SPIN HYDRODYNAMICS FOR RELATIVISTIC HEAVY-ION COLLISIONS – RECENT NUMERICAL DEVELOPMENTS

Radoslaw Ryblewski
The H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences, Kraków, Poland

In collaboration with:
Sushant K. Singh *(Florence U. & VECC Calcutta)*
Wojciech Florkowski *(Jagiellonian U.)*

primary reference: e-Print: 2411.08223 [hep-ph]

"BIAŁASÓWKA"
DECEMBER 6, 2024, AGH, KRAKÓW

NATIONAL SCIENCE CENTRE
POLAND
SONATA BIS 8 Grant No. 2018/30/E/ST2/00432

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

MOTIVATION

QGP EVOLVES HYDRODYNAMICALLY

Observables constructed from **momentum-space distribution of charged hadrons** are being used to **study properties of hot and dense QCD matter**

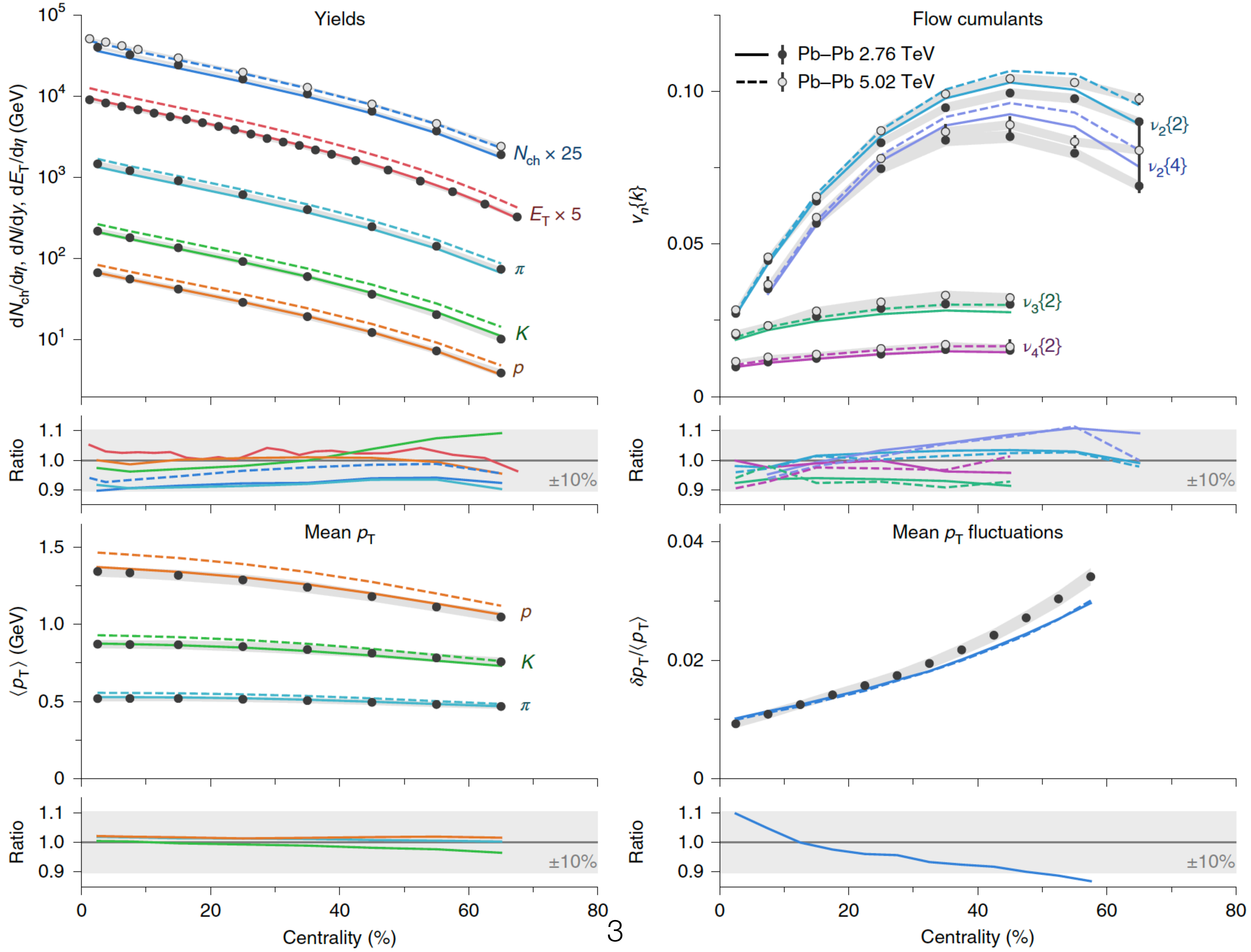


fig: Bernhard, Moreland, Bass, Nature Phys. 15, 1113–1117 (2019)

QGP EVOLVES HYDRODYNAMICALLY

Established properties of the produced matter:

Behaves like **a fluid**

➡ hydrodynamics is applicable

Low viscosity

➡ inclusion of **dissipative effects** is required

➡ **relativistic viscous hydrodynamics** is used

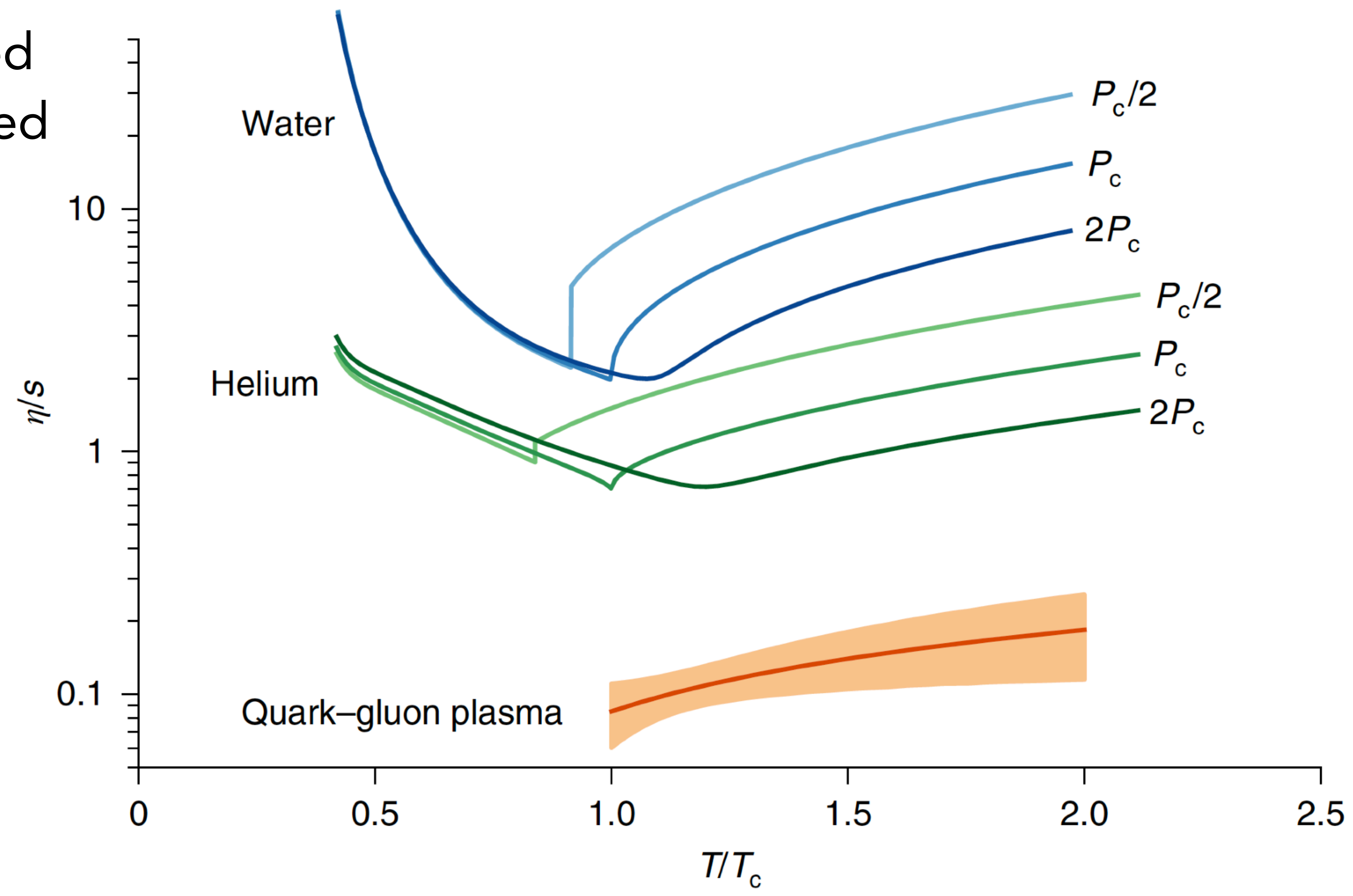


fig: Bernhard, Moreland, Bass, Nature Phys. 15, 1113–1117 (2019)

Non-central heavy-ion collisions

Large initial orbital angular momentum (OAM)

Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

orbital

$\boldsymbol{L}_{\text{init}}$

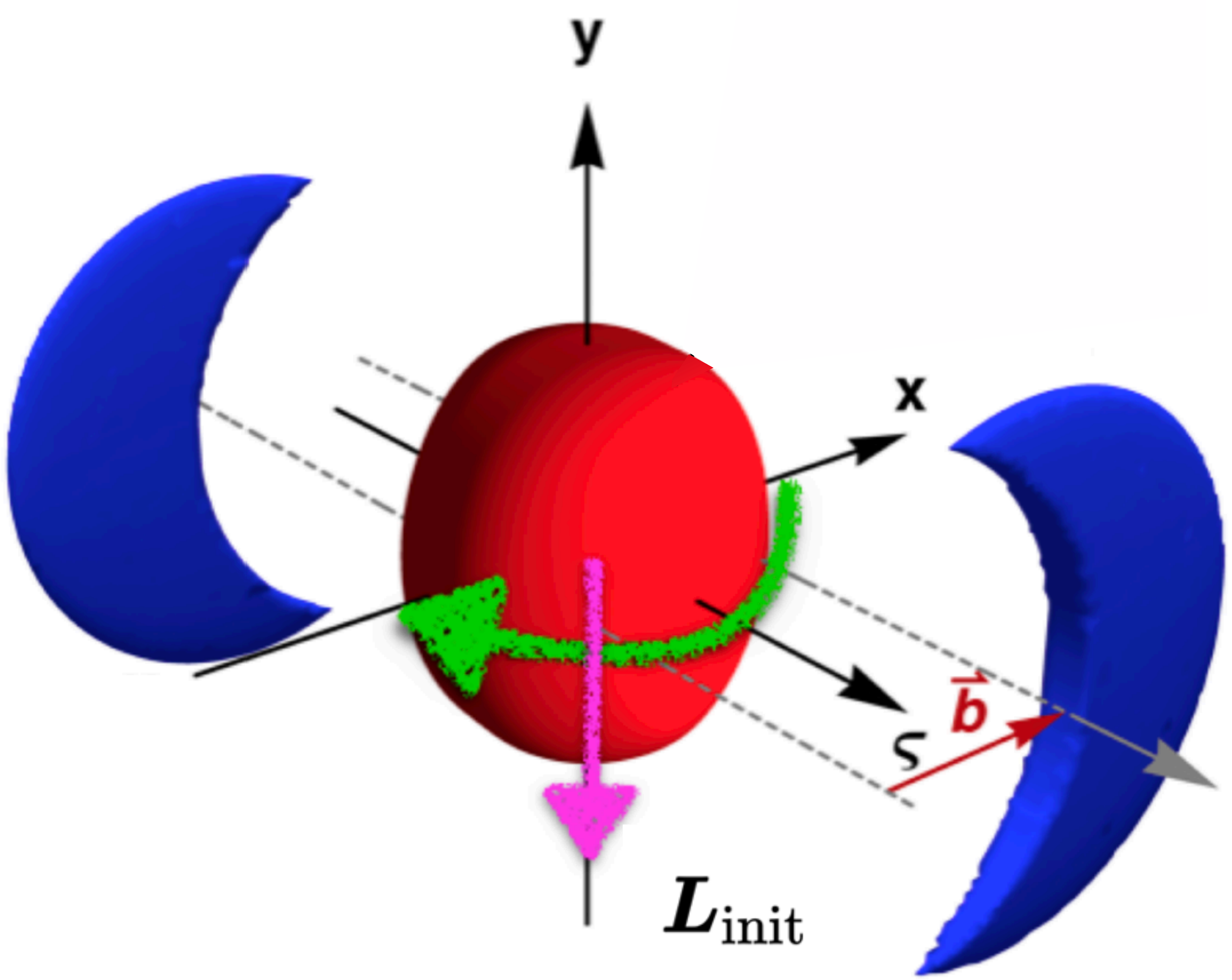


fig: R. R.

NON-CENTRAL HEAVY-ION COLLISIONS

Large initial orbital angular momentum (OAM)

Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

OAM can be transferred to the spin of QGP constituents

orbital

L_{init}

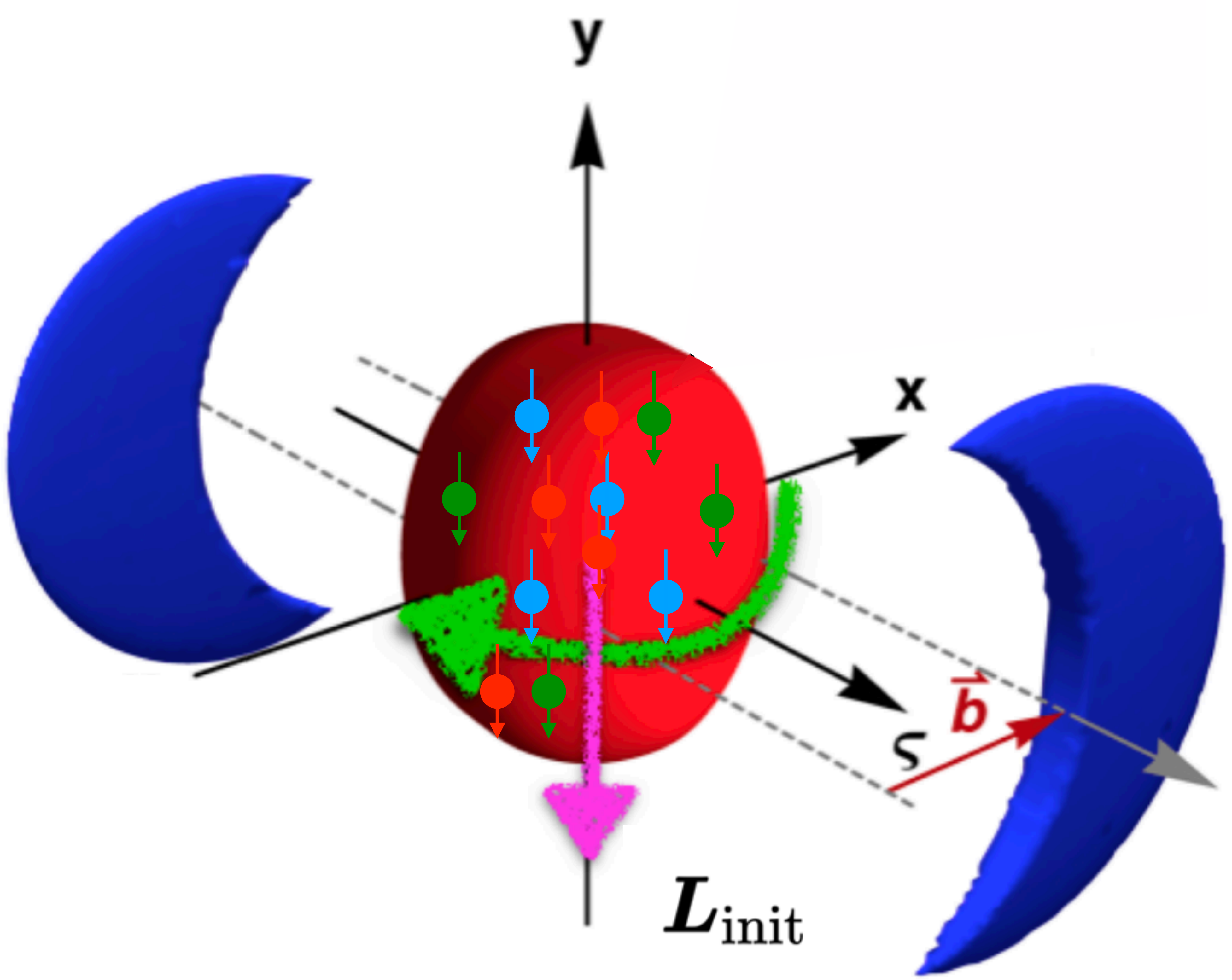


fig: R. R.

NON-CENTRAL HEAVY-ION COLLISIONS

Large initial orbital angular momentum (OAM)

Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

OAM can be transferred to the spin of QGP constituents

Emitted **particles (on average) are expected to be polarized** along the fireball's global angular

Liang, Wang PRL 94:102301 (2005)
Betz, Gyulassy, Torrieri, PRC 76:044901 (2007)
Gao, et al. PRC 77:044902 (2008)
Becattini, Piccinini, et al. J. Phys. G 35:054001 (2008)

orbital orbital + spin

$$\boldsymbol{L}_{\text{init}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

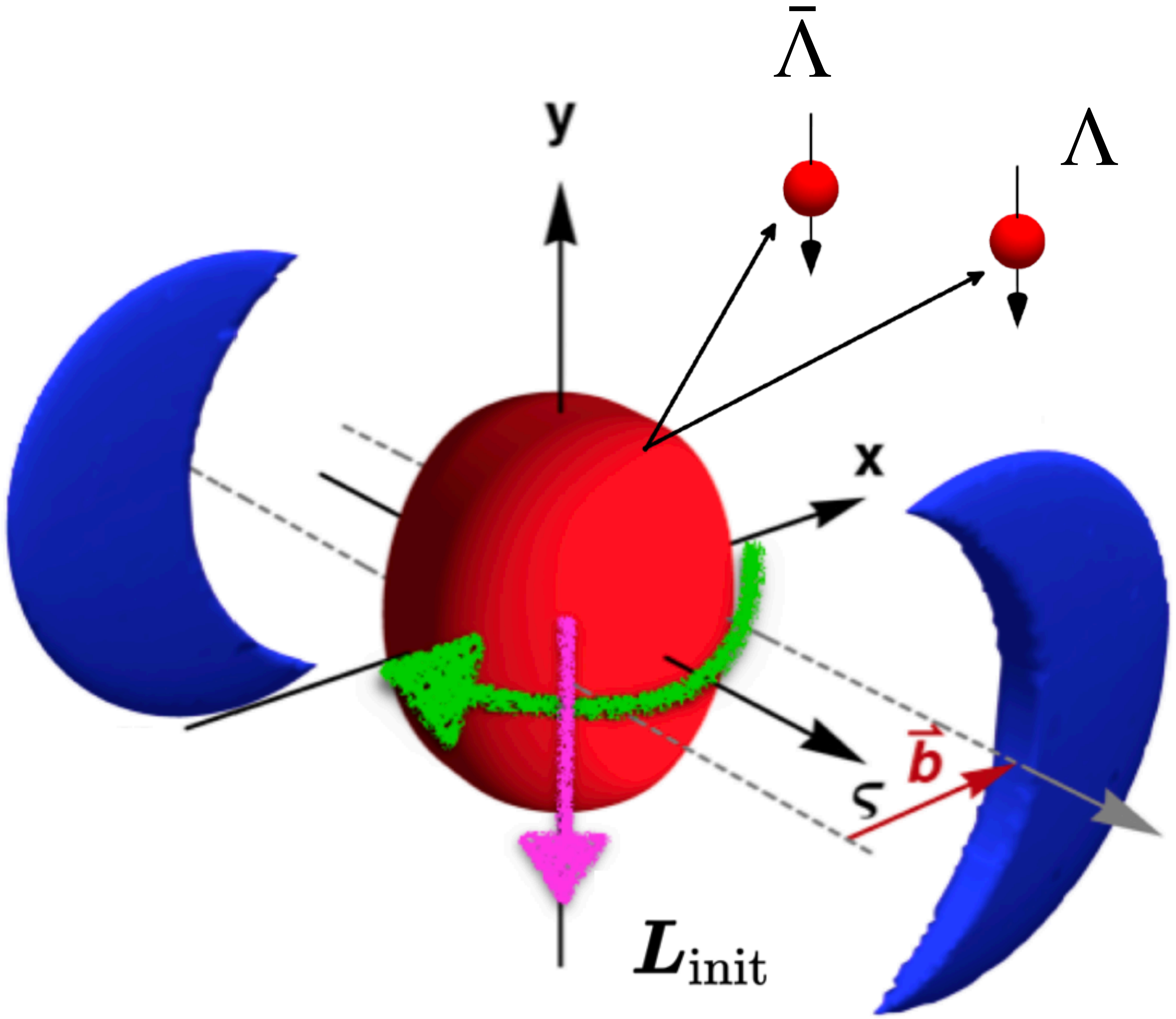


fig: R. R.

Non-central heavy-ion collisions

Large initial orbital angular momentum (OAM)

Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

OAM can be transferred to the spin of QGP constituents

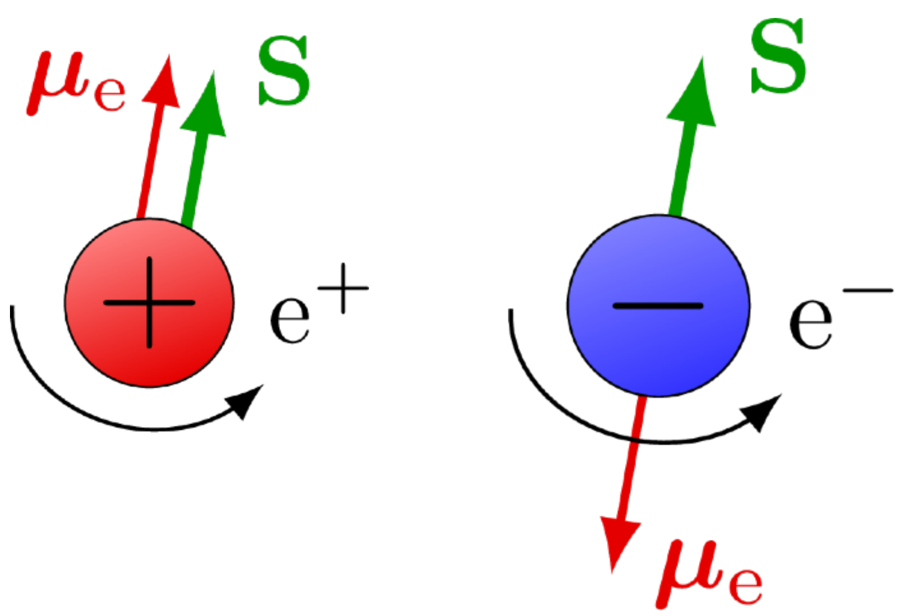
Emitted **particles (on average) are expected to be polarized** along the fireball's global angular

Liang, Wang PRL 94:102301 (2005)
Betz, Gyulassy, Torrieri, PRC 76:044901 (2007)
Gao, et al. PRC 77:044902 (2008)
Becattini, Piccinini, et al. J. Phys. G 35:054001 (2008)

Large magnetic field may be created initially

Bzdak and Skokov, Phys. Lett. B 710 (2012) 171-174

Particle's magnetic moments alignment is possible



$$\boldsymbol{\mu} = \frac{g_s q}{2m}\mathbf{S}$$

$$E = -\boldsymbol{\mu} \cdot \mathbf{B}$$

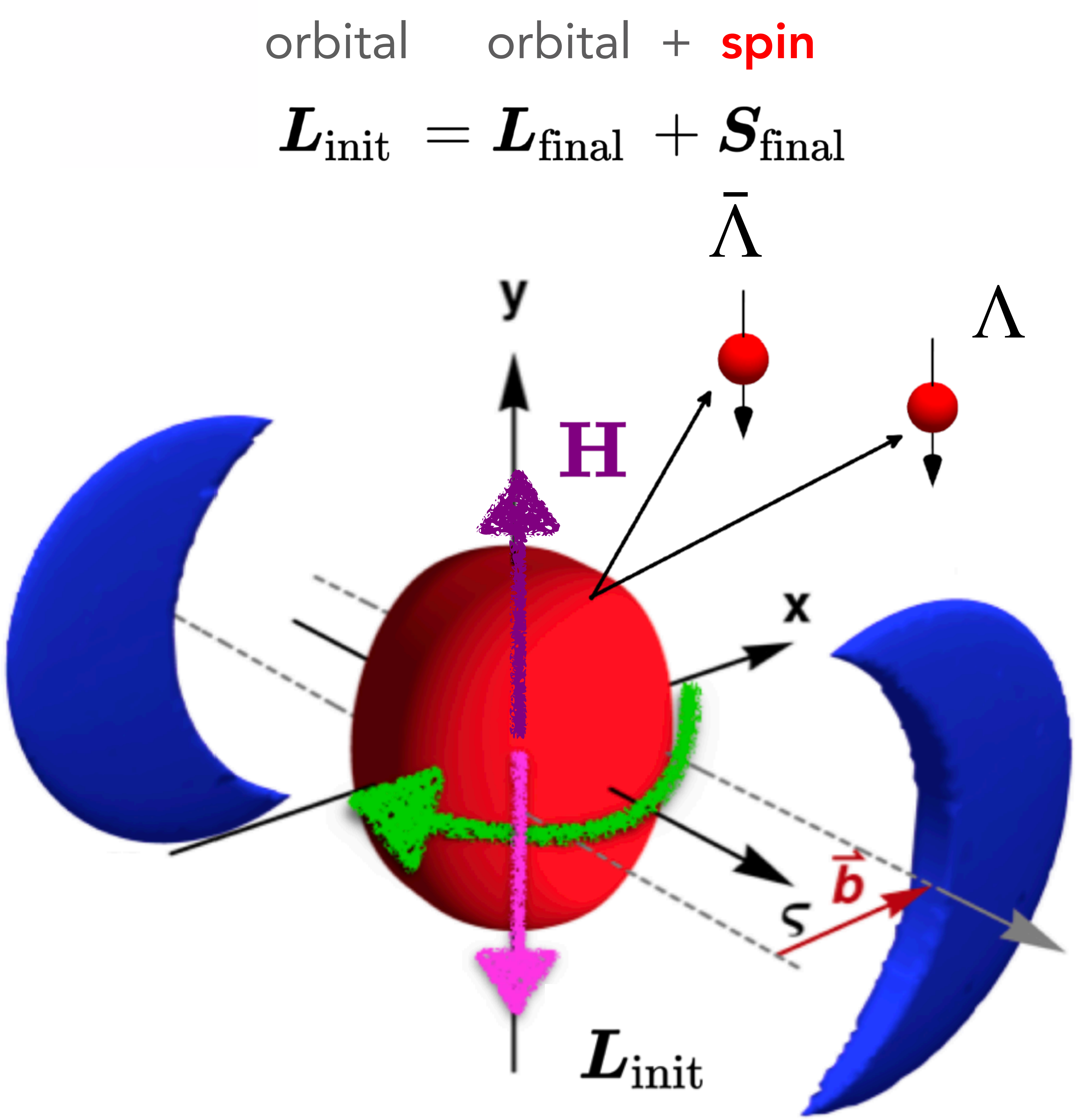


fig: R. R.

Measurement of Λ and $\bar{\Lambda}$ global spin polarization

Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ

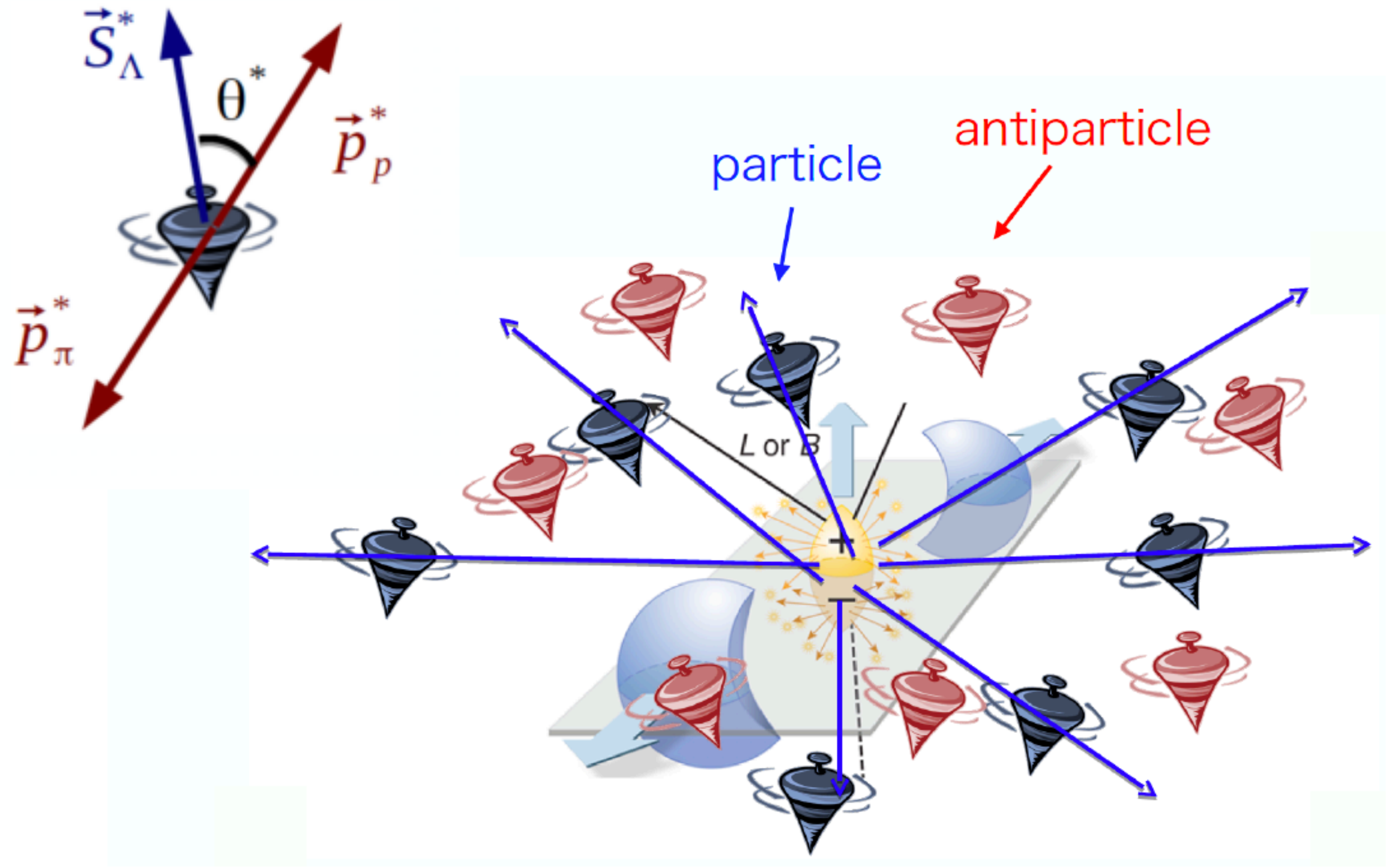


figure: T. Niida

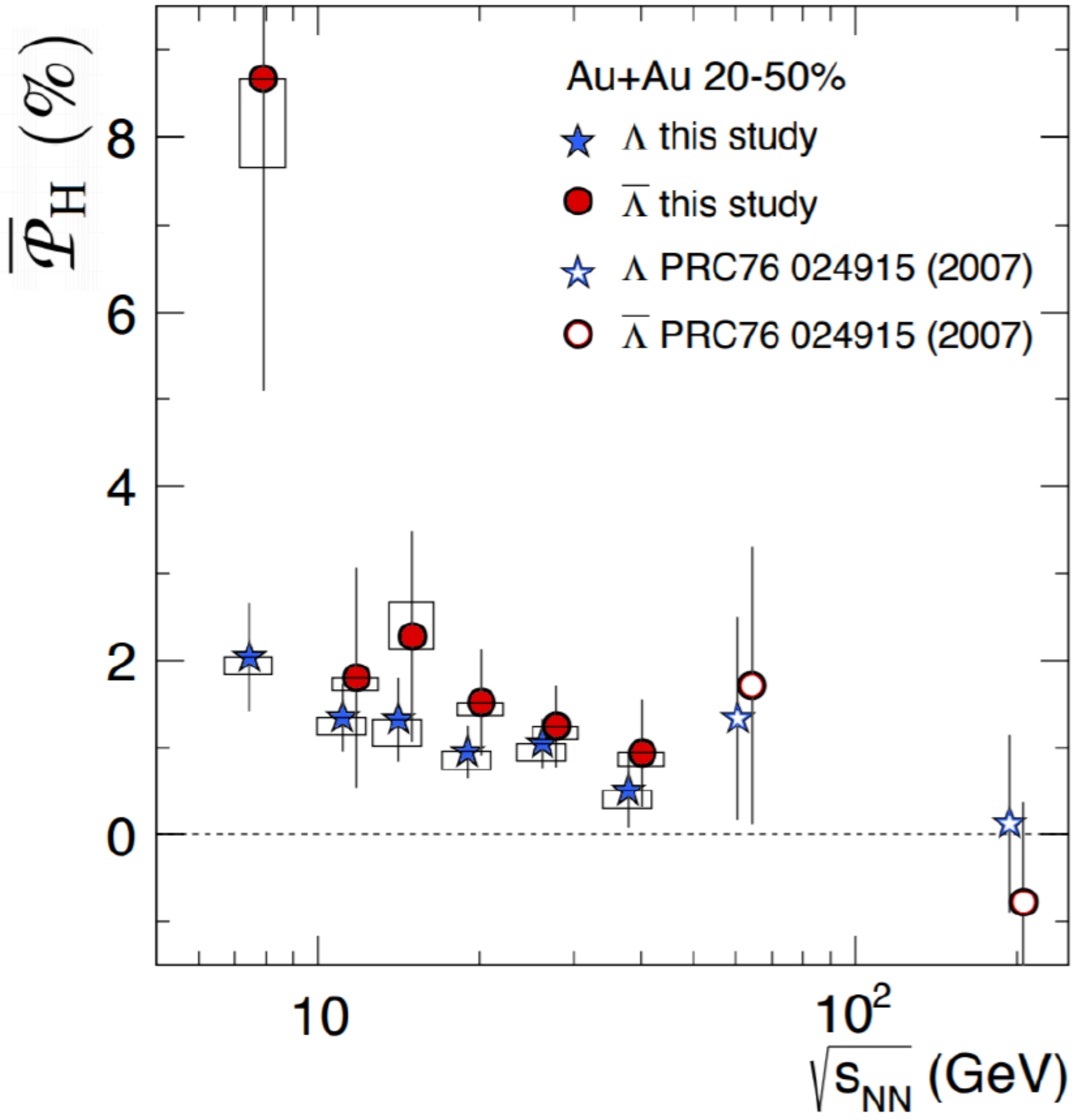
$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*\right)$$

$$(\alpha_\Lambda = 0.732)$$

$$\overline{\mathcal{P}}_H \equiv \langle \vec{\mathcal{P}}_H \cdot \hat{J}_{sys} \rangle = \frac{8}{\pi \alpha_H} \frac{\left\langle \cos\left(\phi^*_p - \phi_{\hat{J}_{sys}}\right)\right\rangle}{R^{(1)}_{EP}}$$

