Magnetic and electric dipole moments of short-lived particles and proposal for their measurement at CERN

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Outline of the talk

- Introduction: orbital and spin magnetic dipole moment (MDM)
- Electric dipole moment (EDM): violation of T- and CP-invariance
- Short-lived charmed baryons with lifetime $\sim 10^{-13}$ s and their MDMs
- Precession of spin in external magnetic and electric fields
- Feasibility of measuring magnetic and electric dipole moments of short-lived baryons using bent crystals
  Application to τ lepton produced in $D_s^+ \to \tau^+ \nu_\tau$ decay
- Proposal for experiments at the LHC

Magnetic moment: Schrodinger equation

First consider non-relativistic quantum mechanics based on **Schrodinger** equation.

Hamiltonian of a particle in external electromagnetic field $A^\mu = (A^0, \vec{A})$ is

$$H = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 + eA^0 = \frac{\vec{p}^2}{2m} + eA^0 - \frac{e}{mc}\vec{A}\,\vec{p} + i\hbar\vec{\nabla}\vec{A} + \frac{e^2}{2mc^2}\vec{A}^2$$

If there is only constant magnetic field $\vec{B}$, then $A^0 = 0$, and $\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$, and

$$H = \frac{\vec{p}^2}{2m} - \vec{\mu}\,\vec{B} + \mathcal{O}(e^2)$$

$$\vec{\mu} = \frac{e}{2m}(\vec{r} \times \vec{p}) = \frac{e}{2m}\vec{L},$$

where $\vec{L}$ is the orbital moment. Like in classical electrodynamics, magnetic moment is determined by mechanical moment.

Note that spin magnetic moment does not appear in non-relativistic description.

Spin magnetic moment: Dirac equation

Unification of special relativity and quantum mechanics.

The Dirac equation (linear in derivatives) for spin-1/2 fermion in external electromagnetic field $A^\mu = (A^0, \vec{A})$

$$i \frac{\partial}{\partial t}\psi = [\, c\vec{\alpha}\,(\vec{p} - \frac{e}{c}\vec{A}) + \beta mc^2 + eA^0 \,]\,\psi,$$

for the 4-component spinor $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, and β and $\vec{\alpha}$ are 4×4 Dirac matrices.

For non-relativistic velocities $v \ll c$ we reduce the Dirac eq-n to the Pauli equation for the 2-component spinor φ:

$$i \frac{\partial}{\partial t}\varphi = \left[\frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 + eA^0 - \frac{e\hbar}{2m\,c}\vec{\sigma}\,\vec{B} \right] \varphi$$

The important feature is appearance of the spin MDM

$$\vec{\mu}_{spin} = 2\,\frac{e}{2m\,c}\,\vec{S}, \qquad \text{where } \vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

Magnetic moments in quantum physics: spin and MDM

Now we add orbital and spin contributions and obtain the total magnetic moment

$$\vec{\mu} = \vec{\mu}_{orb} + \vec{\mu}_{spin} = \frac{e}{2mc}(\vec{L} + 2\vec{S})$$

It is customary to write the spin MDM in the form

$$\vec{\mu}_{spin} = g\frac{e}{2mc}\vec{S} = \frac{g}{2}\frac{e\hbar}{2mc}\vec{\sigma}, \qquad \text{with } g = 2,$$

and g **is called** g**-factor, or gyromagnetic factor**.
Usually the Bohr magneton is introduced

$$\mu_B \equiv \frac{|e|\hbar}{2mc}$$

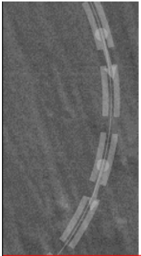
which for the electron is $\mu_B \approx 0.927 \cdot 10^{-20}\,\frac{erg}{Gauss} = 2 \times 10^{-11}\,|e| \cdot cm$,

Then the absolute value of MDM is measured in units of μ_B

$$\frac{\mu_{spin}}{\mu_B} = \frac{e}{|e|}\frac{g}{2}$$

For the point-like Dirac electron g-factor is equal to 2. However there are radiative corrections and possibly New Physics contributions. What do we know for leptons and quarks?

Electron g-factor: experiment vs. theory for $\mu/\mu_B = g/2$



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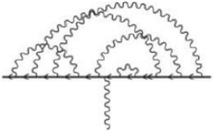
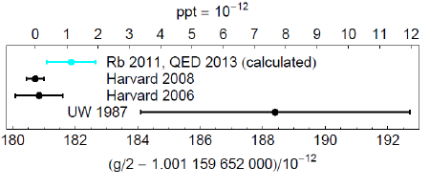


from measured
fine structure constant

Predicted: $\mu/\mu_B = -1.001\,159\,652\,181\,78\,(77)$
Measured: $\mu/\mu_B = -1.001\,159\,652\,180\,73\,(28)$



What can we say about agreement of theory with experiment?

Take the difference of central values and compare with total standard deviation σ

$$\Delta = \mu/\mu_B(exp) - \mu/\mu_B(theor) = 105 \times 10^{-14},$$

$$\sigma = \sqrt{\sigma^2_{theor} + \sigma^2_{exp}} = 82 \times 10^{-14}$$

Since $\Delta = 1.28\,\sigma$ agreement of QED with measurement for electron is indeed good.

Muon g-factor: experiment vs. theory

Introduce $a \equiv \frac{1}{2}(g-2)$, which is anomalous magnetic moment. Below is $a \times 10^{10}$.

	2011		2017	*to be discussed
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*
	HLMNT11		**KNT17**	
LO HVP	694.91 (4.27)	$\longrightarrow$	692.23 **(2.54)**	this work*
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.83 (0.04)	this work*
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	28.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.9σ	this work

To compare theory and experiment calculate

$$\Delta = a(exp) - a(theor) \approx 28.1 \times 10^{-9},$$

$$\sigma = \sqrt{\sigma^2_{theor} + \sigma^2_{exp}} = 7.3 \times 10^{-9}$$

We see that $\Delta = 3.9\,\sigma$ which means a big disagreement for the muon.

