Heavy quarks embedded in glasma

based on: arXiv:2001.05074 (accepted for publication in Nuclear Physics A)

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Krakow, May 8th, 2020

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Introduction

evolution of nuclear matter after heavy ion collision



many approaches/models needed

Properties of QCD medium are known only to some extent and are under active exploration.

Heavy quarks - charm and beauty - due to their large masses are produced with high p_T at the earliest stage of HIC through hard interactions.

Heavy quarks propagate through the QCD medium and test its evolution at all stages.

Heavy quarks

heavy quark in a weakly interacting QCD thermal medium (kinetic theory) a Brownian particle undergoing random kicks from fast-moving particles of the medium (changes of movement, energy loss due to collisions and due to radiation)

$$\begin{array}{c|c} & & \\ \mbox{typical momentum exchange of HQ with the} & q \approx gT & \\ & \mbox{heat bath} & p \gg q & \\ & \mbox{momentum} & p \mbox{ of HQ related to large mass} & \\ & \mbox{thermalization time of the bulk medium} & \tau_{\rm th} & \\ & \mbox{thermal relaxation time of HQ} & \tau_Q \approx \tau_{\rm th} m/T & \end{array} \rightarrow \qquad \tau_Q \gg \tau_{\rm th}$$

What are the effects of pre-equilibrium phase on heavy quarks?

- within kinetic theory (Das et al, J. Phys. G 44, 095102 (2017))
- within CGC Wong equations of motion of HQ solved numerically (Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018), Ruggieri, Das, Phys. Rev. D 98, 094024 (2018))
- HERE: within CGC - analytically tractable approach of HQ interacting with highly- occupied soft gluon fields ⁴

The system

Heavy quarks: $m, p^{\mu}, E_{\mathbf{p}}, \mathbf{v} = \frac{\mathbf{p}}{E_{\mathbf{p}}}$ - mass, four-momentum, energy and velocity of HQ $p_0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ - momenta of heavy quarks obey mass shell constraint $Q(t, \mathbf{r}, \mathbf{p})$ - distribution function of heavy quarks in the medium

Soft classical field (because of the large occupation numbers): $A^{\mu}(x)$

Interaction: ${f F}(t,{f r})\equiv gig({f E}(t,{f r})+{f v} imes{f B}(t,{f r})ig)$ - color Lorentz force

g - coupling constant

 $\mathbf{E}(t,\mathbf{r}),\;\mathbf{B}(t,\mathbf{r})\;$ - chromoelectric and chromomagnetic fields

Towards Fokker-Planck equation

Transport of heavy quarks interacting within a medium is described by the Fokker-Planck equation (usually applied when HQ interact with plasma constituents).

Liouville equation in the Vlasov form: (contains all microscopic information)

$$ig(D^0+\mathbf{v}\cdot\mathbf{D}ig)Q(t,\mathbf{r},\mathbf{p})-rac{1}{2}ig\{\mathbf{F}(t,\mathbf{r}),
abla_pQ(t,\mathbf{r},\mathbf{p})ig\}=0$$

covariant derivative:
$$D^\mu \equiv (D^0, {f D}) \equiv \partial^\mu - ig[A^\mu(x), \cdots]$$

Distribution function can be divided into two parts:

$$egin{aligned} Q(t,\mathbf{r},\mathbf{p}) &= \langle Q(t,\mathbf{r},\mathbf{p})
angle + \delta Q(t,\mathbf{r},\mathbf{p}) \ & ert \ & e$$

averaging: micro -> macro (information about microscopic structure is lost

Towards Fokker-Planck equation

Distribution function satisfies the following conditions:

regular part is much larger than the fluctuating one:

regular part is slowly varying function of time and space:

$$\left|rac{\partial \delta Q}{\partial t}
ight| \gg \left|rac{\partial \langle Q
angle}{\partial t}
ight| \qquad \left|
abla \delta Q
ight| \gg \left|
abla \langle Q
angle
ight|$$