MEASUREMENT OF Λ AND $\bar{\Lambda}$ GLOBAL SPIN POLARIZATION

global polarization ⟷ average over all phasespace



Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

SPIN-THERMAL APPROACH

In **thermodynamic equilibrium** one can establish a link between spin and vorticity

Becattini, Chandra, Del Zanna, Grossi, AP 338:32 (2013)
Becattini, Csernai, and Wang, PRC 88, 034905 (2013)
Fang, Pang, Wang, Wang, PRC 94:024904 (2016)
Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (20

The **polarization vector** of emitted particles is

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

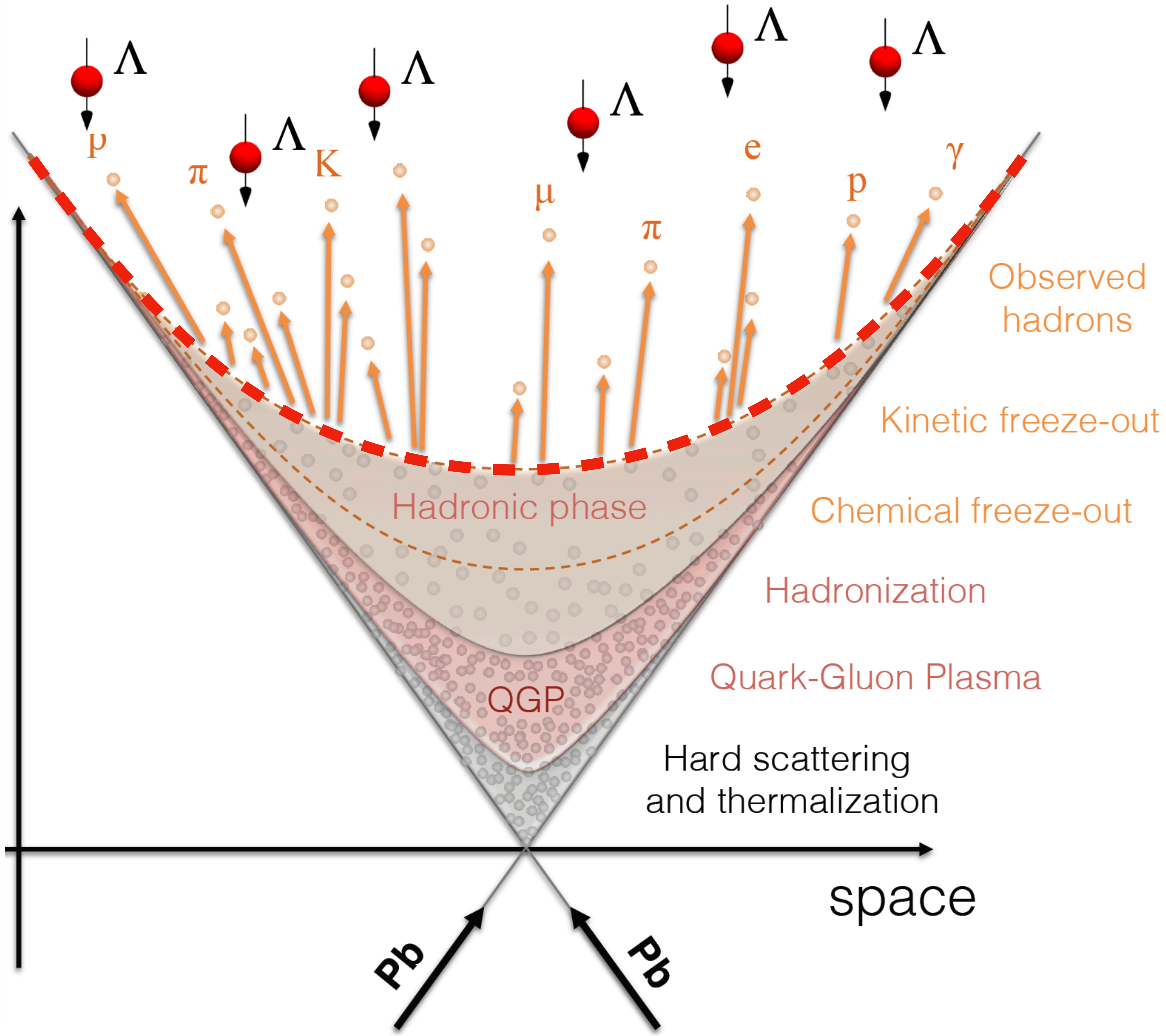


figure: D.D. Chinellato

SPIN-THERMAL APPROACH

In **thermodynamic equilibrium** one can establish a link between spin and vorticity

Becattini, Chandra, Del Zanna, Grossi, AP 338:32 (2013)
Becattini, Csernai, and Wang, PRC 88, 034905 (2013)
Fang, Pang, Wang, Wang, PRC 94:024904 (2016)
Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (20

The **polarization vector** of emitted particles is

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

$$T, \mu_B\, u^{\rho}$$

Which requires the usual hydrodynamic fields at freezeout

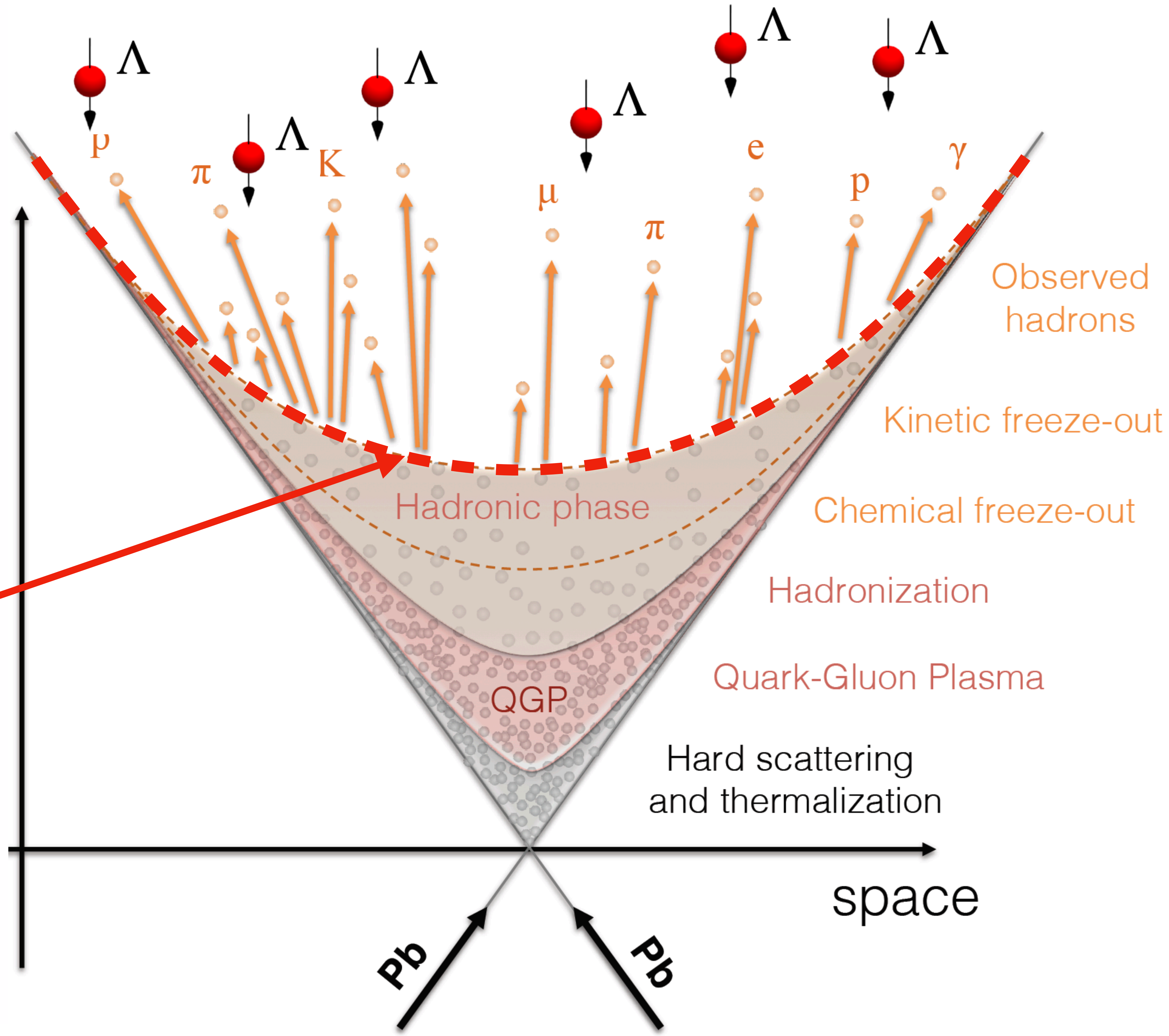


figure: D.D. Chinellato

POLARIZATION VECTOR

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

Polarization vector

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

Here one uses Fermi-Dirac distribution

$$n_F = n_F(T, \mu_B, p\cdot u; m_{\Lambda}) \qquad n_F(z) = \frac{1}{\mathrm{e}^{z}+1}$$

Polarization vector

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

Here one uses Fermi-Dirac distribution

$$n_F = n_F(T, \mu_B, p\cdot u; m_{\Lambda}) \qquad n_F(z) = \frac{1}{\mathrm{e}^z + 1}$$

The integral is performed over the switching hypersurface Σ

POLARIZATION VECTOR

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) \;=\; -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

Here one uses Fermi-Dirac distribution

$$n_F = n_F(T,\mu_B,p\cdot u;m_{\Lambda}) \qquad\qquad n_F(z) = \frac{1}{\mathrm{e}^z+1}$$

The integral is performed over the switching hypersurface Σ

The **thermal vorticity** is

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} \;= -\,\frac{1}{2}\left(\partial_{\mu}\beta_{\nu}-\partial_{\nu}\beta_{\mu}\right) \qquad\qquad \beta^{\mu} \equiv u^{\mu}/T$$

Polarization vector

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu_\varpi(p) \;=\; -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

Here one uses Fermi-Dirac distribution

$$n_F = n_F(T, \mu_B, p\cdot u; m_\Lambda) \qquad\qquad n_F(z) = \frac{1}{\mathrm{e}^z + 1}$$

The integral is performed over the switching hypersurface Σ

The **thermal vorticity** is

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} \;= -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad\qquad \beta^\mu \equiv u^\mu/T$$

Temperature, chemical potential and flow are found from standard viscous hydrodynamics (or any other model)

Polarization vector

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) \; = \; -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

Here one uses Fermi-Dirac distribution

$$n_F = n_F(T, \mu_B, p\cdot u; m_{\Lambda}) \qquad\qquad n_F(z) = \frac{1}{\mathrm{e}^z+1}$$

The integral is performed over the switching hypersurface Σ

The **thermal vorticity** is

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} \; = -\,\frac{1}{2}\left(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu}\right) \qquad\qquad \beta^{\mu} \equiv u^{\mu}/T$$

Temperature, chemical potential and flow are found from standard viscous hydrodynamics (or any other model)

Lambda mass has the PDG fixed value

Global polarization: measurement vs spin-thermal approach

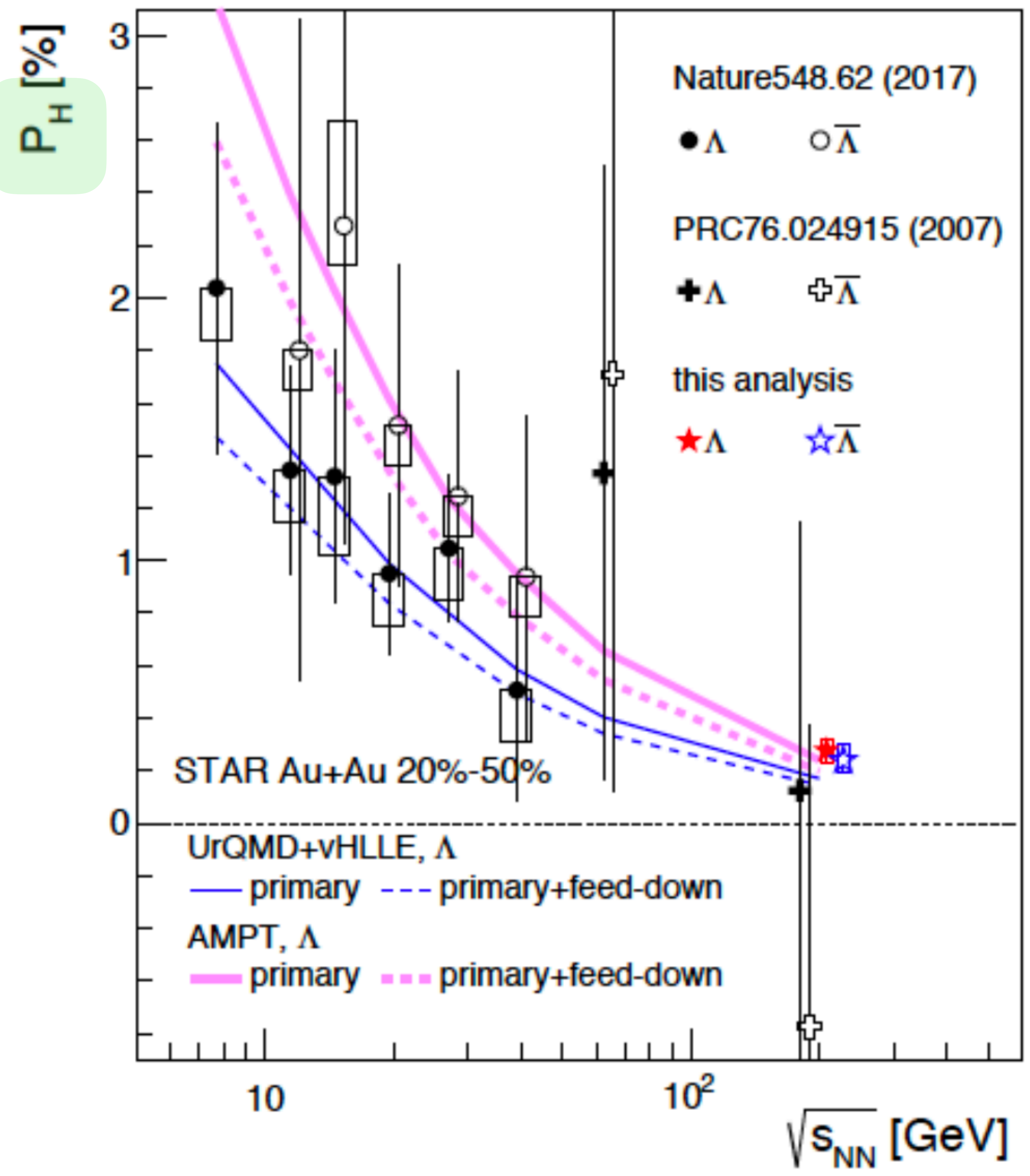
Global polarization data supports spin-thermal approach

Agrees well with predictions of transport and hydrodynamic models

UrQMD+vHLLE: Karpenko, Becattini, EPJC 77, 213 (2017)
AMPT: Li, Pang, Wang, and Xia, PRC 96, 054908 (2017)

$$P_H = -S^y_\varpi$$

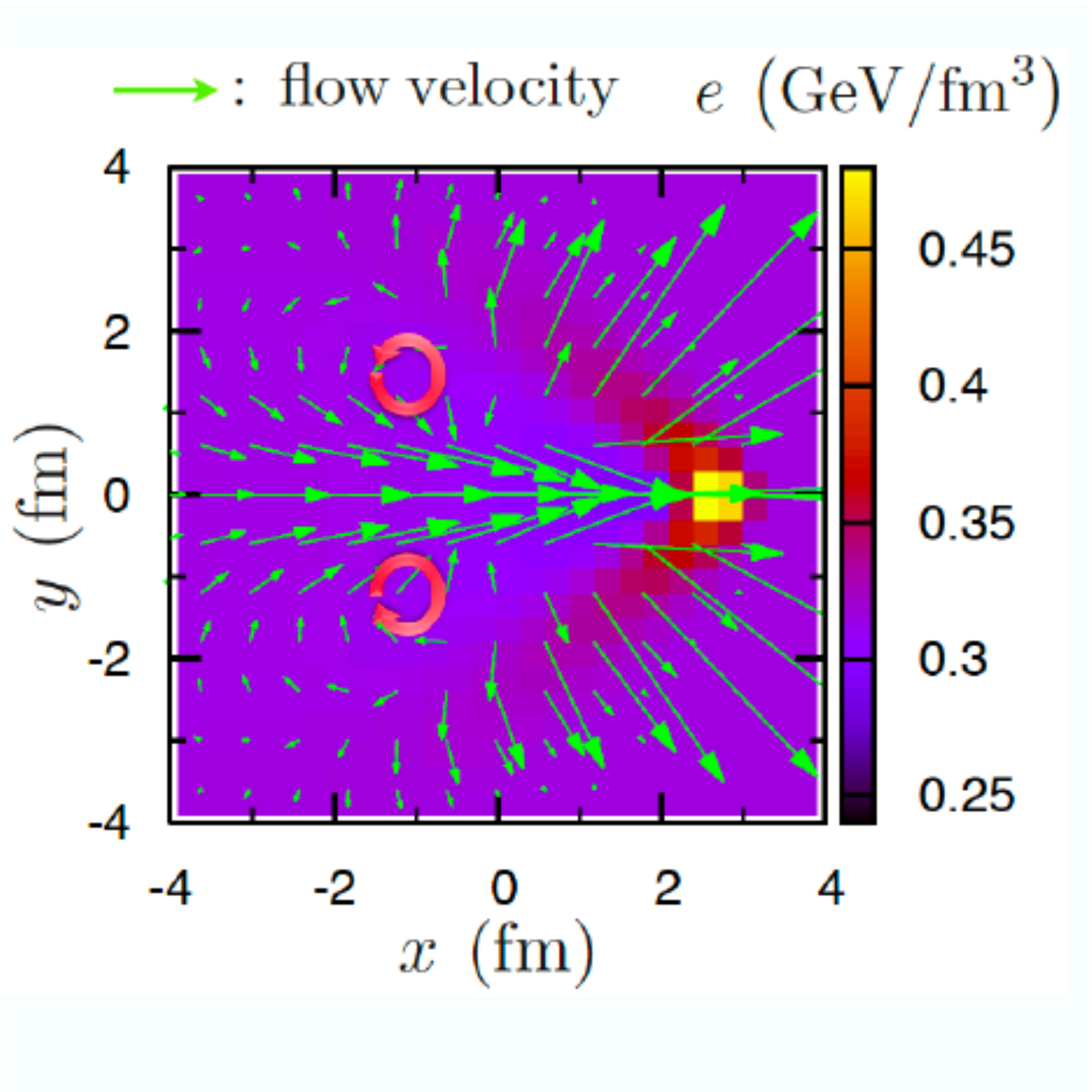
J. Adam et al. (STAR), Phys. Rev. C 98, 014910 (2018)

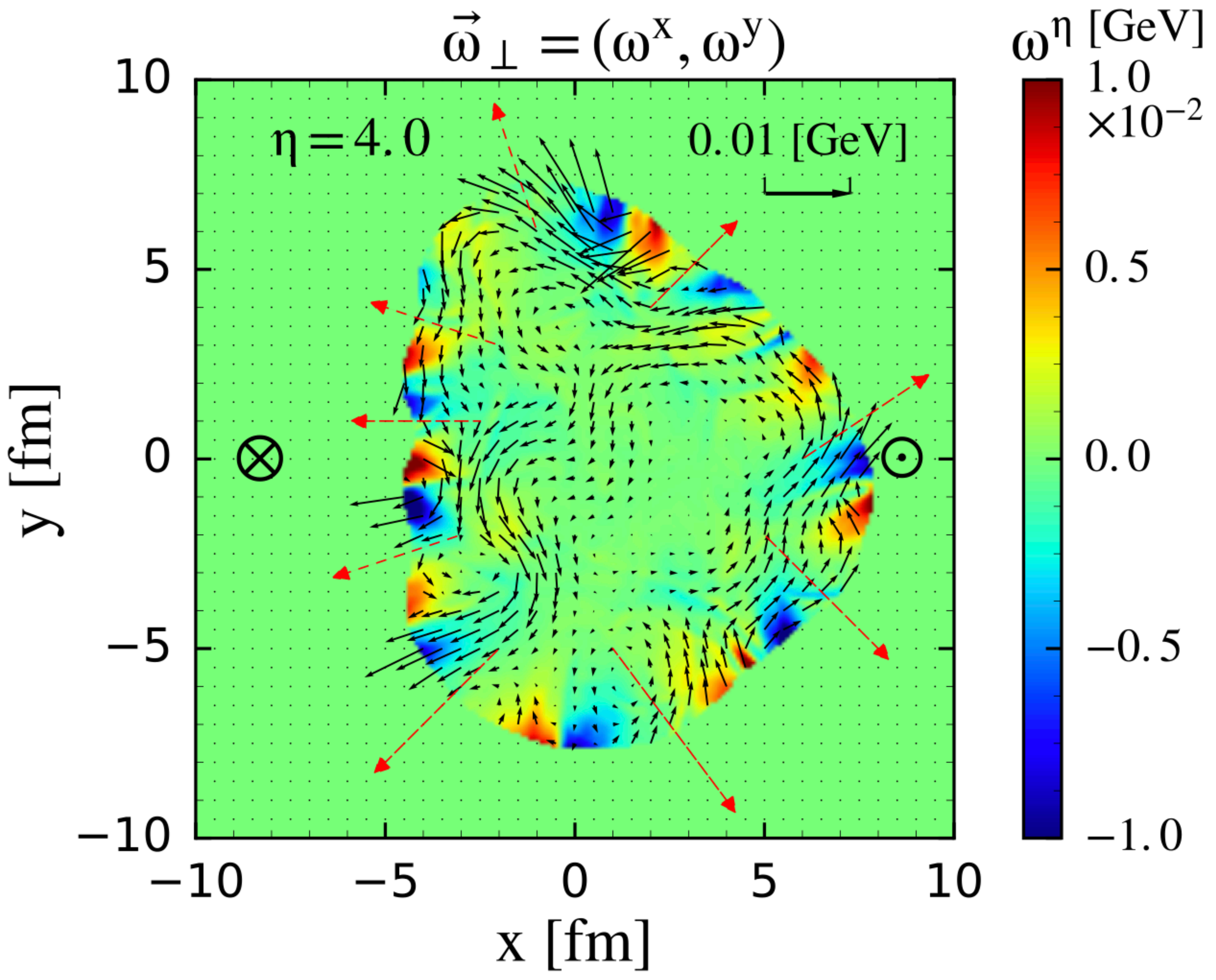


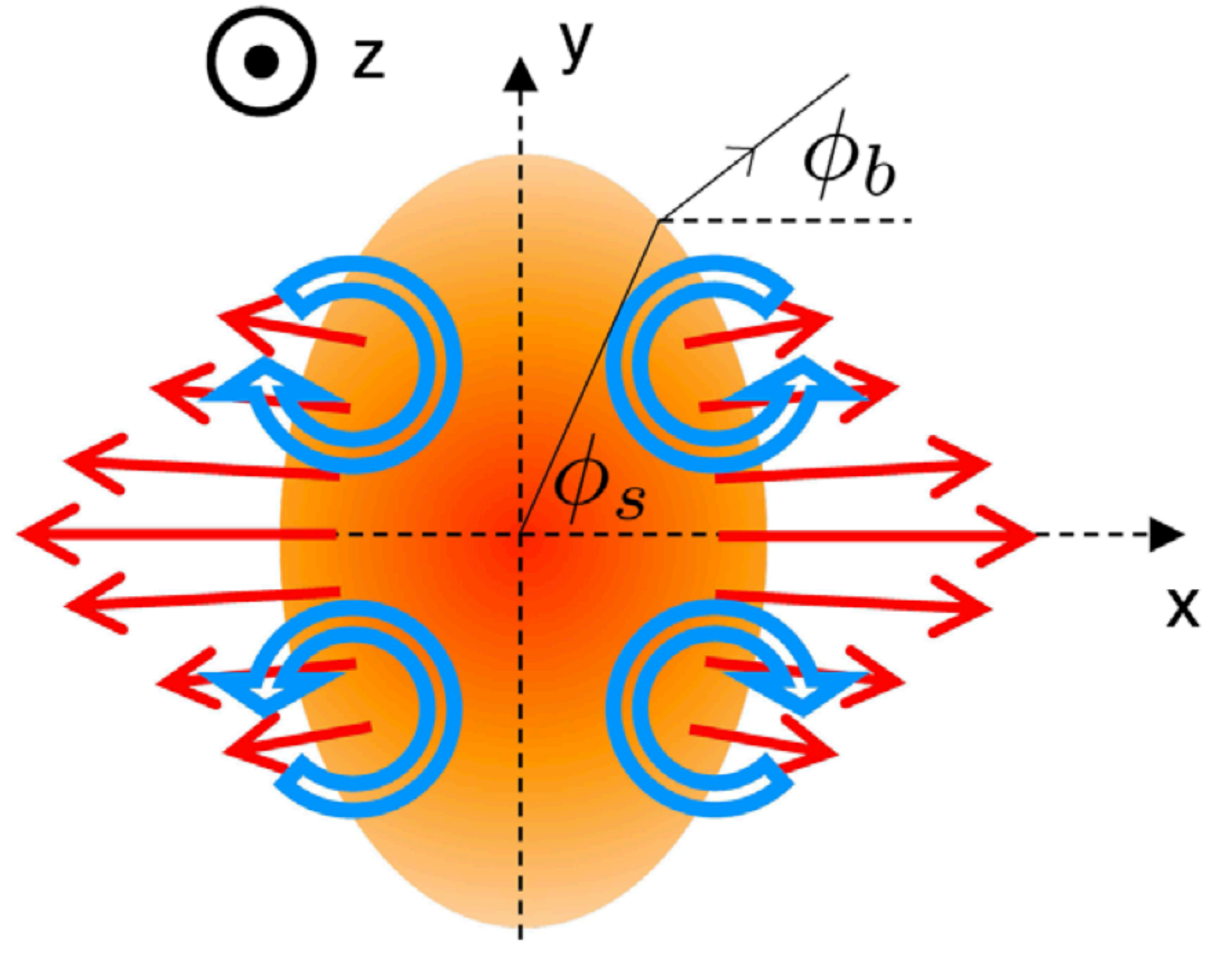
LONGITUDINAL POLARIZATION

Flow structures in the plane transverse to the beam (jets, ebe fluctuations, collision geometry, etc.) lead to longitudinal (beam-direction) polarization

Pang, Petersen, Wang, Wang, Phys.Rev.Lett. 117 (2016) 19, 192301







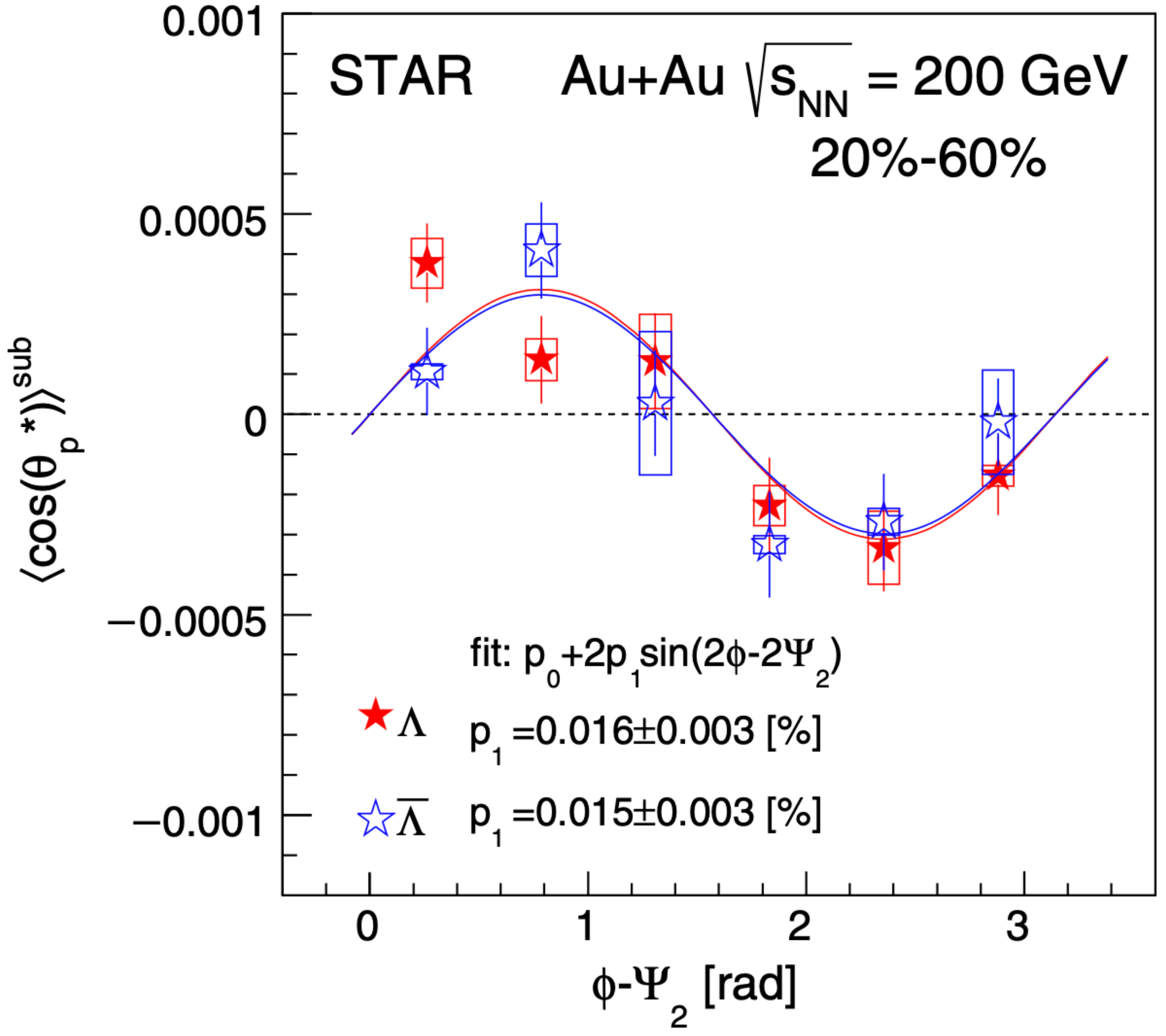
Tachibana, Hirano, Nucl.Phys.A 904-905 (2013) 1023c-1026c

