Entropy production and dissipation in spin hydrodynamics

A. Daher (IFJ PAN)

Collaborators: F. Becattini, X.L. Sheng, D.Wagner (INFN Florence)

References: *Physics Letters* B850 (2024) 138533 + work in progress

NARODOWE CENTRUM NAUKI

THE HENRYK NIEWODNICZAŃSKI INSTITUTE OF NUCLEAR PHYSICS POLISH ACADEMY OF SCIENCES

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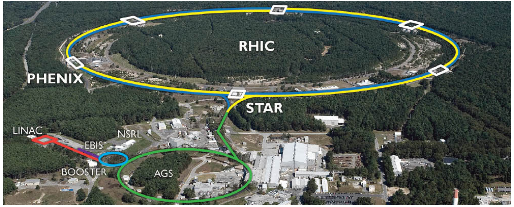
Outline

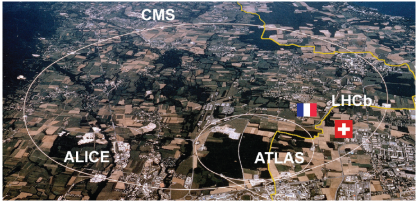
1. **Probes of Quark-Gluon Plasma**
2. **Spin polarization in heavy-ion collisions**
3. **What is spin hydrodynamics?**
4. **Quantum-statistical formulation and entropy production rate**
5. **Dissipative currents: Method and results (Ongoing work)**
6. **Conclusions and outlook**

Probes of Quark-Gluon Plasma

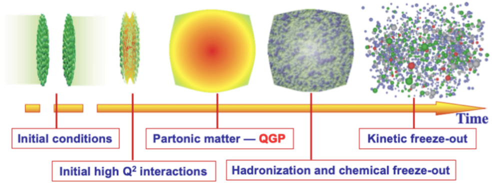
Relativistic heavy ion collisions



RHIC @ BNL



LHC @ CERN



Schematic picture of the evolution stages in relativistic heavy-ion collisions [Nucl.Phys.News30no.2,(2020)10–16]

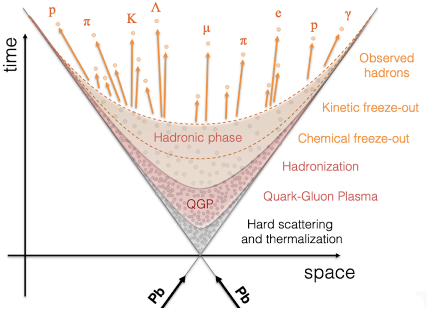
Quark-gluon Plasma

- A thermalized collective deconfined phase of quarks and gluons

Deconfinement

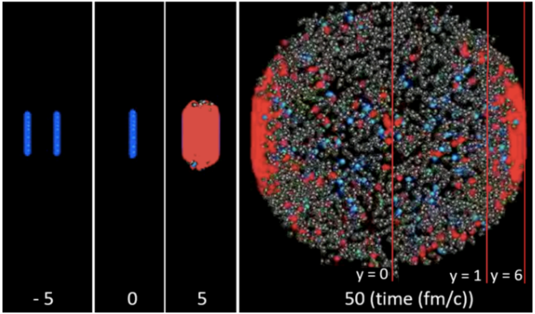
Thermalization

Collective behavior



Schematic spacetime diagram of relativistic heavy-ion collisions [D.D.Chinellato]

Deconfinement



Simulation of Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV, showing hadrons (blue) and QGP (red) [HIC group @ MIT]

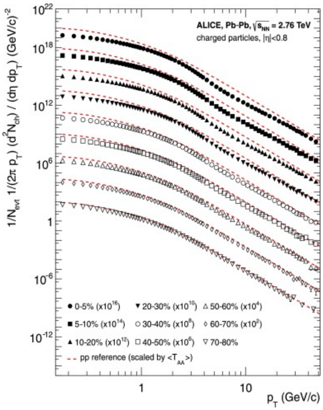
- **At $t = 1\,\mathrm{fm}/c$ after the collision, the estimated average energy density is**

