Longitudinal spin polarization in a thermal model with dissipative corrections

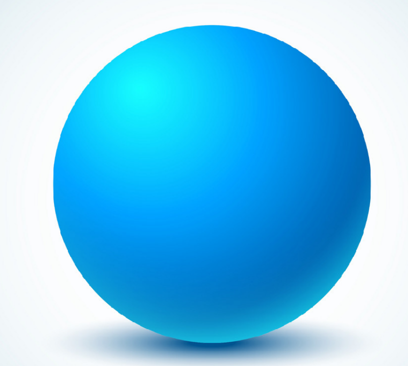
Amaresh Jaiswal

School of Physical Sciences, NISER Bhubaneswar, Jatni, India
Institute of Theoretical Physics, Jagiellonian University, Krakow, Poland

Białasówka

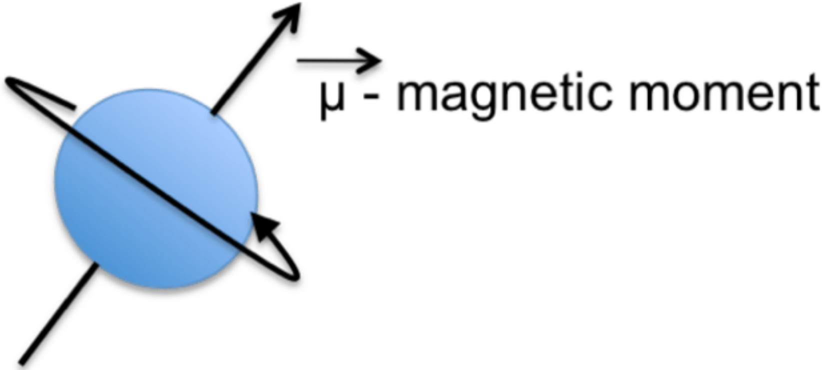
AGH University, Krakow

Decay of scalar particles



No anisotropy in the rest frame: isotropic decay products.

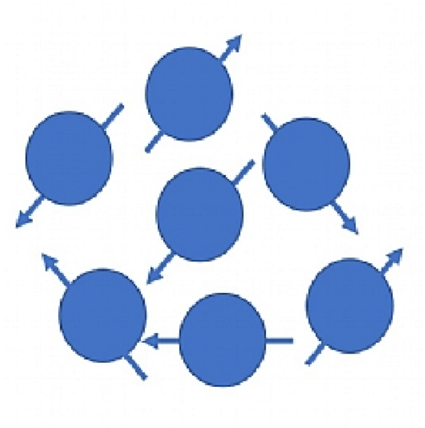
Decay of particles with spin



Preferred direction due to spin: anisotropic decay products

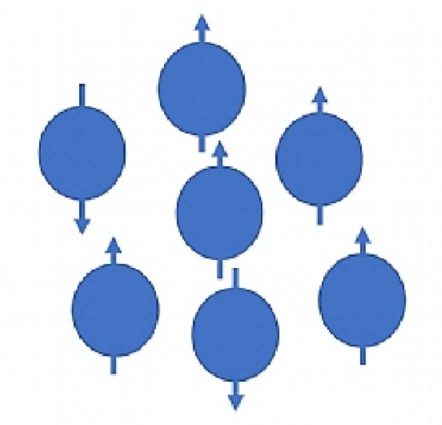
Basis for polarization observables.

Several random decays



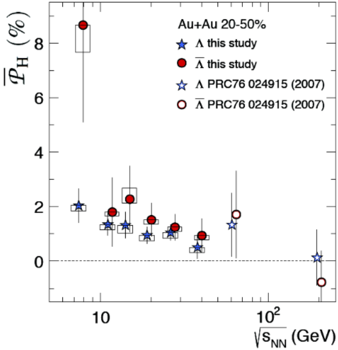
Averaging over random decays should lead to isotropic decay products.

