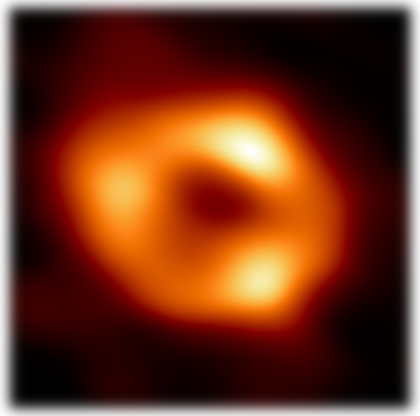
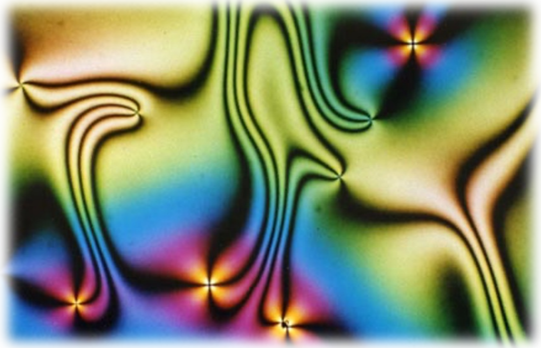


Sagittarius A (EHT)



Liquid crystals (APS)

From black holes to topological defects

A journey at the (fuzzy) frontier between the classical and the quantum realms

with many detours

George Zahariade

JAGIELLONIAN UNIVERSITY IN KRAKÓW

Main question

How to couple quantum and classical degrees to one another?

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- **Description:** states in abstract Hilbert spaces, wavefunction(al)s, Wigner functions…
- **Uncertainty principle:** intrinsic fuzziness due to non-commutative algebraic structure
- **Dynamics:** Schrödinger equation, unitary evolution
- **Measurement:** non-unitary evolution
- **Quantum determinism:** exact predictions of average values, correlation functions, statistical quantities
- **Quantum non-determinism:** probabilistic predictions of results of a measurement

- **Description:** position (configuration space) and momentum (phase space), fields, metrics…
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- **Classical determinism:** exact predictions of the time-evolution of the variables which can be directly measured

VERY DIFFICULT TO DO IN A CONSISTENT WAY…

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BUT DO WE NEED TO?

Just treat everything quantum mechanically!!!

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BUT DO WE NEED TO? YES!

Just treat everything quantum mechanically!!! Not entirely satisfactory…

3 reasons why…

- Macroscopic classicality: *we don't know how/when the transition from quantum to classical happens but it does…*
- Systems/phenomena that behave approximately classically but with quantum corrections, that are classical but originate from quantum mechanics
- GRAVITY

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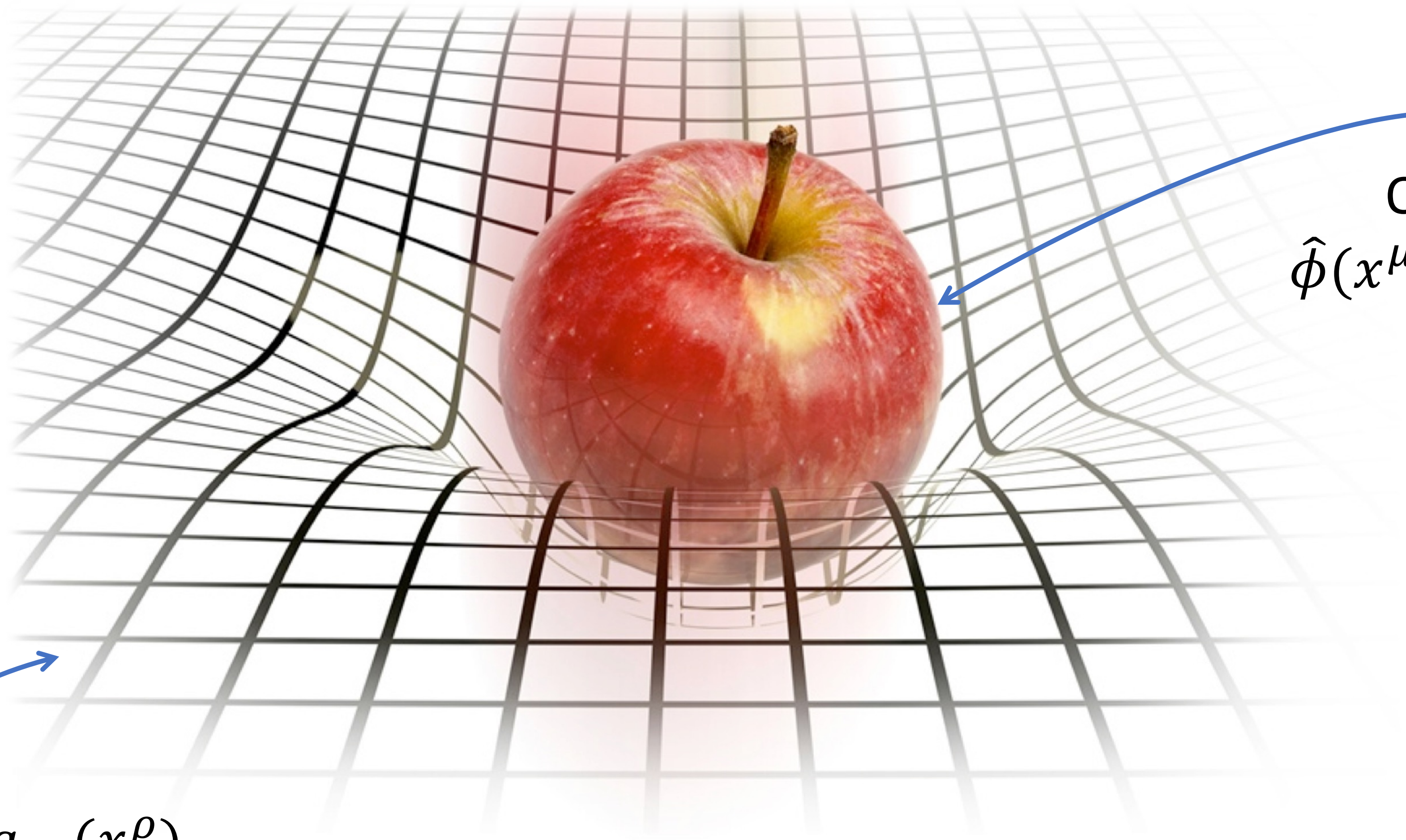
- GRAVITY
    - Progress towards quantizing gravity (loop quantum gravity, causal dynamical triangulations, causal sets, strings, holography…) but no final victor so far
    - Some even argue that gravity may not be fundamentally quantum
    - Famous open problems pertaining to quantum degrees of freedom interacting gravitationally

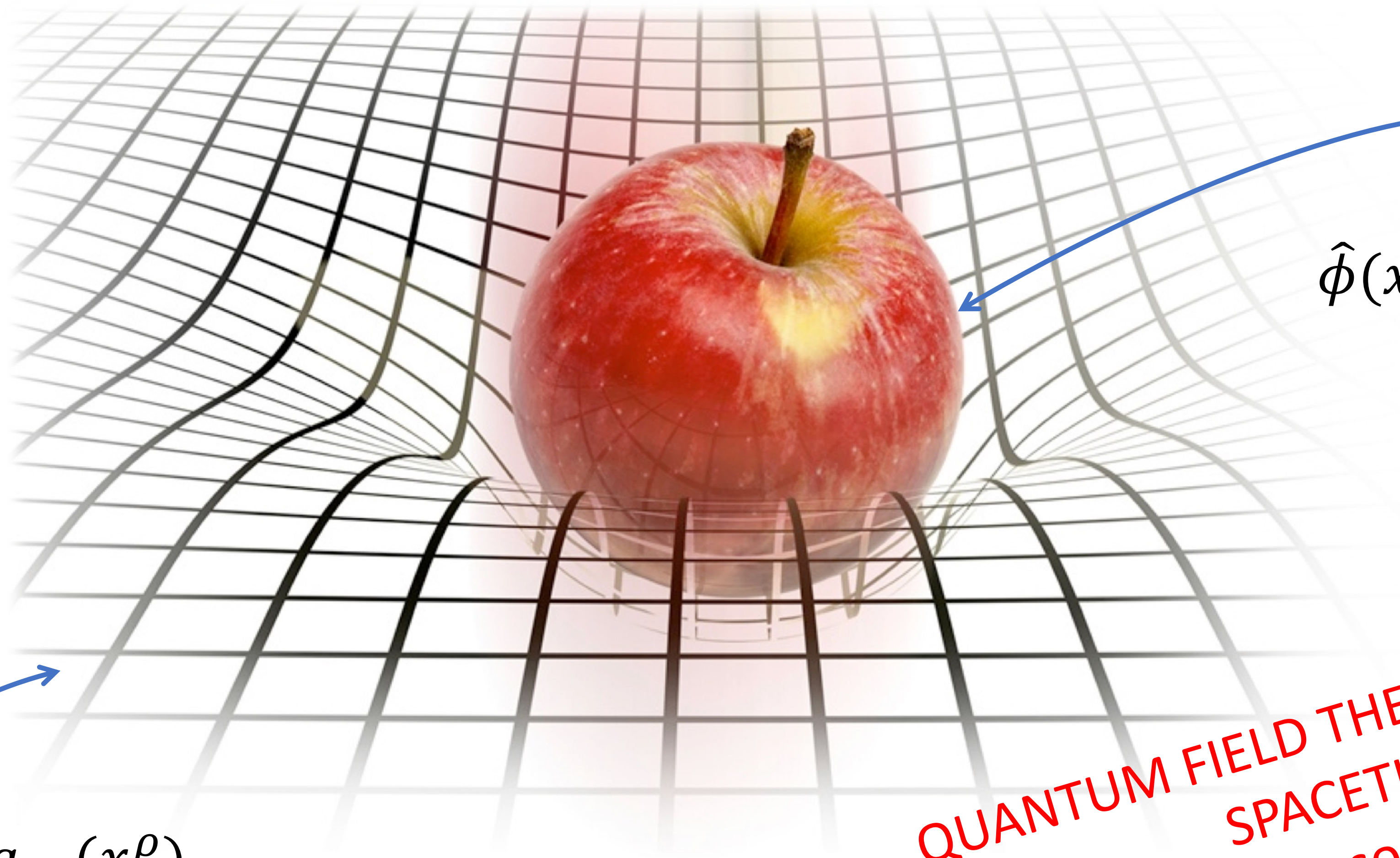
Coupling quantum matter to gravity



Quantum fields
$\hat{\phi}(x^\mu), \hat{\psi}(x^\mu), \hat{A}_\mu(x^\nu)$

Metric tensor $g_{\mu\nu}(x^\rho)$
Riemann tensor $R_{\mu\nu\rho\sigma}(x^\alpha)$

Coupling quantum matter to gravity



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QUANTUM FIELD THEORY IN CURVED SPACETIME:
well-posed (up to some ambiguities) as long as…

Coupling quantum matter to gravity

... quantum backreaction can be neglected (but can it?)