$$\varepsilon \approx 12\,\mathrm{GeV}/\mathrm{fm}^3$$

which is 20 $\times$ the typical hadron energy density of

$$\varepsilon_{\mathrm{hadron}} \approx 500\,\mathrm{MeV}/\mathrm{fm}^3$$

Thermalization

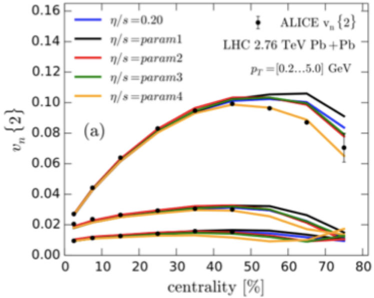


p_T **distribution of charge particles emitted from the QGP measured in Pb–Pb collisions in different centrality intervals [PhysicsLettersB720(2013)52–62]**

- **The QGP formed in the collision is locally equilibrated as suggested from the spectra**

- **Anisotropic distribution signals collectivity**

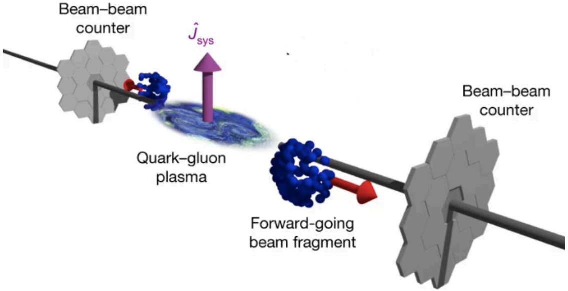
$$dN/d\phi \simeq 1 + 2v_1 cos(\phi - \psi_{RP}) + 2v_2 cos[2(\phi - \psi_{RP})] + \dots$$



A hydrodynamic model of v_2 (top) is compared with ALICE measurements of the anisotropy [Ann.Rev.Nucl.Part.Sci.2018.68:1–49]

Spin polarization in heavy-ion collisions

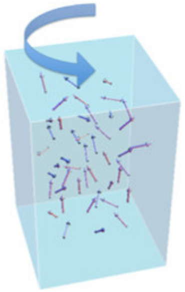
Total angular momentum of the QGP



A sketch of a Au+Au collision in the STAR detector system [Naturevolume548,pages62–65(2017)]

- **Non-central collisions involve substantial global angular momentum J_{sys} of the order of $10^3 - 10^4\hbar$**

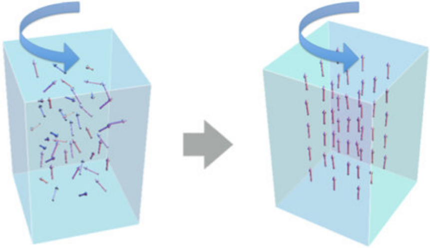
Vortical structures in the QGP?



Vorticity in the local fluid cell

- **Local vortical structures of the created fluid are suggested**
- **The average (over the fluid elements) vorticity points along the direction of the angular momentum of the collision J_{sys}**

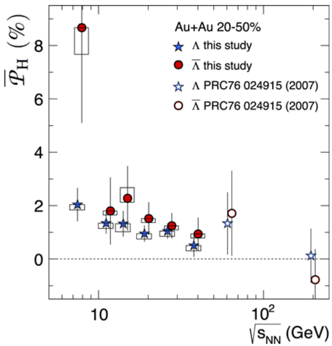
$$\omega_{kin} \simeq \nabla \times v \longrightarrow \omega_{\mu\nu} = \nabla_{(\nu} u_{\mu)} / T$$

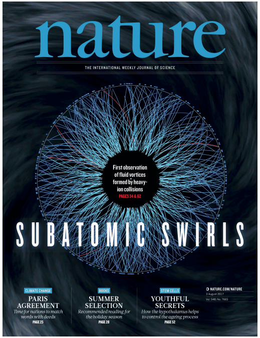


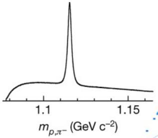
- **If on average the spin of emitted hadrons tends to be polarized along to J_{sys}, signatures of local vortical structures are expected**

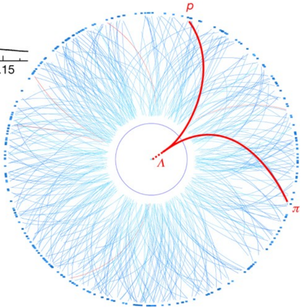
Global Λ polarization

- **STAR at RHIC presented the first measurement in 2017 of an alignment between the global angular momentum in non-central collisions and the spin of emitted particles.**



