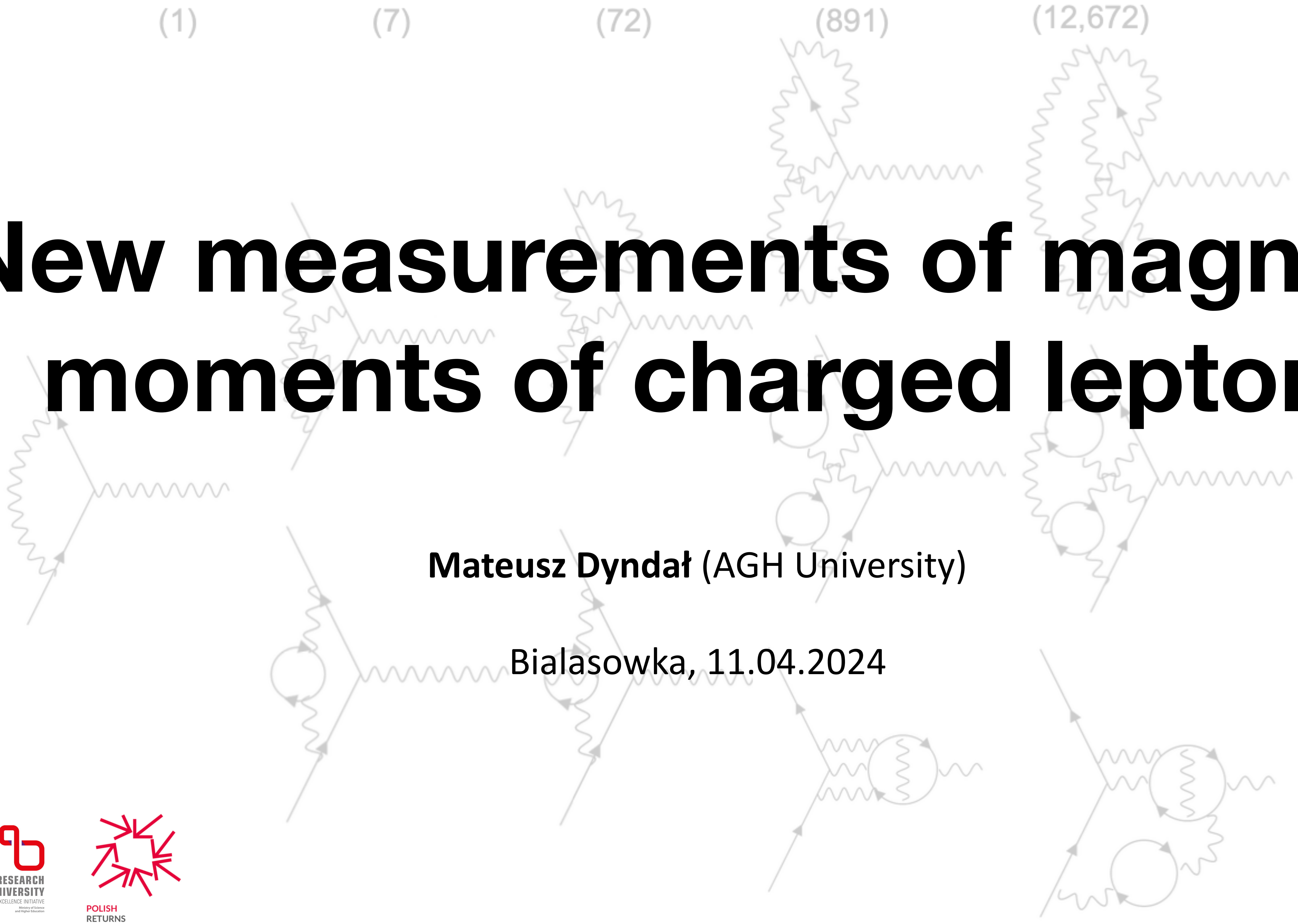
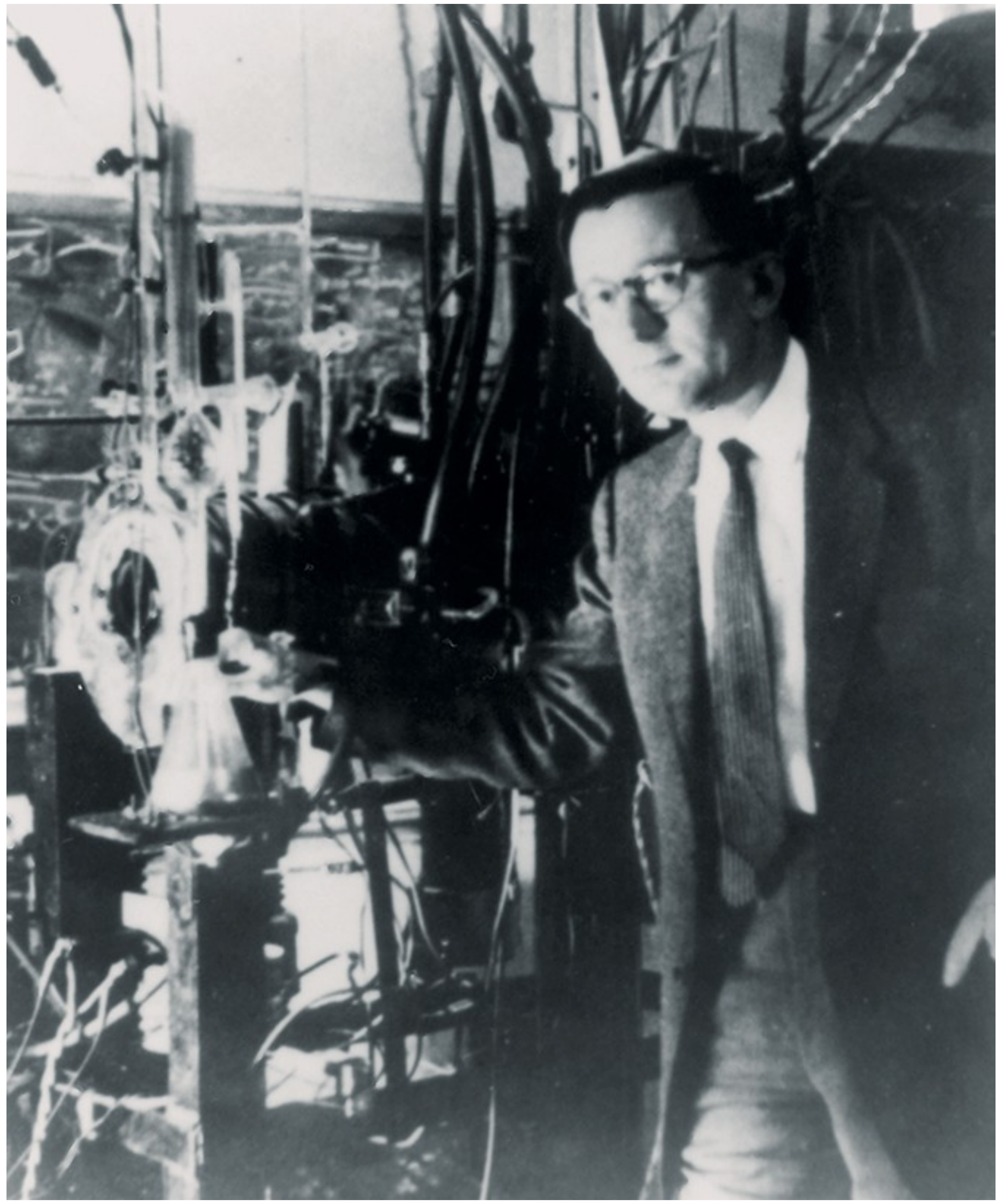
New measurements of magnetic moments of charged leptons

Mateusz Dyndał (AGH University)

Bialasowka, 11.04.2024



Historical perspective



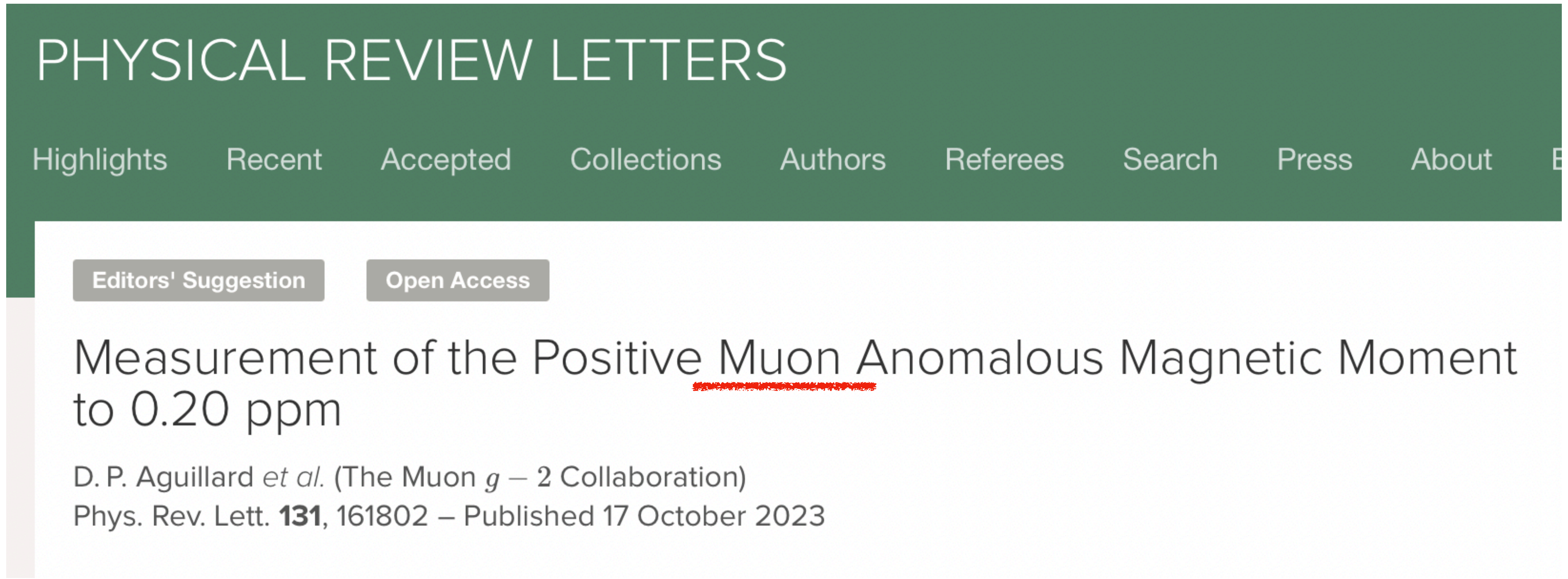
P. Kusch and H. M. Foley (1947)

New papers published in 2023

Measurement of the Electron Magnetic Moment

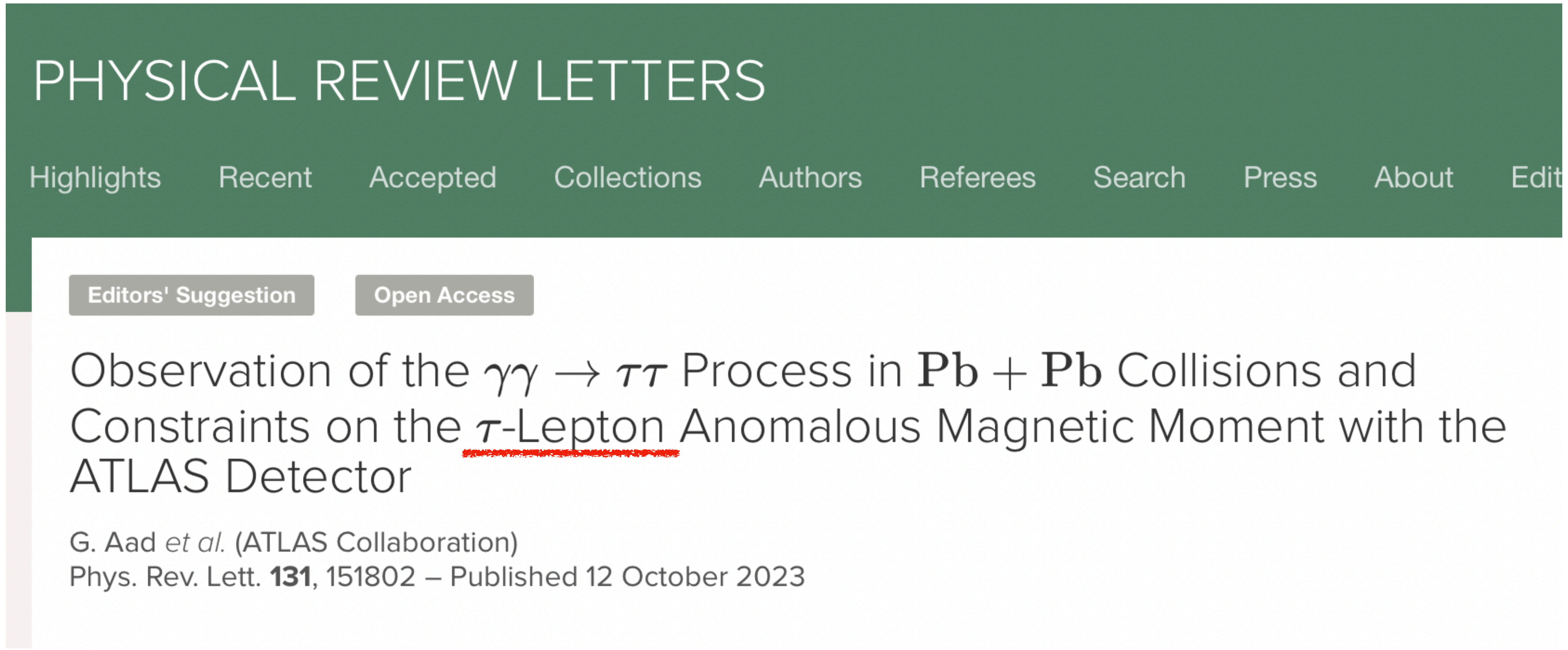
X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse
Phys. Rev. Lett. **130**, 071801 – Published 13 February 2023

Physics See Viewpoint: Searching for New Physics with the Electron’s Magnetic Moment



Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

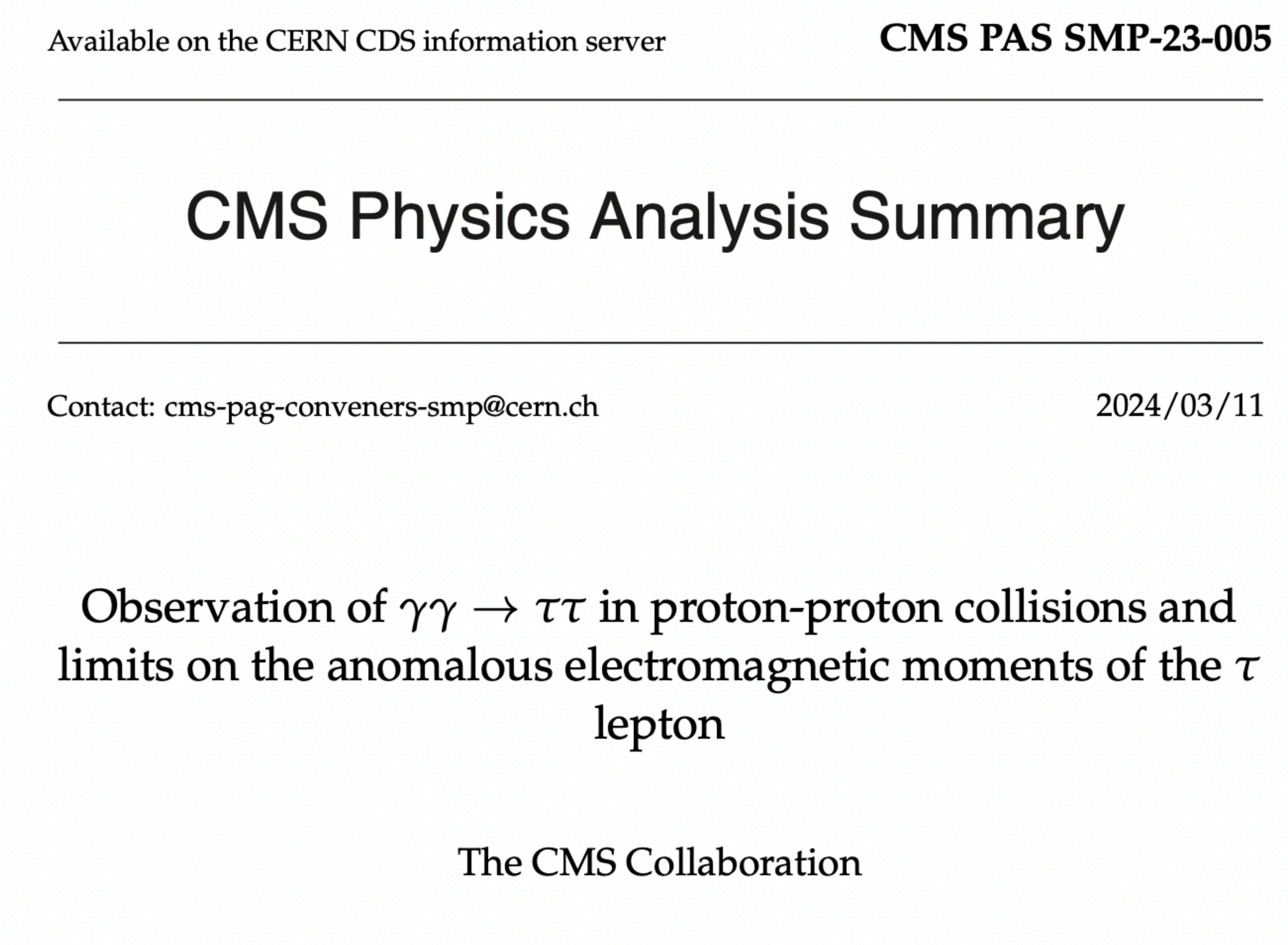
D. P. Aguillard *et al.* (The Muon $g-2$ Collaboration)
Phys. Rev. Lett. **131**, 161802 – Published 17 October 2023



Observation of the $\gamma\gamma \to \tau\tau$ Process in $\mathrm{Pb}+\mathrm{Pb}$ Collisions and Constraints on the τ-Lepton Anomalous Magnetic Moment with the ATLAS Detector