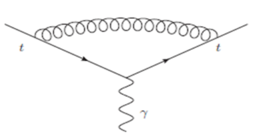
Magnetic moment of a quark

Because of the quark confinement one cannot measure MDM of a free quark, but only for quarks inside baryons or mesons.

In general, for a quark like for any fermion we define

$$\mu_q = \frac{|e|\, Q_q\, \hbar}{2m_q\, c}\, \frac{g_q}{2}$$

g_q - is quark gyromagnetic factor, $Q_q = +2/3$ for u, c, t and $Q_q = -1/3$ for d, s, b. For a point-like Dirac quark $g_q = 2$, however there are radiative corrections similarly to electron and muon, but with the strong coupling constant α_s.



$a_c = \frac{4}{3}\frac{\alpha_s(m_c)}{2\pi}$ + higher orders

For example, for the charm quark $\alpha_s(m_c) = 0.3378 \gg \alpha_{em} \approx 0.0073$, and the radiative corrections to g_c up to 3 loops [Grozin et al., 2008] are

$$a_c = \frac{g_c}{2} - 1 = +0.0717 + 0.07995 + 0.1137 + \mathcal{O}(\alpha_s^4) = 0.2655 + \mathcal{O}(\alpha_s^4)$$

Unfortunately, there is no convergence in α_s expansion, so the result cannot be reliable.

Electric dipole moment (EDM) of elementary particles

EDM is even more interesting and intriguing characteristic of a particle, because for particle at rest the only vector available is its spin:

$$\vec{\mu} \sim \vec{S}, \qquad \vec{d} \sim \vec{S}$$

The magnetic moment and magnetic field $\vec{B}$ behave similarly with respect to space reflection $\hat{P}$:

$$\vec{\mu} \to \vec{\mu}, \qquad \vec{B} \to \vec{B},$$

and time inversion $\hat{T}$:

$$\vec{\mu} \to -\vec{\mu}, \qquad \vec{B} \to -\vec{B},$$

Therefore Hamiltonian $H_{mag} = -\vec{\mu} \cdot \vec{B}$ is invariant under $\hat{P}$ and $\hat{T}$ transformations.

However, the electric dipole moment and electric field behave differently under space reflection $\hat{P}$:

$$\vec{d} \to \vec{d}, \qquad \vec{E} \to -\vec{E},$$

and time inversion $\hat{T}$:

$$\vec{d} \to -\vec{d}, \qquad \vec{E} \to \vec{E},$$

Therefore Hamiltonian $H_{elec} = -\vec{d} \cdot \vec{E}$ violates both $\hat{P}$ and $\hat{T}$.

Parity is violated in weak interactions. But if time inversion $\hat{T}$ is violated, then due to *CPT* theorem [J. Schwinger 1951, G. Luders 1952, W. Pauli 1957], *CP* symmetry should also be violated.

Electric dipole moment (EDM) of particles

Magnetic dipole moment

$$\vec{\mu} = g \frac{Qe}{2m} \vec{s}$$

η is a dimensionless constant, analogous to g

Electric dipole moment

$$\vec{d} = \eta \frac{Qe}{2m\,c} \vec{s}$$

$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

Transformation Properties

	$\vec{B}$	$\vec{E}$	$\vec{\mu}$	$\vec{d}$
C	-	-	-	-
P	+	-	+	+
T	-	+	-	-
CP	-	+	-	-
CPT	+	+	+	+

If CPT valid → EDM would violate CP

Search for sources of *CP* violation is important, in particular, because it is related to the problem of matter-antimatter asymmetry in the Universe.

One of the conditions is violation of *CP* symmetry (in fact, there is *CP* violation in SM due to CKM quark-mixing matrix, but the effect is many orders of magnitude below what is needed [Farrar, Shaposhnikov, 1994]).

EDM of leptons and quarks

For leptons and quarks one can define EDM

$$d = \frac{|e|Q}{2m} \frac{\eta}{2}$$

where η is analogue of g-factor for MDM.

What is known at present? There are no direct measurements of EDM. Theoretically, d for leptons is not zero but extremely small (4-loop diagrams in the Standard Model). EDM scales with mass, i.e.

$$d_\tau \sim d_e \frac{m_\tau}{m_e} \approx 3500\, d_e$$

	d_{exp}, $\|e\| \cdot cm$ [PDG]	d_{theor}, $\|e\| \cdot cm$
electron	$< 0.11 \times 10^{-28}$	$\sim 10^{-38}$
muon	$< 1.8 \times 10^{-19}$	$\sim 2 \times 10^{-36}$
τ lepton	$(-0.22\,, +0.45) \times 10^{-16}$ (real)	$\sim 3.5 \times 10^{-35}$
	$(-0.25\,, +0.0080) \times 10^{-16}$ (imaginary)	
neutron	$< 3 \times 10^{-26}$	$(1-6) \times 10^{-32}$
charm quark	not known	$< 4.4 \times 10^{-17}$

MDM of relatively long-lived baryons with $\tau \sim 10^{-10}$ s

For the light baryons, consisting of u, d, s quarks, the MDMs are given in units of nuclear magneton $\mu_N = \frac{e\hbar}{2m_p c}$, so that $g/2 = \mu/\mu_N$.

Magnetic Moments of Baryons

Baryon	μ/μ_N (Experiment)		Quark model:	μ/μ_N
p	$+2.792\,847\,386$	$\pm\,0.000\,000\,063$	$(4\mu_u - \mu_d)/3$	---
n	$-1.913\,042\,75$	$\pm\,0.000\,000\,45$	$(4\mu_d - \mu_u)/3$	---
Λ^0	-0.613	$\pm\,0.004$	μ_s	---
Σ^+	$+2.458$	$\pm\,0.010$	$(4\mu_u - \mu_s)/3$	$+2.67$
Σ^0			$(2\mu_u + 2\mu_d - \mu_s)/3$	$+0.79$
$\Sigma^0 \to \Lambda^0$	-1.61	$\pm\,0.08$	$(\mu_d - \mu_u)/\sqrt{3}$	-1.63
Σ^-	-1.160	$\pm\,0.025$	$(4\mu_d - \mu_s)/3$	-1.09
Ξ^0	-1.250	$\pm\,0.014$	$(4\mu_s - \mu_u)/3$	-1.43
Ξ^-	$-0.650\,7$	$\pm\,0.002\,5$	$(4\mu_s - \mu_d)/3$	-0.49
Ω^-	-2.02	$\pm\,0.05$	$3\mu_s$	-1.84

The masses of constituent quarks are here: $m_u = m_d = 336$ MeV, $m_s = 538$ MeV

Charmed baryons

Charmed baryons include at least one charm quark with electric charge $\frac{2}{3}|e|$ and mass $m_c = 1.27$ GeV.