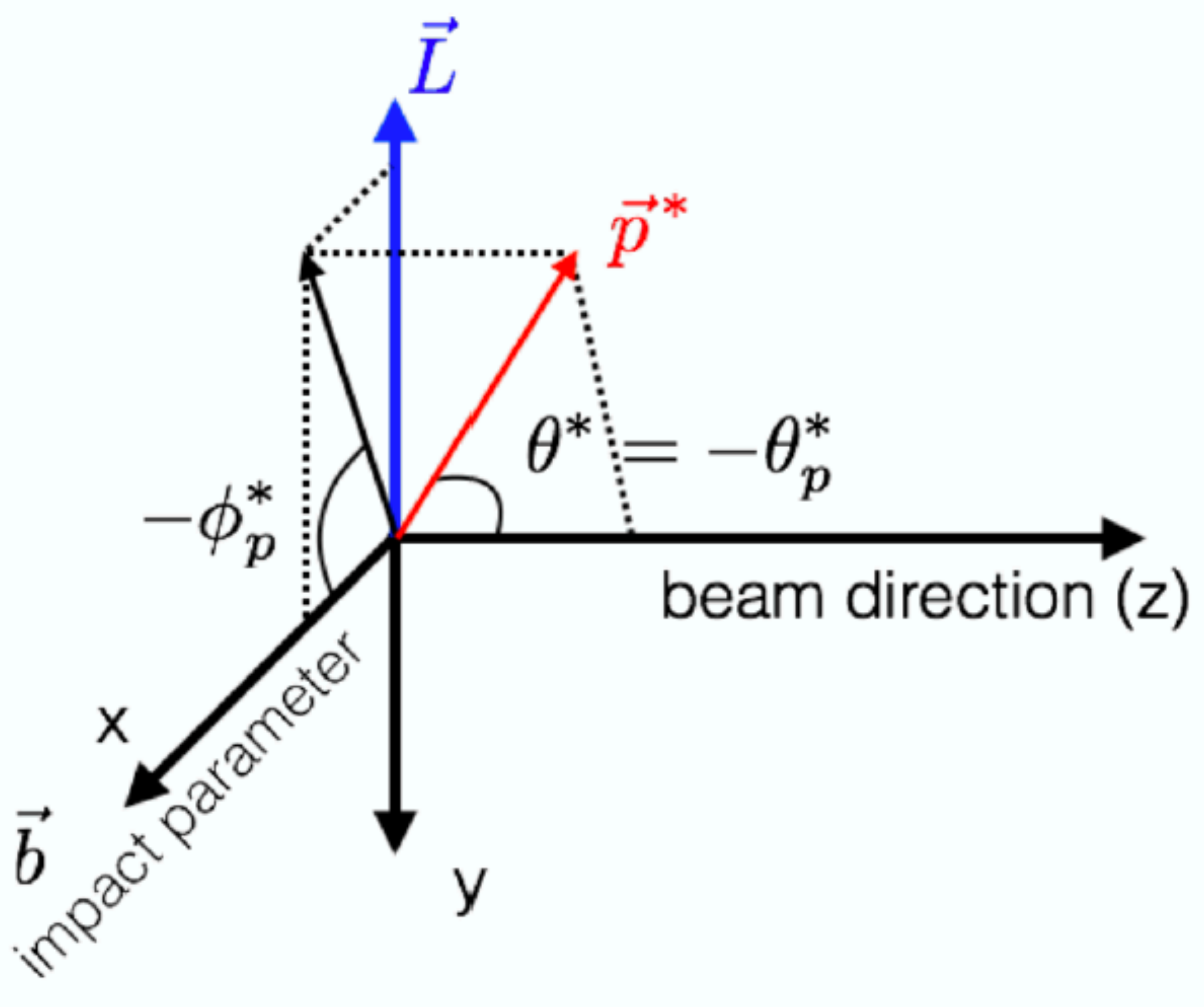
Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

MEASUREMENT OF Λ AND $\bar{\Lambda}$ LONGITUDINAL SPIN POLARIZATION

Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

Niida, The 5th Workshop on Chirality, Vorticity, and Magnetic Field in HIC, '19





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle(\cos\theta_p^*)^2\rangle$$

$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_H\langle(\cos\theta_p^*)^2\rangle}$$

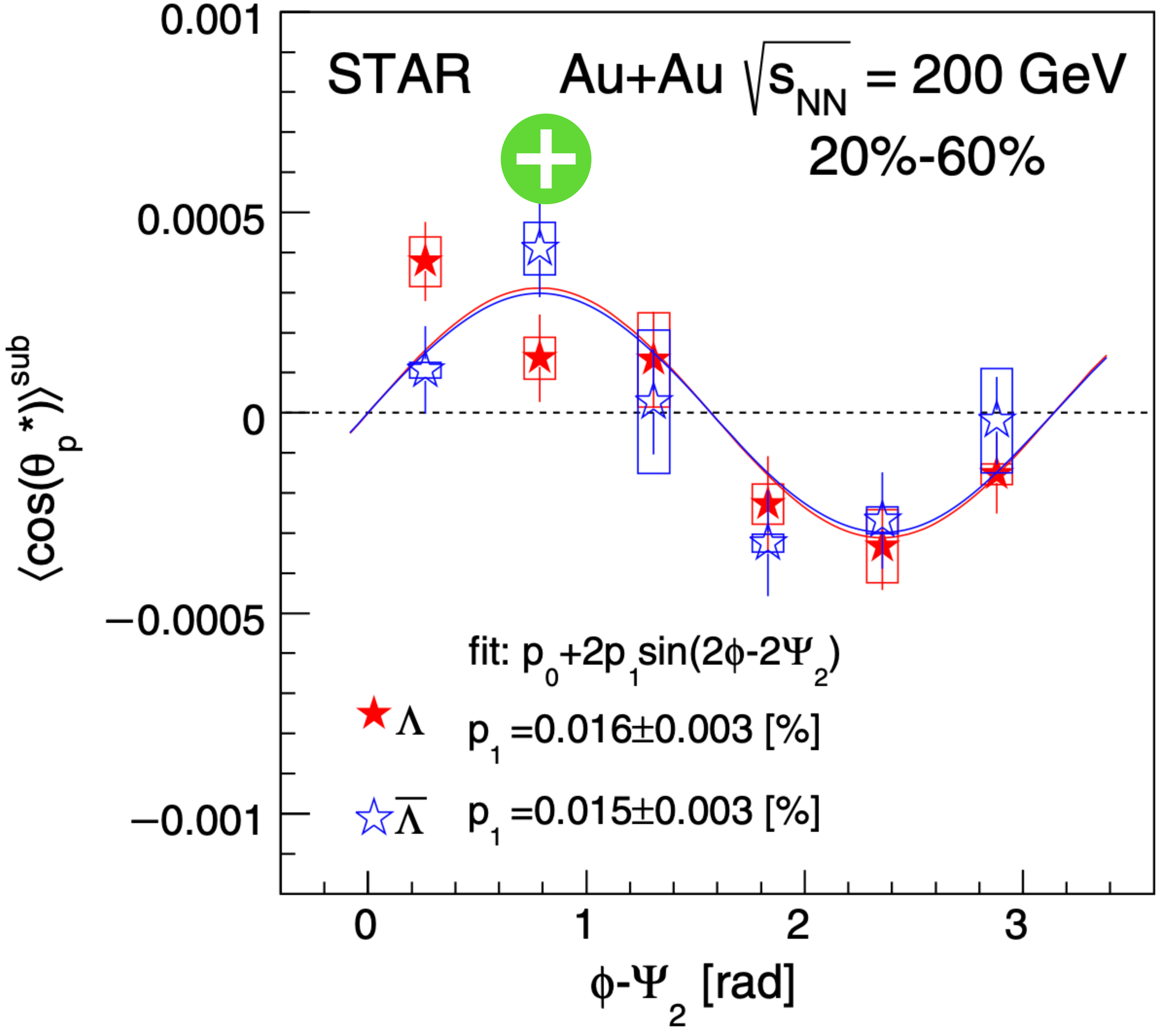
$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_H} \quad \text{(if perfect detector)}$$

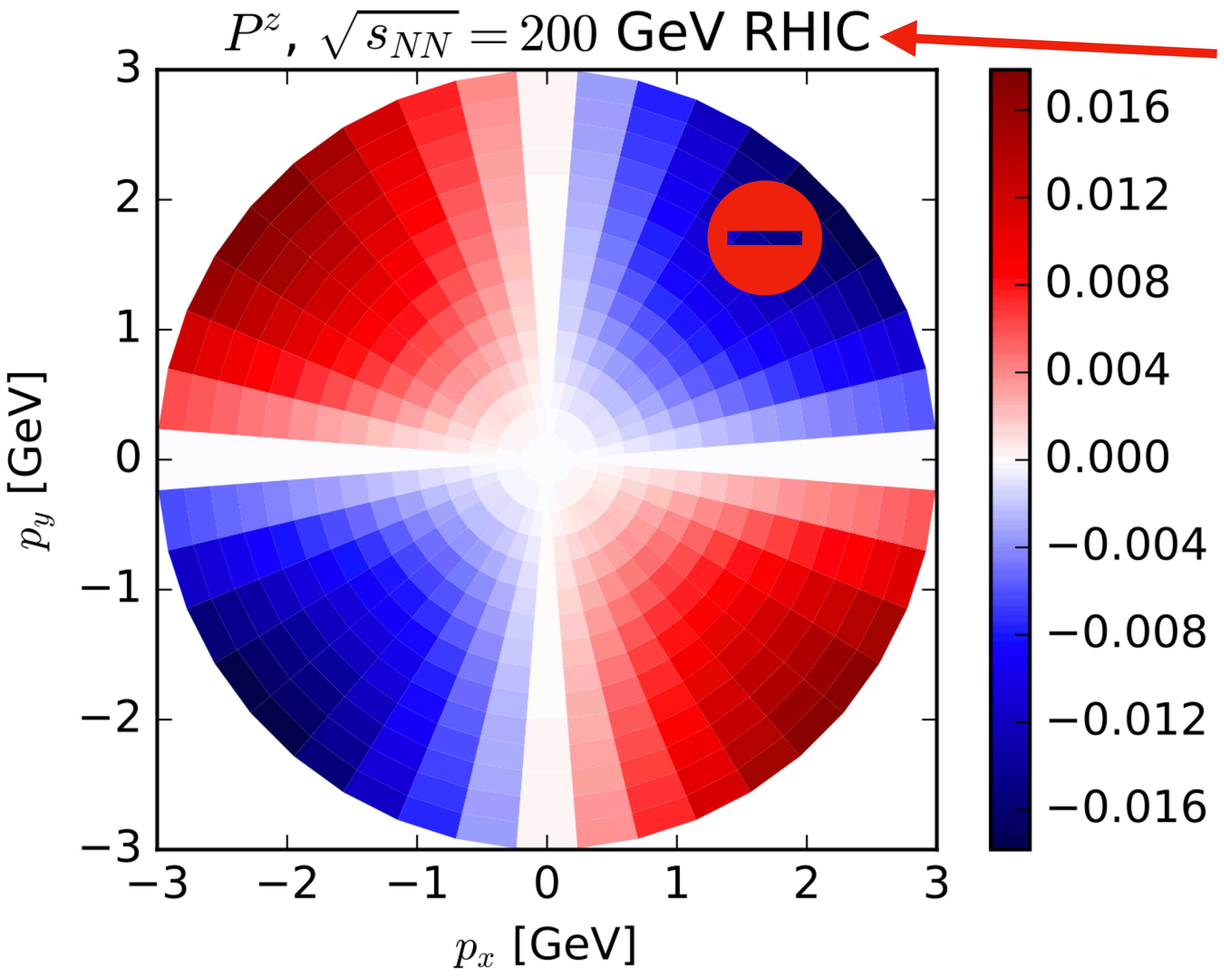
α_H: hyperon decay parameter

θ_p^*: θ of daughter proton in Λ rest frame

Longitudinal polarization – 'Spin sign' puzzle

Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

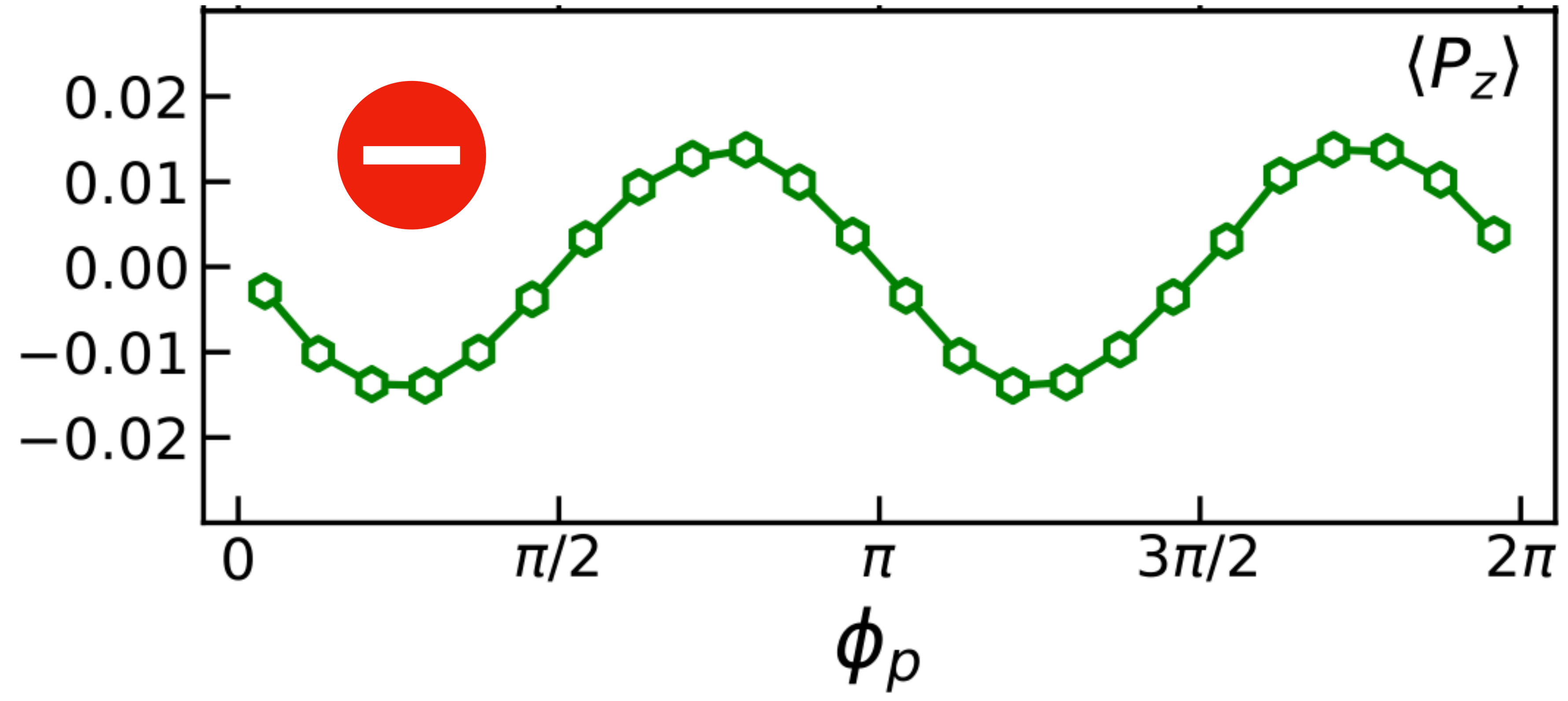




Becattini, Karpenko, Phys.Rev.Lett. 120 (2018) 1, 012302

$P^z = S^z_\varpi$

Hydrodynamics



Transport

Xia, Li, Tang, Wang, Phys.Rev.C 98 (2018) 024905

SPIN HYDRODYNAMICS – CURRENT STATUS

Spin-thermal approach does not describe the data properly

If spin polarization is trully hydrodynamic variable,
it should not be enslaved to thermal vorticity

Perfect spin hydrodynamics was proposed

Florkowski, Friman, Jaiswal, Speranza, Phys. Rev. C97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, RR, Speranza, Phys. Rev. D97 (2018) 116017

Spin hydrodynamics is being actively developed

Montenegro and Torrieri, Phys. Rev. D 100, 056011 (2019)
Bhadury, Florkowski, Jaiswal, Kumar, and R. R, Phys. Rev. Lett. 129, 192301 (2022)
Weickgenannt, Speranza, Sheng, Wang, and Rischke, Phys. Rev. Lett. 127, 052301 (2021)
Li, Stephanov, and Yee, Phys. Rev. Lett. 127, 082302 (2021)
Gallegos, Gursoy, and Yarom, JHEP 05, 139
Hongo, Huang, Kaminski, Stephanov, Yee JHEP 11, 150
…

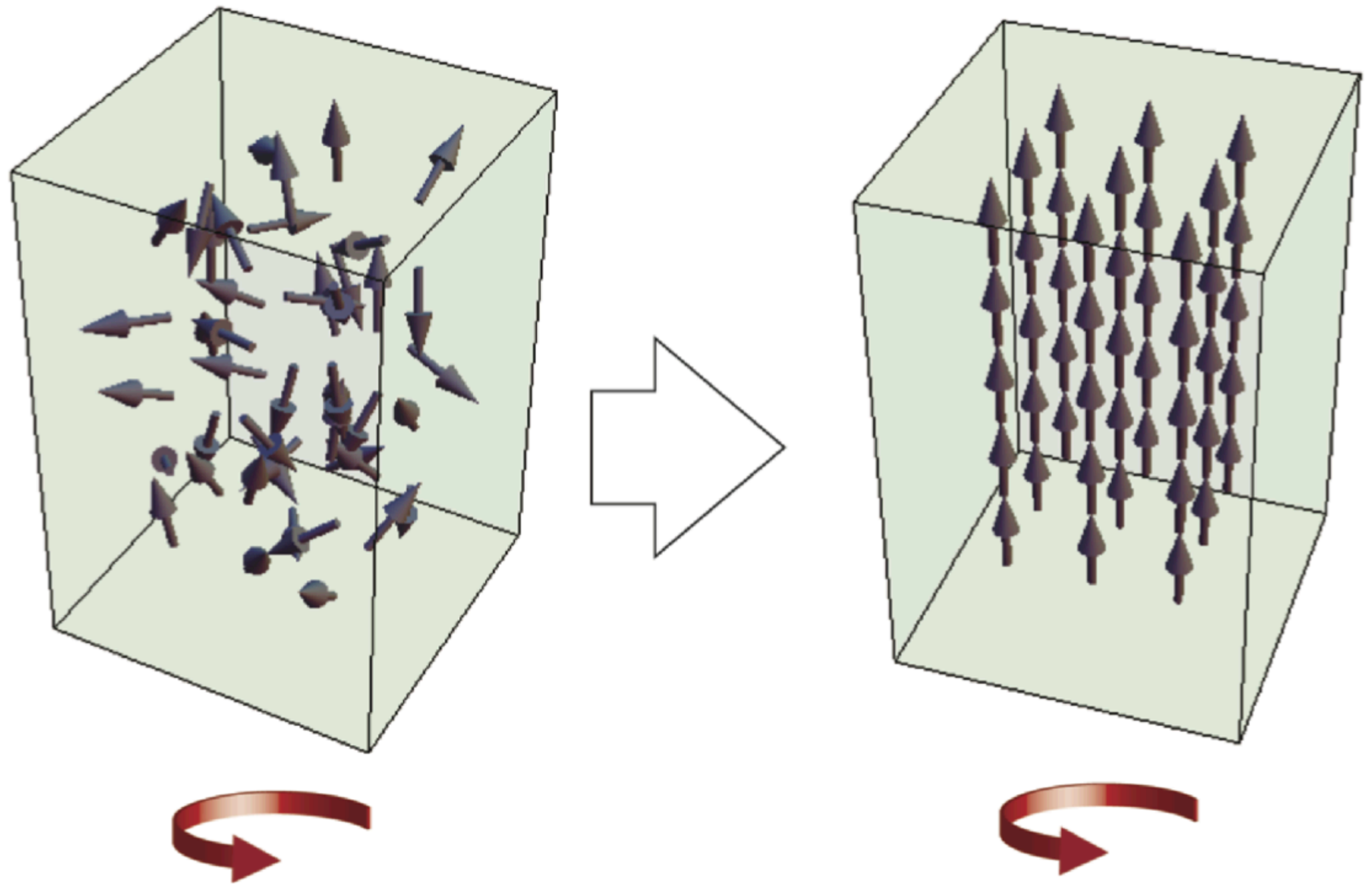
Future measurements are planned

Bondar and Florkowski, Acta Phys. Polon. B 55, 9 (2024)

Perfect spin hydrodynamics was studied in simple systems

No realistic modelling in 3+1D available so far

figure: Journal of the Physical Society of Japan 90, 081003 (2021)



THEORETICAL FRAMEWORK

Conservation Laws and Lagrange Multipliers

conservation laws + (near) local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \Rightarrow \quad \mu \equiv \xi T \qquad \text{(1 eq / charge)}$$

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

Conservation Laws and Lagrange Multipliers

conservation laws + (near) local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \rightarrow \quad \mu \equiv \xi T \qquad \text{(1 eq / charge)}$$

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

- conservation of angular momentum

spin chemical potential ↓ spin polarization tensor ↓

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu} \qquad \text{(6 eqs)}$$

Conservation Laws and Lagrange Multipliers

conservation laws + (near) local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \rightarrow \quad \mu \equiv \xi T \qquad \text{(1 eq / charge)}$$

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

- conservation of angular momentum

spin chemical potential ↓ spin polarization tensor ↓

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu} \qquad \text{(6 eqs)}$$

- constitutive relations

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Perfect spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

Perfect spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

The total angular momentum is decomposed into **orbital angular momentum** and intrinsic **spin tensor**

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

Perfect spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

The total angular momentum is decomposed into **orbital angular momentum** and intrinsic **spin tensor**

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

$$D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

PERFECT SPIN HYDRODYNAMICS

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

The total angular momentum is decomposed into **orbital angular momentum** and intrinsic **spin tensor**

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

$$D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \xrightarrow{T^{\nu\mu} = -T^{\mu\nu}} D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

For conserved **symmetric EMT** implies the conservation of the spin tensor

Perfect spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

The total angular momentum is decomposed into **orbital angular momentum** and intrinsic **spin tensor**

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

$$D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \xrightarrow{T^{\nu\mu} = -T^{\mu\nu}} D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

For conserved **symmetric EMT** implies the conservation of the spin tensor

From Quantum Kinetic Theory at linear order in spin polarization tensor (small polarization limit)
Florkowski, Kumar, and RR, Phys. Rev. C98, 044906 (2018)
Florkowski, RR, and Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$T^{\mu\nu} = T^{\mu\nu}[\beta,\xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta,\omega,\xi], \quad N^\mu = N^\mu[\beta,\xi]$$

Background hydrodynamics decouples from **spin hydrodynamics**!

Theoretical framework

Background hydrodynamics

BACKGROUND HYDRODYNAMICS

Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$\begin{aligned} D_\alpha T^{\alpha\beta}(x) &= 0 \\ D_\alpha N^{\alpha}(x) &= 0 \end{aligned}$$

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}[\beta, \xi], \\ N^{\mu} &= N^{\mu}[\beta, \xi], \end{aligned}$$

BACKGROUND HYDRODYNAMICS

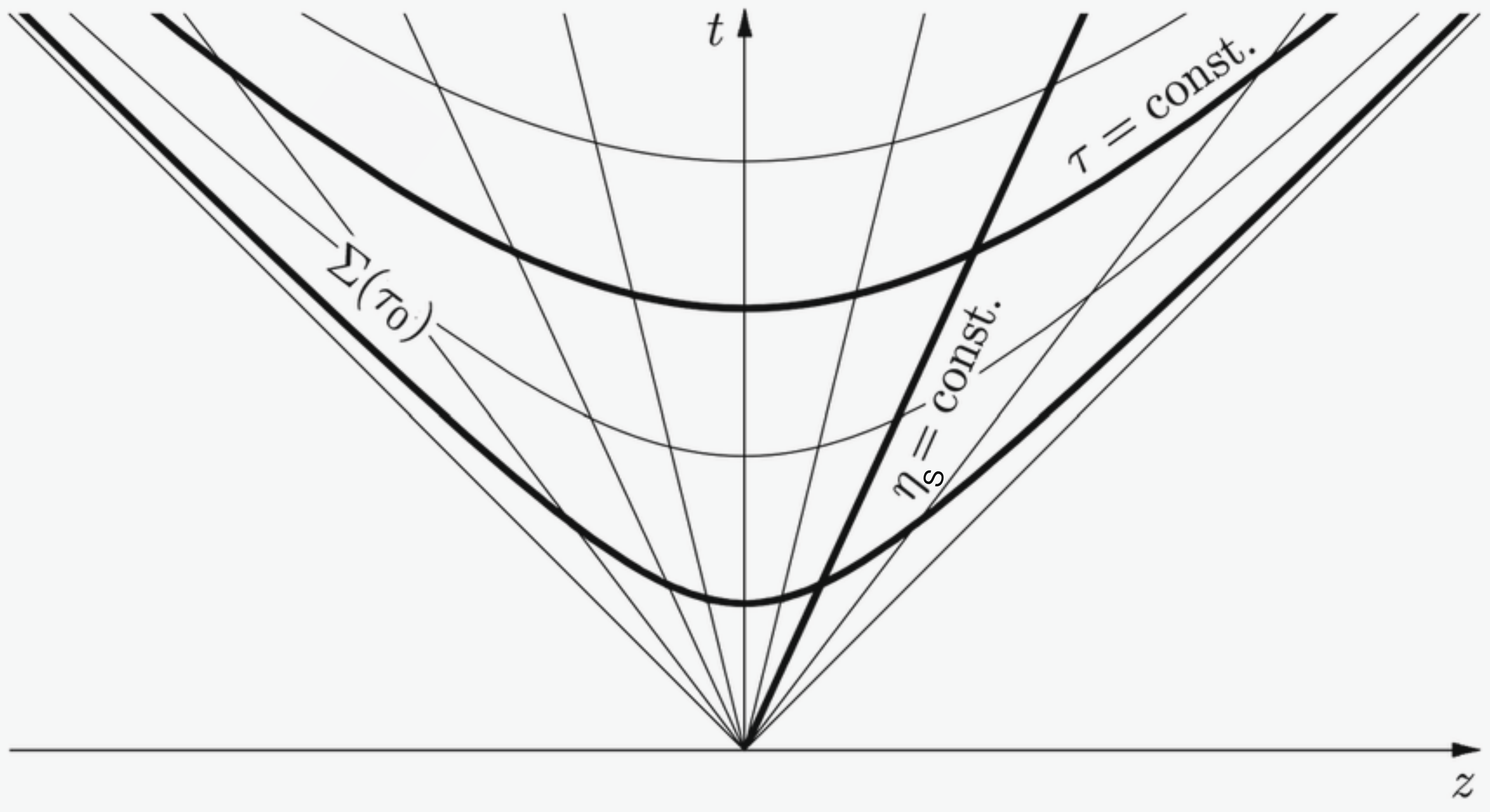
Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

We work in **curvilinear (Milne) coordinates** with **non-zero Christoffel symbols**

$$(t, x, y, z) \rightarrow (\tau, x, y, \eta_s)$$

$$t = \tau \cosh \eta_s \qquad \tau = \sqrt{t^2 - z^2}$$
$$\rightarrow$$
$$z = \tau \sinh \eta_s \qquad \eta_s = \tfrac{1}{2} \log\left(\tfrac{t+z}{t-z}\right)$$



Therefore we use the covariant derivative

$$\partial_\alpha \rightarrow D_\alpha$$

BACKGROUND HYDRODYNAMICS

Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$
\begin{aligned}
D_\alpha T^{\alpha\beta}(x) &= 0 \\
D_\alpha N^{\alpha}(x) &= 0
\end{aligned}
$$

We adopt **Landau's definition of flow four-velocity**

$$T^{\alpha\beta} u_\beta = \varepsilon\, u^\alpha$$

BACKGROUND HYDRODYNAMICS

Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

We adopt **Landau's definition of flow four-velocity**

$$T^{\alpha\beta} u_\beta = \varepsilon\, u^\alpha$$

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta} \quad \text{(energy-momentum tensor, EMT)}$$
$$N^\alpha = n\, u^\alpha + n^\alpha \quad \text{(net baryon current)}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

Our notation is

ε	(energy density)
$P_{\rm eq}$	(equilibrium pressure)
n	(baryon number density)
$\Delta^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta$	(projector onto the subspace orthogonal to flow)

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

Must be supplemented by **equation of state (EOS)** relating pressure to energy density and baryon density

$$P_{\rm eq} = P_{\rm eq}(\varepsilon, n)$$

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

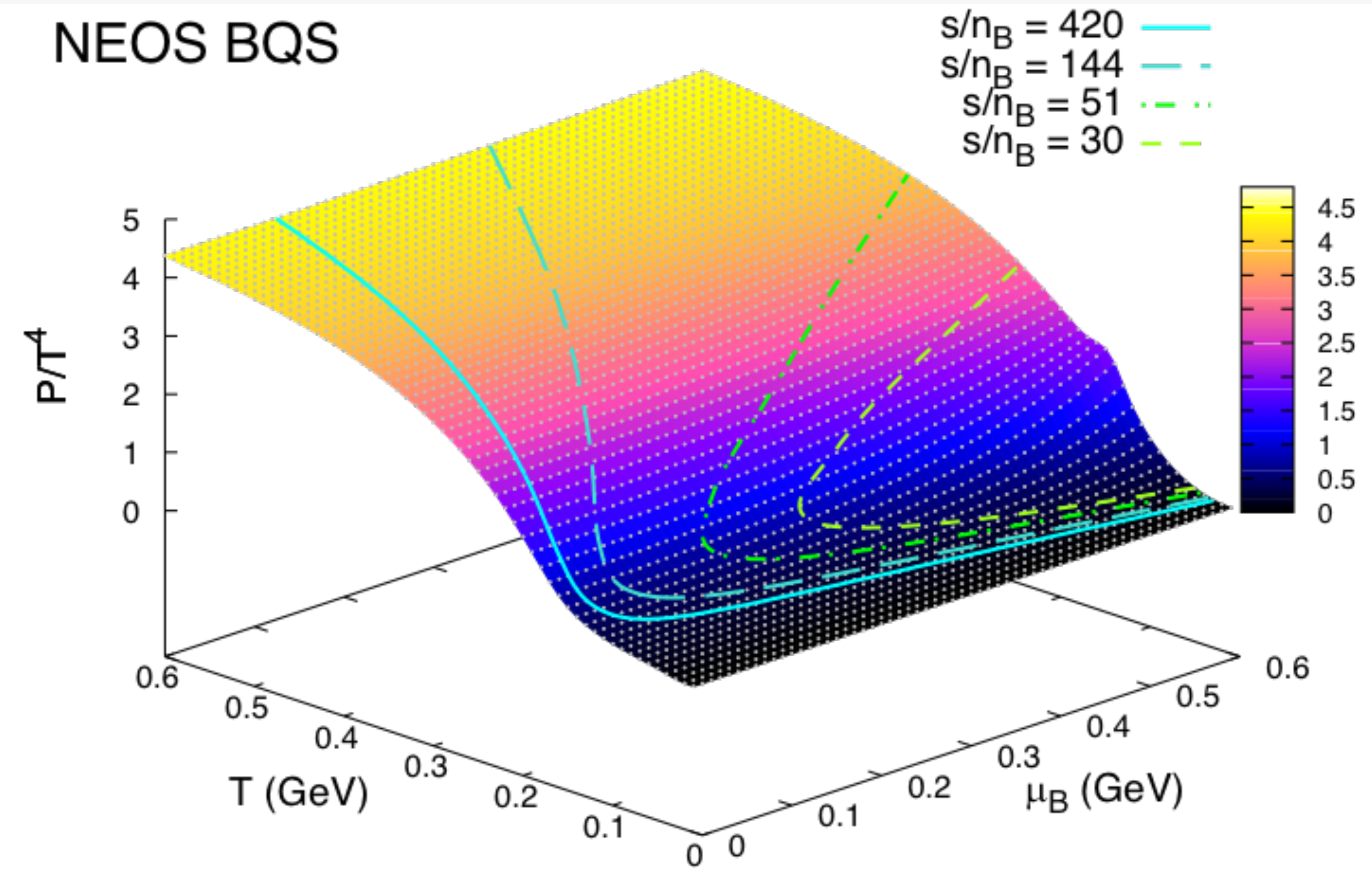
$$N^\alpha = n\, u^\alpha + n^\alpha$$

Must be supplemented by **equation of state (EOS)** relating pressure to energy density and baryon density

$$P_{\rm eq} = P_{\rm eq}(\varepsilon, n)$$

We use **lattice-QCD-based EOS at finite net baryon density** which exhibits a crossover phase transition across the entire parametric space of the phase diagram

Monnai, Schenke, and Shen, Phys. Rev. C 100, 024907 (2019)
Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)



Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **dissipative currents** are

Π (bulk pressure)

$\pi^{\alpha\beta}$ (shear-stress tensor)

n^α (charge diffusion current)

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **dissipative currents** are

Π (bulk pressure)

$\pi^{\alpha\beta}$ (shear-stress tensor)

n^α (charge diffusion current)

We neglect the charge diffusion current!

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\,\sigma^{\alpha\beta}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\,\sigma^{\alpha\beta}$$

They are entirely determined by the spacetime gradients of the flow

$$\theta \equiv D\cdot u$$ (expansion scalar)

$$\sigma^{\alpha\beta} \equiv D^{\langle\gamma}u^{\delta\rangle} \equiv \Delta^{\alpha\beta}_{\gamma\delta} D^\gamma u^\delta$$ (shear-flow tensor)

$$\Delta^{\alpha\beta}_{\gamma\delta} \equiv \frac{1}{2}\left[\Delta^\alpha_\gamma\Delta^\beta_\delta + \Delta^\alpha_\delta\Delta^\beta_\gamma - (2/3)\Delta^{\alpha\beta}\Delta_{\gamma\delta}\right]$$ (projector selecting symmetric, traceless, and orthogonal part relative to flow)

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^{\alpha}\, u^{\beta} - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^{\alpha} = n\, u^{\alpha} + n^{\alpha}$$

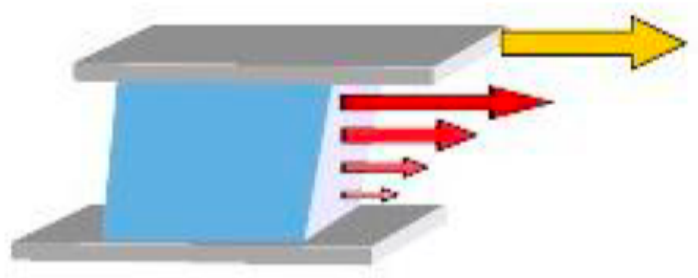
The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

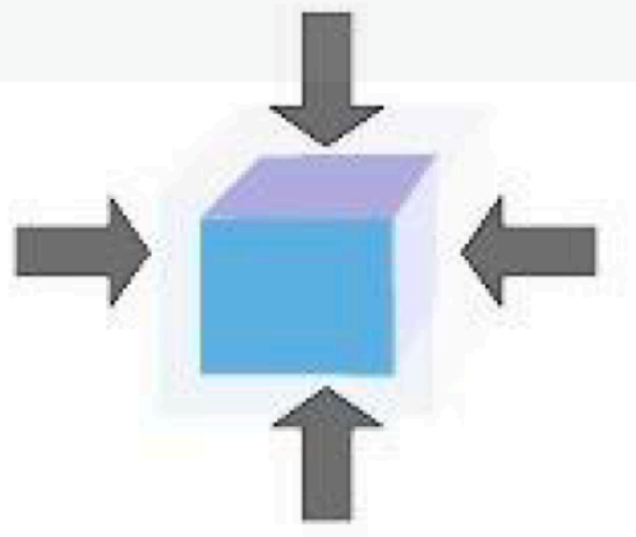
$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\,\sigma^{\alpha\beta}$$

The **bulk and shear viscosities** are given by

Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014);
Denicol, Gale, Jeon, Monnai, Schenke, and Shen, Phys. Rev. C 98, 034916 (2018)

$$\eta = C_\eta \frac{\varepsilon_{\rm eq} + P_{\rm eq}}{T}, \quad \zeta = 75\eta \left(\frac{1}{3} - c_s^2\right)^2 \qquad C_\eta = 0.12$$





BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\, \sigma^{\alpha\beta}$$

The **bulk and shear viscosities** are given by

Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014);
Denicol, Gale, Jeon, Monnai, Schenke, and Shen, Phys. Rev. C 98, 034916 (2018)

$$\eta = C_\eta \frac{\varepsilon_{\rm eq} + P_{\rm eq}}{T}, \quad \zeta = 75\eta \left(\frac{1}{3} - c_s^2\right)^2 \qquad C_\eta = 0.12$$

The **speed of sound** is obtained from

Monnai, Schenke, and Shen, Phys. Rev. C 100, 024907 (2019)

$$c_s^2 = \left.\frac{\partial P_{\rm eq}}{\partial \varepsilon_{\rm eq}}\right|_{n_{\rm eq}} + \frac{n_{\rm eq}}{\varepsilon_{\rm eq} + P_{\rm eq}} \left.\frac{\partial P_{\rm eq}}{\partial n_{\rm eq}}\right|_{\varepsilon_{\rm eq}}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\, \sigma^{\alpha\beta}$$

Landau (relativistic Navier-Stokes) theory is acausal!

