# Heavy quarks embedded in glasma

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# Introduction

evolution of nuclear matter after heavy ion collision



many approaches/models needed

Properties of QCD medium are known only to some extent and are under active exploration.

Heavy quarks - charm and beauty - due to their large masses are produced with high  $p_T$  at the earliest stage of HIC through hard interactions.

Heavy quarks propagate through the QCD medium and test its evolution at all stages.

# Heavy quarks

**heavy quark in a weakly interacting QCD thermal medium (kinetic theory)** a Brownian particle undergoing random kicks from fast-moving particles of the medium (changes of movement, energy loss due to collisions and due to radiation)

$$\begin{array}{c|c} & & \\ \mbox{typical momentum exchange of HQ with the} & q \approx gT & \\ & \mbox{heat bath} & p \gg q & \\ & \mbox{momentum} & p \mbox{ of HQ related to large mass} & \\ & \mbox{thermalization time of the bulk medium} & \tau_{\rm th} & \\ & \mbox{thermal relaxation time of HQ} & \tau_Q \approx \tau_{\rm th} m/T & \end{array} \rightarrow \qquad r_Q \gg \tau_{\rm th}$$

#### What are the effects of pre-equilibrium phase on heavy quarks?

- within kinetic theory (Das et al, J. Phys. G 44, 095102 (2017))
- within CGC Wong equations of motion of HQ solved numerically (Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018), Ruggieri, Das, Phys. Rev. D 98, 094024 (2018))
- HERE: within CGC - analytically tractable approach of HQ interacting with highly- occupied soft gluon fields <sup>4</sup>

# The system

Heavy quarks:  $m, p^{\mu}, E_{\mathbf{p}}, \mathbf{v} = \frac{\mathbf{p}}{E_{\mathbf{p}}}$  - mass, four-momentum, energy and velocity of HQ  $p_0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$  - momenta of heavy quarks obey mass shell constraint  $Q(t, \mathbf{r}, \mathbf{p})$  - distribution function of heavy quarks in the medium

Soft classical field (because of the large occupation numbers):  $A^{\mu}(x)$ 

Interaction:  ${f F}(t,{f r})\equiv gig({f E}(t,{f r})+{f v} imes{f B}(t,{f r})ig)$  - color Lorentz force

g - coupling constant

 ${f E}(t,{f r}),\;{f B}(t,{f r})$  - chromoelectric and chromomagnetic fields

# **Towards Fokker-Planck equation**

Transport of heavy quarks interacting within a medium is described by the Fokker-Planck equation (usually applied when HQ interact with plasma constituents).

Liouville equation in the Vlasov form: (contains all microscopic information)

$$ig(D^0+\mathbf{v}\cdot\mathbf{D}ig)Q(t,\mathbf{r},\mathbf{p})-rac{1}{2}ig\{\mathbf{F}(t,\mathbf{r}),
abla_pQ(t,\mathbf{r},\mathbf{p})ig\}=0$$

covariant derivative: 
$$D^\mu \equiv (D^0, {f D}) \equiv \partial^\mu - ig[A^\mu(x), \cdots]$$

Distribution function can be divided into two parts:

$$egin{aligned} Q(t,\mathbf{r},\mathbf{p}) &= \langle Q(t,\mathbf{r},\mathbf{p}) 
angle + \delta Q(t,\mathbf{r},\mathbf{p}) \ & ert \ & e$$

averaging: micro -> macro (information about microscopic structure is lost

# **Towards Fokker-Planck equation**

Distribution function satisfies the following conditions:

regular part is much larger than the fluctuating one:

regular part is slowly varying function of time and space:

$$\left|rac{\partial \delta Q}{\partial t}
ight| \gg \left|rac{\partial \langle Q 
angle}{\partial t}
ight| \qquad \left|
abla \delta Q
ight| \gg \left|
abla \langle Q 
angle
ight|$$

 $\langle {f E}(t,{f r})
angle = \langle {f B}(t,{f r})
angle = 0$ 

 $|\langle Q 
angle| \gg |\delta Q| \qquad |
abla_p \langle Q 
angle| \gg |
abla_p \delta Q|$ 

regular part of the chromodynamic fields vanishes:

Solving the Vlasov equation in 2 steps:

 $\delta Q$  - obeys Liouville (collisionless) transport equation

the solution of the transport equation for  $\delta Q$  plays the role of the collision term in the transport equation for the regular part - Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks: Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018)

$$egin{aligned} & \left(D-
abla^i_pX^{ij}(\mathbf{v})
abla^j_p-
abla^i_pY^i(\mathbf{v})
ight)n(t,\mathbf{x},\mathbf{p})=0 \ & \langle Q(t,\mathbf{r},\mathbf{p})
angle=n(t,\mathbf{r},\mathbf{p}) & D=rac{\partial}{\partial t}+\mathbf{v}\cdot
abla \ & X^{ij}(\mathbf{v})\equivrac{1}{2N_c}\int_0^t dt'ig\langle F^i_a(t,\mathbf{x})F^j_aig(t',\mathbf{x}-\mathbf{v}(t-t')ig) 
angle \end{aligned}$$

 $Y^i(\mathbf{v}) = X^{ij}(\mathbf{v}) rac{v^j}{T}$  - postulated so that the distribution function satisfies the Fokker-Planck equation in equilibrium

 $T\,$  - temperature of plasma that has the same energy density as in equilibrium

Physical meaning:

$$rac{\langle \Delta p^i 
angle}{\Delta t} = -Y^i ({f v})$$

$$rac{\langle \Delta p^i \Delta p^j 
angle}{\Delta t} = X^{ij}(\mathbf{v}) + X^{ji}(\mathbf{v})$$

## Energy loss and momentum broadening

$$rac{dE}{dx} = -rac{v}{T} rac{v^i v^j}{\mathbf{v}^2} X^{ij}(\mathbf{v})$$

$$\hat{q} = rac{2}{v} \Big( \delta^{ij} - rac{v^i v^j}{\mathbf{v}^2} \Big) X^{ij}(\mathbf{v})$$

# Color Glass Condensate (CGC)

CGC - a framework to describe physics of heavy ion collisions in the earliest stage of the collision



after the collision

before the collision:

- large-*x* partons: valence quarks, the source partons
- small-x partons: soft gluon fields, solutions of classical Yang-Mills equations

- valence quarks fly away
- soft gluon fields interact producing glasma
- glasma solutions of classical Yang-Mills equations given in terms of the single-nucleus solutions

x - the fraction of momentum carried by a parton

Since the description of the longitudinal physics (along the beam) separates from the transverse one (within the area of the nuclei) it is convenient to use the light-cone coordinates

$$(x^+,\ x^-,\ x_\perp) \qquad \qquad x^+ = rac{t+z}{\sqrt{2}},\ x^- = rac{t-z}{\sqrt{2}}$$

Sources given by  $SU(N_{c})$  four-current:

$$J^{\mu}_{1,2}(x^{\mp},ec{x}_{\perp}) = \delta^{\mu\pm}
ho_{1,2}(x^{\mp},ec{x}_{\perp})$$

color-charge surface density

In McLerran-Venugopalan model:

$$ho_{1,2}(x^{\mp},ec{x}_{\perp})=\delta(x^{\mp})
ho_{1,2}(ec{x}_{\perp})$$

Classical Yang-Mills equations:  $\left[D_{\mu},F^{\mu
u}
ight]=J^{
u}$ 



# CGC: before the collision

Solutions of CYM equations:

$$egin{aligned} &A^\pm_{1,2}(x^\pm,ec x_ot)=0\ &A^i_{1,2}(x^\pm,ec x_ot)= heta(x^\mp)A^i_{1,2}(ec x_ot)\ &A^i_{1,2}(ec x_ot)=-rac{1}{ig}U_{1,2}(ec x_ot)\partial^i U^\dagger_{1,2}(ec x_ot)\ & ext{pure gauge transform of vacuum} \end{aligned}$$

 $U(ec{x}_{ot})\equiv U[g,
ho,ec{x}_{ot}]$  - the unitary matrix

Chromoelectric and chromomagnetic fields are given by the respective components of the strength tensor. They are:

$$egin{aligned} E^z_{1,2}(x^\pm,ec x_ot) &= 0 & E^i_{1,2}(x^\pm,ec x_ot) &= -rac{\delta(x^\mp)}{\sqrt{2}}\,A^i_{1,2}(ec x_ot) \ B^z_{1,2}(x^\pm,ec x_ot) &= 0 & B^i_{1,2}(x^\pm,ec x_ot) &= \mp \epsilon^{ij}E^j_{1,2}(x^\pm,ec x_ot) \end{aligned}$$

Only transverse components are different from zero.