Decay of spin polarized particles



Averaging over decay of spin-polarized particles should lead to anisotropic decay products.

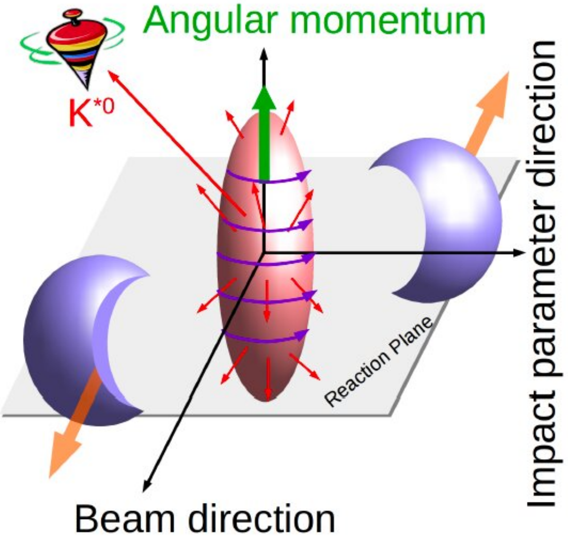
STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017



First evidence of a quantum effect in (relativistic) hydrodynamics

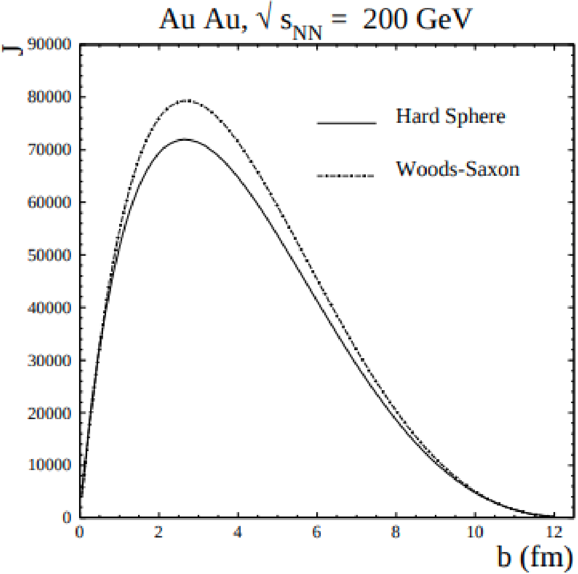
Adapted from F. Becattini
'Subatomic Vortices'

Global angular momentum in heavy ion collisions



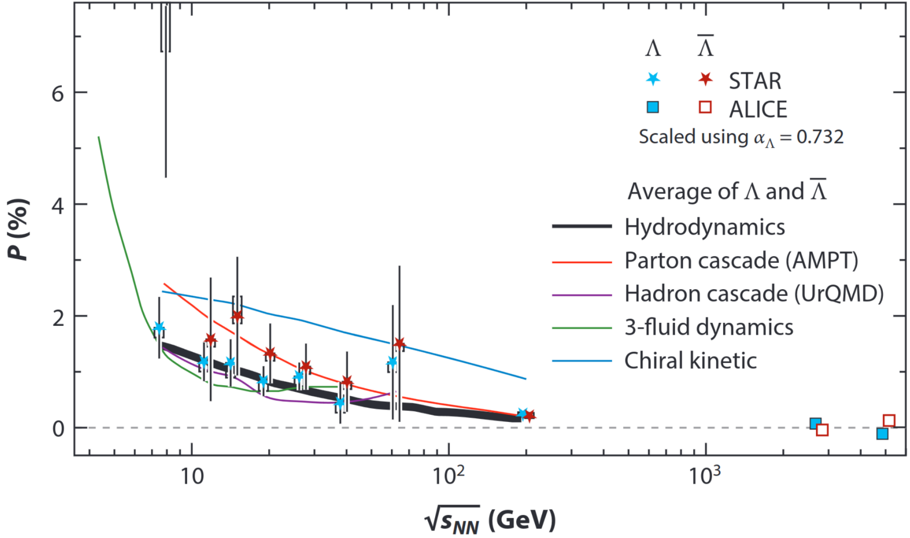
[B. Mohanty, ICTS News 6, 18-20 (2020).]