Hiscock and Lindblom, Annals Phys. 151, 466 (1983).
Denicol, Kodama, Koide, and Mota, J. Phys. G 35, 115102 (2008

Lev Landau, MIPT History Museum

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

At **second order** in spacetime gradients, in DNMR framework, the **time evolution of the dissipative currents** is

Denicol, Niemi, Molnar, and Rischke, Phys. Rev. D 85, 114047 (2012);
Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014).

$$\dot{\Pi} = \frac{\Pi_{\rm NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\alpha\beta}\sigma_{\alpha\beta}$$

$$\dot{\pi}^{\langle\alpha\beta\rangle} = \frac{\pi^{\alpha\beta}_{\rm NS} - \pi^{\alpha\beta}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\alpha\beta}\theta + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\alpha\beta} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\gamma^{\langle\alpha}\sigma^{\beta\rangle\gamma} + \frac{\phi_7}{\tau_\pi}\pi_\gamma^{\langle\alpha}\pi^{\beta\rangle\gamma}$$

Where the comoving derivative is

$$\dot{(\ )} \equiv u \cdot D$$

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

At **second order** in spacetime gradients, in DNMR framework, the **time evolution of the dissipative currents** is

Denicol, Niemi, Molnar, and Rischke, Phys. Rev. D 85, 114047 (2012);
Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014).

$$\dot{\Pi} = \frac{\Pi_{\rm NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\alpha\beta}\sigma_{\alpha\beta}$$

$$\dot{\pi}^{\langle\alpha\beta\rangle} = \frac{\pi^{\alpha\beta}_{\rm NS} - \pi^{\alpha\beta}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\alpha\beta}\theta + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\alpha\beta} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\gamma^{\langle\alpha}\sigma^{\beta\rangle\gamma} + \frac{\phi_7}{\tau_\pi}\pi_\gamma^{\langle\alpha}\pi^{\beta\rangle\gamma}$$

The **second-order transport coefficients** are

Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014);

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}, \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) \quad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}, \quad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}, \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}, \quad \frac{\phi_7}{\tau_\pi} = \frac{9}{70 P_{\rm eq}\tau_\pi}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^\alpha = n\, u^\alpha + n^\alpha$$

At **second order** in spacetime gradients, in DNMR framework, the **time evolution of the dissipative currents** is

Denicol, Niemi, Molnar, and Rischke, Phys. Rev. D 85, 114047 (2012);
Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014).

$$\dot{\Pi} = \frac{\Pi_{\rm NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\alpha\beta}\sigma_{\alpha\beta}$$

$$\dot{\pi}^{\langle\alpha\beta\rangle} = \frac{\pi^{\alpha\beta}_{\rm NS} - \pi^{\alpha\beta}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\alpha\beta}\theta + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\alpha\beta} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\gamma^{\langle\alpha}\sigma^{\beta\rangle\gamma} + \frac{\phi_7}{\tau_\pi}\pi_\gamma^{\langle\alpha}\pi^{\beta\rangle\gamma}$$

Relaxation times are chosen as follows

$$\tau_\pi = \tau_\Pi = \frac{5C_\eta}{T}$$

Theoretical framework

Spin hydrodynamics

SPIN HYDRODYNAMICS

The **conservation law for total angular momentum** is

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

Spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

In the **de Groot—van Leeuven—van Weert (GLW) pseudogauge** the spin tensor for spin 1/2 is expressed as

Florkowski, Kumar, and R. R., Phys. Rev. C98, 044906 (2018)
de Groot, van Leeuwen, and van Weert, Relativistic Kinetic Theory: Principles and Applications

$$S^{\alpha,\beta\gamma} = \mathcal{A}_1 u^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 u^\alpha u^{[\beta}\omega^{\gamma]\delta} u_\delta + \mathcal{A}_3 \left(u^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\omega^{\gamma]\delta} u_\delta \right)$$

Spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

In the **de Groot—van Leeuven—van Weert (GLW) pseudogauge** the spin tensor for spin 1/2 is expressed as

Florkowski, Kumar, and R. R., Phys. Rev. C98, 044906 (2018)
de Groot, van Leeuwen, and van Weert, Relativistic Kinetic Theory: Principles and Applications

$$S^{\alpha,\beta\gamma} = \mathcal{A}_1 u^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 u^\alpha u^{[\beta}\omega^{\gamma]\delta} u_\delta + \mathcal{A}_3 \left(u^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\omega^{\gamma]\delta} u_\delta\right)$$

$$\mathcal{A}_1 = \cosh\left(\frac{\mu_B}{T}\right)\frac{T^3}{\pi^2}\left[\left(4+\frac{z^2}{2}\right)K_2(z) + zK_1(z)\right],$$

$$\mathcal{A}_2 = 2\cosh\left(\frac{\mu_B}{T}\right)\frac{T^3}{\pi^2}\left[\left(12+\frac{z^2}{2}\right)K_2(z) + 3zK_1(z)\right], \qquad z \equiv m/T$$

$$\mathcal{A}_3 = \frac{1}{2}\left(\mathcal{A}_1 - \frac{\mathcal{A}_2}{2}\right),$$

$$\mathcal{A}_i = \mathcal{A}_i(\mu_B, T; m)$$

Spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

In the **de Groot—van Leeuven—van Weert (GLW) pseudogauge** the spin tensor for spin 1/2 is expressed as

Florkowski, Kumar, and R. R., Phys. Rev. C98, 044906 (2018)
de Groot, van Leeuwen, and van Weert, Relativistic Kinetic Theory: Principles and Applications

$$S^{\alpha,\beta\gamma} = \mathcal{A}_1 u^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 u^\alpha u^{[\beta}\omega^{\gamma]\delta}u_\delta + \mathcal{A}_3 \left(u^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\omega^{\gamma]\delta}u_\delta\right)$$

$$\omega^{\beta\gamma} = -\omega^{\gamma\beta}$$ (spin polarization tensor)

Numerical framework

Background hydrodynamics

BACKGROUND HYDRODYNAMICS

We use **(Milne) coordinates**

$$(t, x, y, z) \quad \rightarrow \quad (\tau, x, y, \eta_s)$$

$$t = \tau \cosh \eta_s \qquad \tau = \sqrt{t^2 - z^2}$$

$$\rightarrow$$

$$z = \tau \sinh \eta_s \qquad \eta_s = \frac{1}{2} \log \left(\frac{t+z}{t-z} \right)$$

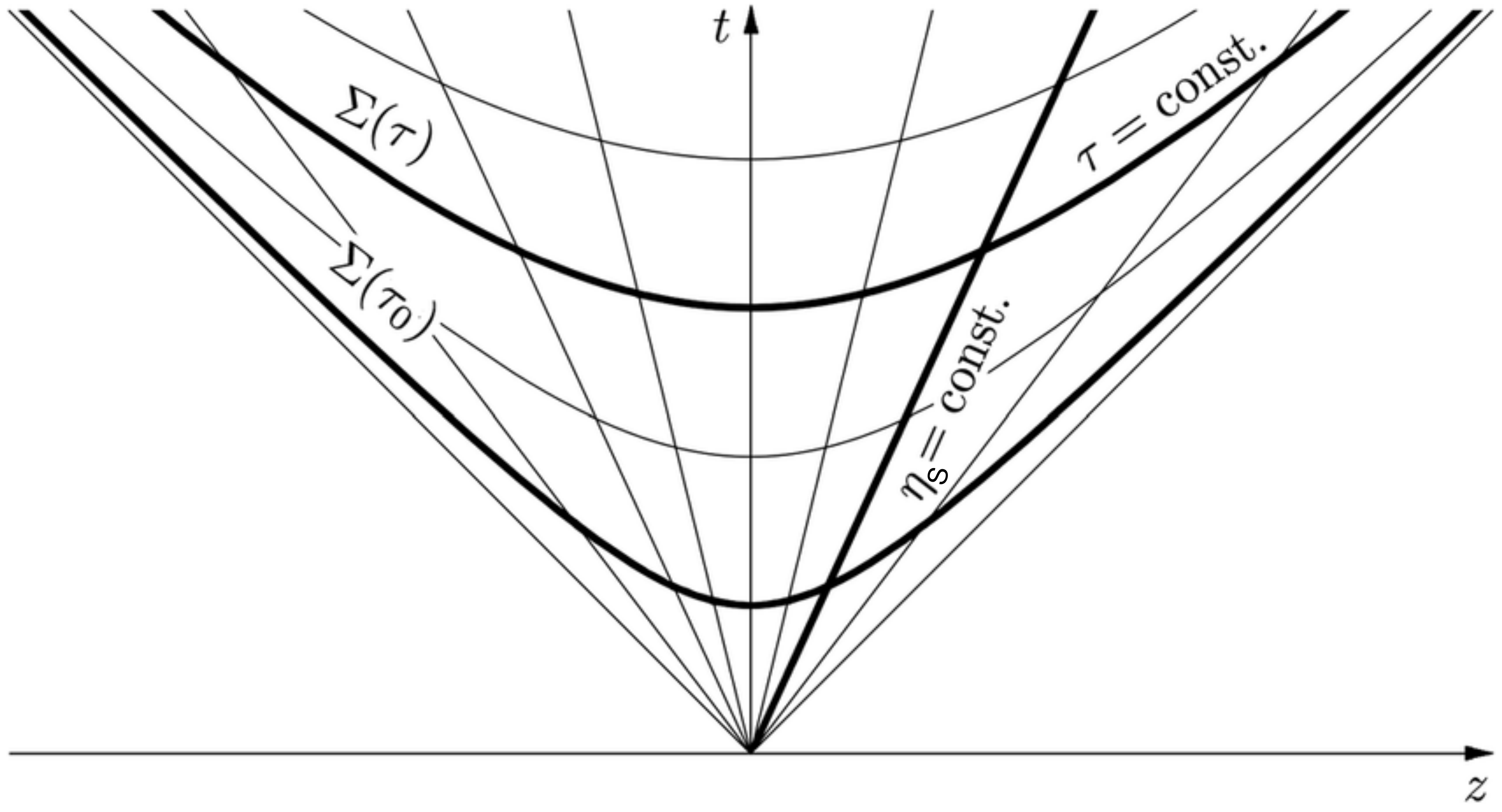


fig: Rindori, Tinti, Becattini, Rischke, Phys.Rev.D 105 (2022) 5, 056003

Background hydrodynamics

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

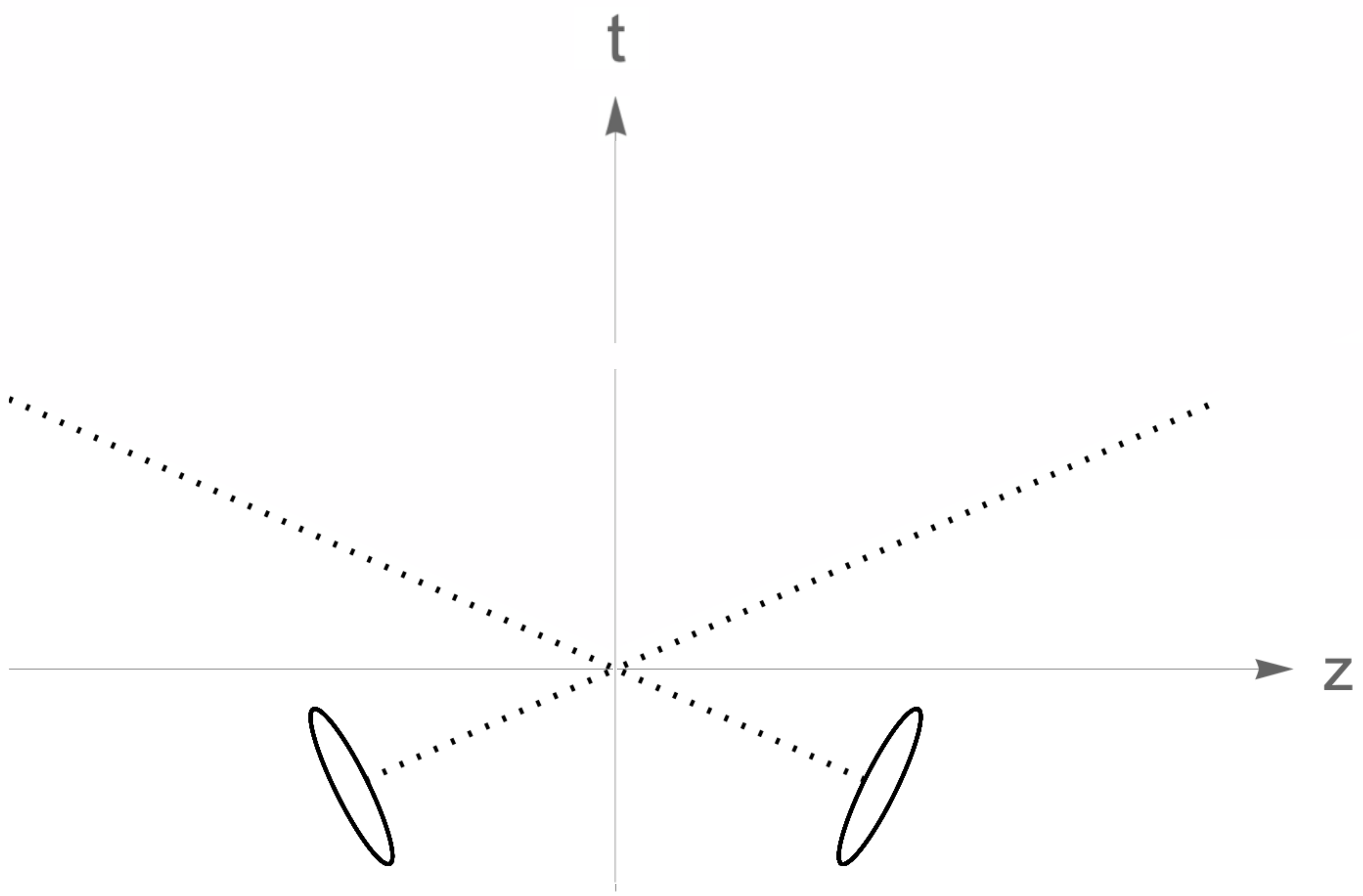


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

Initialize the background evolution at the proper time $\tau_0 = 1\ \mathrm{fm}$

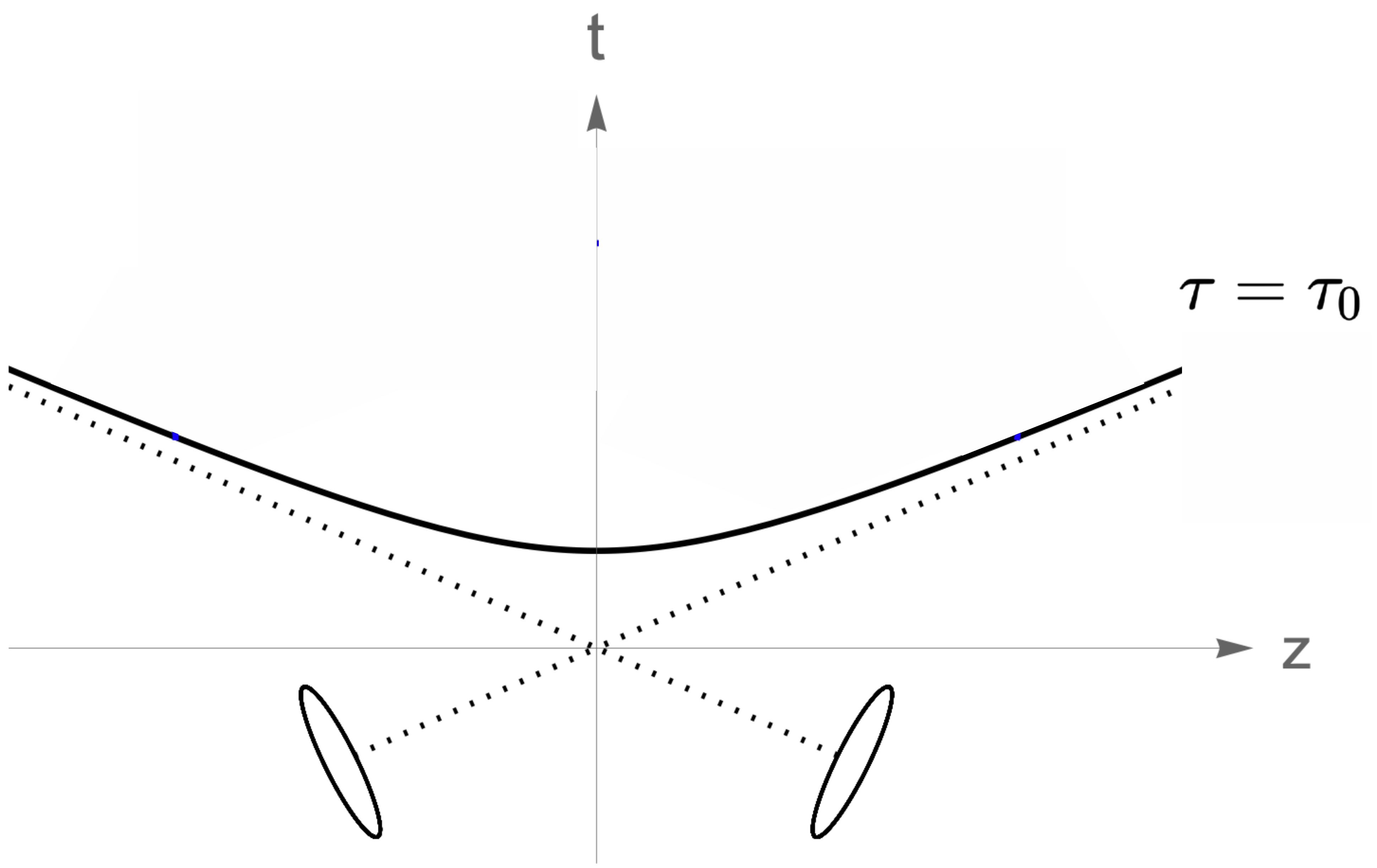


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

Initialize the background evolution at the proper time $\tau_0 = 1\ \mathrm{fm}$

The initial **energy density and baryon density profiles** are set according to the model based on the Glauber collision geometry with local energy-momentum conservation

Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)
Ryu, Jupic, and Shen, Phys. Rev. C 104, 054908 (2021)

However, to compute the thickness functions and wounded nucleon densities we use the **optical limit of the Glauber model**

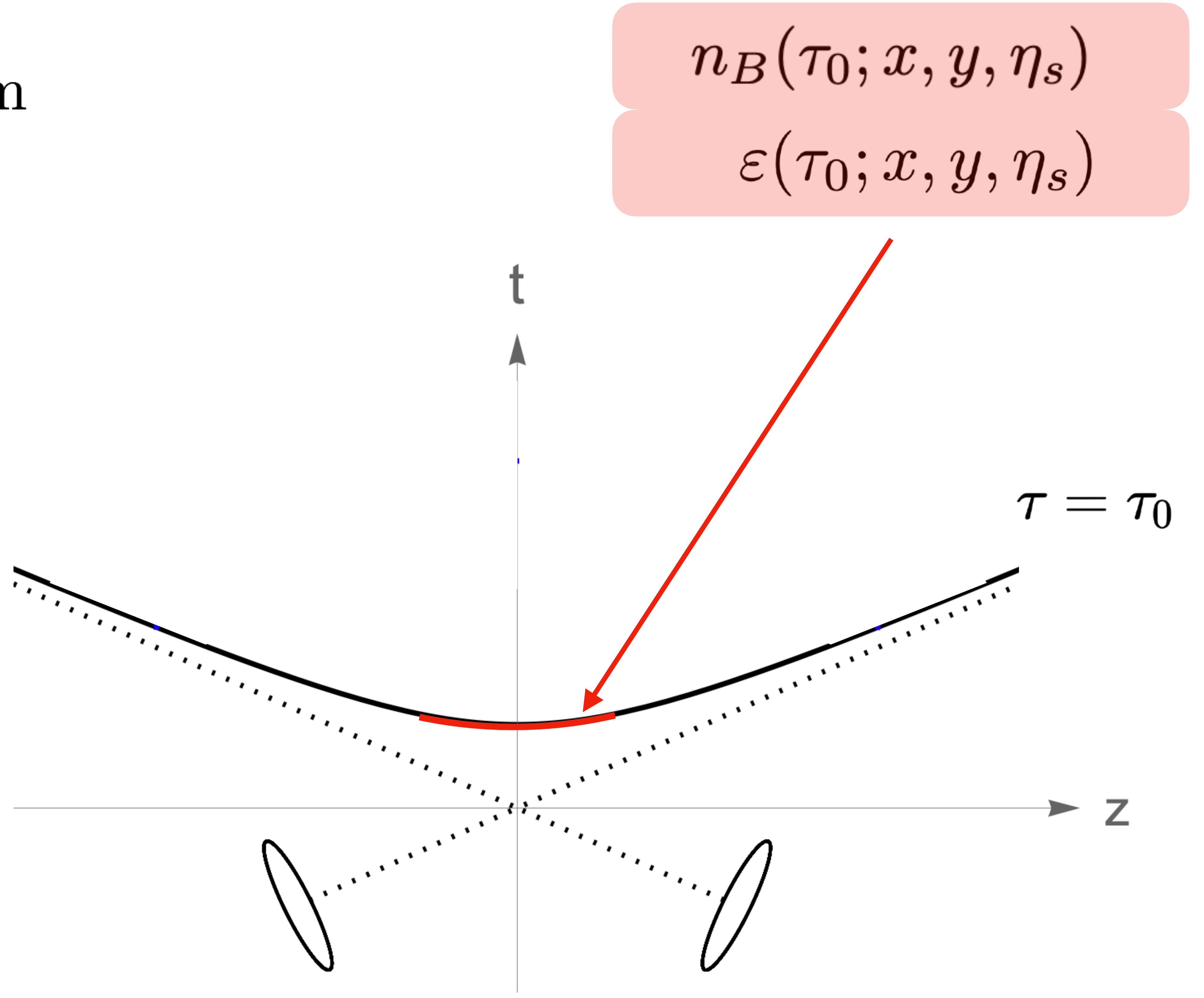


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

Initialize the background evolution at the proper time $\tau_0 = 1\ \mathrm{fm}$

The initial **energy density and baryon density profiles** are set according to the model based on the Glauber collision geometry with local energy-momentum conservation

Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)
Ryu, Jupic, and Shen, Phys. Rev. C 104, 054908 (2021)

However, to compute the thickness functions and wounded nucleon densities we use the **optical limit of the Glauber model**

The **longitudinal flow** is numerically determined from the initial energy-momentum tensor components

The **initial transverse flow components are zero**

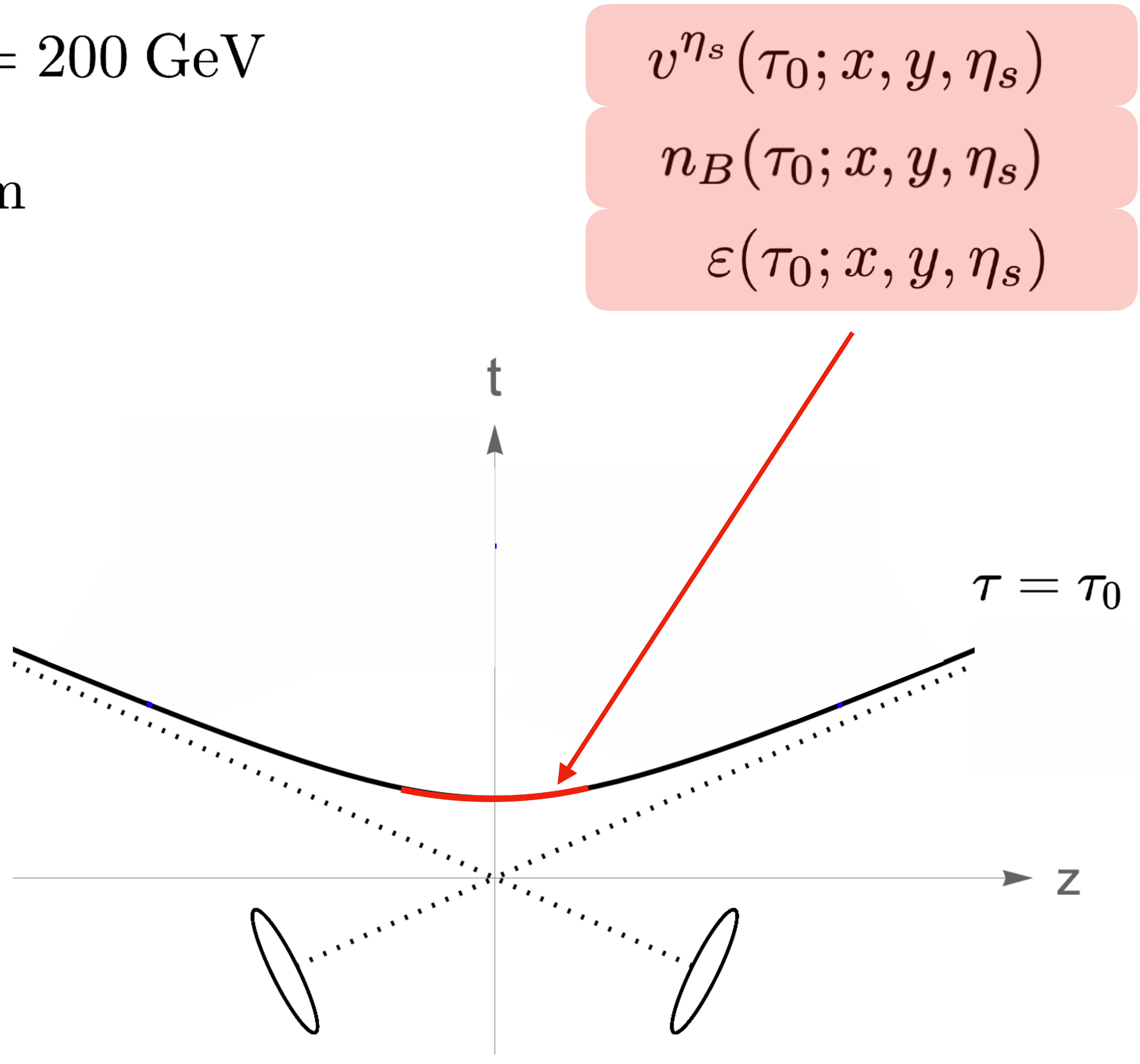


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

Initialize the background evolution at the proper time $\tau_0 = 1\ \mathrm{fm}$

The initial **energy density and baryon density profiles** are set according to the model based on the Glauber collision geometry with local energy-momentum conservation

Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)
Ryu, Jupic, and Shen, Phys. Rev. C 104, 054908 (2021)

However, to compute the thickness functions and wounded nucleon densities we use the **optical limit of the Glauber model**

The **longitudinal flow** is numerically determined from the initial energy-momentum tensor components

The **initial transverse flow components are zero**

The **dissipative corrections initially are zero**

$v^{\eta_s}(\tau_0; x, y, \eta_s)$

$n_B(\tau_0; x, y, \eta_s)$

$\varepsilon(\tau_0; x, y, \eta_s)$

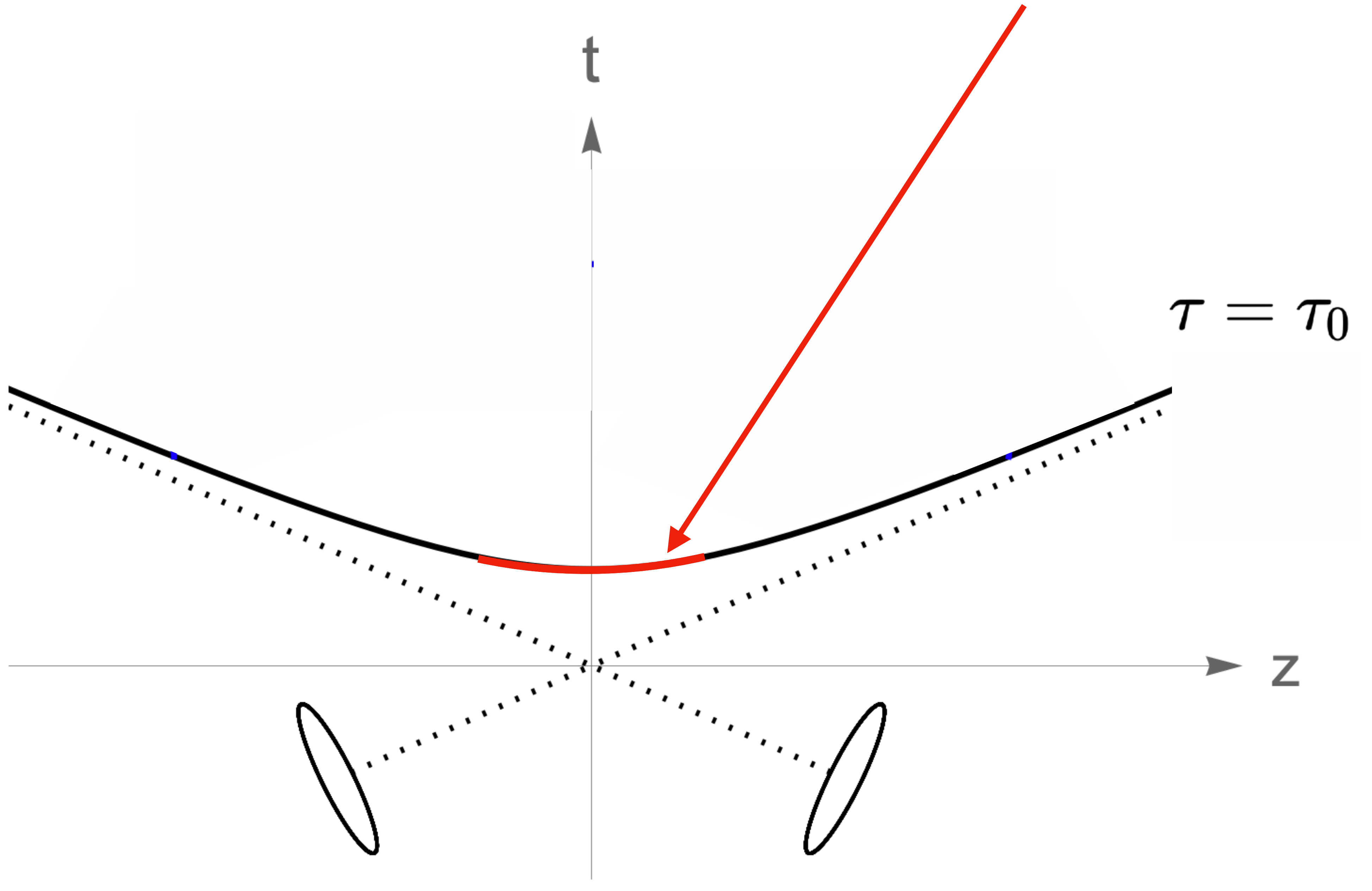


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve background EOMs in 3+1 dimensions in τ

EOMs constitute **11 PDEs** for **11 DOFs**

Use Godunov-type relativistic Harten-Lax-van Leer-Einfeldt (HLLE) approximate Riemann solver

Karpenko, Huovinen, and Bleicher, Comput. Phys. Commun. 185, 3016 (2014)
Singh and Alam, The European Physical Journal C 83, 585 (2023)