 $\langle {f E}(t,{f r})
angle = \langle {f B}(t,{f r})
angle = 0$

 $|\langle Q
angle| \gg |\delta Q| \qquad |
abla_p \langle Q
angle| \gg |
abla_p \delta Q|$

regular part of the chromodynamic fields vanishes:

Solving the Vlasov equation in 2 steps:

 δQ - obeys Liouville (collisionless) transport equation

the solution of the transport equation for δQ plays the role of the collision term in the transport equation for the regular part - Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks: Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018)

$$egin{aligned} & \left(D-
abla^i_pX^{ij}(\mathbf{v})
abla^j_p-
abla^i_pY^i(\mathbf{v})
ight)n(t,\mathbf{x},\mathbf{p})=0 \ & \langle Q(t,\mathbf{r},\mathbf{p})
angle=n(t,\mathbf{r},\mathbf{p}) & D=rac{\partial}{\partial t}+\mathbf{v}\cdot
abla \ & X^{ij}(\mathbf{v})\equivrac{1}{2N_c}\int_0^t dt'ig\langle F^i_a(t,\mathbf{x})F^j_aig(t',\mathbf{x}-\mathbf{v}(t-t')ig) ig
angle \end{aligned}$$

 $Y^i(\mathbf{v}) = X^{ij}(\mathbf{v}) rac{v^j}{T}$ - postulated so that the distribution function satisfies the Fokker-Planck equation in equilibrium

 $T\,$ - temperature of plasma that has the same energy density as in equilibrium

Physical meaning:

$$rac{\langle \Delta p^i
angle}{\Delta t} = -Y^i ({f v})$$

$$rac{\langle \Delta p^i \Delta p^j
angle}{\Delta t} = X^{ij}(\mathbf{v}) + X^{ji}(\mathbf{v})$$

Energy loss and momentum broadening

$$rac{dE}{dx} = -rac{v}{T} rac{v^i v^j}{\mathbf{v}^2} X^{ij}(\mathbf{v})$$

$$\hat{q}=rac{2}{v}\Big(\delta^{ij}-rac{v^iv^j}{\mathbf{v}^2}\Big)X^{ij}(\mathbf{v})$$

Color Glass Condensate (CGC)

CGC - a framework to describe physics of heavy ion collisions in the earliest stage of the collision



after the collision

before the collision:

- large-*x* partons: valence quarks, the source partons
- small-x partons: soft gluon fields, solutions of classical Yang-Mills equations

- valence quarks fly away
- soft gluon fields interact producing glasma
- glasma solutions of classical Yang-Mills equations given in terms of the single-nucleus solutions

x - the fraction of momentum carried by a parton

Since the description of the longitudinal physics (along the beam) separates from the transverse one (within the area of the nuclei) it is convenient to use the light-cone coordinates

$$(x^+, \ x^-, \ x_\perp) \qquad \qquad x^+ = rac{t+z}{\sqrt{2}}, \ x^- = rac{t-z}{\sqrt{2}}$$

Sources given by $SU(N_{c})$ four-current:

$$J^{\mu}_{1,2}(x^{\mp},ec{x}_{\perp}) = \delta^{\mu\pm}
ho_{1,2}(x^{\mp},ec{x}_{\perp})$$

color-charge surface density

In McLerran-Venugopalan model:

$$ho_{1,2}(x^{\mp},ec{x}_{\perp})=\delta(x^{\mp})
ho_{1,2}(ec{x}_{\perp})$$

Classical Yang-Mills equations: $\left[D_{\mu},F^{\mu
u}
ight]=J^{
u}$



CGC: before the collision

Solutions of CYM equations:

$$egin{aligned} &A^\pm_{1,2}(x^\pm,ec x_ot)=0\ &A^i_{1,2}(x^\pm,ec x_ot)= heta(x^\mp)A^i_{1,2}(ec x_ot)\ &A^i_{1,2}(ec x_ot)=-rac{1}{ig}U_{1,2}(ec x_ot)\partial^i U^\dagger_{1,2}(ec x_ot)\ & ext{pure gauge transform of vacuum} \end{aligned}$$