Baryon	Flavor content	$SU(3)_f$	Charm	Mass (MeV)	Cross section, μb fixed targ.	collider	Life-length $c\tau$, or width Γ
Λ_c^+	$[ud]c$	$\bar{3}$	1	2286.5 ± 0.1	10.13	758.1	$60.0 \pm 1.2\ \mu$m
Ξ_c^+	$[us]c$	$\bar{3}$	1	2467.9 ± 0.2	0.588	65.5	$132.5 \pm 7.8\ \mu$m
Ξ_c^0	$[ds]c$	$\bar{3}$	1	2470.9 ± 0.3	0.510	65.6	$33.6 \pm 3.6\ \mu$m
Σ_c^{++}	uuc	6	1	2454.0 ± 0.1	0.863	42.0	1.9 ± 0.1 MeV
Σ_c^+	$\{ud\}c$	6	1	2452.9 ± 0.4	0.697	42.2	< 4.6 MeV
Σ_c^0	ddc	6	1	2453.8 ± 0.1	0.461	41.6	1.8 ± 0.1 MeV
$\Xi_c^{\prime +}$	$\{us\}c$	6	1	2578.4 ± 0.5	0.083	6.3	–
$\Xi_c^{\prime 0}$	$\{ds\}c$	6	1	2579.2 ± 0.5	0.072	6.6	–
Ω_c^0	ssc	6	1	2695.2 ± 1.7	0.028	3.0	$80.3 \pm 10\ \mu$m
Ξ_{cc}^{++}	ccu	3	2	3621.4 ± 0.8	$< 10^{-4}$	$\sim 10^{-3}$	$76.7 \pm 10\ \mu$m
Ξ_{cc}^+	ccd	3	2	3518.9 ± 0.9	$< 10^{-4}$	$< 10^{-3}$	–
Ω_{cc}^+	ccs	3	2	–	$< 10^{-4}$	$\sim 10^{-3}$	–

We would like to study baryons which decay due to weak interaction with lifetime $\sim 10^{-13}$ s, or $c\,\tau \sim 100\ \mu$m:
– have also the largest production cross sections,
– are positively charged,
– their magnetic moment is equal to magnetic moment μ_c of the charm quark.

MDM of charmed baryons in quarks model

For calculation of MDM of charmed baryons, only the spin-flavor wave functions are important:

$$|\Lambda_c^+([ud]c); \tfrac{1}{2}, \uparrow\rangle = \frac{1}{\sqrt{2}}(ud - du)c \times \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow$$

$$|\Sigma_c^+(\{ud\}c); \tfrac{1}{2}, \uparrow\rangle = \frac{1}{\sqrt{2}}(ud + du)c \times \frac{1}{\sqrt{6}}[2\uparrow\uparrow\downarrow - (\downarrow\uparrow + \uparrow\downarrow)\uparrow], \quad \text{etc. for all baryons}$$

These wave functions allow one to find MDM of all charmed baryons:

$$\mu(\Lambda_c^+) = \mu(\Xi_c^+) = \mu(\Xi_c^0) = \mu_c,$$

$$\mu(\Sigma_c^{++}) = \frac{1}{3}(4\mu_u - \mu_c), \qquad \mu(\Sigma_c^+) = \frac{1}{3}(2\mu_u + 2\mu_d - \mu_c),$$

$$\mu(\Sigma_c^0) = \frac{1}{3}(4\mu_d - \mu_c), \qquad \mu(\Xi_c'^+) = \frac{1}{3}(2\mu_u + 2\mu_s - \mu_c), \quad \text{etc. for all}$$

We see that MDM for some baryons are equal to MDM of the charm quark:

$$\mu\,(\Lambda_c^+) = \mu(\Xi_c^+) = \mu_c = \frac{|e|\hbar}{3m_c\,c}\,\frac{g_c}{2}$$

that can yield information on the MDM and g-factor of the charm quark (approximately).

Precession of spin in external magnetic and electric fields

How to measure MDM/EDM of particles which live so short time $\tau_{\Lambda_c} \sim 2 \times 10^{-13}$ s?

We need to accelerate it to increase its lifetime and the distance it passes, $L = \gamma\, v\, \tau_0$. For the LHC, the energy is a few TeV, then Lorentz factor $\gamma = E/m_{\Lambda_c} \sim 10^3$, and the length can be macroscopic, $L \sim 10$ cm.

Then one can use phenomenon of spin precession in external fields.

In the rest frame of particle the vector of spin (or one can say about polarization $\vec{\mathcal{P}} = \frac{2}{\hbar} < \vec{S} >$) satisfies

$$\frac{d\vec{S}}{dt^*} = \vec{\mu} \times \vec{B}^\star + \vec{d} \times \vec{E}^\star,$$

where $\vec{B}^\star$ and $\vec{E}^\star$ are magnetic and electric fields in the **rest frame** and t^* is the proper time. Both MDM and EDM are proportional to vector $\vec{S}$.

If, for example, only $\vec{B}^\star \neq 0$, then the spin rotates around the magnetic field with the angular velocity

$$\omega = \frac{eB^\star}{mc}\,\frac{g}{2}$$

Precession of spin in external fields

One needs description of spin precession for a fermion moving with ultrarelativistic energies with $\gamma \gg 1$.