G. Aad *et al.* (ATLAS Collaboration)
Phys. Rev. Lett. **131**, 151802 – Published 12 October 2023

..plus a new preliminary result from last month



Available on the CERN CDS information server **CMS PAS SMP-23-005**

CMS Physics Analysis Summary

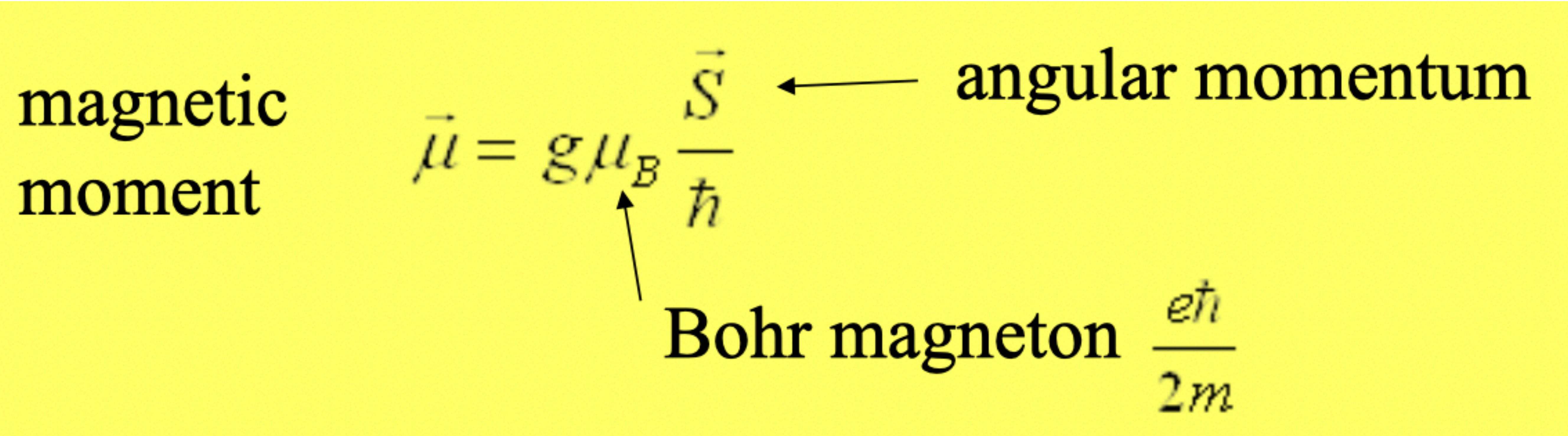
Contact: cms-pag-conveners-smp@cern.ch 2024/03/11

Observation of $\gamma\gamma \rightarrow \tau\tau$ in proton-proton collisions and limits on the anomalous electromagnetic moments of the τ lepton

The CMS Collaboration

QM: Magnetic (dipole) moment and the g-factor

- **g** = proportionality constant between **spin** and **magnetic moment**

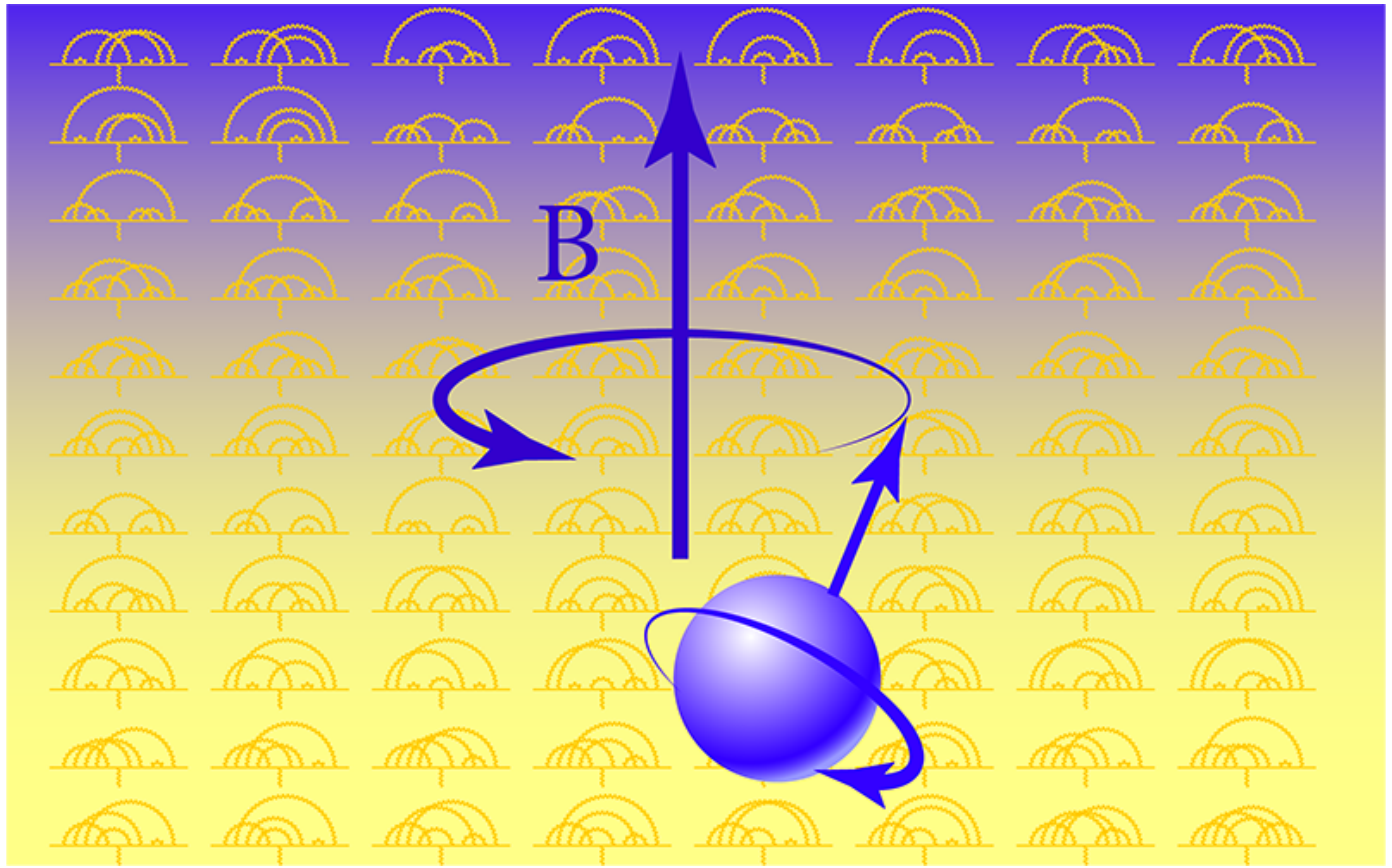


magnetic moment

$$\vec{\mu} = g\mu_B \frac{\vec{S}}{\hbar}$$

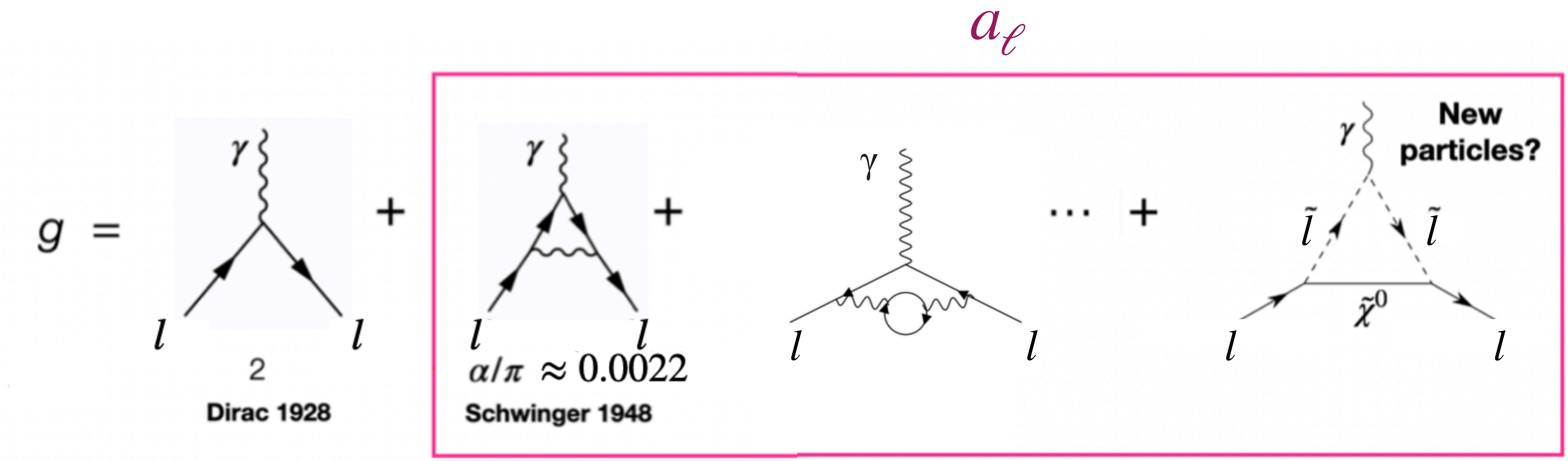
angular momentum

Bohr magneton $\frac{e\hbar}{2m}$



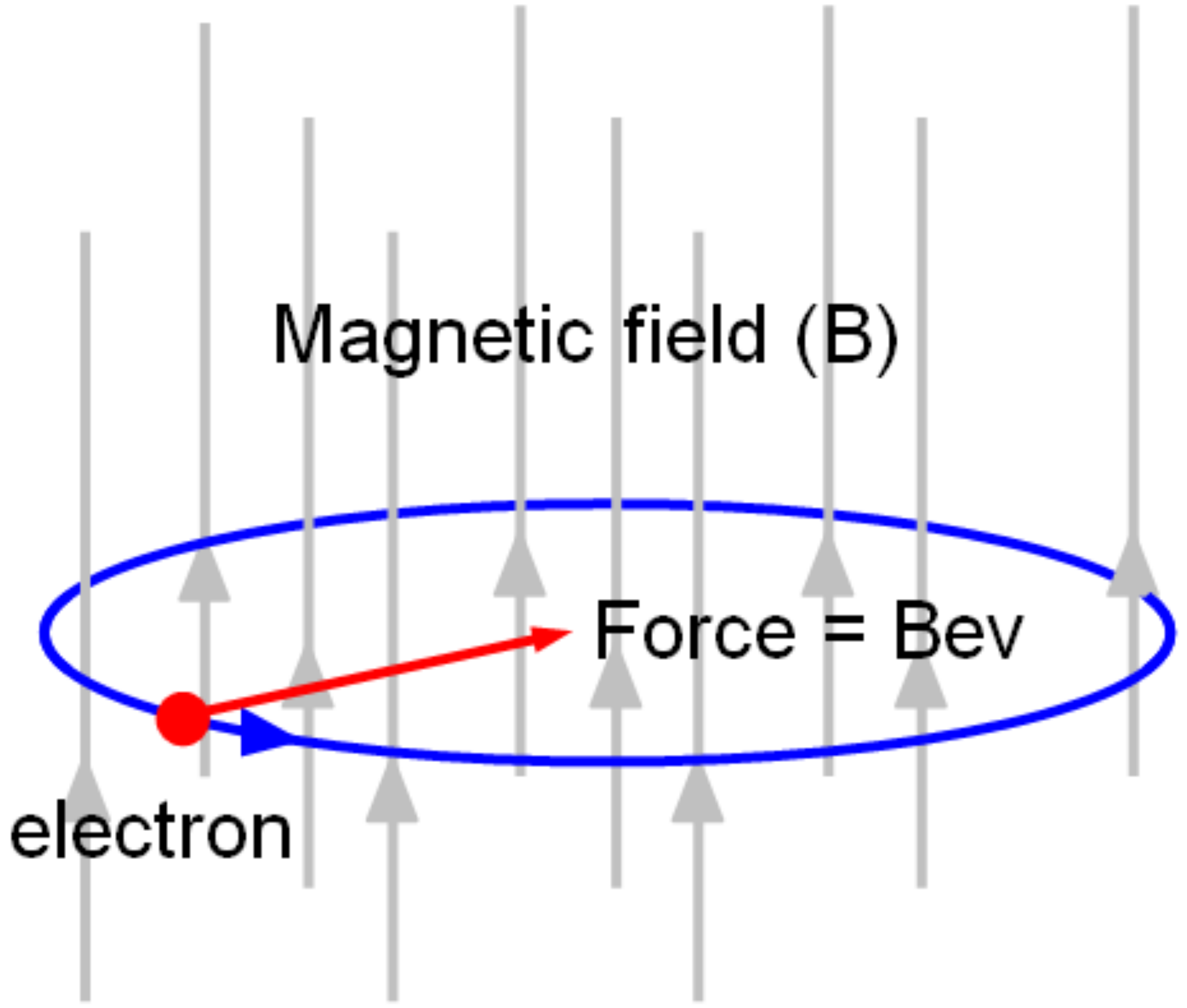
Magnetic (dipole) moment of charged leptons

- For electrons/muons/taus Standard Model predicts **g ≈ 2**
- Note: quantum fluctuations make it slightly larger than 2…
- Anomalous magnetic moment
  - $a_\ell = (g_\ell - 2)/2$
  - Shows how much g differs fractionally from 2!



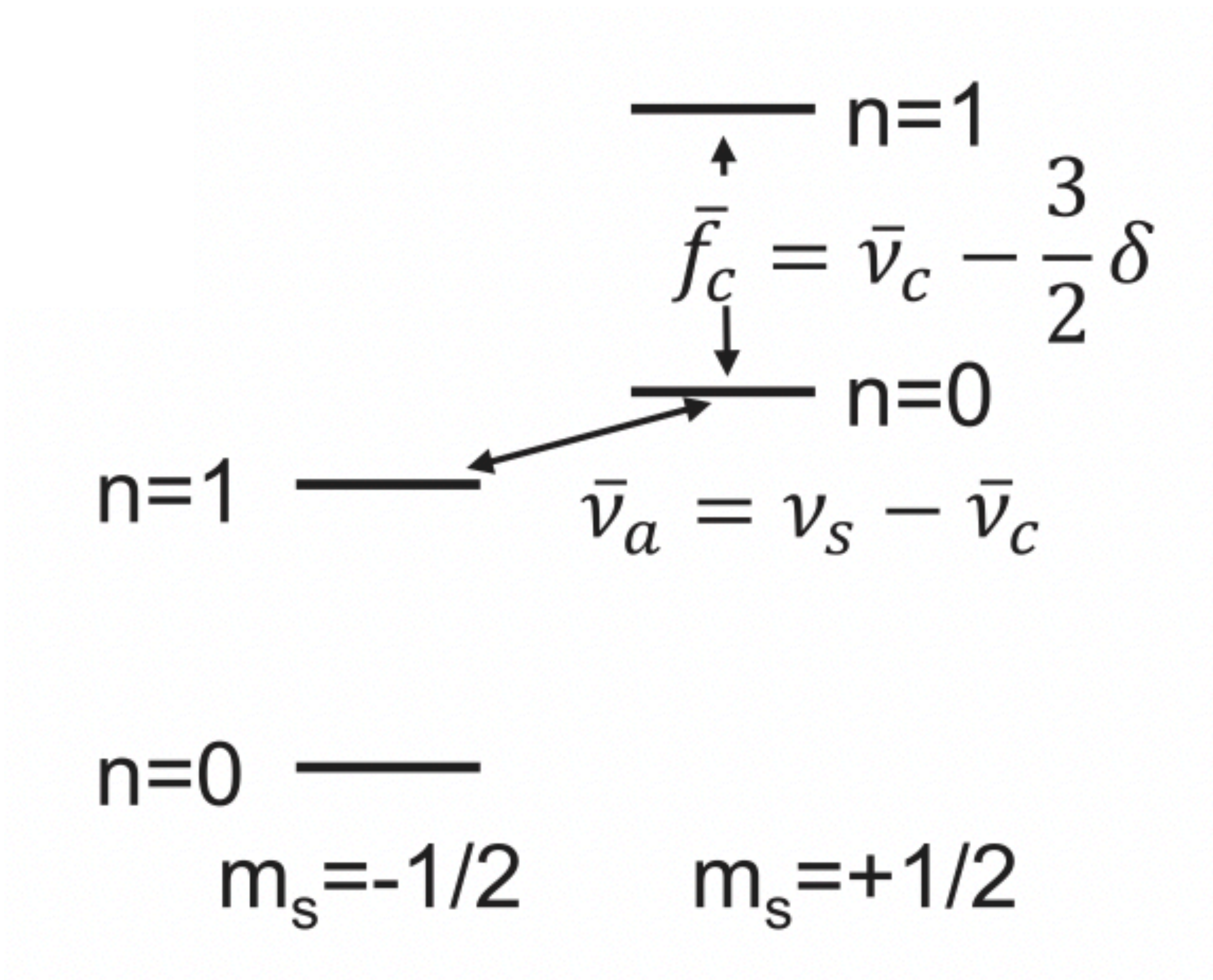
Electron in a magnetic field

Classically



$$\frac{mv^2}{r} = qBv \rightarrow \nu_c = eB/(2\pi m)$$

In QM



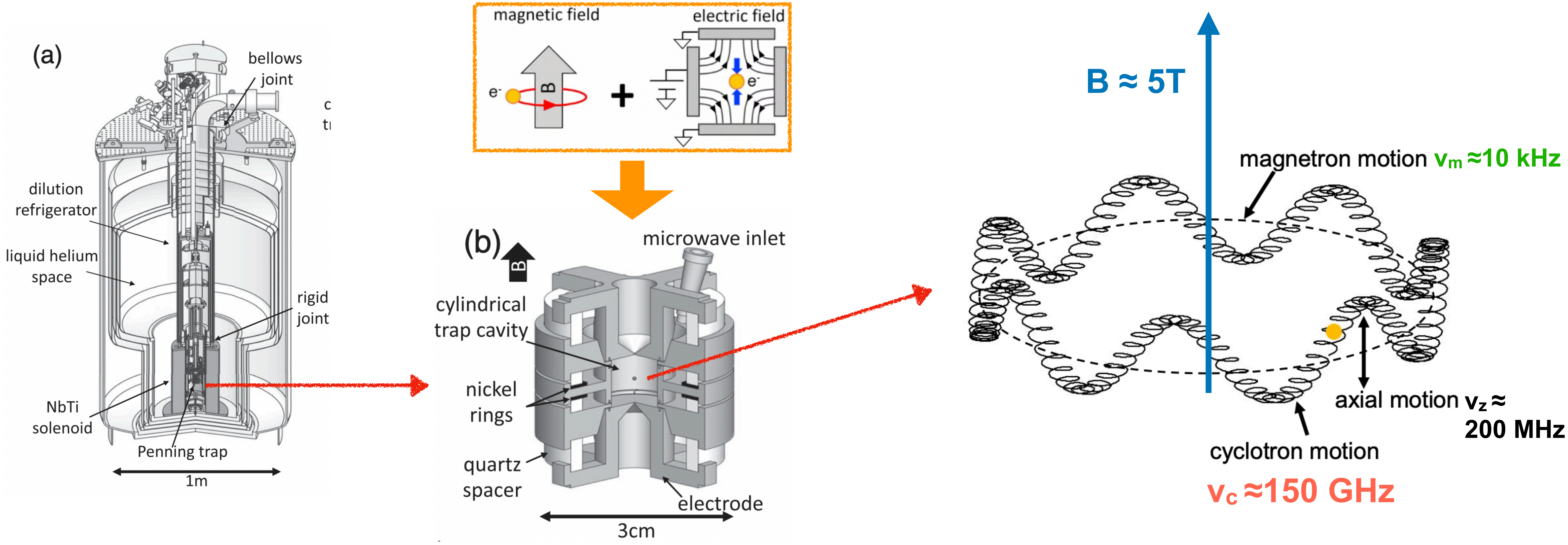
$$E = h\nu_s m_s + h\nu_c\left(n + \frac{1}{2}\right)$$

$$\boxed{\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}}$$

Note there is no B depencence!