Coupling quantum matter to gravity

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Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci tensor

Ricci scalar

Stress-energy tensor

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CONTRIBUTION FROM QUANTUM FIELDS (semiclassical)

2 (in)famous open problems in gravity

- Cosmological constant problem: zero-point energy of quantum fields sources curvature of spacetime via $\langle 0 | \hat{T}_{\mu\nu} | 0 \rangle \propto \Lambda g_{\mu\nu}$

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W. Pauli

WAY TOO BIG

the radius of curvature of the universe
“would not even reach the moon”

2 (in)famous open problems in gravity

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- Quantum evaporation of black holes: information loss, violation of unitarity

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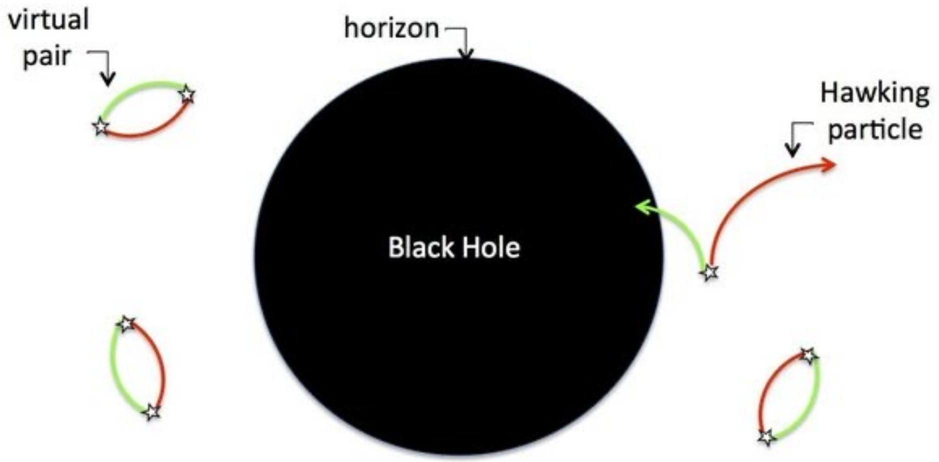
S. Hawking

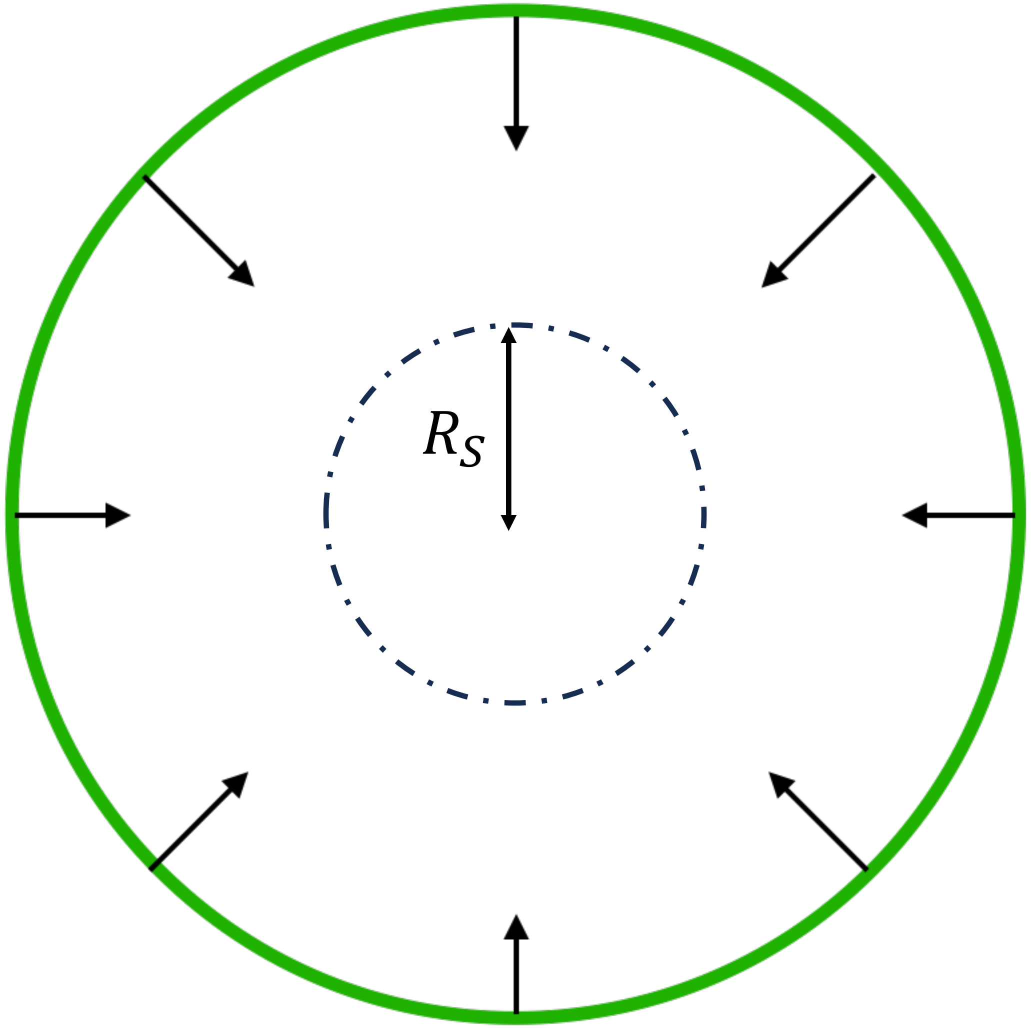
$$R_S = \frac{2GM}{c^2}$$



Thermal radiation: $T_H = \frac{\hbar c^3}{8\pi G M k_B}$

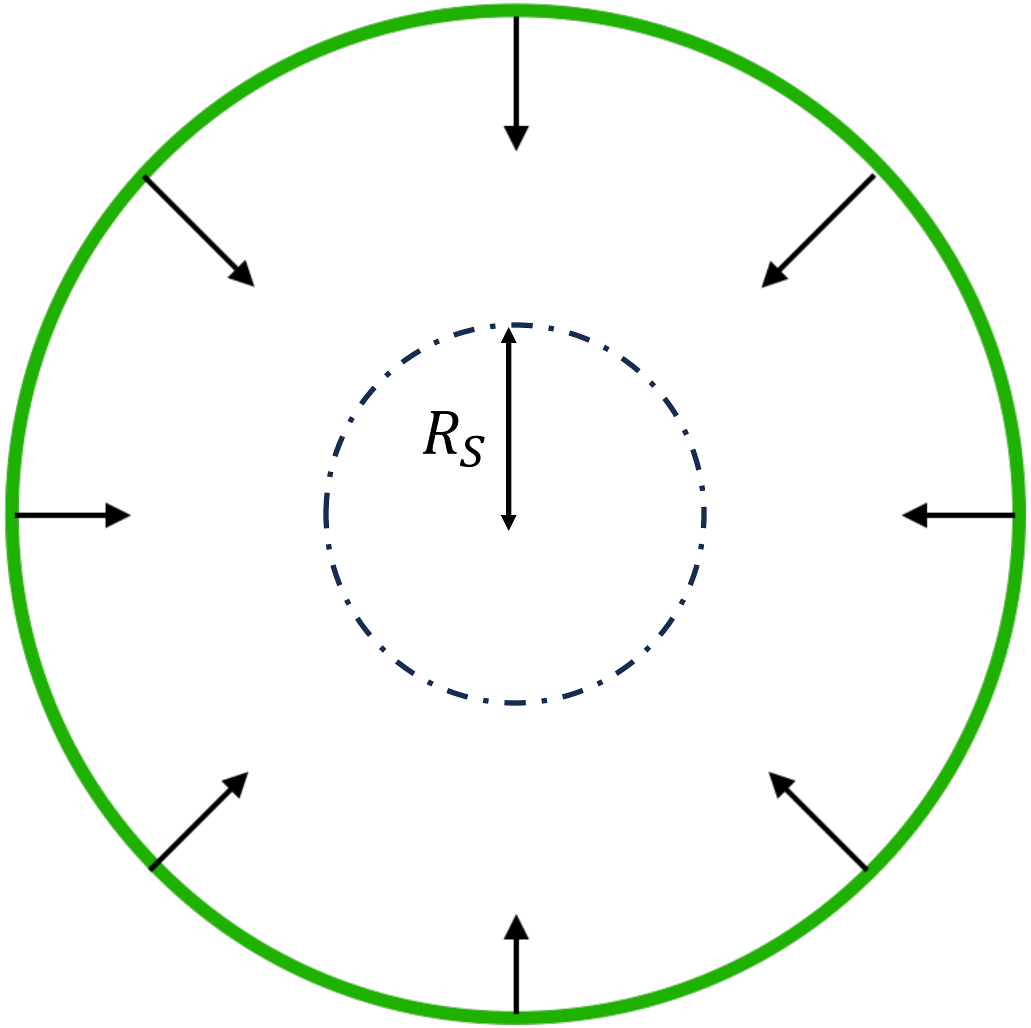
Black hole evaporation

Hawking evaporation revisited



Gravitationally collapsing object:
dying star, matter shell…

Hawking evaporation revisited



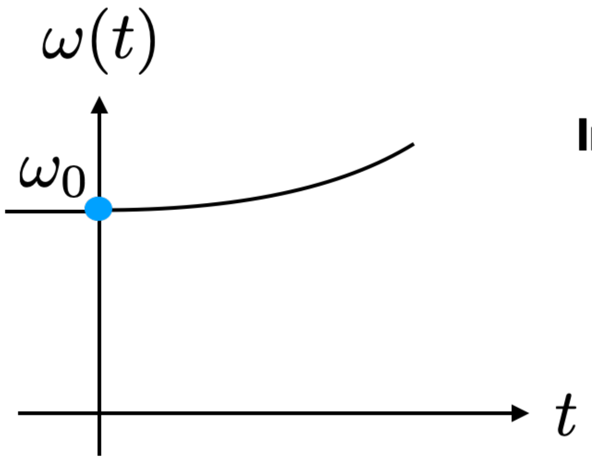
Gravitationally collapsing object:
dying star, matter shell…

- Time dependent metric $g_{\mu\nu}(t)$
- *Eventual* formation of a black hole
- Quantum fields initially in *their vacuum* get excited: particle production
- At late times, power spectrum becomes thermal: Hawking radiation

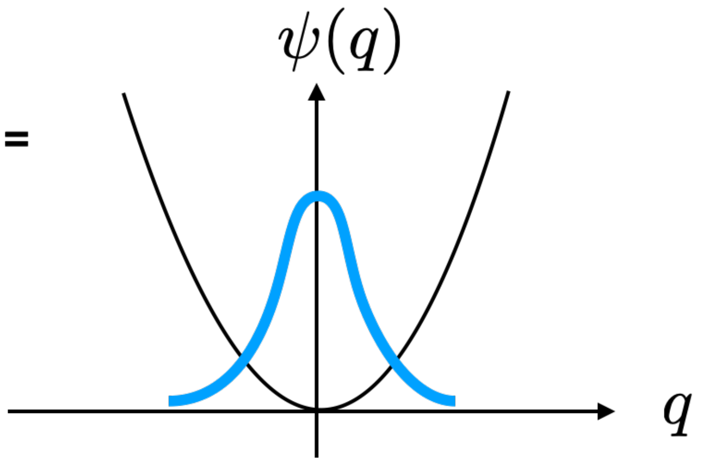
First detour: harmonic oscillators

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega(t)^2 \hat{q}^2$$

Explicit time-dependence ⇒ Excited states get populated



Initial quantum state = ground state of harmonic oscillator



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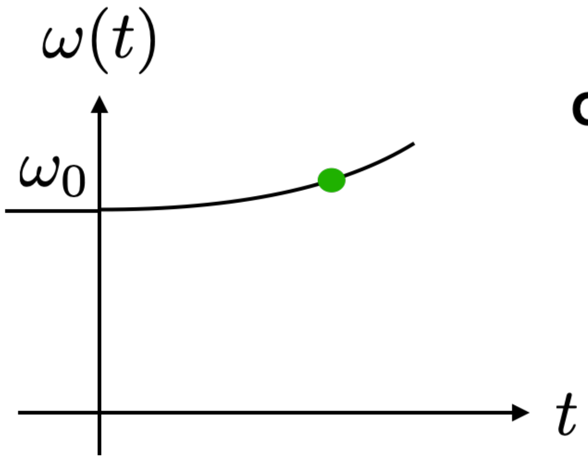
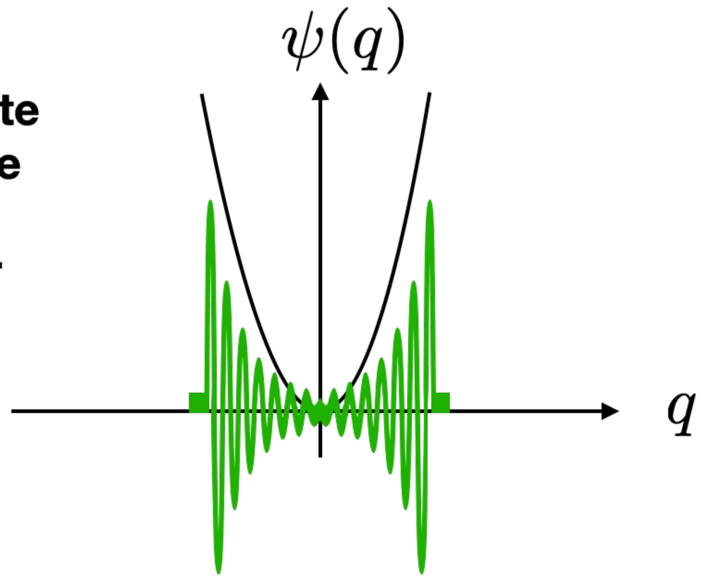