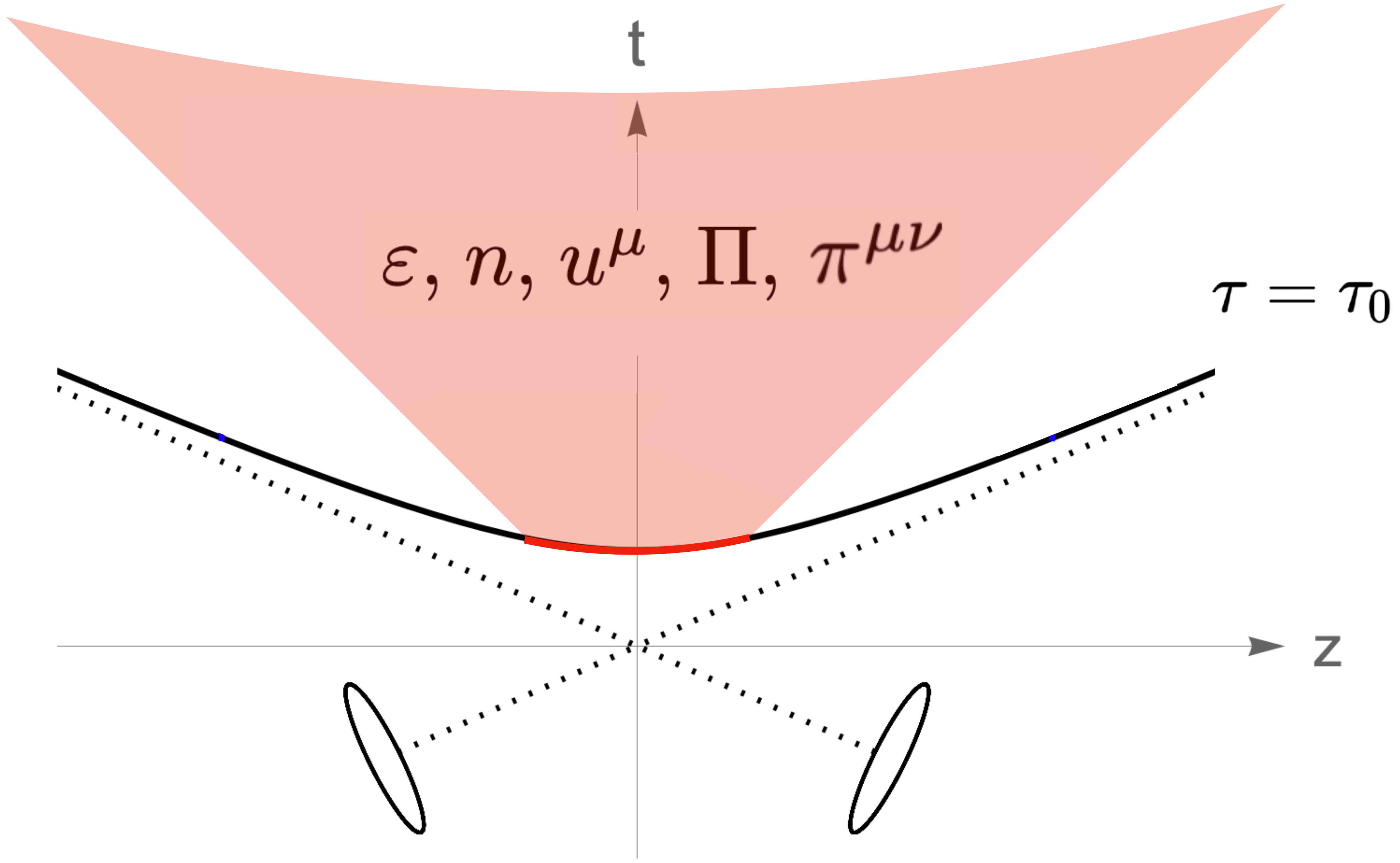


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\rm sw} = 0.5\ {\rm GeV/fm^3}$

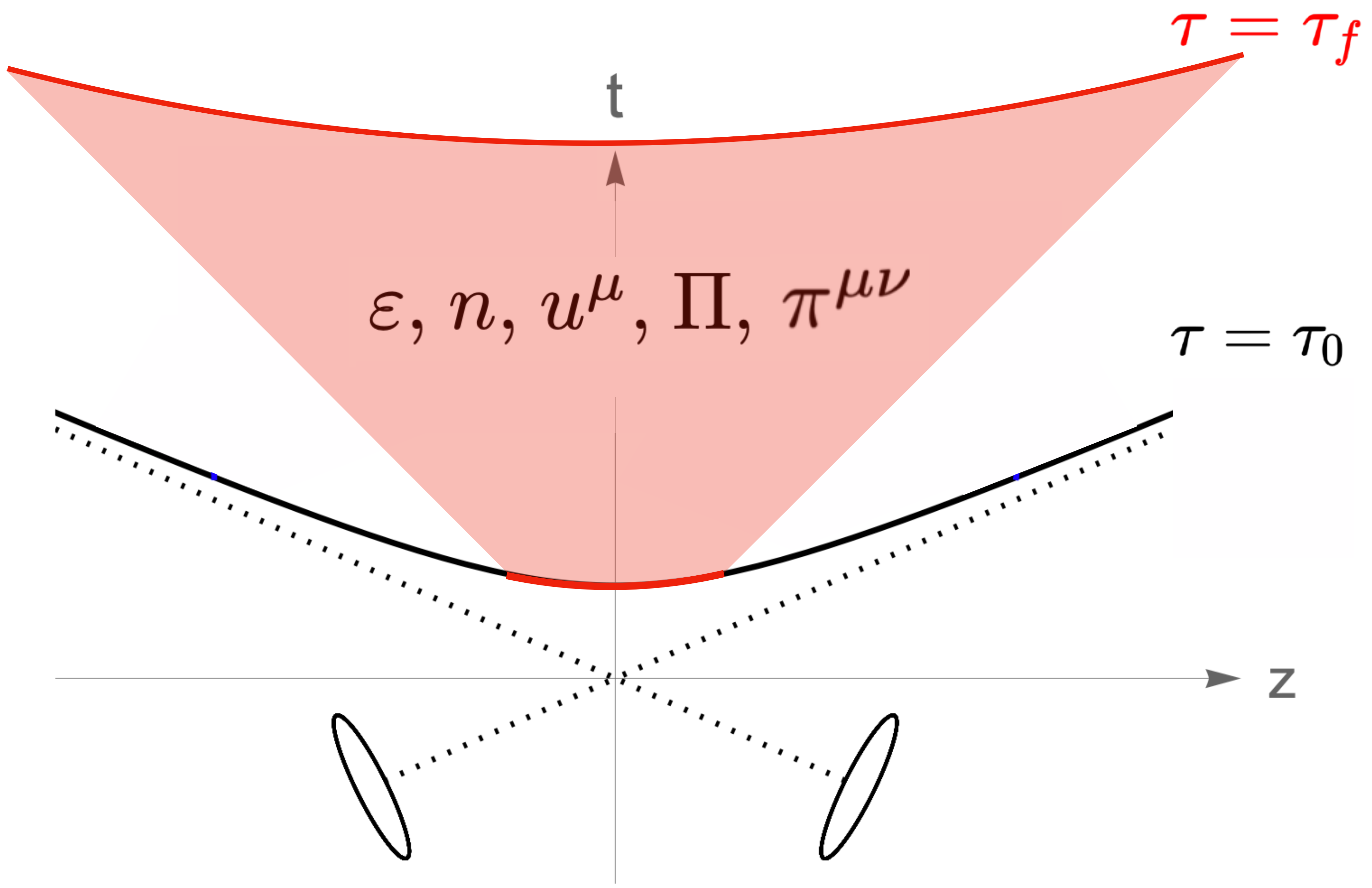


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\rm sw} = 0.5~\mathrm{GeV/fm^3}$

The switching hypersurface Σ is extracted with the CORNELIUS code using the condition $\varepsilon(T, \mu_B) = \varepsilon_{\rm sw}$

Huovinen, Petersen, Eur.Phys.J.A 48 (2012) 171

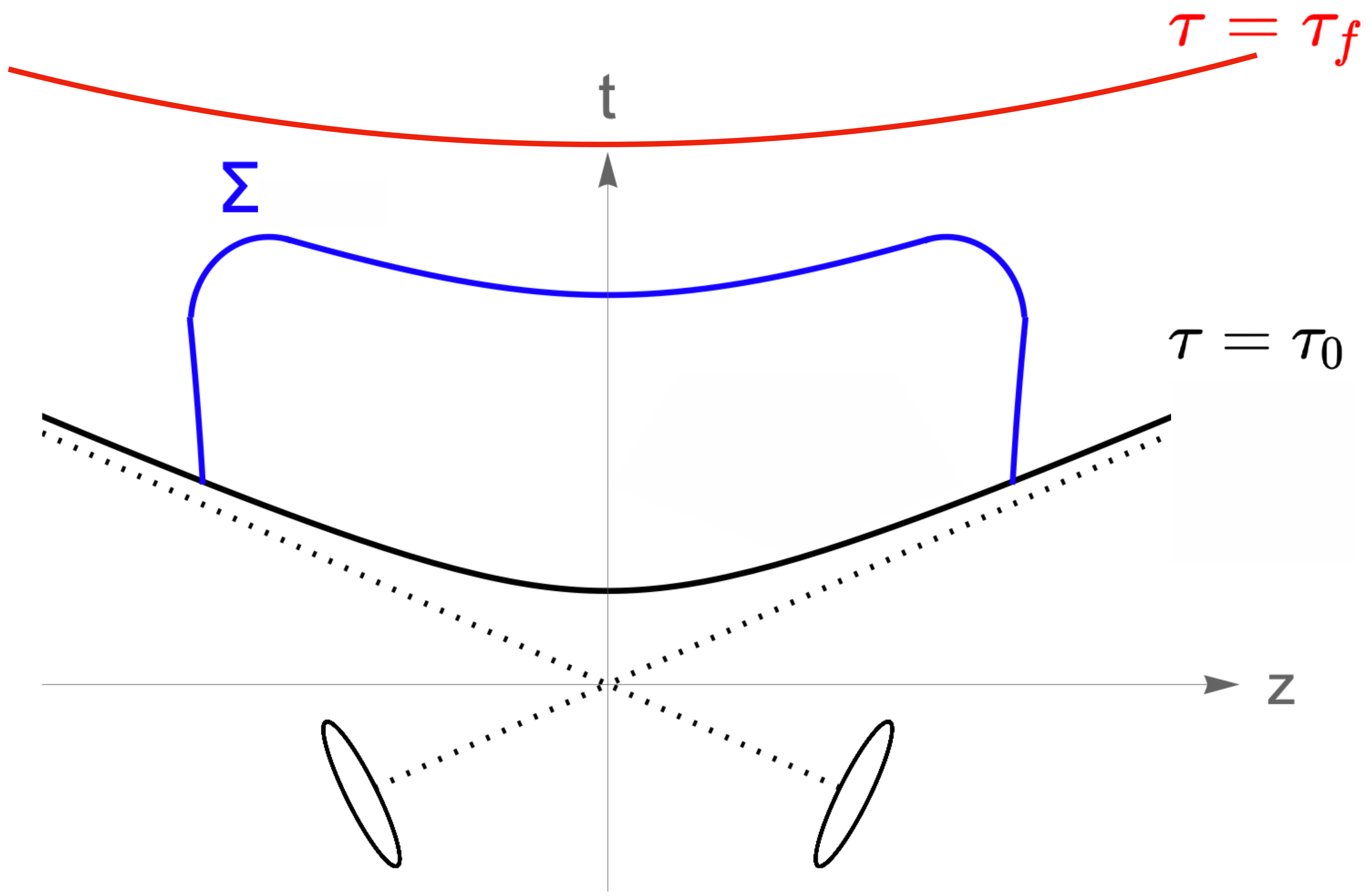


fig: https://arxiv.org/pdf/2407.12130 (modified)

BACKGROUND HYDRODYNAMICS

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\mathrm{sw}} = 0.5 \ \mathrm{GeV/fm^3}$

The switching hypersurface Σ is extracted with the CORNELIUS code using the condition $\varepsilon(T, \mu_B) = \varepsilon_{\mathrm{sw}}$

Huovinen, Petersen, Eur.Phys.J.A 48 (2012) 171

The hydrodynamic fields on Σ are passed to a hadron sampler

Karpenko, Huovinen, Petersen, and Bleicher, Phys. Rev. C 91, 064901 (2015),
Schafer, Karpenko, Wu, Hammelmann, and Elfner, The European Physical Journal A 58, 230 (2022).

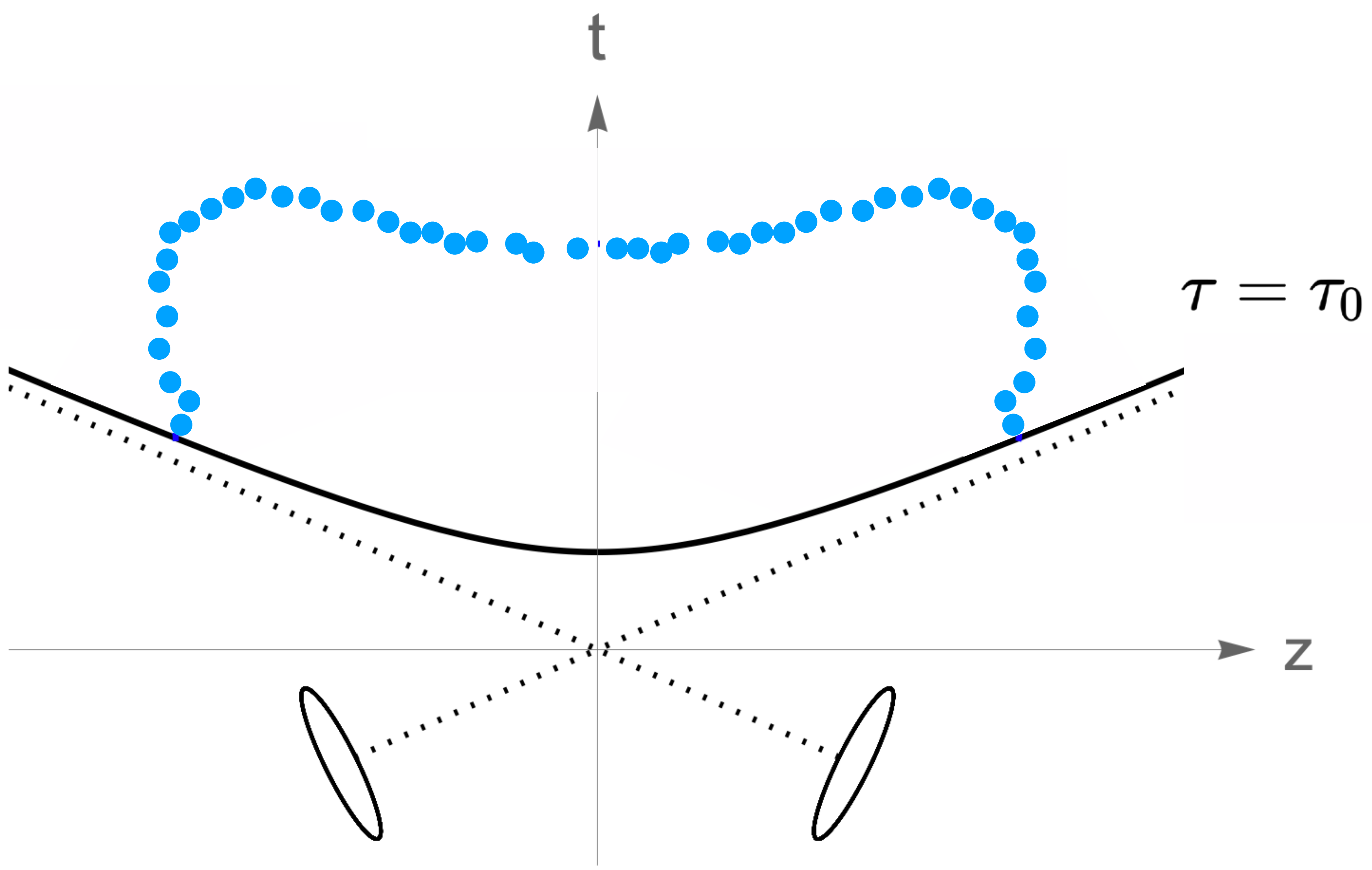


fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\rm sw} = 0.5 \ {\rm GeV/fm^3}$

The switching hypersurface Σ is extracted with the CORNELIUS code using the condition $\varepsilon(T, \mu_B) = \varepsilon_{\rm sw}$

Huovinen, Petersen, Eur.Phys.J.A 48 (2012) 171

The hydrodynamic fields on Σ are passed to a hadron sampler

Karpenko, Huovinen, Petersen, and Bleicher, Phys. Rev. C 91, 064901 (2015),
Schafer, Karpenko, Wu, Hammelmann, and Elfner, The European Physical Journal A 58, 230 (2022).

The resulting particle set serves as input to the SMASH transport model, which describes subsequent hadron interactions and decays

Weil et al., Phys. Rev. C 94, 054905 (2016)

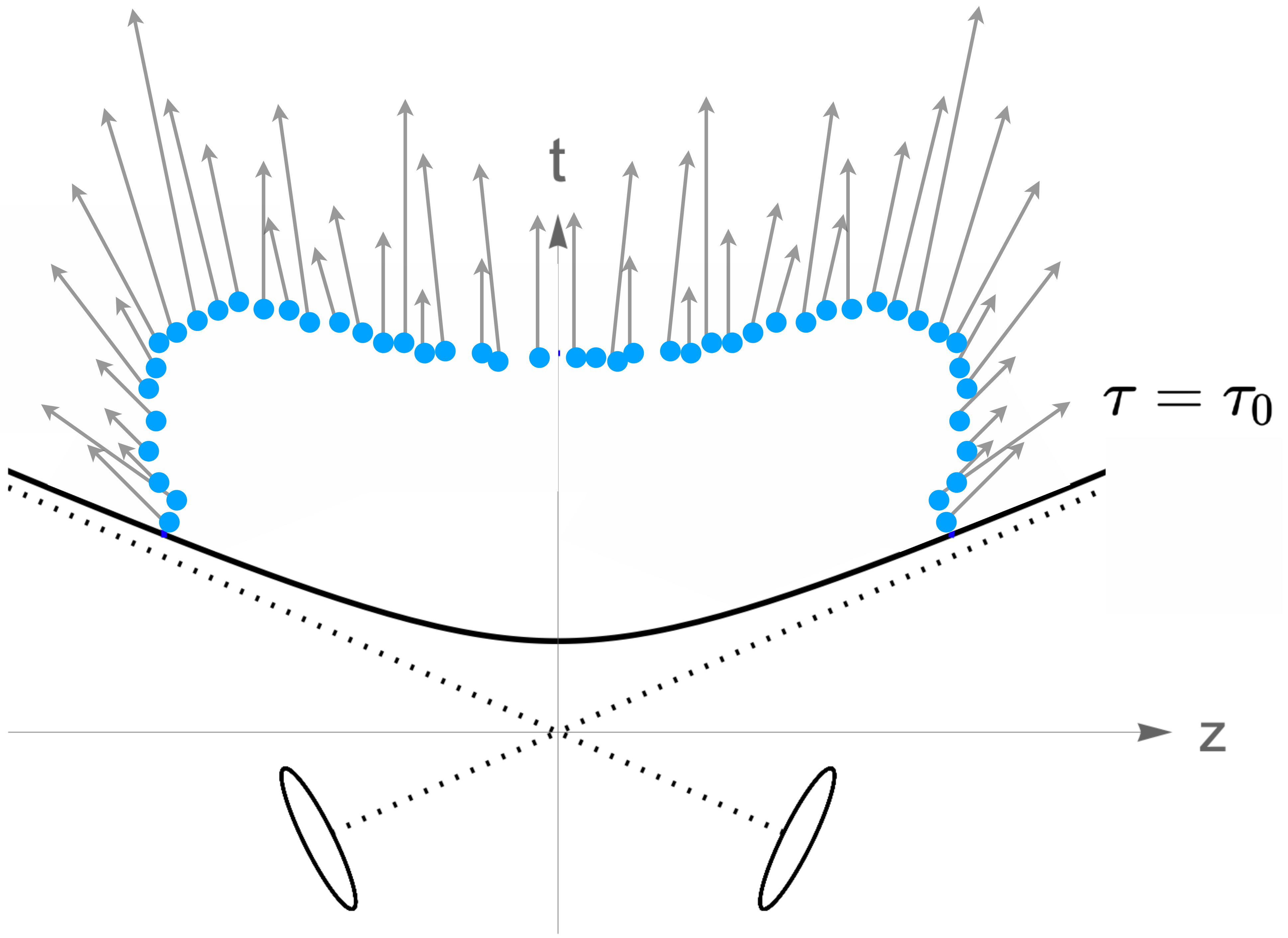
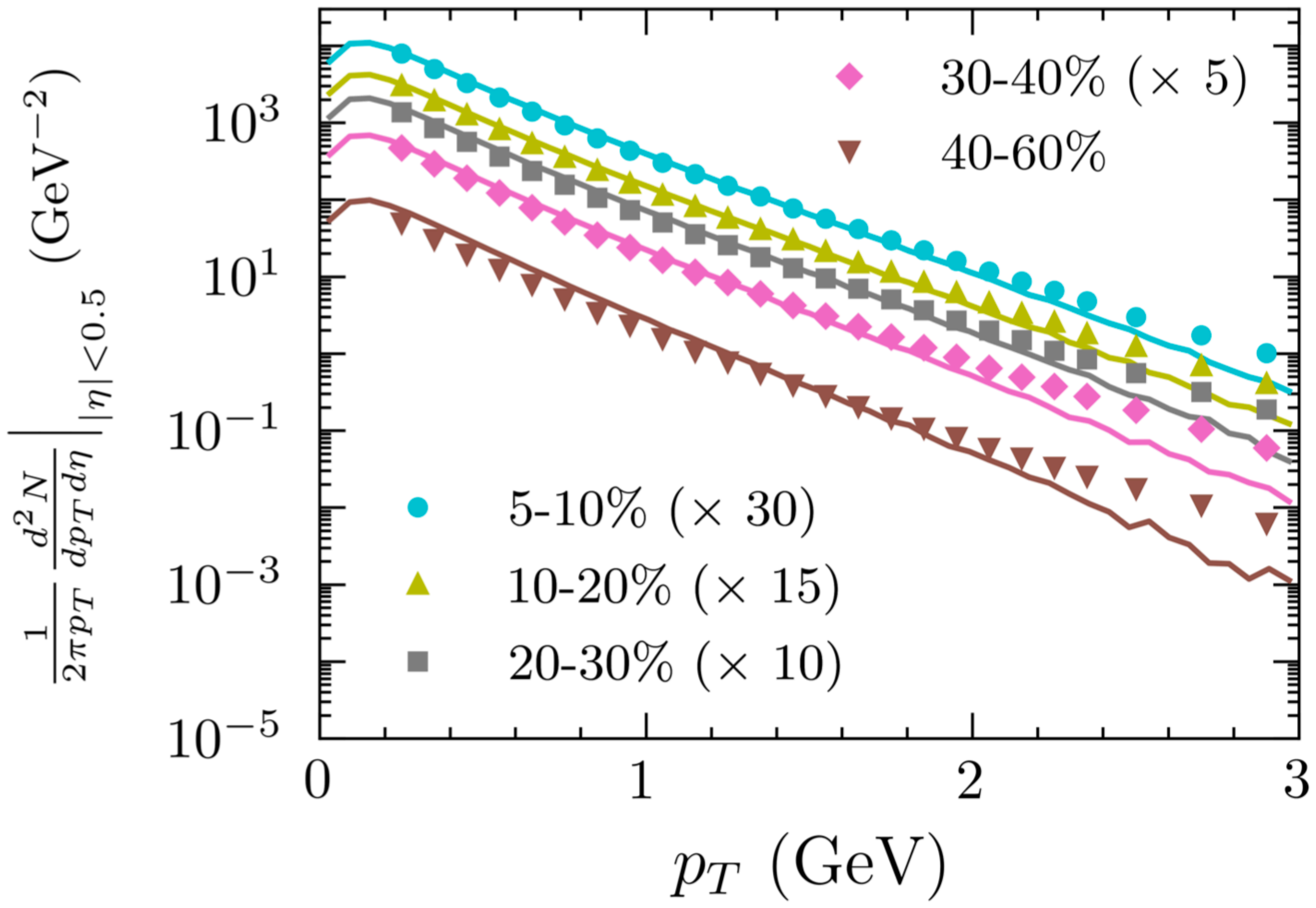


fig: https://arxiv.org/pdf/2407.12130 (modified)

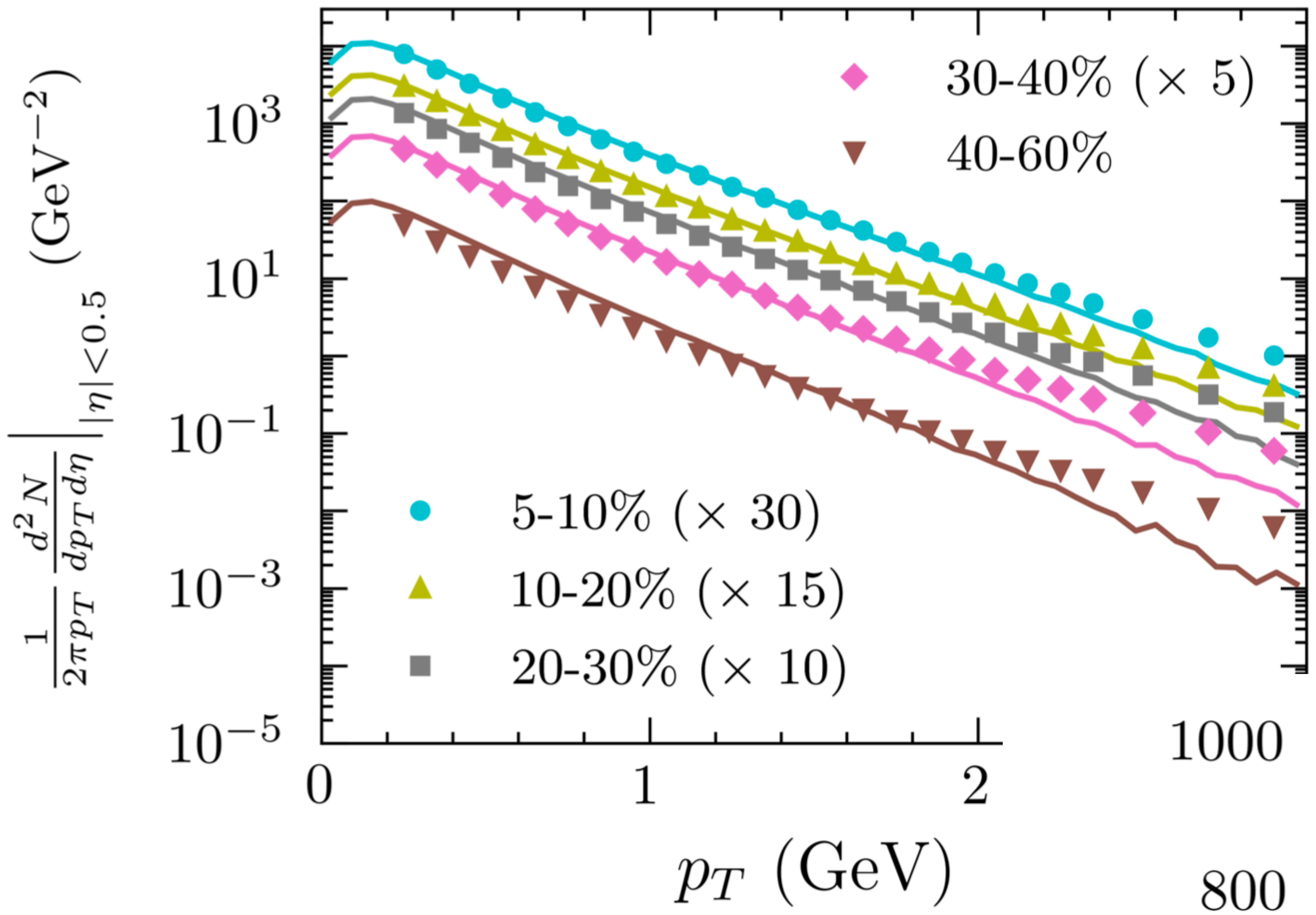
Results for background

RESULTS FOR BACKGROUND HYDRODYNAMICS



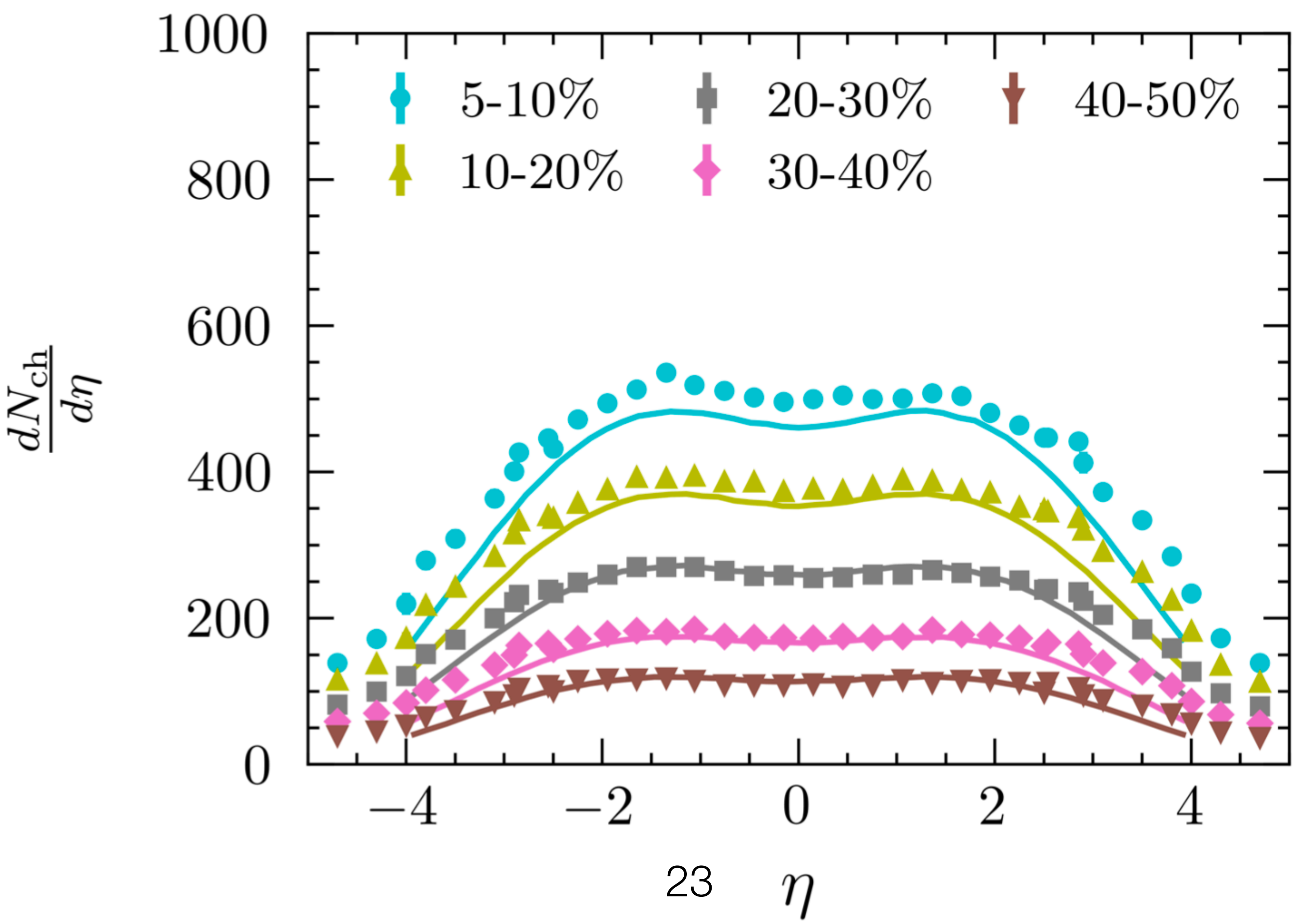
Adams et al. (STAR Collaboration),
Phys. Rev. Lett. 91, 172302 (2003)

RESULTS FOR BACKGROUND HYDRODYNAMICS

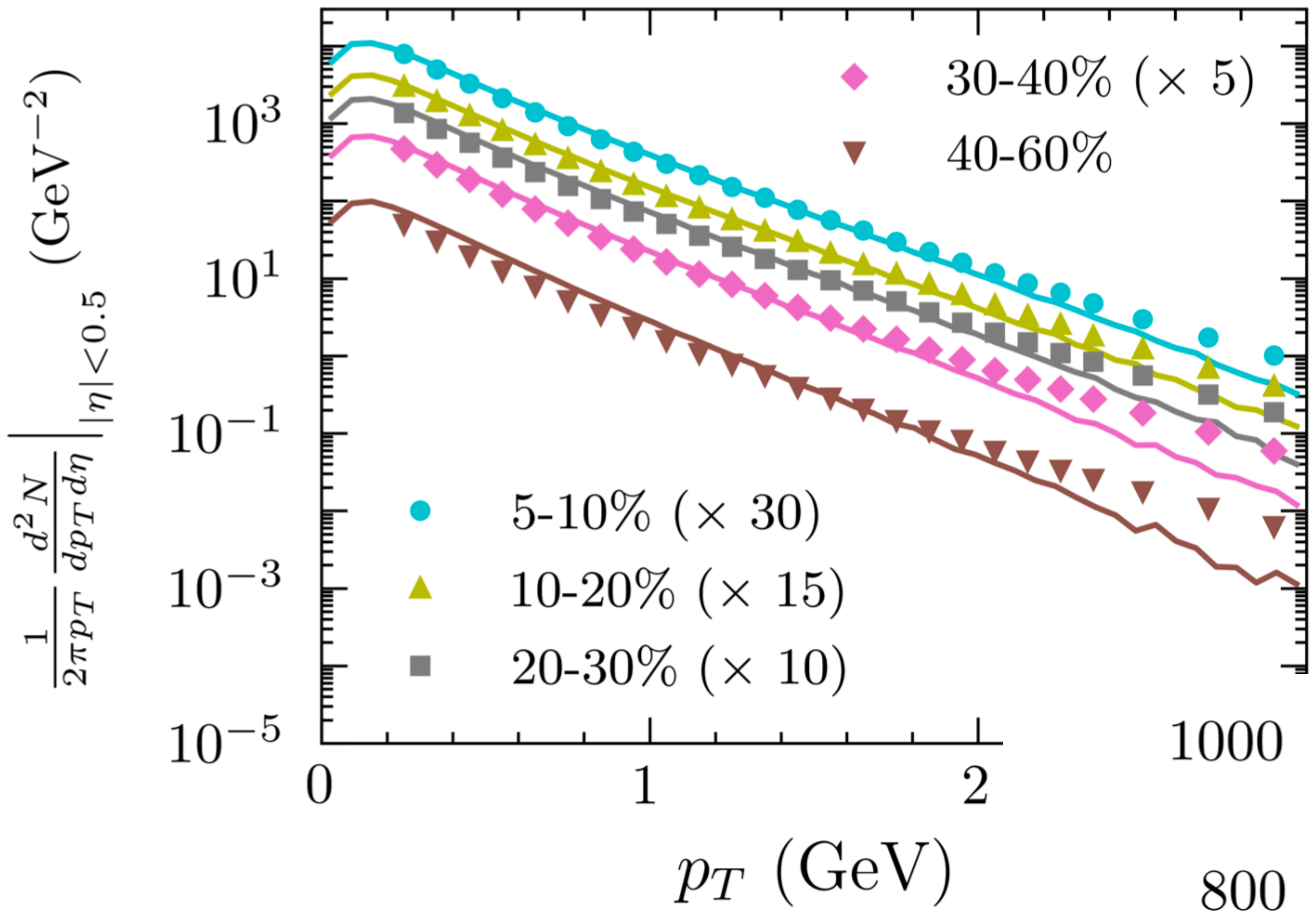


Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 172302 (2003)

Bearden et al. (BRAHMS Collaboration), Phys. Rev. Lett. 88, 202301 (2002).

