Exclusive production of χ_c charmonia at the EIC: can we observe Odderon?

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NATIONAL SCIENCE CENTRE POLAND
Project is supported by National Science Centre, grant no. 2021/43/D/ST2/03375.

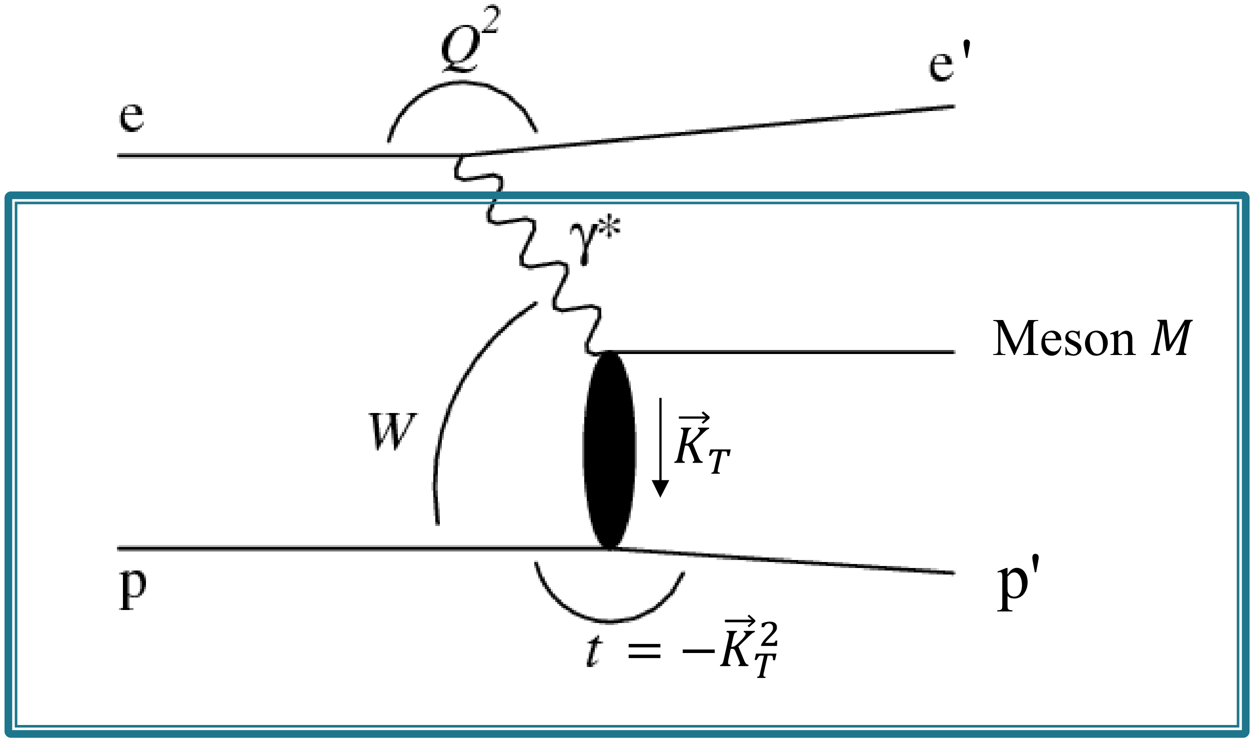
with Sanjin Benić, Adrian Dumitru, Leszek Motyka

Based on:
Phys.Rev.D 110 (2024) 1, 014025 and *2407.04968*

Białasówka
25.10.2024

Exclusive meson electroproduction in ep scattering

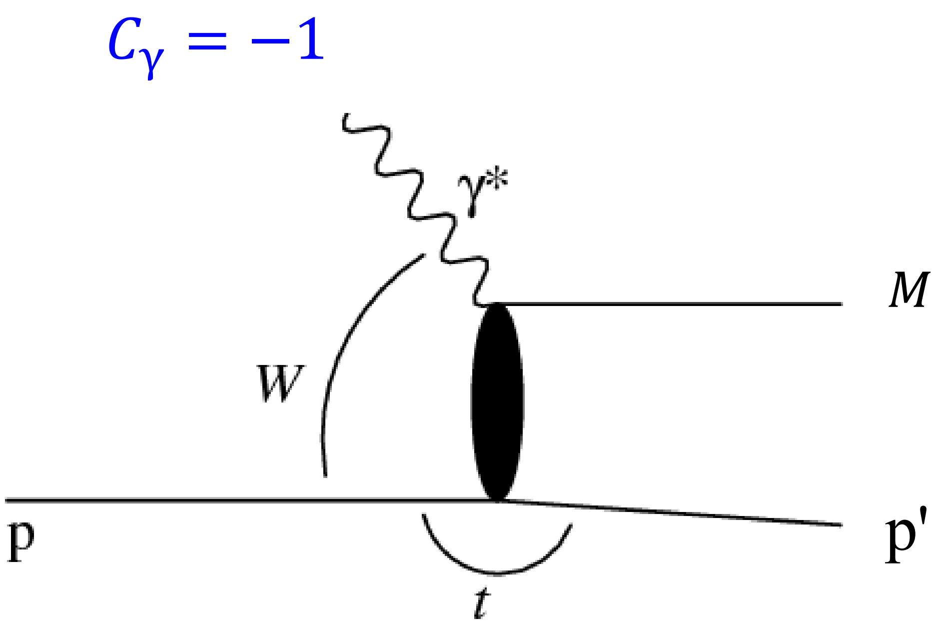
$$e + p \rightarrow e' + p' + M$$



$$\gamma^* + p \rightarrow p' + M$$

Exclusive meson production in $\gamma^* - p$ scattering

Consider C−parity of particles:



Meson with given C−parity:

$C_M = -1$, eg. $\rho, \phi, J/\psi \ldots$
(pomeron exchange)

or

$C_M = +1$, eg. $\pi^0, \eta_c, \chi_c \ldots$
(odderon exchange)

For small W other exchanges also possible.

L. Lukaszuk and B. Nicolescu, Lett. Nuovo Cim. 8, 405 (1973) J. Bartels, Nucl. Phys. B 175, 365 (1980) J. Kwiecinski and M. Praszalowicz, Phys. Lett. 94B, 413 (1980)

R. A. Janik and J. Wosiek, Phys. Rev. Lett. 82, 1092 (1999) J. Bartels, L. N. Lipatov and G. P. Vacca, Phys. Lett. B 477, 178 (2000)

Pomeron and Odderon

Regge theory: $\sigma_{tot} \sim s^{\alpha(0)-1}$

Intercept of Regge trajectory:
$\alpha(t) = \alpha(0) + \alpha' t$

POMERON

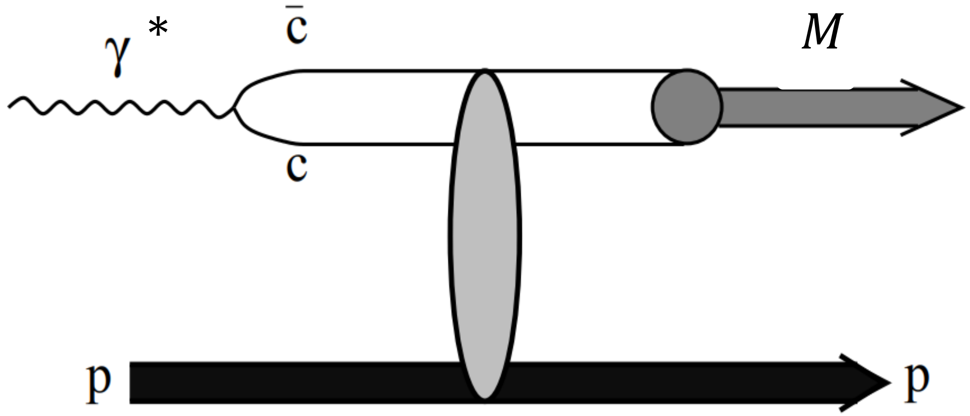
- $C = 1,\ P = 1$
- $\alpha_P(0) > 1$
- Leading logarithms $\alpha_S^n (\log s)^n$ resummed by Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation
- Observed experimentally in many processes