Glasma fields develop only in the forward light-cone region.

The general form of the gauge potential is:

 $A^{\mu}$ 

$$egin{aligned} A^+(x) &= x^+ A( au, ec{x}_ot) \ A^-(x) &= -x^- A( au, ec{x}_ot) \ A^i(x) &= A^i_ot( au, ec{x}_ot) \end{aligned}$$



they are dynamic: note the dependence on  $\tau=\sqrt{t^2-z^2}=\sqrt{2x^+x^-}$  they are boost-invariant: no dependence on  $\eta=rac{1}{2} ln rac{x^+}{x^-}$ 

$$\begin{array}{l} & \frac{1}{\tau}\partial_{\tau}\frac{1}{\tau}\partial_{\tau}\tau^{2}A - [D^{i},[D^{i},A]] = 0 \\ \rightarrow F^{\mu\nu} \rightarrow \text{ YM equations:} & ig\tau\left[A,\partial_{\tau}A\right] - \frac{1}{\tau}\left[D^{i},\partial_{\tau}A_{\perp}^{i}\right] = 0 \\ & \frac{1}{\tau}\partial_{\tau}\tau\partial_{\tau}A_{\perp}^{i} - ig\tau^{2}[A,[D^{i},A]] - [D^{j},F^{ji}] = 0 \end{array}$$

No coupling to any current! The current enters through boundary conditions.

General solutions to CYM equations are not known.

analytical approach to solve CYN proposed in: Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

CGC looses applicability soon after the collision  $\longrightarrow$  proper time of such an evolving system is small au -acts as an expansion parameter

$$egin{aligned} A^i_ot( au,ec{x}_ot) &= \sum_{n=0}^\infty au^n A^i_{ot(n)}(ec{x}_ot) \ A( au,ec{x}_ot) &= \sum_{n=0}^\infty au^n A_{(n)}(ec{x}_ot) \end{aligned}$$

 $egin{aligned} |A^i_{ot(n)}| &\sim Q^n_s |A| \ |A_{(n)}| &\sim Q^{n+1}_s |A| \ |A| &= \sqrt{A^i_1 A^i_1} \ \end{aligned}$ 

radius of convergence is set by the only time scale saturation momentum scale as

$$1/Q_s$$
  $au Q_s$  -dimensionless

Boundary conditions connect different light-cone sectors. In the forward light cone they are:

$$egin{aligned} A^i_{ot}( au=0,ec{x}_{ot}) &= A^i_1(ec{x}_{ot}) + A^i_2(ec{x}_{ot}) \ A( au=0,ec{x}_{ot}) &= -rac{ig}{2}[A^i_1(ec{x}_{ot}),A^i_2(ec{x}_{ot})] \end{aligned}$$

Using the boundary conditions the system of coupled YM equations can be solved recursively.

## Expansion in the proper time

Solutions:

$$\begin{split} A^{i}_{\perp(0)} &= A^{i}_{1} + A^{i}_{2} \\ A^{i}_{\perp(2)} &= \frac{1}{4} \epsilon^{ij} [D^{j}_{(0)}, B_{0}] \\ A^{i}_{\perp(4)} &= \frac{ig}{64} \left[ \left[ D^{i}_{(0)}, B_{0} \right], B_{0} \right] + \frac{ig}{16} \left[ A_{(0)}, \left[ D^{i}_{(0)}, A_{(0)} \right] \right] \\ &+ \frac{1}{64} \left( \epsilon^{ik} \left[ D^{j}_{(0)}, \left[ D^{j}_{(0)}, \left[ D^{k}_{(0)}, B_{0} \right] \right] \right] - \epsilon^{jk} \left[ D^{j}_{(0)}, \left[ D^{i}_{(0)}, \left[ D^{k}_{(0)}, B_{0} \right] \right] \right] \right) \\ A_{(0)} &= -\frac{ig}{2} [A^{i}_{1}, A^{i}_{2}] \\ A_{(2)} &= \frac{1}{8} [D^{j}_{(0)}, [D^{j}_{(0)}, A_{(0)}]] \\ A_{(4)} &= \frac{1}{192} [D^{k}_{(0)}, [D^{k}_{(0)}, \left[ D^{j}_{(0)}, \left[ D^{j}_{(0)}, A_{(0)} \right] \right] \right] + \frac{ig}{48} \epsilon^{ij} \left[ [D^{i}_{(0)}, A_{(0)}], [D^{j}_{(0)}, B_{0}] \right] \end{split}$$

Similarly: 
$$F^{\mu
u} = F^{\mu
u}_{(0)} + au F^{\mu
u}_{(1)} + au^2 F^{\mu
u}_{(2)} + \mathcal{O}( au^3)$$

$$egin{aligned} B_0 &= B^z_{( au=0)} \ E_0 &= E^z_{( au=0)} \end{aligned}$$

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# Expansion in the proper time

Expansion of chromodynamic fields

$$egin{aligned} \mathbf{E} &= \mathbf{E}_{(0)} + au \mathbf{E}_{(1)} + au^2 \mathbf{E}_{(2)} + \dots \ \mathbf{B} &= \mathbf{B}_{(0)} + au \mathbf{B}_{(1)} + au^2 \mathbf{B}_{(2)} + \dots \end{aligned}$$

 $egin{aligned} E^z_{(0)} &= ig[A^i_1,A^i_2] \ B^z_{(0)} &= ig\epsilon^{ij}[A^i_1,A^j_2] \end{aligned}$ 

#### Oth order fields are purely longitudinal

(superposition of two pure-gauge potentials is not pure gauge due to non-linear character of non-Abelian theory)

$$egin{aligned} E^i_{(1)} &= -rac{1}{2} igg( \sinh \eta \left[ D^i_{(0)}, E_0 
ight] + \cosh \eta \epsilon^{ij} \left[ D^j_{(0)}, B_0 
ight] igg) \ & ext{1st order fields are purely transverse} \ B^i_{(1)} &= rac{1}{2} igg( \cosh \eta \epsilon^{ij} \left[ D^j_{(0)}, E_0 
ight] - \sinh \eta \left[ D^i_{(0)}, B_0 
ight] igg) \ & ext{(induced by the decrease of longitudinal fields after a short time)} \end{aligned}$$

To compute the energy loss and momentum broadening we need the correlators of fields:

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^2}{2N_c} \int_0^t dt' \Big[ ig\langle E^i_a(t,\mathbf{x}) E^j_a(t',\mathbf{x}') ig
angle + \epsilon^{jkl} v^k ig\langle E^i_a(t,\mathbf{x}) B^l_a(t',\mathbf{x}') ig
angle \ &+ \epsilon^{ikl} v^k ig\langle B^l_a(t,\mathbf{x}) E^i_a(t,\mathbf{x}') ig
angle + \epsilon^{ikl} \epsilon^{jmn} v^k v^m ig\langle B^l_a(t,\mathbf{x}) B^n_a(t',\mathbf{x}') ig
angle \Big] \end{aligned}$$

# Correlators of gauge potentials

Correlators of gauge potentials determine correlators of chromodynamic fields.