Theory:

L.H. Thomas, Nature 117, 514 (1926)
V. Bargmann, L. Michel, V.L. Telegdi, Phys. Rev. Lett. 2, 435 (1959)
J.D. Jackson, "Classical electrodynamics", sec. 11.11, John Wiley, 3rd ed., 1999
V.B. Beresteckii, E.M. Lifshitz, L.P. Pitaevskii, "Quantum electrodynamics", sec. 41, Pergamon Press, 1982
V. Lyuboshits, Yad. Fiz. 31 (1980) 986; I. Kim, Nucl. Phys. B229 (1983) 251
V.G. Baryshevsky, Phys. Lett. B757 (2016) 426

One transforms $\vec{B}^* \to \vec{B}$ and $\vec{E}^* \to \vec{E}$ from the rest frame to Lab frame:

$$\vec{B}^\star = \gamma(\vec{B} - \frac{\vec{v}\times\vec{E}}{c}) - \frac{\gamma^2}{1+\gamma}\frac{\vec{v}(\vec{v}\vec{B})}{c^2},$$

$$\vec{E}^\star = \gamma(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}) - \frac{\gamma^2}{1+\gamma}\frac{\vec{v}(\vec{v}\vec{E})}{c^2}$$

and also transforms the time $t^* \to t = \gamma t^*$. In addition, a non-inertial frame of moving particle is accounted for by the Thomas correction [L.H. Thomas, 1927]:

$$\frac{\gamma^2}{1+\gamma}\frac{\vec{S}\times(\vec{v}\times\vec{a})}{c^2}$$

Precession of spin in external fields

Now let us write equation of motion in external fields of a charged particle with charge Q:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{Q}{m\gamma}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c} - \frac{\vec{v}(\vec{v}E)}{c^2}\right) = \vec{\omega}_0 \times \vec{v} + \frac{Q}{m\gamma}\frac{1}{\gamma^2-1}\frac{\vec{v}(\vec{v}E)}{c^2},$$

$$\vec{\omega}_0 = \frac{Q}{mc\gamma}\left(\frac{\gamma^2}{\gamma^2-1}\frac{\vec{v}\times\vec{E}}{c} - \vec{B}\right) \quad \text{angular velocity of rotation of particle velocity}$$

The final equations for the spin precession in Lab frame

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S},$$

$$\vec{\Omega} = \vec{\omega}_{MDM} + \vec{\omega}_{EDM}, \qquad \text{angular velocity of spin rotation}$$

$$\vec{\omega}_{MDM} = \vec{\omega}_B + \vec{\omega}_E,$$

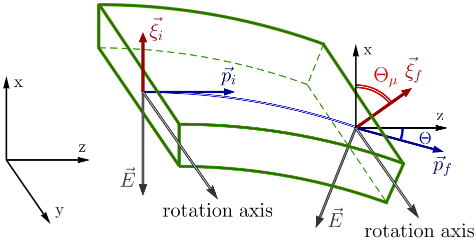
$$\vec{\omega}_B = -\frac{Q}{mc}\left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2} - 1\right)\frac{\gamma}{1+\gamma}\frac{\vec{v}\,(\vec{B}\,\vec{v})}{c^2}\right],$$

$$\vec{\omega}_E = -\frac{Q}{mc}\left(\frac{g}{2} - \frac{\gamma}{1+\gamma}\right)\frac{\vec{E}\times\vec{v}}{c},$$

$$\vec{\omega}_{EDM} = -\frac{\eta Q}{2mc}\left[\vec{E} + \frac{\vec{v}\times\vec{B}}{c} - \frac{\gamma}{1+\gamma}\frac{\vec{v}(\vec{v}E)}{c^2}\right]$$

Rotation of spin (polarization vector) in a bent crystal

Here $\vec{\xi}_i$ is initial polarization, $\vec{\xi}_f$ is final polarization,
Θ_μ is rotation angle of the polarization and Θ is rotation angle of the momentum.



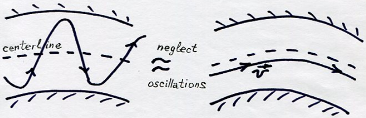
The gradient of the inter-plane electric field of a silicon crystal reaches the maximum value about 5 GeV/cm. This corresponds to the induced magnetic field in the instantaneous rest frame of a particle

$$\vec{B}^* = \gamma\left(\frac{\vec{v}}{c} \times \vec{E}\right) \sim 1000 \text{ Tesla},$$

if the particle moves with relativistic energies about TeV.

With Lorentz factor $\gamma \sim 10^3$ the particle can move about ~ 10 cm in the crystal before decaying to observed particles.

Spin precession in electric field of bent crystal



Spin precession in electric field of bent crystal

In a bent crystal there is strong electric field $\vec{E}$, perpendicular to velocity of particle, $\vec{E} \perp \vec{v}$, so that the energy of particle is conserved and it moves with constant velocity $|\vec{v}| \sim c$.
The component of electric field is along OX and particle velocity along OZ, then momentum of particle rotates with angular velocity around OY:

$$\omega_0 = \frac{QE}{m\gamma v} = \frac{v}{R},$$

where R is the curvature radius of a crystal.
Then the spin rotation velocity due to MDM is also along OY and is equal to

$$\omega_{MDM} = \gamma\,\omega_0\,(\frac{g}{2} - 1 - \frac{g}{2\gamma^2} + \frac{1}{\gamma}) \approx \gamma\,\omega_0\,(\frac{g}{2} - 1) = \gamma\,\omega_0\,a$$

where $a \equiv g/2 - 1$ is anomalous magnetic moment.

The spin rotation velocity due to EDM is along different axis, OX:

$$\omega_{EDM} = \gamma\,\omega_0\,\frac{\eta q}{2c}$$

So that the total vector of angular velocity of the spin rotation is

$$\vec{\Omega} = \vec{n}_x\,\omega_{EDM} - \vec{n}_y\,\omega_{MDM}$$

Spin precession in electric field of bent crystal

Integration over time leads to relations for the angle of rotation of polarization

$$\vec{\Phi} = \theta' \, \vec{n}_x - \theta \, \vec{n}_y,$$

$$\theta \approx \gamma \, \theta_0 \, a, \qquad \theta' = \gamma \, \theta_0 \, \frac{\eta \, v}{2c}$$

The rotation angle of the velocity is $\theta_0 = L/R$, where L is the arc length that baryon passes in the channeling regime, and R is curvature of the crystal.