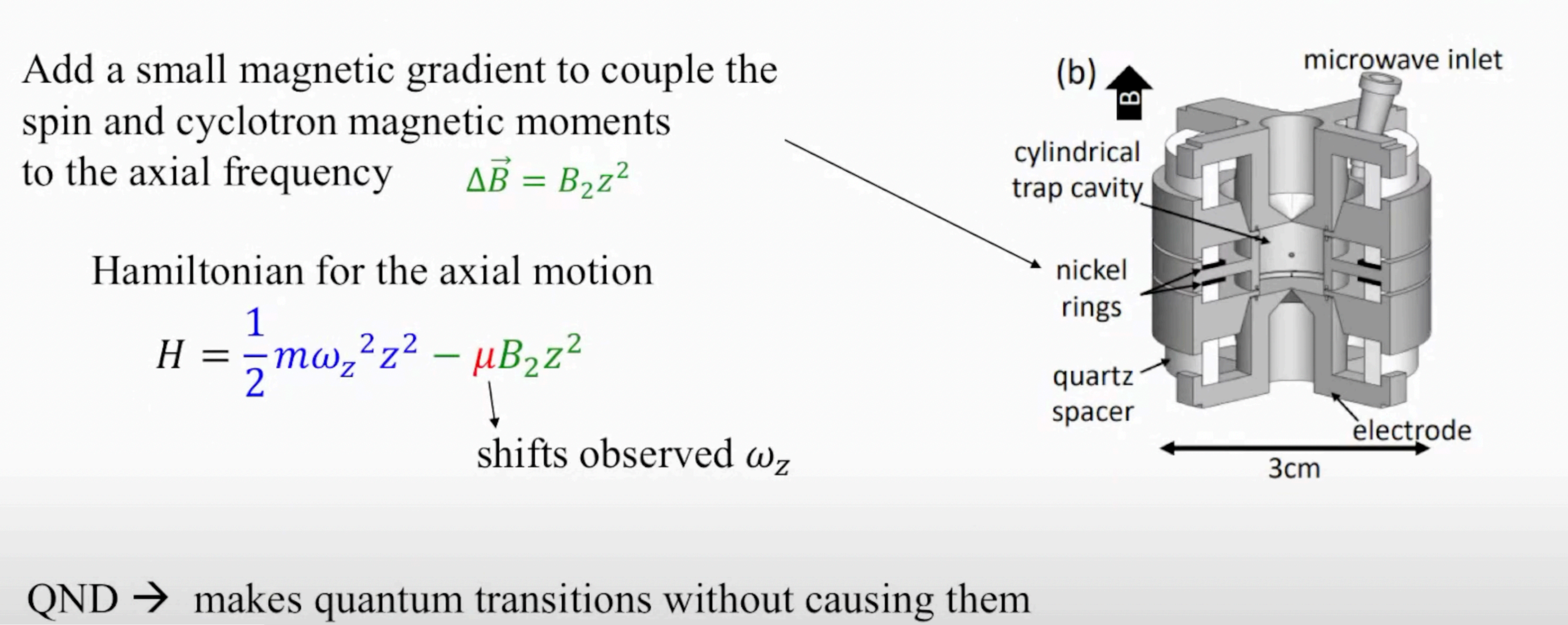
Electron g-2: measurement idea

- Using one-electron quantum cyclotron (Penning trap), with T<100 mK



X. Fan, PhD thesis

Electron g-2: quantum non-demolition (QND)



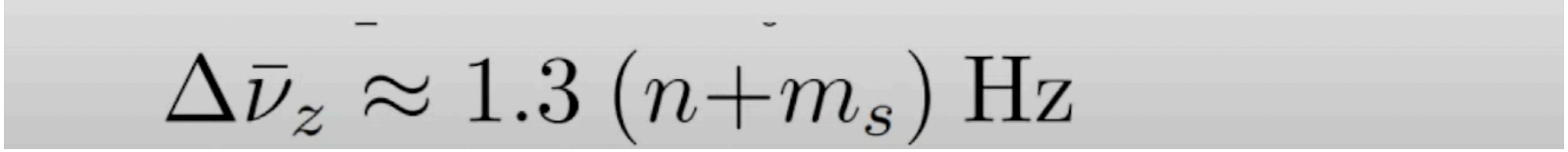
Add a small magnetic gradient to couple the spin and cyclotron magnetic moments to the axial frequency $\Delta\vec{B} = B_2 z^2$

Hamiltonian for the axial motion

$$H = \frac{1}{2} m\omega_z{}^2 z^2 - \mu B_2 z^2$$

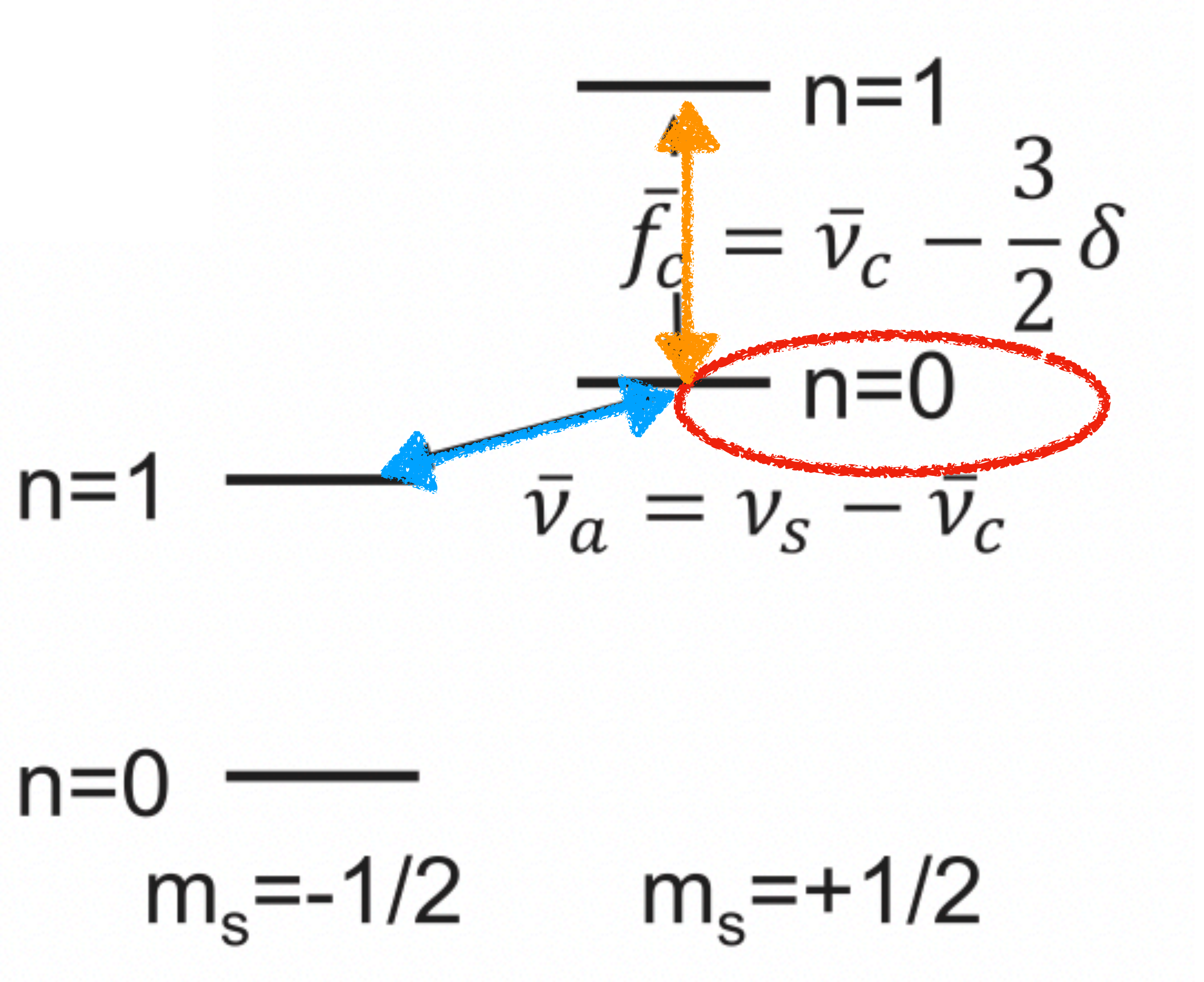
shifts observed ω_z

QND → makes quantum transitions without causing them



$$\Delta\bar{\nu}_z \approx 1.3\,(n+m_s)\ \mathrm{Hz}$$

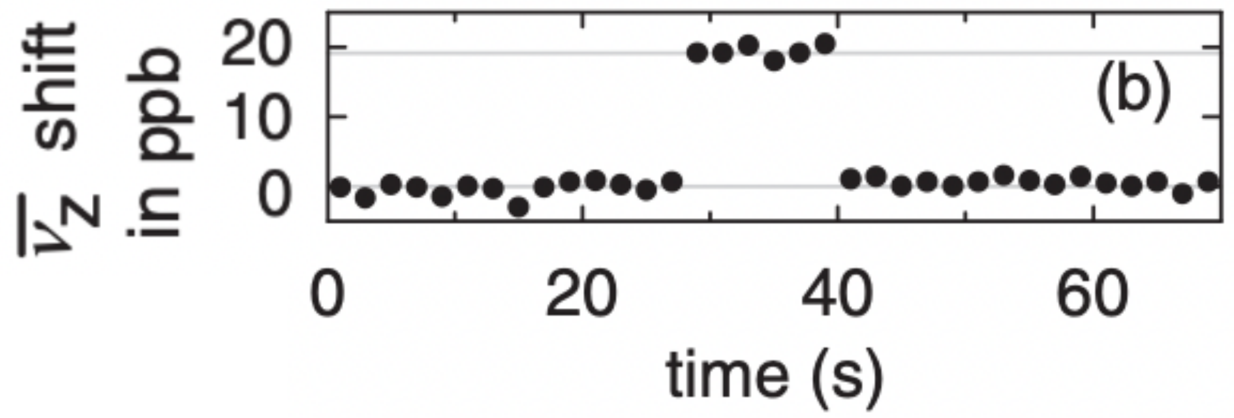
Electron g-2: Measurement procedure



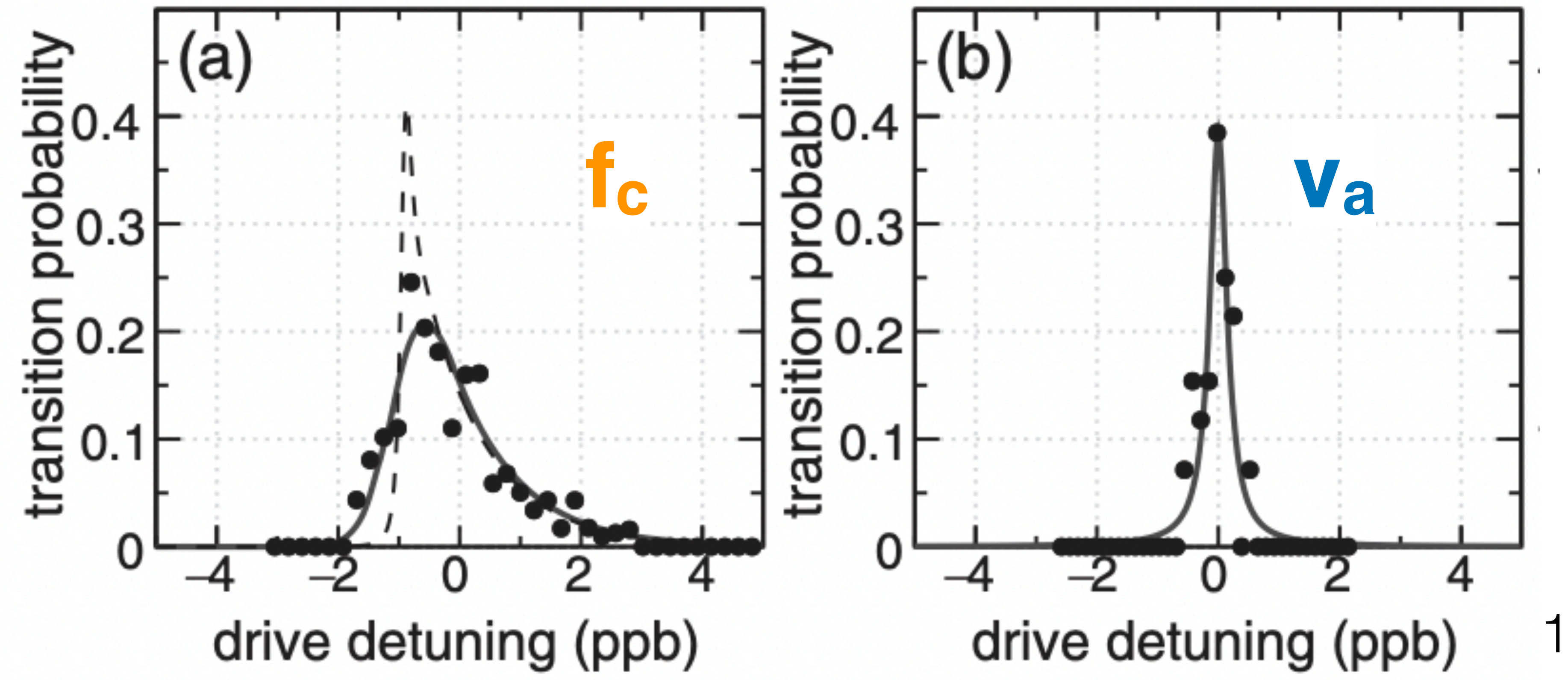
- Cyclotron quantum jump spectroscopy

- Each cyclotron and anomaly quantum jump trial starts with preparing the electron in the spin-up ground state, **|n=0, m_S=+1/2>**

- Quantum jumps detected via shifts in the axial frequency



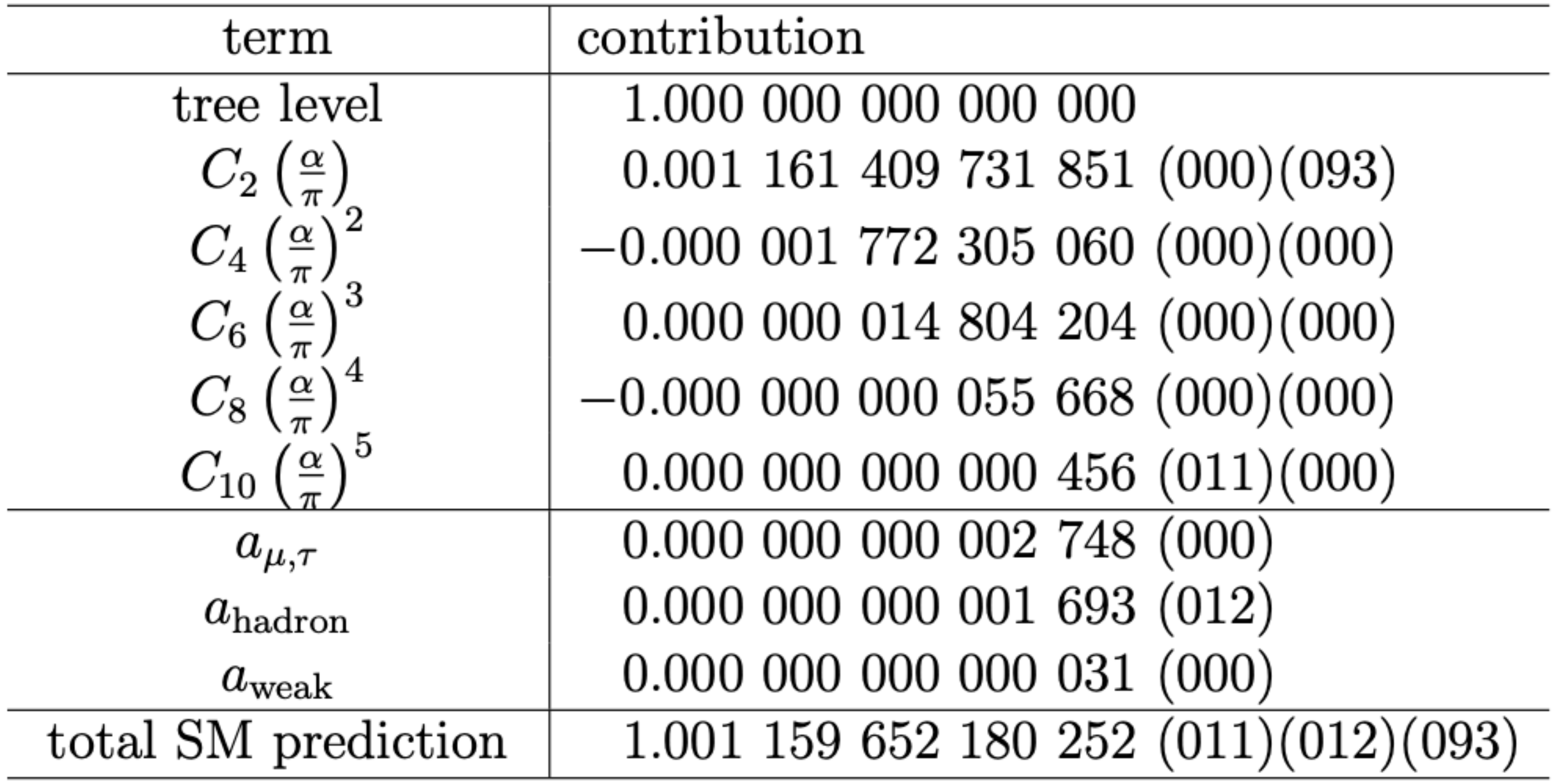
- Multiple attempts at different frequencies are binned in a histograms to reveal the frequency lines:



Electron g-2: status of theory calculations

- QED provides the asymptotic series in powers of α, along with the muon and tau contributions ($a_{\mu\tau}$)
- The constants C_2, C_4, C_6, C_8 calculated exactly, but require measured lepton mass ratios as input
- The measurement is so precise that a numerically calculated tenth order C_{10} is required
- Alternative theory evaluation of C_{10} differs slightly for reasons not yet understood

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \cdots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}}.$$



term	contribution
tree level	1.000 000 000 000 000
$C_2\left(\frac{\alpha}{\pi}\right)$	0.001 161 409 731 851 (000)(093)
$C_4\left(\frac{\alpha}{\pi}\right)^2$	−0.000 001 772 305 060 (000)(000)
$C_6\left(\frac{\alpha}{\pi}\right)^3$	0.000 000 014 804 204 (000)(000)
$C_8\left(\frac{\alpha}{\pi}\right)^4$	−0.000 000 000 055 668 (000)(000)
$C_{10}\left(\frac{\alpha}{\pi}\right)^5$	0.000 000 000 000 456 (011)(000)
$a_{\mu,\tau}$	0.000 000 000 002 748 (000)
a_{hadron}	0.000 000 000 001 693 (012)
a_{weak}	0.000 000 000 000 031 (000)
total SM prediction	1.001 159 652 180 252 (011)(012)(093)