Angular momentum generation in non-central collisions



[F. Becattini, et al., Phys. Rev. C77, 024906 (2008)]

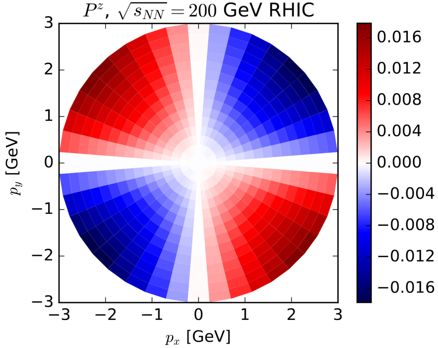
Global polarization



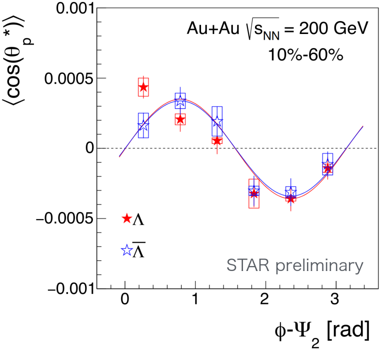
[F. Becattini and M. Lisa, Annu. Rev. Nucl. Part. Sci. 2020. 70:395-423]

Local polarization and sign problem

vHLLE+Glissando IS



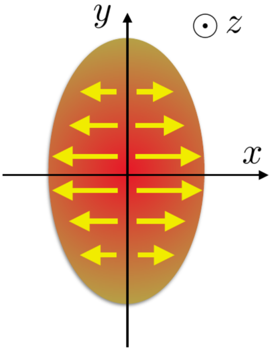
Preliminary STAR data: Takafumi Niida, talk at Chirality workshop 2018

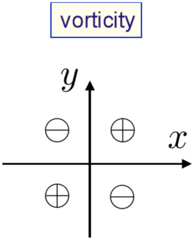


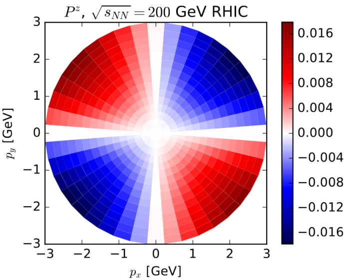
Similar $\sin(2\phi)$ structure is observed, with opposite sign!

[Iurii Karpenko, Lambda polarization from RHIC BES to LHC]

(c) Sergei Voloshin, SQM2017







Polarization depends on the the thermal vorticity:

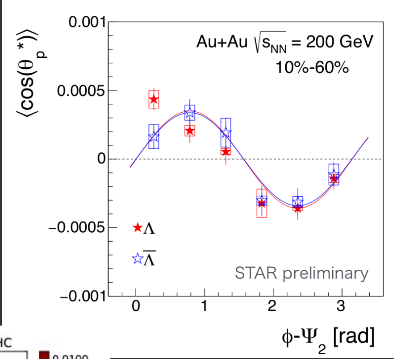
$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

[Iurii Karpenko, Lambda polarization from RHIC BES to LHC]

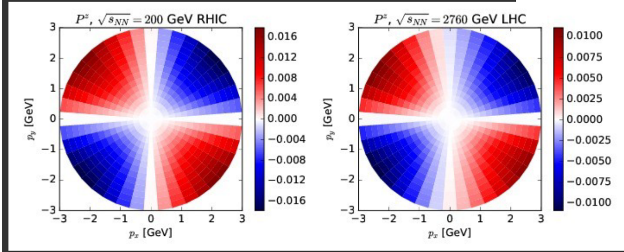
A *sign problem* for the longitudinal component

Quadrupolar structure of longitudinal polarization in the transverse momentum plane, as predicted.
Spectacular confirmation of hydro predictions... yet with a flipped sign!

