# Associated production of top-antitop and massive gauge bosons: new precision results


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# Origin of the study

- **Goal:** estimate resummed soft gluon corrections to t-tbar-H, t-tbar-Z, t-tbar-W hadroproduction and improve theoretical precision
- **Reasons:**

– process with the heaviest Standard Model particles that probes directly the value of top Yukawa coupling and coupling to electroweak bosons – potentially sensitive to new physics

– extension of the soft gluon resummation technique to more than two final state particles

- Report on results of a long-term research line by

**Anna Kulesza, Tomasz Stebel, Vincent Theeuwes, Daniel Schwartlander, Roger Balsach, LM**

*[JHEP 1603 (2016) 065, Phys.Rev. D97 (2018) 114007,*

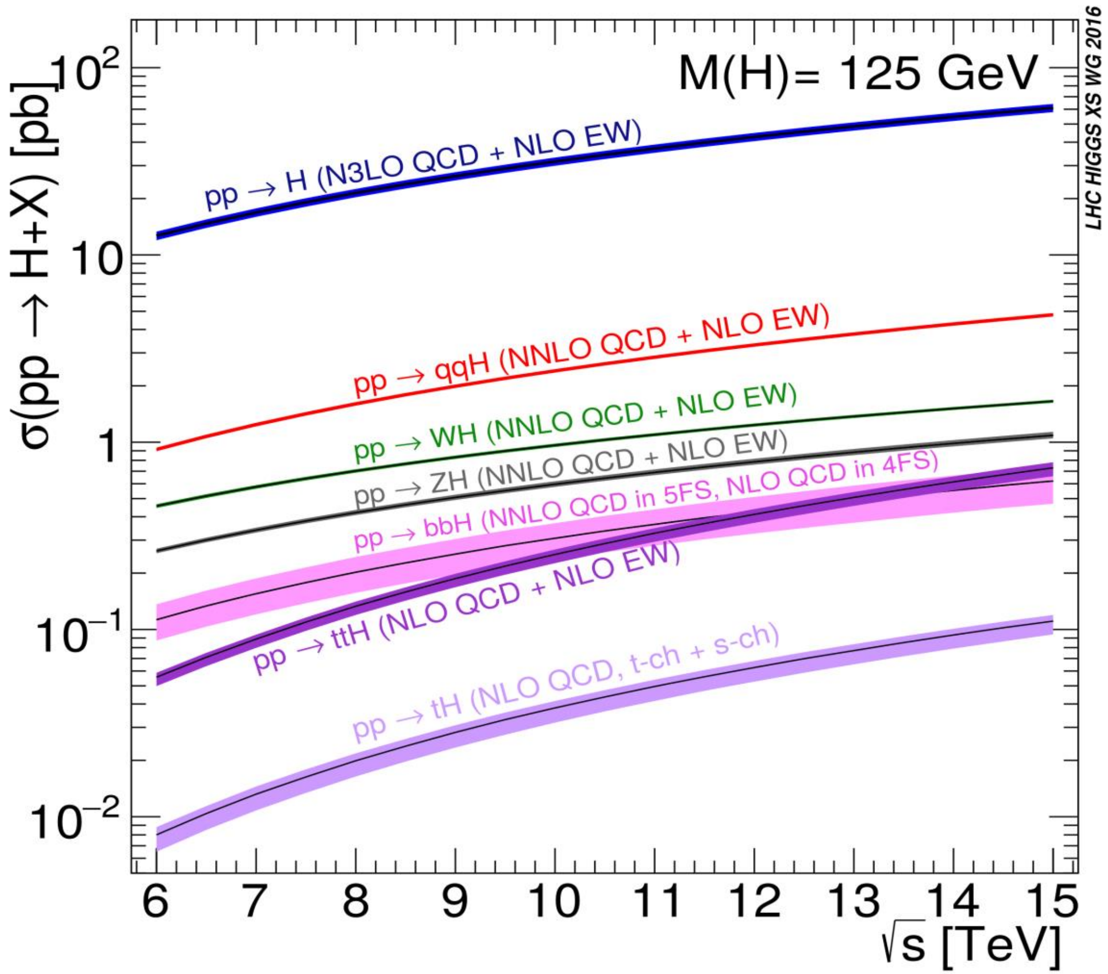
*Eur.Phys.J. C79 (2019), 249] , Eur.Phys.J.C 80 (2020) 5, 428*

# Outline

- LHC context
- Motivation
- Current status of the cross-section
- Threshold corrections at NLO
- Soft gluon resummation
- Soft anomalous dimension
- Resummed cross-section
- Theoretical precision / scale dependence
- Comparison to data
- Discussion and conclusions

# Higgs boson cross-sections at the LHC

Context: Higgs boson discovery and productions channels:




**[LHC Higgs Cross Section Working Group 2016]**

# The precision path explored at the LHC

- Calculation based on perturbative quantum field theory. Need for 2 or 3 loop calculations in QCD and EW. QCD corrections are usually larger than EW
- The most robust calculational technique: fixed order calculations. Difficulty grows quickly with increasing number of external legs and with the number of important mass scales.
- With more than 2 final state particles dimension of phase space grows from 2 to 5 (or more) at the LO. Also more complicated topologies appear and higher order calculations become much more difficult

- The current theoretical frontier for calculations of cross sections for hadroproduction of massive particles:
  - NNNLO calculations for 2 → 1 e.g.: gg → H (since 2015): precision O(3%)
  - NNLO calculations for 2 → 2 e.g.: gg→ t-tbar: precision O(10%)
  - NLO calculation for 2 → 3 e.g.: pp → t-tbar-H: precision O(20%)

- Possible improvement of precision: all order resummation of enhanced corrections in perturbative series → soft gluon resummation

# Why t-tbar-Higgs?

- Process with the heaviest Standard Model particles that probes value of top Yukawa coupling

- Enhanced precision in the Higgs boson sector

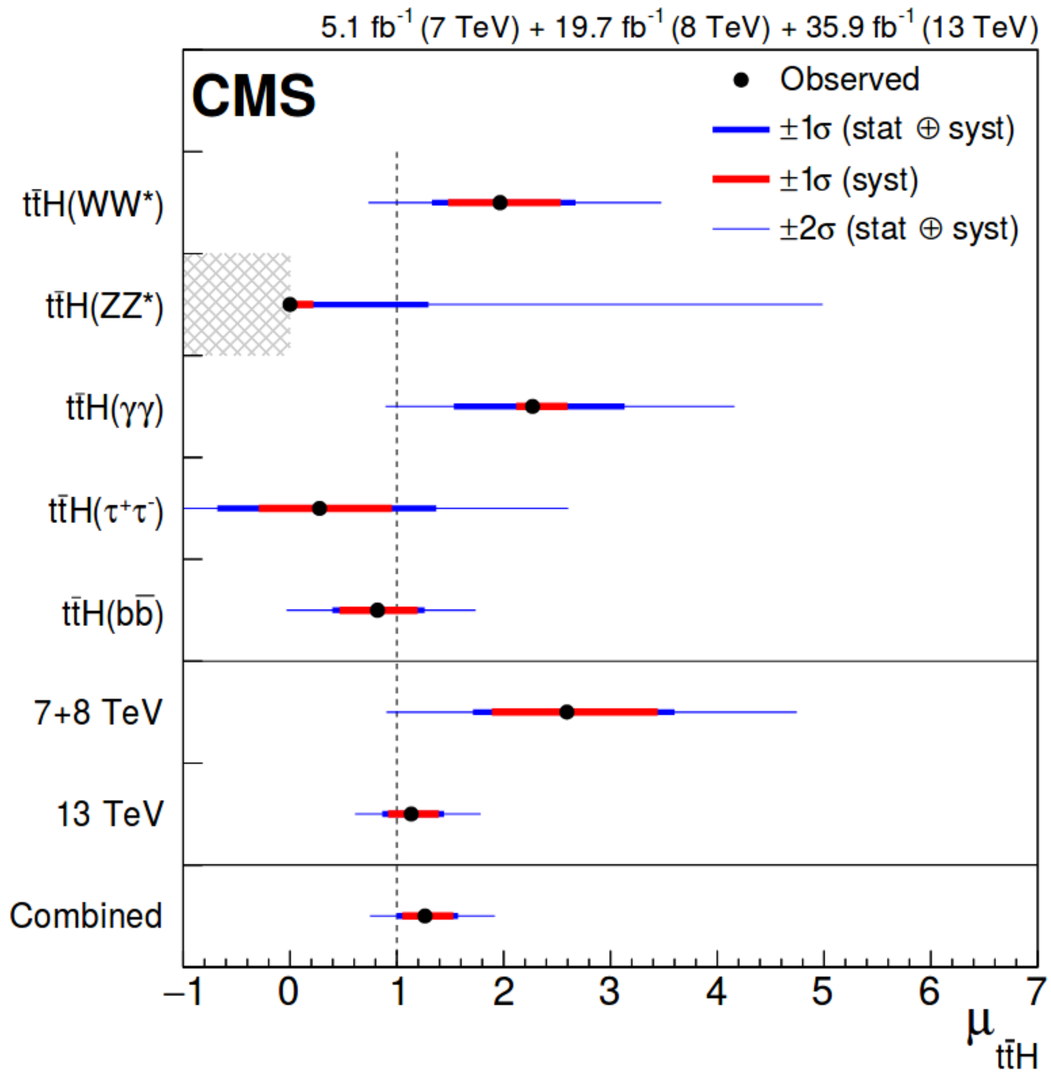
- Only two colored final state particles**:** kinematics and phase space goes beyond two-particle final state, but color structures and Soft Anomalous Dimension matrices stay similar to the very well known case of t-tbar hadroproduction

- t-tbar hadroproduction cross-section known at NNLL level – and provides useful reference for t-tbar-Higgs

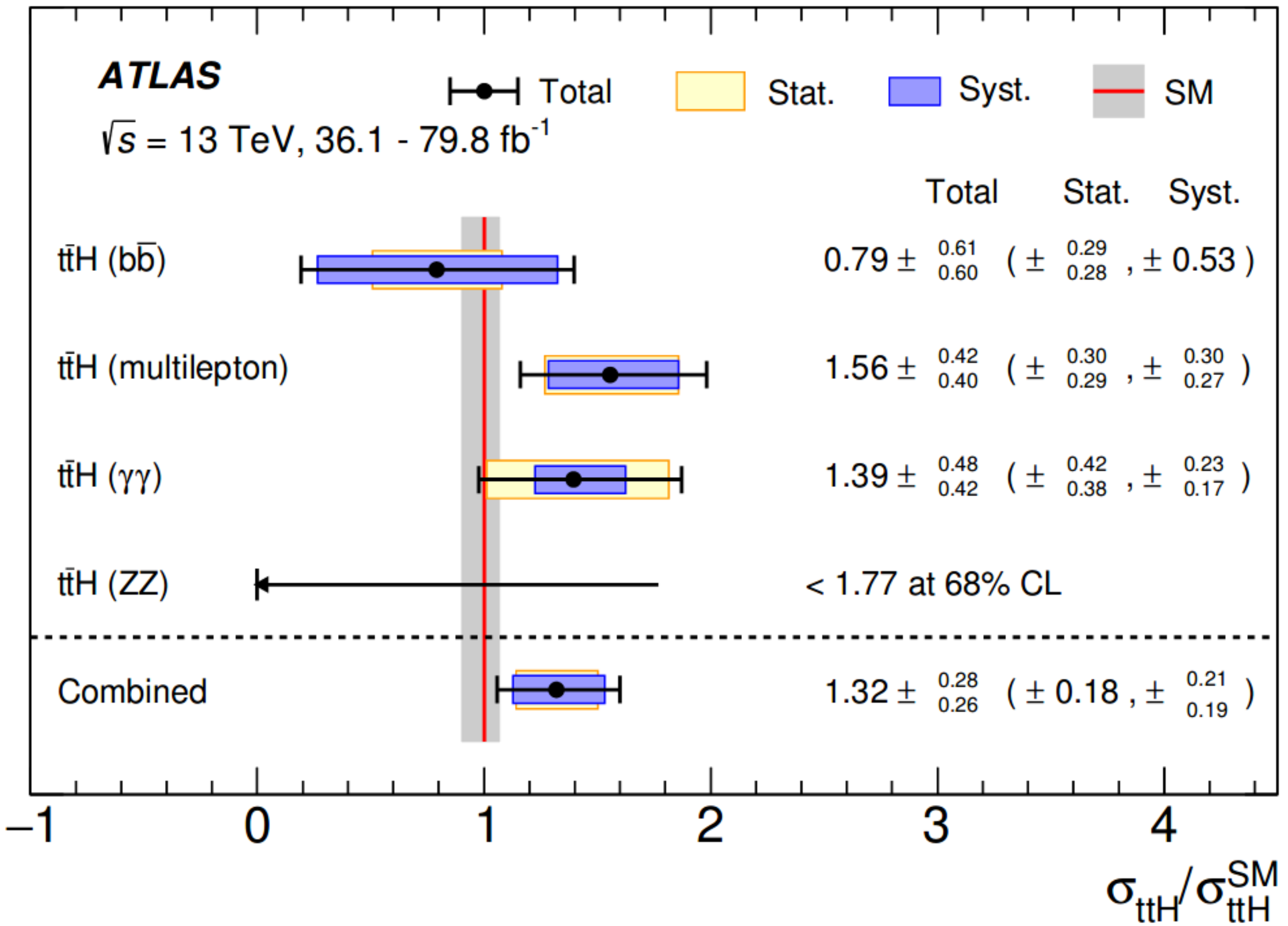
- Dominant soft gluon corrections to inclusive cross-section expected to come from simple kinematical domain of the absolute threshold of