ODDERON

- $C = -1,\ P = -1$
- $\alpha_O(0) \lesssim 1$
- Leading logarithms $\alpha_S^n (\log s)^n$ resummed by Bartels–Kwieciński–Praszałowicz (BKP) equation
- Evidence for existence (comparison pp vs. $p\bar{p}$) by TOTEM, G. Antchev et al. Eur.Phys.J.C 80 (2020) 2, 91)

Charmonia

- Charmonia = $c\bar{c}$ states



1) charm mass is a hard scale: $m_c \sim 1.3\ \mathrm{GeV} \gg \Lambda_{QCD}$

2) relatively easy to measure

In this talk we consider :

χ_{cJ}: $J = 0$ scalar, $J = 1$ axial, $J = 2$ tensor
P–wave meson with $C_{\eta_c} = +1$

Odderon–type exchange

Why χ_c?

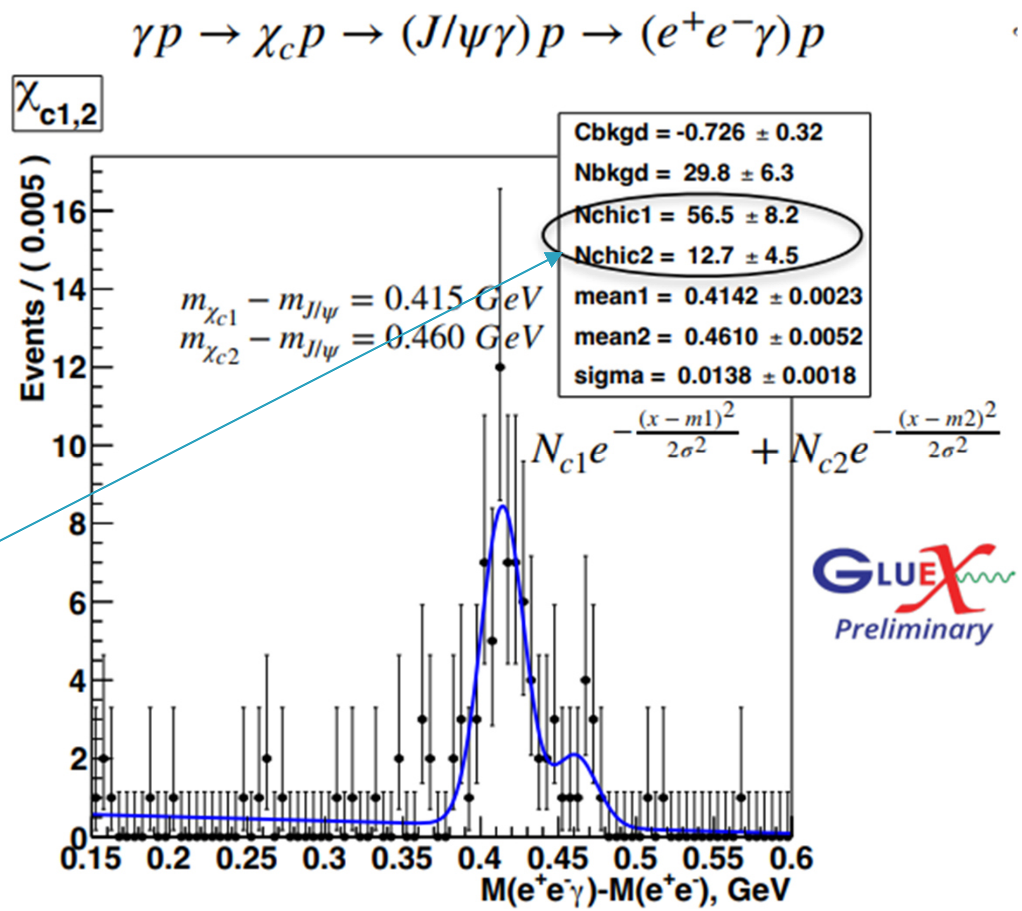
From late 90's theorists focus on exclusive η_c production as a evidence of odderon exchange.

J. Czyzewski, J. Kwiecinski, L. Motyka, and M. Sadzikowski, Phys. Lett. B 398, 400 (1997),
R. Engel, D. Y. Ivanov, R. Kirschner, and L. Szymanowski, Eur. Phys. J. C 4, 93 (1998)

At HERA no evidence for exclusive η_c production was found. It is hard to measure:

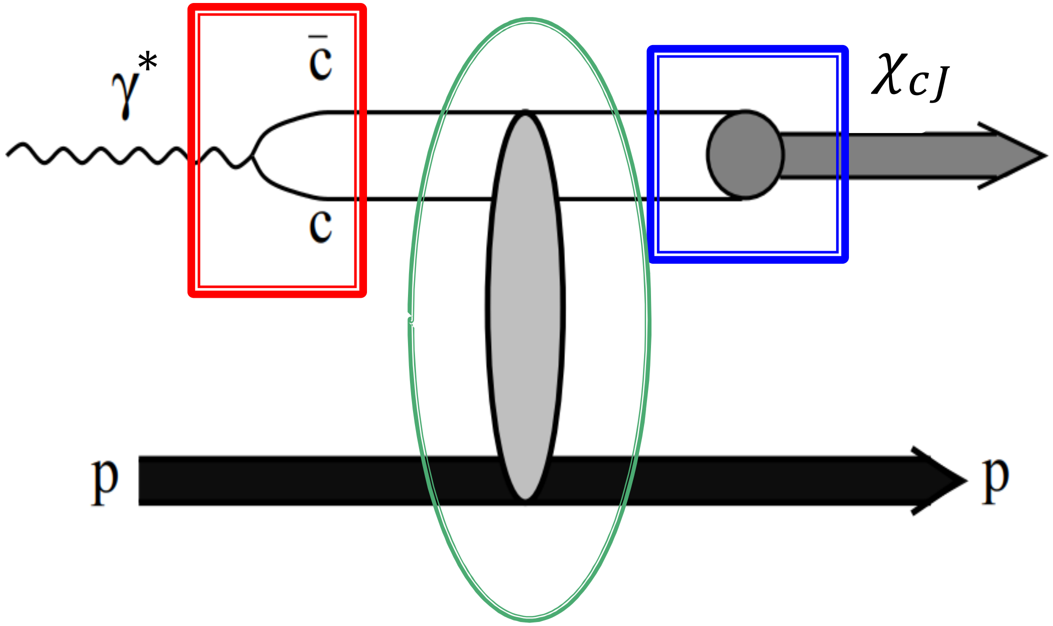
small branching ratio $\text{Br}(\eta_c \to \gamma\gamma) \sim 10^{-4}$ &
feedown from J/ψ.