Generic quantum state = excited state of the instantaneous harmonic oscillator



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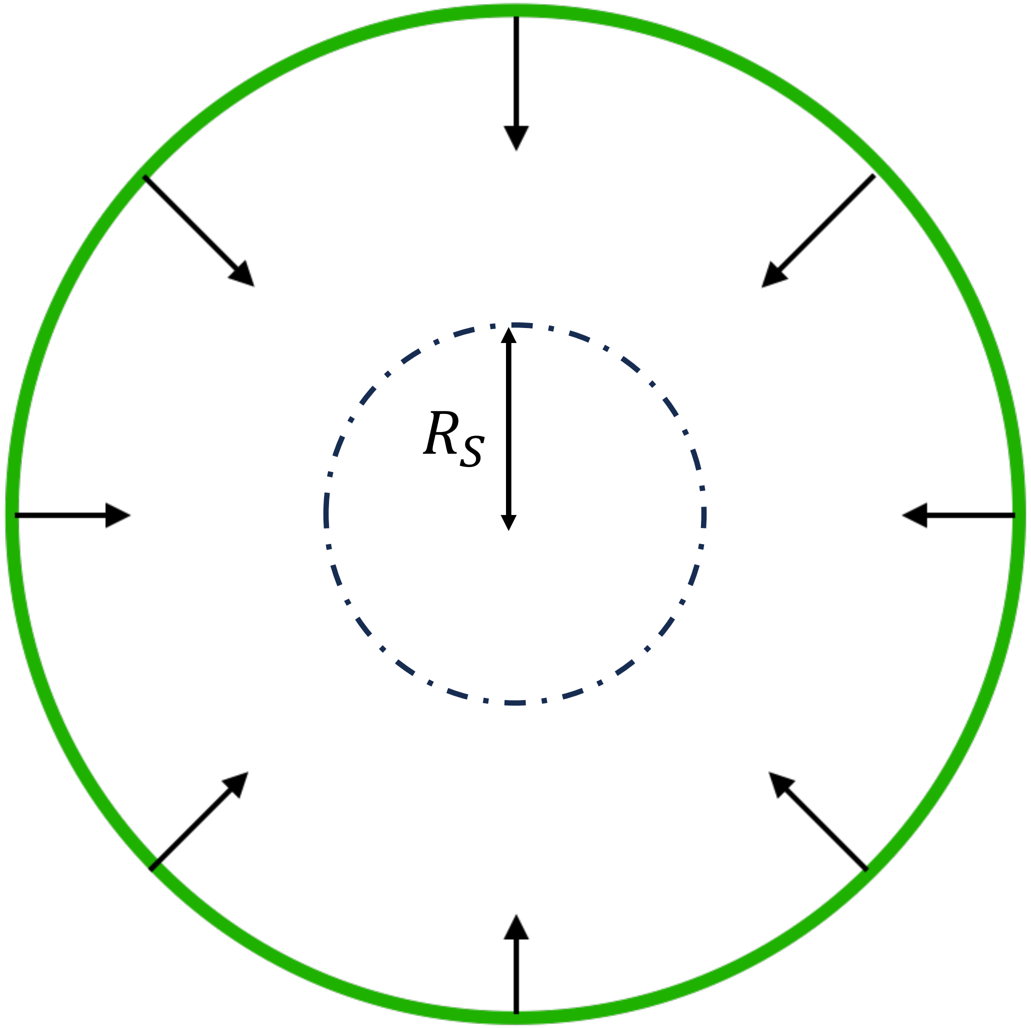

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HOW DOES IT EVAPORATE?
SEMICLASSICAL QUANTUM BACKREACTION

Toy model: collapsing shell

- Thin spherically symmetric domain wall: classical degrees of freedom radius $R(t)$, mass $M(t)$ obeying classical equations of motion
- Metric (dynamic via dependence on $R(t)$ and $M(t)$): *Minkowski* inside, *Schwarzschild* outside
- Quantum scalar field mode coupled to $R(t)$ and $M(t)$ via time-dependent vacuum expectation values

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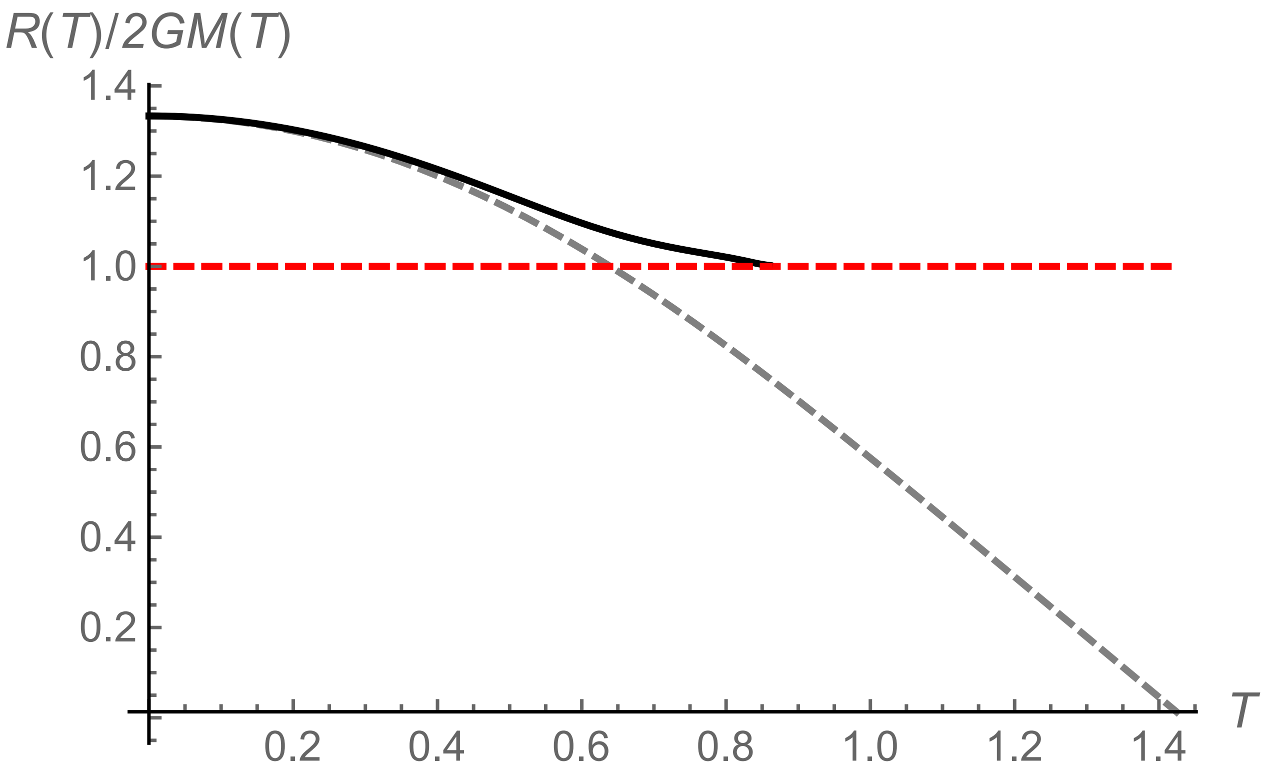

T. Vachaspati, GZ (2018)

COLLAPSE SEEMS TO SLOW DOWN….

but model is too simplistic

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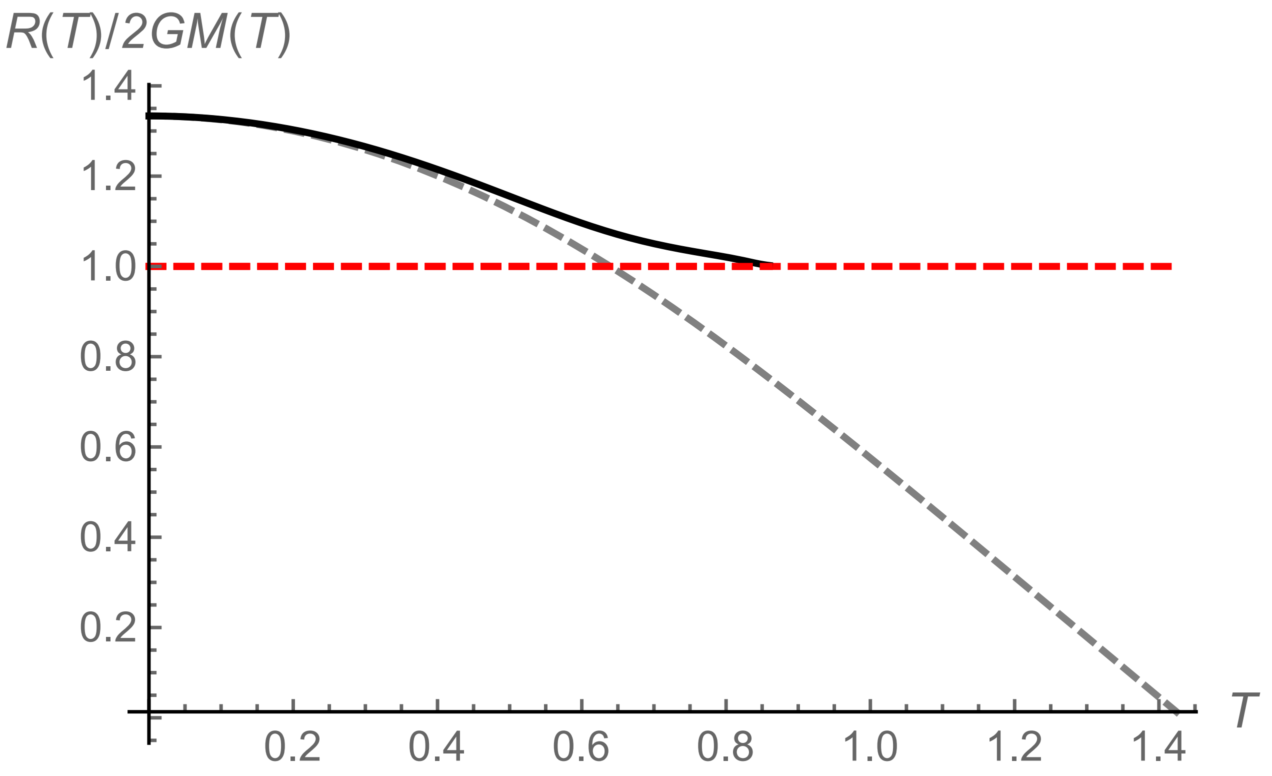

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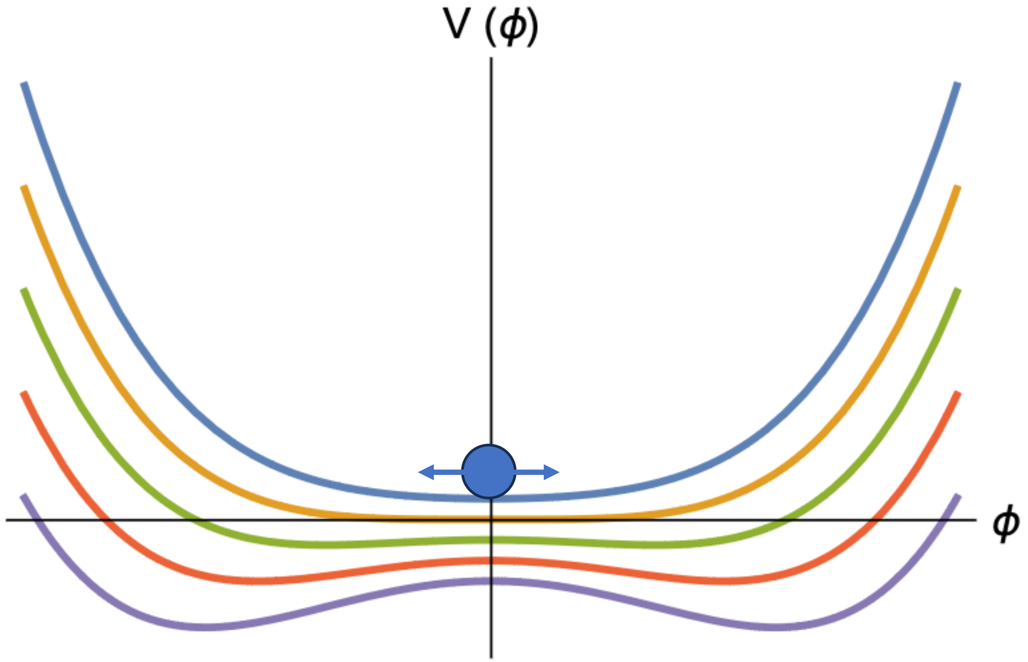
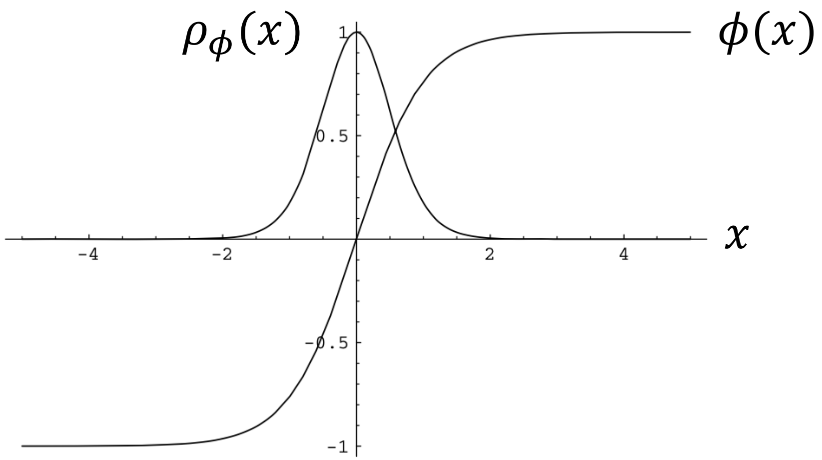
Second detour: domain walls