potentials of different nuclei are uncorrelated:

$$\langle A^i_{1a} A^j_{2b} 
angle = 0$$

potentials of the same nuclei are correlated

$$egin{aligned} &\langle 
ho_a(x^{\mp},ec{x}_{\perp})
ho_b(y^{\mp},ec{y}_{\perp})
angle = rac{g^2}{N_c^2-1}\delta_{ab}\lambda(x^{\mp},ec{x}_{\perp})\delta(x^{\mp}-y^{\mp})\delta^2(ec{x}_{\perp}-ec{y}_{\perp})\ &\int dx^{\mp}\lambda(x^{\mp},ec{x}_{\perp}) = \mu(ec{x}_{\perp})\ &\checkmark \end{aligned}$$

volume density of sources

area density of sources

- → solution of YM equations starting from a covariant gauge  $\Delta \alpha_{\text{COV}}(x^-, \vec{x}_\perp) = -\rho_{\text{COV}}(x^-, \vec{x}_\perp)$
- $\rightarrow$  a path through different gauges, IR cut-off, retarded boundary conditions

# Correlators of gauge potentials

potentials of the same nuclei:

$$\langle A^i_a({f x}_\perp) A^j_b({f x}'_\perp) 
angle = \delta^{ab} \Big( \delta^{ij}_\perp C_1(r) - \hat{r}^i \hat{r}^j C_2(r) \Big)$$

$$rac{\mathbf{x}}_{\perp}-\mathbf{x}'_{\perp}, \ \ r\equiv |\mathbf{r}|, \ \ \hat{r}^i\equiv r^i/r$$

 $K_0,\;K_1$  MacDonald functions

 $mpprox\Lambda_{
m QCD}pprox200~{
m MeV}$  infrared regulator

 $\mu = g^{-4} \, (N_c^2 - 1) Q_s^2$  charge density per unit transverse area

# Correlators of electric and magnetic fields

$$egin{aligned} M_E(r) &\equiv 2C_1^2(r) - 2C_1(r)\,C_2(r) + C_2^2(r) \ M_B(r) &\equiv 2C_1^2(r) - 2C_1(r)\,C_2(r) \end{aligned}$$

#### **Oth order correlators**

$$egin{aligned} &\langle E^z_a(\mathbf{x}_ot)\,E^z_b(\mathbf{x}_ot^\prime)
angle &= g^2 N_c \delta^{ab}\,M_E(r) \ &\langle B^z_a(\mathbf{x}_ot)\,B^z_b(\mathbf{x}_ot^\prime)
angle &= g^2 N_c \delta^{ab}\,M_B(r) \ &\langle E^z_a(\mathbf{x}_ot)\,B^z_b(\mathbf{x}_ot^\prime)
angle &= 0 \end{aligned}$$

**1st order correlators** 

$$M'_E(r), \; M'_B(r) \qquad igstarrow au' \langle E^0_a({f x}_ot) \, E^i_{(1)b}(t',{f x}'_ot,z') 
angle = -rac{g^2}{2} \, N_c \delta^{ab} \, \hat{r}^i z' \, M'_E(r) \ \ldots$$

(correlators of three gluon fields vanish)

all possible combinations of  $~E^0,~B^0,~E^i_{(1)},~B^i_{(1)}$  at order of  $\tau$ 

#### Energy loss and momentum broadening

 $rac{dE}{dx}$ 

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^2}{2N_c} \int_0^t dt' \Big[ ig\langle E^i_a(t,\mathbf{x}) E^j_a(t',\mathbf{x}') ig
angle + \epsilon^{jkl} v^k ig\langle E^i_a(t,\mathbf{x}) B^l_a(t',\mathbf{x}') ig
angle \ &+ \epsilon^{ikl} v^k ig\langle B^l_a(t,\mathbf{x}) E^i_a(t,\mathbf{x}') ig
angle + \epsilon^{ikl} \epsilon^{jmn} v^k v^m ig\langle B^l_a(t,\mathbf{x}) B^n_a(t',\mathbf{x}') ig
angle \Big] \end{aligned}$$