  $s_{part} \to 2m_t + M_H$

# Observation of t-t-Higgs at the LHC (2018)

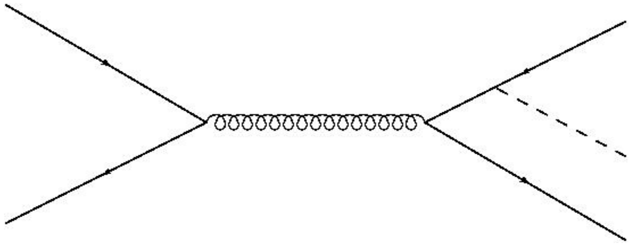


# Observation of t-tbar-Higgs at the LHC (2018)

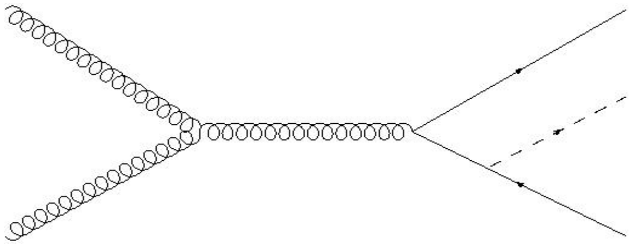


# Associated Higgs boson production at LO: topologies, channels, colours

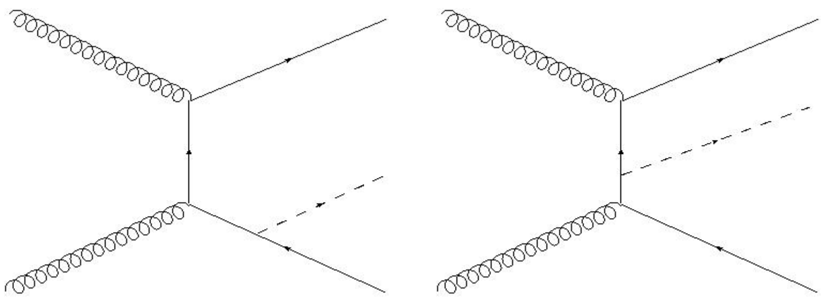
- Higgs radiation off top – anti-top pair



- Flavour singlet quark-antiquark
- channel and gluon channel

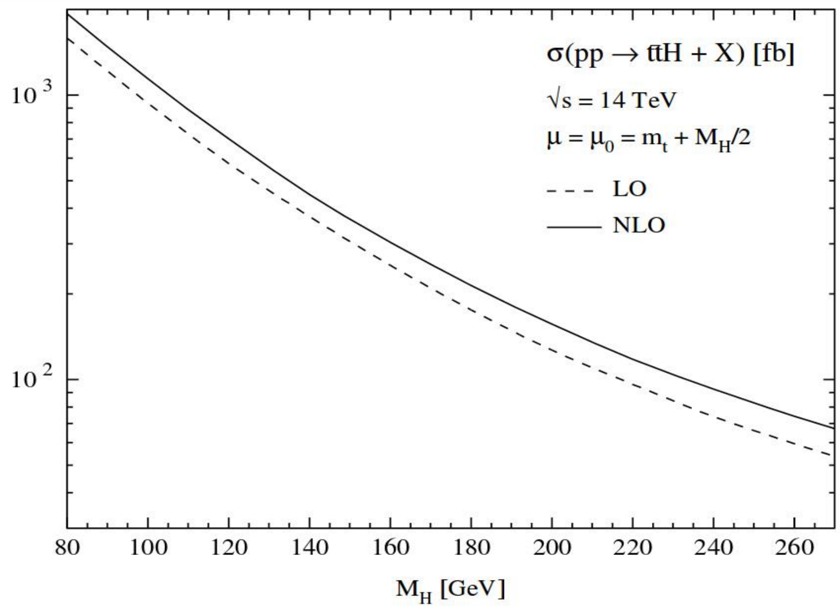


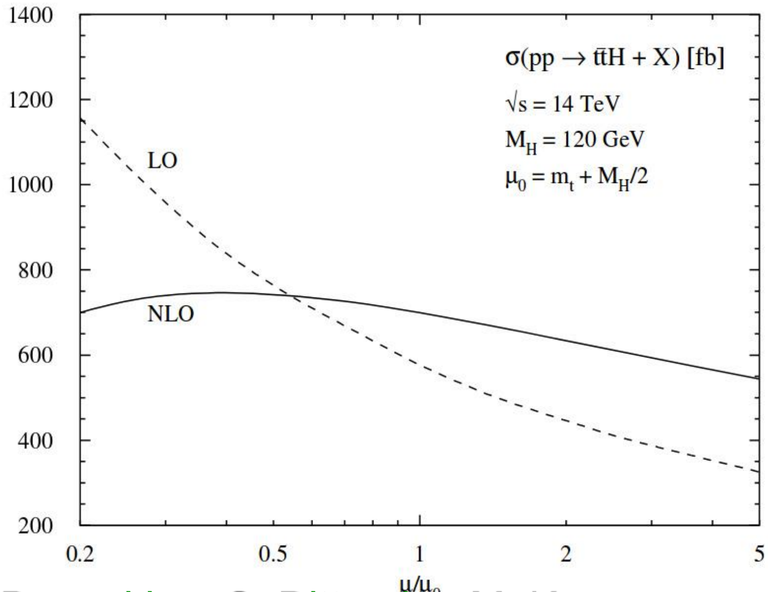
- Color structure:
- qq channel – color octet only
- gg channel – color singlet,
- symmetric color octet and
- antisymmetric color octet



- 3-body phase space – makes higher order precision calculations more demanding. Generalised Mandelstam invariants enter

# Higher order corrections and uncertainties




W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper, M. Spira, P.M. Zerwas, 2001-2002, also Dawson, Reina, Wackeroth, Orr, Jackson 2001-2003

- K-factor NLO/LO for 14 TeV: 1.22,
- Higgs mass 125 GeV
- NLO scale uncertainty: +6.2% -9.4% (obtained by independent variations of $\mu_R$ and $\mu_F$ in (½, 2) $\mu_0$ range)
- NLO electroweak corrections at 1 – 2% level

• S. Frixione, V. Hirshi, D. Pagani, H.G. Shao, M. Zaro, 2014; Y. Zhang, W.-G. Ma, R.-Y. Zhang, C. Chen, L. Guo, 2014

# Parallel efforts in soft gluon resummation in ttH

- Currently two active groups:
- **Direct QCD – Mellin space:** A. Kulesza, D. Schwartlander, T. Stebel, V. Theeuwes, R. Balsach, LM
- **Soft Collinear Effective Theory (SCET)** A. Broggio, A. Ferroglia, B.D. Pecjak, A. Signer and L.L.Yang
- *Soft gluon NLL resummation at the absolute threshold* (direct) arXiv:1509.02780
- *Approximate NNLO from soft gluon NNLL resummation formula in Soft Collinear Effective Theory: invariant mass dependent resummation* (SCET) arXiv:1510.01914
- Invariant mass dependent resummation at NLL (direct) arXiv:1609.01619
- Invariant mass dependent resummation at NNLL (SCET) arXiv:1611.00049
- Invariant mass dependent resummation at NNLL (direct) arXiv:1704.03363
- For W,Z: Differential distributions at NNLL + EW (direct) arXiv:1812.08622, arXiv:2001.03031

- **Approximate NNLO calculation:** S. Catani et al. *Phys.Rev.Lett.* 130 (2023) 11, 111902]

# Soft gluon corrections at NLO

- Close to the absolute threshold: $s_{part} \to s_0 = 2m_t + M_H$ enhanced soft gluon corrections: cancellation of collinear and soft IR singular terms from virtual corrections and real emissions only in phase space region squeezed by kinematics: emergence of double and single logarithms of $\beta = (1\text{-}s_0 / s_{part})^{1/2}$ :

$$\delta\hat{\sigma}_{NLO}\big|_{\log} = \hat{\sigma}_{\rm Born}\frac{2\alpha_s}{\pi}\left\{C_{ab}\left[2\log^2\beta - 3\log\beta - 2\log\beta\log\left(\frac{\mu_F}{2m_t + M_H}\right) - C^{ab}_{\rm FSR}\ln\beta\right]\right\}$$

- In addition, Sommerfeld – Coulomb corrections found:

$\sim (\alpha_s \kappa_I / \beta_{34})\ \sigma_{Born} \to$ promoted to $\sim (\alpha_s / \beta)\, \sigma_{Born}$ corrections to the total cross-section, where $\beta_{34} = (1 - 4m_t^2 / s_{34})^{1/2}$ , due to Coulomb exchanges between massive quarks

- Soft gluon effects at absolute threshold less relevant than usually in the total NLO cross-section as the approach to the threshold $\boldsymbol{\sigma_{Born} \sim \beta^4}$ due to volume measure of 3 massive particle phase space

# Soft gluon resummation

- Framework based on proofs of hard factorization by Collins, Soper and Sterman and by Catani and Trentadue (1980-s)

- Logarithmically enhanced soft gluon corrections may factorized and resummed to all order of perturbation theory keeping:

  $(\alpha_s \log^2 \beta)^n$ at LL, $(\alpha_s^n \log^{2n-1} \beta)$ at NLL, $(\alpha_s^n \log^{2n-2} \beta)$ at NNLL accuracy

- We work up to NNLL accuracy: hard scattering at NLO, soft-collinear logs at NNLO + collinear logs (NNLL) + soft logs (NNLL)

  Collinear logs → incoherent, redefine massless partons

  Soft wide angle logs → coherent, depend on total color current

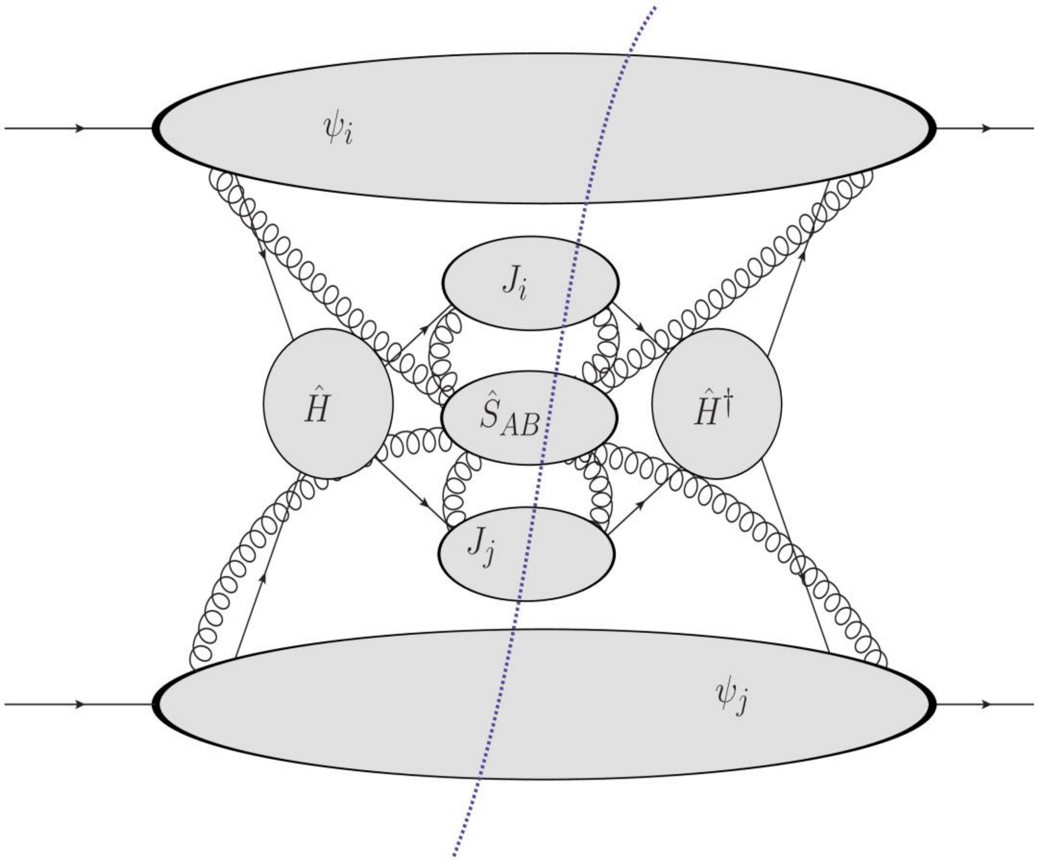
- Resummation may be performed using Renormalization Group technique