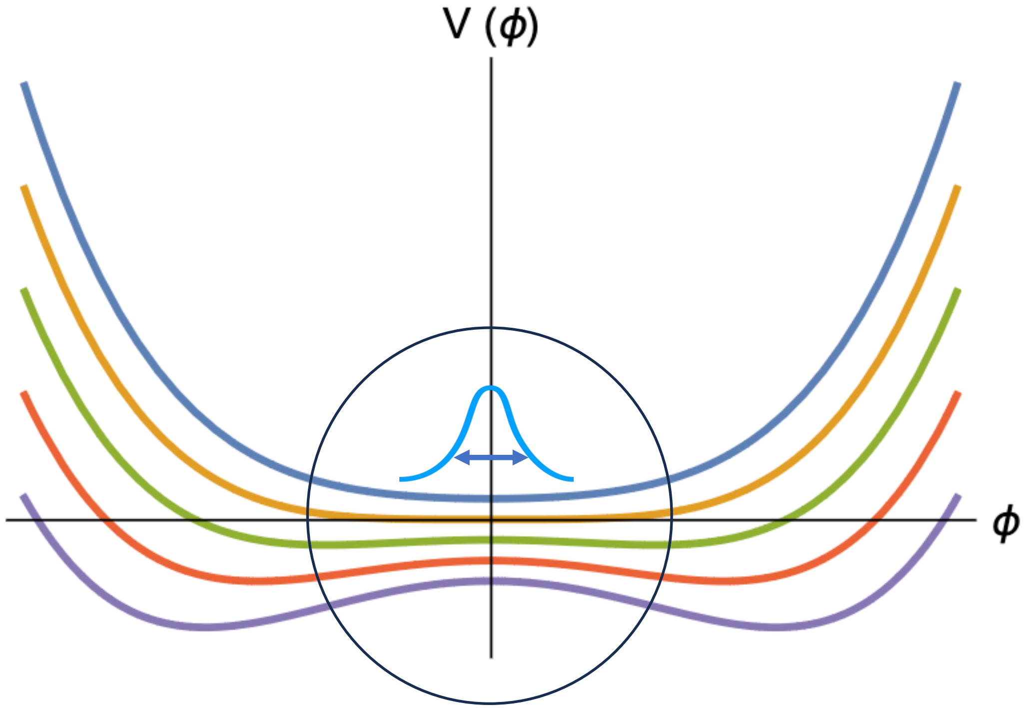


Spontaneous symmetry breaking of a discrete symmetry (e.g. phase transition): degenerate vacua

- Field *picks* a vacuum independently at different points in space
- Regions with different vacua separated by *domain walls*
- Stable *classical* field configurations interpolating between the two vacua



Second detour: domain walls



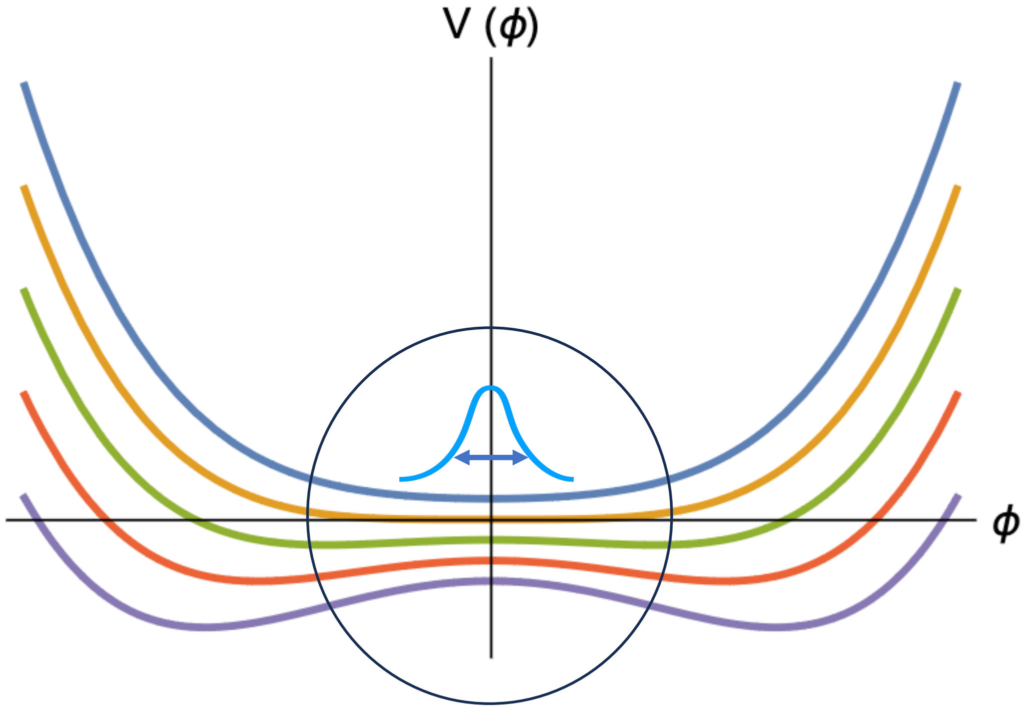
Spontaneous symmetry breaking of a discrete symmetry (e.g. phase transition): degenerate vacua

Formation of these objects during quantum phase transitions?

How do they become classical?

NOT A QUESTION WE CAN STRICTLY ASK

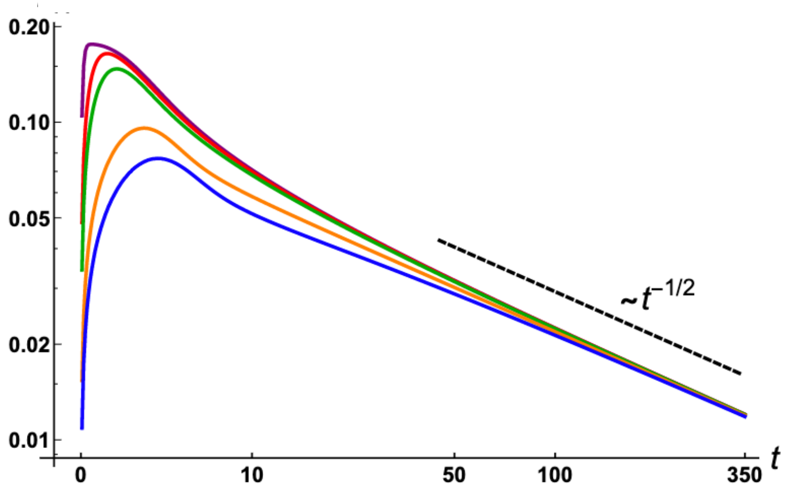
Second detour: domain walls



Spontaneous symmetry breaking of a discrete symmetry (e.g. phase transition): degenerate vacua

Formation of these objects during quantum phase transitions?

NEW QUESTION:
How does their average *abundance* evolve?



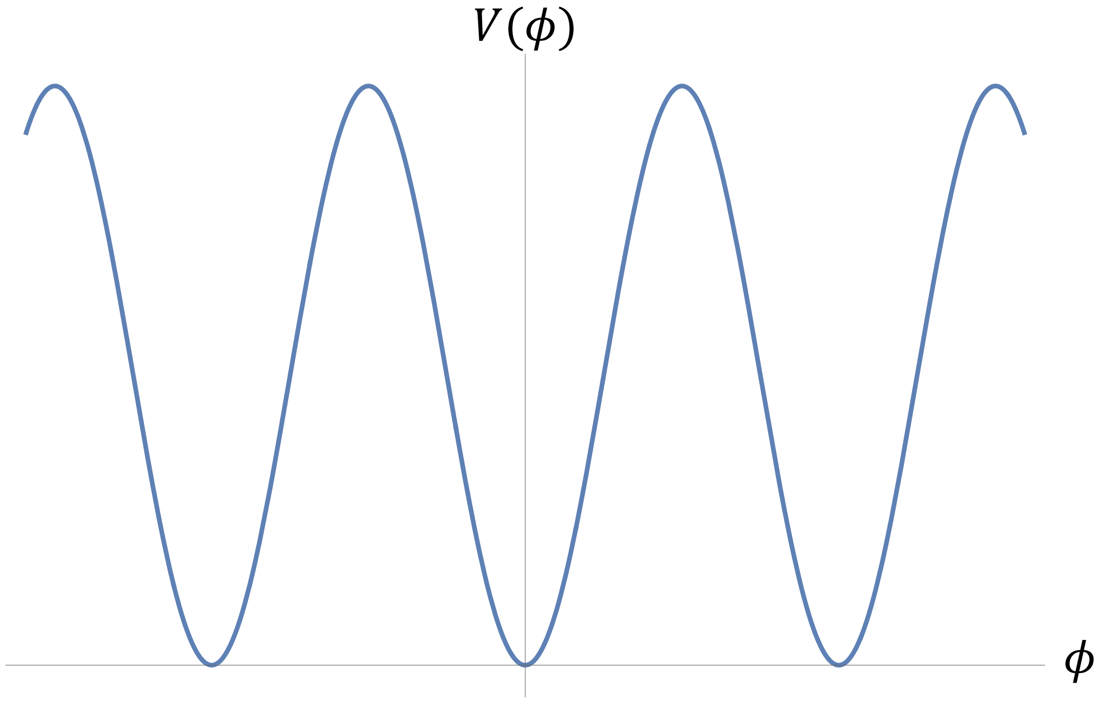
M. Mukhopadhyay, T. Vachaspati, GZ (2020)

More complex toy model

sine-Gordon zoology

- Laboratory: sine-Gordon model in one spatial dimension

$$\mathcal{L}_{SG} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi'^2 - m^2(1 - \cos\phi)$$



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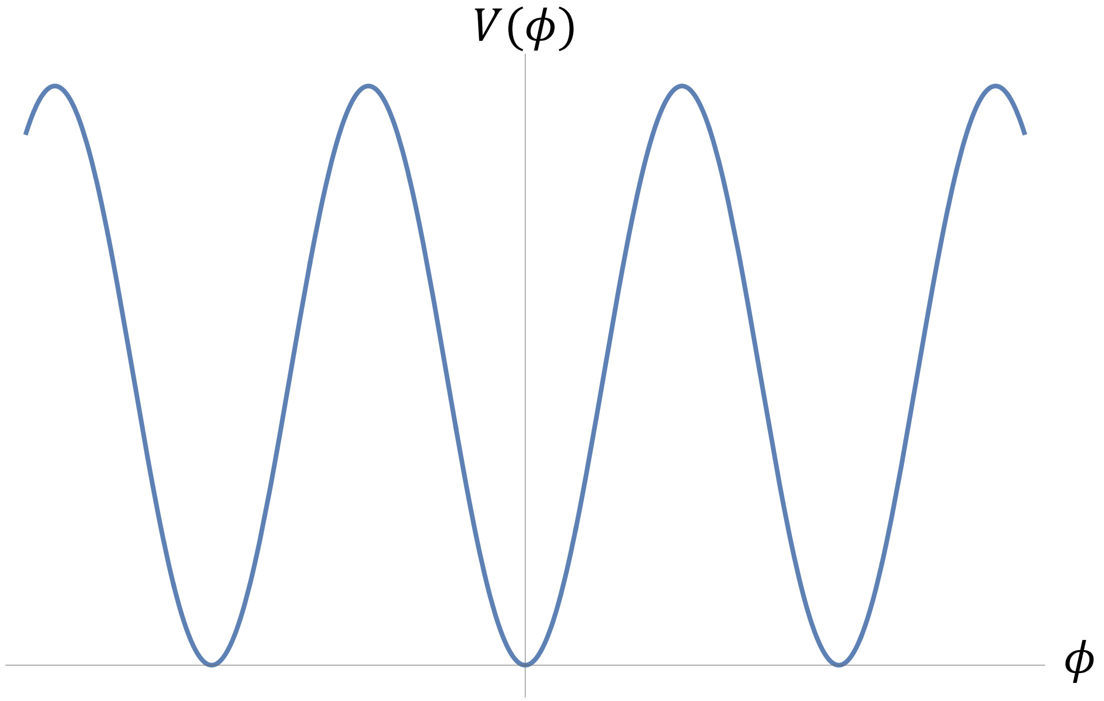
Degenerate vacuum manifold



