable uniformity! (good enough to fit! ☺)

reduced cross sections

Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o



summary

Summary and conclusion

The nCTEQ collaboration has revisited the theory and phenomenology TMCs in DIS off nuclear targets

- **extended** formalism for protons to nuclei
- **pedagogical appendix** that fills in gaps in literature/texts
- **lots of phenomenology, numbers, and plots** (... so many plots)
- hope this work **guides future discussions**
- lots not covered (ACOT, uncertainties, $x_N > 1$, fit results), so see the paper! [2301.07715]

Thank you!