Last year results from GlueX



L. Pentchev et al. (GlueX), Exclusive threshold J/ψ photoproduction with GlueX (2023), presented at DIS2023.

Amplitude for $\gamma^* + p \to p' + \chi_{cJ}$



Odderon exchange

$$\langle \mathcal{M}_{\lambda\bar{\lambda}}(\gamma^* p \to \mathcal{H} p)\rangle = 2q^- N_c \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp}\, \mathrm{i}\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\, \mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp)$$

$$\mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp) = \int_z \int_{\boldsymbol{l}_\perp \boldsymbol{l}'_\perp} \sum_{h\bar{h}} \Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{l}_\perp, z)\, \Psi^{\mathcal{H}*}_{\bar{\lambda},h\bar{h}}(\boldsymbol{l}'_\perp - z\boldsymbol{\Delta}_\perp, z)\, \mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp + \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \boldsymbol{r}_\perp}$$

Photon wave function

Meson wave function

Wave functions

Photon wave function:

$$\Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{k}_\perp, z) \equiv \sqrt{z\bar{z}}\,\frac{\bar{u}_h(k) e q_c \not{\epsilon}(\lambda, q) v_{\bar{h}}(q-k)}{\boldsymbol{k}_\perp^2 + \varepsilon^2}$$

Meson wave function (covariant):

$$\Psi^{\mathcal{H}}_{\bar{\lambda},h\bar{h}}(\boldsymbol{k}_\perp, z) \equiv \frac{1}{\sqrt{z\bar{z}}}\bar{u}_h(k)\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k')v_{\bar{h}}(k')\phi_{\mathcal{H}}(\boldsymbol{k}_\perp, z)$$

where:

$$\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k') = \begin{cases} 1\ , & \mathcal{H} = \mathcal{S}\,, \\ \mathrm{i}\gamma_5 \not{E}(\bar{\lambda}, \Delta_0)\ , & \mathcal{H} = \mathcal{A}\,, \\ \frac{1}{2}\left(\gamma_\mu(k_\nu - k'_\nu) + \gamma_\nu(k_\mu - k'_\mu)\right) E^{\mu\nu}(\bar{\lambda}, \Delta_0)\ , & \mathcal{H} = \mathcal{T}\,. \end{cases}$$

Final results for amplitudes (scalar quarkonium χ_{c0}):

$$\mathcal{A}_L(r_\perp) \equiv -\frac{2}{\pi} m_c Q(z-\bar{z}) K_0(\varepsilon r_\perp)\phi_{\mathcal{S}}(r_\perp, z)\,,$$

$$\mathcal{A}_T(r_\perp) \equiv \frac{\mathrm{i}\sqrt{2}}{2\pi}\frac{m_c}{z\bar{z}}\left[(z-\bar{z})^2 \varepsilon K_1(\varepsilon r_\perp)\phi_{\mathcal{S}}(r_\perp, z) - K_0(\varepsilon r_\perp)\frac{\partial \phi_{\mathcal{S}}}{\partial r_\perp}\right]$$

Similar expressions for axial χ_{c1} and tensor χ_{c2}

Boosted Gaussian

- Scalar part of meson w.f. needs to be modeled. We use:

$$\phi_{\mathcal{H},B}(r_\perp, z) = \mathcal{N}_{\mathcal{H},B}\, z\bar{z} \exp\left(-\frac{m_c^2 \mathcal{R}_\mathcal{H}^2}{8z\bar{z}} - \frac{2z\bar{z}r_\perp^2}{\mathcal{R}_\mathcal{H}^2} + \frac{1}{2} m_c^2 \mathcal{R}_\mathcal{H}^2 \right)$$

Free parameters, $\mathcal{R}_H$ and $\mathcal{N}_{\mathcal{H},B}$ we obtained from:

1) normalization condition:

$$1 = N_c \sum_{h\bar{h}} \int_z \int_{\boldsymbol{r}_\perp} \left| \Psi^{\mathcal{H}}_{\lambda, h\bar{h}}(\boldsymbol{r}_\perp, z) \right|^2$$

2) decay into $\gamma\gamma$:

$$\Gamma(\mathcal{S} \to \gamma\gamma) = \frac{\pi \alpha^2}{4} M_\mathcal{S}^3 F_\mathcal{S}^2 \,,$$

$$F_\mathcal{S} \equiv 4 q_c^2 m_c N_c \int_z \int_{\boldsymbol{k}_\perp} \frac{\boldsymbol{k}_\perp^2 + (z-\bar{z})^2 m_c^2}{(\boldsymbol{k}_\perp^2 + m_c^2)^2} \frac{\phi_\mathcal{S}(\boldsymbol{k}_\perp, z)}{z\bar{z}}$$

Exception: χ_{c1}

Landau-Yung theorem: massive particle with spin 1 cannot decay into two real photons.

We assume $\mathcal{R}_{\chi_{c1}} = \mathcal{R}_{\chi_{c2}}$

Dipole distribution evolution

Dipole distribution in terms of Wilson lines:

$$\mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \equiv \frac{1}{N_c} \mathrm{tr} \left\langle V^\dagger \left(\boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2} \right) V \left(\boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2} \right) \right\rangle$$

Small-x evolution is given by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial \mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{D}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}) \mathcal{D}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}) - \mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \right]$$

We also have decomposition: $\mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = 1 - \mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) + i\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$

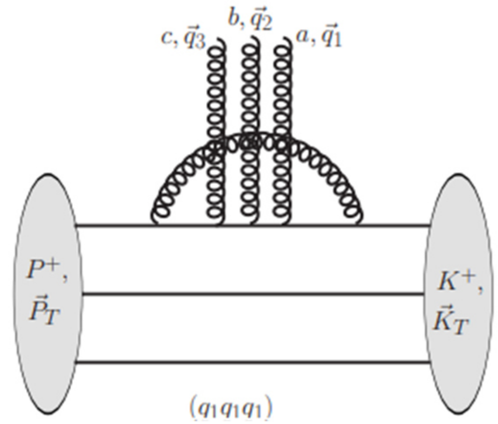
Pomeron Odderon

$$\frac{\partial \mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \int_{\boldsymbol{r}_{1\perp}} \mathcal{K}_{\mathrm{Bal}}(\boldsymbol{r}_\perp, \boldsymbol{r}_{1\perp}, \boldsymbol{r}_{2\perp}) \big[\mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) \mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) \mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) \big]$$

$$\frac{\partial \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \int_{\boldsymbol{r}_{1\perp}} \mathcal{K}_{\mathrm{Bal}}(\boldsymbol{r}_\perp, \boldsymbol{r}_{1\perp}, \boldsymbol{r}_{2\perp}) \big[\mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) - \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) \mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) \mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) \big]$$