- Hydro initial conditions? (polarization is a sensitive probe of the initial flow)
- Incomplete local thermodynamic equilibrium for the spin degrees of freedom (spin kinetic theory)?
- Effect of spin dissipative transport coefficients?
- Effect of initial state fluctuations?
- Effect of decays?
- Error in the calculation



Z. Ye, T. Niida, Quark Matter 2018



F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302
S. Voloshin, in SQM 2017

Same pattern found in AMPT+thermal vorticity calculation X. L. Xia, H. Li, Z. B. Tang and Q. Wang, 1803.00867

Relativistic spin-hydrodynamics

Angular momentum conservation: particles

- Angular momentum of a particle with momentum $\vec{p}$:

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

- One can obtain the dual tensor:

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

- We know that both definitions are equivalent.
- In absence of external torque, $\dfrac{d\vec{L}}{dt} = 0$, we also have: $\partial_i L_{ij} = 0$.
- Relativistic generalization: $L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ and $\partial_\mu L^{\mu\nu} = 0$.
- This treatment valid for point particles.
- For fluids, particle momenta $\rightarrow$ "generalized fluid momenta"

The energy-momentum tensor

Angular momentum conservation: fluid

- The orbital angular momentum for relativistic fluids is defined as

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

- Keeping in mind the energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$:

$$\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$$

- Obviously, for symmetric $T^{\mu\nu}$, orbital angular momentum is automatically conserved. Classically $T^{\mu\nu}$ symmetric.
- For medium constituent with intrinsic spin, different story

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- Ensure total angular momentum conservation: $\partial_\lambda J^{\lambda,\mu\nu} = 0$.
- Basis for formulation of spin Hydrodynamics.
  [Florkowski et. al., Prog.Part.Nucl.Phys. 108 (2019) 103709; Bhadury et. al., Eur.Phys.J.ST 230 (2021) 3, 655-672]

Pseudo-gauge transformations

- Total angular momentum is

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- With $\partial_\mu T^{\mu\nu} = 0$, and $\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$,

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \quad \Longrightarrow \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Hence the final hydrodynamic equations can be written as

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Also holds with the following redefinition

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} - \Phi^{\mu,\lambda\nu} - \Phi^{\nu,\lambda\mu}\right)$$

$$\tilde{S}^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu}$$

- Freedom due to space-time symmetry; including torsion fixes this.

  [Gallegos et. al., SciPost Phys. 11, 041 (2021); Hongo et. al., JHEP 11 (2021) 150]

Formulation within relativistic Kinetic Theory

Relativistic kinetic theory

- Kinetic theory: calculation of macroscopic quantities by means of statistical description in terms of distribution function.
- Let us consider a system of relativistic particles of rest mass m with momenta $\mathbf{p}$ and energy p^0

$$p^0 = \sqrt{\mathbf{p}^2 + m^2}$$

- For large no. of particles, $f(x,p)$ gives a distribution of the four-momenta $p = p^\mu = (p^0, \mathbf{p})$ at each space-time point.
- $f(x,p)\Delta^3 x\, \Delta^3 p$ gives average no. of particles in the volume element $\Delta^3 x$ at point x with momenta in the range $(\mathbf{p}, \mathbf{p} + \Delta\mathbf{p})$.
- Statistical assumptions:
  - No. of particles contained in $\Delta^3 x$ is large ($N \gg 1$).
  - $\Delta^3 x$ is small compared to macroscopic volume ($\Delta^3 x/V \ll 1$).
- The equilibrium distribution: $f_{eq}(x,p,s) = \left[\exp\left(\beta \cdot p - \xi\right) \pm 1\right]^{-1}$