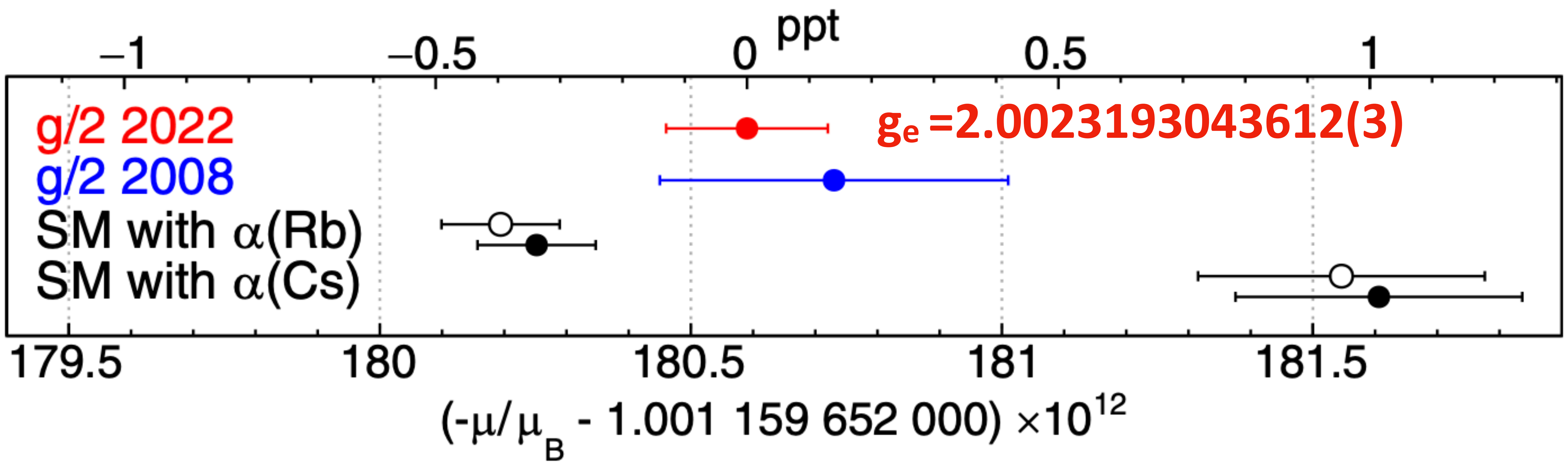
X. Fan, PhD thesis

Electron g-2: Results

Phys.Rev.Lett. 130 (2023) 7, 071801

New Measurement ->

Predictions ->



$g_e = 2.0023193043612(3)$

- Significant uncertainty reduction wrt previous measurement (blue), reaching **0.13 ppt** relative precision (ppt is 10^{-12})
- SM predictions (solid and open black points for slightly differing C_{10} calculations) are functions of discrepant α measurements (independent measurements use Rb or Cs atom recoil)
- New measurement with positron (CPT symmetry test) underway

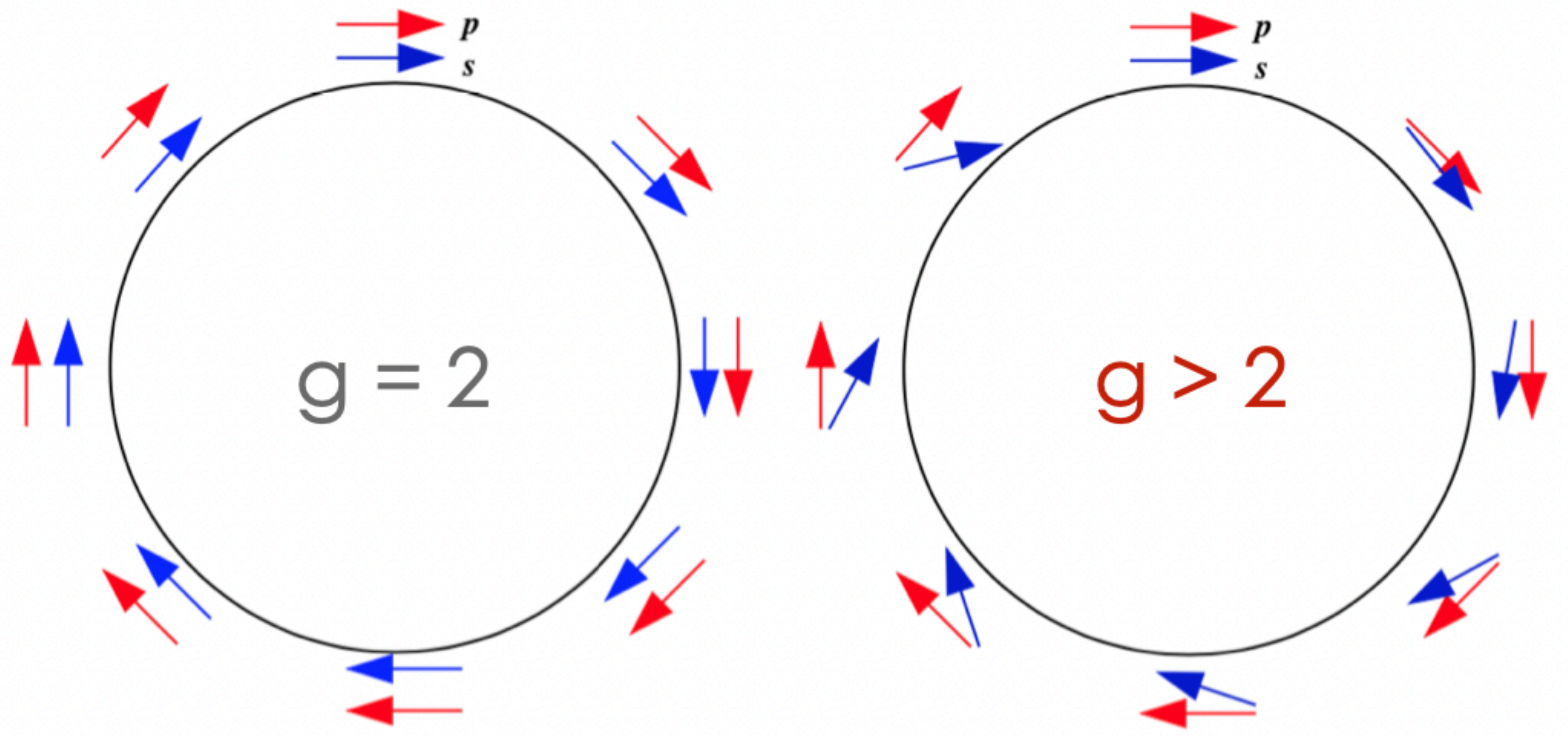
How to measure muon g-2?

- Why muon g-2?
  - The muon is 200 times more massive than electron → **$(m_e/m_\mu)^2$ ~40 000** times more sensitive to new massive particles

- In a **magnetic storage ring**, the muon spin precesses slightly faster than the cyclotron frequency:

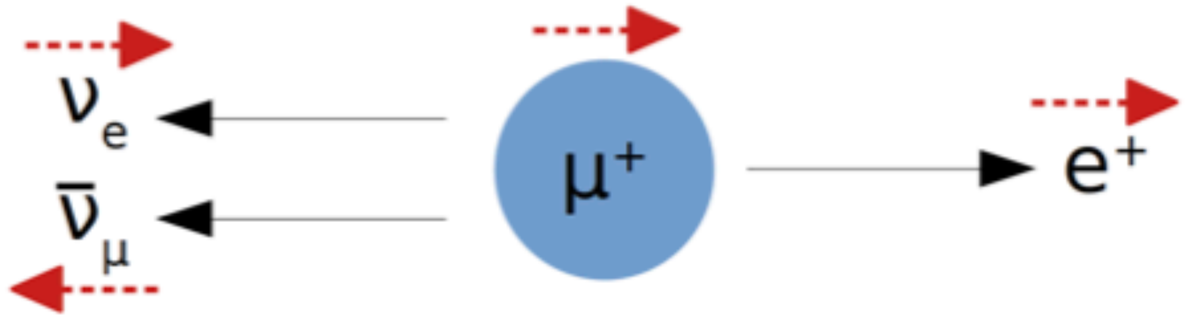
$$\underline{\vec{\omega}_s} = -\frac{ge\vec{B}}{2m} - (1-\gamma)\frac{e\vec{B}}{m\gamma} \qquad \underline{\vec{\omega}_c} = -\frac{e\vec{B}}{m\gamma}$$

$$\vec{\omega}_a = \underline{\vec{\omega}_s} - \underline{\vec{\omega}_c} = -\left(\frac{g-2}{2}\right)\frac{e\vec{B}}{m} \equiv -\boxed{a_\mu}\frac{e\vec{B}}{m}$$

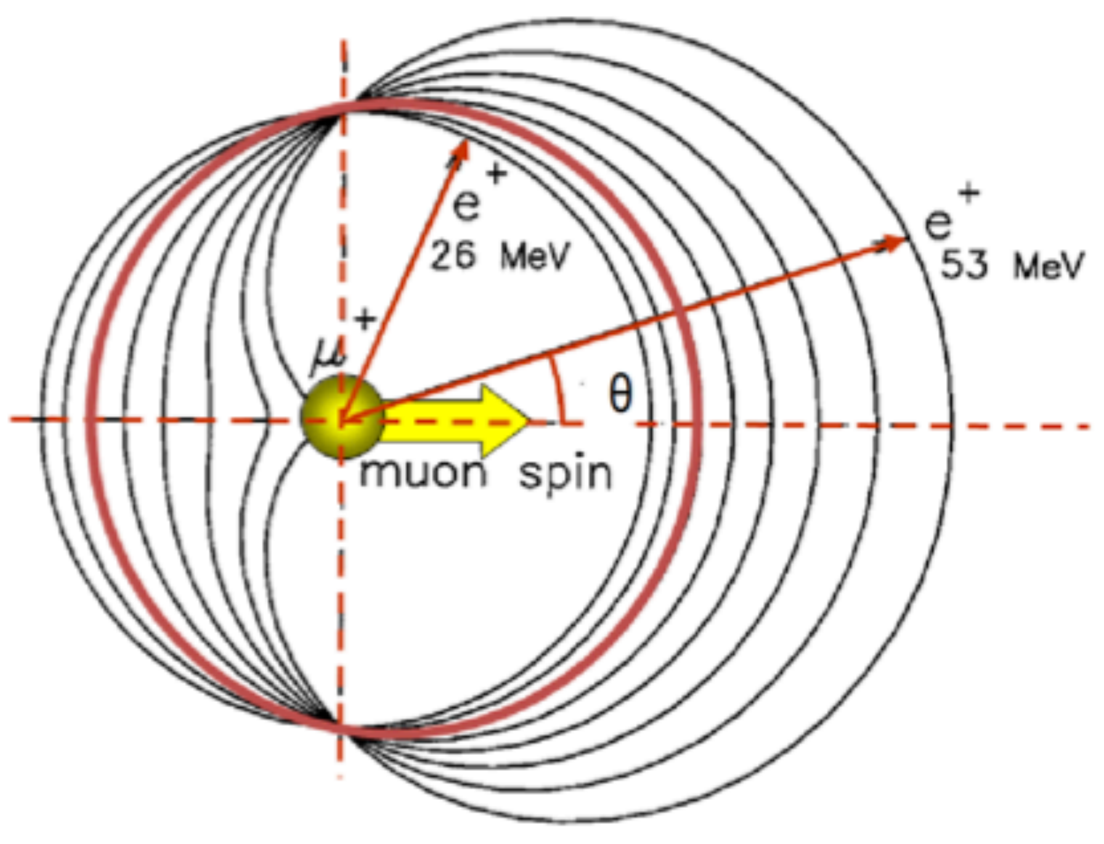


- Measure **ω_a** and **B** → obtain a_μ

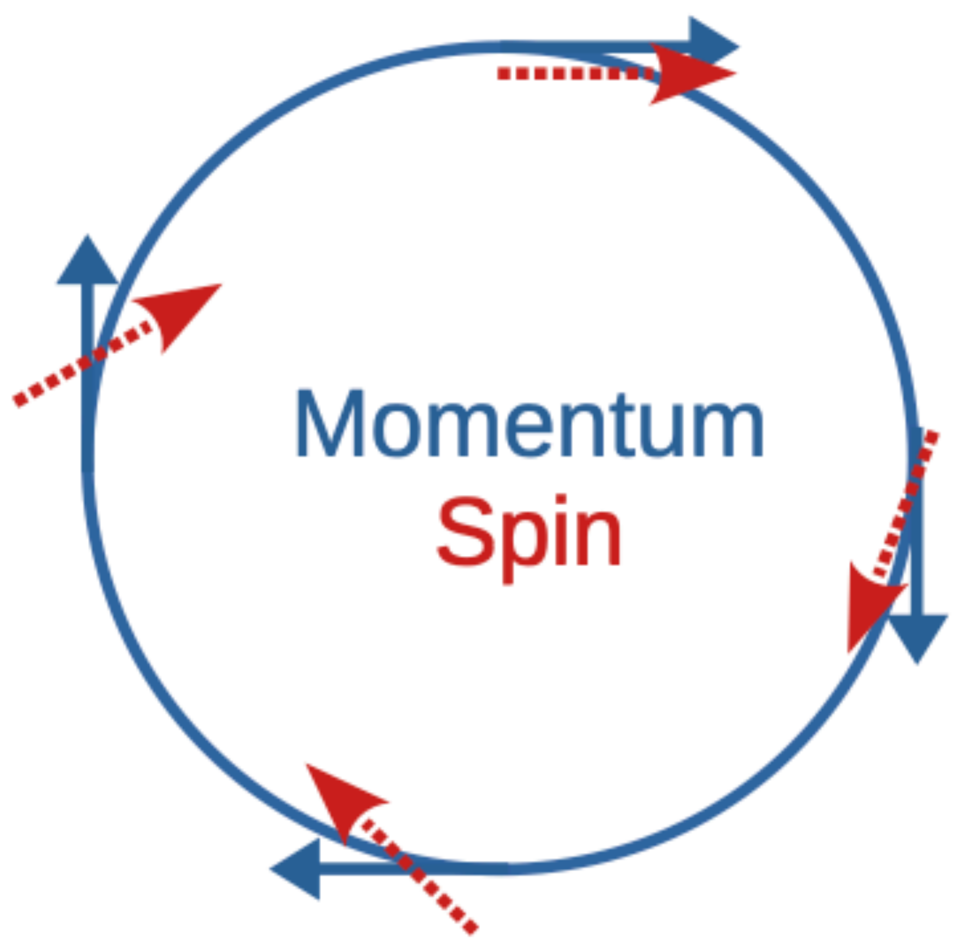
How to measure ω_a for muons?



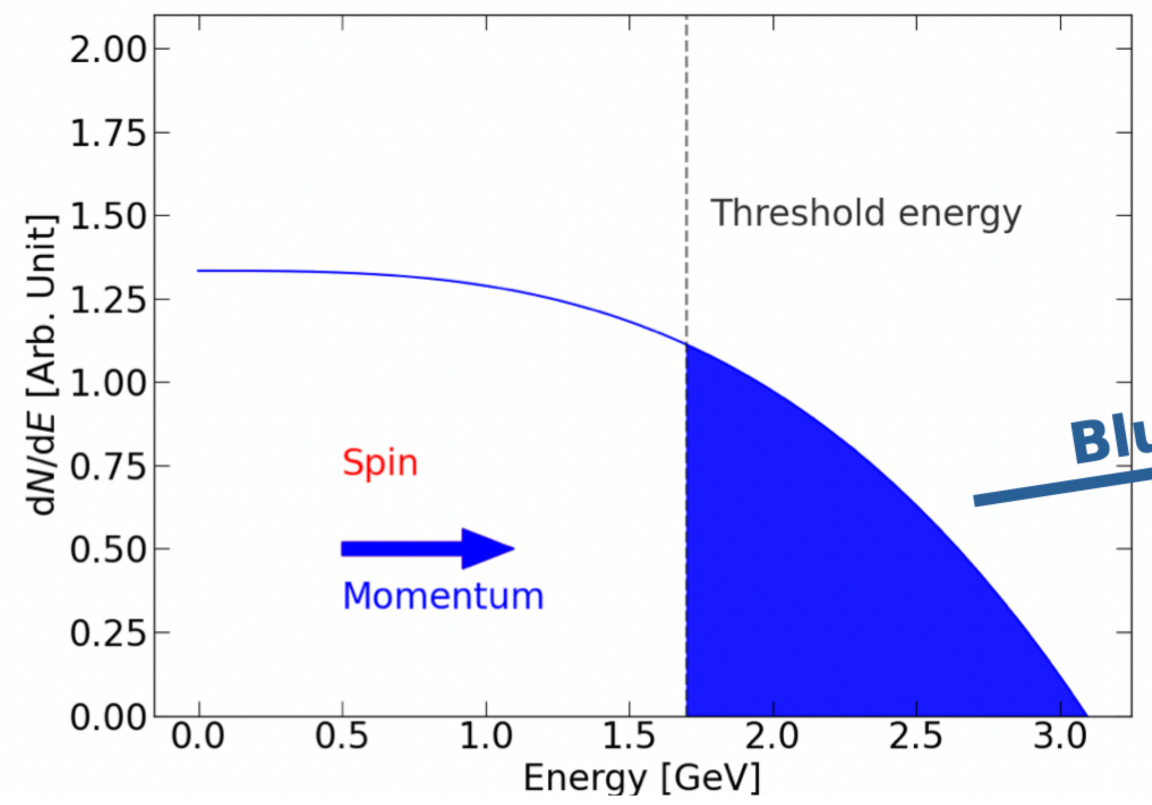
Muon decays in a
positron and 2 neutrinos



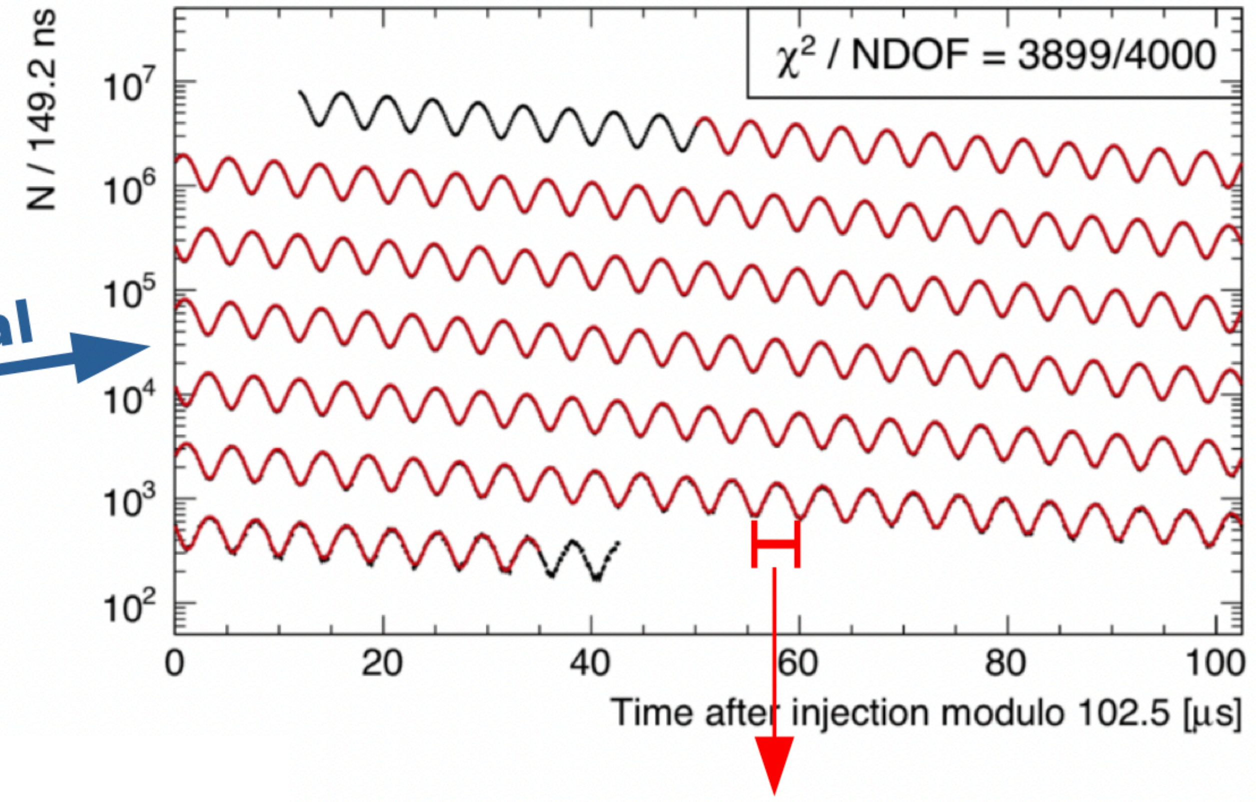
Parity violation → positrons
in CM preferably in the
direction of the muon spin