- Those corrections may be factorized in the Mellin moment space:

  particularly simple picture of resummation → multiplicative factors at given value of the Mellin moment $N$

# General factorization procedure

$$\hat{\sigma} = \hat{H}^\dagger \otimes \hat{S} \otimes \hat{H} \otimes \psi_i \otimes \psi_j \otimes J_i \otimes J_j$$




$S_{AB}$ — soft gluon matrix

$H$ — hard amplitude matrix

$\psi_i$ — initial state "jet factors" collinear radiation of incoming partons

$J_i$ — collinear "jet factors" for final state massless partons

# Scale evolution of the soft gluon matrix: renormalization group equations

- The anomalous dimension matrix in color tensor space governs the Soft Matrix evolution:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\hat{S}(\mu,g)=-\hat{\Gamma}_S^\dagger\hat{S}(\mu,g)-\hat{S}(\mu,g)\hat{\Gamma}_S$$

$$\hat{\Gamma}_S(g)=-\frac{g}{2}\frac{\partial}{\partial g}\,\mathrm{Res}_{\epsilon\to 0}\hat{Z}_S(g,\epsilon)$$

$$\beta(g)=-g^3\frac{\beta_0}{(4\pi)^2}-g^5\frac{\beta_1}{(4\pi)^4}-\ldots$$

$$\hat{\Gamma}_S=\frac{\hat{\gamma}^{(0)}}{(4\pi)^2}+\frac{\hat{\gamma}^{(1)}}{(4\pi)^4+\ldots}$$

- The evolution between the process scale and the lower cutoff scale on the soft gluon energy: generation of logs of the scale ratio
- Solution in the Mellin space

# Compact summary of the resummation formalism

**Ingredients of the NLL calculation:**

- LO hard matrix in color tensor basis (computed)
- LO soft matrix at initial scale (trivial)
- One loop soft anomalous dimension matrix (computed)
- NLL collinear factors (known)

**NNLL calculation:**

- Hard matrix in color tesor basis at one loop (extracted from NLO QCD calculation)
- One-loop soft matrix at one initial scale (computed)
- Two-loop soft anomalous dimension matrix
- NNLL collinear factors (known)

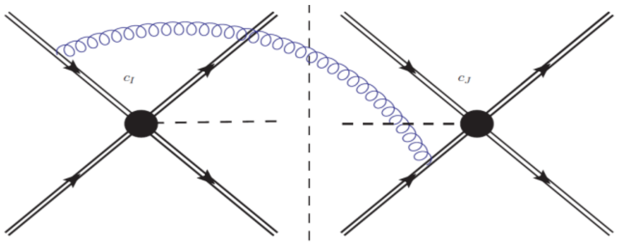
# Soft anomalous dimension – computation

- Colour structures: standard $2 \to 2$ s-channel basis of color tensors is sufficient:

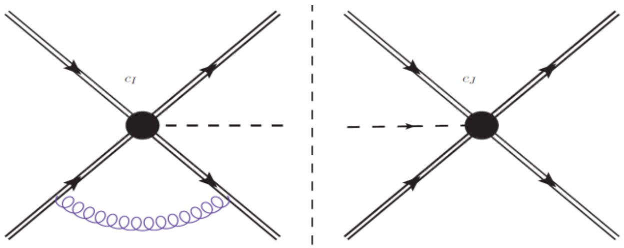
$$q\bar{q}: \qquad 1_{\alpha_1\alpha_2} \otimes 1_{\alpha_3\alpha_4} \qquad t^a_{\alpha_1\alpha_2} \otimes t^a_{\alpha_3\alpha_4}$$

$$gg: \qquad 1^{a_1a_2} \otimes 1_{\alpha_3\alpha_4}\,, \qquad i f^{a_1a_2b} \otimes t^b_{\alpha_3\alpha_4}\,, \qquad d^{a_1a_2b} \otimes t^b_{\alpha_3\alpha_4}$$

- Eikonal integrals (up to power corrections in soft gluon energy)



- Soft anomalous dimension matrices determined from IR singularities (in dimensional regularization) of the virtual diagrams



- Mixing between the color tensors: anomalous dimension matrices

## Organizing the perturbative series:

- In Mellin representation $\log\beta$ -s translate into Mellin moments logs: log $N$

- **Hence the LL corresponds to $(\alpha_s \log^2 N)^n$ ,**
  **NLL to $\alpha_s \log N\, (\alpha_s \log^2 N)^{n-1}$ , NNLL to $\alpha_s\, (\alpha_s \log^2 N)^{n-1}$**

- The LL terms, come from the LO soft–collinear corrections

- The soft (wide angle) gluon matrix enters at NLL (exponentiating result), and its first correction at NNLL

- The Mellin space resumming factors organized into functions at given level of accuracy: $g_1(\alpha_s ,N)$, $g_2(\alpha_s ,N)$ *and* $g_3(\alpha_s ,N)$

# Matching to NLO

- In order to make full use of available information:
  matching of soft gluon resummation to the existing NLO calculations

- The NLO cross-section implemented in MC codes:
  PowHEG BOX, aMC@NLO and SHERPA

- From the cross-section at the NLL / NNLL accuracy the part beyond fixed order NLO expansion is taken and combined with the exact NLO result

  → **NLL / NNLL cross-section matched to NLO**

- Matching to NNLO fixed order results also possible

# Beyond NLO, towards NNLL: NLO hard matrix element

- The available NLO calculations give access to the value of NLO correction in the threshold limit

$$\mathbf{H}_{ij\to klB} = \mathbf{H}^{(0)}_{ij\to klB} + \frac{\alpha_{\mathrm{s}}}{\pi}\mathbf{H}^{(1)}_{ij\to klB} + \dots$$

- Part of this correction coincides with the LL and NLL soft gluon logarithms → taken care by the soft gluon resummation

- The reminder of the NLO correction constant at the threshold limit (as function of the Mellin moment N) → hard matrix at NLO

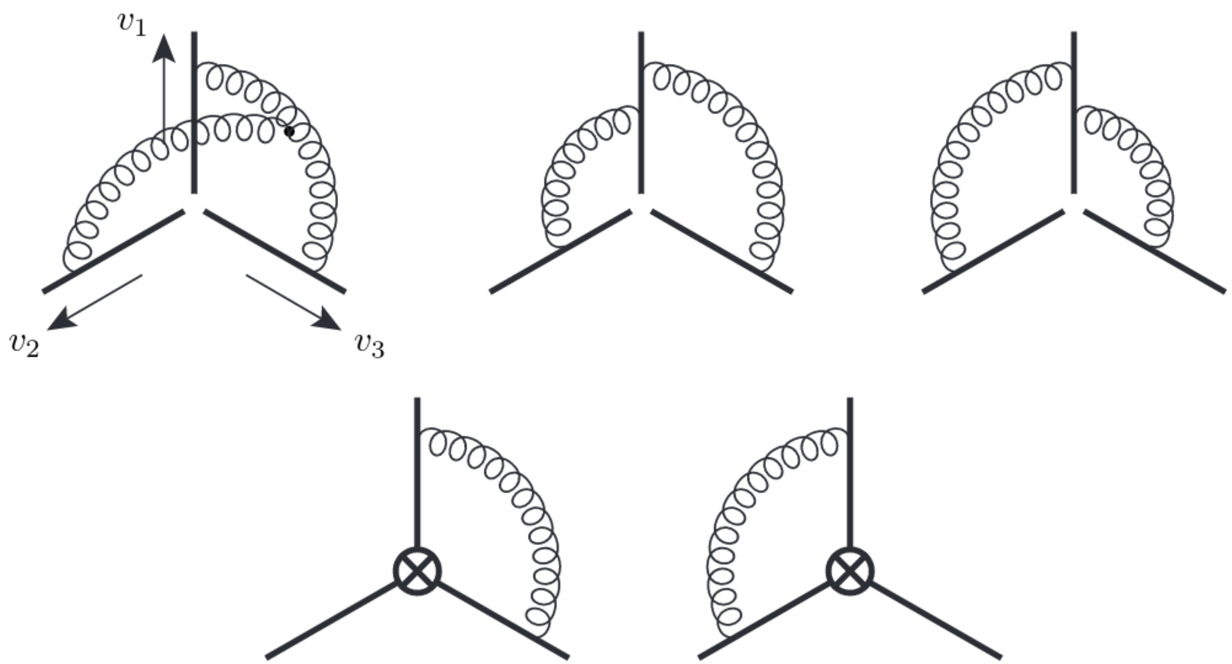
- Inclusion of NLO correction in the hard matrix element → necessary part of NNLL resummation, but it may be also used as an improvement of the NLL resummation → customary in soft gluon resummation in Higgs boson physics

# Soft anomalous dimension at NNLL

- Soft anomalous dimension – new topologies with three eikonal lines

- Color structures containing 3 SU(3) generators: $T^a T^b T^c$
  [A. Ferroglia, M. [Neubert, B. Pecjak, L. L. Yang, Phys.Rev.Lett. 103 (2009) 201601,

JHEP 0911 (2009) 062]



# Resummation for differential distributions: Q-variable

- Picture: resummation of differential distributions picks soft (collinear, and soft wide angle) logarithms depending on the section of the phase space

- The simplest variable: total invariant mass Q of final state particles:

$$Q^2 = (p_t^2 + p_{\bar{t}}^2 + p_H^2)$$

- Emergence of logs: cut-off on the available emitted gluon energy is given by difference of the invariant masses of the initial partonic and the final state

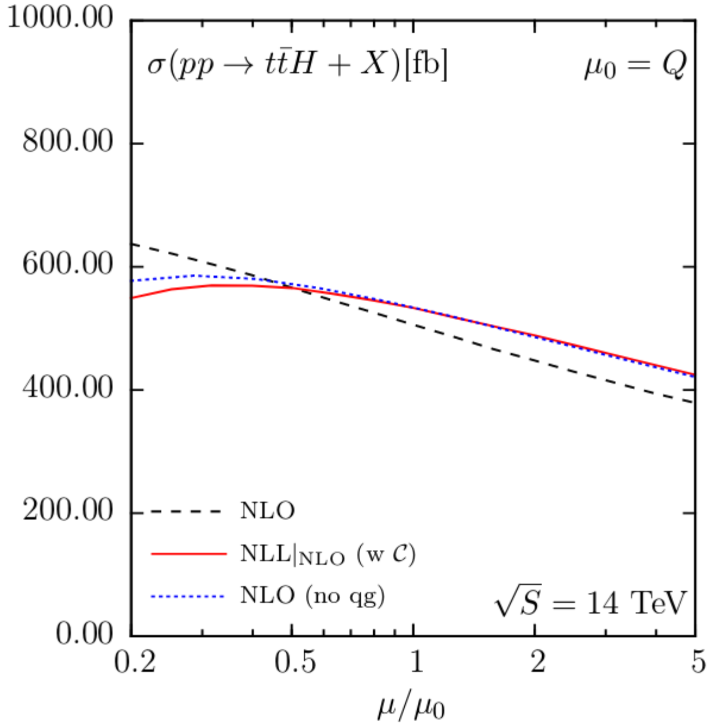
- Invariant mass resummation: coefficients $c_1$ and $c_2$ of the collinear weights are equal to 1