Extended phase-space for spin degrees of freedom

- The phase-space for single particle distribution function gets extended $f(x, p, s)$.
- The equilibrium distribution for Fermions is given by

$$f_{eq}(x,p,s) = \frac{1}{\exp\left[\beta \cdot p - \xi - \frac{1}{2}\,\omega : s\right] + 1} \qquad \begin{cases} \beta \cdot p \equiv \beta_\mu p^\mu \\ \omega : s \equiv \omega_{\mu\nu} s^{\mu\nu} \end{cases}$$

- Quantities $\beta^\mu = u^\mu/T,\ \xi = \mu/T,\ \omega_{\mu\nu}$ are functions of x.
- $\xi,\ \beta^\mu,\ \omega^{\mu\nu}$: Lagrange multipliers for conserved quantities.
- $s^{\mu\nu}$: Particle spin, similar to particle momenta p^μ.
- Hydrodynamics: average over particle momenta and spin.
- Classical treatment of spin.

Bhadury et. al., PLB 814, 136096 (2021); PRD 103, 01430 (2021).

Conserved currents and spin-hydrodynamics

- Express hydrodynamic quantities in terms of $f(x,p,s)$.

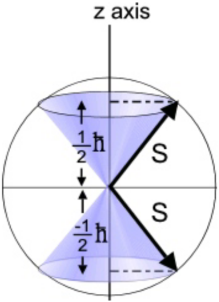
$$T^{\mu\nu}(x) = \int dPdS\ p^\mu p^\nu \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$N^{\mu}(x) = \int dPdS\ p^\mu \left[f(x,p,s) - \bar{f}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu}(x) = \int dPdS\ p^\lambda s^{\mu\nu} \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$dP \equiv \frac{d^3p}{E_p(2\pi)^3}, \quad dS \equiv m\frac{d^4s}{\pi\,\mathfrak{s}}\,\delta(s\cdot s + \mathfrak{s}^2)\,\delta(p\cdot s)$$

$$\int dS = 2; \quad \mathfrak{s}^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4}; \quad s^\mu \equiv \frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}p_\nu s_{\alpha\beta}$$



- Classical treatment of spin: internal angular momentum.
- Equations of motion: $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0.$
- Non-dissipative spin hydrodynamics: $f(x,p,s) = f_{eq}(x,p,s)$.
- Dissipative spin-hydrodynamics: Boltzmann equation for $f(x,p,s)$.

Dissipative spin-hydrodynamics Bhadury et. al., PLB 814, 136096 (2021)

- Introduce out-of-equilibrium distribution function $f(x, p, s)$.
- Use Boltzmann equation for evolution of $f = f_{eq} + \delta f$.

$$p^\mu \partial_\mu f = C[f]$$

- Employ relaxation-time approximation for collision kernel.

$$C[f] = -(u \cdot p) \frac{f - f_{eq}}{\tau_R}$$

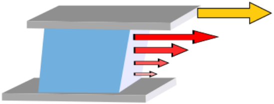
- Solve assuming small departure from equilibrium, $\delta f / f_{eq} \ll 1$.
- First order dissipative spin hydrodynamics for $\delta f = \delta f_1$.

$$\delta f = \frac{\tau_R}{u \cdot p} e^{-p\cdot\beta} \left[p^\mu p^\nu \partial_\mu \beta_\nu \left(1 + \frac{1}{2} s^{\alpha\beta} \omega_{\alpha\beta} \right) - \frac{1}{2} p^\mu s^{\alpha\beta} (\partial_\mu \omega_{\alpha\beta}) \right]$$

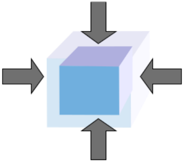
- Relativistic Navier-Stokes analog of spin-hydrodynamics.