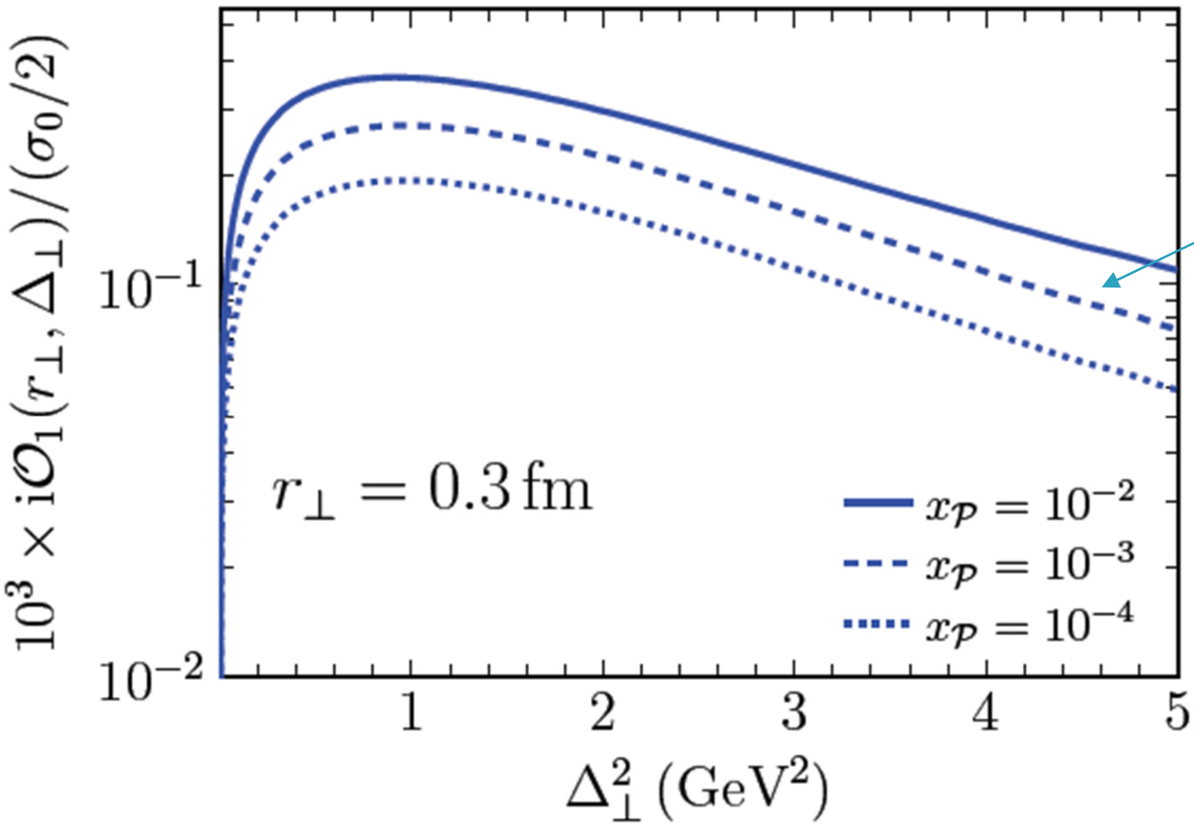
Benić, Horvatić, Kaushik, Vivoda, Phys. Rev. D 108, 074005 (2023),

Odderon evolution



Dumitru, H. Mantysaari, and R. Paatelainen, Phys. Rev. D 107, L011501 (2023),

At x=0.01 we use initial condition from 3-gluon exchange model & gluon emission.

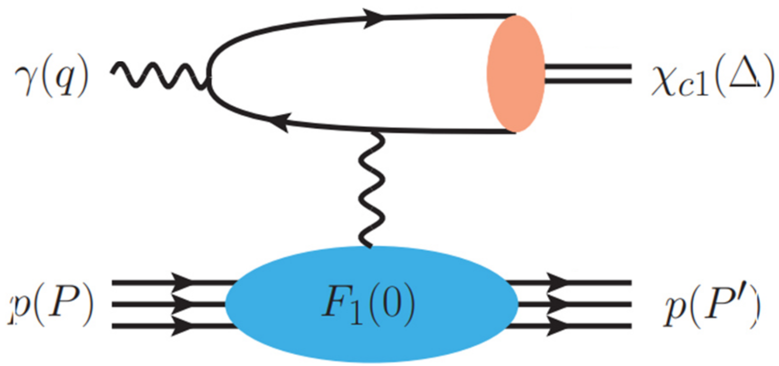


Decrease with energy

Backgroud

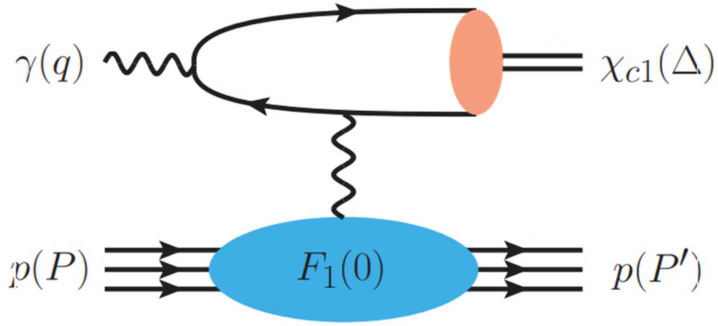
- We have two main sources of background for this process:

  - Primakoff process
    (photon exchange)



  - Feeddown from $\psi(2s) \rightarrow \chi_{cJ} + \gamma$ (this is not covered in this talk).

Primakoff process



$$\mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp) = \int_z \int_{\boldsymbol{l}_\perp \boldsymbol{l}'_\perp} \sum_{h\bar{h}} \Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{l}_\perp, z) \Psi^{\mathcal{H}*}_{\bar{\lambda},h\bar{h}}(\boldsymbol{l}'_\perp - z\boldsymbol{\Delta}_\perp, z) \mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp + \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \boldsymbol{r}_\perp}$$

$$\langle \mathcal{M}_{\lambda\bar{\lambda}}(\gamma^* p \to \mathcal{H}p)\rangle = 2q^- N_c \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \mathrm{i}\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp)$$

Just replace:

Dirac form factor.

$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp) \to 8\pi \mathrm{i} q_c \alpha \sin\left(\frac{\boldsymbol{\Delta}_\perp \cdot \boldsymbol{r}_\perp}{2}\right) \frac{F_1(\Delta_\perp)}{\boldsymbol{\Delta}_\perp^2}$$

Adding Pauli form factor:

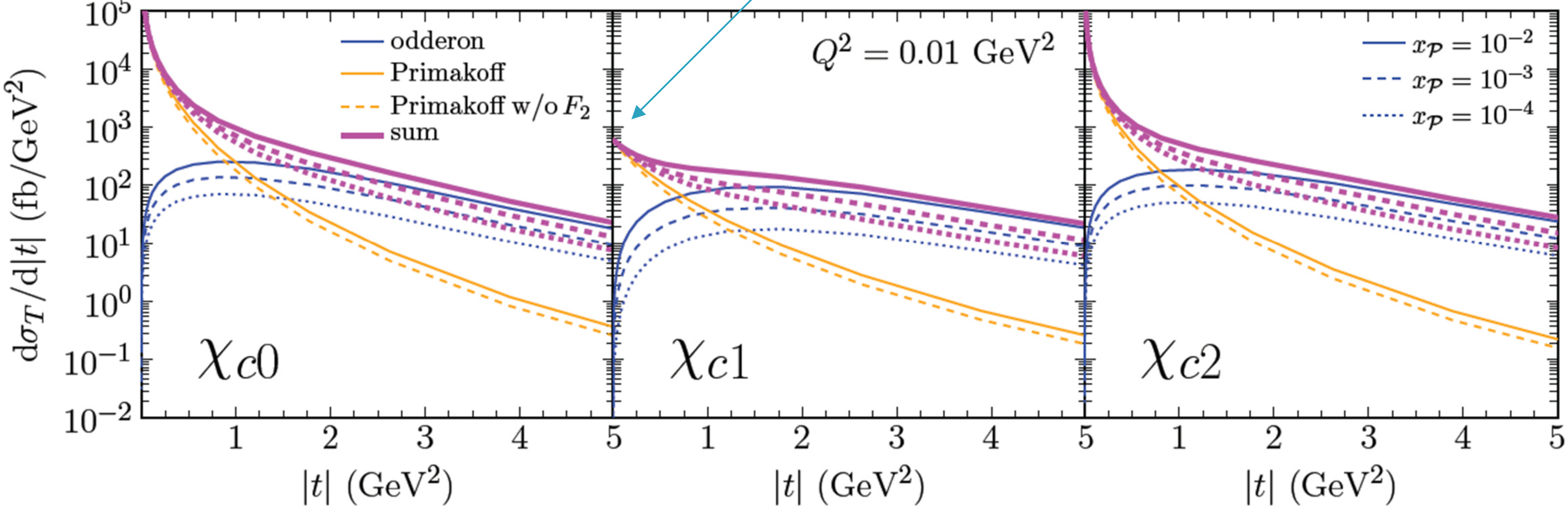
$$F_1^2(\ell_\perp) \to F_1^2(\ell_\perp) + \frac{\boldsymbol{\ell}_\perp^2}{4m_N^2} F_2^2(\ell_\perp)$$