Domain walls!!!
(or *kinks*)

sine-Gordon zoology

- Laboratory: sine-Gordon model in one spatial dimension

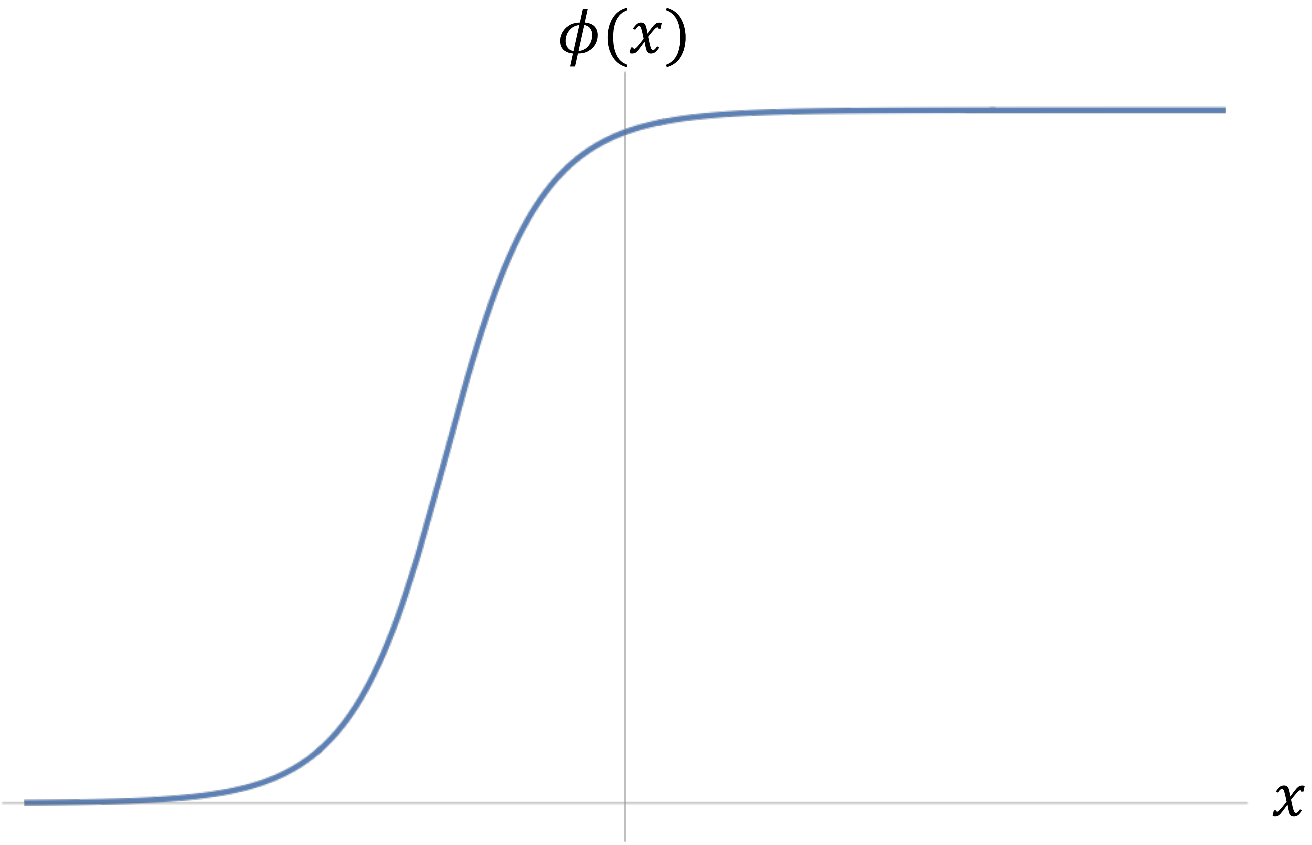
$$\mathcal{L}_{SG} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi'^2 - m^2(1-\cos\phi)$$

- Build classical configuration *analog* to collapsing domain wall
- Free quantum field ψ coupled to this ϕ background via

$$\psi^2(1-\cos\phi)$$

- Semi-classical backreaction $\psi^2 \to \langle\psi^2\rangle$
- No gravity, but full field-theoretic model…

Building the “collapsing” background



Kink solution $\phi = +\,4\ \text{Arctan}\ e^{m(x-x_0)}$

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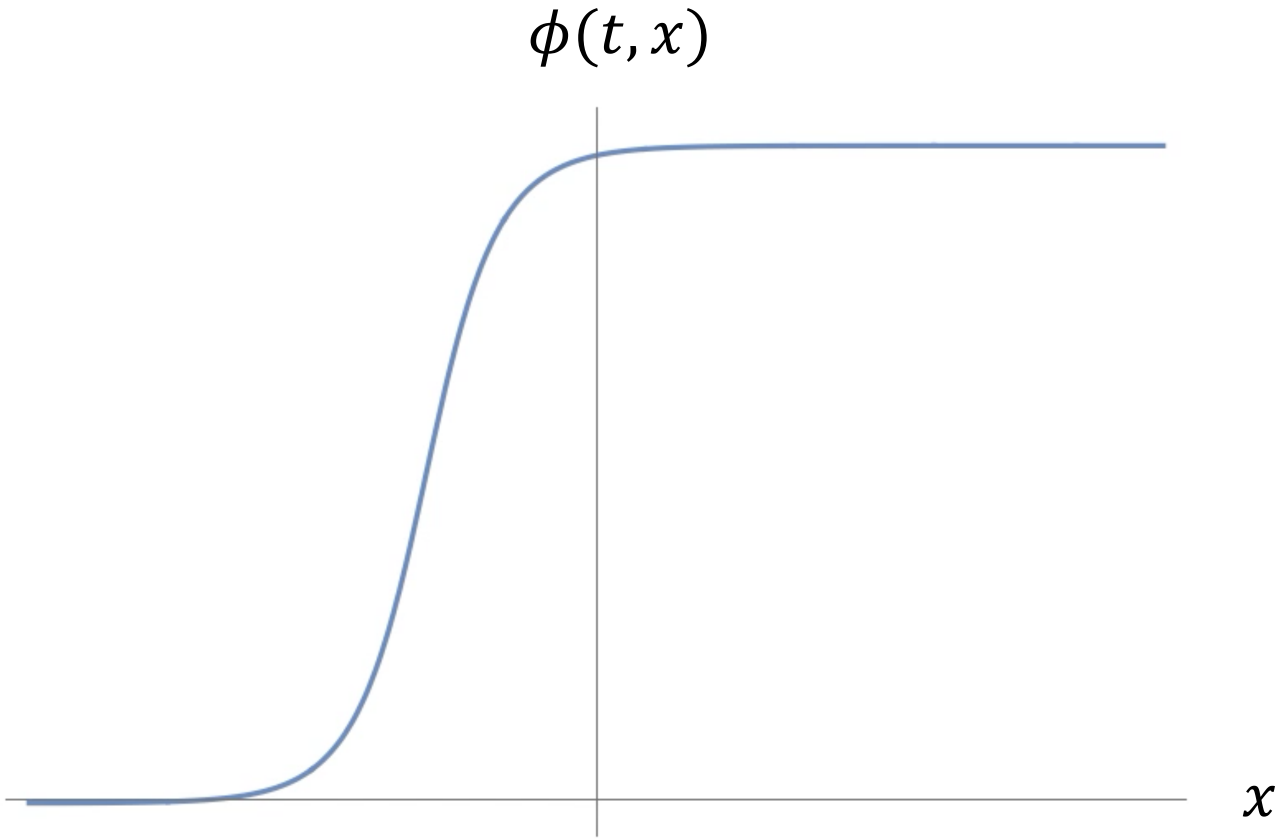

Kink solution $\phi = +\,4\ \mathrm{Arctan}\ e^{\gamma m(x - x_0 - vt)}$

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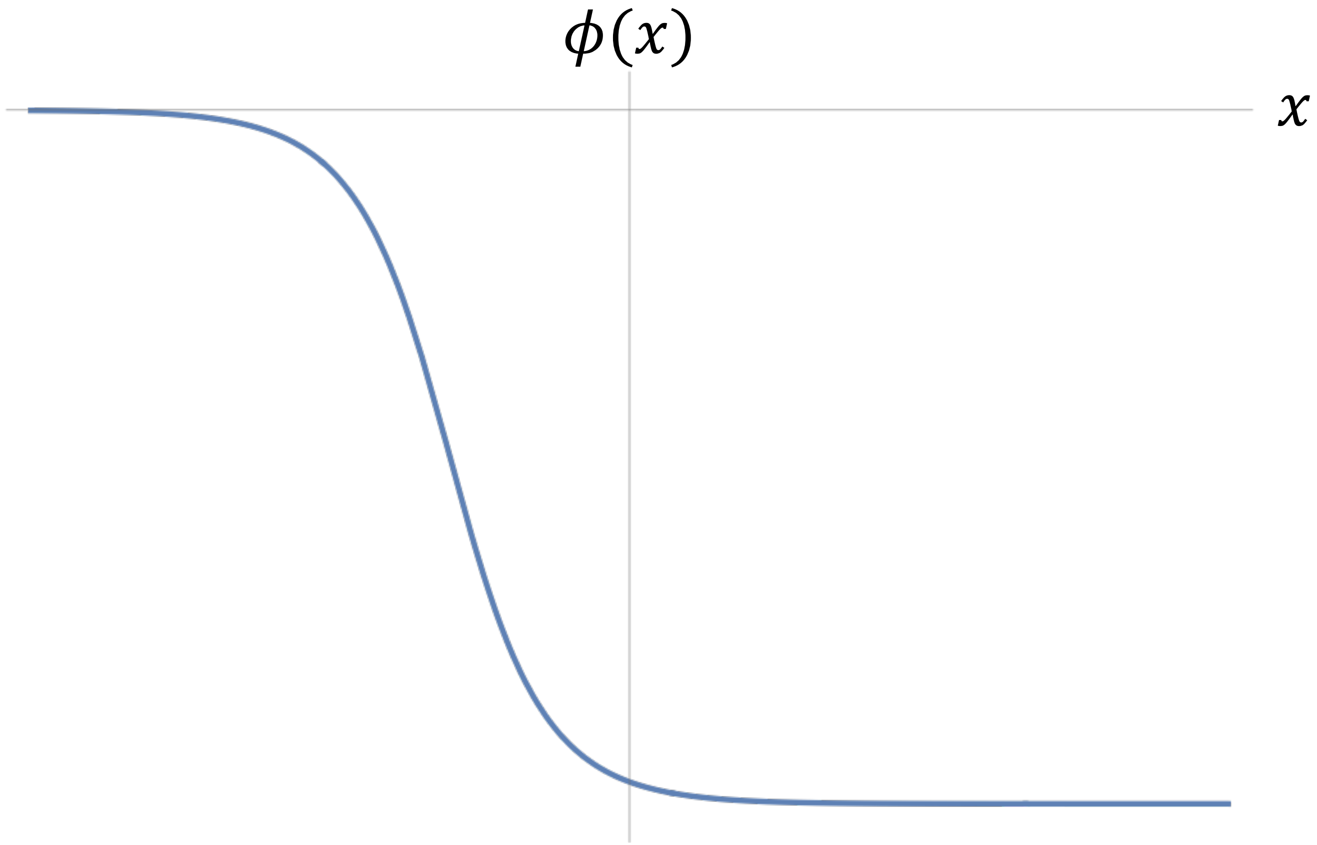
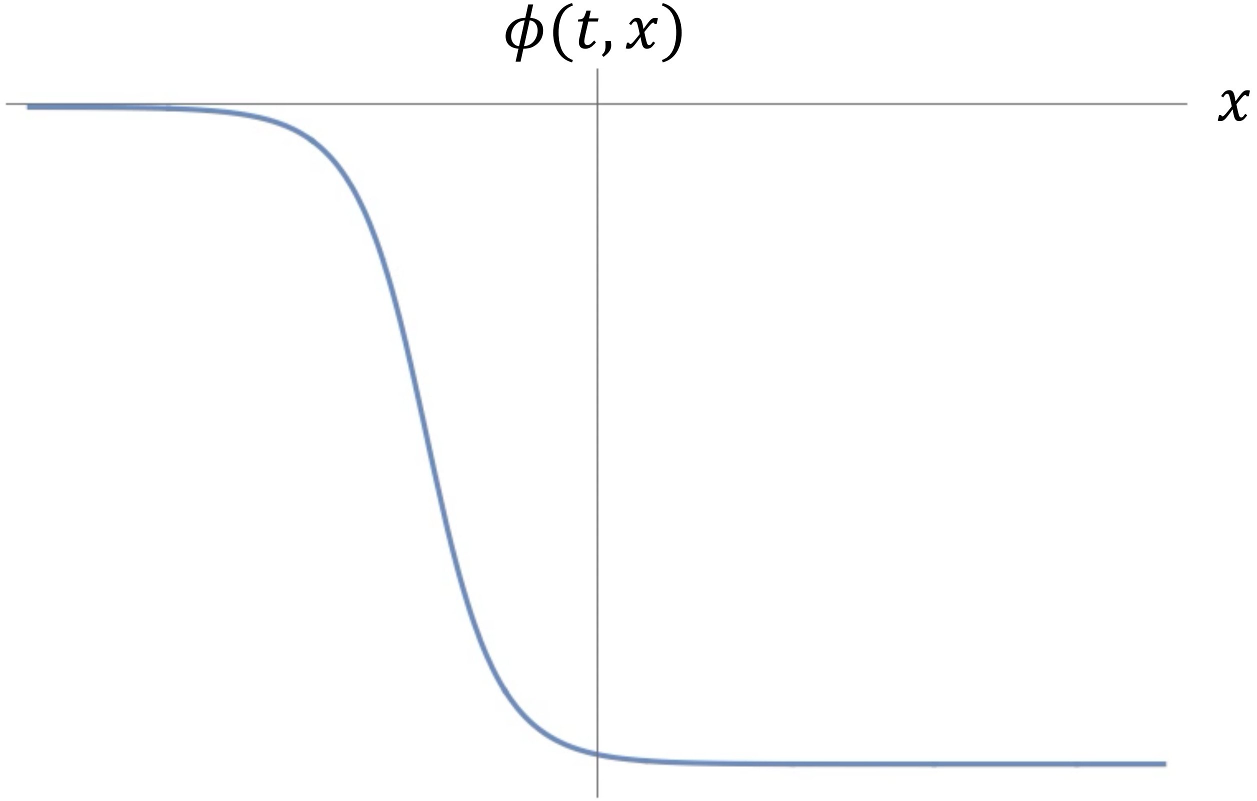