Dissipative effects

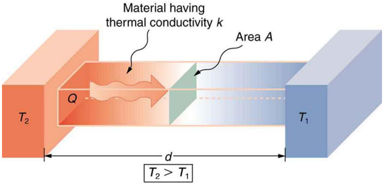
- Shear viscosity: fluid's resistance to shear forces



- Bulk viscosity: fluid's resistance to compression



- Charge/heat conductivity: fluid's resistance to flow of charge/heat.



- Dissipation to spin current new:

Bhadury et. al., PLB 814, 136096 (2021); PRD 103, 01430 (2021).

Calculation of spin-polarization

Pauli-Lubanski and Polarization

- On freeze-out hypersurface: $\langle P(\phi_p)\rangle = \dfrac{\int p_T dp_T E_p \frac{d\Pi^z(p)}{d^3p}}{\int d\phi_p p_T dp_T E_p \frac{dN(p)}{d^3p}}$
- $E_p \dfrac{dN(p)}{d^3p} = \dfrac{4\cosh\xi}{(2\pi)^3}\displaystyle\int \Delta\Sigma_\lambda p^\lambda e^{-\beta.p}, \qquad \xi = \mu/T,\ \beta^\mu = u^\mu/T$
- $E_p \dfrac{d\Delta\Pi_\tau(x,p)}{d^3p} = -\dfrac{1}{2}\epsilon_{\tau\mu\nu\beta}\,\Delta\Sigma_\lambda E_p \dfrac{dS^{\lambda,\mu\nu}(\omega)}{d^3p}\dfrac{p^\beta}{m}$

  [Florkowski et. al., Prog.Part.Nucl.Phys. 108 (2019) 103709]
- In absence of hydrodynamic evolution, we use the ansatz: $\omega \to \varpi$.
- Non-relativistic approx: $\varpi_{0i} = 0$. Necessary to cure sign-problem.

Thermal Vorticity

$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$

Thermal vorticity and Thermal Shear

Thermal vorticity is given by

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

which can be written as:

$$\varpi_{\mu\nu} = -\frac{1}{2T}(\partial_\mu u_\nu - \partial_\nu u_\mu) + \frac{1}{2T^2}(u_\nu \partial_\mu T - u_\mu \partial_\nu T)$$

The thermal Shear is defined as:

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

Which can be written as:

$$\xi_{\mu\nu} = \underbrace{\frac{1}{2T}(\partial_\mu u_\nu + \partial_\nu u_\mu)}_{\xi^{I}_{\mu\nu}} \underbrace{- \frac{1}{2T^2}(u_\nu \partial_\mu T + u_\mu \partial_\nu T)}_{\xi^{II}_{\mu\nu}}$$

$\partial^\alpha T = T(D\, u^\alpha - c_s^2 u^\alpha \partial_\mu u^\mu)$

[S. Banerjee et. al., arXiv:2405.05089]

Thermal Model

- Single freeze-out model.
- $\tau_f^2 = t^2 - x^2 - y^2 - z^2$
  with $x^2 + y^2 \leq r_{max}^2$.
- Asymmetry of the fireball boundary $x = r_{\rm max}\sqrt{1-\epsilon}\,\cos\phi$
  $y = r_{\rm max}\sqrt{1+\epsilon}\,\sin\phi$
- Asymmetry of the hydrodynamic flow.
  $u^\mu = \frac{1}{N}(\,t,\, x\sqrt{1+\delta}\,,\, y\sqrt{1-\delta}\,, z\,)$
  $N = \sqrt{\tau^2 - (x^2 - y^2)\delta}$

[Baran, Broniowski, Florkowski, Acta Phys.Polon.B 35 (2004) 779-798]

Parametrizing the Flow and Hypersurface

c %	ϵ	δ	τ_f [fm]	r_{max} [fm]
0-15	0.055	0.12	7.666	6.540
15-30	0.097	0.26	6.258	5.417
30-60	0.137	0.37	4.266	3.779

Freeze-out Temperature is $T_f = 165\text{MeV}$.