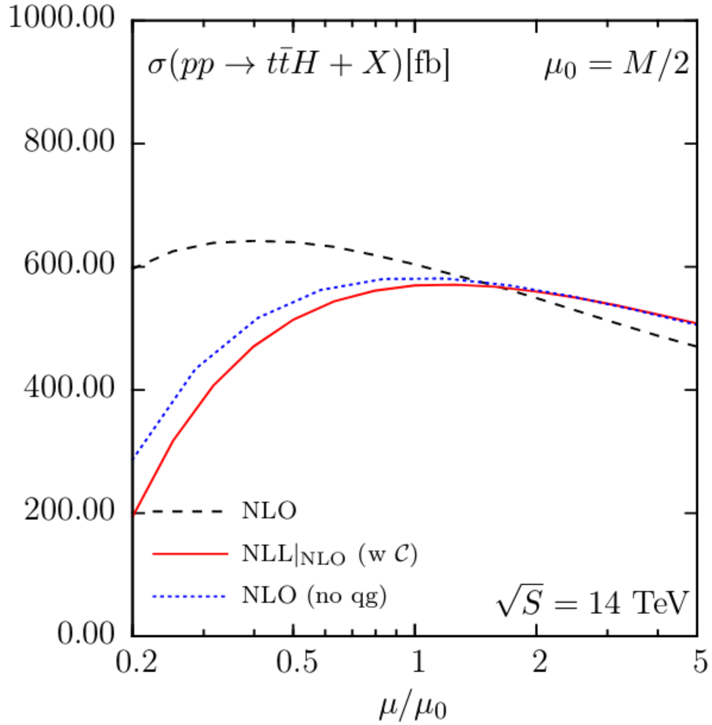
- In differential distributions terms $\sim \log^n(z) / (1-z)_+$ appear, that lead to absolute threshold $\log^{n+1}\beta$ after integration

- Nomenclature: absolute threshold: M-scheme, invariant mass: Q-scheme

# Results: NLO approximate vs exact

- Quality test of the approach: comparison of expanded NNLL result to exact NLO result

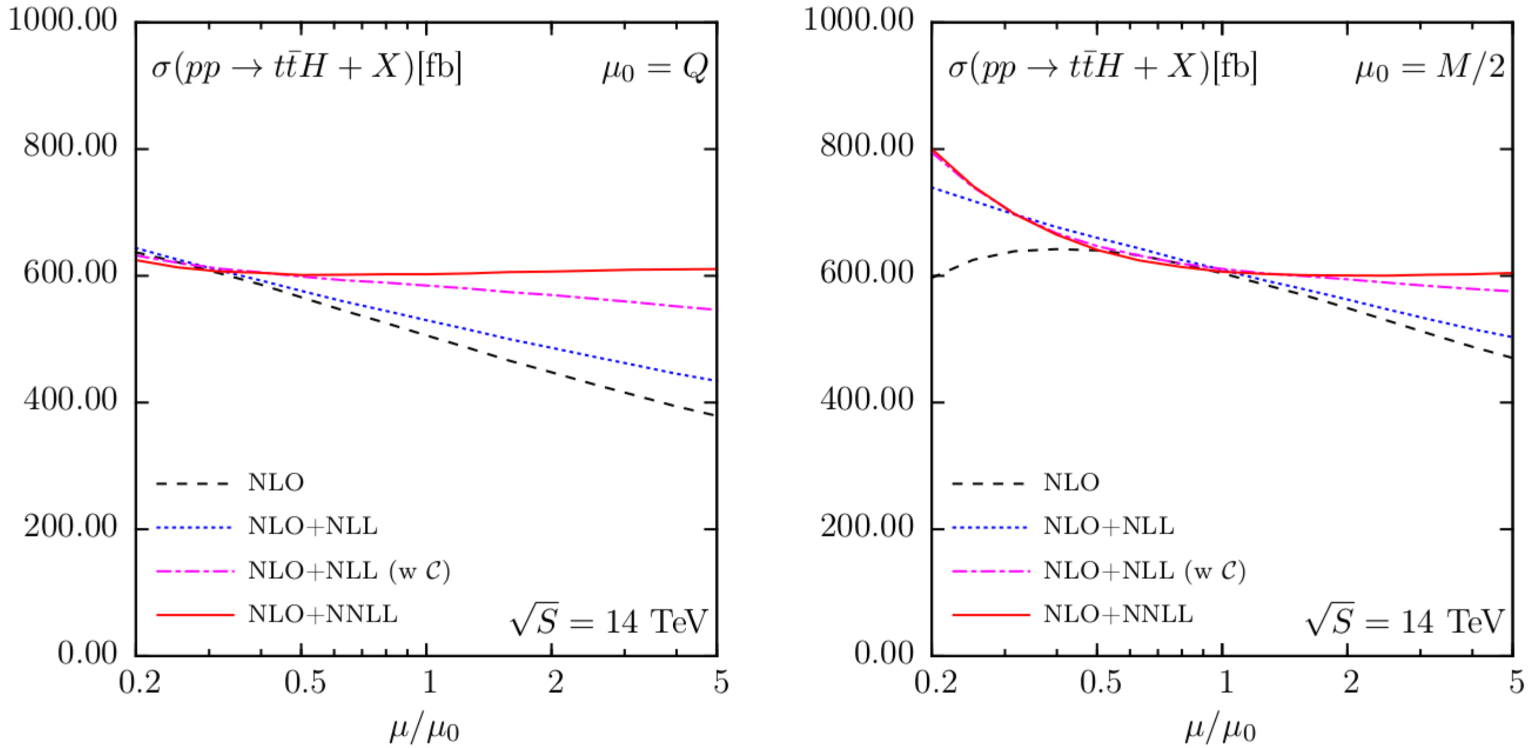



- In the plot the qg channel that enters at NLO as a subleading piece is subtracted from the NLO as it is not generated in the soft gluon resummation
- The NLO result is well approximated

# Results: Q-dependent threshold resummation at NNLL: renormalization scale = factorization scale

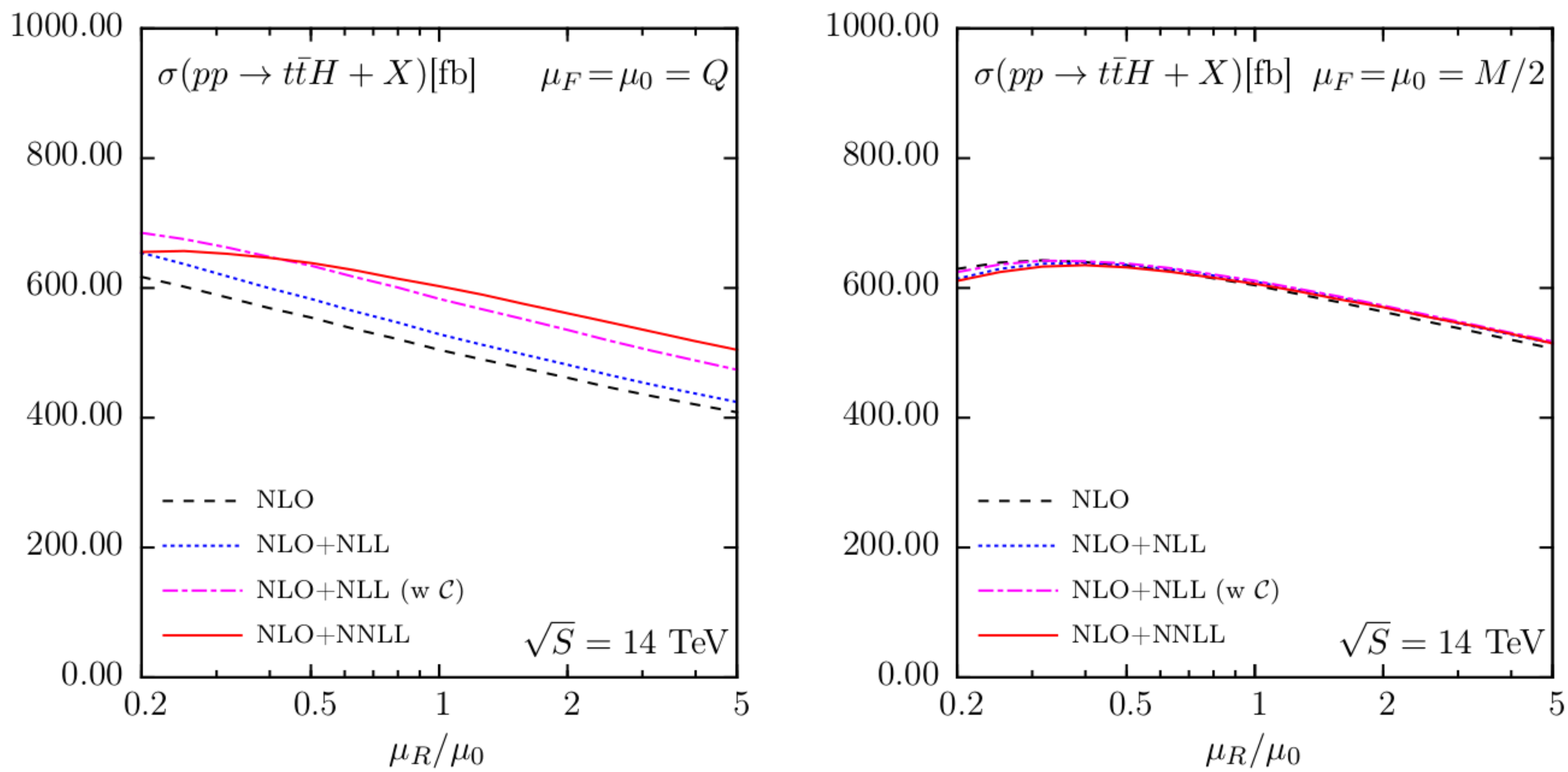
- Scale dependence of total cross-section




- Sizable improvement of the scale stability of NLO+NLLL w.r.t. the NLO
- Increasing theoretical accuracy: NLO → NLO+NLL → NLO+NLLwC → NLO+NNLL leads to increasing stability of predictions

# Results: Q-dependent threshold resummation at NNLL: renormalization scale dependence

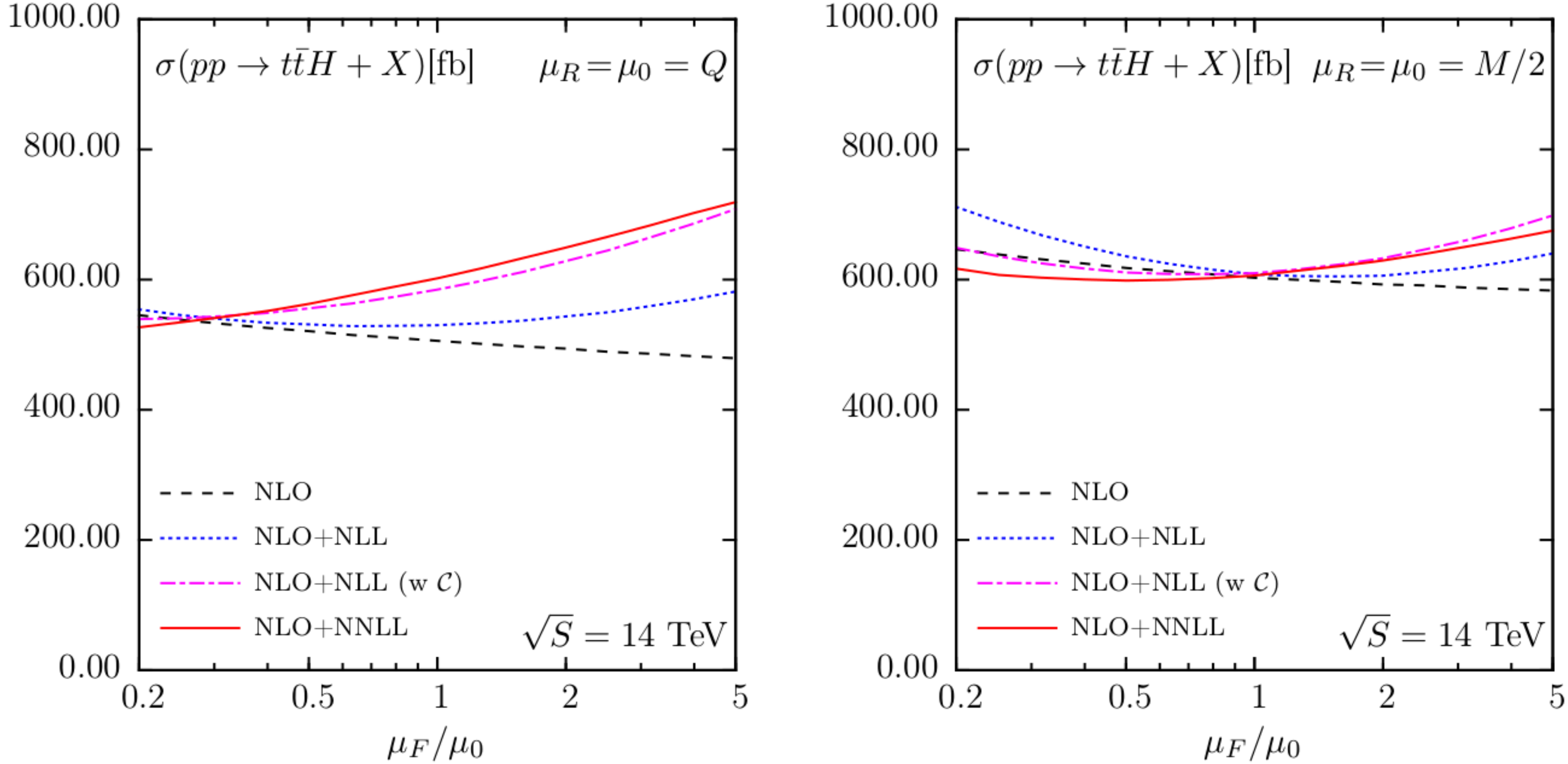
- Renormalization scale dependence of total cross-section