What is the typical rotation angle θ_0 of the particle trajectory, if velocity $v \sim c$?

Take $L \approx 10$ cm and curvature radius of crystal $R \approx 10$ m. Then

$$\theta_0 = \frac{L}{R} = \frac{v\,t}{R} = \frac{v\,\gamma\,\tau_0}{R} \sim \frac{10\,cm}{10\,m} \sim 10\,\mathrm{m}rad \approx 0.6^\circ$$

Now we estimate the rotation angle of the spin. Assume that there is MDM (no EDM) and take $a \sim 0.01$. Then

$$\theta \approx \gamma \, \theta_0 \, a \sim 10^3 \times 0.6^\circ \times 0.01 \approx 6^\circ$$

Even small a is enhanced by very large Lorentz factor γ. If we measure θ and know θ_0, then g-factor can be found.

Rotation of spin in electric field of bent crystal

After particle's passing the crystal, the polarization vector acquires the components which depend on initial polarization $\vec{\xi}_i$ and rotation angles θ and θ'.
If crystal is oriented perpendicular to initial polarization and $\theta' \ll \theta$, then initial polarization is along OX, and final one has the components

$$\vec{\xi}_i = \xi_i \, \vec{n}_x \quad \Longrightarrow \vec{\xi}_f \approx \xi_i \, (\cos\theta \vec{n}_x + \sin\theta \, \vec{n}_z) \approx \xi_i \, (\vec{n}_x + \theta \, \vec{n}_z), \qquad (\textit{if } \theta \textit{ is small})$$

which can be used to determine MDM and $a = g/2 - 1 \sim \theta$.

Measurement of EDM is more tricky, because this effect is expected to be much smaller than effect of MDM.

One possibility would be to rotate the crystal with respect to the beam, then component OY of initial polarization arises

$$\vec{\xi}_i = \xi_i' \, \vec{n}_y \quad \Longrightarrow \vec{\xi}_f \approx \xi_i' \left(\frac{\theta'}{\theta} (\cos\theta - 1) \, \vec{n}_x + \vec{n}_y + \frac{\theta'}{\theta} \sin\theta \, \vec{n}_z \right) \approx \xi' \, (\vec{n}_y + \theta' \, \vec{n}_z)$$

which can be convenient for measurement of EDM and $\eta \sim \theta'$.

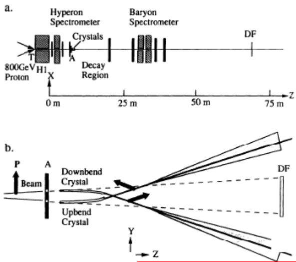
Fermilab experiment of 1992

This idea is not new. It was tested at Fermilab for $\Sigma^+(uus)$ with lifetime $\approx 10^{-10}$ s.

E761 Collaboration. Measurement of the Σ+ magnetic moment - 1

VOLUME 69, NUMBER 23 PHYSICAL REVIEW LETTERS 7 DECEMBER 1992

First Observation of Magnetic Moment Precession of Channeled Particles in Bent Crystals



Proton (800GeV/c) + Cu → Σ^+ n particles

$\Sigma^+ \rightarrow p\, \pi^0$

As illustrated in Fig. 1, a vertically polarized Σ^+ beam [14] was produced by directing the Fermilab Proton Center extracted 800-GeV/*c* proton beam onto a Cu target (T). The resulting Σ^+ were produced alternately at a +3.7- or −3.7-mrad horizontal targeting angle relative to the incident proton beam direction. This allowed the polarization direction to be periodically reversed. The mean

The two bending crystals. Each crystal precess the channelled particle's spin in opposite direction

The deflection of the channeled particles was measured to be $\omega = 1.649 \pm 0.043$ and -1.649 ± 0.030 mrad for the up- and down-bending crystals, respectively. For 375-GeV/*c* Σ^+ this corresponds to an effective magnetic field of $B_x \approx 45$ T in the crystals. The magnetic moment [6] of the Σ^+ should precess by $\varphi \approx 1$ rad in such a field.

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Concept of possible experiment at CERN

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Feasibility of measuring the magnetic dipole moments of the charm baryons at the LHC using bent crystals

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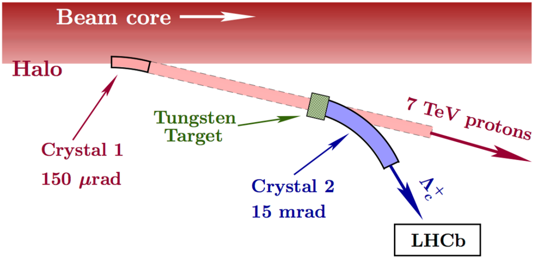

Polarization of final Λ_c^+

Slide from presentation of Achille Stocchi

Polarisation ($\mathcal{P}$) of Λc and weak asymmetry decay parameter (α)

$\mathcal{P}$ (Λc)

The polarisation $\mathcal{P}$ of Λc has not been yet measured precisely.

There are some old experiment and the indicative values are

$\mathcal{P}(\Lambda_c)$ ~[0.4-0.6] ($\mathcal{P}(\Lambda_c)$ = 0.6 (e.g. Bis-2)

$\mathcal{P}$ (Λc) ~ 0.6
To be also measured by this experiment

α

The parameter α is decay dependent

channel	Br		α
$(\Lambda_c \to \Lambda\pi)$ X Br$(\Lambda \to p\,\pi)$	1,07% x 64 % ~ 0,007	~ 1	0.59
$(\Lambda_c \to \Lambda\pi)$ X Br$(\Lambda \to n\,\pi^0)$	1,07% x 35,8 % ~ 0,004	~ 0.6	0.59
$(\Lambda_c \to \Sigma^+\pi^0)$ X Br$(\Sigma^+ \to p\,\pi^0)$	1,00% x 51,5 % ~ 0,005	~ 0.7	0.44
$(\Lambda_c \to \Sigma^+\pi^0)$ X Br$(\Sigma^+ \to n\,\pi^+)$	1,00% x 48,3 % ~ 0,005	~ 0.6	~0
$(\Lambda_c \to \Lambda e\nu)$ X Br$(\Lambda \to p\,\pi)$	2,00% x 64 % ~ 0,0128	~ 1.8	0.60
$(\Lambda_c \to \Lambda\mu\nu)$ X Br$(\Lambda \to p\,\pi)$	2,00% x 64 % ~ 0,0128	~ 1.8	0.60
$(\Lambda_c \to p\,K^-\pi^+)$	**5,00% ~ 0,05**	**~12.5**	**not known**