[Baran, Broniowski, Florkowski]

Polarization for 30-60% Centrality

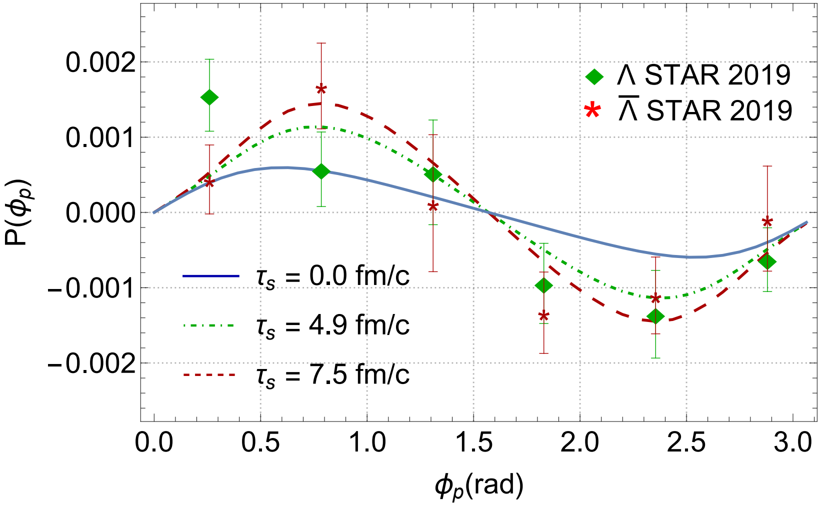


Figure: $\tau_s = 7.5$ fm for $\overline{\Lambda}$ hyperons ($\chi^2_{\mathrm{r}} = 0.6$) and with $\tau_s = 4.9$ fm for Λ hyperons ($\chi^2_{\mathrm{r}} = 1.5$). Data from STAR, 2019. [S. Banerjee et. al., arXiv:2405.05089]

Polarization for different centralities

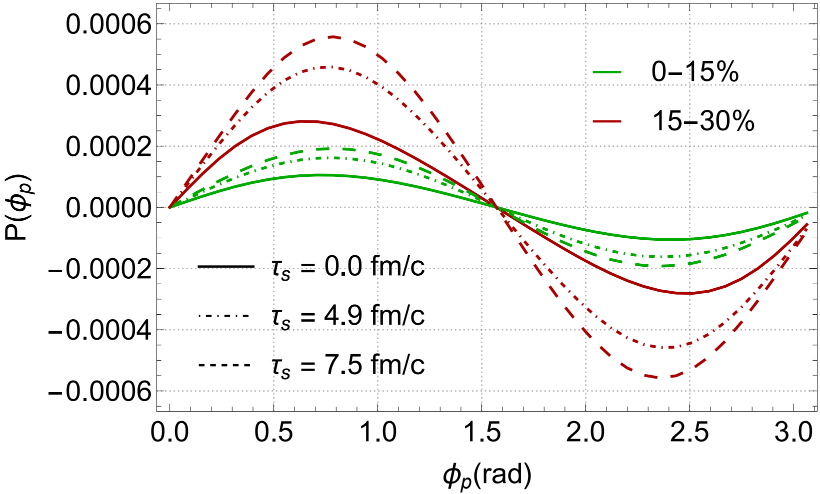


Figure: Predictions for Λ and $\bar{\Lambda}$ polarization.

[S. Banerjee et. al., arXiv:2405.05089]

Summary

- Dissipative terms does play a significant role in polarization.
- We have extracted the spin relaxation time.
- Spin relaxation time is large signifying non-equilibration of spin degrees of freedom.
- Thermal vorticity ansatz for polarization tensor: not good.
- Necessary to perform evolution within spin-hydrodynamics.
- Sign problem in longitudinal polarization needs concrete solution.
- Polarization and spin hydrodynamics: exciting new developments.
- Much work needed in this direction.

Thank you!