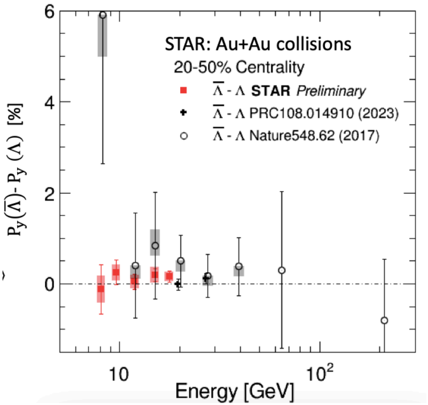






[Naturevolume548,pages62–65(2017)]

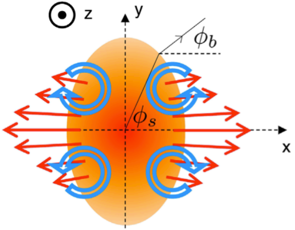
Λ-$\bar{\Lambda}$ splitting: recent result



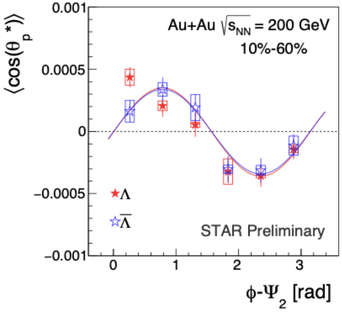
[SQM conference 2024]

- **Recent polarization results shows no splitting between Λ-$\bar{\Lambda}$**
- **This result suggest that vortical structures does not differentiate between Λ and $\bar{\Lambda}$**

Angle-dependent polarization along beam-direction



Vorticities along the beam direction (open arrows) induced by anisotropic flow (solid arrows) in the (x-y)-plane [Phys. Rev. Lett. 123, 132301]



Pz of Λ hyperons as a function of azimuthal angle ϕ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (Local Polarization) [Phys.Rev.Lett.123,132301]

What is spin hydrodynamics?

- **Spin hydrodynamics, emerging as an effective limit of quantum field theory, is a theoretical tool for describing the evolution of the collective partonic medium produced in non-central heavy-ion collisions throughout its lifetime, from its formation to the point at which it cools enough to hadronize into particles**

[1] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, "Relativistic fluid dynamics with spin," *Phys. Rev. C* **97** no. 4, (2018) 041901, arXiv:1705.00587 [nucl-th].

[2] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, "Fate of spin polarization in a relativistic fluid: An entropy-current analysis," *Phys. Lett. B* **795** (2019) 100–106, arXiv:1901.06615 [hep-th].

[3] K. Fukushima and S. Pu, "Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –," *Phys. Lett. B* **817** (2021) 136346, arXiv:2010.01608 [hep-th].

[4] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," *JHEP* **11** (2021) 150, arXiv:2107.14231 [hep-th].

[5] D. She, A. Huang, D. Hou, and J. Liao, "Relativistic viscous hydrodynamics with angular momentum," *Sci. Bull.* **67** (2022) 2265–2268, arXiv:2105.04060 [nucl-th].

[6] A. D. Gallegos, U. Gürsoy, and A. Yarom, "Hydrodynamics of spin currents," *SciPost Phys.* **11** (2021) 041, arXiv:2101.04759 [hep-th].

[7] N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, "Relativistic second-order dissipative spin hydrodynamics from the method of moments," *Phys. Rev. D* **106** no. 9, (2022) 096014, arXiv:2203.04766 [nucl-th].

[8] R. Biswas, A. Daher, A. Das, W. Florkowski, and R. Ryblewski, "Relativistic second-order spin hydrodynamics: An entropy-current analysis," *Phys. Rev. D* **108** no. 1, (2023) 014024, arXiv:2304.01009 [nucl-th].