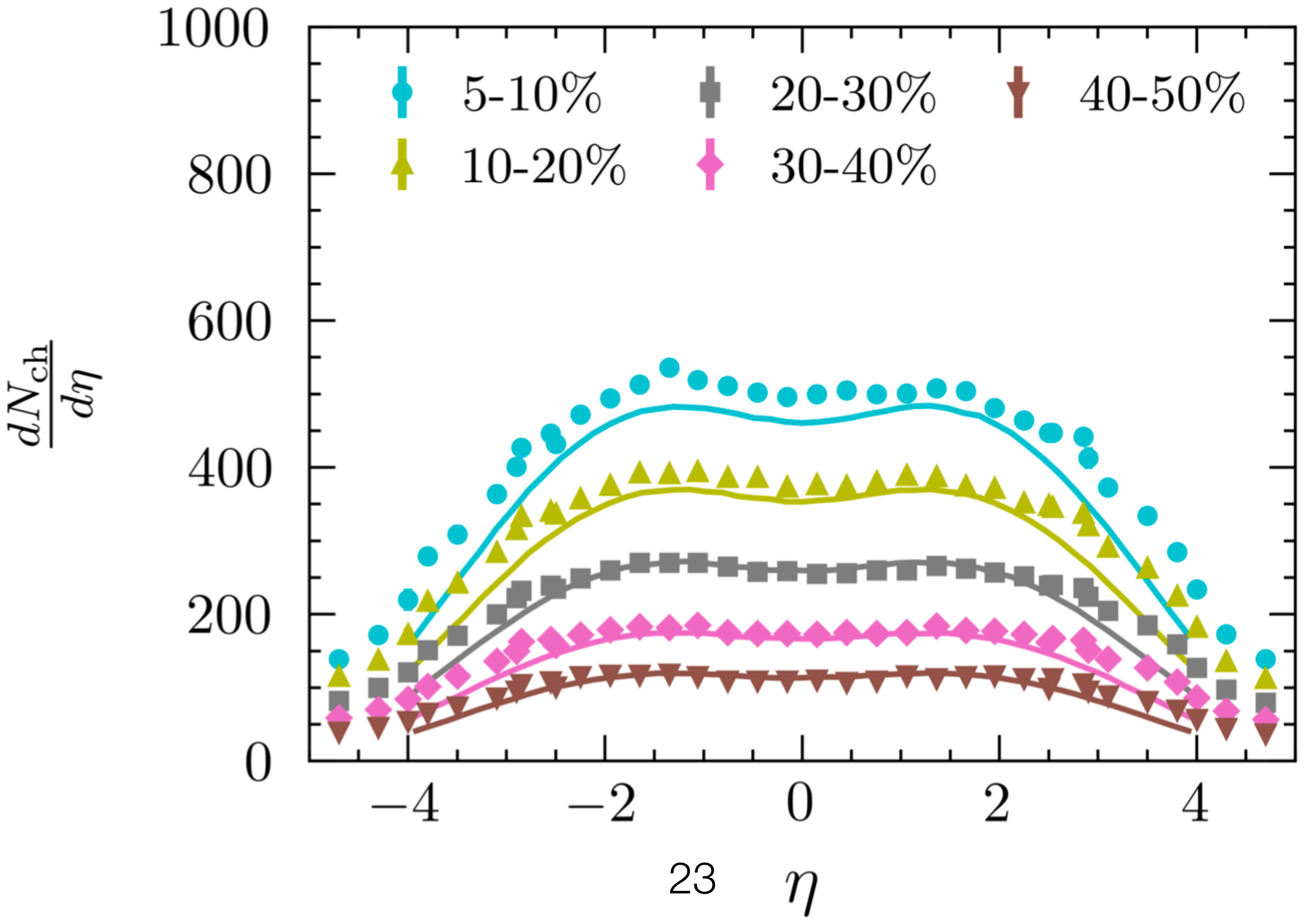


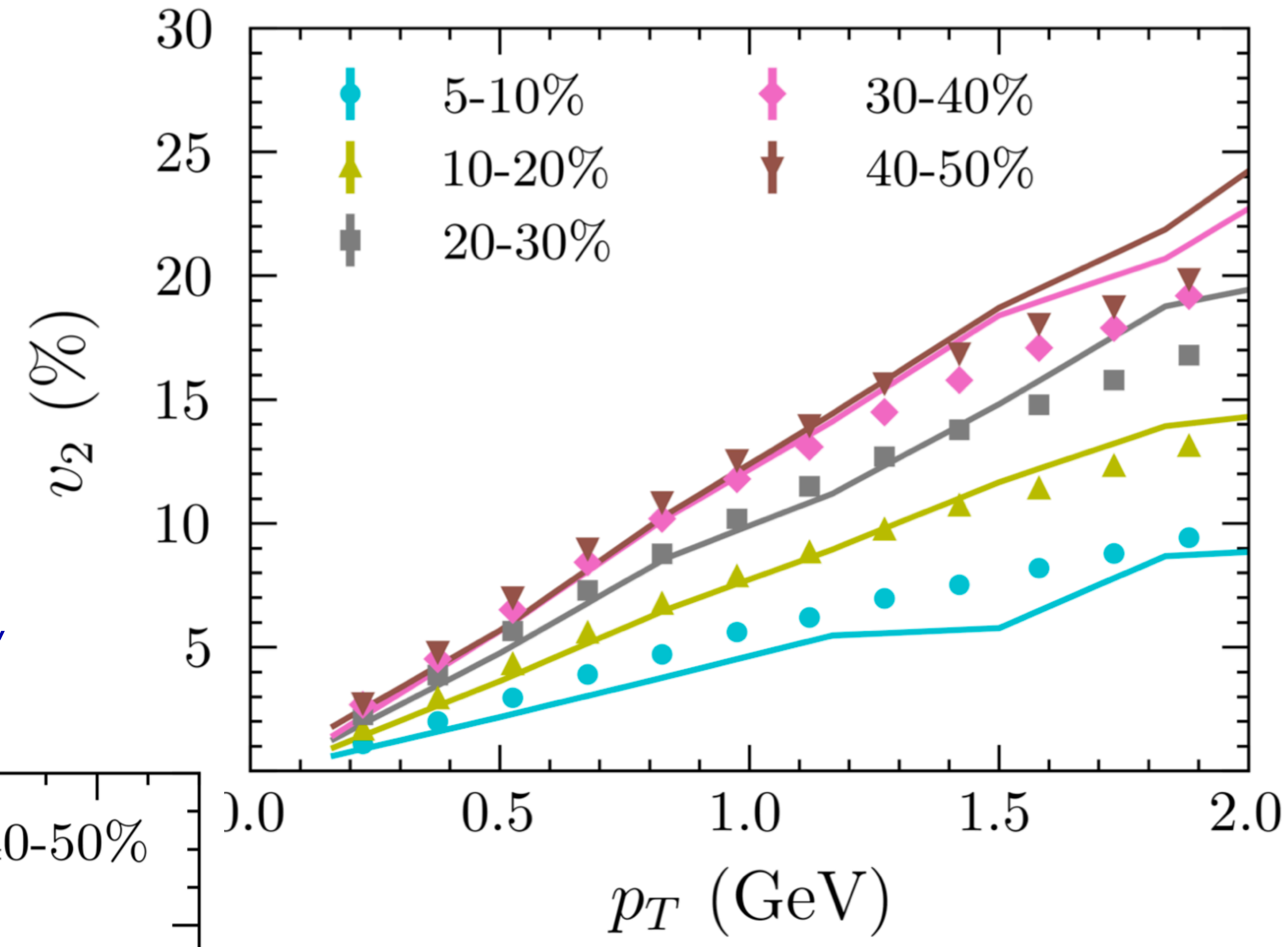
Results for background hydrodynamics



Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 172302 (2003)

Bearden et al. (BRAHMS Collaboration), Phys. Rev. Lett. 88, 202301 (2002).





Adams et al. (STAR and STAR-RICH Collaboration), Phys. Rev. C 72, 014904 (2005)

Numerical framework

Spin hydrodynamics

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

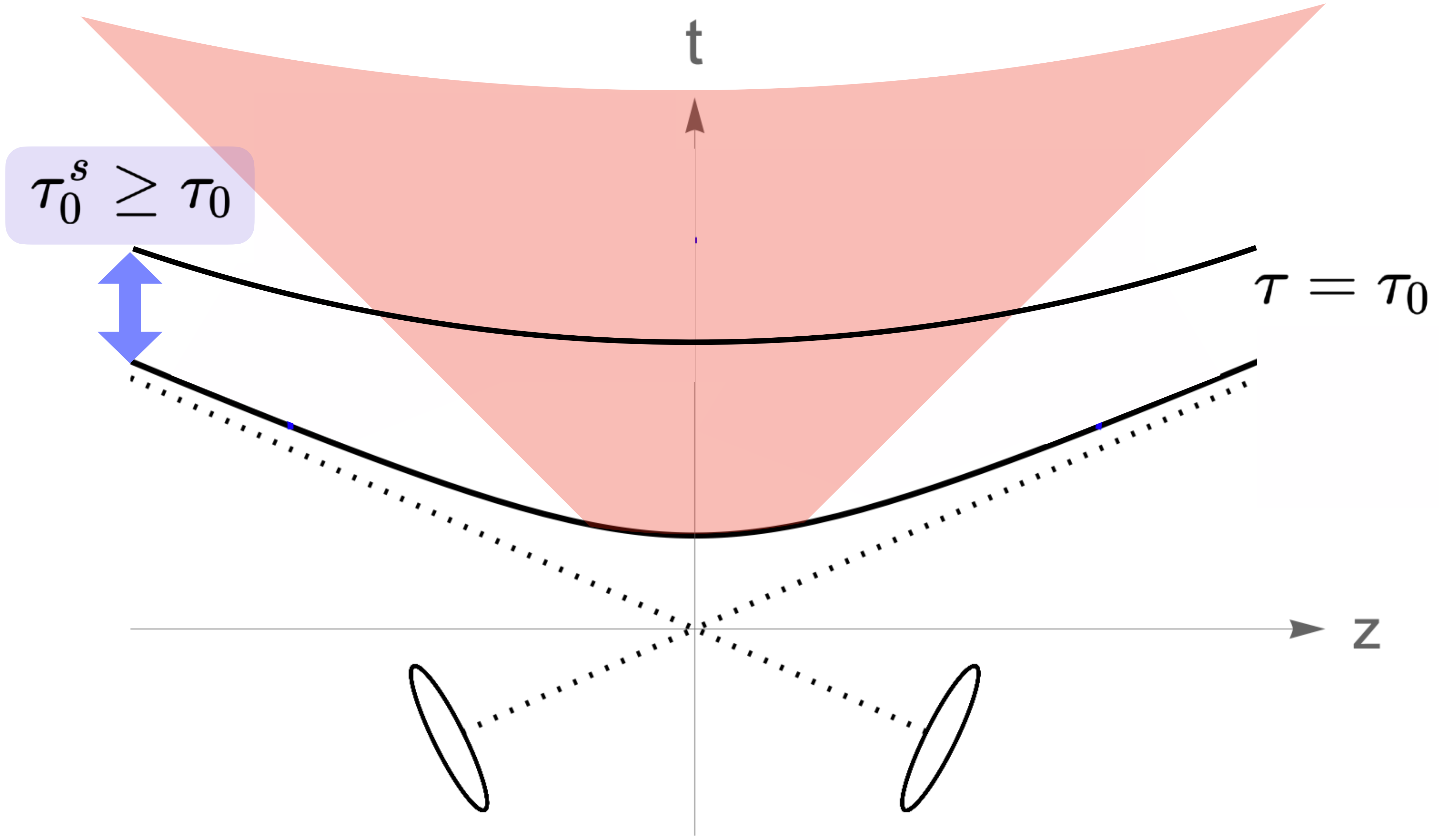


fig: https://arxiv.org/pdf/2407.12130 (modified)

Spin hydrodynamics

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

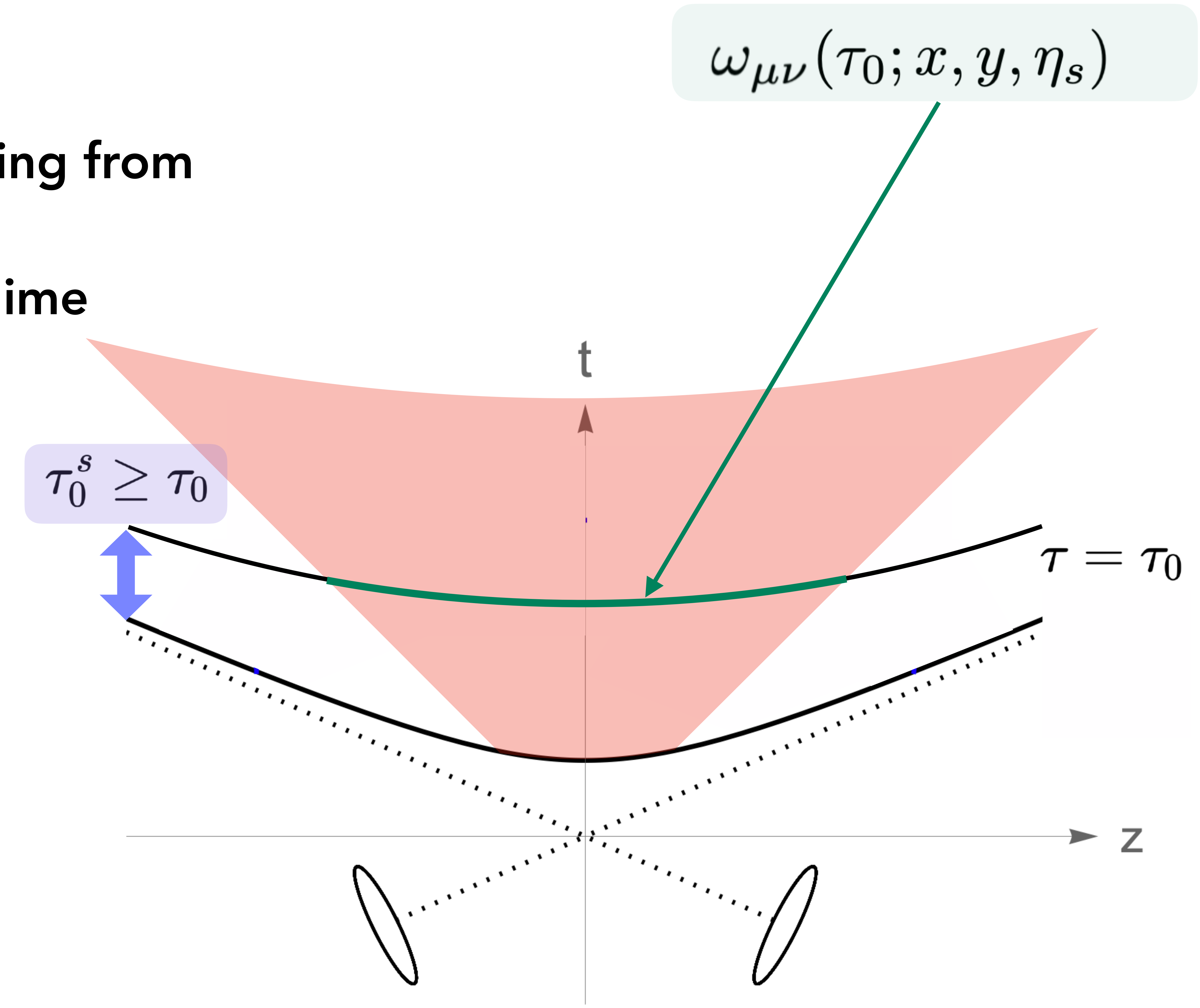


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

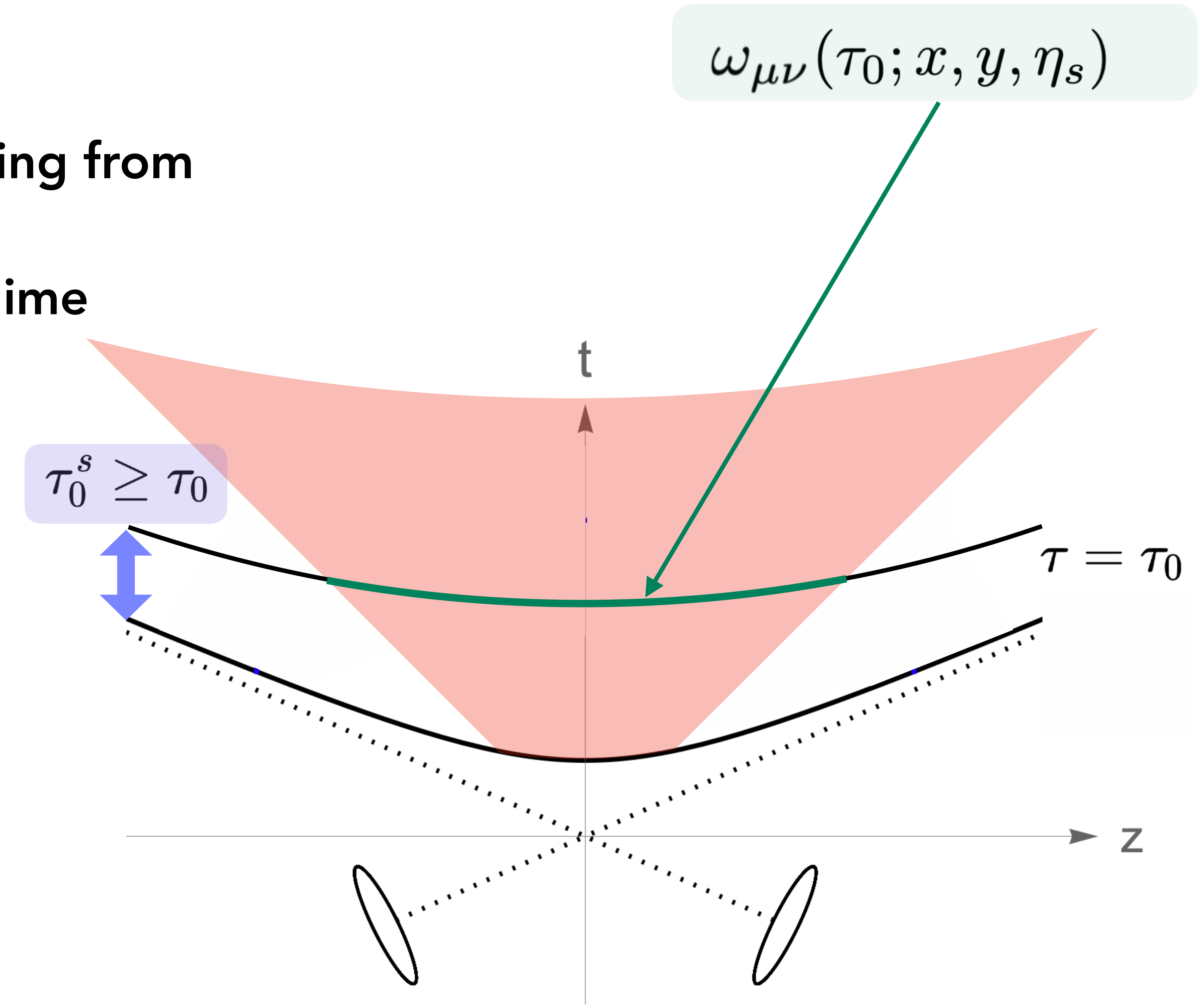


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

$$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$$

$$\varpi^{\rm iso}_{\mu\nu} = \frac{1}{T}\partial_{[\nu}u_{\mu]} \qquad \varpi^{\rm T}_{\mu\nu} = \frac{1}{T}u_{[\nu}\partial_{\mu]}\ln T$$

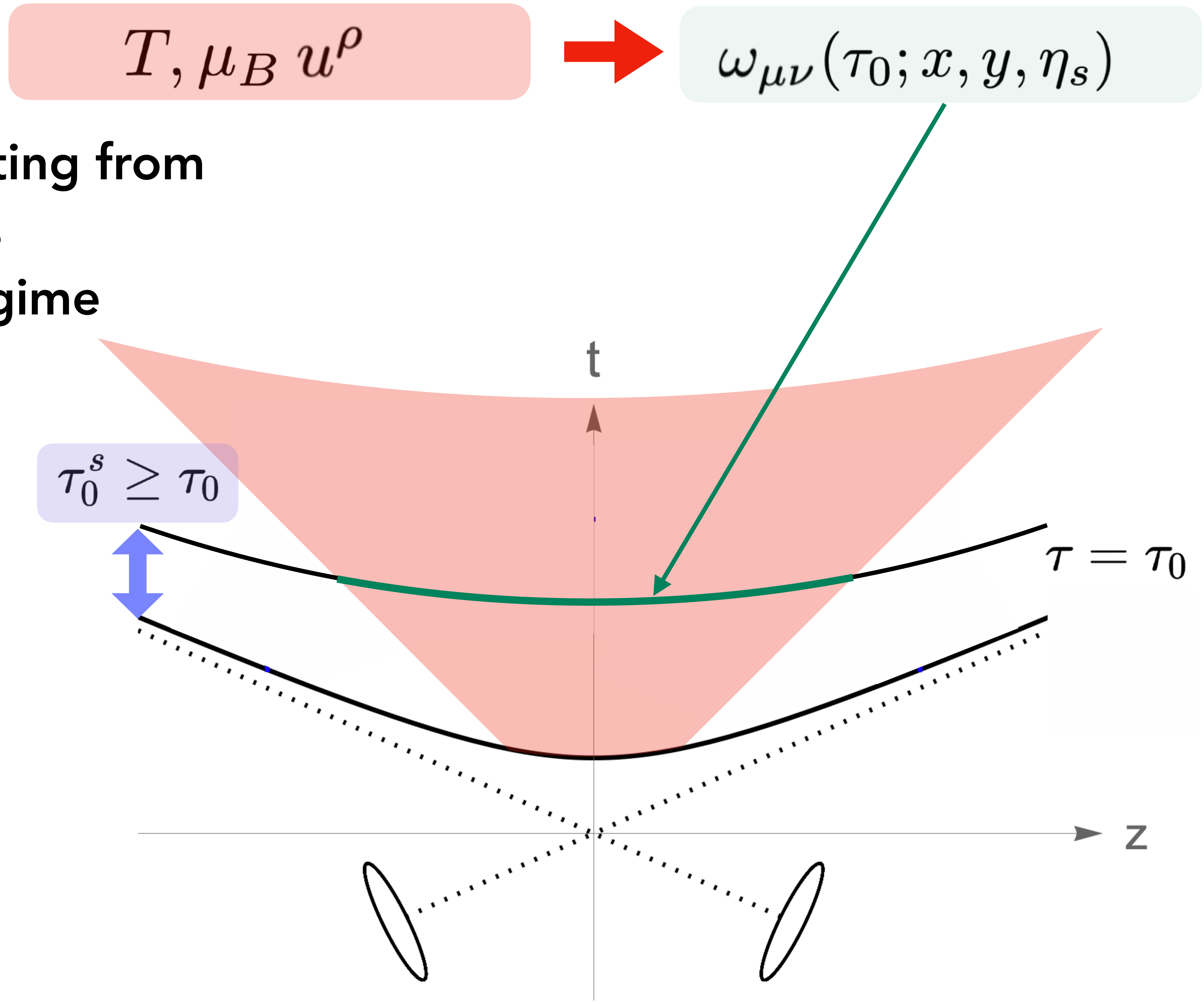


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

$$\varpi_{\mu\nu}^{\rm iso} = \frac{1}{T}\partial_{[\nu}u_{\mu]} \qquad \varpi_{\mu\nu}^{\rm T} = \frac{1}{T}u_{[\nu}\partial_{\mu]}\ln T$$

$$\hat{\tau}^\mu = (1,0,0,0)$$

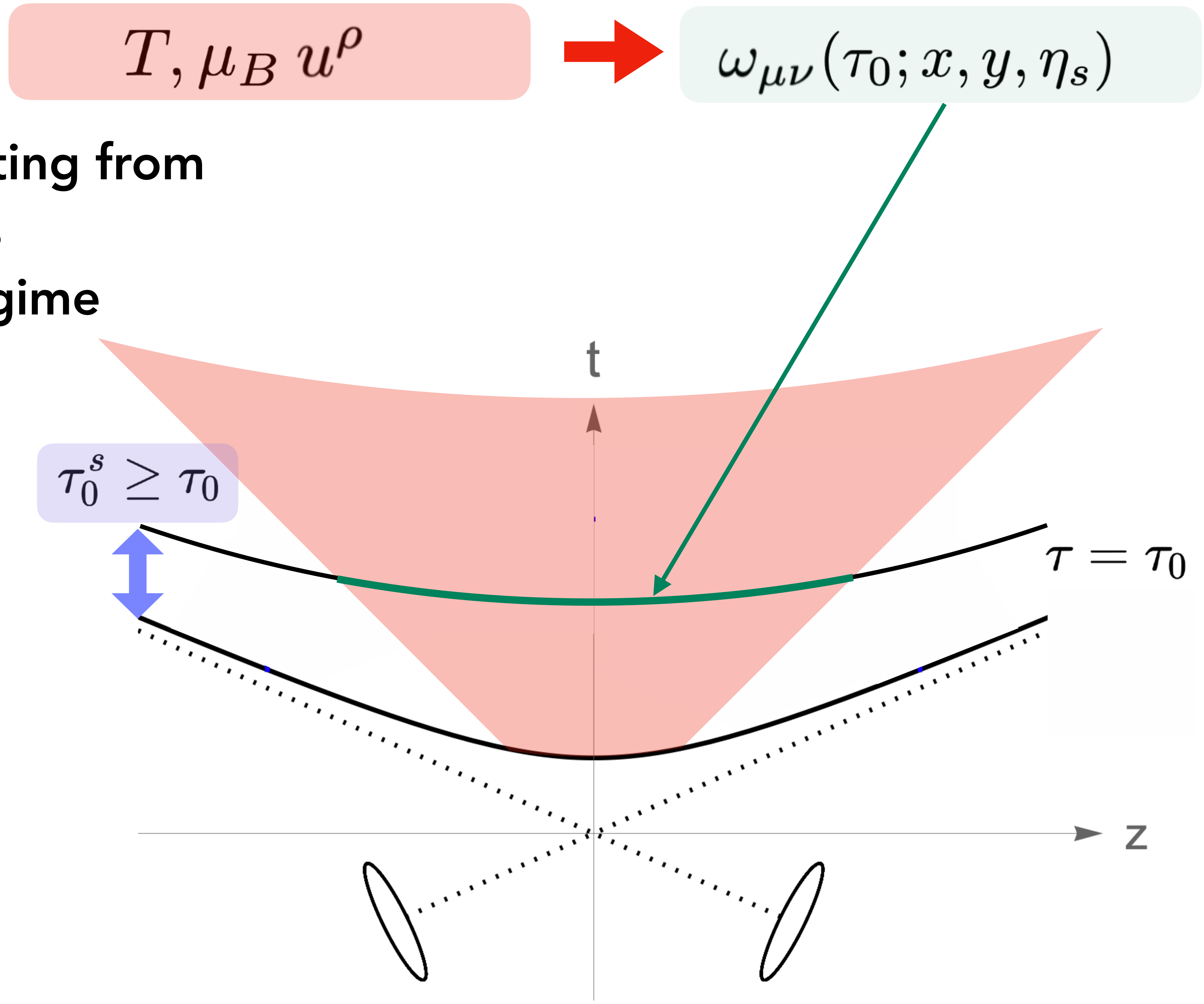


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

$T, \mu_B\, u^\rho$ → $\omega_{\mu\nu}(\tau_0; x, y, \eta_s)$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

$$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$$

$$\varpi^{\rm iso}_{\mu\nu} = \frac{1}{T}\partial_{[\nu}u_{\mu]} \qquad \varpi^{\rm T}_{\mu\nu} = \frac{1}{T}u_{[\nu}\partial_{\mu]}\ln T$$

$$\hat{\tau}^\mu = (1, 0, 0, 0)$$

$$\omega_{\mu\nu}(\tau_0^s) = \varpi^{\rm iso}_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi^{\rm iso}_{\nu]\rho}u^\rho$$

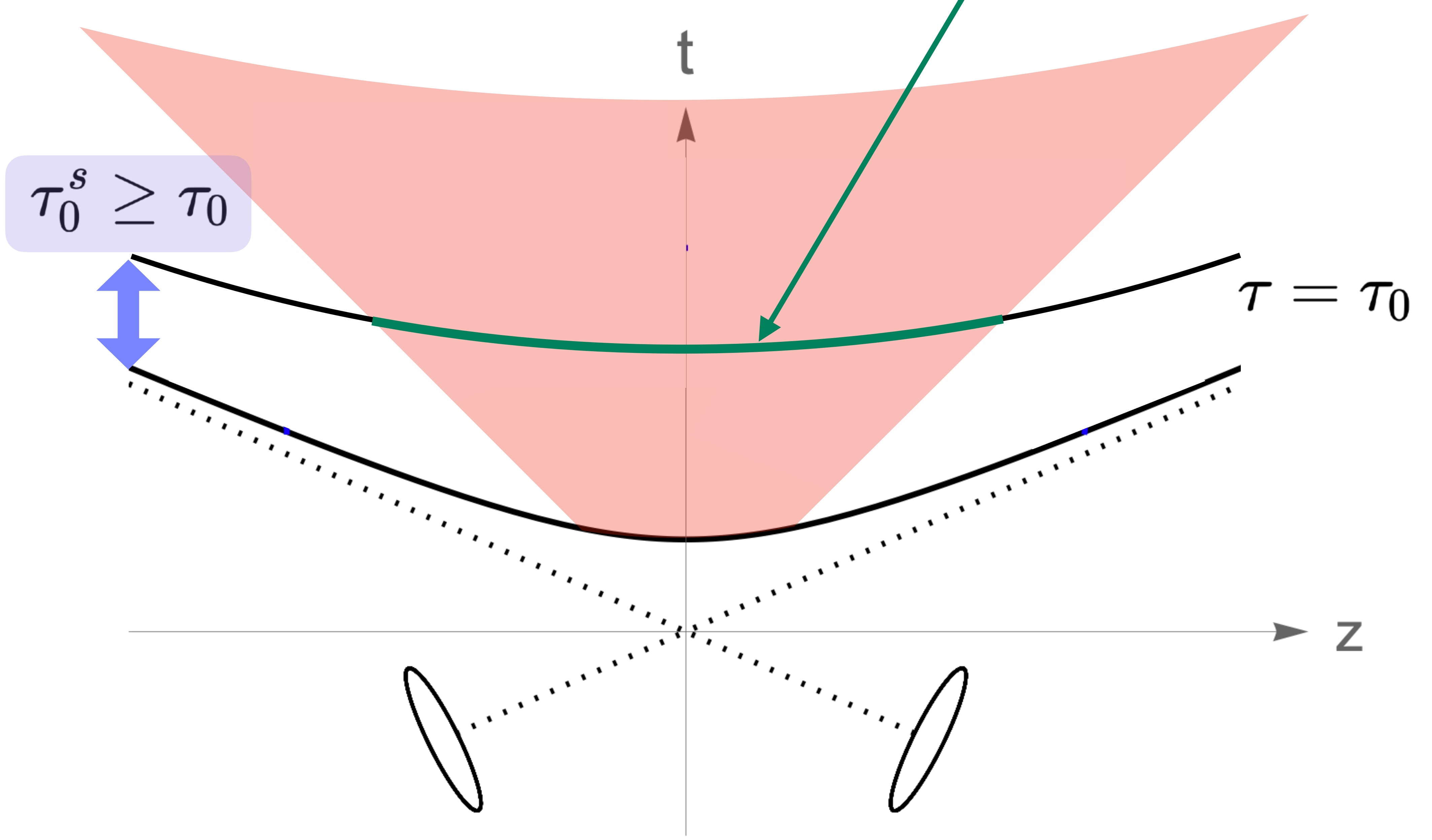


fig: https://arxiv.org/pdf/2407.12130 (modified)