Spin precession → the energy
spectrum in the lab frame
oscillates through time



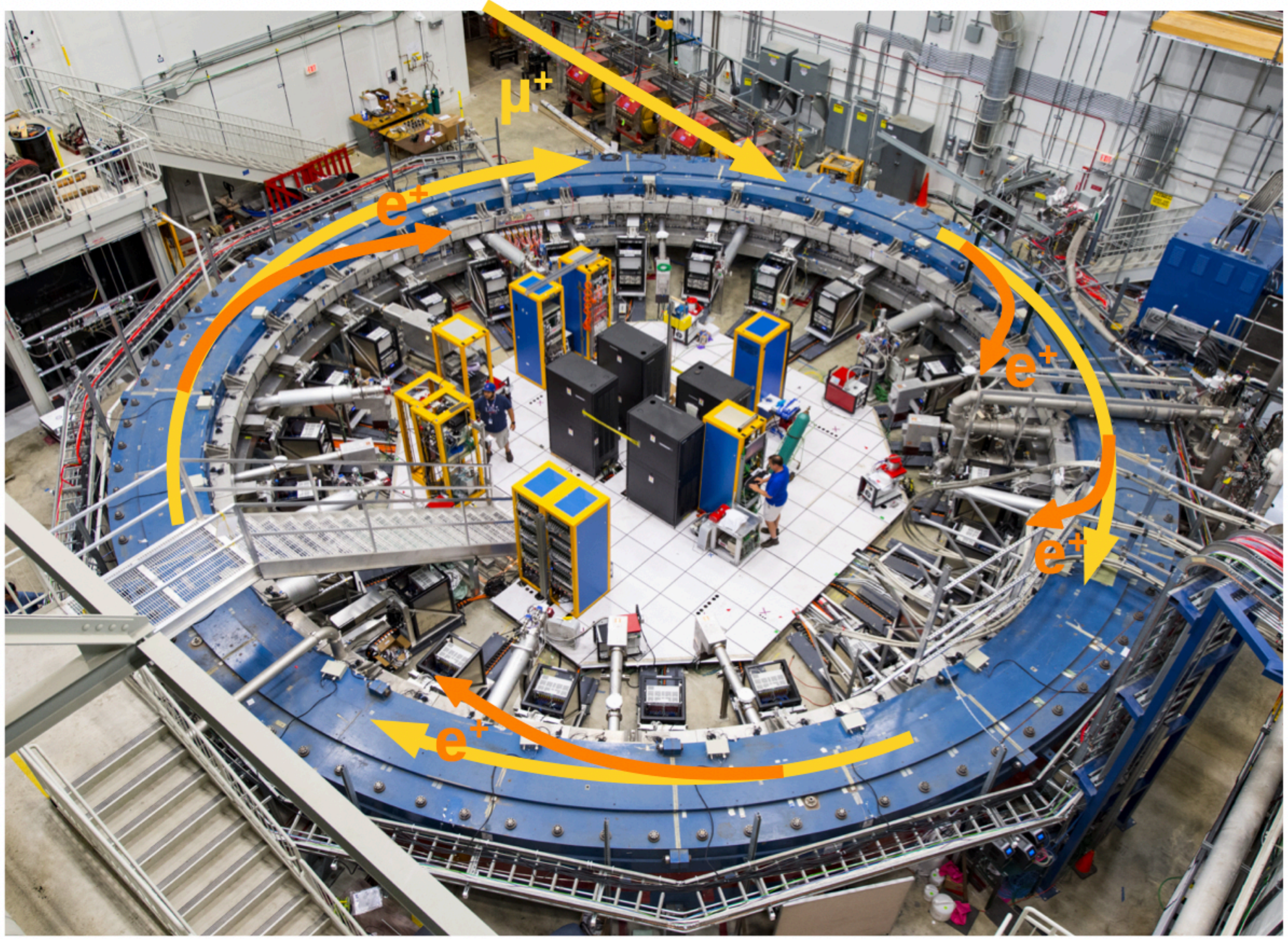
Blue integral



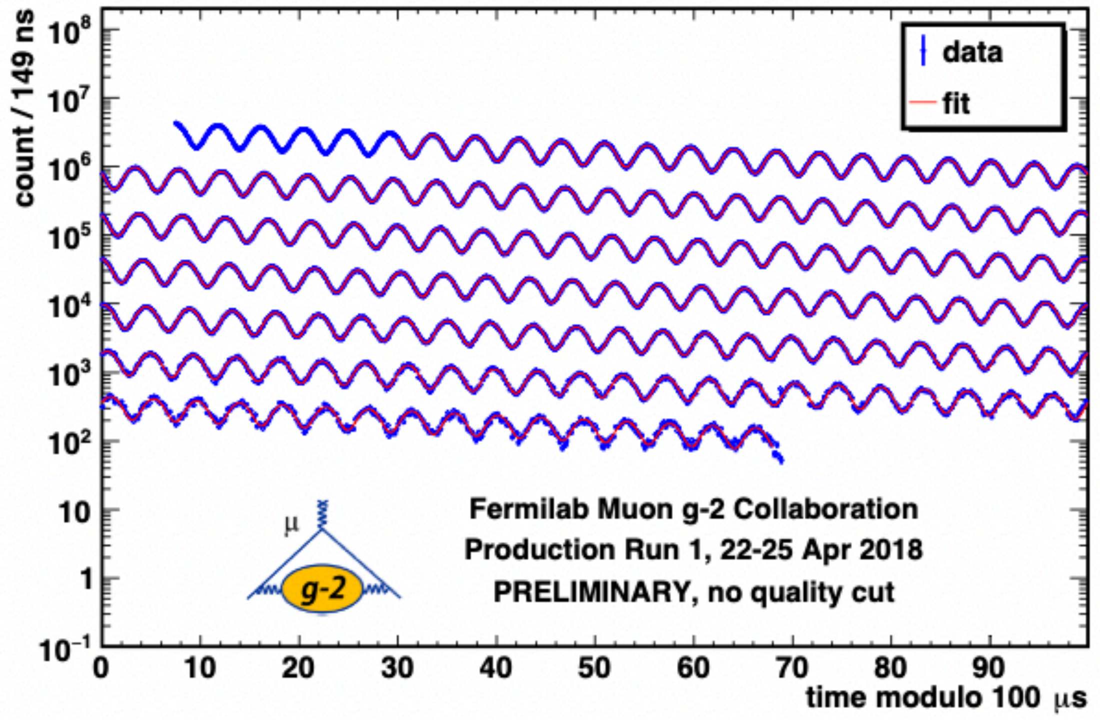
g-2 frequency!

The Muon g-2 Experiment

- Located in Fermilab, Chicago (continuing the experiment conducted at Brookhaven)
- **15 m-diameter superconducting magnet** with an exceptionally uniform magnetic field,
  used as a storage ring
- **24 EM calorimeters** to measure decay positrons (on the inside of the storage ring)
- B-field value actively mapped using an **NMR probe**



Muon g-2: extracting a_μ



ω_a

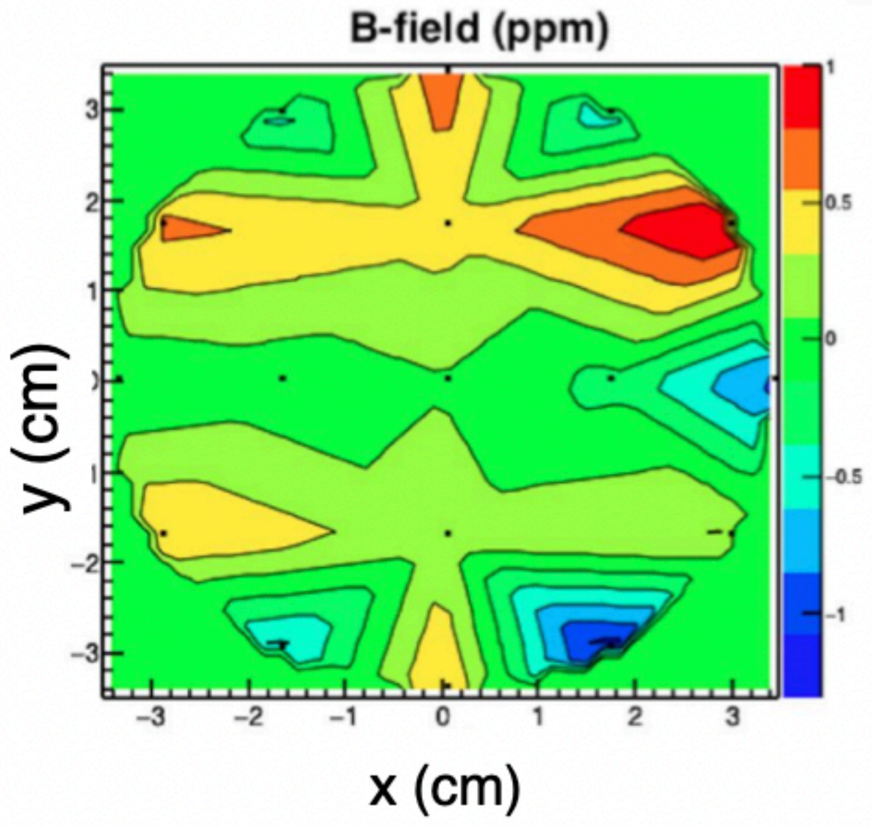
Extract from decay positron time spectra

$$N(t) = N_0 e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \phi)]$$

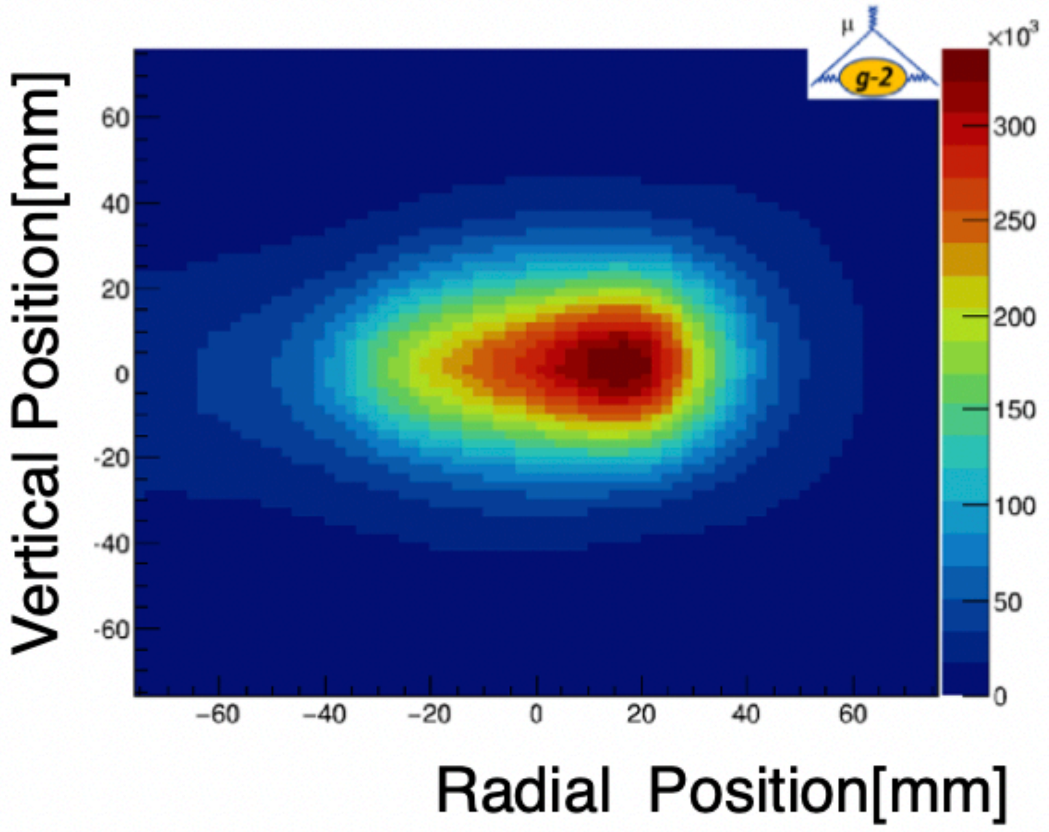
$$a_\mu = \left(\frac{g_e}{2}\right)\left(\frac{\omega_a}{\langle\omega_p\rangle}\right)\left(\frac{\mu_p}{\mu_e}\right)\left(\frac{m_\mu}{m_e}\right)$$

	Relative error (ppb)	Experiment
g_e	0.000 26	Quantum electron cyclotron. Hanneke et al. 2008.
μ_e/μ_p	3.0	Hydrogen spectroscopy. Winkler et al. 1972.
m_μ/m_e	22	Muonium hyperfine splitting. Liu et al. 1999.

0.26 ppt 3 ppb 22 ppb ⇨ 2017 CODATA



Map the magnetic field



Obtain muon distribution In the storage ring

$$\langle\omega_p\rangle \approx \omega_p \otimes \rho(r)$$

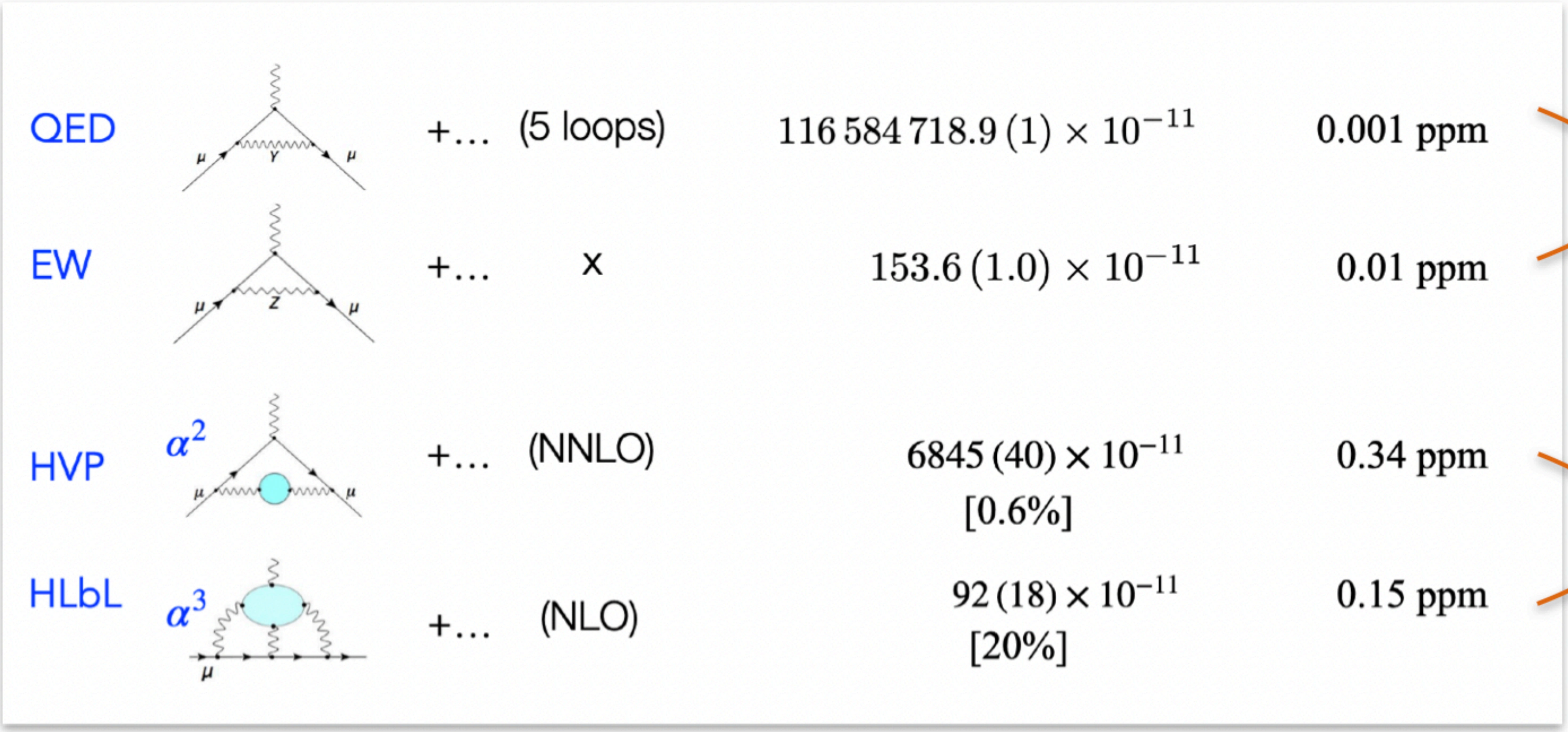
Average magnetic field weighted by muon distribution

ω_p: free proton precession frequency

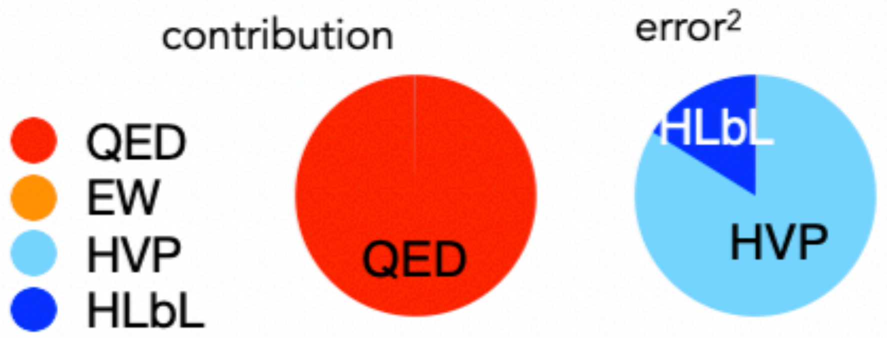
Using proton NMR $\hbar\omega_p = 2\mu_p B$

Muon g-2: status of theory calculations

$$a_\mu = a_\mu(QED) + a_\mu(EW) + a_\mu(hadronic)$$



QED		+… (5 loops)	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm
EW		+… x	$153.6\,(1.0) \times 10^{-11}$	0.01 ppm
HVP	α^2	+… (NNLO)	$6845\,(40) \times 10^{-11}$ [0.6%]	0.34 ppm
HLbL	α^3	+… (NLO)	$92\,(18) \times 10^{-11}$ [20%]	0.15 ppm



Well-known

Non-perturbative
(Data-driven & lattice QCD)

- QED and EW contributions are very well-known with small uncertainties
- **Hadronic vacuum polarisation (HVP)** contribution error dominates the uncertainty budget

Muon g-2: most recent result

- Theory calculation differ in HVP calculations:
  - lattice QCD (Ab-initio)
  - dispersive (e+e-) method (data-driven)
  - this results in “theory-theory” tensions…