- **Evolution of :**

1. $\mathbf{T}^{\mu\nu} \equiv$ **Energy-momentum current,**

$$\partial_\mu \mathbf{T}^{\mu\nu} = 0 \ , \quad \mathbf{T}^{\mu\nu} = \mathbf{T}_0^{\mu\nu} + \delta \mathbf{T}^{\mu\nu} \ , \quad \mathbf{P}^{\nu} = \int \mathbf{d}\, \Sigma_\mu \, \mathbf{T}^{\mu\nu} \ , \quad \mathbf{T}^{\mu\nu} = \langle \hat{\rho}\, \widehat{\mathbf{T}}^{\mu\nu} \rangle$$

2. $\mathbf{j}^{\mu} \equiv$ **Particle number current,**

$$\partial_\mu \mathbf{j}^{\mu} = 0 \ , \quad \mathbf{j}^{\mu} = \mathbf{j}_0^{\mu} + \delta \mathbf{j}^{\mu} \ , \quad \mathbf{J} = \int \mathbf{d}\, \Sigma_\mu \, \mathbf{j}^{\mu} \ , \quad \mathbf{j}^{\mu} = \langle \hat{\rho}\, \widehat{\mathbf{j}}^{\mu} \rangle$$

3. $\mathbf{S}^{\lambda\mu\nu} \equiv$ **Spin current,**

$$\partial_\lambda \mathbf{S}^{\lambda\mu\nu} = \mathbf{T}^{\nu\mu} - \mathbf{T}^{\mu\nu} \ , \quad \mathbf{S}^{\lambda\mu\nu} = \mathbf{S}_0^{\lambda\mu\nu} + \delta \mathbf{S}^{\lambda\mu\nu} \ , \quad \mathbf{S}^{\mu\nu} = \int \mathbf{d}\Sigma_\lambda \mathbf{S}^{\lambda\mu\nu} \ , \quad \mathbf{S}^{\lambda\mu\nu} = \langle \hat{\rho}\, \widehat{\mathbf{S}}^{\lambda\mu\nu} \rangle$$

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The goal of this work is to determine the dissipative currents:

$$\delta \mathbf{T}^{\mu\nu} \ , \quad \delta \mathbf{j}^{\mu} \ , \quad \delta \mathbf{S}^{\lambda\mu\nu}$$

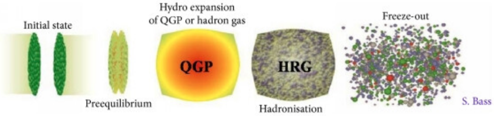
- Once the evolution is analytically determined:

For given initial conditions

Solve numerically

Calculate Observables at freeze-out

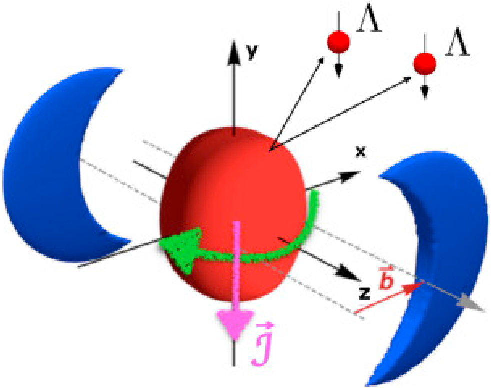
Compare with experiments



http://qgp.phy.duke.edu

Quantum-statistical formulation and entropy production rate

Initial state : 2 nuclei colliding forming a strongly interacting system described by mixed state $\widehat{\rho}_{(0)}$. How to evolve $\widehat{\rho}_{(0)}$? (Typical QFT problem)



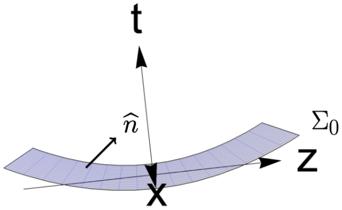
R.Ryblewski et al. [Prog.Part.Nucl.Phys.108(2019)103709]

Local equilibrium is achieved at initial hypersurface Σ_0, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin densities are their actual values:

$$S = -\,\mathrm{Tr}(\widehat{\rho}\log\widehat{\rho})$$

$$F[\widehat{\rho}] = -\,Tr\,[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\; n_\mu\,(T^{\mu\nu}_{\mathrm{LE}} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\; n_\mu\,(j^{\mu}_{\mathrm{LE}} - j^{\mu})\,\zeta(x)$$
$$- \int d\Sigma_0\; n_\mu \left(S^{\mu\lambda\nu}_{\mathrm{LE}} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\mathrm{LE}} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$