α
input parameter

For the numerical study we use
$\mathcal{P}$ (Λc)x α~ 0.6x0.59 ~ 0.35

Two observations :

1) Consider that the sensitivity of the analysis goes as $(\mathcal{P} \times \alpha)^2$
2) More decay channels can be use. In particular if the α parameter of $\Lambda_c \to p\,K^-\pi^+$ decay mode is measured and happened to be large, it would allow to give access to much larger statistics. **Possible at LHCb !**

Feasibility of MDM/EDM measurement for τ lepton

JHEP

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Feasibility of τ-lepton electromagnetic dipole moments measurement using bent crystal at the LHC

A.S. Fomin,[a,b] A.Yu. Korchin,[a,c] A. Stocchi,[d] S. Barsuk[d] and P. Robbe[d]

JHEP03(

Tau-lepton is a short-lived fermion with lifetime 2.9×10^{-13} s.
How to produce polarized τ? One can take charm-strange meson $D_s^+ = (c\,\bar{s})$ with sizable branching fraction (5.5%) of the decay

$$D_s^+ \to \tau^+ + \nu_\tau$$

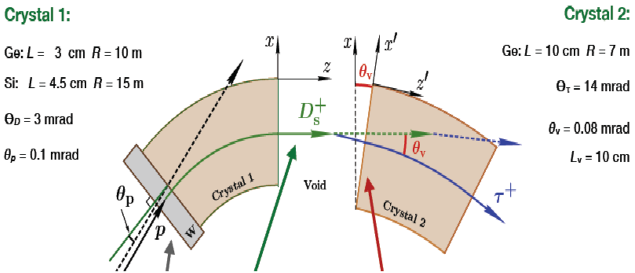
D_s^+ mesons are produced in the pp collisions at the LHC with very high energies, of a few TeV, and then decay to **100 % polarized τ leptons.**

Measurement of MDM and EDM for τ-lepton

Then τ leptons can be directed into a bent crystal, get in the channeling regime, and the direction of τ polarization after the spin precession in the crystal can be determined from angular analysis of its decay products. Schematically

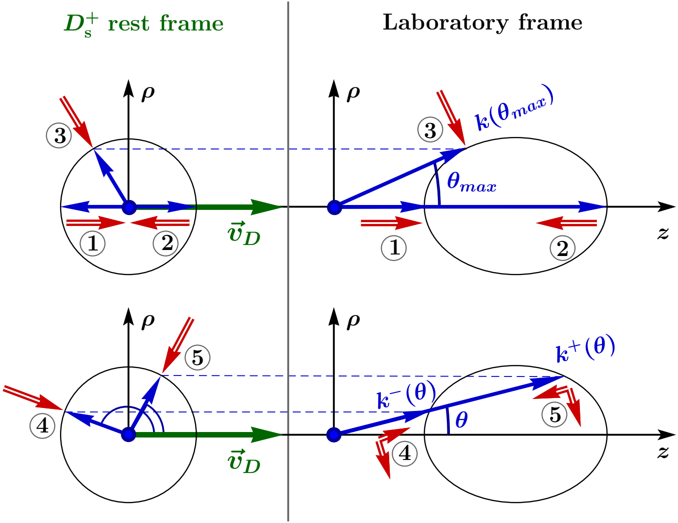
$$p + p \to D_s^+ + X \to \tau^+ + \nu_\tau + X \to \tau^+ \text{ in a crystal} \to \pi^+ + \pi^+ + \pi^- + \nu_\tau$$

Double-crystal setup is proposed by Alex Fomin (IJClab & CERN). Optimal parameters: 1st crystal – silicon (L = 4.5 cm, R = 15 m) or germanium (L = 3 cm, R = 10 m); 2nd crystal – germanium (L = 10 cm, R = 7 m).



Polarization of τ in the weak decay $D_s^+ \to \tau^+ \nu_\tau$

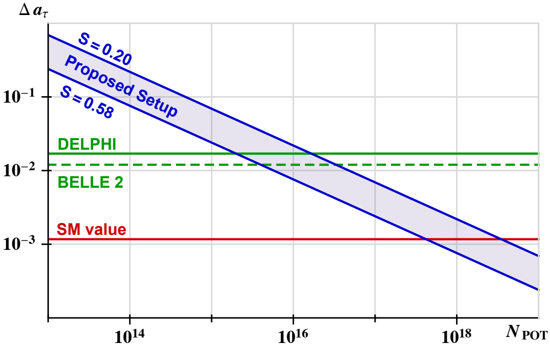
Behavior of polarization vector of τ in Lab frame is complicated, depending on kinematics.



What can we expect in measurements?

Absolute statistical error of the measured anomalous MDM of the τ lepton $a_\tau = \frac{1}{2}(g_\tau - 2)$ as a function of the total number of protons on target N_{POT}.

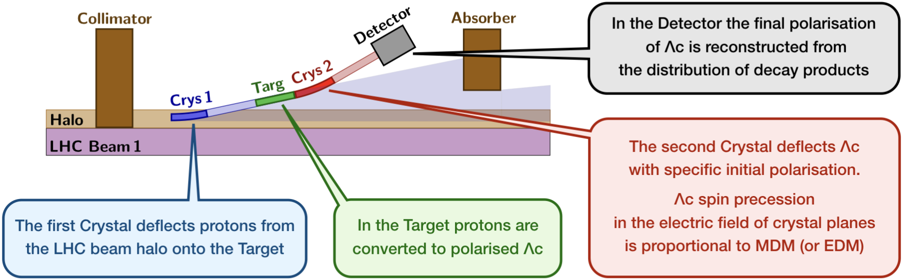
The green lines show the limits of the DELPHI collaboration (LEP) and expected for the experiment at BELLE 2.