- Moderate improvement due to soft gluon resummation

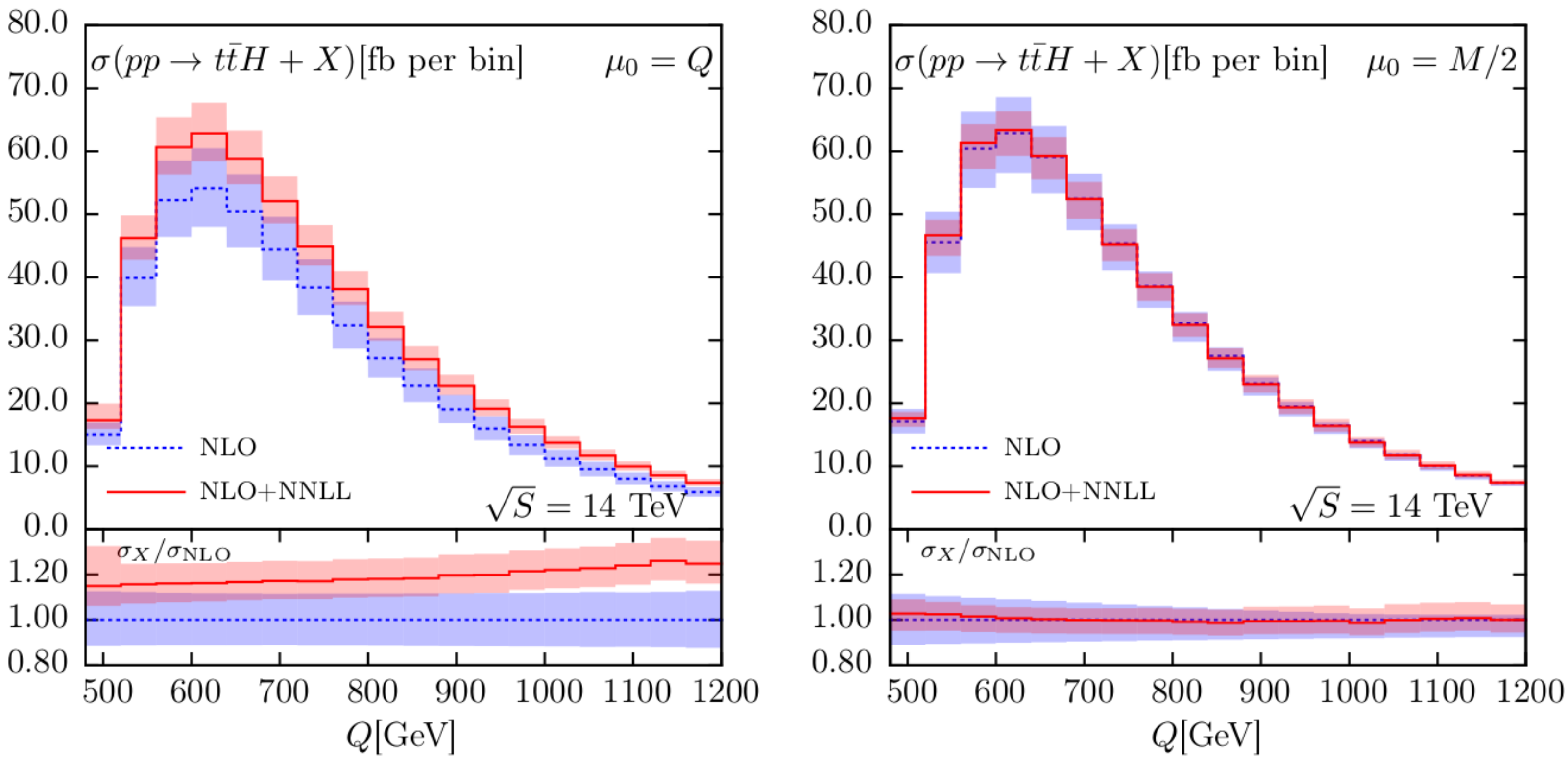
# Results: Q-dependent threshold resummation at NNLL: factorization scale dependence

- Factorization scale dependence of total cross-section




- Stronger effects of resummation on factorization scale dependence

# Results: effects of the NNLL resummation on the invariant mass (Q) distribution




- Sizable improvement of theoretical precision of NLO+NNLL w.r.t. the NLO
- Improvement of the stability of results w.r.t. the choice of the central scale

# Total cross sections at NLO+NNLL

- Scales $\mu_F$ and $\mu_R$ varied independently between ½$\mu_0$ and 2$\mu_0$

| $\sqrt{S}$ [TeV] | $\mu_0$ | NLO+NNLL [fb] | $K_{\rm NNLL}$ factor |
|---|---|---|---|
| 13 | $Q$ | $499^{+7.6\%+2.9\%}_{-6.9\%-2.9\%}$ | 1.19 |
| | $Q/2$ | $498^{+6.0\%+3.0\%}_{-6.3\%-3.0\%}$ | 1.06 |
| | $M/2$ | $502^{+5.3\%+3.1\%}_{-6.0\%-3.1\%}$ | 1.01 |
| 14 | $Q$ | $603^{+7.8\%+2.8\%}_{-6.9\%-2.8\%}$ | 1.22 |
| | $Q/2$ | $602^{+6.0\%+2.9\%}_{-6.4\%-2.9\%}$ | 1.06 |
| | $M/2$ | $607^{+5.7\%+3.0\%}_{-6.1\%-3.0\%}$ | 1.01 |

- K-factors (NNLL+NLO) / NLO close to one for the central scale M/2 → the optimal scale choice for NLO calculations

- Our results are consistent with studies of the same process within SCET formalism: A. Broggio, A. Ferroglia, B.D. Pecjak, A. Signer and L.L.Yang, [JHEP 1603 (2016) 124]

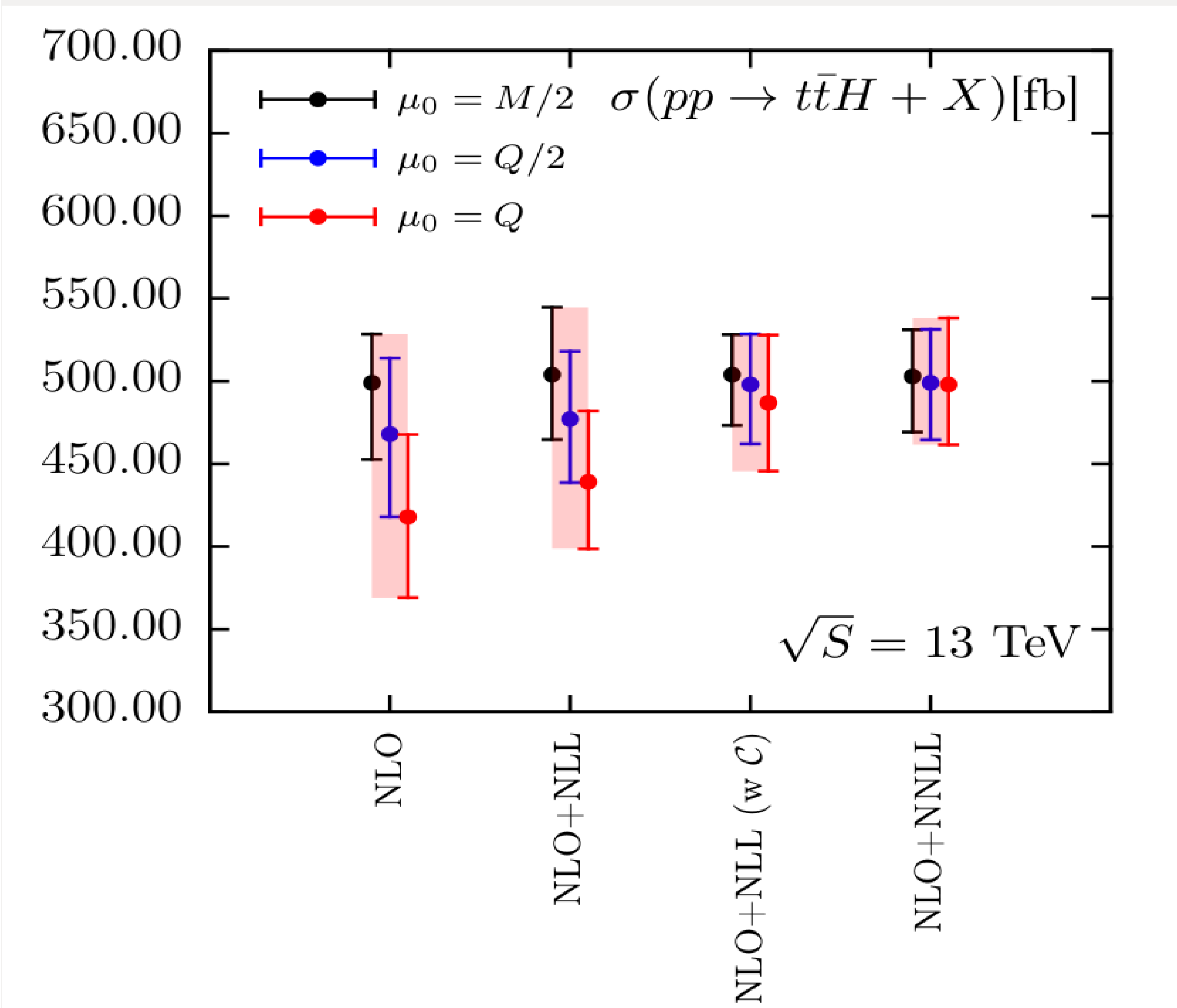
# Predictions: Q-dependent NNLL resummation matched to NLO

- Scales $\mu_F$ and $\mu_R$ varied independently between ½$\mu_0$ and 2$\mu_0$

| $\sqrt{S}$ [TeV] | $\mu_0$ | NLO [fb] | NLO+NLL[fb] | NLO+NLL with $\mathcal{C}$ [fb] | NLO+NNLL[fb] |
|---|---|---|---|---|---|
| 13 | $Q$ | $418^{+11.9\%}_{-11.7\%}$ | $439^{+9.8\%}_{-9.2\%}$ | $484^{+8.2\%}_{-8.5\%}$ | $499^{+7.6\%}_{-6.9\%}$ |
| | $Q/2$ | $468^{+9.8\%}_{-10.7\%}$ | $477^{+8.6\%}_{-8.0\%}$ | $496^{+6.0\%}_{-7.2\%}$ | $498^{+6.0\%}_{-6.3\%}$ |
| | $M/2$ | $499^{+5.9\%}_{-9.3\%}$ | $504^{+8.1\%}_{-7.8\%}$ | $505^{+5.7\%}_{-6.1\%}$ | $502^{+5.3\%}_{-6.0\%}$ |
| 14 | $Q$ | $506^{+11.8\%}_{-11.5\%}$ | $530^{+9.8\%}_{-9.2\%}$ | $585^{+8.3\%}_{-8.5\%}$ | $603^{+7.8\%}_{-6.9\%}$ |
| | $Q/2$ | $566^{+9.9\%}_{-10.6\%}$ | $576^{+8.7\%}_{-8.0\%}$ | $599^{+6.2\%}_{-7.3\%}$ | $602^{+6.0\%}_{-6.4\%}$ |
| | $M/2$ | $604^{+6.1\%}_{-9.2\%}$ | $609^{+8.4\%}_{-7.8\%}$ | $611^{+6.0\%}_{-6.3\%}$ | $607^{+5.7\%}_{-6.1\%}$ |