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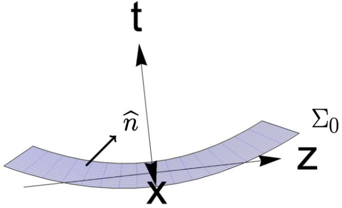


$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\, n_\mu\, (T^{\mu\nu}_{\mathrm{LE}} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\, n_\mu\,(j^\mu_{\mathrm{LE}} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\, n_\mu \left(S^{\mu\lambda\nu}_{\mathrm{LE}} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\mathrm{LE}} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$

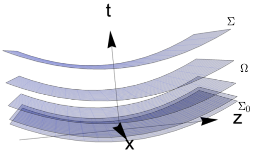
$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z}\exp\left[-\int_{\Sigma_0} \mathrm{d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{S}^{\mu\lambda\nu}\right)\right]$$

The Lagrange multipliers are obtained by solving the constraint equations at Σ_0. Their evolution is determined by solving the conservation equations:

- $\beta^\mu \to u^\mu = \beta^\mu/\sqrt{\beta^2} \quad T = 1/\sqrt{\beta^2}$
- $\zeta = \mu/T$
- $\Omega_{\mu\nu} = \omega_{\mu\nu}/T$
- Thermal Shear: $\xi_{\mu\nu} = \frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu)$ Thermal Vorticity: $\varpi_{\mu\nu} = \frac{1}{2}(\nabla_\nu\beta_\mu - \nabla_\mu\beta_\nu)$

Near local equilibrium at the hypersuface Σ, the entropy is defined as:

$$
\begin{aligned}
S &= -\operatorname{Tr}\left[\widehat{\rho}_{\mathrm{LE}}(t)\log\widehat{\rho}_{\mathrm{LE}}(t)\right] \\
&= \log Z_{\mathrm{LE}} + \int_{\Sigma} \mathrm{d}\Sigma_{\mu}\left[\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{T}^{\mu\nu}\right)\beta_{\nu} - \zeta\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{j}^{\mu}\right) - \frac{1}{2}\Omega_{\lambda\nu}\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]
\end{aligned}
$$



Can we define an entropy current out of S ? In other words, is it possible to show that $\log Z_{\mathrm{LE}}$ is an extensive quantity?

Therefore, entropy current exists:

$$S = \int_\Sigma \mathrm{d}\Sigma_\mu\, \phi^\mu + T^{\mu\nu}_{\mathrm{LE}}\beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}$$

$$s^\mu_{LE} = \phi^\mu + T^{\mu\nu}_{\mathrm{LE}}\beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}\,.$$

In quantum theory, we only have the total entropy, not the entropy current. We need to construct an entropy current through an integral. However, this introduces ambiguities, as several fields can lead to the same integral. However if $s^\mu - s^\mu_{LE} \perp n^\mu$,

$$s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \zeta j^\mu - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu} \qquad \phi^\mu = \int_0^T \frac{\mathrm{d}T'}{T'^2}\left(T^{\mu\nu}[T']u_\nu - \mu j^\mu[T'] - \frac{1}{2}\omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}[T']\right)$$

Using the entropy current $s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu}$, we obtain:

$$\partial_\mu s^\mu = \left(T_S^{\mu\nu} - T_{S(\mathrm{LE})}^{\mu\nu}\right)\xi_{\mu\nu} - (j^\mu - j^\mu_{\mathrm{LE}})\,\partial_\mu\zeta + \left(T_A^{\mu\nu} - T_{A(\mathrm{LE})}^{\mu\nu}\right)(\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2}\left(\mathcal{S}^{\mu\lambda\nu} - \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}\right)\partial_\mu\Omega_{\lambda\nu}$$

$\varpi_{\mu\nu}$: is the thermal vorticity

This formula is a generalization of what was obtained ***C. Van Weert*** **without spin:**

C. van Weert, "Maximum entropy principle and relativistic hydrodynamics," *Annals of Physics, Volume 140, Issue 1,1982* .