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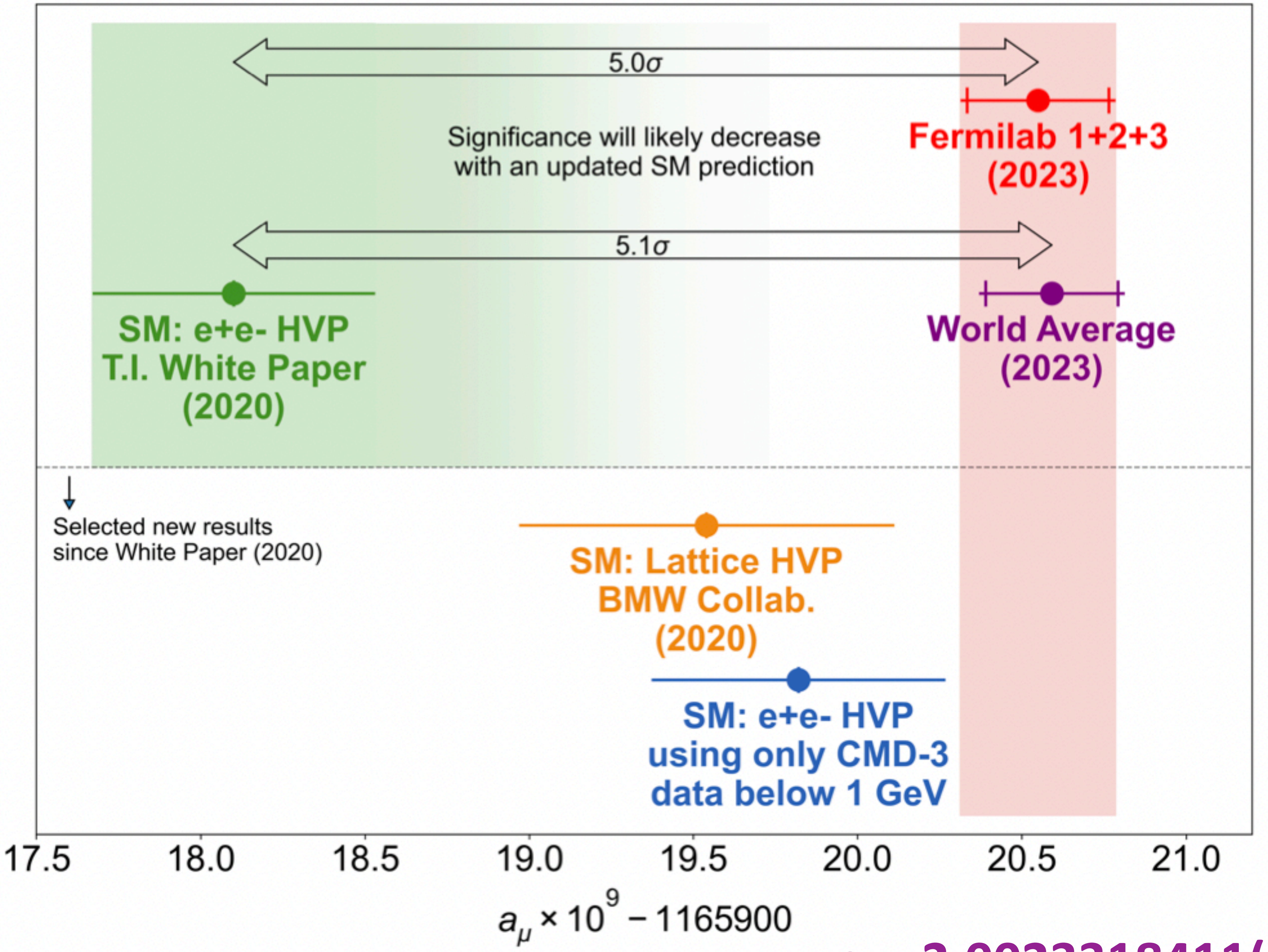
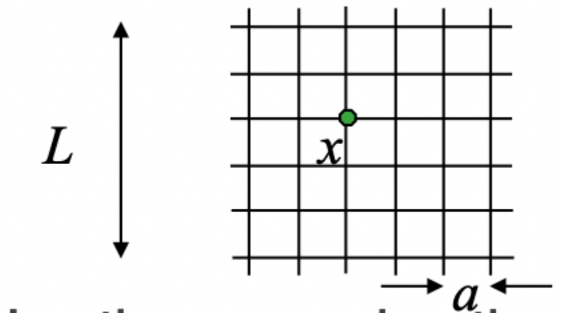
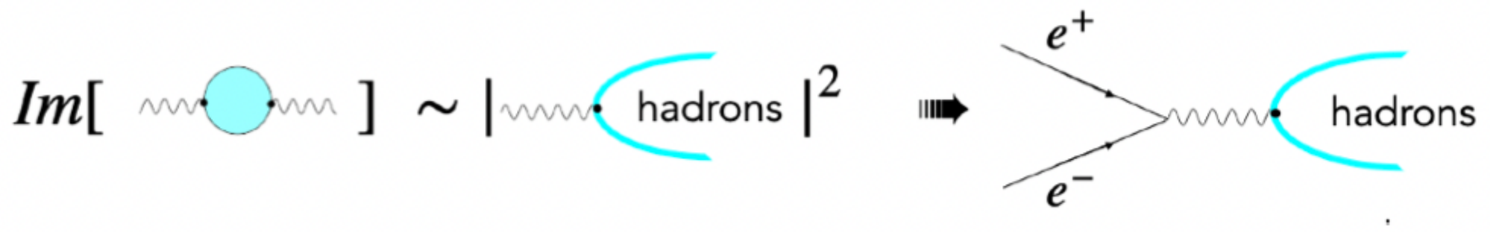


lattice



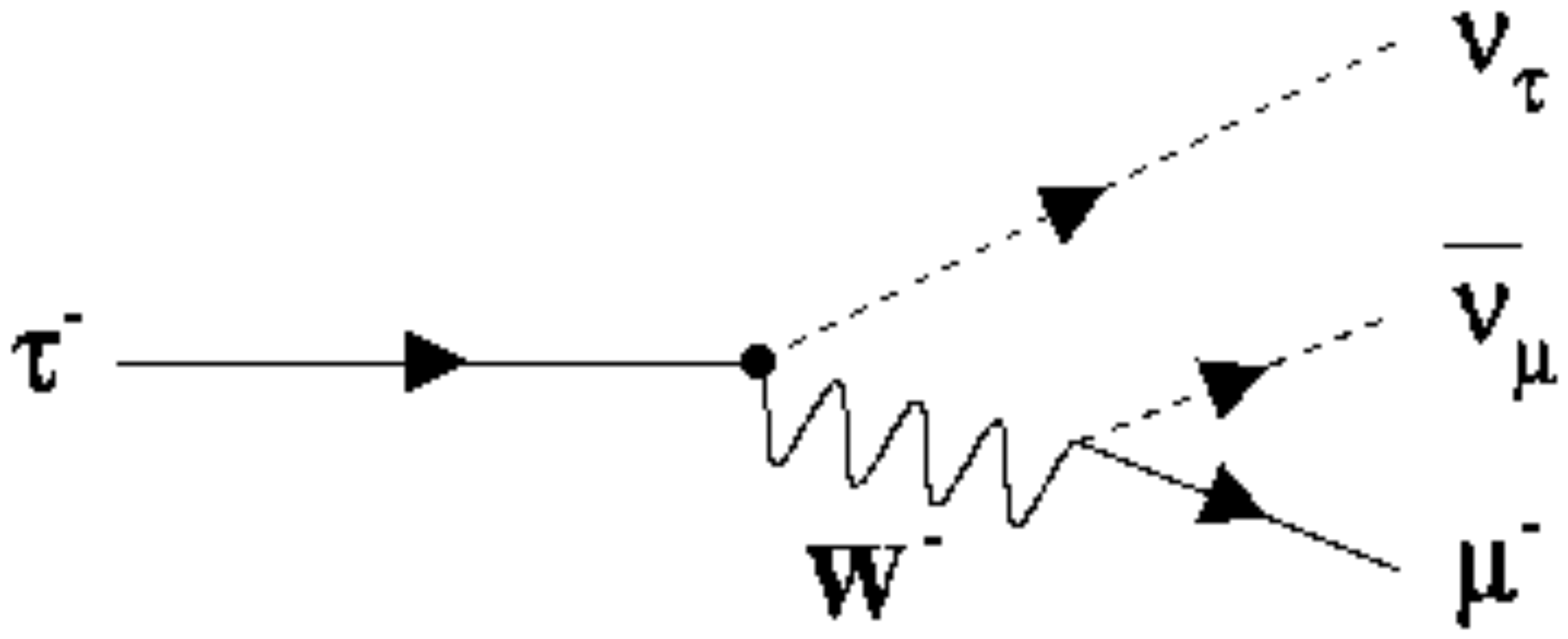
dispersive



- g_μ =2.0023318411(5)

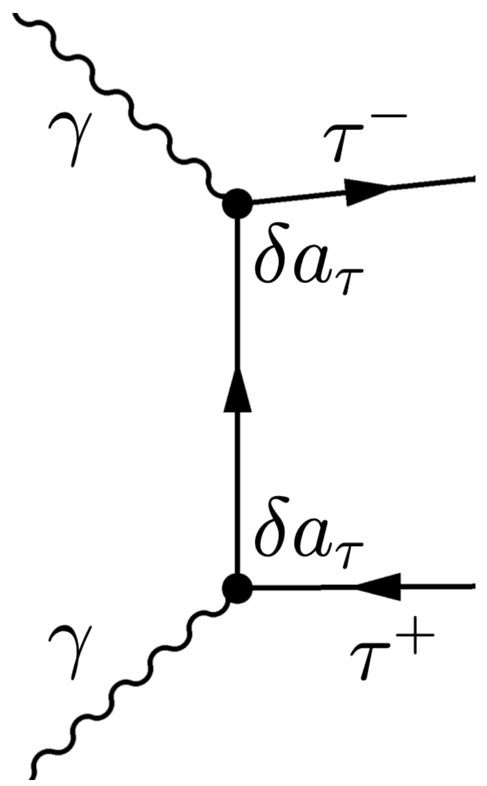
The tau lepton - a recap

- Discovered in 1970's, it's the heaviest charged lepton
  - ≈ 2000 heavier than the electron
  - Due to large mass, it decays almost immediately (lifetime of **3×10^{-13} s**)
- Because of extremely short lifetime impossible to make spin-precession experiments

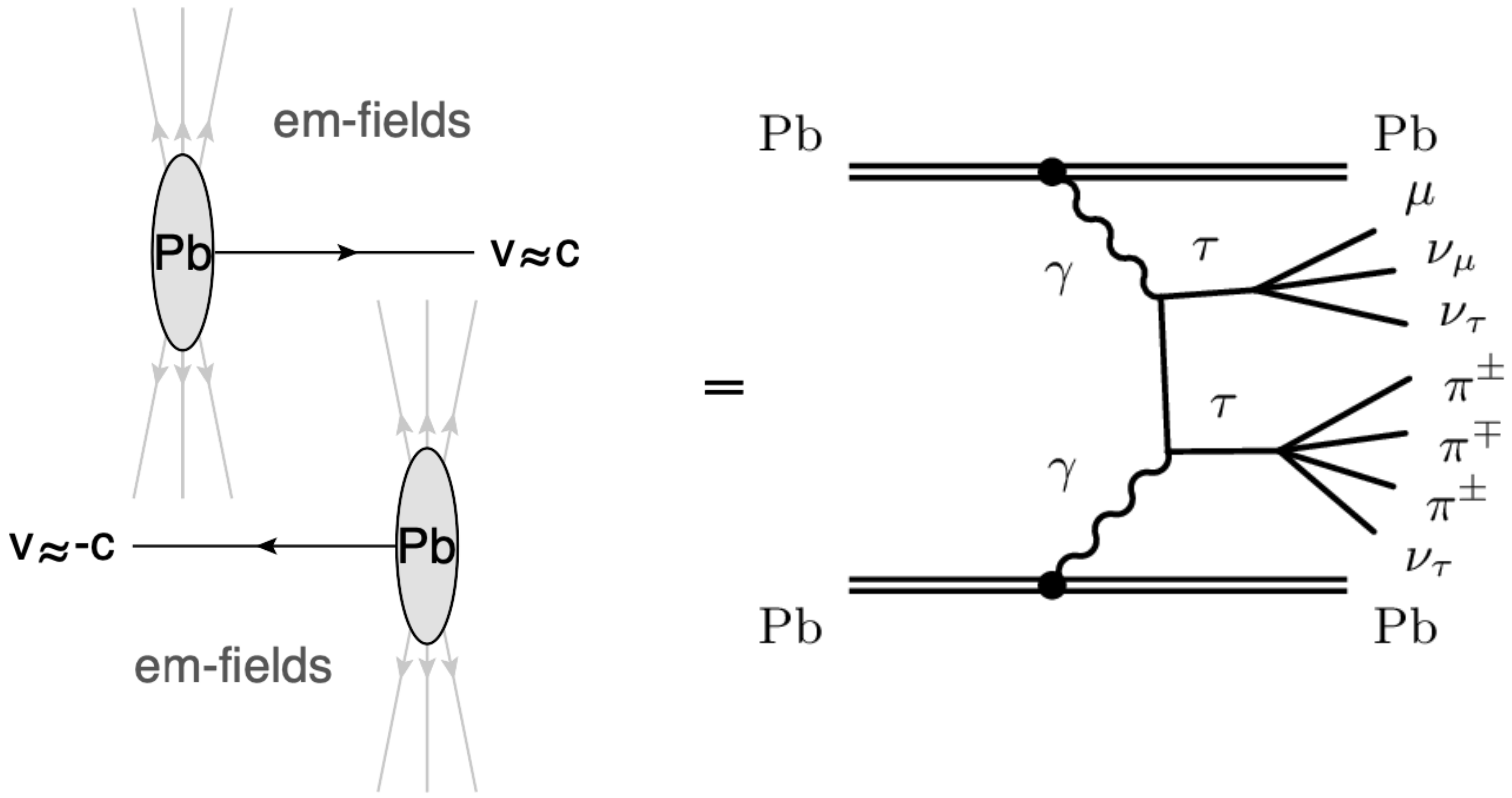


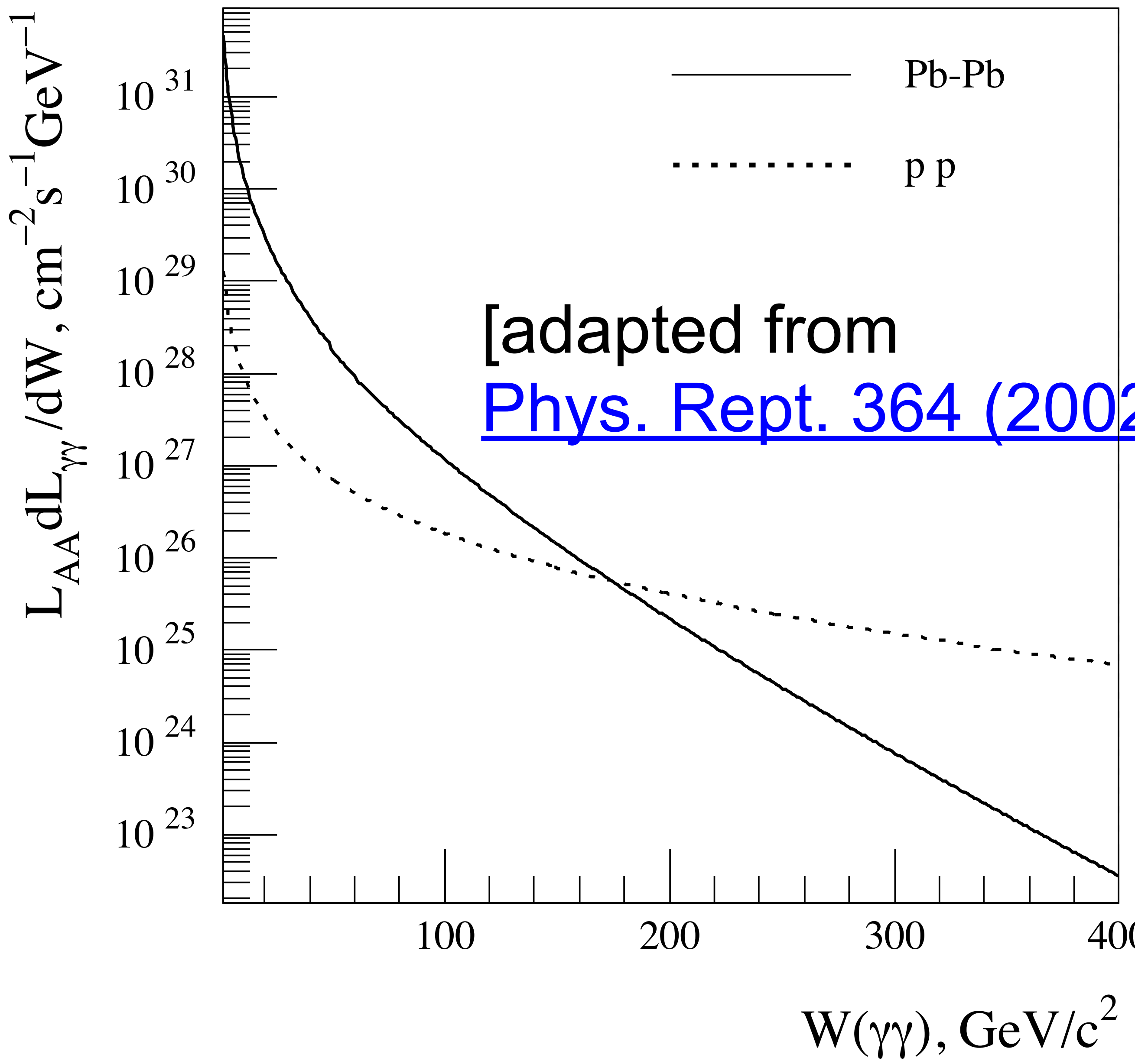
Example tau decay into neutrinos and a muon

Tau lepton EM interactions at the LHC



- We can measure the strength of EM interaction with tau lepton by studying the following process:
  - But: a powerful source of high-energy photons is needed…
- Heavy (charged) ions are intense source of photons
  - Fortunately we collide protons and lead ions at the Large Hadron Collider!