Red line is the SM prediction: $a_\tau = 1.17721(5) \times 10^{-3}$ [Eidelman, Passera, 2007].

Possible layout of experiment at the LHC

Feasibility of measuring the magnetic and electric dipole moments of the short-lived particles at the LHC



- A.S. Fomin, A.Yu. Korchin, A. Stocchi, O.A. Bezshyyko, L. Burmistrov, S.P. Fomin, I.V. Kirillin, L. Massacrier, A. Natochii, P. Robbe, W. Scandale, N.F. Shul'ga, [arXiv:1705.03382] JHEP 08 **(2017)** 120 [inSPIRE
- A.S. Fomin, Ph.D. thesis, Paris-Sud University, Paris France, **(2017)** [full text]
- A.S. Fomin, A. Yu. Korchin, A. Stocchi, S. Barsuk, P. Robbe, [arXiv:1810.06699] **(2018),** JHEP 1903 **(2019)** 156 [inSPIRE]
- D. Mirarchi, A.S. Fomin, S. Redaelli, W. Scandale, [arXiv:1906.08551] **(2019)** (submitted to EPJ C) [inSPIRE]
- A.S. Fomin, S. Barsuk, A.Yu. Korchin, V.A. Kovalchuk, E. Kou, M. Liul, A. Natochii, E. Niel, P. Robbe, A. Stocchi, [arXiv:1909.04654] **(2019),** (submitted to EPJ C) [inSPIRE]

Our proposal

Any direct measurement of MDM/EDM of short-lived charmed baryons, beauty baryons, or τ lepton, would be the first one.

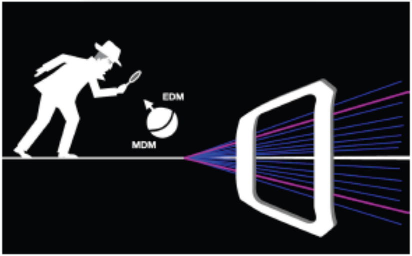
Collaboration 2019-2020: Orsay (LAL and Paris-Sud University) & CERN & Kharkiv (KIPT and V.N. Karazin University) & Kyiv (T. Shevchenko University) & Rome (INFN)

- A.S. Fomin, A.Yu. Korchin, A. Stocchi, O.A. Bezshyyko, L. Burmistrov, S.P. Fomin, I.V. Kirillin, L. Massacrier, A. Natochii, P. Robbe, W. Scandale, N.F. Shul'ga, *JHEP 08 (2017) 120*
- A.S. Fomin, A.Yu. Korchin, A. Stocchi, S. Barsuk, P. Robbe, *JHEP 03 (2019) 156*
- A.S. Fomin, S. Barsuk, A.Yu. Korchin, V.A. Kovalchuk, E. Kou, M. Liul, A. Natochii, E. Niel, P. Robbe, A. Stocchi, *Eur. Phys. J. C **80** (2020) 358*
- A.Yu Korchin, V.A. Kovalchuk. *Int. J. Mod. Phys. A **35** (2020) 11n12, 2050060*
- D. Mirarchi, A.S. Fomin, S. Redaelli, W. Scandale, *Eur. Phys. J C **80** (2020) 10, 929*
- A.S. Fomin, Ph.D Thesis, Paris-Sud University, France, 2017
- A.S. Fomin et al. *"Electromagnetic dipole moments of unstable particles", Milano, Italy, 2-4 October 2019*
- A.S. Fomin et al. *FTE@LHC and NLOAccess STRONG 2020 joint kick-off Meeting, CERN, Geneva, 7-8 November 2019*
- ...

Status of proposal in 2024

Slide from presentation of Nicola Neri in the CERN meeting on 28-29 Feb 2024

Direct measurement of dipole moments of short-lived particles at LHC



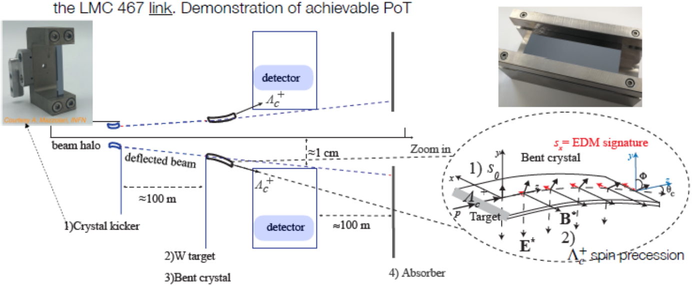
Nicola Neri
UniMi and INFN Milano/CERN

on behalf of the proto-collaboration

LHCC meeting
CERN, 28-29 February 2024

Slide from presentation of Nicola Neri in the CERN meeting on 28-29 Feb 2024

- Experiment for direct measurement of dipole moments of charm baryons [1-9] at LHC. Letter of Intent (LoI) in preparation. [Idea to explore τ lepton [10-11]]
- Expected precision 4 % on Λ_c^+, Ξ_c^+ MDM (EDM at $3 \cdot 10^{-16} e$ cm) with $1.4 \cdot 10^{13}$ PoT (2 years). Exploit particle channeling and spin precession in bent crystals
- Proof-of-Principle (PoP) test in 2025 (TWOCRYST project, installation in next YETS) approved by the LMC 467 link. Demonstration of achievable PoT



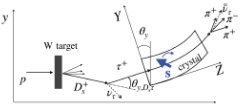
- Reuse of existing hardware: Roman Pot (RP) stations, warm magnet corrector for spectrometer, collimators. Machine layout designed and simulated [12]. No civil engineering needed