Both form factors are very well constrained experimentally.

Numerical results for EIC

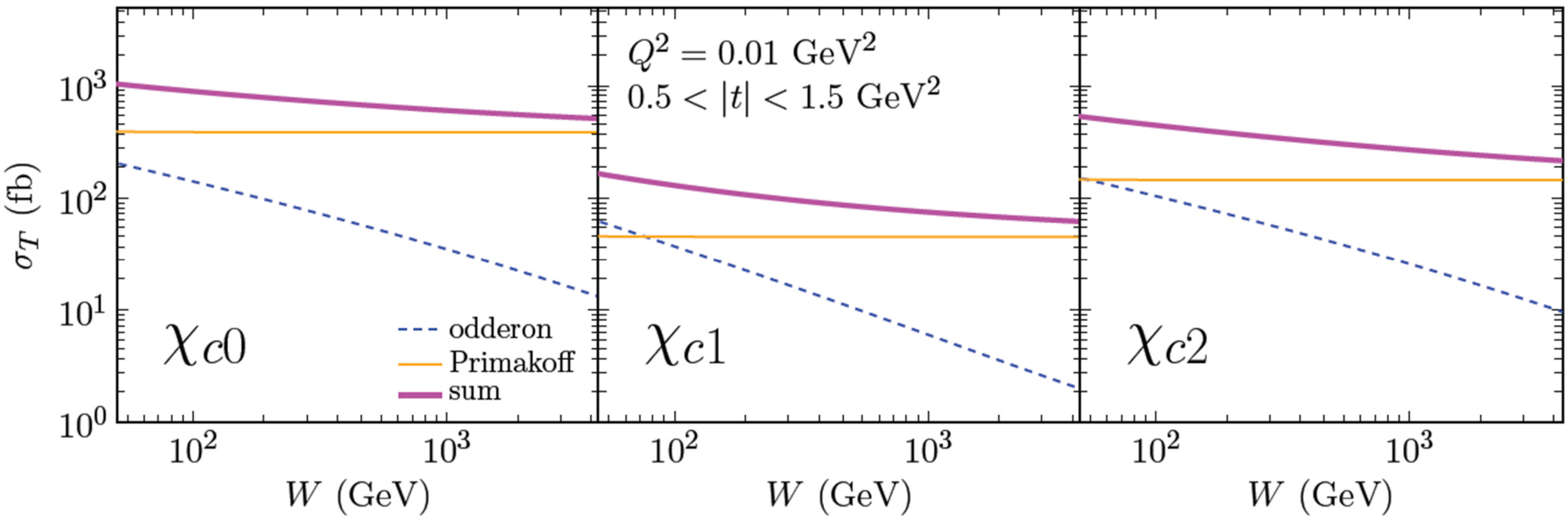
$\gamma^* + p \to p' + \chi_{cJ}$: momentum transfer dependence

Finite Primakoff
(Landau-Yung theorem)



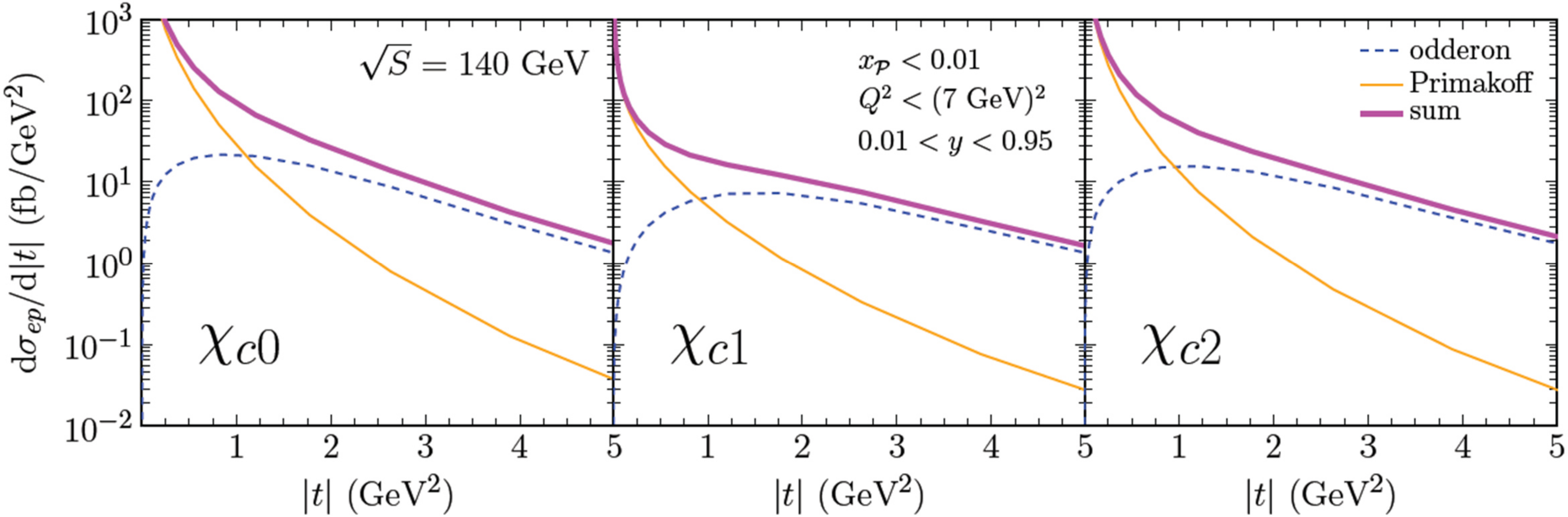
Positive interference between odderon and Primakoff.