Antikink solution $\phi = -4\,\mathrm{Arctan}\, e^{m(x-x_0)}$

Building the “collapsing” background



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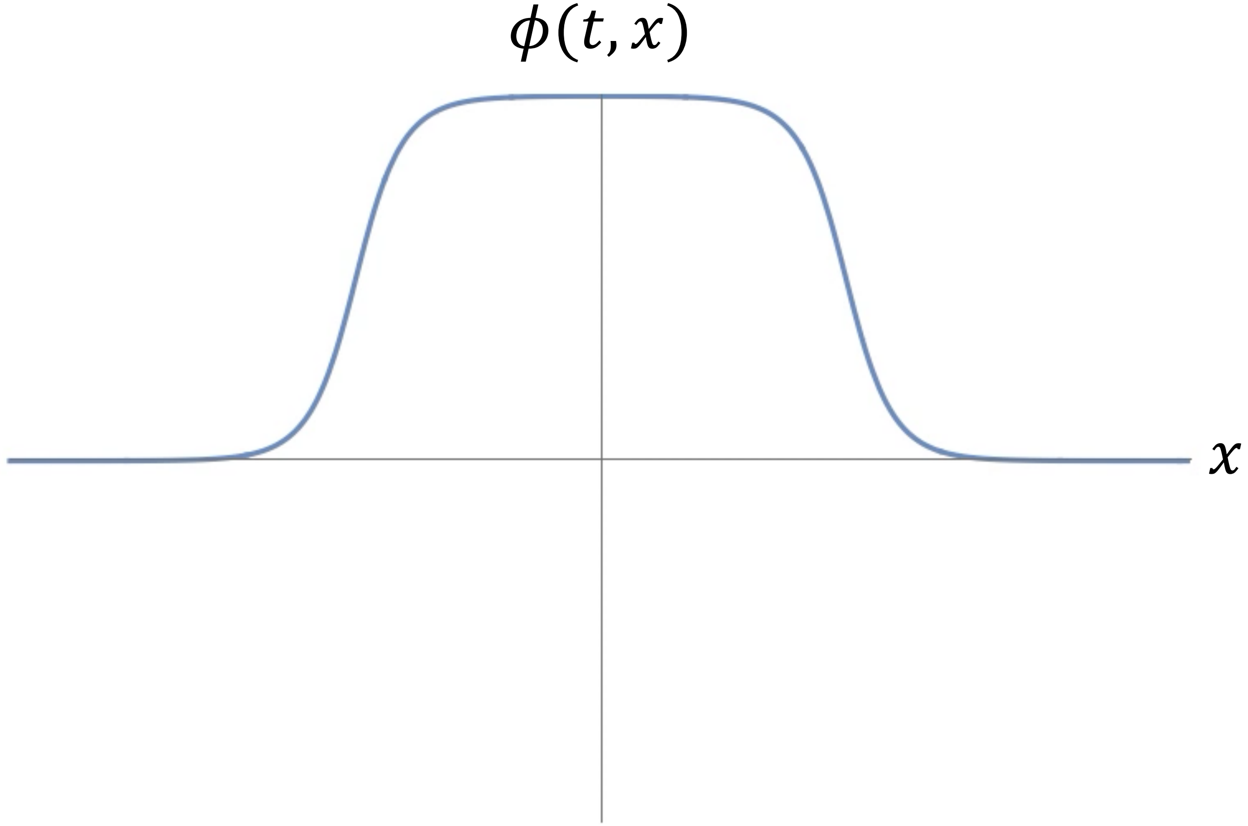
Building the “collapsing” background

$$\eta = \sqrt{\left(\frac{m}{\omega}\right)^2 - 1} \gg 1$$



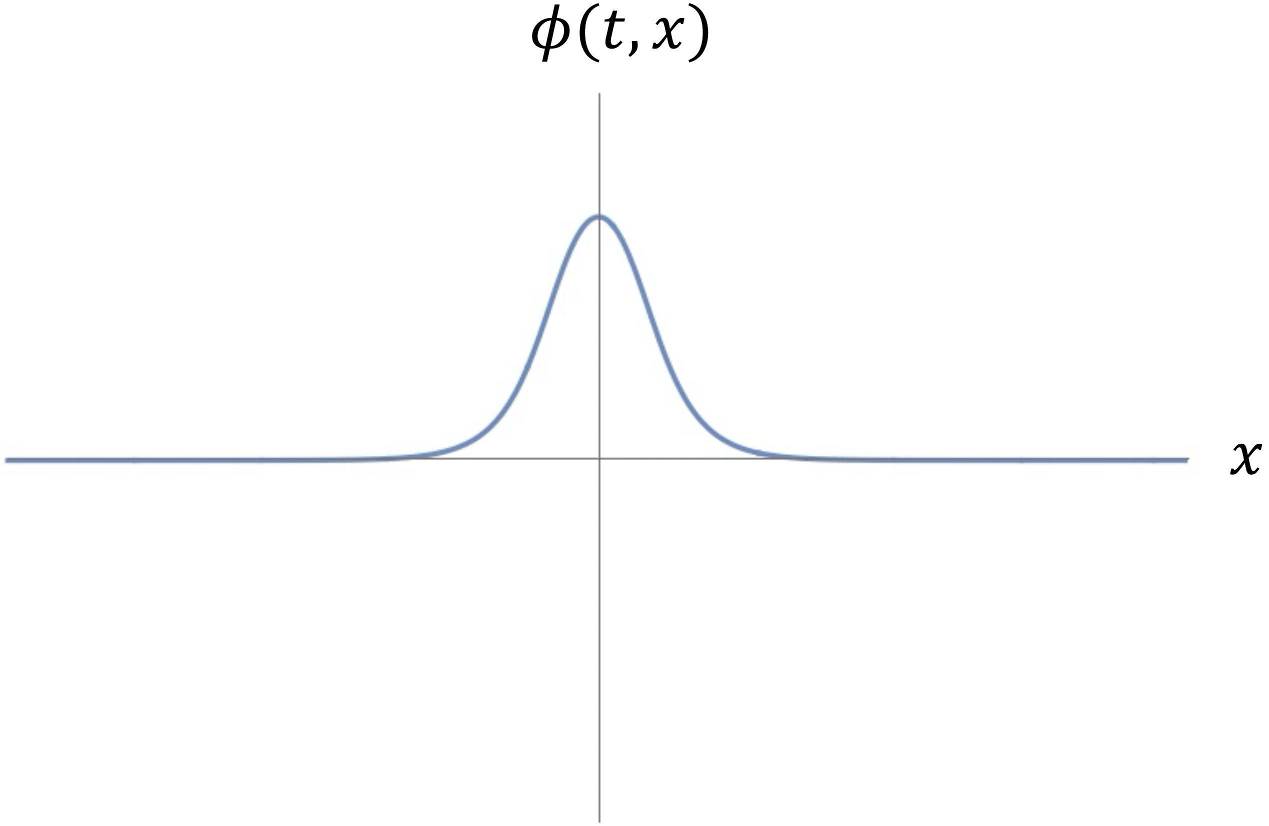
Large breather

Kink-antikink bound state: breather

$$\phi = +\,4\ \mathrm{Arctan}\left(\frac{\eta\cos(\omega t)}{\cosh(\eta\omega x)}\right)$$

Building the "collapsing" background

$$\eta = \sqrt{\left(\frac{m}{\omega}\right)^2 - 1} \ll 1$$



Small breather

Kink-antikink bound state: breather

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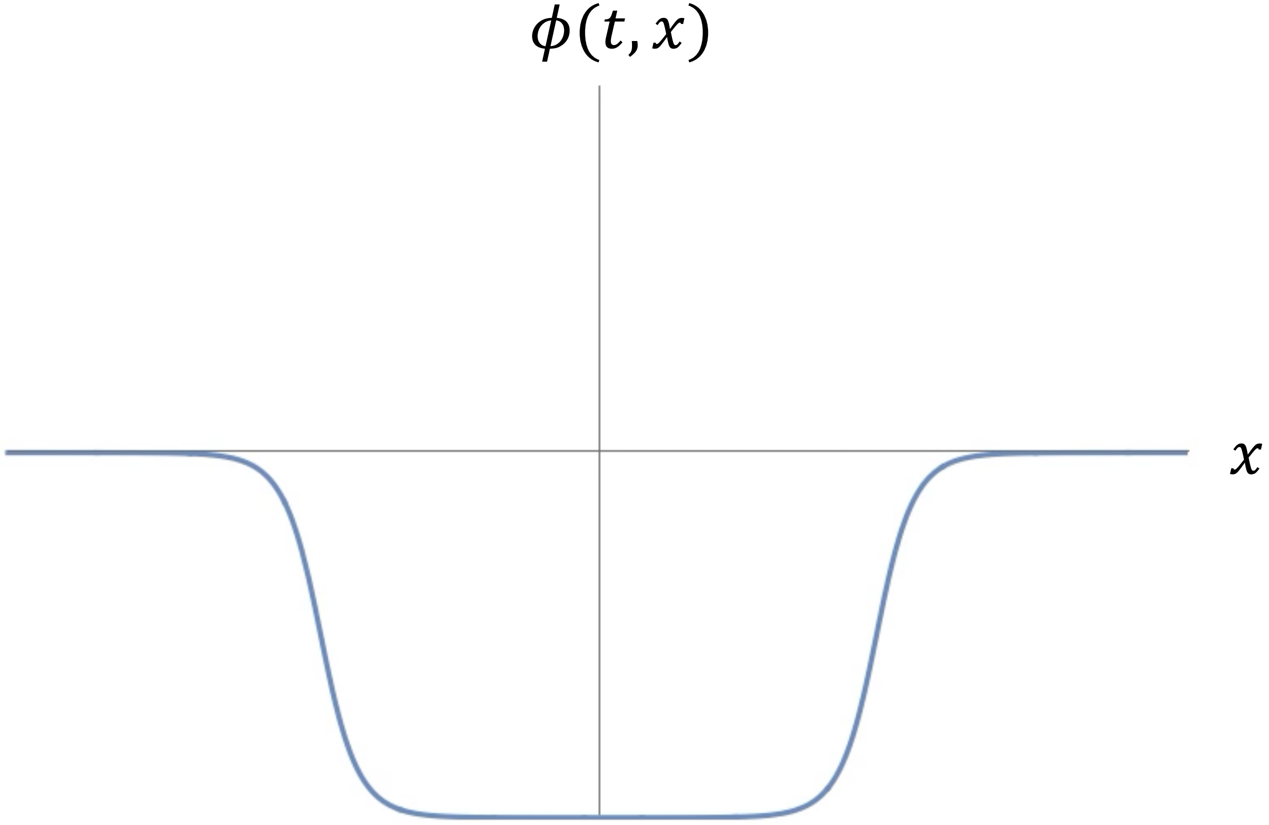
Building the “collapsing” background