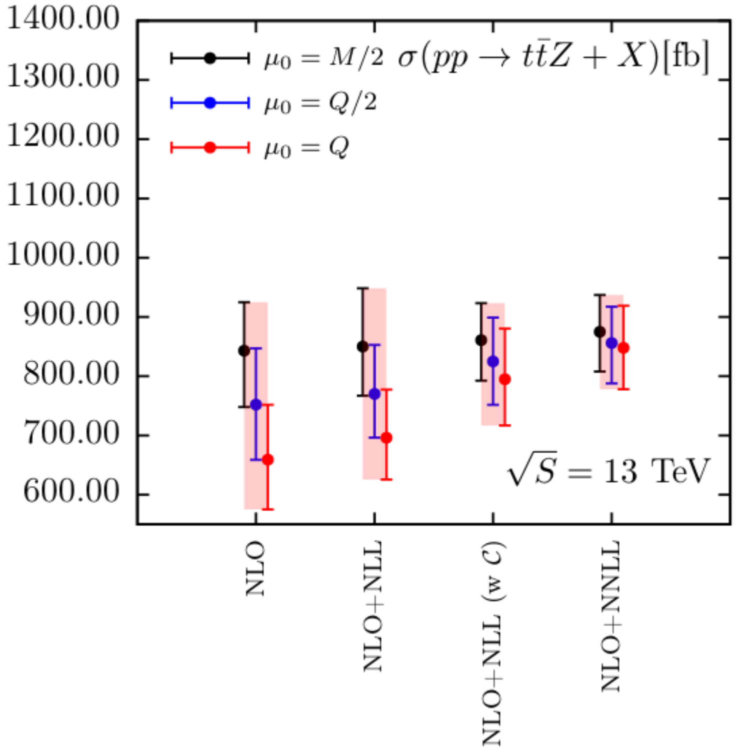
- Improvement of theoretical accuracy at given scale
- Improved stability w.r.t. the central scale variation

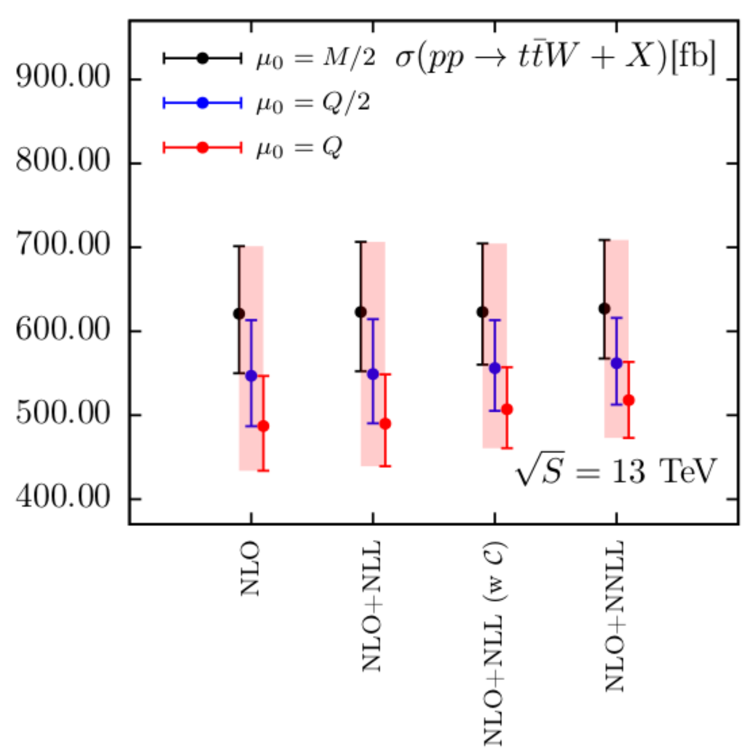
# Predictions: NLO + NNLL invariant mass dependent resummation



# Predictions for t-tbar-Z and t-tbar-W: NLO + NNLL invariant mass dependent resummation

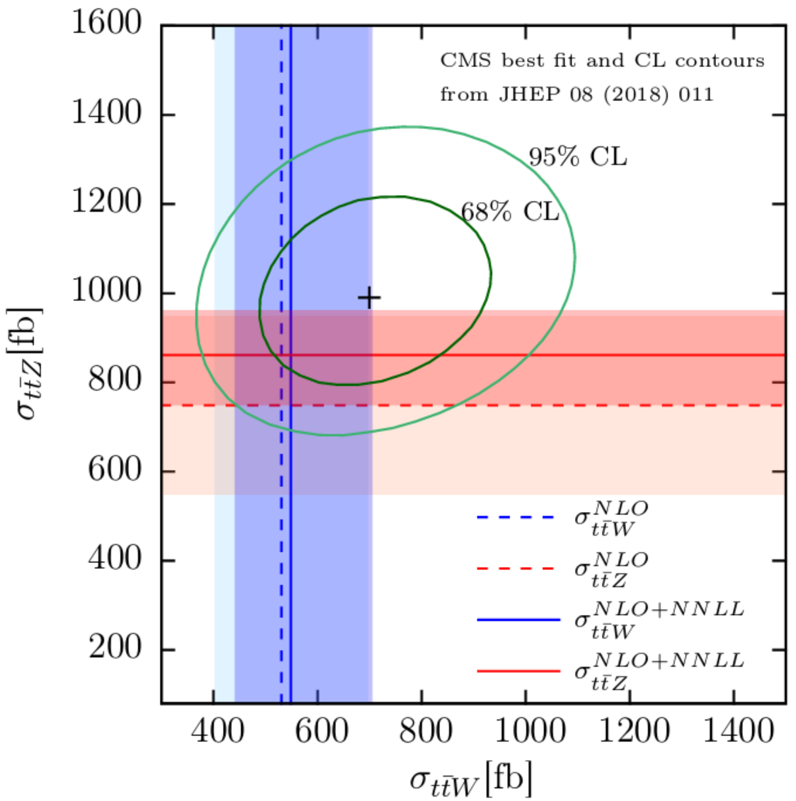
- The same framework as for associated Higgs boson production

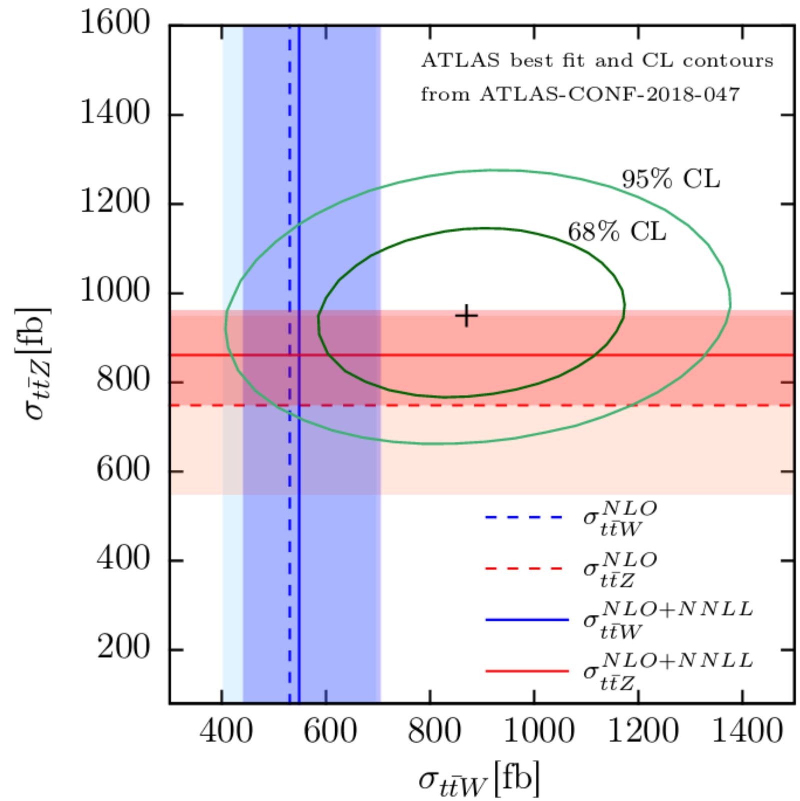



- In the case of Z boson – similar pattern to the Higgs boson
- For the W boson – only small improvement due to resummation → reason: lack of gluon – gluon channel

# NLO + NNLL invariant mass dependent resummation for t-tbar-Z and t-tbar-W: vs CMS and ATLAS data

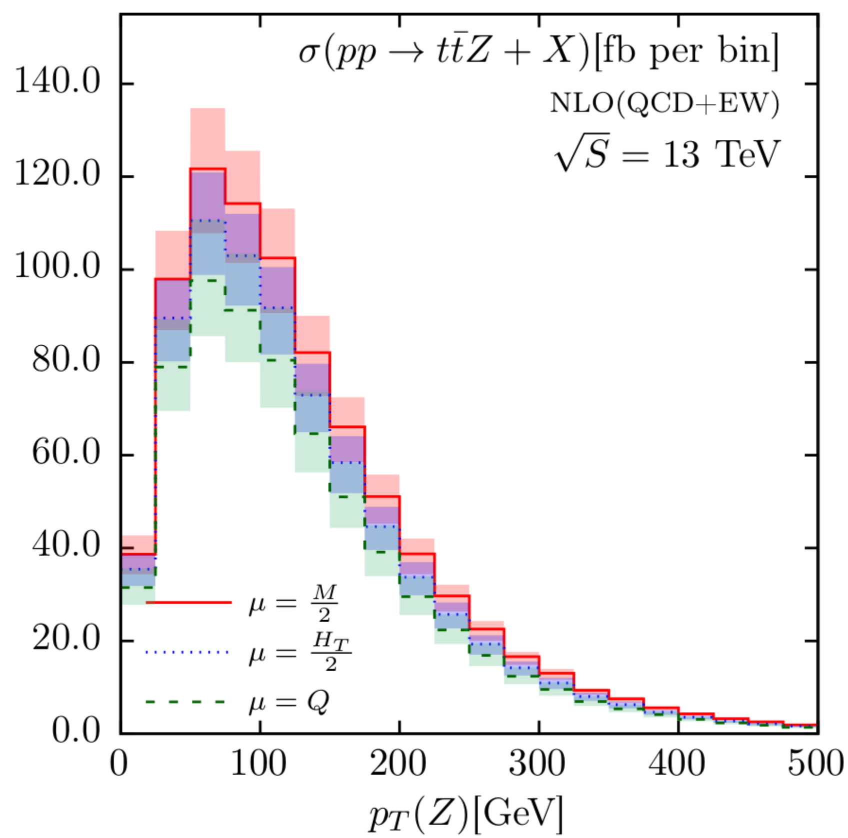



- Theory errors reduced, and for Z made lower than experimental ones
- Measurements consistent with the Standard Model predictions