[adapted from Phys. Rept. 364 (2002) 359]

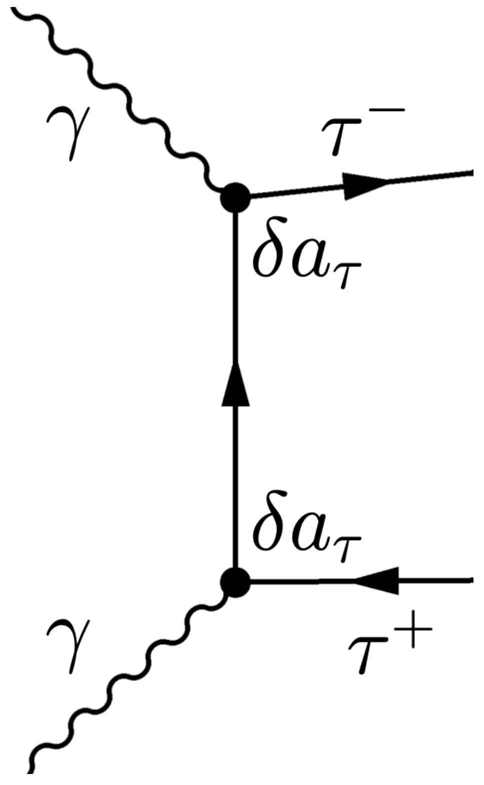
Tau g-2: a_τ parametrisation

- Elementary γγ→τ τ cross section has explicit dependence on photon-τ vertex function:

$$\boxed{i\Gamma_\mu^{(\gamma\ell\ell)}(p',p) = -ie\left[\gamma_\mu F_1(q^2) + \frac{i}{2m_\ell}\sigma_{\mu\nu}\underline{q}^\nu \boxed{F_2(q^2)} + \frac{1}{2m_\ell}\gamma^5\sigma_{\mu\nu}\underline{q}^\nu \boxed{F_3(q^2)}\right]}$$

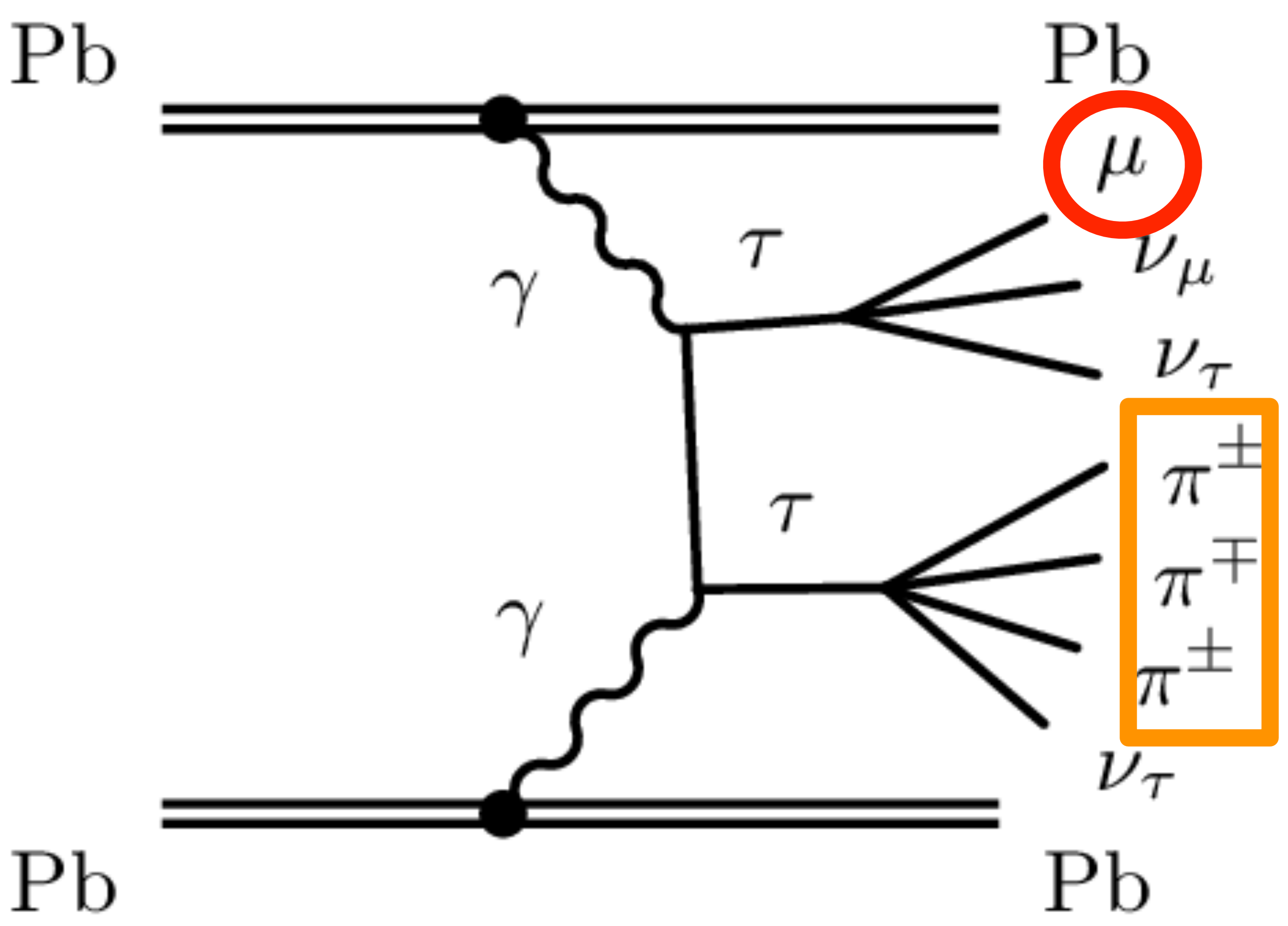
$=a_\tau$ ($q^2=0$)

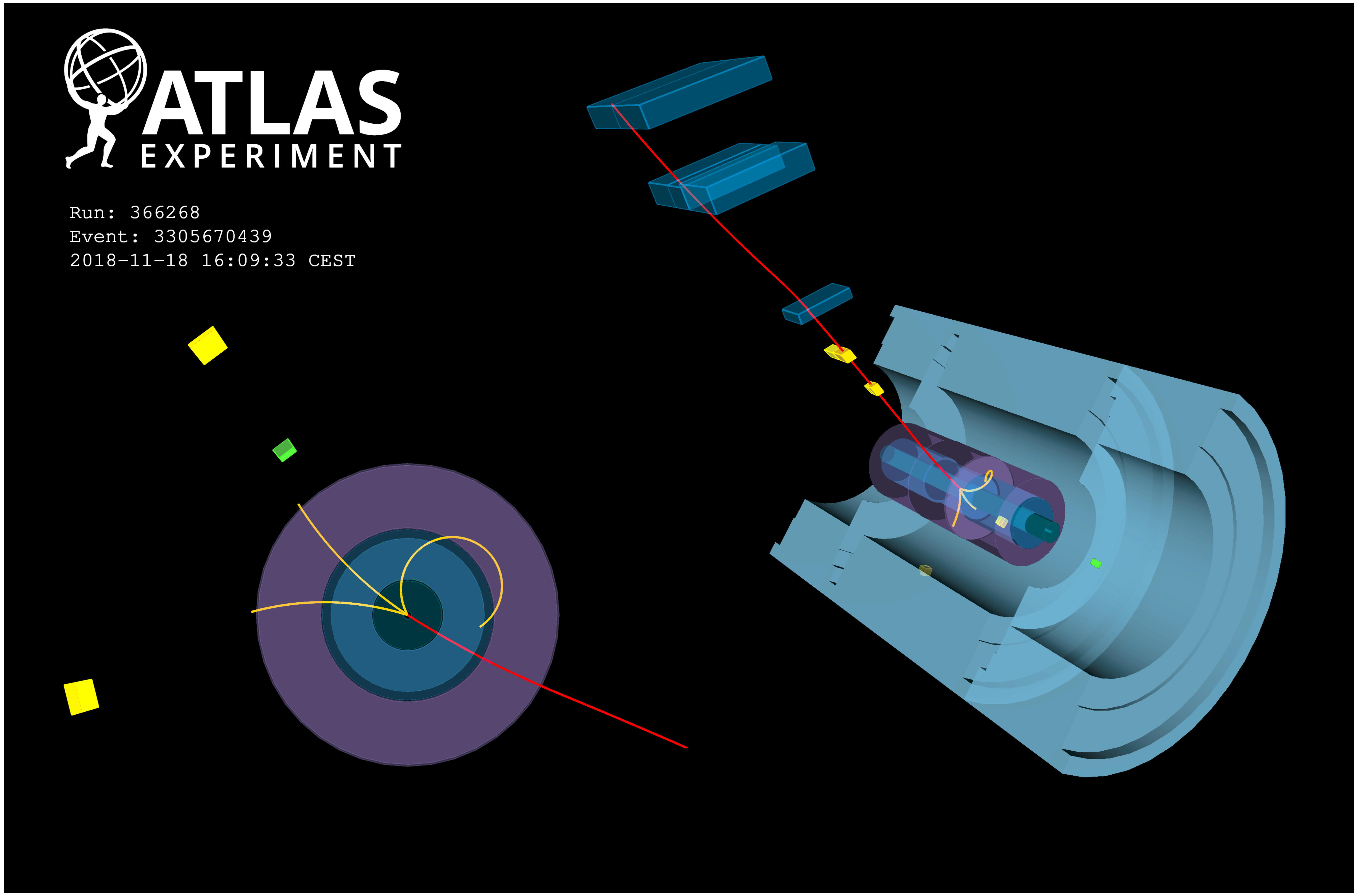
$=d_\tau * 2m_\tau/e$ ($q^2=0$)



Recording tau pairs at LHC experiments

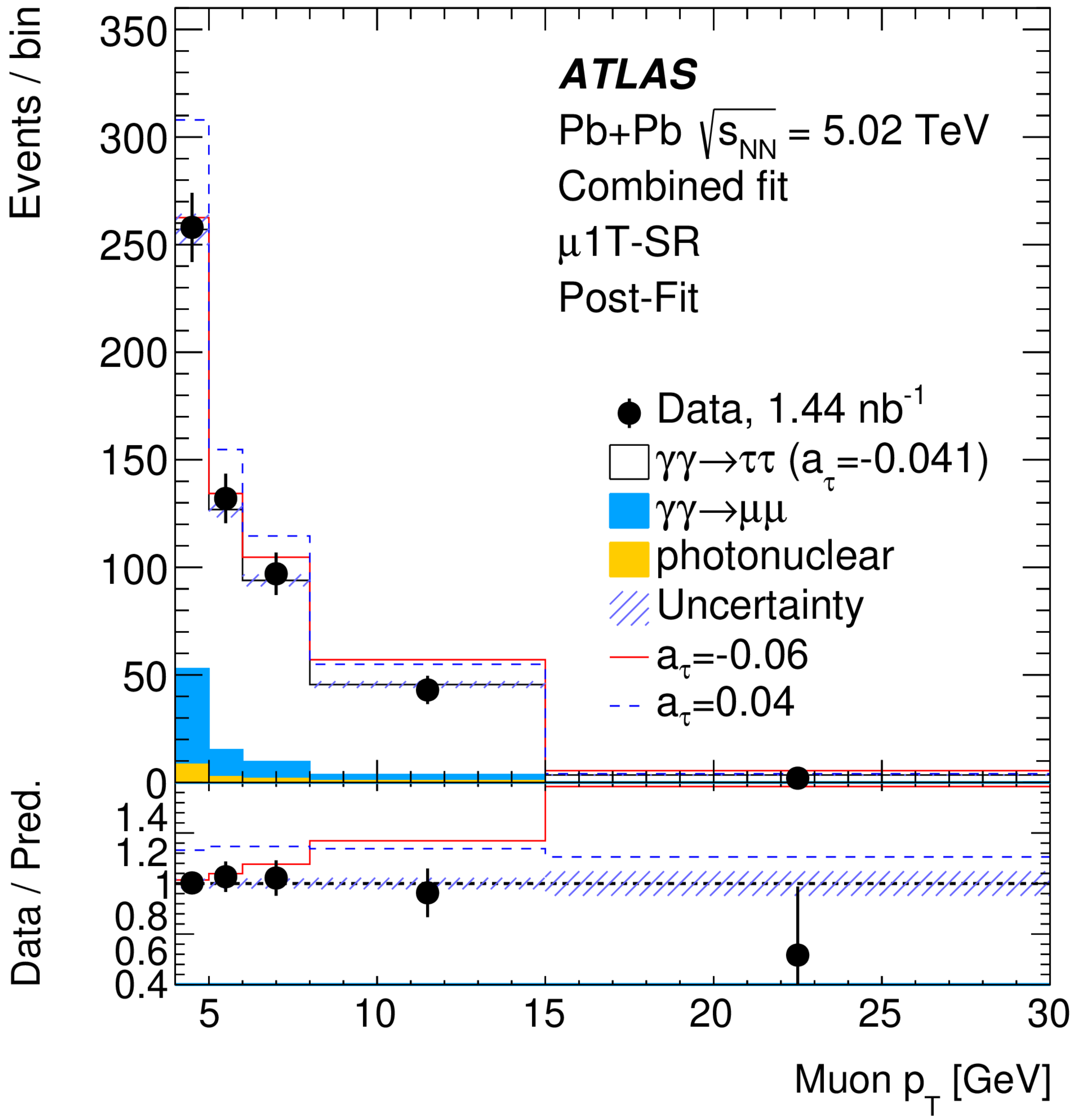
- At collider experiments we reconstruct decay products of taus
- We observe taus **for the first time** in ion collisions!

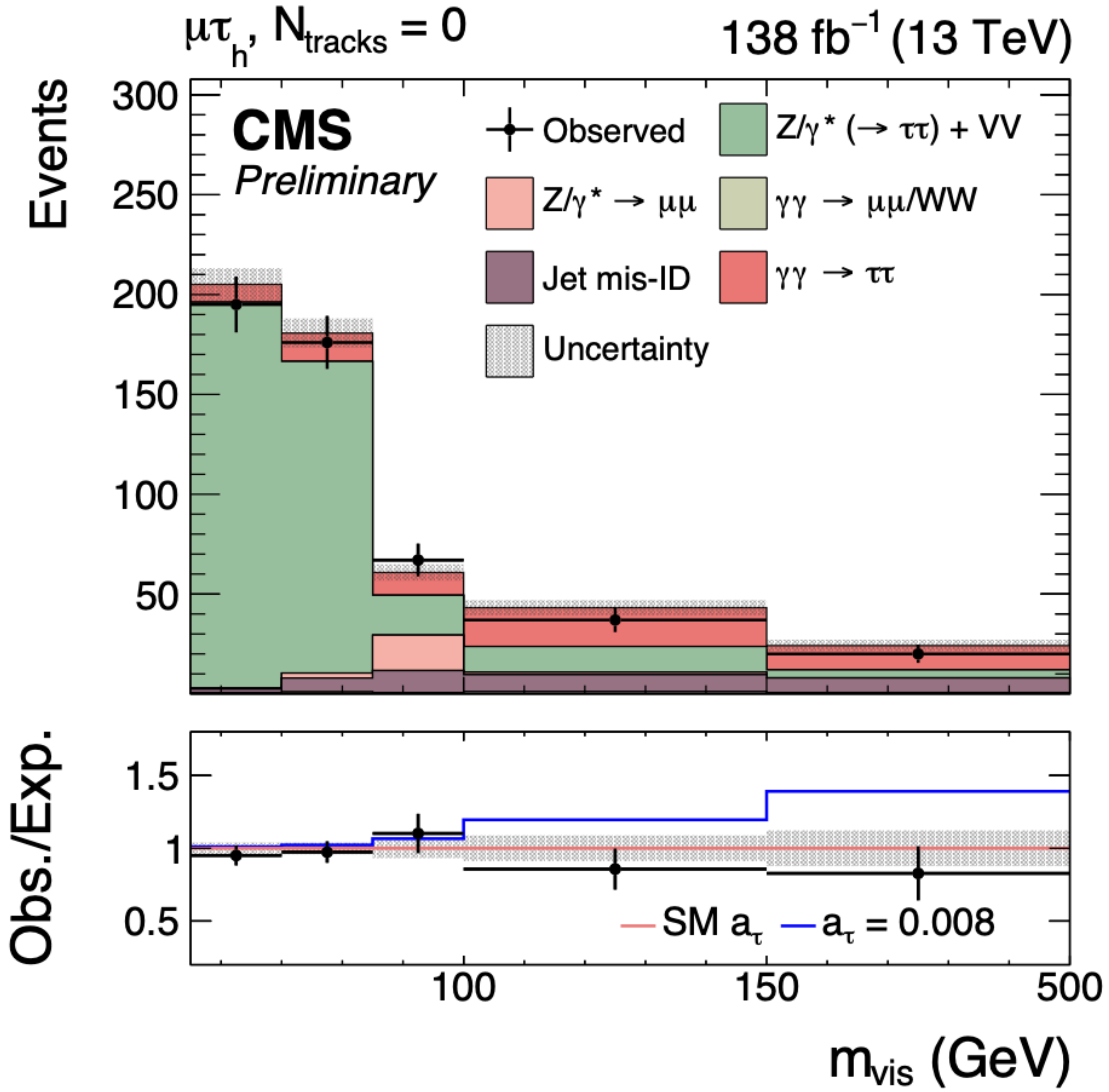




Tau g-2 measured in ATLAS and CMS

- The value of $a_\tau = (g_\tau - 2)/2$ is sensitive to both cross-section variations and shapes of kinematic distributions
- CMS (pp) measurement seems to have better sensitivity due to harder incoming photon fluxes (p size vs Pb size)





Tau g-2 measured in ATLAS and CMS

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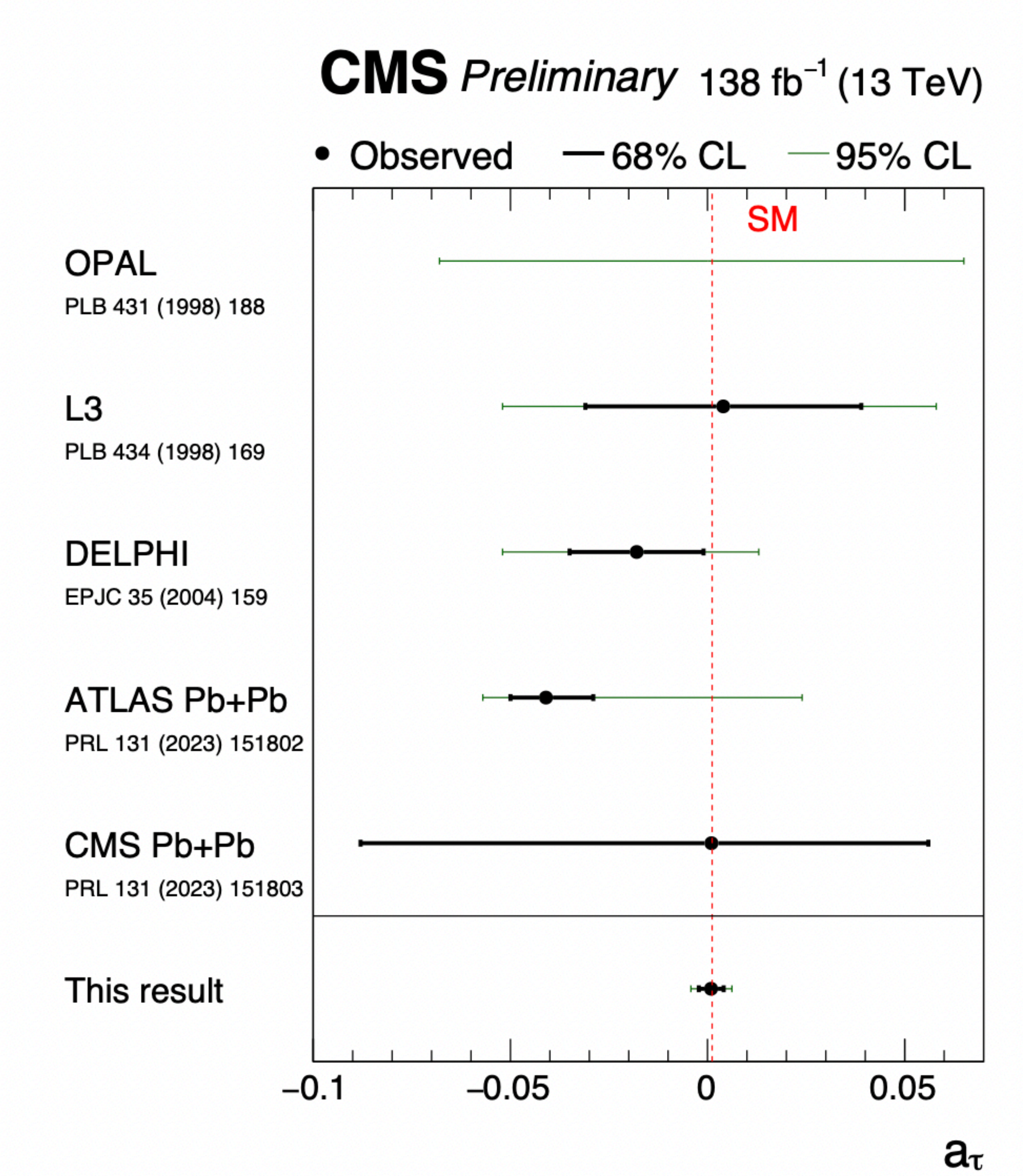
CMS PAS SMP-23-005

- We measure $a_\tau = (g_\tau - 2)/2$
  - g_τ is found to be consistent with “2”