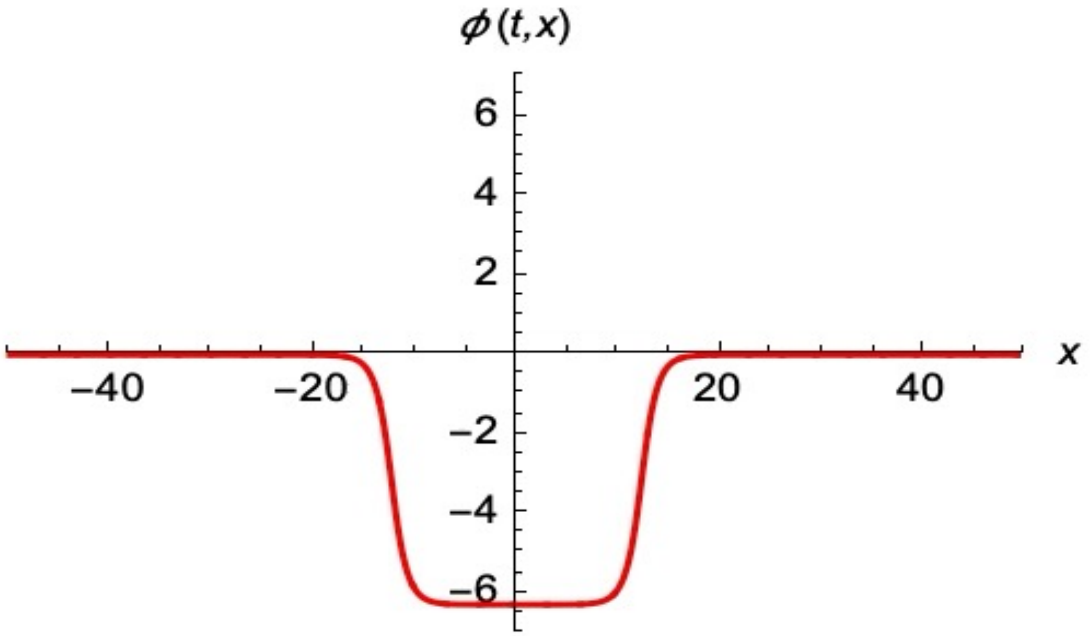
Kink-antikink scattering state

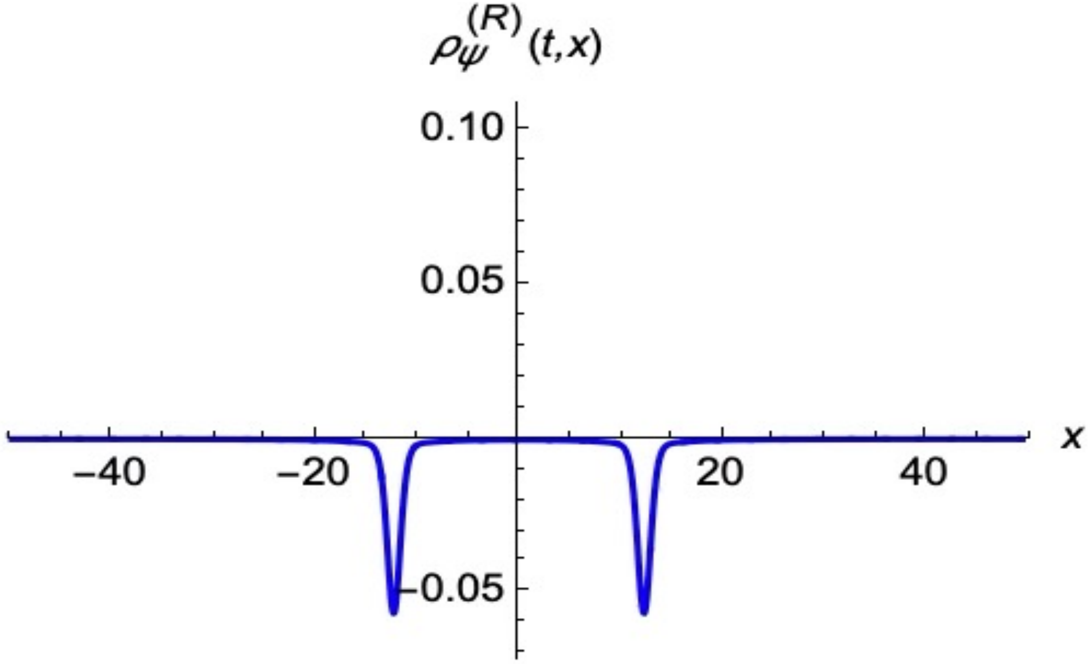
$$\phi = +\,4\ \mathrm{Arctan}\left(\frac{\sinh(\gamma m v t)}{v\cosh(\gamma m x)}\right)$$