$\gamma^* + p \to p' + \chi_{cJ}$: energy dependence



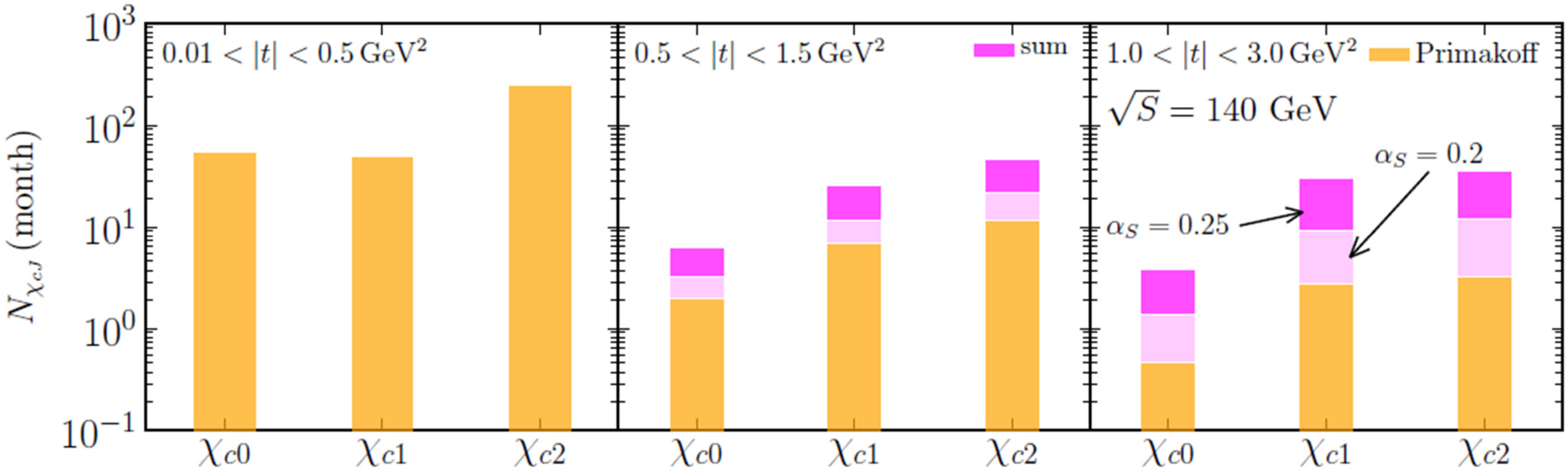
Electroproduction $ep \to \chi_{cJ} ep$

$$\frac{\mathrm{d}\sigma_{ep}}{\mathrm{d}x_{\mathcal{P}}\mathrm{d}Q^2\mathrm{d}|t|} = \frac{\alpha}{2\pi Q^2 x_{\mathcal{P}}}\left\{2(1-y)\frac{\mathrm{d}\sigma_L}{\mathrm{d}|t|} + \left(1+(1-y)^2 - 2(1-y)\frac{Q^2_{\mathrm{min}}}{Q^2}\right)\frac{\mathrm{d}\sigma_T}{\mathrm{d}|t|}\right\}$$



Electroproduction: number of events

number of exclusive $\chi_{cJ} \to J/\psi\gamma \to l^+l^-\gamma$ events



Branching ratios for decays included.
Top EIC luminosity is assumed.

Odderon and TMD

Odderon and TMD

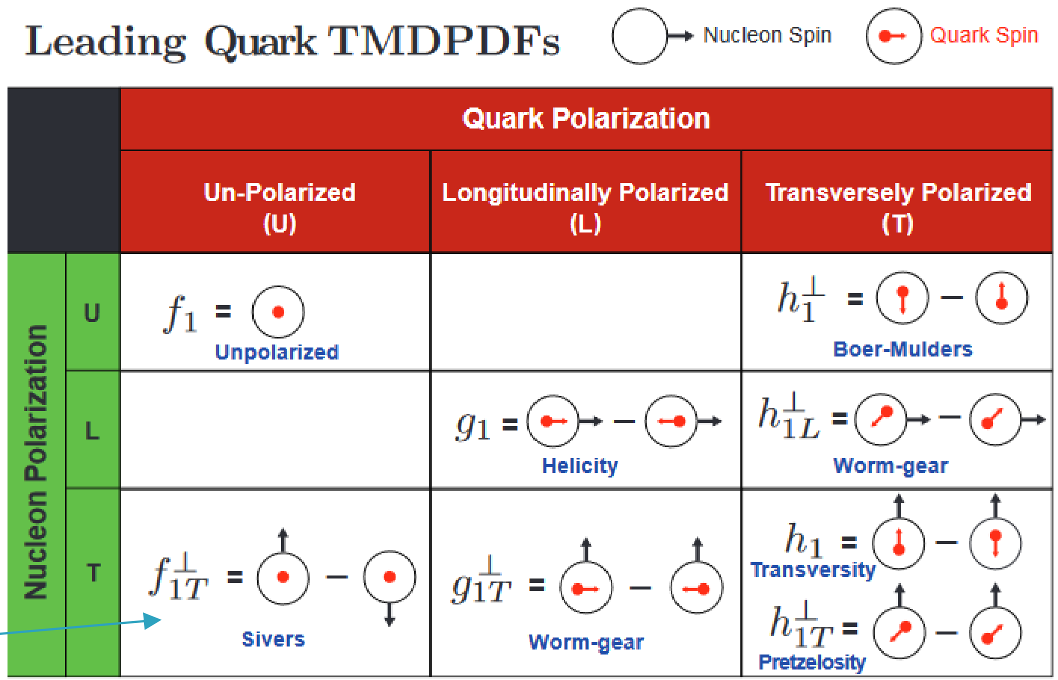
$$\mathcal{O}_{\lambda_p \lambda_p'}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \int \mathrm{d}^2\boldsymbol{r}_\perp \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \times \frac{1}{N_c} \frac{\mathrm{Im}\left\{\langle P'\lambda_p' | \mathrm{tr}\left[V^\dagger(\boldsymbol{x}_\perp)V(\boldsymbol{y}_\perp)\right] | P\lambda_p\rangle\right\}}{\langle P\lambda_p | P\lambda_p\rangle}$$

Protons' helicities

Let consider forward limit: $\Delta_\perp \rightarrow 0$ (zero momentum transfer), then:

$$\mathcal{O}_{\lambda_p \lambda_p'}(\boldsymbol{k}_\perp, 0) = \lambda_p \delta_{\lambda_p, -\lambda_p'} (\boldsymbol{k}_\perp \times \boldsymbol{\epsilon}_\perp^{\lambda_p}) \times \frac{(2\pi)^3 \mathrm{i} g^2}{2\sqrt{2}N_c} \frac{x f_{1T}^{\perp g}(x, \boldsymbol{k}_\perp)}{\boldsymbol{k}_\perp^2 M_p},$$