(1.94< g_τ < 2.02 @95% CL **-> ATLAS Pb+Pb)**

(1.995< g_τ < 2.007 @95% CL **-> CMS pp)**



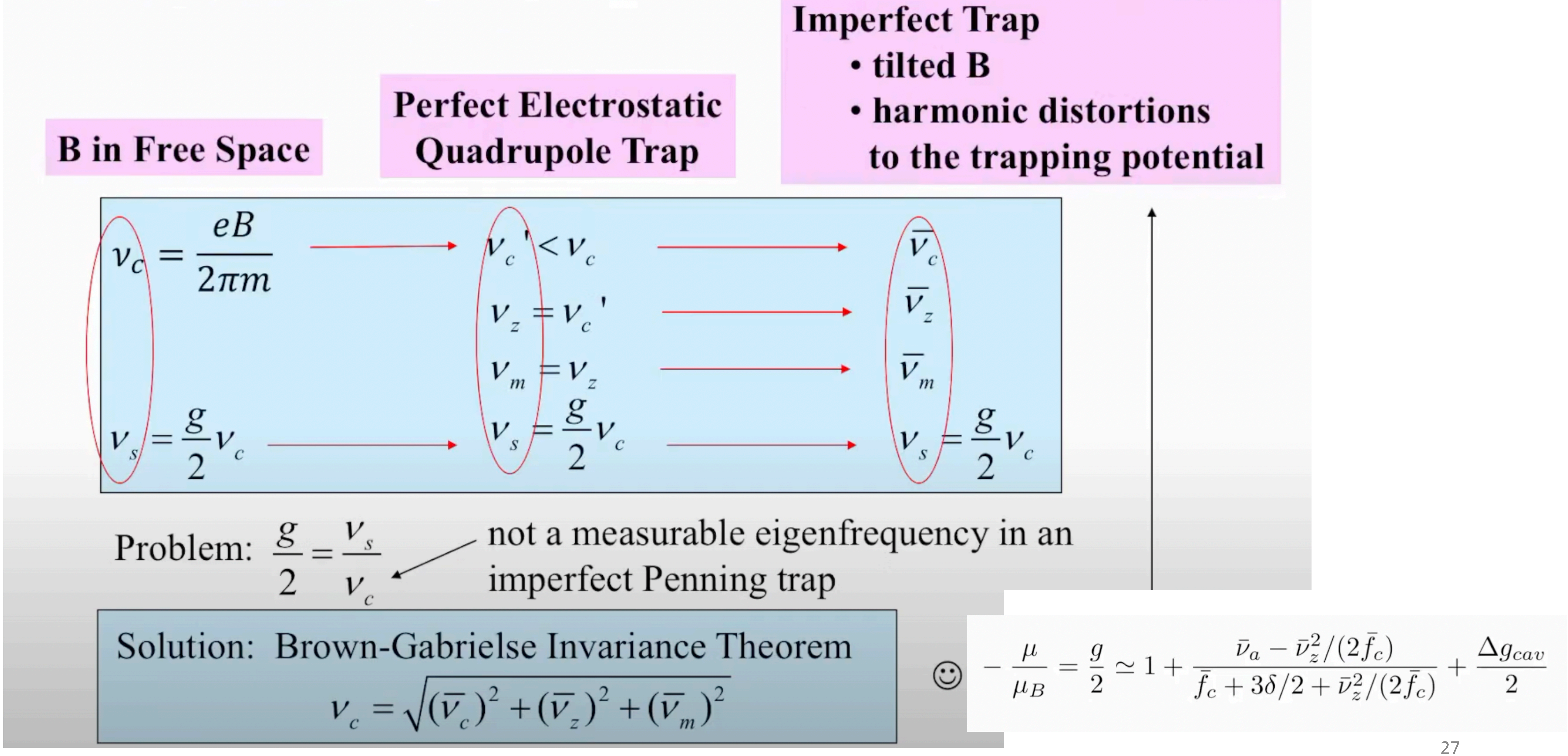
Summary

- Magnetic dipole moments of charged leptons can be measured with high precision
  - Sensitive to 'new' particles via quantum fluctuations
  - Unprecedented accuracy achieved for electrons and muons:
    g_e =2.0023193043612(3), **g_μ =2.0023318411(5)**
  - Waiting for a clarification (of the theory): discrepant α measurements (electrons), hadronic contributions (muons)

- Challenging to measure tau lepton magnetic dipole moment
  - ATLAS and CMS experiments have measured this recently (for the first time by using hadron collisions)
    (1.94< g_τ < 2.02 @95% CL -> ATLAS Pb+Pb)
    (1.995< g_τ < 2.007 @95% CL -> CMS pp)
    -> precision will be improved by studying more data

Backup

Electron g-2: trap imperfections



B in Free Space

Perfect Electrostatic Quadrupole Trap

Imperfect Trap
- **tilted B**
- **harmonic distortions to the trapping potential**

$$\nu_c = \frac{eB}{2\pi m} \longrightarrow \nu_c' < \nu_c \longrightarrow \bar{\nu}_c$$

$$\nu_z = \nu_c' \longrightarrow \bar{\nu}_z$$

$$\nu_m = \nu_z \longrightarrow \bar{\nu}_m$$

$$\nu_s = \frac{g}{2}\nu_c \longrightarrow \nu_s = \frac{g}{2}\nu_c \longrightarrow \nu_s = \frac{g}{2}\nu_c$$

Problem: $\frac{g}{2} = \frac{\nu_s}{\nu_c}$ ← not a measurable eigenfrequency in an imperfect Penning trap

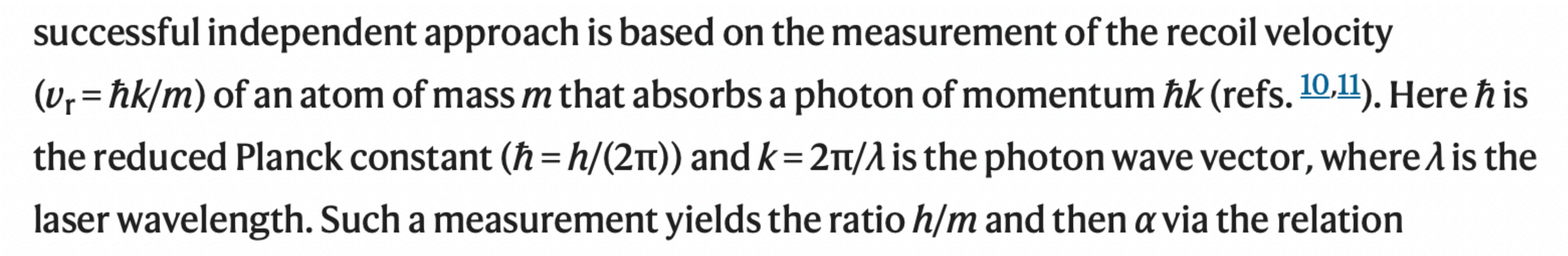
Solution: Brown-Gabrielse Invariance Theorem

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$

☺

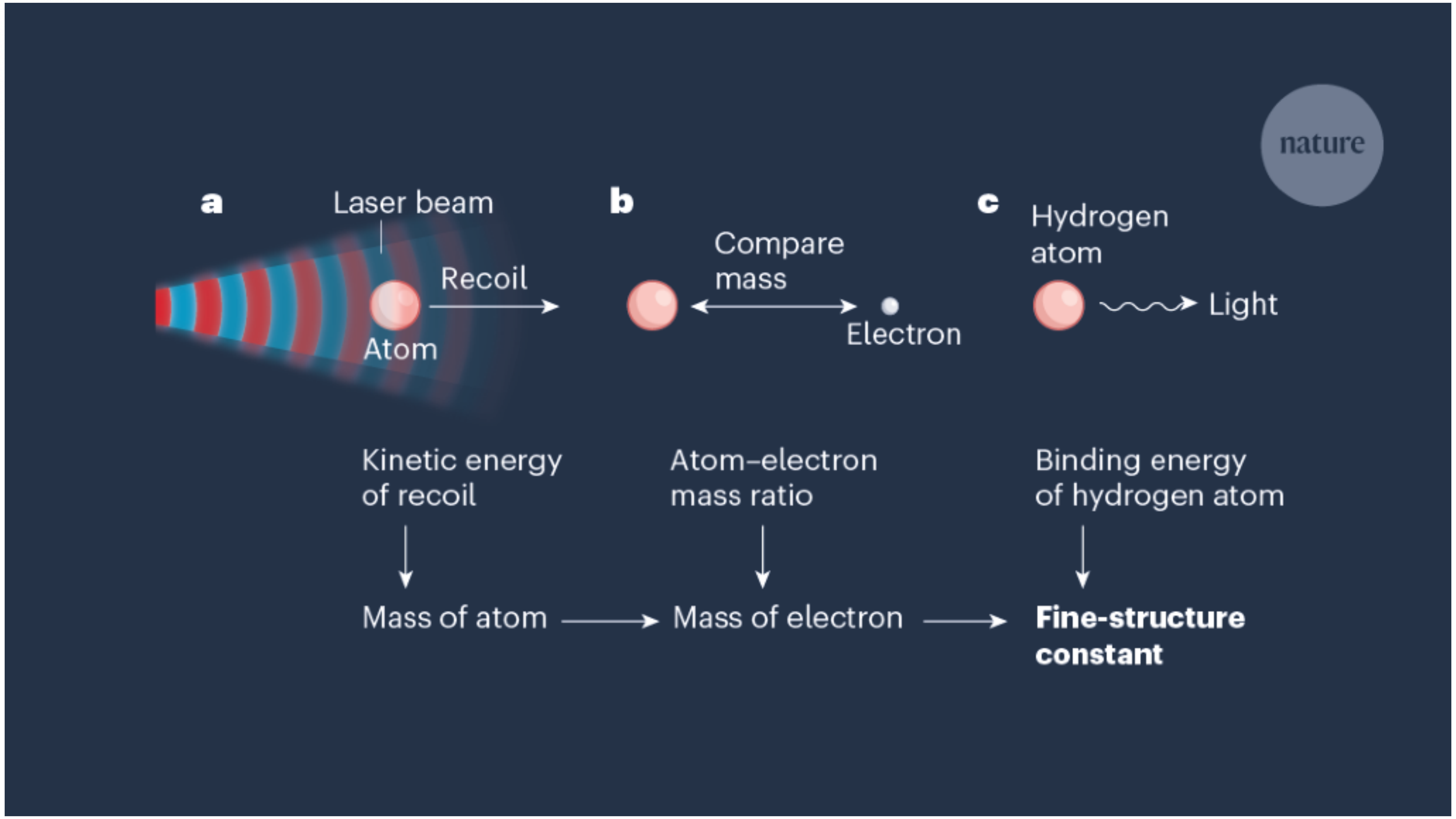
$$-\frac{\mu}{\mu_B} = \frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \bar{\nu}_z^2/(2\bar{f}_c)}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)} + \frac{\Delta g_{cav}}{2}$$

Fine structure constant measurement



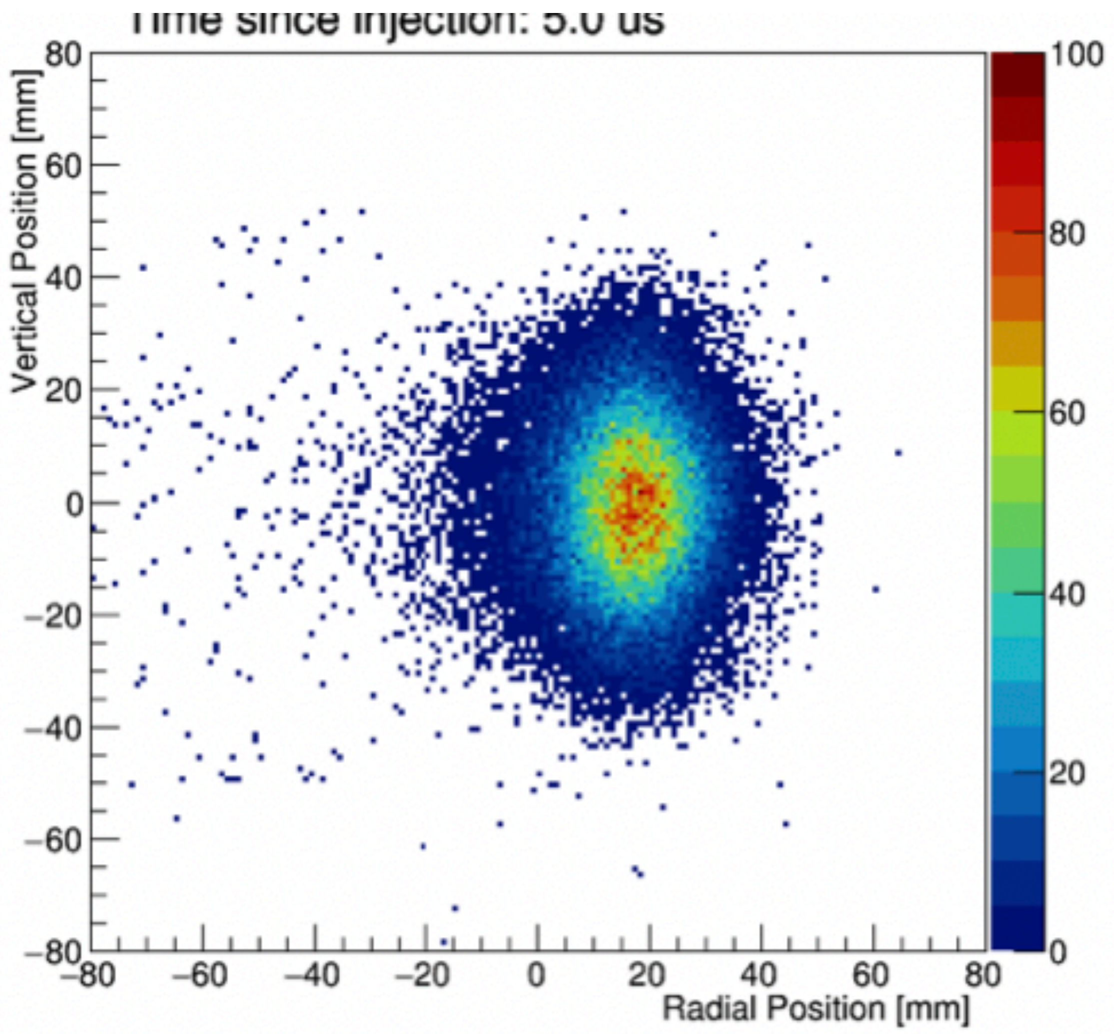
successful independent approach is based on the measurement of the recoil velocity ($v_r = \hbar k/m$) of an atom of mass m that absorbs a photon of momentum $\hbar k$ (refs. [10,11]). Here $\hbar$ is the reduced Planck constant ($\hbar = h/(2\pi)$) and $k = 2\pi/\lambda$ is the photon wave vector, where λ is the laser wavelength. Such a measurement yields the ratio h/m and then α via the relation

$$\alpha^2 = \frac{2R_\infty}{c} \times \frac{m}{m_e} \times \frac{h}{m}.$$



Muon g-2: beam-dynamics effects

- The muon beam oscillates and breathes as a whole
- The full equation is more complex and corrections due to radial (x) and vertical (y) beam motion are needed



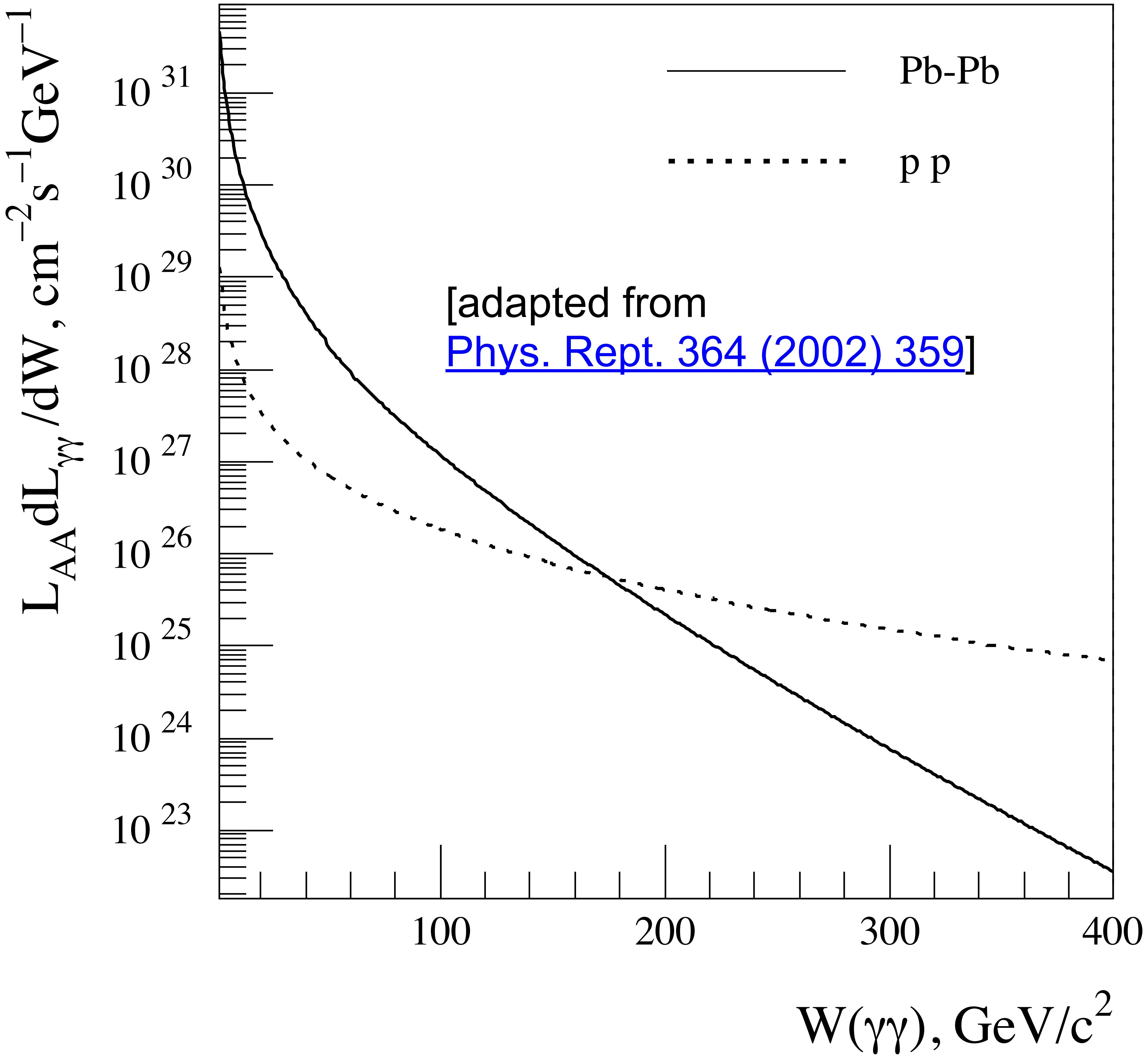
$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c =$$

$$= -\frac{e}{mc}\left[a_\mu \vec{B} - \boxed{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}} - \boxed{a_\mu\left(\frac{\gamma}{\gamma + 1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}}\right]$$

- Running at γ_{magic}=29.3 (p=3.094 GeV/c) this coefficient is null
- Because of momentum spread (<0.2%)→ **E-field Correction**

- Vertical beam oscillation → **Pitch correction**

Photon-photon collisions: pp vs Pb+Pb



[adapted from
Phys. Rept. 364 (2002) 359]