Spin hydrodynamics

Spin EOMs constitute **6 PDEs** for **6 DOFs**

We extended the code (using also HLLE algorithm) to incorporate the spin EOMs

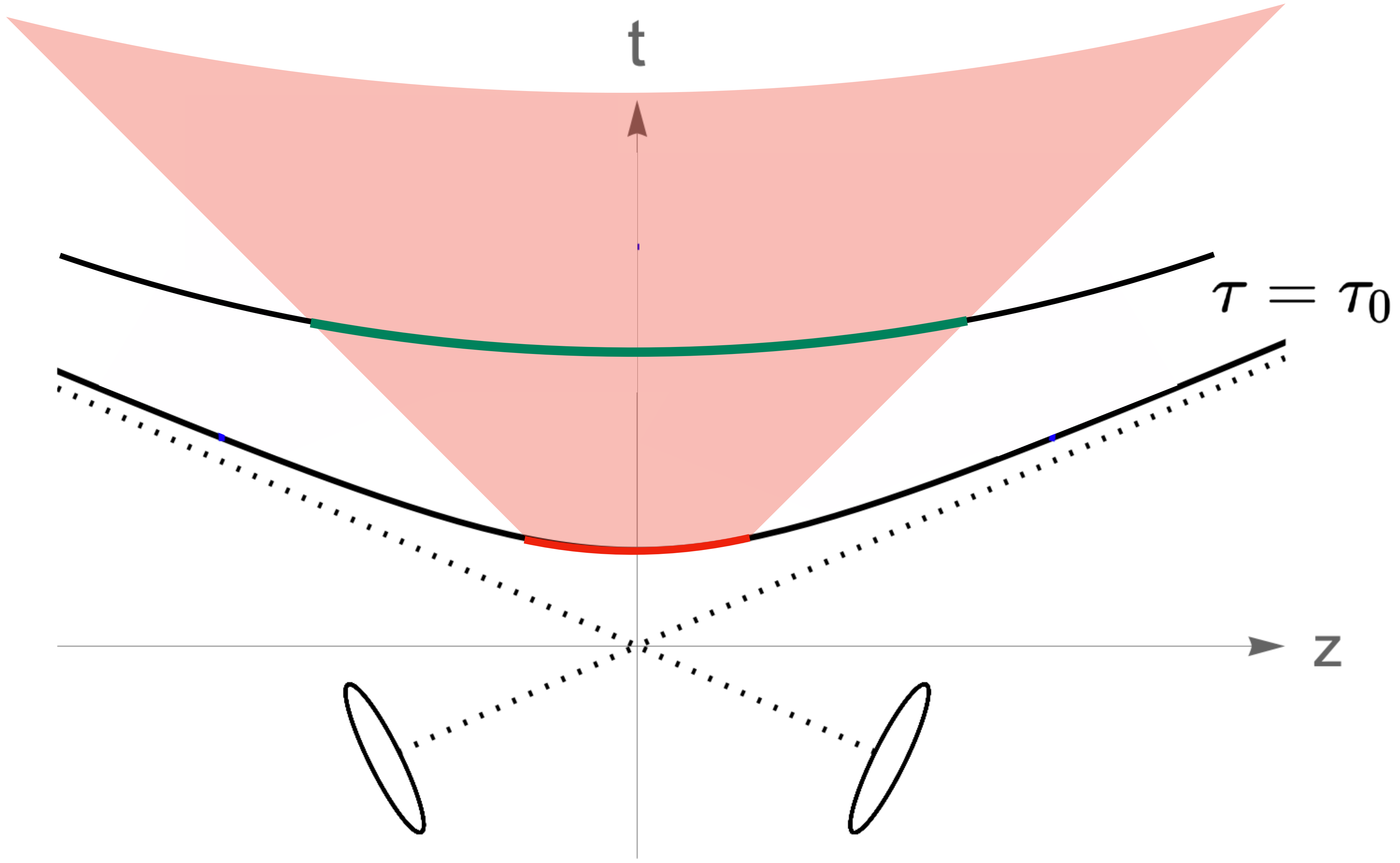


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Spin EOMs constitute **6 PDEs** for **6 DOFs**

We extended the code (using also HLLE algorithm) to incorporate the spin EOMs

Evolve spin EOMs in 3+1 dimensions in τ

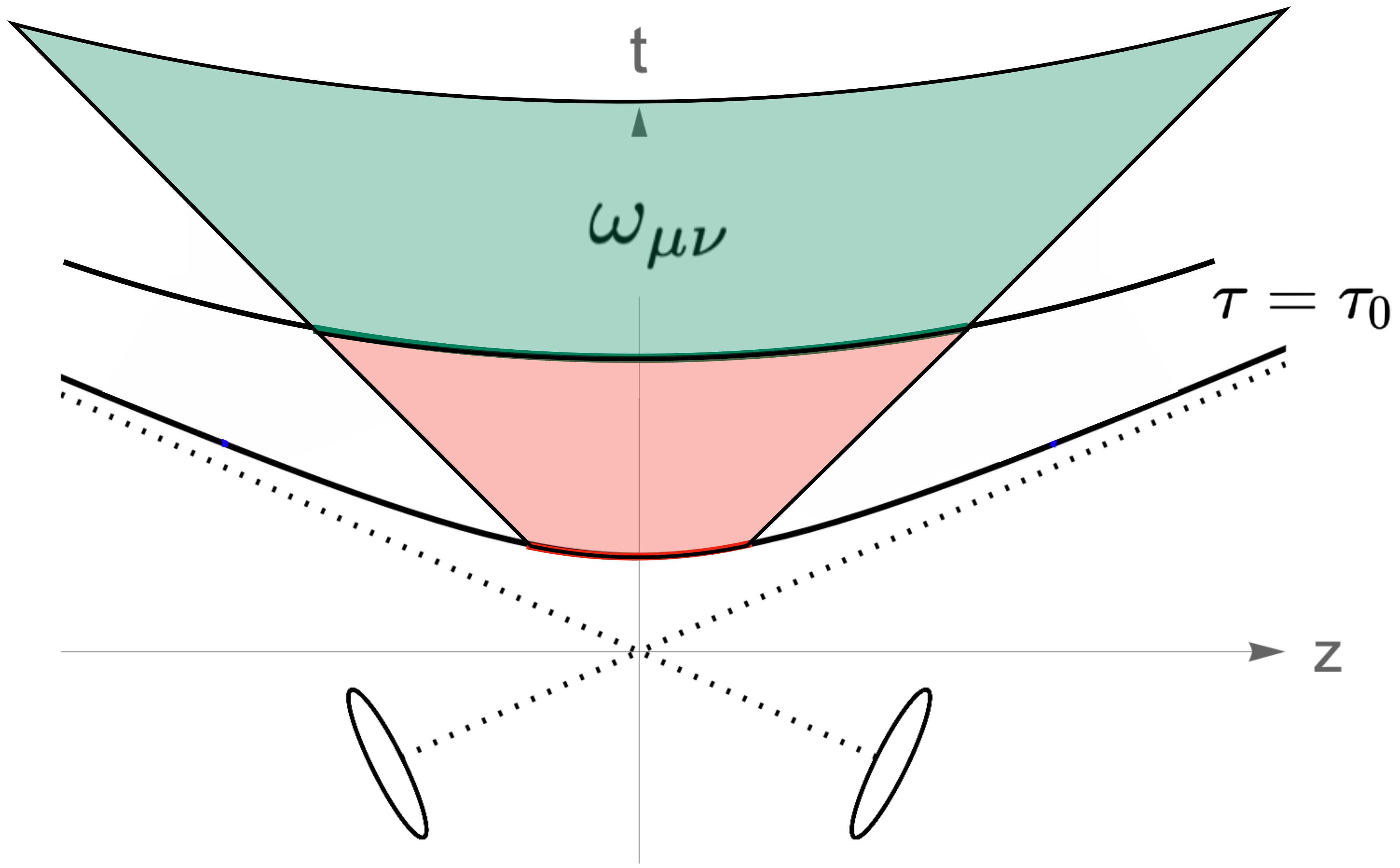


fig: https://arxiv.org/pdf/2407.12130 (modified)

SPIN HYDRODYNAMICS

Spin EOMs constitute **6 PDEs** for **6 DOFs:** $\omega_{\mu\nu}$

We extended the code (using also HLLE algorithm) to incorporate the spin EOMs

Evolve spin EOMs in 3+1 dimensions in τ

We **calculate spin observables for Λ hyperons at Σ**

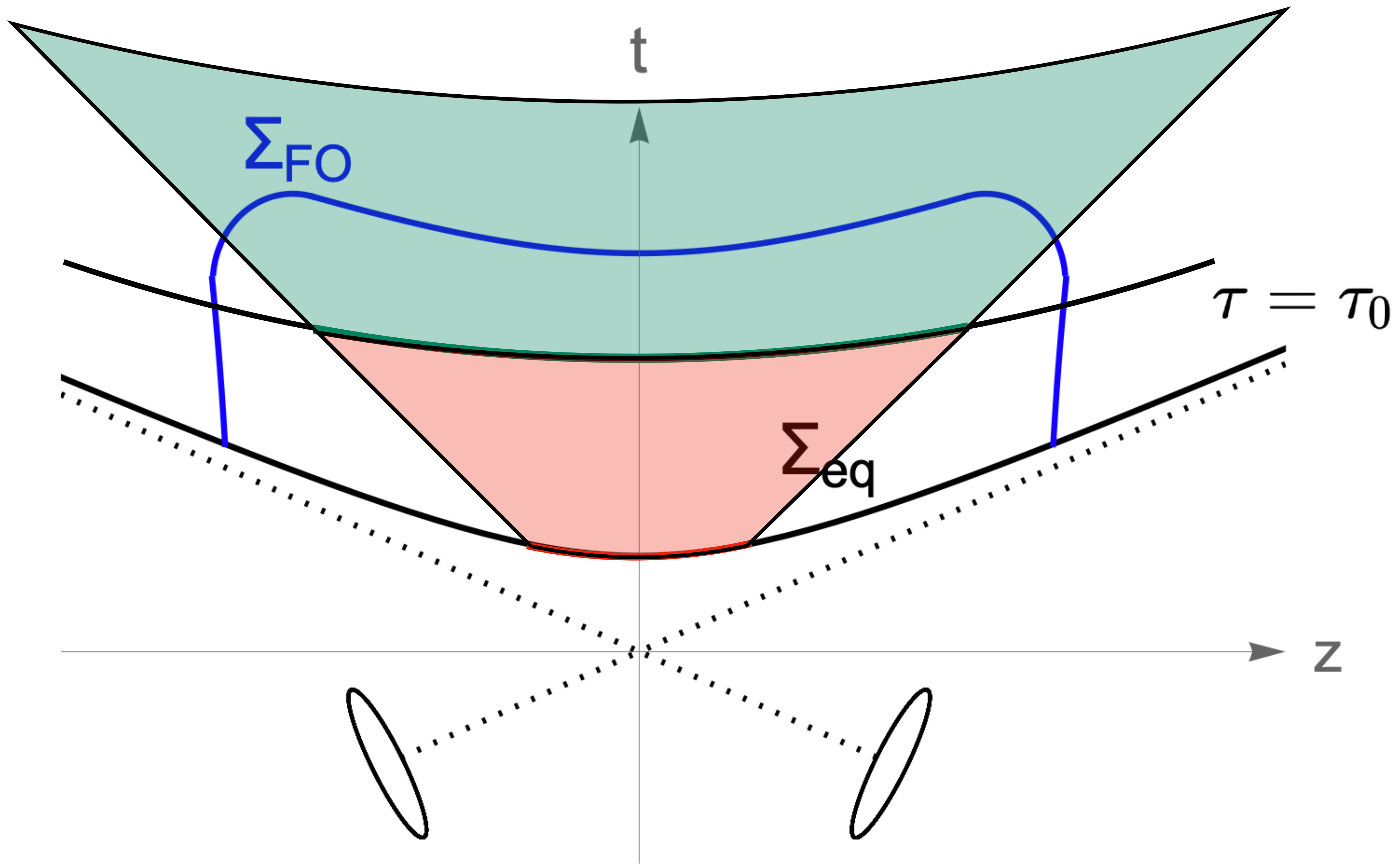


fig: https://arxiv.org/pdf/2407.12130 (modified)

Results for spin

POLARIZATION VECTOR

We calculate the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma \cdot p \, n_F(1-n_F)\omega_{\nu\rho}}{\int d\Sigma \cdot p \, n_F},$$

Polarization vector

We calculate the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma \cdot p \; n_F(1-n_F)\omega_{\nu\rho}}{\int d\Sigma \cdot p \; n_F},$$

Spin polarization tensor is determined from spin hydrodynamics

Polarization vector

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

POLARIZATION VECTOR

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

Where the first term is

Becattini, Inghirami, Rolando, Beraudo, DelZanna, DePace, Nardi, Pagliara, and Chandra, Eur. Phys. J. C 75, 406 (2015),

$$S^\mu_\varpi(p) \; = \; -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} \; = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$$

POLARIZATION VECTOR

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently **two prescriptions**

POLARIZATION VECTOR

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently two prescriptions

First is **BBP**

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$S^\mu_{\xi,\mathrm{BBP}}(p) = -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}\frac{p_\sigma p^\lambda}{p\cdot\hat{t}}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\lambda\rho}}{\int d\Sigma\cdot p\, n_F}$$

Polarization vector

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently two prescriptions

First is **BBP**

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$S^\mu_{\xi,\mathrm{BBP}}(p) = -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}\frac{p_\sigma p^\lambda}{p\cdot\hat{t}}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\lambda\rho}}{\int d\Sigma\cdot p\; n_F}$$

$$\xi_{\mu\nu} = \partial_{(\nu}\beta_{\mu)} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

POLARIZATION VECTOR

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently two prescriptions

First is **BBP**

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$S^\mu_{\xi,\mathrm{BBP}}(p) = -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}\frac{p_\sigma p^\lambda}{p\cdot\hat{t}}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\lambda\rho}}{\int d\Sigma\cdot p\; n_F}$$

$$\xi_{\mu\nu} = \partial_{(\nu}\beta_{\mu)} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

$$\hat{t} = (1,0,0,0)$$

POLARIZATION VECTOR

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently two prescriptions

Second is **LY**

Liu and Yin, JHEP 07, 188,

$$S^\mu_{\xi,\mathrm{LY}}(p) \;=\; -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\frac{p_\perp^\lambda u_\nu}{p\cdot u}\xi_{\rho\lambda}}{\int d\Sigma\cdot p\; n_F}$$

$$p^\perp_\mu \equiv \Delta_\mu^{\ \nu} p_\nu$$

STANDARD HYDRODYNAMICS VS SPIN HYDRODYNAMICS

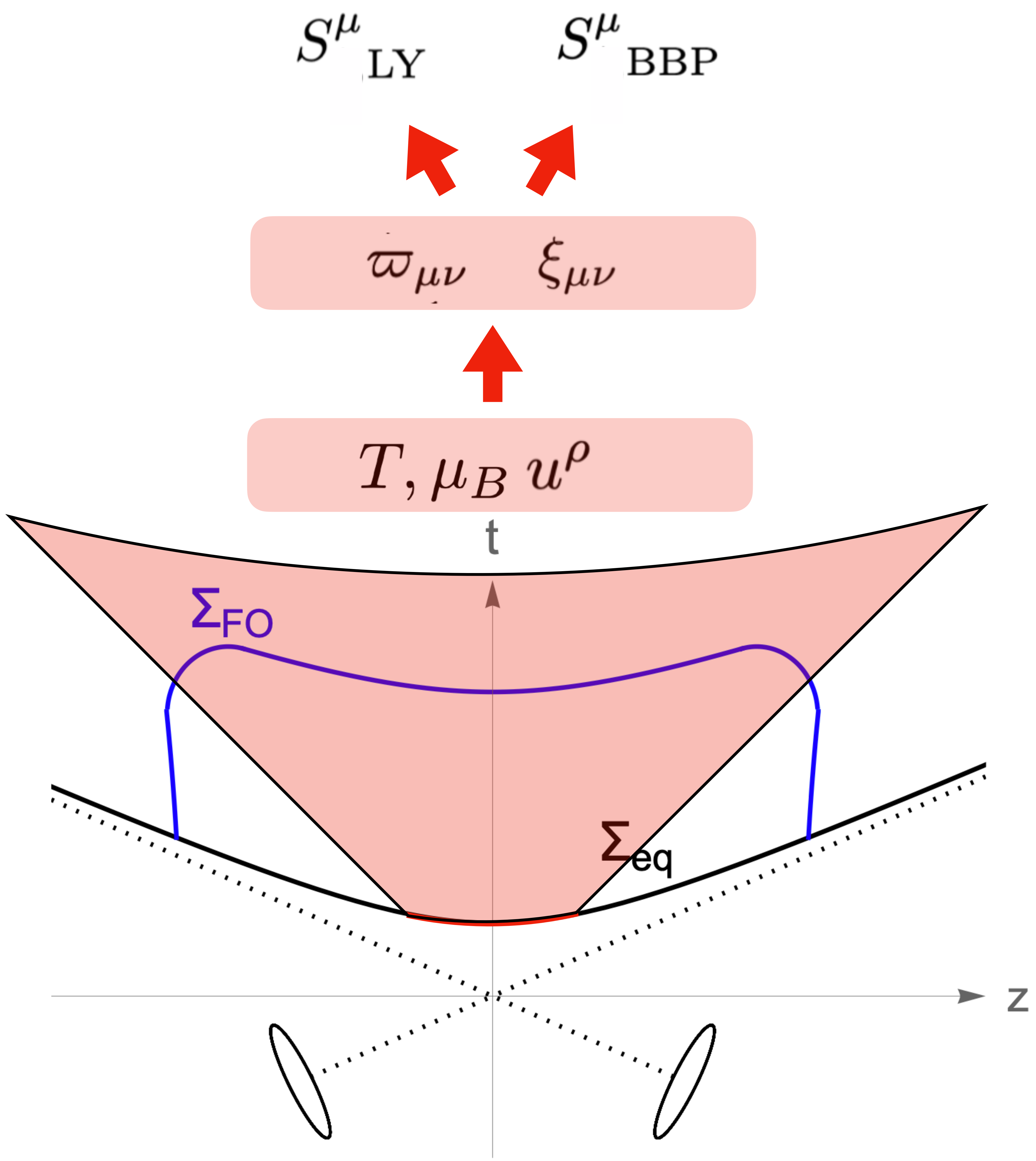


fig: https://arxiv.org/pdf/2407.12130 (modified)

Standard hydrodynamics vs Spin hydrodynamics

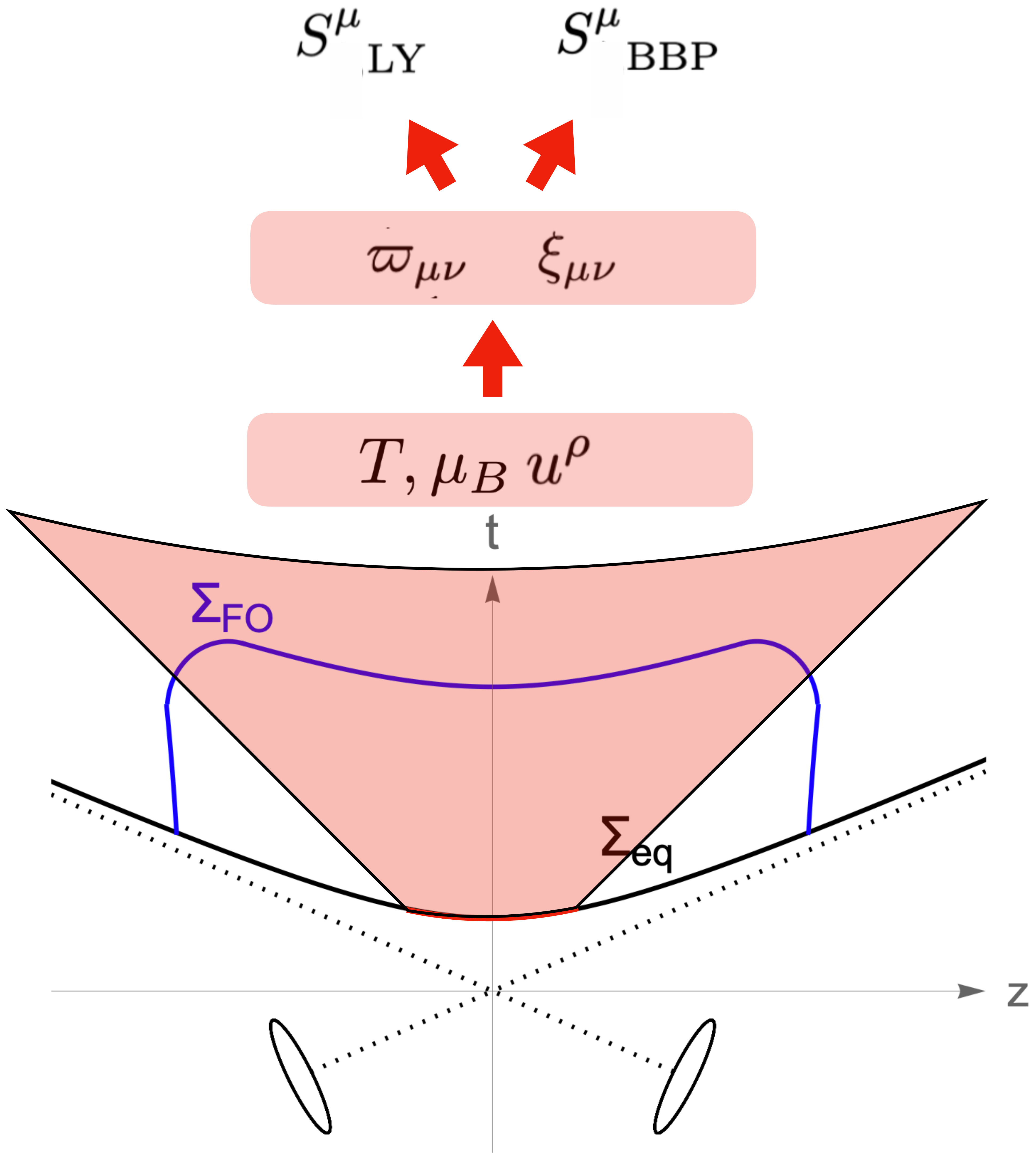


fig: https://arxiv.org/pdf/2407.12130 (modified)

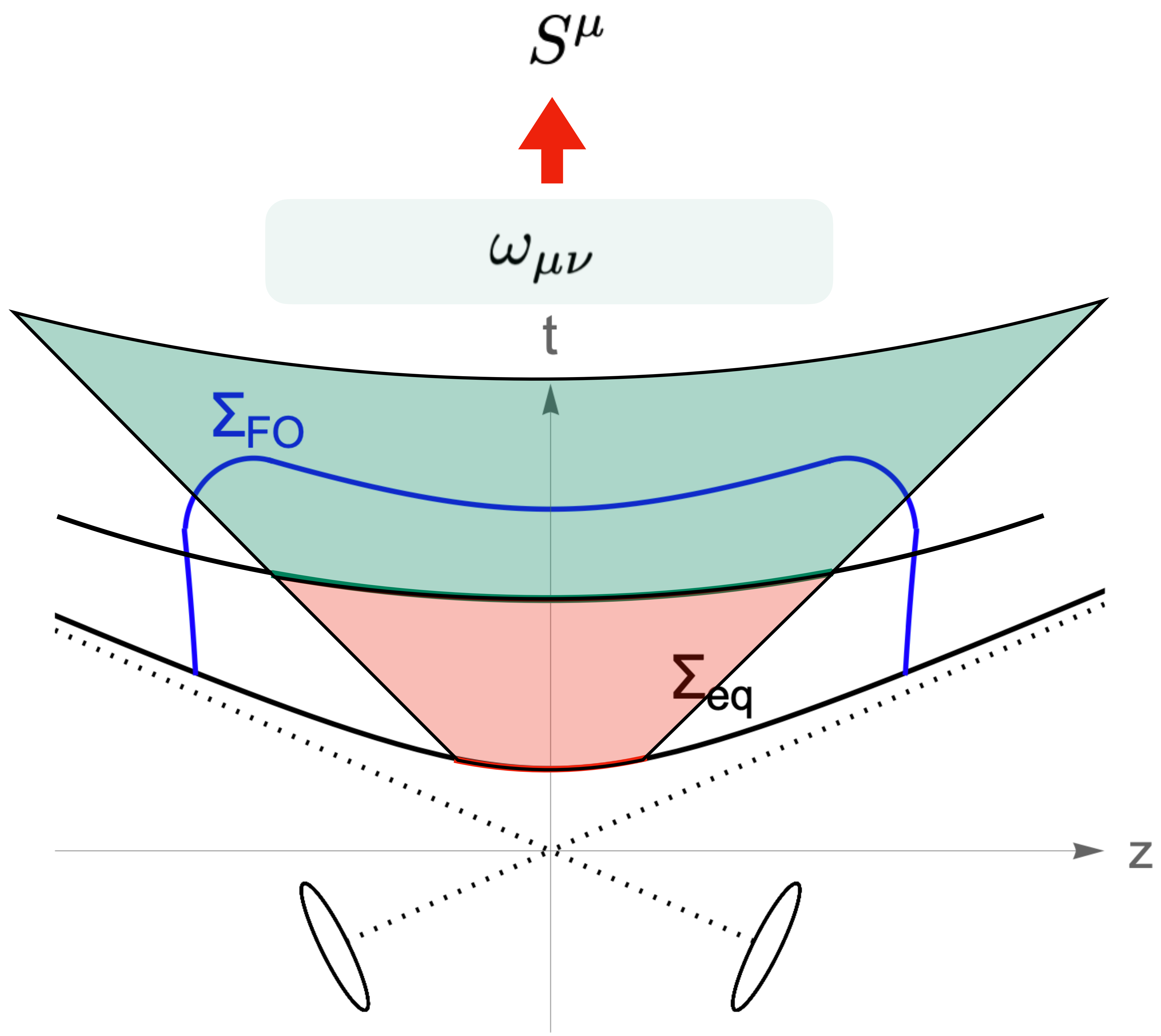
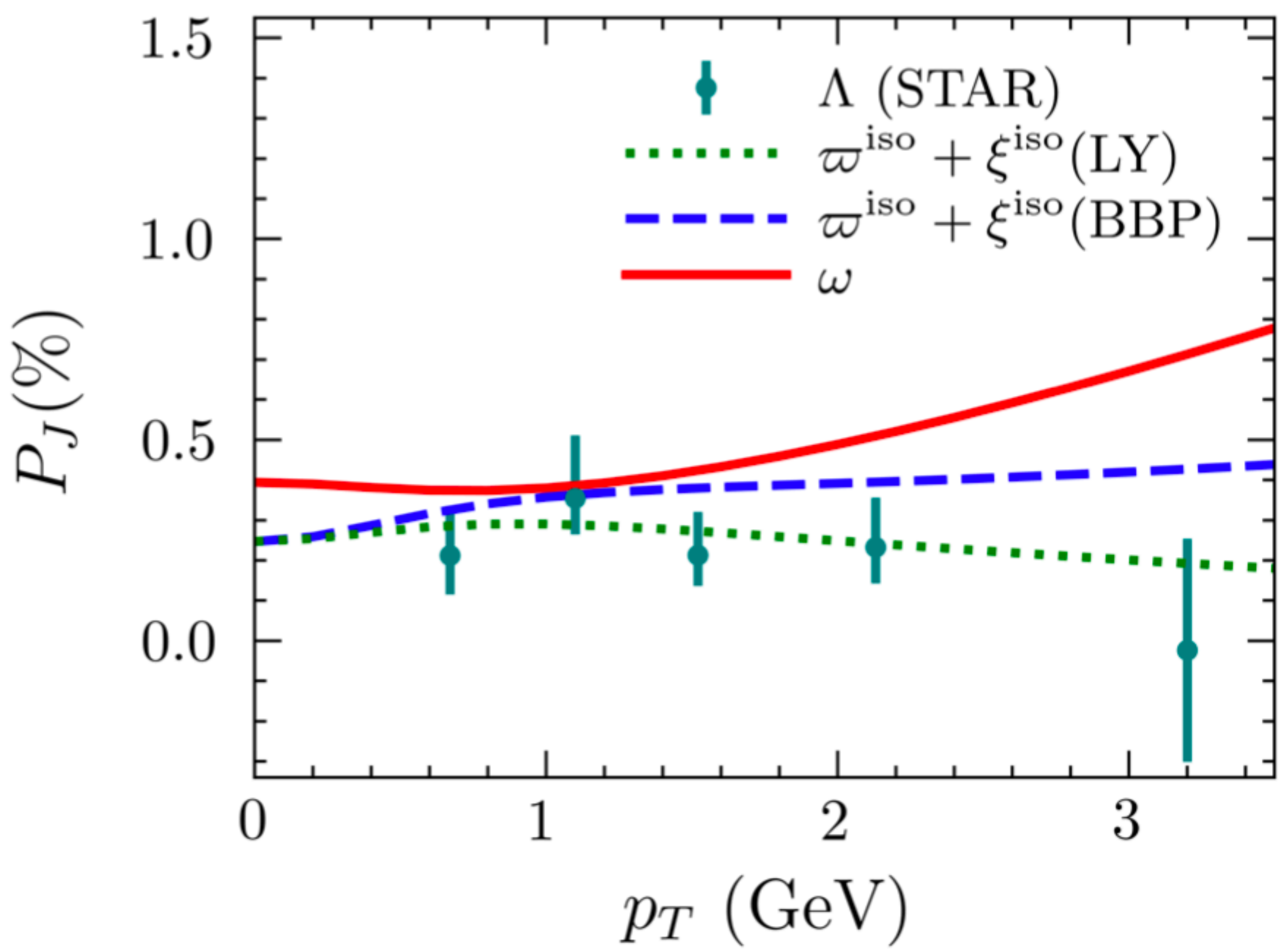


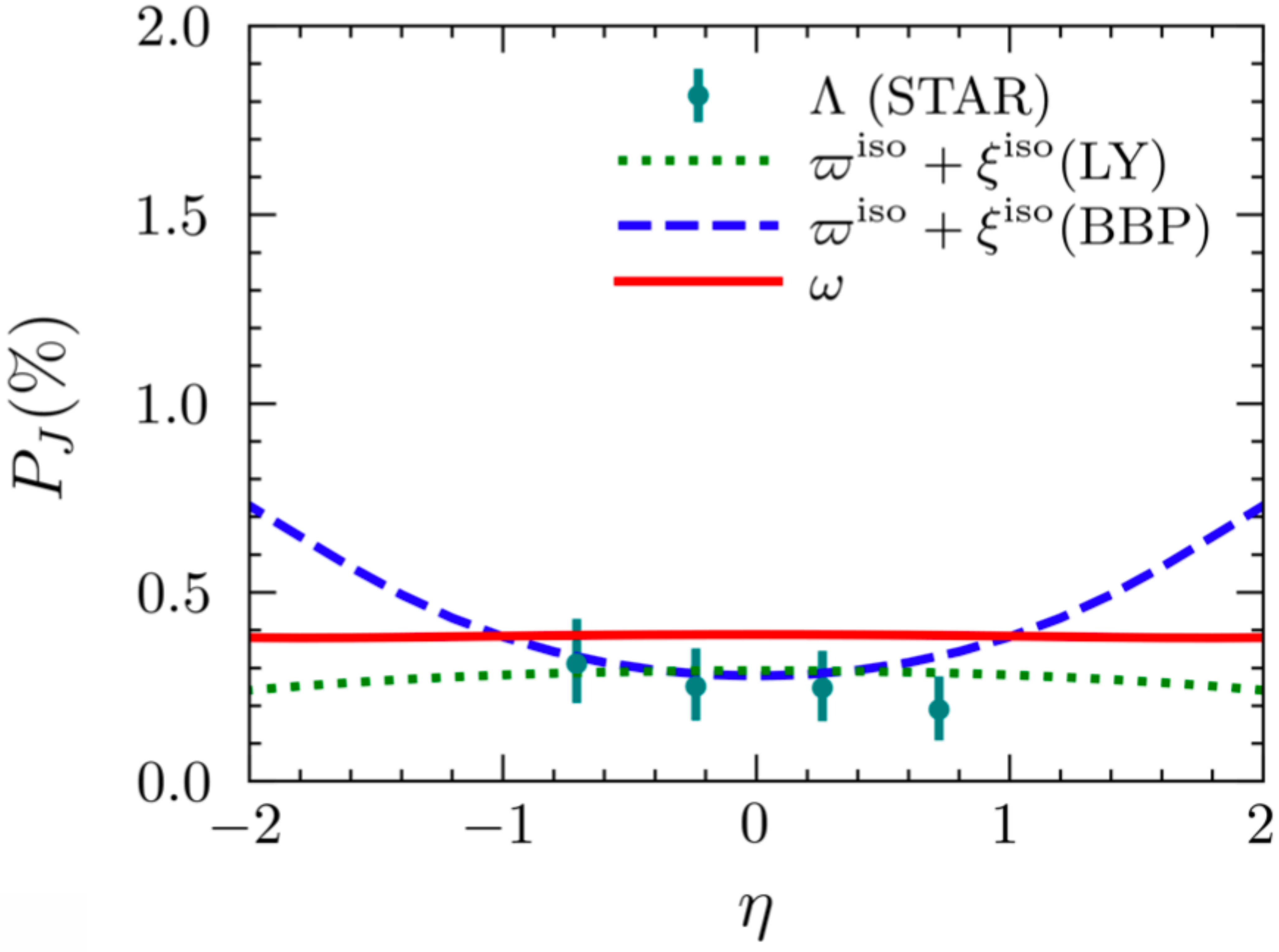
fig: https://arxiv.org/pdf/2407.12130 (modified)

RESULTS FOR SPIN HYDRODYNAMICS

Adam et al. (STAR Collaboration), Phys. Rev. C 98, 014910 (2018)



Adam et al. (STAR Collaboration), Phys. Rev. C 98, 014910 (2018)