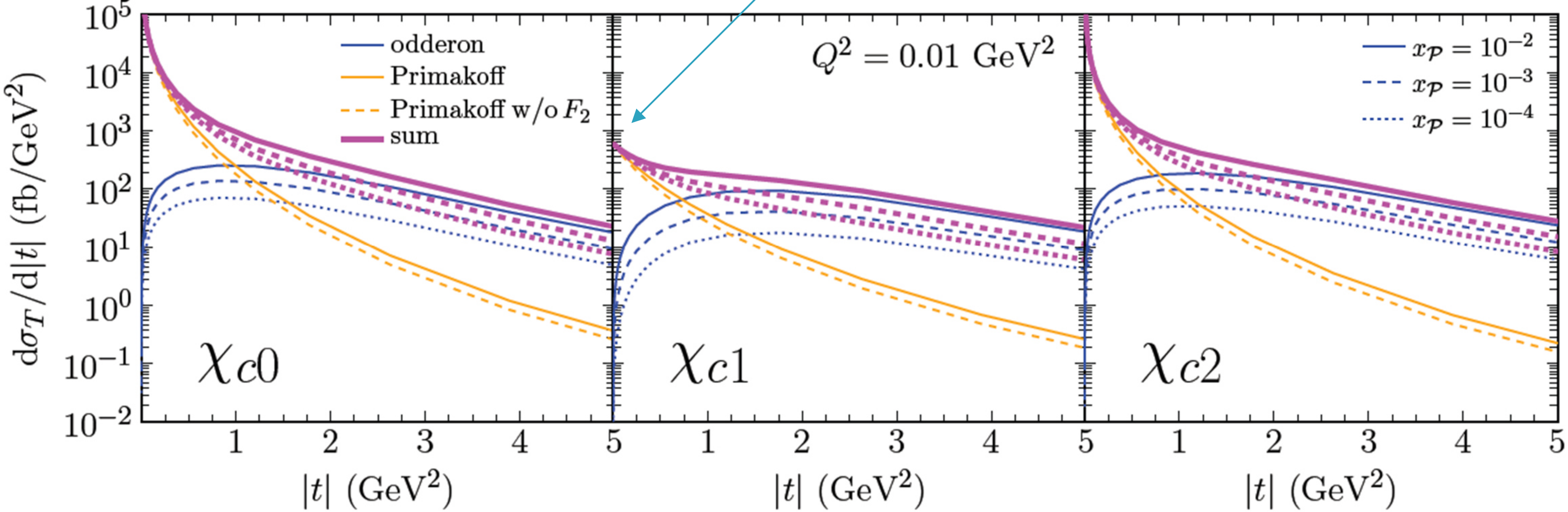
Gluon Sivers function



Measuring TMDs is one of the main objective of the EIC.

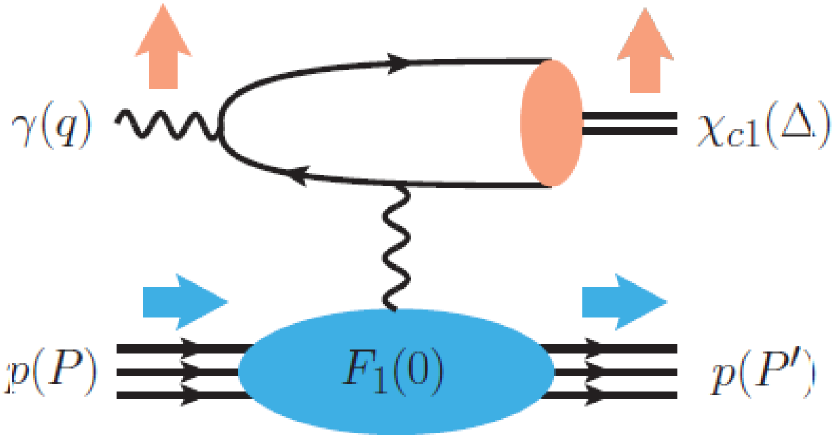
$\gamma^* + p \to p' + \chi_{cJ}$: momentum transfer dependence

Finite Primakoff
(Landau-Yung theorem)



χ_{c1} from Primakoff and Sivers: $Q^2 \to 0$ and $t \to 0$ limit

Primakoff:

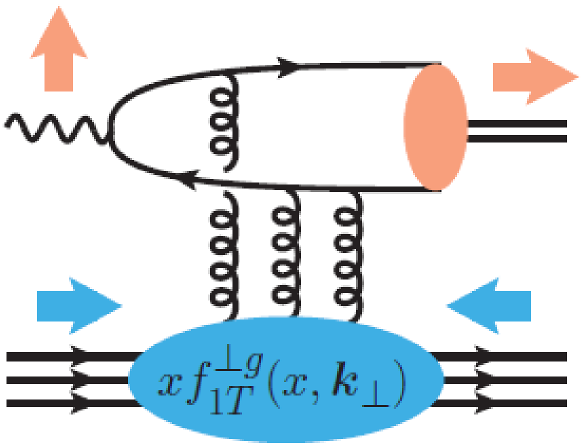


Nonrelativistic (NR) results:

$$\lim_{t\to 0} \frac{d\sigma_{\rm Prim}}{d|t|} = \frac{3\pi q_c^4 \alpha^3 N_c |R'(0)|^2 |F_1(0)|^2}{m_c^9}$$

Derivative of NR wave function

Sivers:



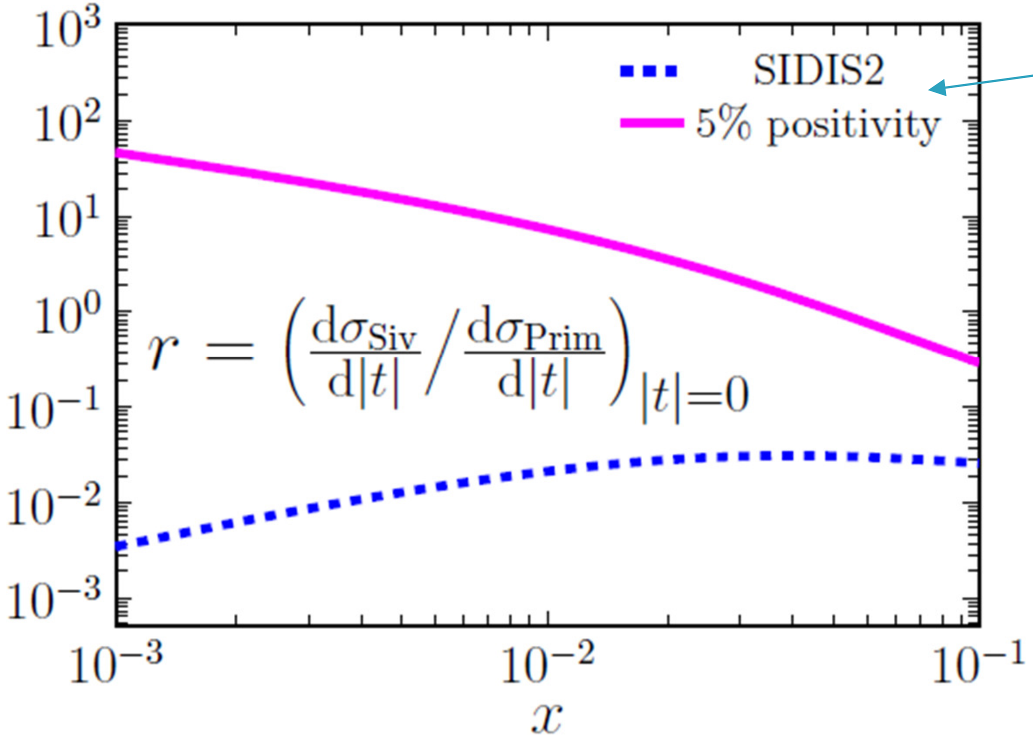
$$\lim_{t\to 0} \frac{d\sigma_{\rm Siv}}{d|t|} = \frac{3\pi^3 q_c^2 \alpha \alpha_S^2 M_p^2 |R'(0)|^2 |x f_{1T}^{\perp(1)g}(x)|^2}{N_c m_c^{11}}$$

$$x f_{1T}^{\perp(1)g}(x, \mu^2) \equiv \int^{\mu^2} d^2 k_\perp \frac{k_\perp^2}{2M_p^2} x f_{1T}^{\perp g}(x, k_\perp),$$