Slide from presentation of Nicola Neri in the CERN meeting on 28-29 Feb 2024

Physics reach

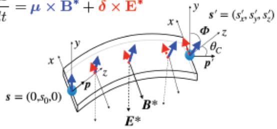
- **First measurements** of **charm baryon dipole moments** in 2 year data taking assuming 10^6 p/s on target. Only a small fraction of the beam halo is deflected towards the target
- Sensitivity on **MDM** $2 \cdot 10^{-2} \mu_N$ and **EDM** $3 \cdot 10^{-16} e$ cm with $1.4 \cdot 10^{13}$ PoT
- Two alternatives: **i) dedicated experiment at IR3 (baseline)**; **ii) LHCb detector at IP8**
  - **Pros/cons**: **i)** optimal experiment and detector, PID / More resources needed. New detector + services (long cables, cooling) in IR3 ; **ii)** use existing tracking detector and infrastructure. Experimental area. Less resources needed / No PID for p>100 GeV/c, potential interference with LHCb core program
- Exploration **of τ g-2 and EDM** (improvements are required)

PRL 123, 011801 (2019)



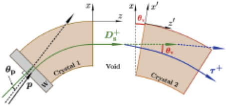
MDM μ and EDM δ precession in a bent crystal

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\mu} \times \mathbf{B}^* + \boldsymbol{\delta} \times \mathbf{E}^*$$



PRD 103, 072003 (2021)

Proposed setup for τ precession



JHEP 03 (2019) 156

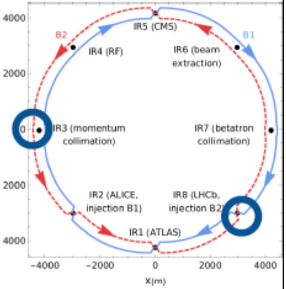
Schedule and organization

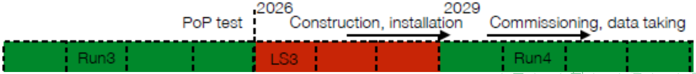
Slide from presentation of Nicola Neri in the CERN meeting on 28-29 Feb 2024

Schedule and organisation

- **Active collaboration of accelerator and experimental physicists about 50 people**: CERN, IJCLab, INFN Genova, INFN Milano, INFN Milano Bicocca, INFN Padova, INFN Pisa, UCAS, IFIC Univ. of Valencia, Univ. of Bonn (other institutions have expressed interest). Proto-collaboration for LoI being finalised
- Series of **topical workshops**: 1st, 2nd, 3rd workshop

View of the IR3 region identified for the experiment





Collaboration in 2024

Slide from presentation of Nicola Neri in the CERN meeting on 28-29 Feb 2024

Proponents of the LoI (being finalised)

M. Benettoni[10], R. Cardinale[7], S. Cesare[8], M. Citterio[8], S. Coelli[8], A. S. Fomin[3], R. Forty[1], J. Fu[6], P. Gandini[8], M. A. Giorgi[11], J. Grabowski[5], S. J. Jaimes Elles[2], A. Yu. Korchin[4], E. Kou[3], S. Libralon[2], G. Lamanna[11], C. Maccani[1,10], D. Marangotto[8], F. Martinez Vidal[2], J. Mazorra de Cos[2], A. Merli[8], H. Miao[6], N. Neri[1,8], S. Neubert[5], A. Petrolini[7], J. Pinzino[11], M. Prest[9], P. Robbe[3], L. Rossi[8], J. Ruiz Vidal[2], I. Sanderswood[2], A. Sergi[7], G. Simi[10], M. Sorbi[8], M. S. Sozzi[11], E. Spadaro Norella[7], A. Stocchi[3], G. Tonani[2,8], N. Turini[12], E. Vallazza[9], S. Vico Gil[2], M. Wang[8], Z. Wang[8], T. Xing[8], M. Zanetti[10], F. Zangari[8], Y. Zheng[6]

[1]CERN, [2]IFIC Univ. of Valencia - CSIC, [3]IJCLab, [4]NSC KIPT, Karkhiv, [5]Univ. of Bonn, [6]UCAS , [7]UniGe & INFN Genova, [8]UniMi & INFN Milano, [9]Uninsubria & INFN Milano Bicocca, [10]UniPd & INFN Padova, [11]UniPi & INFN Pisa, [12]UniSi & INFN Pisa

With support from PBC and TWOCRYST collaborators for the machine PoP

8

Thank you for attention!

Do we need relativism for the spin MDM?

Actually no, one can obtain spin MDM from **nonrelativistic** Hamiltonian $H = \vec{p}^2/2m + V(\vec{r})$:

$$\frac{1}{2m}\vec{p}^2 = \frac{1}{2m}(\vec{\sigma}\vec{p})\,(\vec{\sigma}\vec{p}) \;\Rightarrow\; \frac{1}{2m}\vec{\sigma}(\vec{p} - \frac{e}{c}\vec{A})\,\vec{\sigma}(\vec{p} - \frac{e}{c}\vec{A})$$
$$= \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 + i\vec{\sigma}\,[(\vec{p} - \frac{e}{c}\vec{A}) \times (\vec{p} - \frac{e}{c}\vec{A})]$$
$$= \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 - \frac{e\hbar}{2mc}\vec{\sigma}\,\vec{B} = \frac{1}{2m}\vec{p}^2 - \frac{e}{2mc}(\vec{L} + 2\vec{S})\,\vec{B} + \mathcal{O}(e^2)$$

where we use: $(\vec{\sigma}\,\vec{a})\,(\vec{\sigma}\,\vec{b}) = \vec{a}\,\vec{b} + i\vec{\sigma}(\vec{a} \times \vec{b})$, and magnetic field is $\vec{B} = \vec{\nabla} \times \vec{A}$.
This very surprising result was mentioned by J.J. Sakurai (with reference to idea of R.P. Feynman).
Even more surprising result can obtained if we write identity for arbitrary λ

$$\vec{p}^2 = (1-\lambda)\,\vec{p}^2 + \lambda\,(\vec{\sigma}\vec{p})\,(\vec{\sigma}\vec{p})$$

and make minimal substitution: $\vec{p} \Rightarrow \vec{p} - \frac{e}{c}\vec{A}$, then we get

$$\frac{1}{2m}\vec{p}^2 - \frac{e}{2mc}(\vec{L} + 2\lambda\,\vec{S})\,\vec{B}$$

Obviously, the method of minimal substitution is ambiguous, when we deal with interaction of the magnetic type $\vec{\mu}\,\vec{B}$.