 $U(ec{x}_{ot})\equiv U[g,
ho,ec{x}_{ot}]$ - the unitary matrix

Chromoelectric and chromomagnetic fields are given by the respective components of the strength tensor. They are:

$$egin{aligned} E^z_{1,2}(x^\pm,ec x_ot) &= 0 & E^i_{1,2}(x^\pm,ec x_ot) &= -rac{\delta(x^\mp)}{\sqrt{2}}\,A^i_{1,2}(ec x_ot) \ B^z_{1,2}(x^\pm,ec x_ot) &= 0 & B^i_{1,2}(x^\pm,ec x_ot) &= \mp \epsilon^{ij}E^j_{1,2}(x^\pm,ec x_ot) \end{aligned}$$

Only transverse components are different from zero.

Glasma fields develop only in the forward light-cone region.

The general form of the gauge potential is:

 A^{μ}

$$egin{aligned} A^+(x) &= x^+ A(au, ec{x}_ot) \ A^-(x) &= -x^- A(au, ec{x}_ot) \ A^i(x) &= A^i_ot(au, ec{x}_ot) \end{aligned}$$



they are dynamic: note the dependence on $\tau=\sqrt{t^2-z^2}=\sqrt{2x^+x^-}$ they are boost-invariant: no dependence on $\eta=rac{1}{2} ln rac{x^+}{x^-}$

$$\begin{array}{l} & \frac{1}{\tau}\partial_{\tau}\frac{1}{\tau}\partial_{\tau}\tau^{2}A - [D^{i},[D^{i},A]] = 0 \\ \rightarrow F^{\mu\nu} \rightarrow \text{ YM equations:} & ig\tau\left[A,\partial_{\tau}A\right] - \frac{1}{\tau}\left[D^{i},\partial_{\tau}A_{\perp}^{i}\right] = 0 \\ & \frac{1}{\tau}\partial_{\tau}\tau\partial_{\tau}A_{\perp}^{i} - ig\tau^{2}[A,[D^{i},A]] - [D^{j},F^{ji}] = 0 \end{array}$$

No coupling to any current! The current enters through boundary conditions.

General solutions to CYM equations are not known.

analytical approach to solve CYN proposed in: Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

CGC looses applicability soon after the collision \longrightarrow proper time of such an evolving system is small au -acts as an expansion parameter

$$egin{aligned} A^i_ot(au,ec x_ot) &= \sum_{n=0}^\infty au^n A^i_{ot(n)}(ec x_ot) \ A(au,ec x_ot) &= \sum_{n=0}^\infty au^n A_{(n)}(ec x_ot) \end{aligned}$$

 $egin{aligned} |A^i_{ot(n)}| &\sim Q^n_s |A| \ |A_{(n)}| &\sim Q^{n+1}_s |A| \ |A| &= \sqrt{A^i_1 A^i_1} \ \end{aligned}$

radius of convergence is set by the only time scale saturation momentum scale as

$$1/Q_s$$
 $au Q_s$ -dimensionless

Boundary conditions connect different light-cone sectors. In the forward light cone they are:

$$egin{aligned} A^i_{ot}(au=0,ec{x}_{ot}) &= A^i_1(ec{x}_{ot}) + A^i_2(ec{x}_{ot}) \ A(au=0,ec{x}_{ot}) &= -rac{ig}{2}[A^i_1(ec{x}_{ot}),A^i_2(ec{x}_{ot})] \end{aligned}$$

Using the boundary conditions the system of coupled YM equations can be solved recursively.

Expansion in the proper time

Solutions:

$$\begin{split} A^{i}_{\perp(0)} &= A^{i}_{1} + A^{i}_{2} \\ A^{i}_{\perp(2)} &= \frac{1}{4} \epsilon^{ij} [D^{j}_{(0)}, B_{0}] \\ A^{i}_{\perp(4)} &= \frac{ig}{64} \left[\left[D^{i}_{(0)}, B_{0} \right], B_{0} \right] + \frac{ig}{16} \left[A_{(0)}, \left[D^{i}_{(0)}, A_{(0)} \right] \right] \\ &+ \frac{1}{64} \left(\epsilon^{ik} \left[D^{j}_{(0)}, \left[D^{j}_{(0)}, \left[D^{k}_{(0)}, B_{0} \right] \right] \right] - \epsilon^{jk} \left[D^{j}_{(0)}, \left[D^{i}_{(0)}, \left[D^{k}_{(0)}, B_{0} \right] \right] \right] \right) \\ A_{(0)} &= -\frac{ig}{2} [A^{i}_{1}, A^{i}_{2}] \\ A_{(2)} &= \frac{1}{8} [D^{j}_{(0)}, [D^{j}_{(0)}, A_{(0)}]] \\ A_{(4)} &= \frac{1}{192} [D^{k}_{(0)}, [D^{k}_{(0)}, \left[D^{j}_{(0)}, \left[D^{j}_{(0)}, A_{(0)} \right] \right] \right] + \frac{ig}{48} \epsilon^{ij} \left[[D^{i}_{(0)}, A_{(0)}], [D^{j}_{(0)}, B_{0}] \right] \end{split}$$

Similarly:
$$F^{\mu
u} = F^{\mu
u}_{(0)} + au F^{\mu
u}_{(1)} + au^2 F^{\mu
u}_{(2)} + \mathcal{O}(au^3)$$

$$egin{array}{lll} B_0 &= B^z_{(au=0)} \ E_0 &= E^z_{(au=0)} \end{array}$$

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Expansion in the proper time

Expansion of chromodynamic fields

$$egin{aligned} \mathbf{E} &= \mathbf{E}_{(0)} + au \mathbf{E}_{(1)} + au^2 \mathbf{E}_{(2)} + \dots \ \mathbf{B} &= \mathbf{B}_{(0)} + au \mathbf{B}_{(1)} + au^2 \mathbf{B}_{(2)} + \dots \end{aligned}$$

 $egin{aligned} E^z_{(0)} &= ig[A^i_1,A^i_2] \ B^z_{(0)} &= ig\epsilon^{ij}[A^i_1,A^j_2] \end{aligned}$

Oth order fields are purely longitudinal

(superposition of two pure-gauge potentials is not pure gauge due to non-linear character of non-Abelian theory)

$$egin{aligned} E^i_{(1)} &= -rac{1}{2} igg(\sinh \eta \left[D^i_{(0)}, E_0
ight] + \cosh \eta \epsilon^{ij} \left[D^j_{(0)}, B_0
ight] igg) \ & ext{1st order fields are purely transverse} \ B^i_{(1)} &= rac{1}{2} igg(\cosh \eta \epsilon^{ij} \left[D^j_{(0)}, E_0
ight] - \sinh \eta \left[D^i_{(0)}, B_0
ight] igg) \ & ext{(induced by the decrease of longitudinal fields after a short time)} \end{aligned}$$

To compute the energy loss and momentum broadening we need the correlators of fields:

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^2}{2N_c} \int_0^t dt' \Big[ig\langle E^i_a(t,\mathbf{x}) E^j_a(t',\mathbf{x}') ig
angle + \epsilon^{jkl} v^k ig\langle E^i_a(t,\mathbf{x}) B^l_a(t',\mathbf{x}') ig
angle \ &+ \epsilon^{ikl} v^k ig\langle B^l_a(t,\mathbf{x}) E^i_a(t,\mathbf{x}') ig
angle + \epsilon^{ikl} \epsilon^{jmn} v^k v^m ig\langle B^l_a(t,\mathbf{x}) B^n_a(t',\mathbf{x}') ig
angle \Big] \end{aligned}$$

Correlators of gauge potentials

Correlators of gauge potentials determine correlators of chromodynamic fields.