First moment of Sivers gluon function

Ratio Sivers/Primakoff

$$r \equiv \left(\frac{\mathrm{d}\sigma_{\mathrm{Siv}}}{\mathrm{d}|t|} \Big/ \frac{\mathrm{d}\sigma_{\mathrm{Prim}}}{\mathrm{d}|t|} \right)_{t=0} \qquad \to \frac{4\pi^2}{q_c^2 N_c^2} \frac{\alpha_S^2}{\alpha^2} \frac{M_p^2}{M_\chi^2} |x f_{1T}^{\perp(1)g}(x)|^2$$



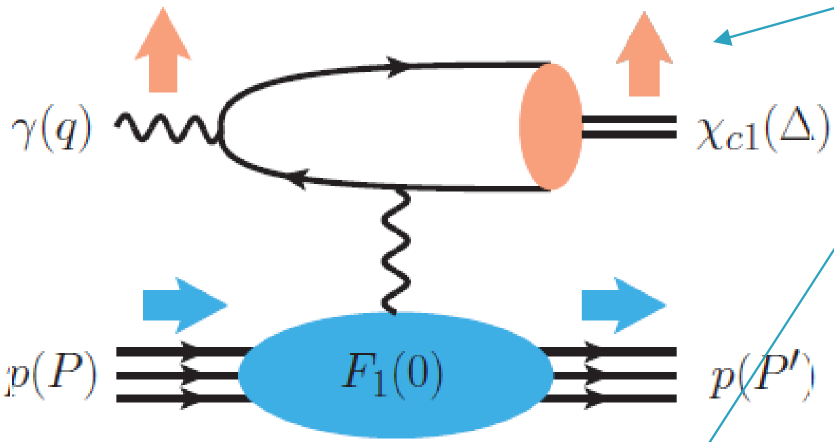
some models of Sivers TMD available in the literature

Remark:
Sivers function is very poorly constrained.

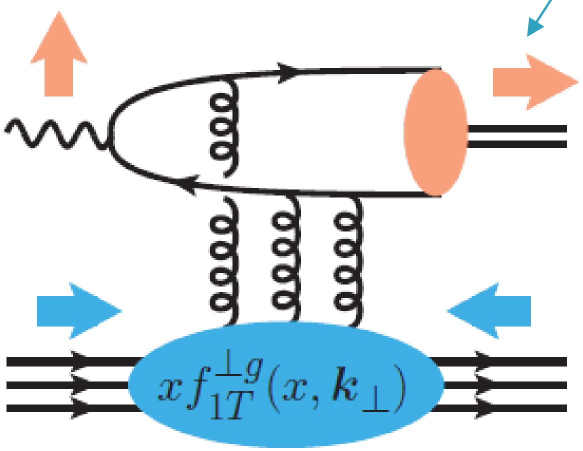
Exclusive χ_{c1} production can be used to constrain it.

χ_{c1} polarization

Primakoff:



Sivers:



Different polarization of χ_{c1}

Consider decay $\chi_{c1} \to J/\psi + \gamma$

then the angular distribution of J/ψ:

$$W(\cos\vartheta, \varphi) \propto \frac{1}{(3+\lambda_\vartheta)}(1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi)$$

When $t \to 0$:

$$\lambda_\theta = \frac{2r-1}{2r+3} \qquad r \equiv \left(\frac{\mathrm{d}\sigma_{\mathrm{Siv}}}{\mathrm{d}|t|} \Big/ \frac{\mathrm{d}\sigma_{\mathrm{Prim}}}{\mathrm{d}|t|} \right)_{t=0}$$

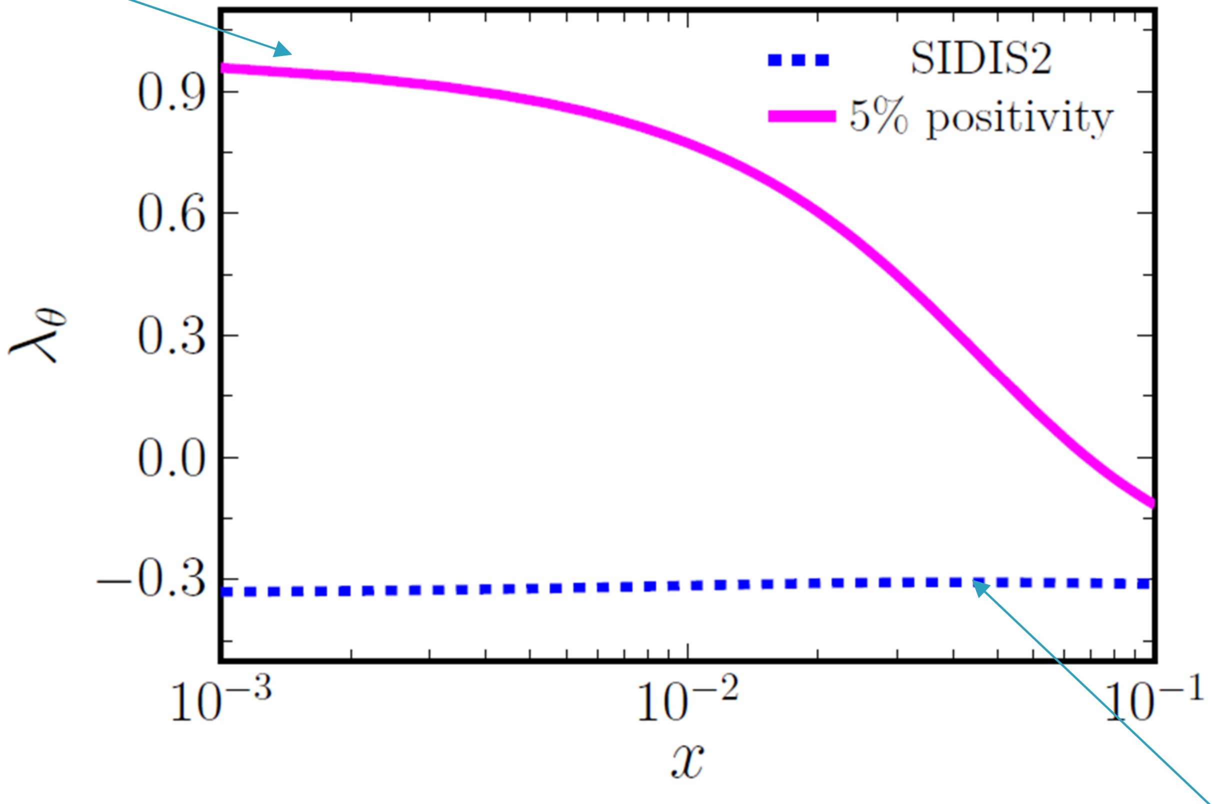
and $\lambda_\varphi = \lambda_{\theta\varphi} = 0$

χ_{c1} polarization

$$\lambda_\theta = \frac{2r-1}{2r+3}$$

$$r \equiv \left(\frac{d\sigma_{\mathrm{Siv}}}{d|t|} \Big/ \frac{d\sigma_{\mathrm{Prim}}}{d|t|} \right)_{t=0}$$

Sivers dominance



Primakoff dominance

Summary

- We calculated exclusive production of χ_c.
- Finding odderon may be possible at EIC.
- χ_{c1} production at $t = 0$ can be used to constrain the Sivers TMD.

Thank you!