Some results

M. Mukhopadhyay, E. Sfakianakis, T. Vachaspati, GZ (2021)

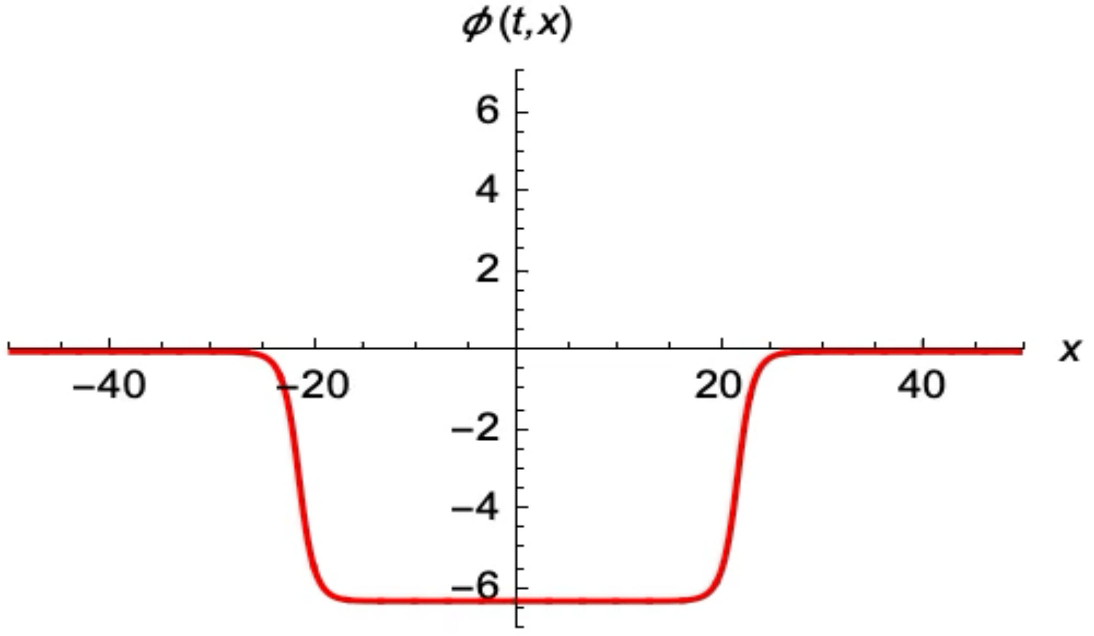
Kink-antikink scattering

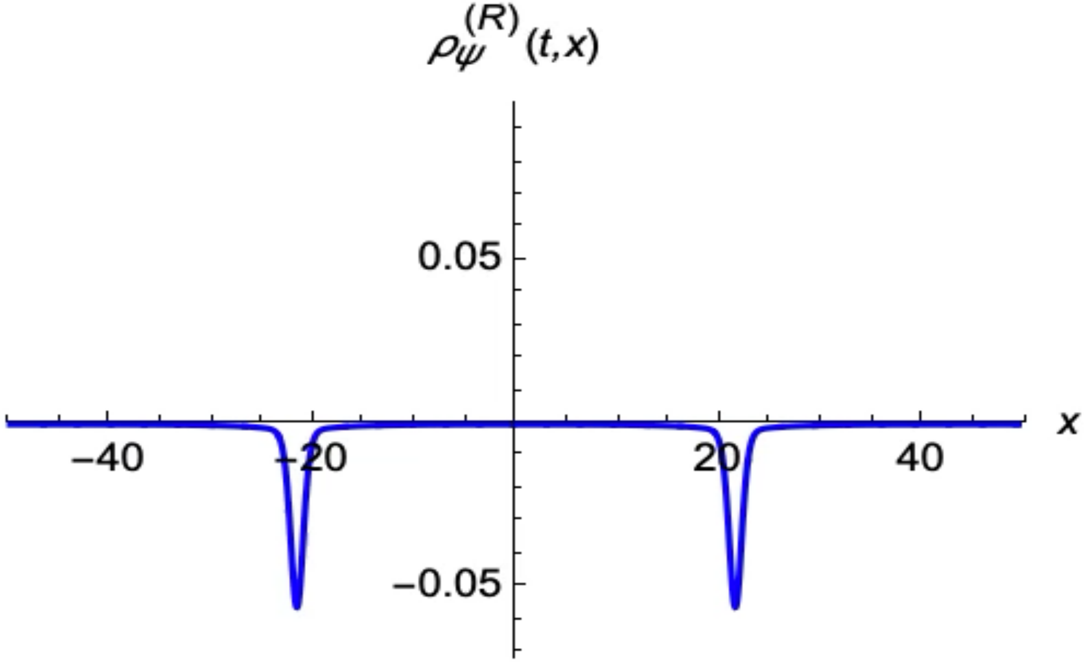




No backreaction of quantum radiation

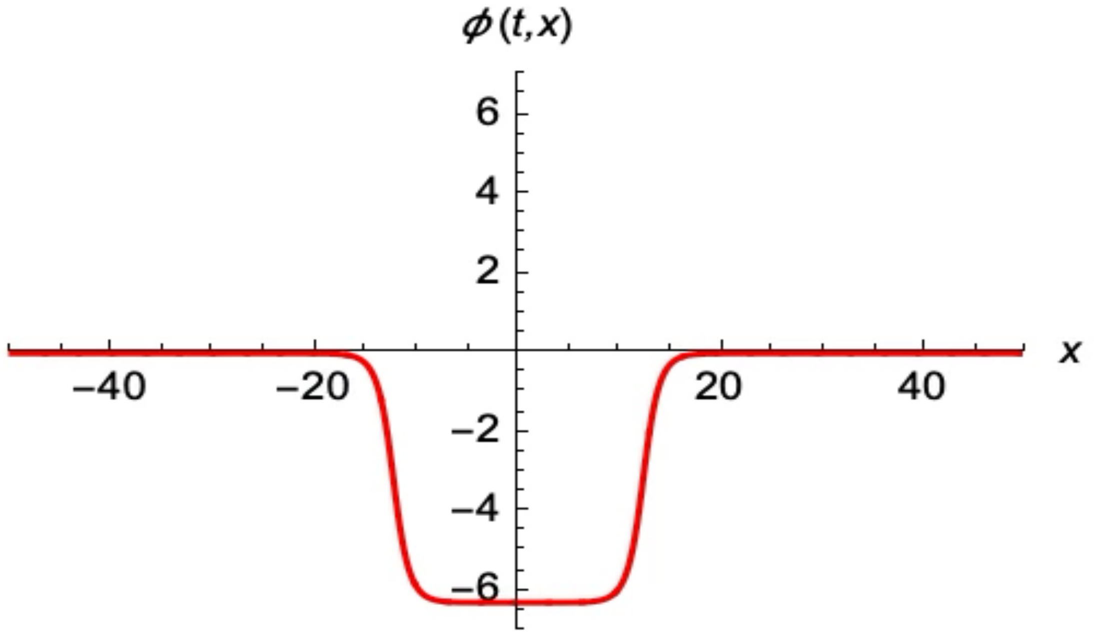
Kink-antikink scattering

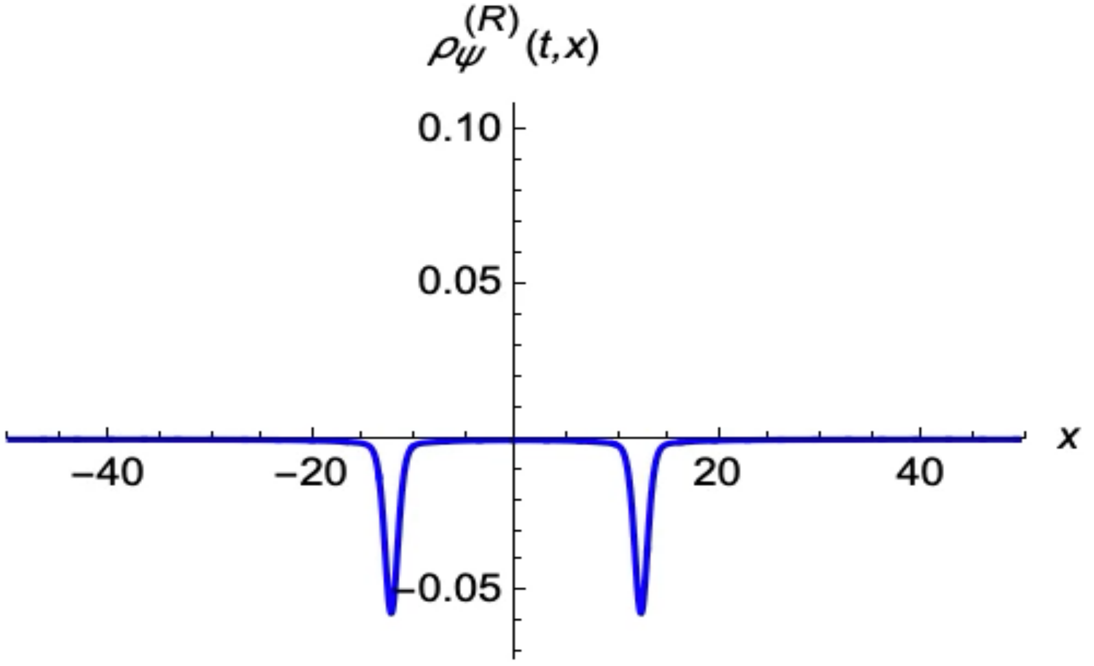




Semiclassical backreaction of quantum radiation

Breather formation and evaporation





Semiclassical backreaction of quantum radiation

Future prospects

- 3 dimensional field theoretic description of collapsing domain wall
- Include gravity (with all its renormalization problems)…
- Add a free quantum field minimally coupled to gravity (only indirectly coupled to the domain wall)
- Include semiclassical backreaction

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COMPUTATIONALLY INSTENSIVE

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- 3 dimensional field theoretic description of collapsing domain wall
- Include gravity (with all its renormalization problems)…
- Add a free quantum field minimally coupled to gravity (only indirectly coupled to the domain wall)
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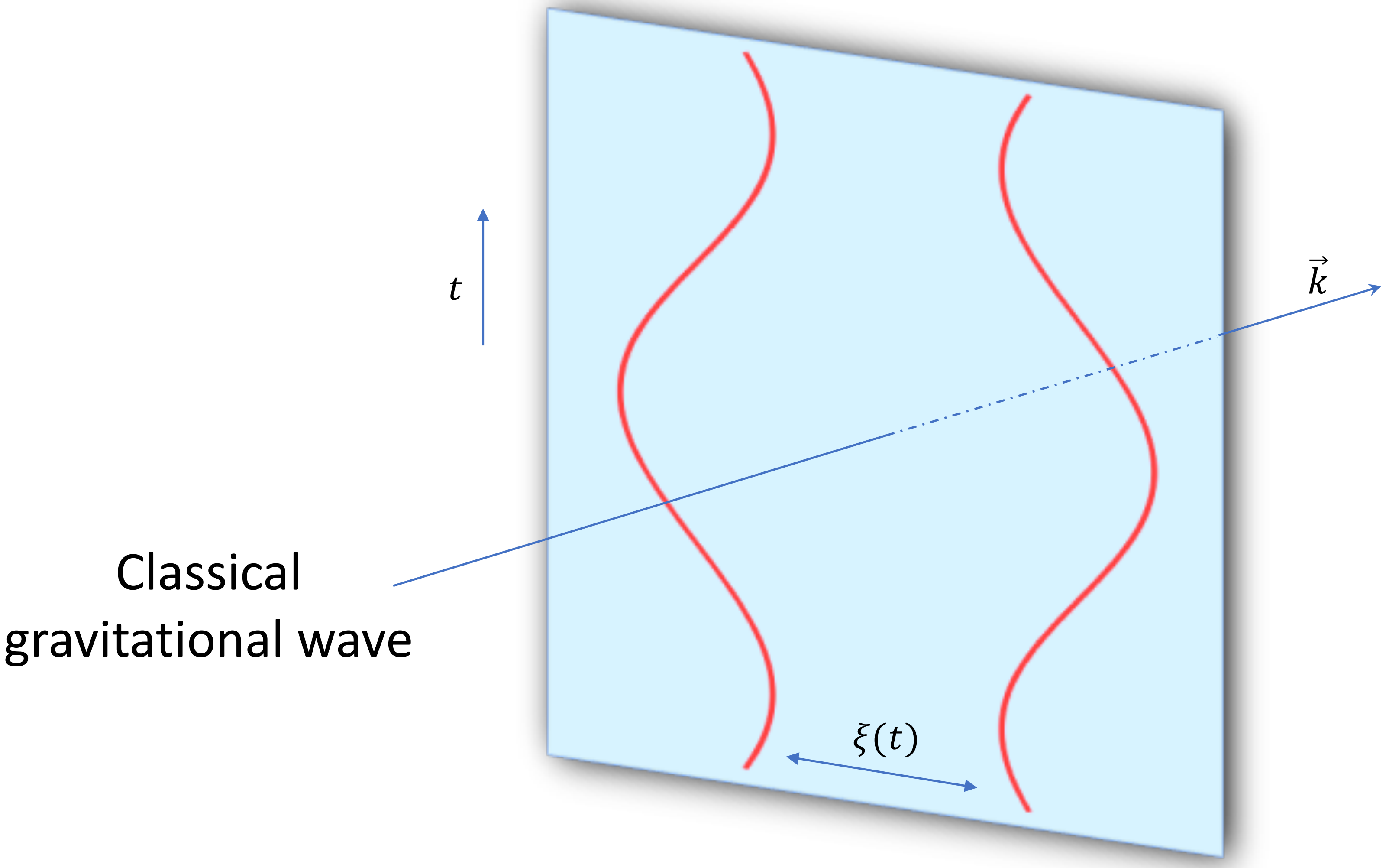
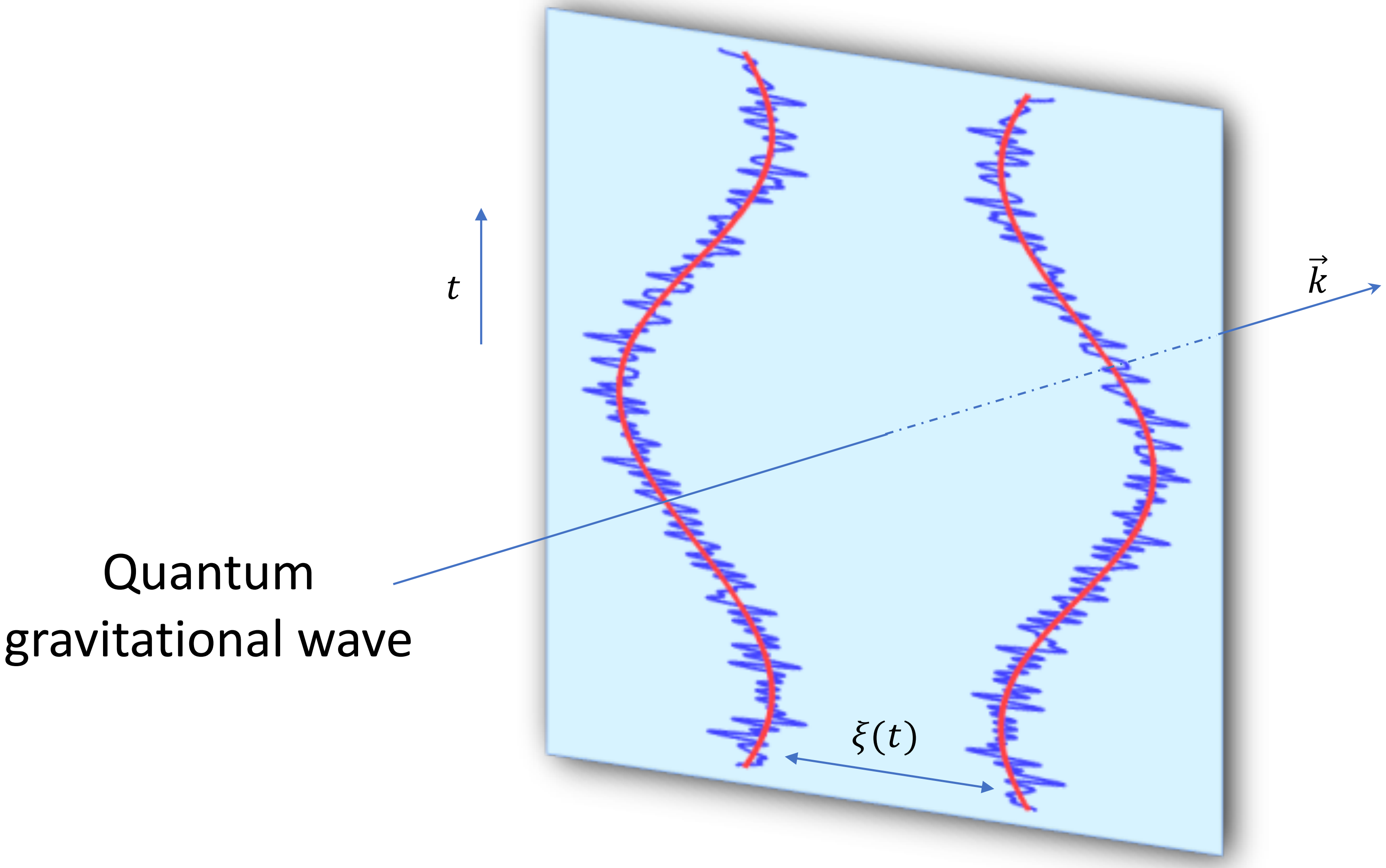
COMPUTATIONALLY INSTENSIVE

- Exploration of other ways to include quantum backreaction (e.g. statistical ensembles, Wigner function truncations, stochastic gravity…)

Final detour: the poor man's quantum gravity



Final detour: the poor man's quantum gravity



Final detour: the poor man's quantum gravity

$$\ddot{\xi}(t) - \frac{1}{2}\left[\ddot{h}_{\rm cl}(t) + \ddot{N}(t) - \frac{mG}{c^5}\frac{d^5}{dt^5}\xi^2(t)\right]\xi(t) = 0$$

Classical wave profile

Vacuum fluctuations

Radiation reaction

Effective equation of motion for the detector including quantum effects

M. Parikh, F. Wilczek, GZ (2020)

Personal perspective

Semiclassical methods: peek through the keyhole

Strings, canonical quantization methods… : pick the lock

Door to quantum gravity