potentials of different nuclei are uncorrelated:

$$\langle A^i_{1a} A^j_{2b}
angle = 0$$

potentials of the same nuclei are correlated

$$egin{aligned} &\langle
ho_a(x^{\mp},ec{x}_{\perp})
ho_b(y^{\mp},ec{y}_{\perp})
angle = rac{g^2}{N_c^2-1}\delta_{ab}\lambda(x^{\mp},ec{x}_{\perp})\delta(x^{\mp}-y^{\mp})\delta^2(ec{x}_{\perp}-ec{y}_{\perp})\ &\int dx^{\mp}\lambda(x^{\mp},ec{x}_{\perp}) = \mu(ec{x}_{\perp})\ &\checkmark \end{aligned}$$

volume density of sources

area density of sources

- → solution of YM equations starting from a covariant gauge $\Delta \alpha_{\text{COV}}(x^-, \vec{x}_\perp) = -\rho_{\text{COV}}(x^-, \vec{x}_\perp)$
- \rightarrow a path through different gauges, IR cut-off, retarded boundary conditions

Correlators of gauge potentials

potentials of the same nuclei:

$$\langle A^i_a(\mathbf{x}_\perp) A^j_b(\mathbf{x}'_\perp)
angle = \delta^{ab} \Big(\delta^{ij}_\perp C_1(r) - \hat{r}^i \hat{r}^j C_2(r) \Big)$$

$$rac{\mathbf{x}}_{\perp}-\mathbf{x}'_{\perp}, \ \ r\equiv |\mathbf{r}|, \ \ \hat{r}^i\equiv r^i/r$$

 $K_0,\;K_1$ MacDonald functions

 $mpprox\Lambda_{
m QCD}pprox200~{
m MeV}$ infrared regulator

 $\mu = g^{-4} \, (N_c^2 - 1) Q_s^2$ charge density per unit transverse area

Correlators of electric and magnetic fields

$$egin{aligned} M_E(r) &\equiv 2C_1^2(r) - 2C_1(r)\,C_2(r) + C_2^2(r) \ M_B(r) &\equiv 2C_1^2(r) - 2C_1(r)\,C_2(r) \end{aligned}$$

Oth order correlators

$$egin{aligned} &\langle E^z_a(\mathbf{x}_ot)\,E^z_b(\mathbf{x}_ot^\prime)
angle &= g^2 N_c \delta^{ab}\,M_E(r) \ &\langle B^z_a(\mathbf{x}_ot)\,B^z_b(\mathbf{x}_ot^\prime)
angle &= g^2 N_c \delta^{ab}\,M_B(r) \ &\langle E^z_a(\mathbf{x}_ot)\,B^z_b(\mathbf{x}_ot^\prime)
angle &= 0 \end{aligned}$$

1st order correlators

$$M'_E(r), \; M'_B(r) \qquad igstarrow au' \langle E^0_a({f x}_ot) \, E^i_{(1)b}(t',{f x}'_ot,z')
angle = -rac{g^2}{2} \, N_c \delta^{ab} \, \hat{r}^i z' \, M'_E(r) \ \ldots$$

(correlators of three gluon fields vanish)

all possible combinations of $~E^0,~B^0,~E^i_{(1)},~B^i_{(1)}$ at order of τ

Energy loss and momentum broadening

 $rac{dE}{dx}$

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^2}{2N_c} \int_0^t dt' \Big[ig\langle E^i_a(t,\mathbf{x}) E^j_a(t',\mathbf{x}') ig
angle + \epsilon^{jkl} v^k ig\langle E^i_a(t,\mathbf{x}) B^l_a(t',\mathbf{x}') ig
angle \ &+ \epsilon^{ikl} v^k ig\langle B^l_a(t,\mathbf{x}) E^i_a(t,\mathbf{x}') ig
angle + \epsilon^{ikl} \epsilon^{jmn} v^k v^m ig\langle B^l_a(t,\mathbf{x}) B^n_a(t',\mathbf{x}') ig
angle \Big] \end{aligned}$$

We were able to obtain an explicit analytic form of the tensor:

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^4(N_c^2-1)}{4} \int_0^t dt' \Big\{ 2n^i n^j M_E(r) - \Big(n^i \hat{r}^j z' - n^j \hat{r}^i z \Big) M'_E(r) \ &+ \epsilon^{jkl} v^k \Big(n^i n^n \epsilon^{lmn} \hat{r}^m t' + n^l n^n \epsilon^{imn} \hat{r}^m t \Big) M'_E(r) \ &+ \epsilon^{ikl} v^k \Big[\epsilon^{jmn} v^m \Big(2n^l n^n M_B(r) - ig(n^l \hat{r}^n z' - n^n \hat{r}^l z ig) M'_B(r) \Big) & \mathbf{n} = (0,0,1) \ &- \Big(n^l n^n \epsilon^{jmn} \hat{r}^m t' + n^j n^n \epsilon^{lmn} \hat{r}^m t \Big) M'_B(r) \Big] \Big\}. \ &= - rac{v}{T} rac{v^i v^j}{\mathbf{v}^2} X^{ij} ig(\mathbf{v} ig) & \hat{q} = rac{2}{v} ig(\delta^{ij} - rac{v^i v^j}{\mathbf{v}^2} ig) X^{ij}(\mathbf{v}) \end{aligned}$$

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Energy loss and momentum broadening

$$egin{aligned} rac{dE}{dx} &= -rac{v_{\parallel}^2}{vT} \Big[f^0_E(v_{\perp}) + v_{\perp} f^1_E(v_{\perp}) \Big] \ \hat{q} &= rac{2v_{\perp}^2}{v} \Big[rac{f^0_E(v_{\perp})}{v^2} + f^0_B(v_{\perp}) + rac{v_{\perp}}{v^2} f^1_E(v_{\perp}) + rac{(1-v_{\parallel}^2)}{v_{\perp}} f^1_B(v_{\perp}) \Big] \end{aligned}$$

$$egin{aligned} f^0_{E,B}(v_\perp) &\equiv rac{g^4(N_c^2-1)}{2} \int_0^t dt' M_{E,B}(r) \ f^1_{E,B}(v_\perp) &\equiv rac{g^4(N_c^2-1)}{4} \int_0^t dt' (t-t') M'_{E,B}(r) \end{aligned}$$

 $M_{E,B}(r)$ diverges when $\,r
ightarrow 0\,$ (CGC breaks down at small distances.)

r

regularization procedure: $M^{
m reg}(r)\equiv\Theta(r_s-r)\,M(r_s)+\Theta(r-r_s)\,M(r)$

$$q_s=Q_s^{-1}$$

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Functions $f_{E,B}$

$$m=200{
m MeV},\;N_c=3,\;g=1,\;Q_s=2{
m GeV}$$



Saturation is reached before t=1fm/c.

 $f^0_{E,B} > f^1_{E,B} ~~{
m for}~~ v_\perp > 0.73$

Energy loss and momentum broadening

$$m=200{
m MeV},\;N_c=3,\;g=1,\;Q_s=2{
m GeV}$$



Energy loss and momentum broadening



heta -angle between the the heavy-quark velocity and the collision axis

energy loss is minimal when HQ moves perpendicularly to the axis

momentum broadening is maximal when HQ moves perpendicularly to the axis

Remarks

saturation value at $v=0.9,\;\cos heta=0$

$${\hat q}_{
m LO}=42{
m GeV}^2/{
m fm}$$
 ${\hat q}_{
m LO+NLO}=8.5{
m GeV}^2/{
m fm}$

momentum broadening inferred from a jet quenching of heavy ion collisions

$${\hat q}_{
m JQ}:~1.5-7.0~{
m GeV^2/fm}$$

glasma may provide a significant contribution to jet quenching

- regularization procedure: we checked a few possibilities - results are not very sensitive to them
- higher order terms and convergence validity of CGC work in progress

Summary and conclusions

- Collision terms of the Fokker-Planck equation for HQ transported through glasma were derived
- Energy loss and momentum broadening of HQ were computed
- Both quantities are strongly directionally dependent
- Energy loss is maximal when the heavy quark moves along the collision axis
- Momentum broadening is maximal when the heavy quark moves perpendicularly to the axis
- The values of both transport coefficients are sizeable so the glasma phase may have a large effect on the jet quenching observed in HIC
- Higher order terms have to be carefully studied to draw a firm conclusion

NOTE: many papers use only LO terms, while here we obtained NLO corrections!

Regularization