# Predictions: NLO+NNLL resummation for Z-boson pT-distribution in hadroproduction of t-tbar-Z

- Included: NLO+NNLL QCD + NLO EW




- NLO results show quite some sensitivity to the choice of central scale

# Predictions: NLO+NNLL resummation for Z-boson pT-distribution in hadroproduction of t-tbar-Z

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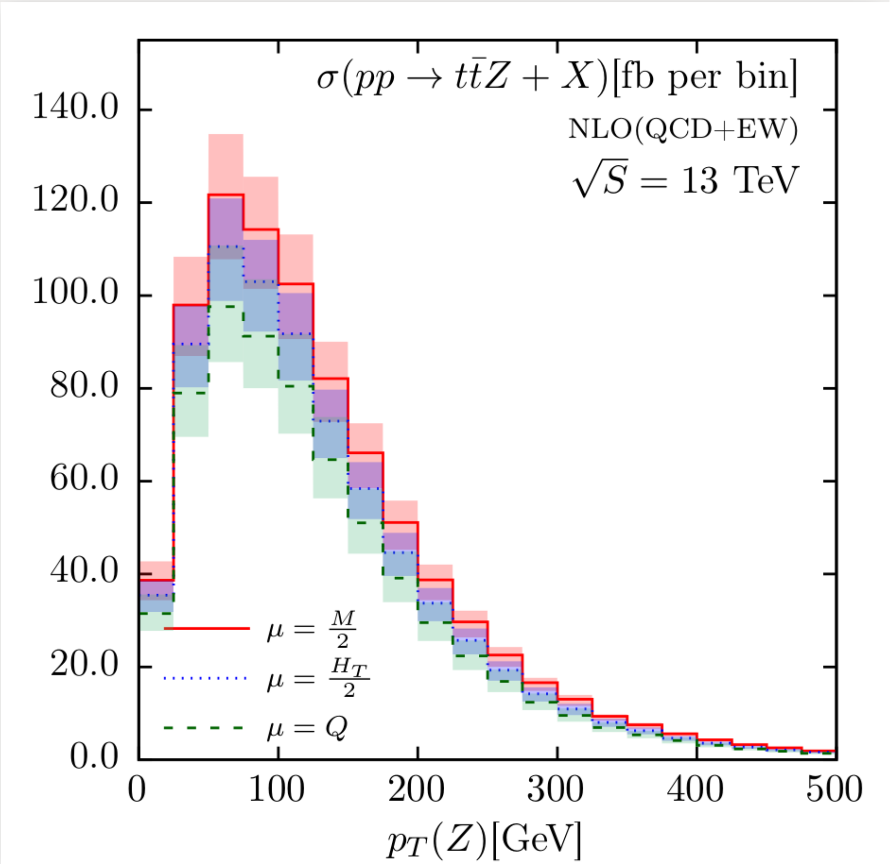
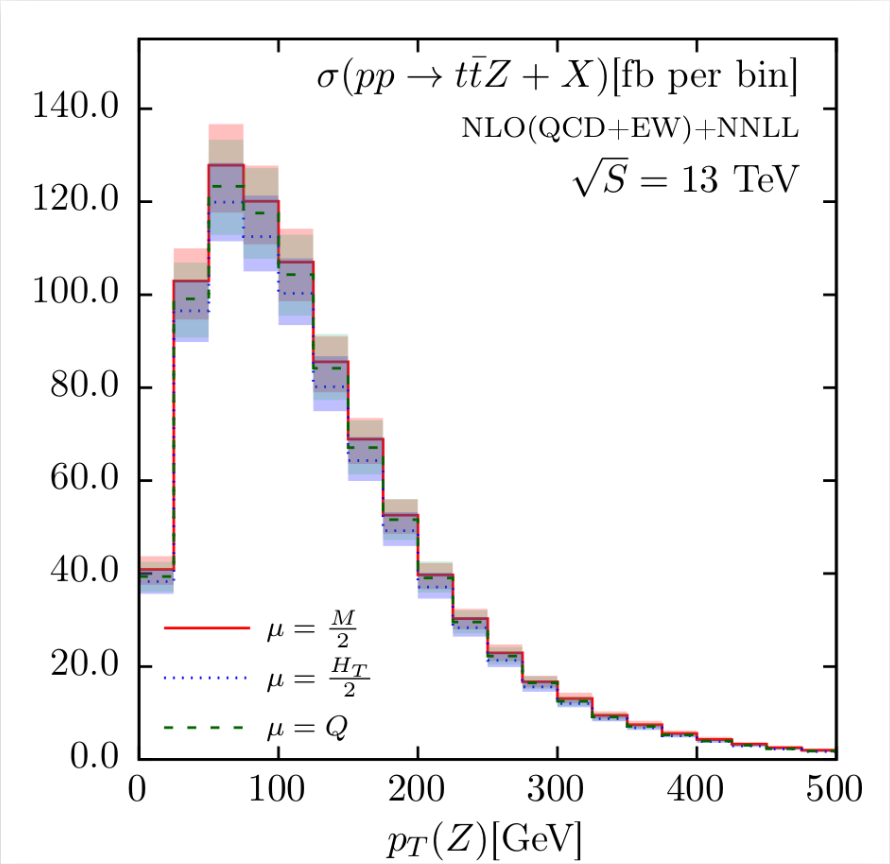
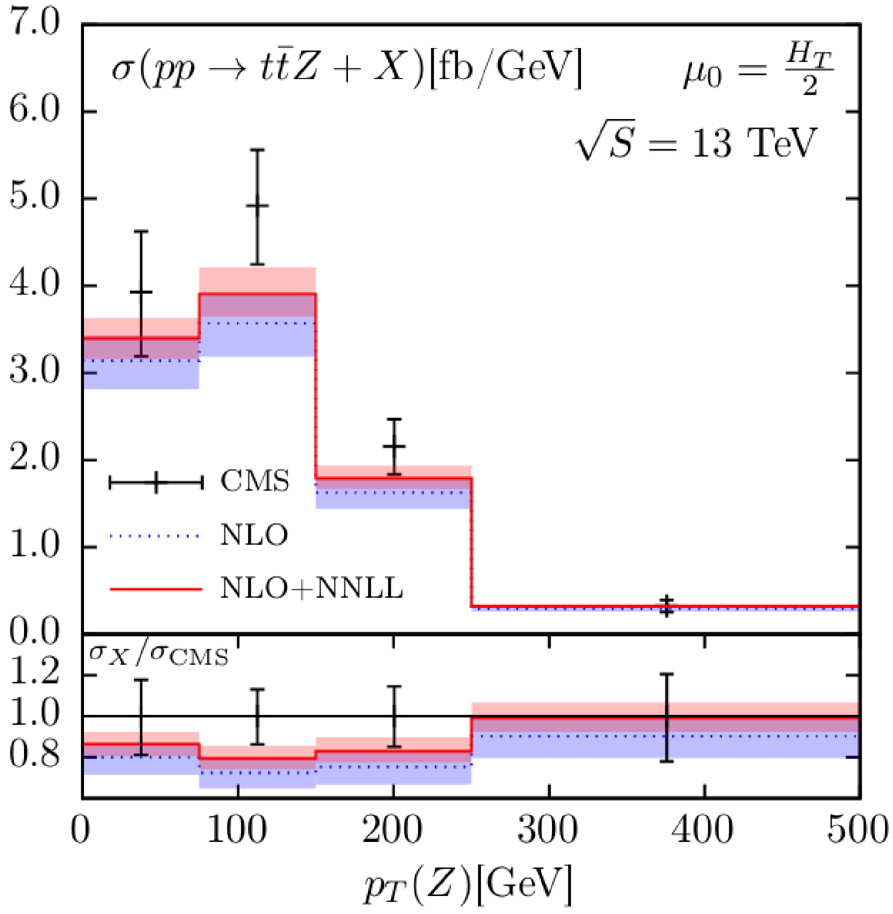



- NLO results show quite some sensitivity to the choice of central scale
- NLO + NNLL results lead to smaller theoretical uncertainty

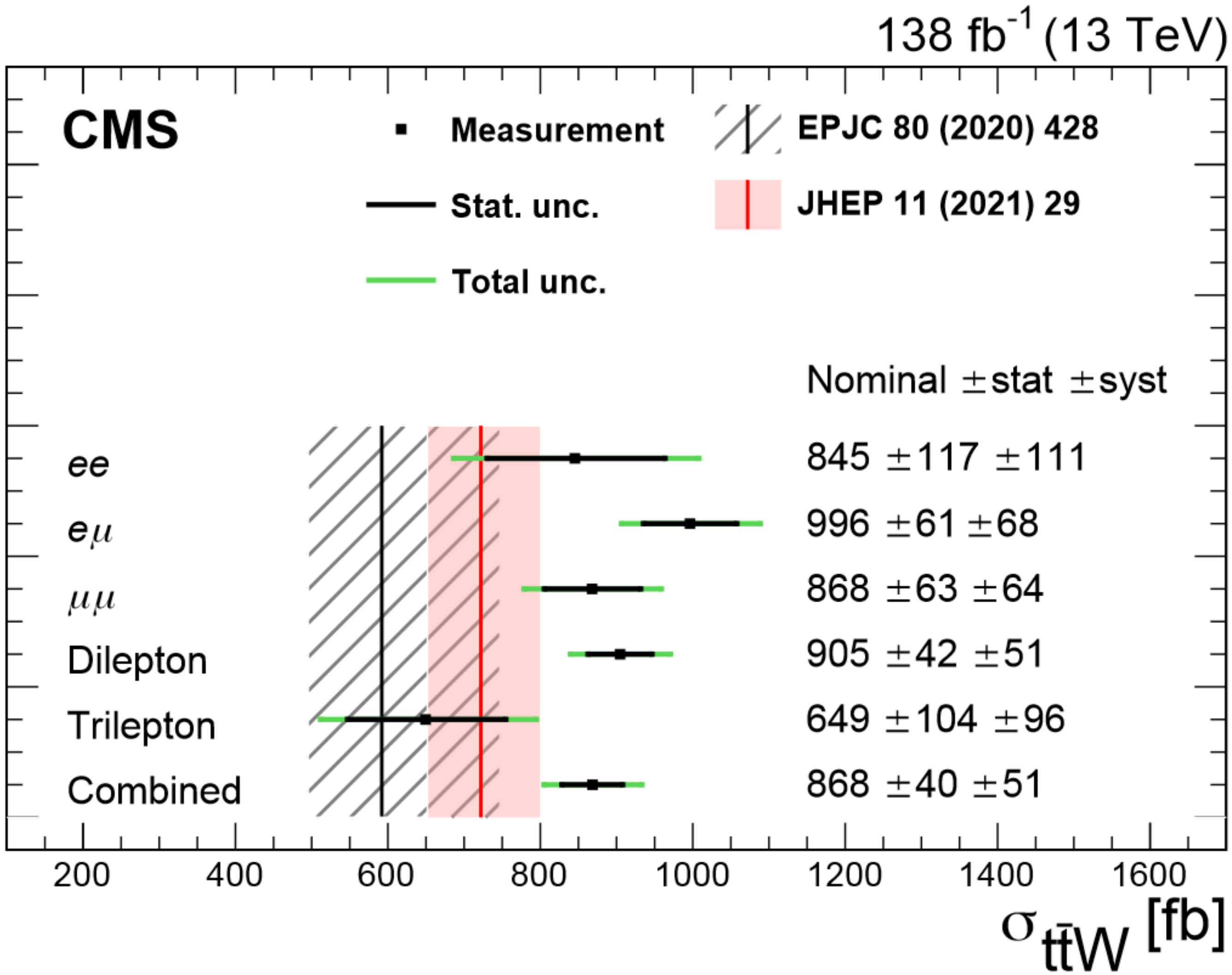
# NLO + NNLL resummation for Z boson pT distribution in t-tbar-Z hadroproduction vs CMS data