Dissipative currents: Method and results (Ongoing work)

$$\delta T_S^{\mu\nu} = H^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + K^{\mu\nu\rho}\partial_\rho\zeta + L^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + M^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta T_A^{\mu\nu} = N^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + P^{\mu\nu\rho}\partial_\rho\zeta + Q^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + R^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta j^\mu = G^{\mu\rho\sigma}\xi_{\rho\sigma} + I^{\mu\rho}\partial_\rho\zeta + O^{\mu\rho\sigma}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + F^{\mu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta S^{\mu\lambda\nu} = T^{\mu\lambda\nu\rho\sigma}\xi_{\rho\sigma} + U^{\mu\lambda\nu\rho}\partial_\rho\zeta + V^{\mu\lambda\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + W^{\mu\lambda\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau}.$$

Hence the goal reduces to determining the coefficient tensors:

$$H^{\mu\nu,\rho,\sigma}\ ,\quad K^{\mu\nu\rho}\ ,\quad L^{\mu\nu\rho\sigma}\ ,\quad M^{\mu\nu\rho\sigma\tau}$$

$$N^{\mu\nu\rho\sigma}\ ,\quad P^{\mu\nu\rho}\ ,\quad Q^{\mu\nu\rho\sigma}\ ,\quad R^{\mu\nu\rho\sigma\tau}$$

$$G^{\mu\rho\sigma}\ ,\quad I^{\mu\rho}\ ,\quad O^{\mu\rho\sigma}\ ,\quad F^{\mu\rho\sigma\tau}$$

$$T^{\mu\lambda\nu\rho\sigma}\ ,\quad U^{\mu\lambda\nu\rho}\ ,\quad V^{\mu\lambda\nu\rho\sigma}\ ,\quad W^{\mu\lambda\nu\rho\sigma\tau}$$

Using irreducible representation of $\mathbb{SO}(3)$ group,

$$\text{Vector:} \quad V^{\mu} = (0 \oplus 1)$$

$$\text{Symmetric 2-tensor:} \quad B^{\mu\nu} = (0 \oplus 0 \oplus 1 \oplus 2)$$

$$\text{Antisymmetric 2-tensor:} \quad A^{\mu\nu} = (1 \oplus 1)$$

Our hydrodynamics "tools" existing at global equilibrium,

$$u^{\mu}\ , \quad \Delta^{\mu\nu}\ , \quad \epsilon^{\mu\nu\alpha\beta}$$

Therefore, the irreducible representation, interms of our hydrodynamic variables:

$$\text{Vector:}\ \ V^{\mu} = (u^{\mu} \oplus \Delta^{\mu}_{\alpha})$$

$$\text{Symmetric 2-tensor:}\ \ B^{\mu\nu} = (u^{\mu}u^{\nu} \oplus \Delta^{\mu\nu} \oplus u^{\mu}\Delta^{\nu}_{\alpha} + u^{\nu}\Delta^{\mu}_{\alpha} \oplus \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}),$$

$$\text{Antisymmetric 2-tensor:}\ \ A^{\mu\nu} = (u^{\mu}\Delta^{\nu}_{\alpha} - u^{\nu}\Delta^{\mu}_{\alpha} \oplus \epsilon^{\mu\nu\tau\alpha}u_{\tau}).$$

Hence we can decompose any tensor interms of its $\mathbb{SO}(3)$ irreducible components, by using V^{μ} , $B^{\mu\nu}$, $A^{\mu\nu}$:

1. **Any rank**
2. **Any symmetry**

$$
W^{\mu\lambda\nu\rho\sigma\tau} : \begin{cases}
w_1\, u^\mu u^{[\lambda} u^\rho u^{[\sigma} \Delta^{\nu]\tau]} \\
w_2\, u^\mu u^{[\lambda} u^\rho u_\beta \epsilon^{\sigma\tau\beta\nu]} \\
w_3\, u^\mu u^\rho \Delta^{[\lambda[\tau} \Delta^{\nu]\sigma]} \Leftrightarrow u^\mu u_\beta u^\rho u_\gamma \epsilon^{\lambda\nu\beta\alpha} \epsilon^{\sigma\tau\gamma}{}_\alpha \\
w_4\, u^\mu u_\beta u^\rho u^{[\sigma} \epsilon^{\lambda\nu\beta\tau]} \\
w_5\, u^{[\lambda} u^{[\sigma} \Delta^{\mu\nu]} \Delta^{\tau]\rho} \\
w_6\, u^{[\lambda} u_\beta \Delta^{\mu\nu]} \epsilon^{\sigma\tau\beta\rho} \\
w_7\, u_\beta u^{[\sigma} \Delta^{\tau]\rho} \epsilon^{\lambda\nu\beta\mu} \\
w_8 (\Delta^{[\lambda\rho} \Delta^{\mu[\sigma} \Delta^{\nu]\tau]} + \tfrac{1}{2} \Delta^{[\lambda[\tau} \Delta^{\nu]\sigma]} \Delta^{\mu\rho}) \Leftrightarrow u_\beta u_\gamma \epsilon^{\lambda\nu\beta\mu} \epsilon^{\sigma\tau\gamma\rho} \\
w_9\, u^\rho u^{[\sigma} u^{[\lambda} u_\alpha \epsilon^{\mu\nu]\alpha\tau]} \\
w_{10}\, u^\rho u^{[\sigma} \Delta^{[\lambda\tau]} \Delta^{\mu\nu]} \Leftrightarrow u^\rho u^\sigma u_a u_c \epsilon^{\mu\beta\tau a} \epsilon^{\lambda\nu c}{}_\beta - u^\rho u^\tau u_a u_c \epsilon^{\mu\beta\sigma a} \epsilon^{\lambda\nu c}{}_\beta \\
w_{11}\, u^\rho \Delta^{\mu[\tau} \epsilon^{\lambda\nu\sigma]\alpha} u_\alpha \Leftrightarrow u^\rho u_z u_a u_c \epsilon^{\sigma\tau zy} \epsilon^{\mu\beta}{}_y{}^a \epsilon^{\lambda\nu c}{}_\beta \\
w_{12}\, u^{[\lambda} u^\rho \Delta^{\mu[\tau} \Delta^{\nu]\sigma]} \Leftrightarrow u^\rho u^\lambda u_a u_z \epsilon^{\sigma\tau zy} \epsilon^{\mu\nu a}{}_y - u^\rho u^\nu u_a u_z \epsilon^{\sigma\tau zy} \epsilon^{\mu\lambda a}{}_y \\
w_{13}\, u^{[\sigma} u_w u^\mu u^{[\lambda} \epsilon^{\rho\tau]w\nu]} \\
w_{14}\, u^\mu u^{[\sigma} \Delta^{[\lambda\tau]} \Delta^{\nu]\rho} \Leftrightarrow u^\sigma u_w u^\mu \epsilon^{\rho\tau w\beta} \epsilon^{\lambda\nu c}{}_\beta u_c - u^\tau u_w u^\mu \epsilon^{\rho\sigma w\beta} \epsilon^{\lambda\nu c}{}_\beta u_c \\
w_{15}\, (u^{[\lambda} u^{[\sigma} \Delta^{\mu\tau]} \Delta^{\nu]\rho} - u^{[\lambda} u^{[\sigma} \Delta^{\nu]\tau]} \Delta^{\mu\rho}) \Leftrightarrow u^\sigma u_w u^\lambda u_a \epsilon^{\rho\tau wv} \epsilon^{\mu\nu a}{}_v - u^\sigma u_w u^\nu u_a \epsilon^{\rho\tau wv} \epsilon^{\mu\lambda a}{}_v - u^\tau u_w \epsilon^{\rho\sigma wv} \epsilon^{\mu\nu a}{}_v u_a u^\lambda + \imath \\
w_{16}\, (u^{[\sigma} \Delta^{\mu\tau]} \epsilon^{\lambda\nu\rho\alpha} u_\alpha - u^{[\sigma} \epsilon^{\lambda\nu\tau]\alpha} u_\alpha \Delta^{\mu\rho}) \Leftrightarrow u^\sigma u_w u_a u_c \epsilon^{\rho\tau wb} \epsilon^{\mu\beta}{}_b{}^a \epsilon^{\lambda\nu c}{}_\beta - u^\tau u_w u_a u_c \epsilon^{\rho\sigma wb} \epsilon^{\mu\beta}{}_b{}^a \epsilon^{\lambda\nu c}{}_\beta \\
w_{17}\, u^\mu u^{[\nu} \Delta^{\lambda][\tau} \Delta^{\rho\sigma]} \Leftrightarrow u_w u_z u^\mu u^\lambda \epsilon^{\rho y\nu w} \epsilon^{\sigma\tau z}{}_y - u_w u_z u^\mu u^\nu \epsilon^{\rho y\lambda w} \epsilon^{\sigma\tau z}{}_y \\
w_{18}\, u^\mu \Delta^{\rho[\sigma} \epsilon^{\lambda\nu\tau]\alpha} u_\alpha \; u_w u_z u_c u^\mu \epsilon^{\rho y\beta w} \epsilon^{\sigma\tau z}{}_y \epsilon^{\lambda\nu c}{}_\beta \\
w_{19}\, (u^{[\lambda} \Delta^{\nu]\rho} \epsilon^{\mu\sigma\tau\alpha} u_\alpha - u^{[\lambda} \Delta^{\mu\rho} \epsilon^{\nu]\sigma\tau\alpha} u_\alpha) \Leftrightarrow u_w u_z u^\lambda u_a \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\mu\nu a}{}_b - u_w u_z u^\nu u_a \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\mu\lambda a}{}_b \\
w_{20}\, \Delta^{[\lambda\mu} \Delta^{\nu][\tau} \Delta^{\rho\sigma]} \Leftrightarrow u_w u_z u_a u_c \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\lambda\nu c}{}_\beta \epsilon^{\mu\beta}{}_b{}^a \\
w_{21}\, u^{[\sigma} u^{[\lambda} \Delta^{\tau]\nu]} \Delta^{\rho\mu} \\
w_{22}\, u^{[\sigma} u_\alpha \epsilon^{\lambda\nu\alpha\tau]} \Delta^{\mu\rho} \\
w_{23}\, u^{[\lambda} \epsilon^{\sigma\tau\alpha\nu]} u_\alpha \Delta^{\rho\mu} \\
w_{24}\, \Delta^{\lambda[\tau} \Delta^{\nu\sigma]} \Delta^{\rho\mu} \Leftrightarrow \epsilon^{\sigma\tau zy} u_z \epsilon^{\lambda\nu c}{}_y u_c \Delta^{\rho\mu} + \epsilon^{\sigma\tau zx} u_z \Delta^{\rho\mu} u_c \epsilon^{\lambda\nu c}{}_x
\end{cases}
$$