We were able to obtain an explicit analytic form of the tensor:

$$egin{aligned} X^{ij}(\mathbf{v}) &= rac{g^4(N_c^2-1)}{4} \int_0^t dt' \Big\{ 2n^i n^j M_E(r) - \Big( n^i \hat{r}^j z' - n^j \hat{r}^i z \Big) M'_E(r) \ &+ \epsilon^{jkl} v^k \Big( n^i n^n \epsilon^{lmn} \hat{r}^m t' + n^l n^n \epsilon^{imn} \hat{r}^m t \Big) M'_E(r) \ &+ \epsilon^{ikl} v^k \Big[ \epsilon^{jmn} v^m \Big( 2n^l n^n M_B(r) - ig( n^l \hat{r}^n z' - n^n \hat{r}^l z ig) M'_B(r) \Big) & \mathbf{n} = (0,0,1) \ &- \Big( n^l n^n \epsilon^{jmn} \hat{r}^m t' + n^j n^n \epsilon^{lmn} \hat{r}^m t \Big) M'_B(r) \Big] \Big\}. \ &= - rac{v}{T} rac{v^i v^j}{\mathbf{v}^2} X^{ij} ig( \mathbf{v} ig) & \hat{q} = rac{2}{v} ig( \delta^{ij} - rac{v^i v^j}{\mathbf{v}^2} ig) X^{ij}(\mathbf{v}) \end{aligned}$$

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#### Energy loss and momentum broadening

$$egin{aligned} rac{dE}{dx} &= -rac{v_{\parallel}^2}{vT} \Big[ f^0_E(v_{\perp}) + v_{\perp} f^1_E(v_{\perp}) \Big] \ \hat{q} &= rac{2v_{\perp}^2}{v} \Big[ rac{f^0_E(v_{\perp})}{v^2} + f^0_B(v_{\perp}) + rac{v_{\perp}}{v^2} f^1_E(v_{\perp}) + rac{(1-v_{\parallel}^2)}{v_{\perp}} f^1_B(v_{\perp}) \Big] \end{aligned}$$

$$egin{aligned} f^0_{E,B}(v_\perp) &\equiv rac{g^4(N_c^2-1)}{2} \int_0^t dt' M_{E,B}(r) \ f^1_{E,B}(v_\perp) &\equiv rac{g^4(N_c^2-1)}{4} \int_0^t dt' (t-t') M'_{E,B}(r) \end{aligned}$$

 $M_{E,B}(r)$  diverges when  $\,r
ightarrow 0\,$  (CGC breaks down at small distances.)

r

regularization procedure:  $M^{
m reg}(r)\equiv\Theta(r_s-r)\,M(r_s)+\Theta(r-r_s)\,M(r)$ 

$$q_s=Q_s^{-1}$$

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# Functions $f_{E,B}$

$$m=200{
m MeV},\;N_c=3,\;g=1,\;Q_s=2{
m GeV}$$



Saturation is reached before t=1fm/c.

 $f^0_{E,B} > f^1_{E,B} ~~{
m for}~~ v_\perp > 0.73$ 

#### Energy loss and momentum broadening

$$m=200{
m MeV},\;N_c=3,\;g=1,\;Q_s=2{
m GeV}$$



# Energy loss and momentum broadening



heta -angle between the the heavy-quark velocity and the collision axis

energy loss is minimal when HQ moves perpendicularly to the axis

momentum broadening is maximal when HQ moves perpendicularly to the axis

## Remarks

saturation value at  $v=0.9,\;\cos heta=0$ 

$${\hat q}_{
m LO}=42{
m GeV}^2/{
m fm}$$
  ${\hat q}_{
m LO+NLO}=8.5{
m GeV}^2/{
m fm}$ 

momentum broadening inferred from a jet quenching of heavy ion collisions

$${\hat q}_{
m JQ}:~1.5-7.0~{
m GeV^2/fm}$$

#### glasma may provide a significant contribution to jet quenching

- regularization procedure: we checked a few possibilities - results are not very sensitive to them
- higher order terms and convergence validity of CGC work in progress

# Summary and conclusions

- Collision terms of the Fokker-Planck equation for HQ transported through glasma were derived
- Energy loss and momentum broadening of HQ were computed
- Both quantities are strongly directionally dependent
- Energy loss is maximal when the heavy quark moves along the collision axis
- Momentum broadening is maximal when the heavy quark moves perpendicularly to the axis
- The values of both transport coefficients are sizeable so the glasma phase may have a large effect on the jet quenching observed in HIC
- Higher order terms have to be carefully studied to draw a firm conclusion

#### NOTE: many papers use only LO terms, while here we obtained NLO corrections!

# Regularization