- Accuracy: NLO QCD + NLO EW / NLO+NNLL QCD + NLO EW



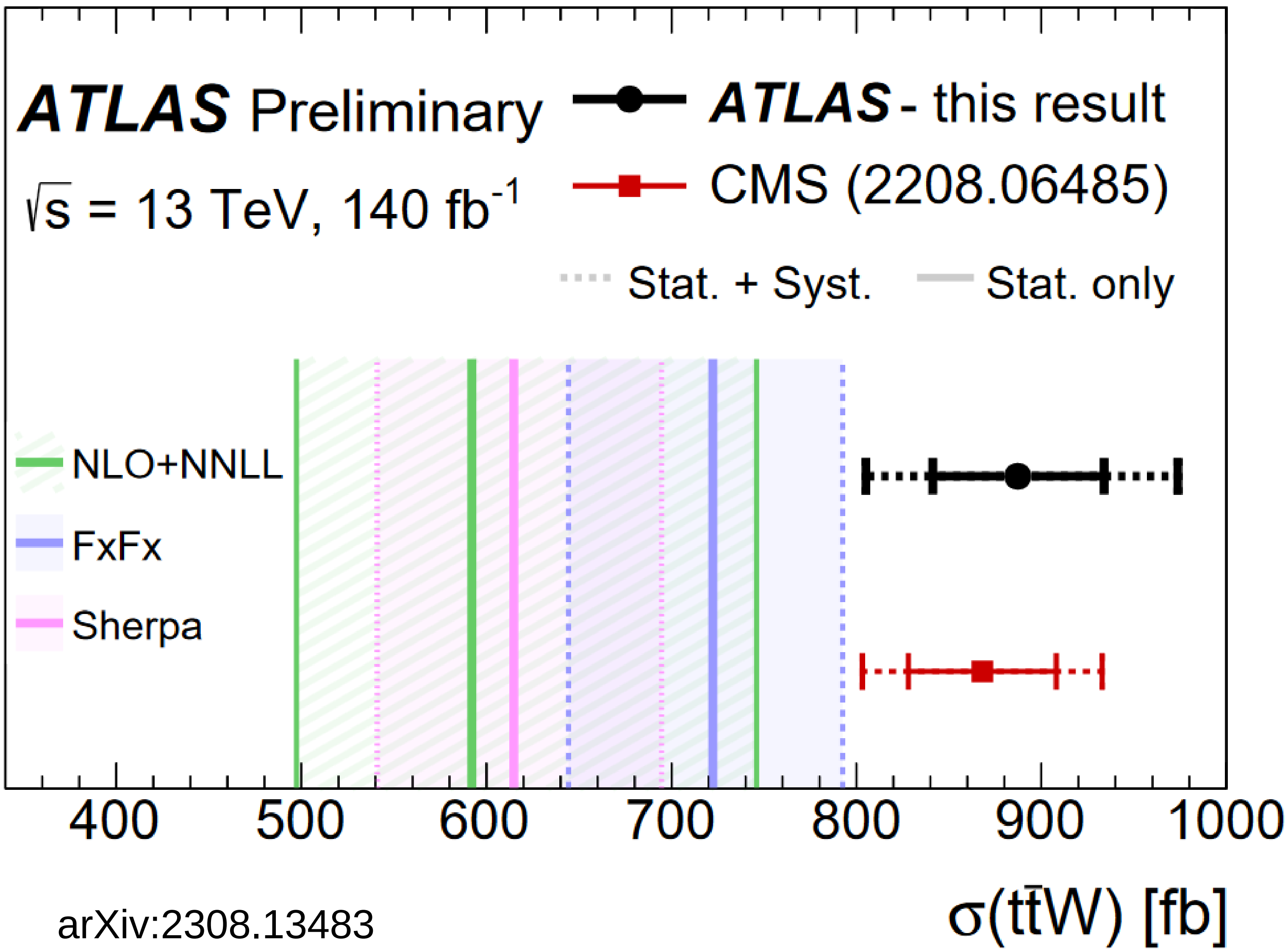

- NLO + NNLL results: closer to data, smaller theoretical uncertainty

# NLO + NNLL resummation for t-tbar-W




arXiv:2208.06485

# NLO + NNLL resummation for t-tbar-W




arXiv:2308.13483

# Most recent news for t-tbar-Higgs: approximate NNLO calculation

- The exact two-loop calculation for t-t-Higgs is not yet in reach, but an approximate NNLO calculation improves the theoretical accuracy

[S. Catani et al. *Phys.Rev.Lett.* 130 (2023) 11, 111902] (Oct. 2022)

- The key ingredient: approximation the Higgs boson as a soft particle

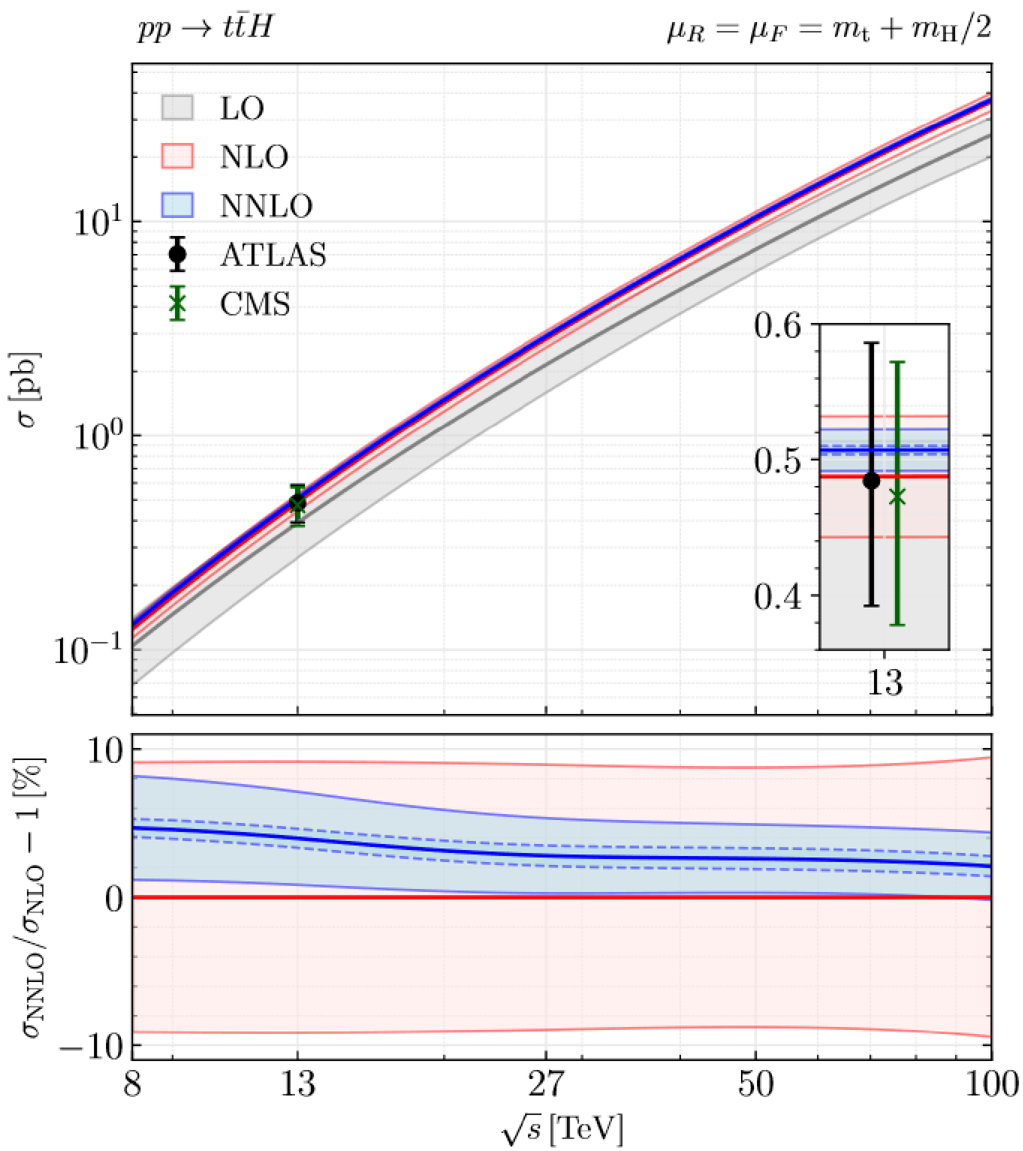
$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R);\tfrac{m_t}{\mu_R})\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\,\mathcal{M}(\{p_i\})$$

- Accuracy of this approximation was tested against LO and NLO calculations, the overall effect estimated to results with ~ 0.6% additional uncertainty
- Results:

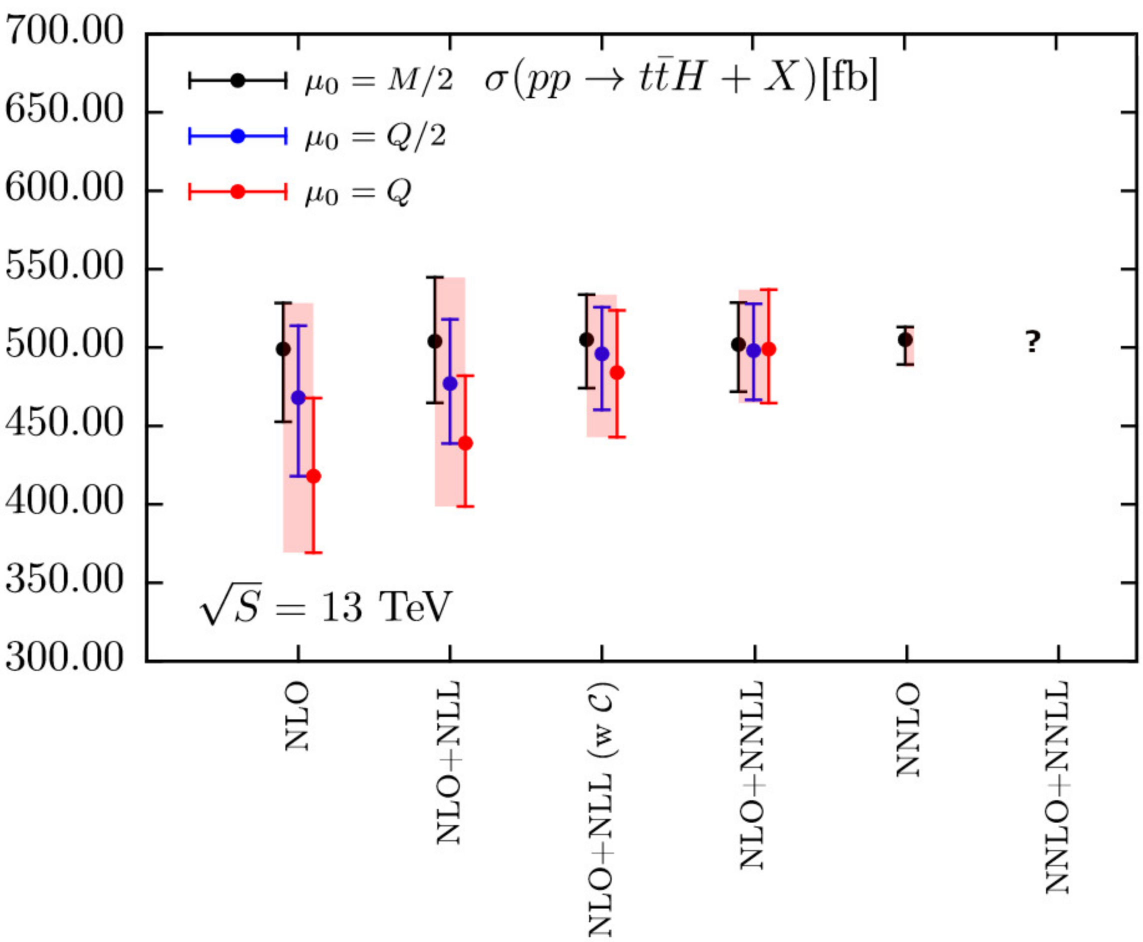
| $\sigma$ [pb] | $\sqrt{s}=13$ TeV | $\sqrt{s}=100$ TeV |
|---|---|---|
| $\sigma_{\rm LO}$ | $0.3910\,^{+31.3\%}_{-22.2\%}$ | $25.38\,^{+21.1\%}_{-16.0\%}$ |
| $\sigma_{\rm NLO}$ | $0.4875\,^{+5.6\%}_{-9.1\%}$ | $36.43\,^{+9.4\%}_{-8.7\%}$ |
| $\sigma_{\rm NNLO}$ | $0.5070\,(31)\,^{+0.9\%}_{-3.0\%}$ | $37.20(25)\,^{+0.1\%}_{-2.2\%}$ |

# Approximate NNLO predictions

[S. Catani et al. *Phys.Rev.Lett.* 130 (2023) 11, 111902]



# Approximate NNLO + (NNLL?)




- Approximate NNLO greatly improves the theoretical precision
- Consistency of approximate NNLO results with NLO+NNLL
- Room for further improvement:

  NNLO + NNLL

## Summary

- Soft gluon resummation in t-tbar-H, Z and W production in proton – proton collisions have been performed for absolute threshold and mass dependent schemes at NNLL accuracy matched to NLO QCD
- Extension performed of the soft gluon resummation to three body final state with non-trivial colour structure
- Significant improvement of the NLO+NNLL theoretical precision w.r.t. the NLO results
- Recently approximate NNLO calculation become available, consistent with NLO+NNLL predictions, high precision, the theory uncertainty at +- 3.5%
- Further improvement of the theoretical precision by resummation possible

**THANK YOU!**