- **Matching conditions**

$$n_\mu(\delta T_S^{\mu\nu} + \delta T_A^{\mu\nu}) = 0, \quad n_\mu \delta j^\mu = 0, \quad n_\mu \delta S^{\mu\lambda\nu} = 0.$$

- $\partial_\mu s^\mu$ **is** $\mathbb{SO}(3)$ **invariant, and is a true scalar** $\longrightarrow$ **parity invariant**

- $\partial_\mu s^\mu \geq 0$

- **This allows us to cancel out all the non-physical coefficients**

Results

1. **We have retrieved the standard expressions of $\delta T_S^{\mu\nu}$ and δj^μ in relativistic hydrodynamics.**

2. **The expressions for $\delta T_S^{\mu\nu}$, $\delta T_A^{\mu\nu}$, and δj^μ include contributions proportional to $(\Omega_{\mu\nu} - \varpi_{\mu\nu})$ and $\partial_\lambda \Omega_{\mu\nu}$. Such new contributions are not corrections or only valid in a specific limit!**

3. **The expression for $\delta S^{\lambda\mu\nu}$.**

Conclusions and outlook

Conclusions

1. **We used a first-principle quantum-statistical methods to derive the entropy current and the entropy production rate.**

2. **We developed a new method based on the $\mathbb{SO}(3)$ irreducible representation that allows for the decomposition of any rank-tensor given any symmetry in terms of its rotation-invariant components.**

3. **We showed that $\delta T_S^{\mu\nu}$, $\delta T_A^{\mu\nu}$, δj^{μ} admit contributions proportional to $(\Omega_{\mu\nu} - \varpi_{\mu\nu})$ and $\partial_\lambda \Omega_{\mu\nu}$, and obtained the expression of $\delta S^{\lambda\mu\nu}$ (to be finalized).**

Outlook

1. **Develop a second-order theory based on the form of dissipative currents obtained from our method.**

2. **Numerically solve the conservation laws.**

3. **Calculate observables at the freeze-out hypersurface and compare them with experimental data.**

NARODOWE CENTRUM NAUKI

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THANK YOU