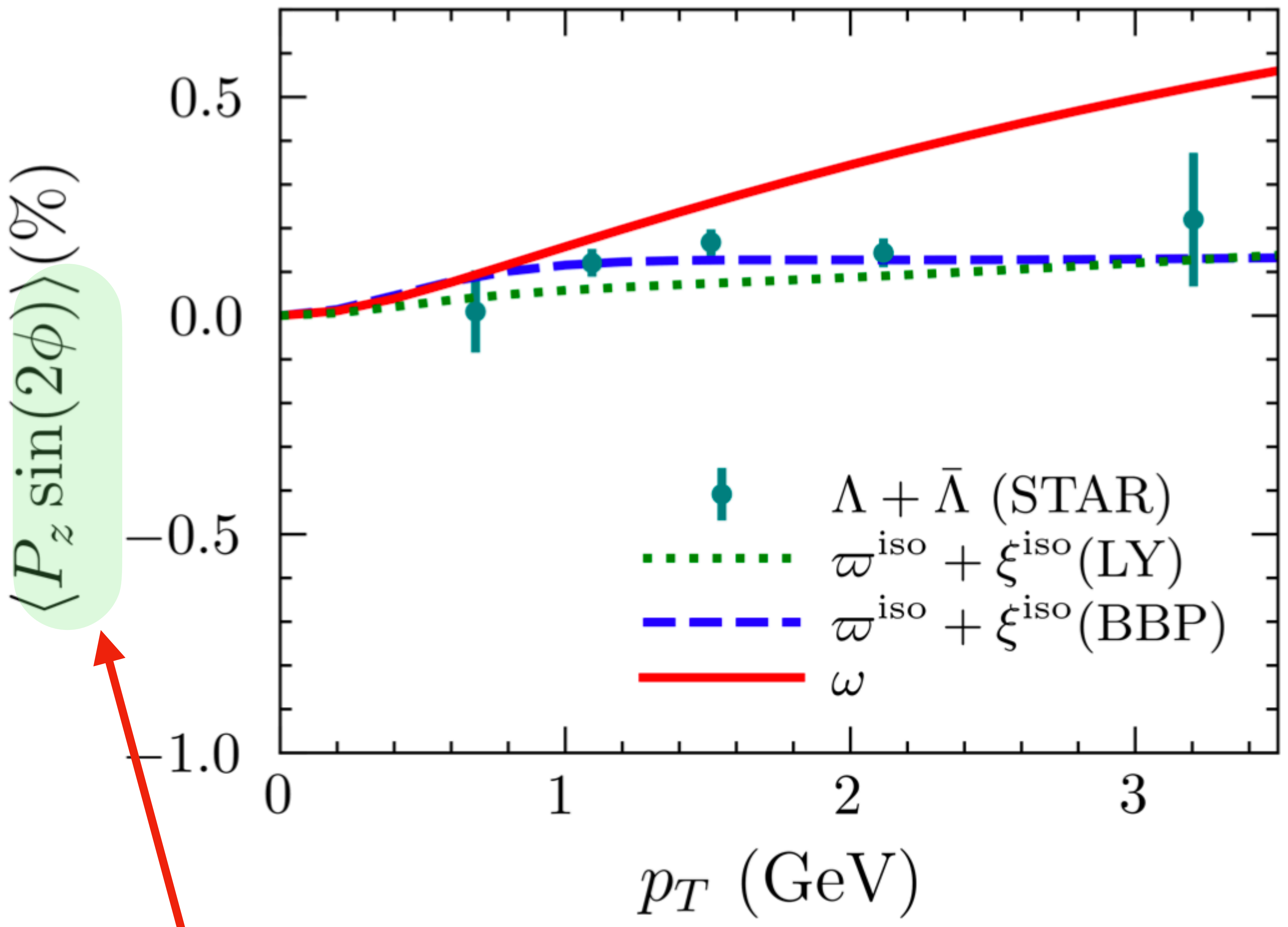


$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

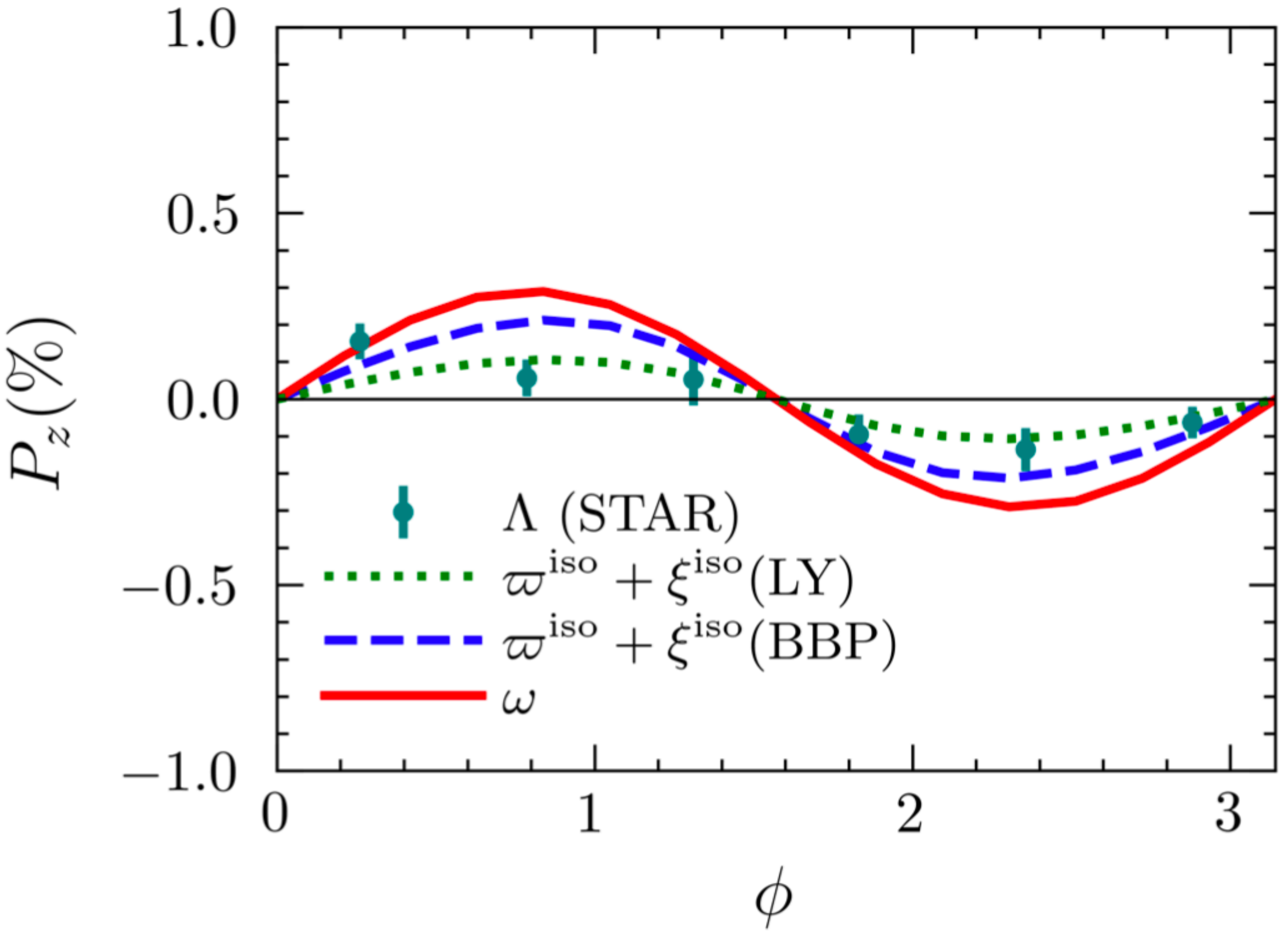
$$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$$

Results for spin hydrodynamics

Adam et al. (STAR Collaboration), Phys. Rev. Lett. 123, 132301 (2019)



Adam et al. (STAR Collaboration), Phys. Rev. Lett. 123, 132301 (2019)

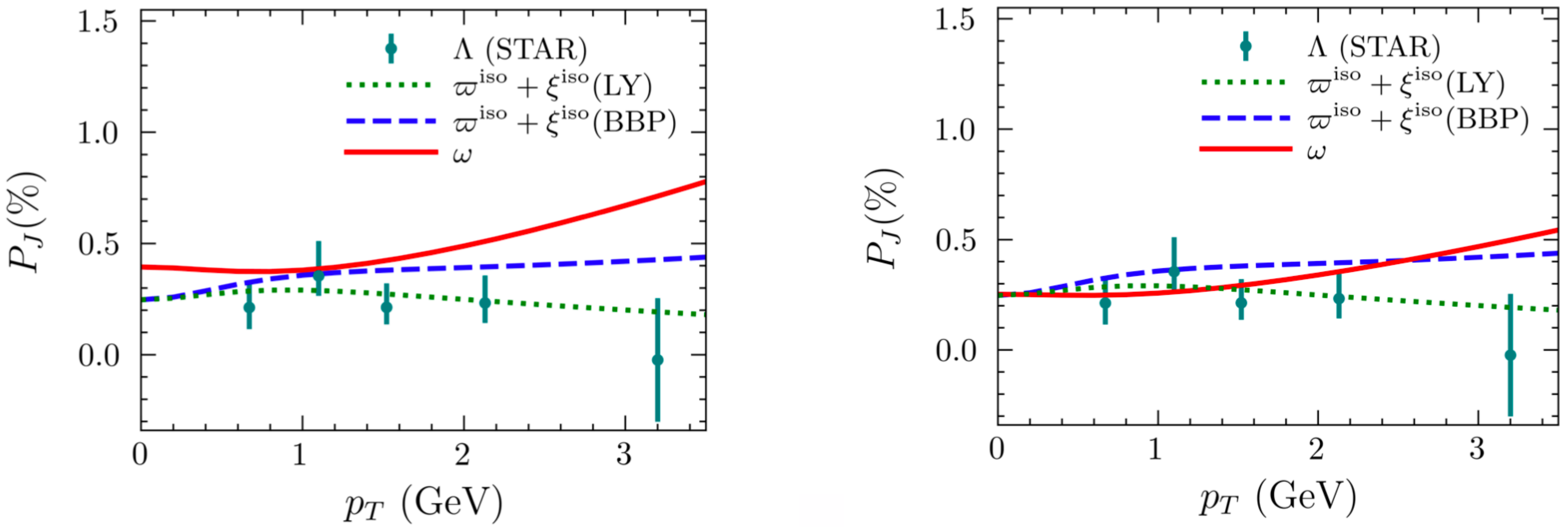


$$\langle P_z \sin(2\phi)\rangle \equiv \frac{\int P_z \sin(2\phi) d\phi dy}{\int E \frac{dN}{d^3p} d\phi} dy$$

$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

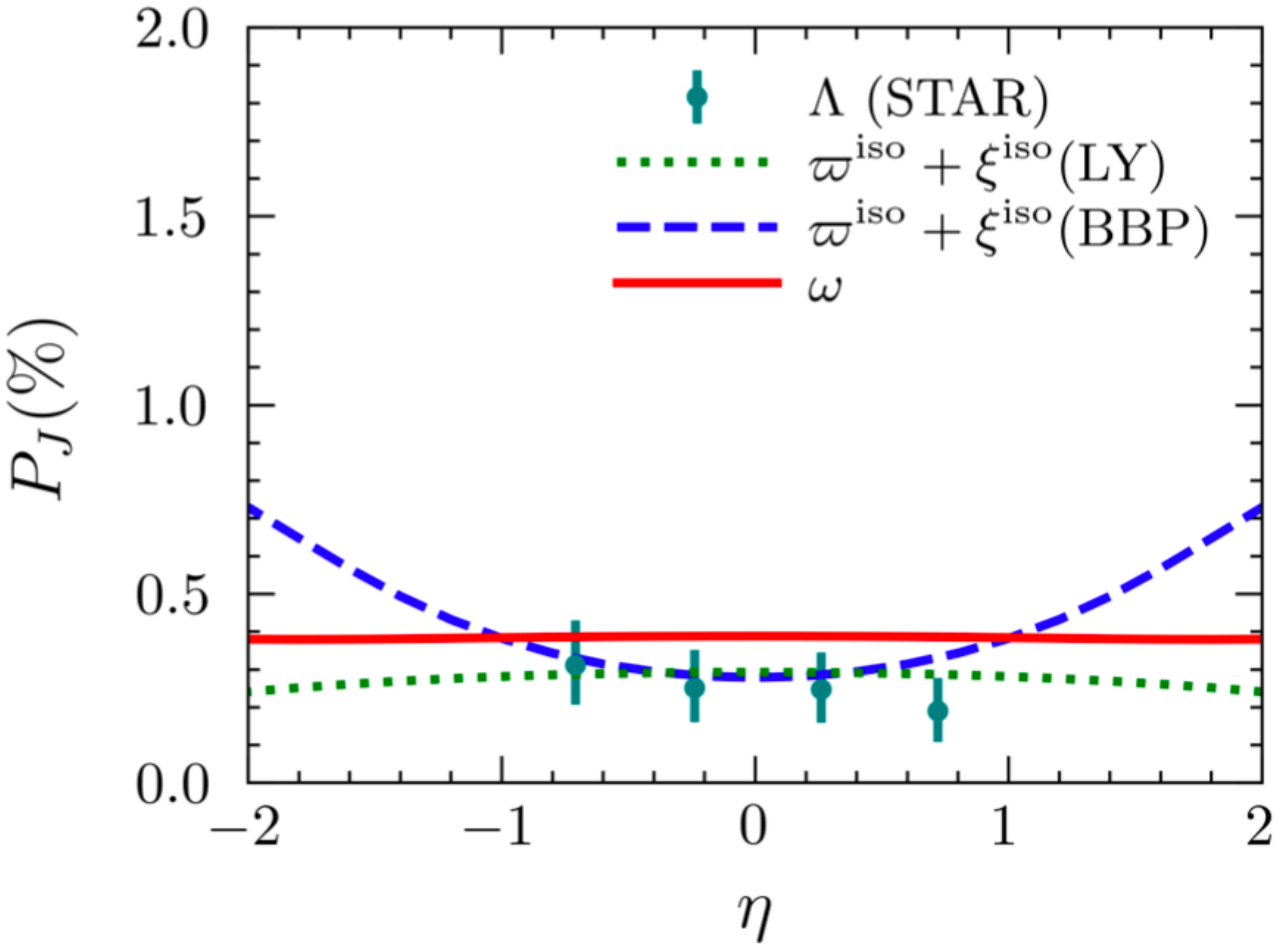
Results for spin hydrodynamics



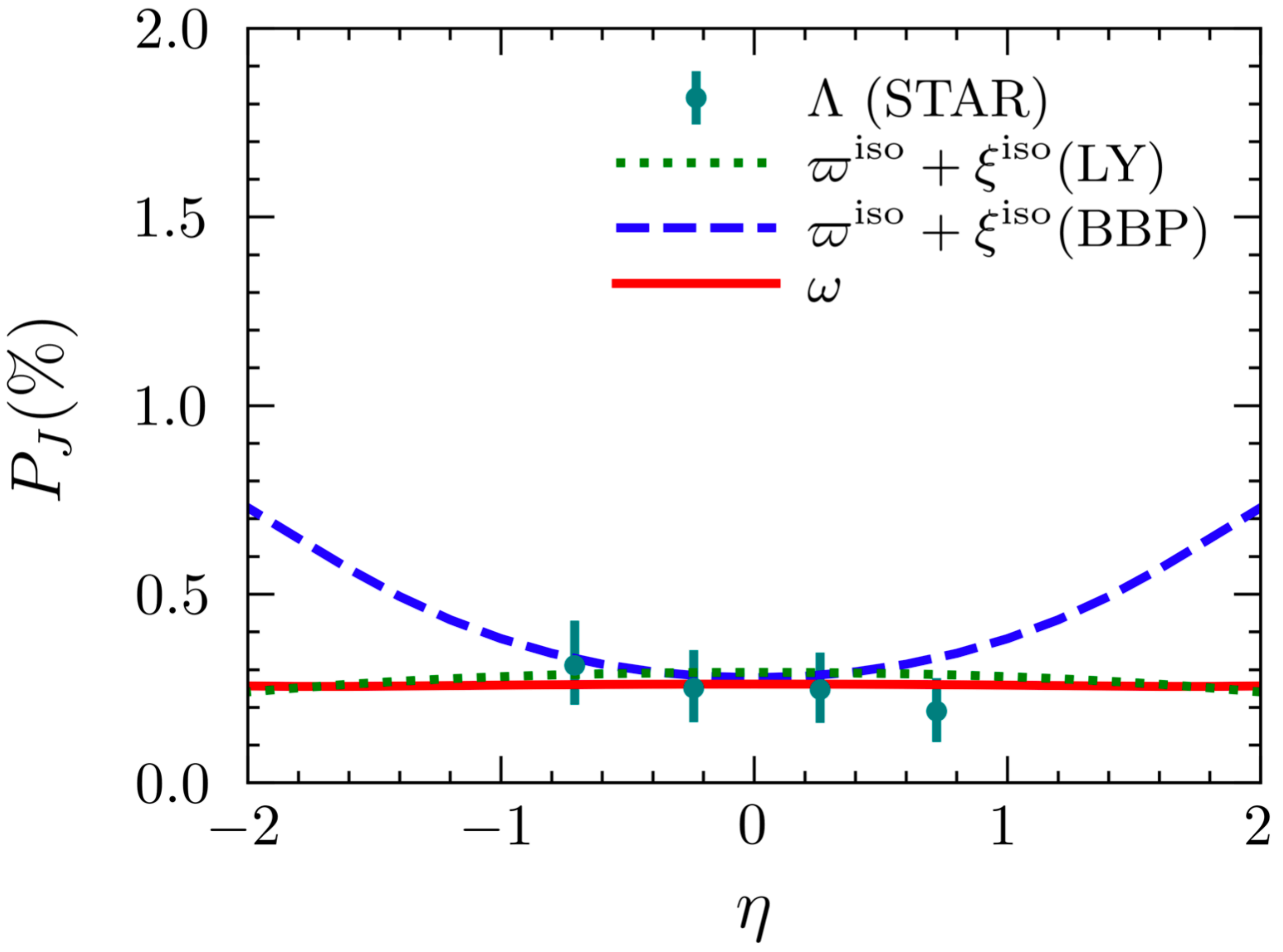
$\tau_0^s = 4$ fm and $m = m_\Lambda$ $\qquad$ $\tau_0^s = 4$ fm and $m = 300$ MeV

$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T}$ $\quad$ $\xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$

RESULTS FOR SPIN HYDRODYNAMICS



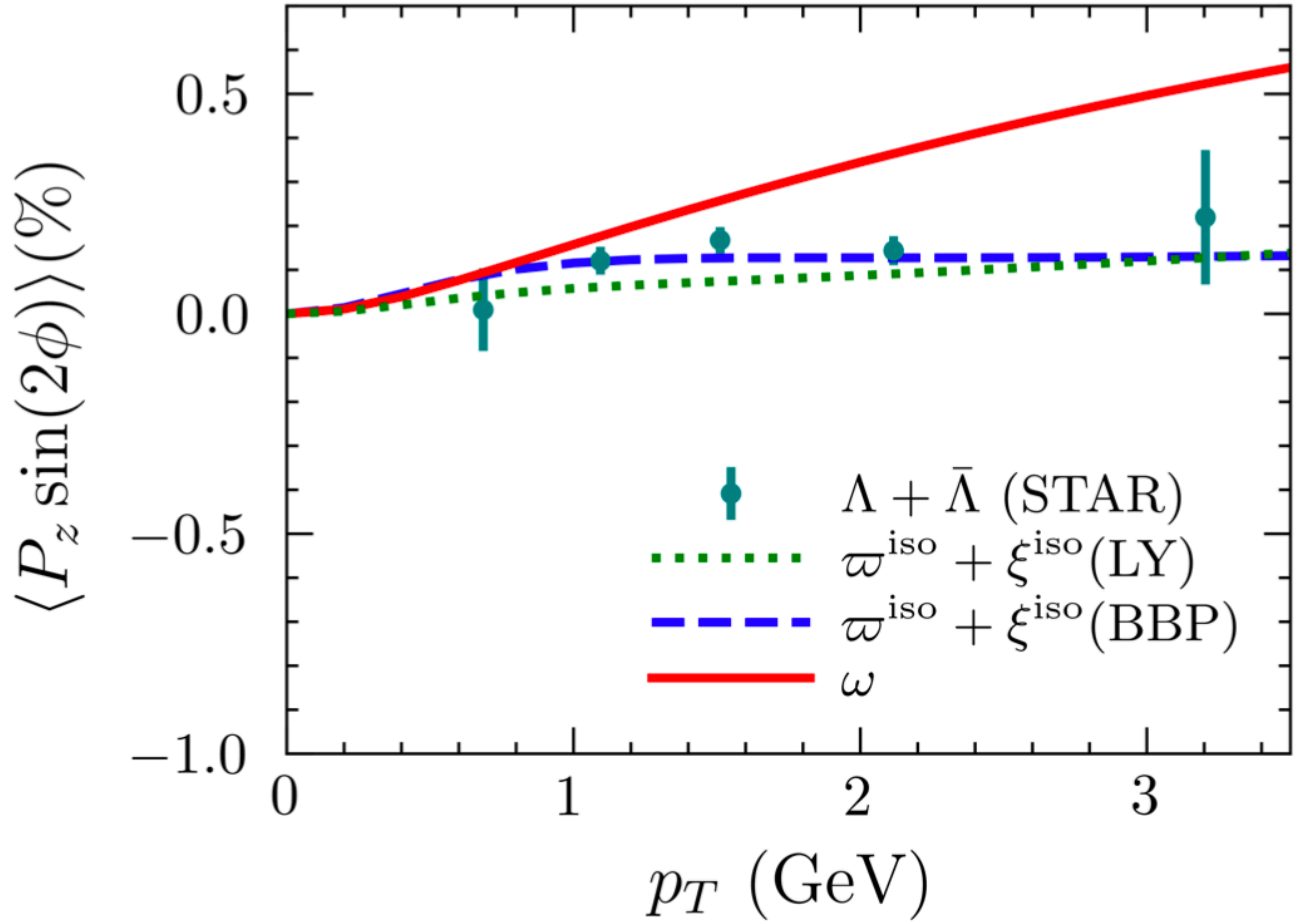
$\tau_0^s = 4$ fm and $m = m_\Lambda$



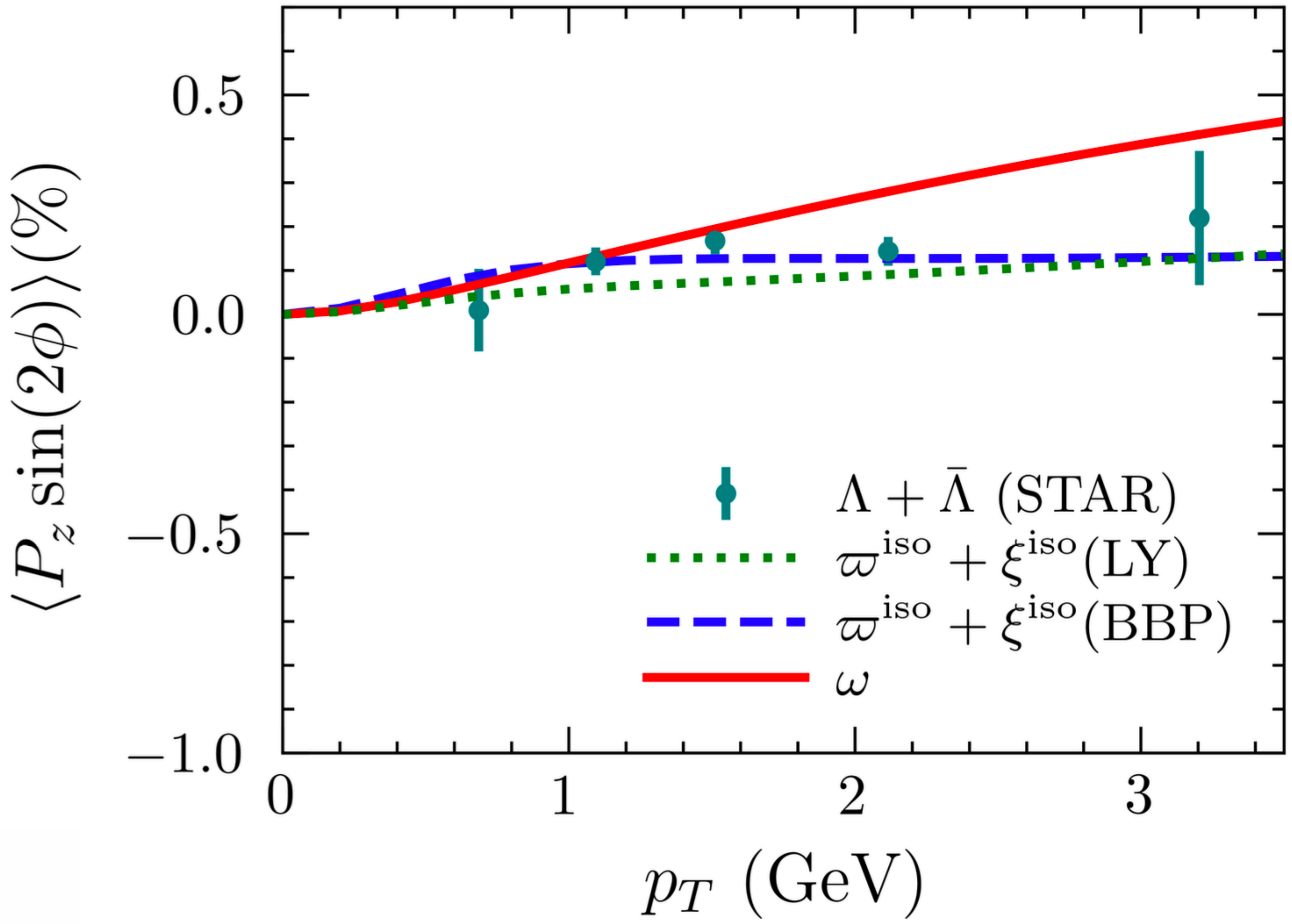
$\tau_0^s = 4$ fm and $m = 300$ MeV

$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu}$ $\quad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$

Results for spin hydrodynamics



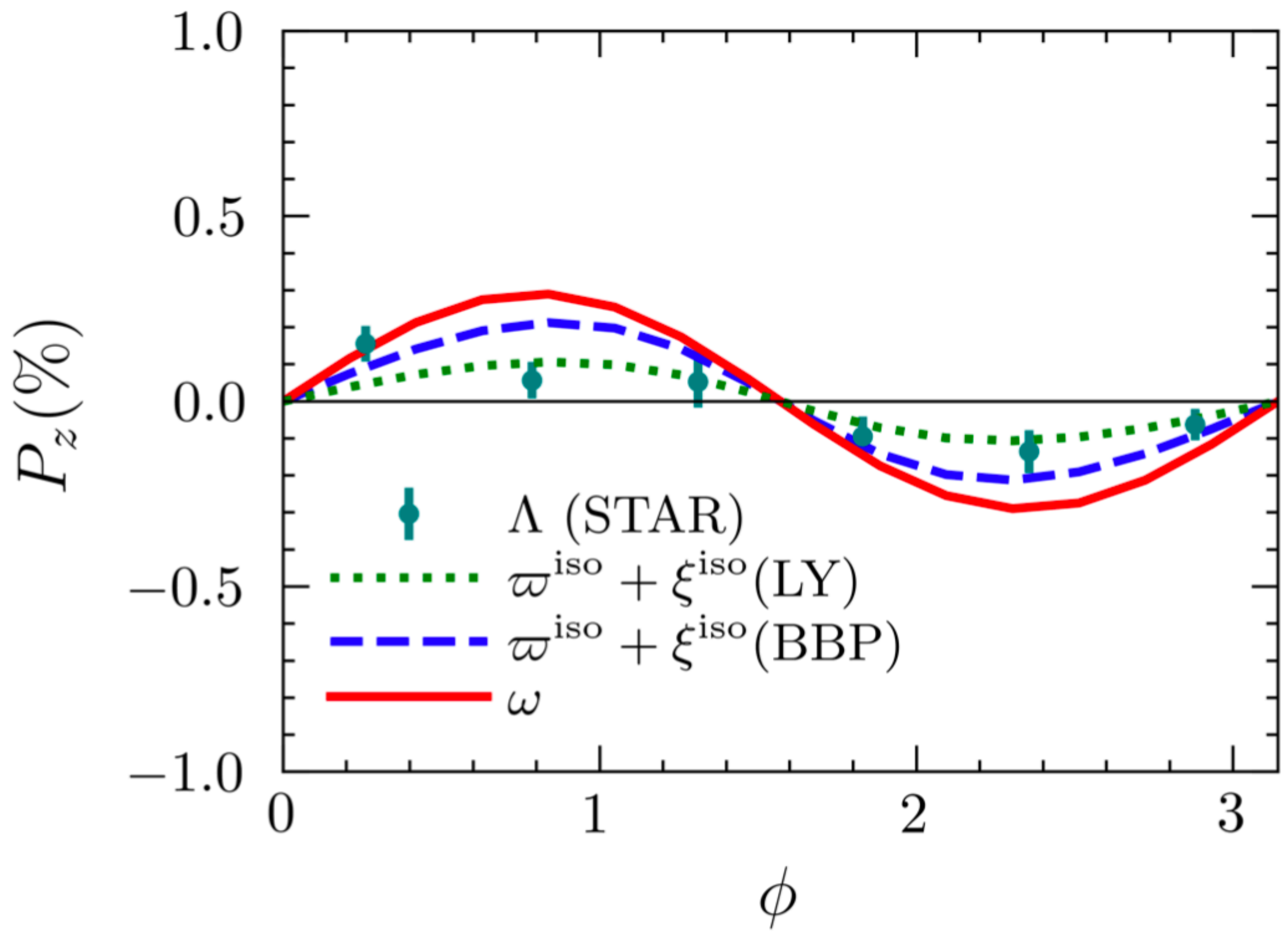
$\tau_0^s = 4$ fm and $m = m_\Lambda$



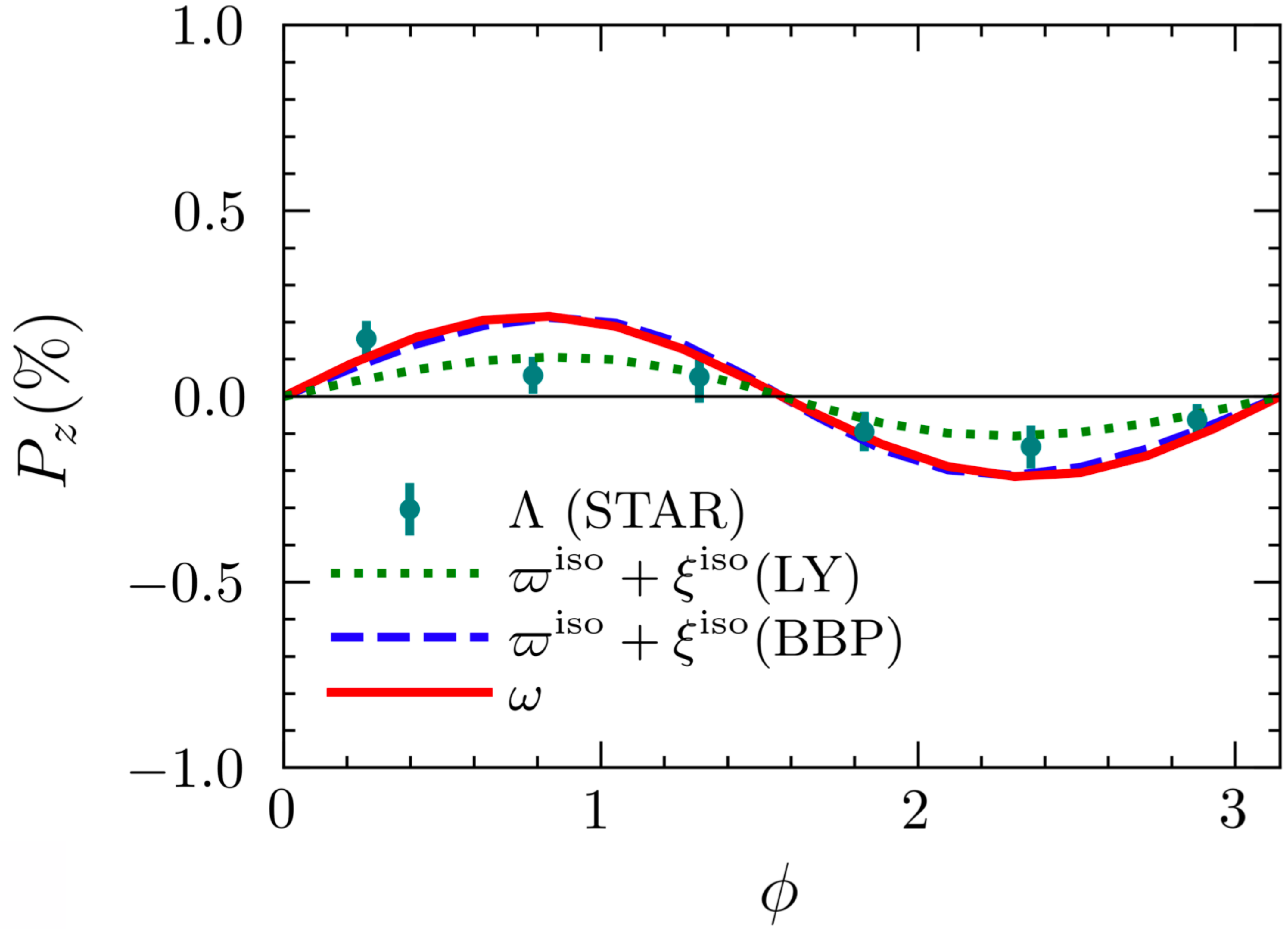
$\tau_0^s = 4$ fm and $m = 300$ MeV

$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T}$ $\quad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$

Results for spin hydrodynamics



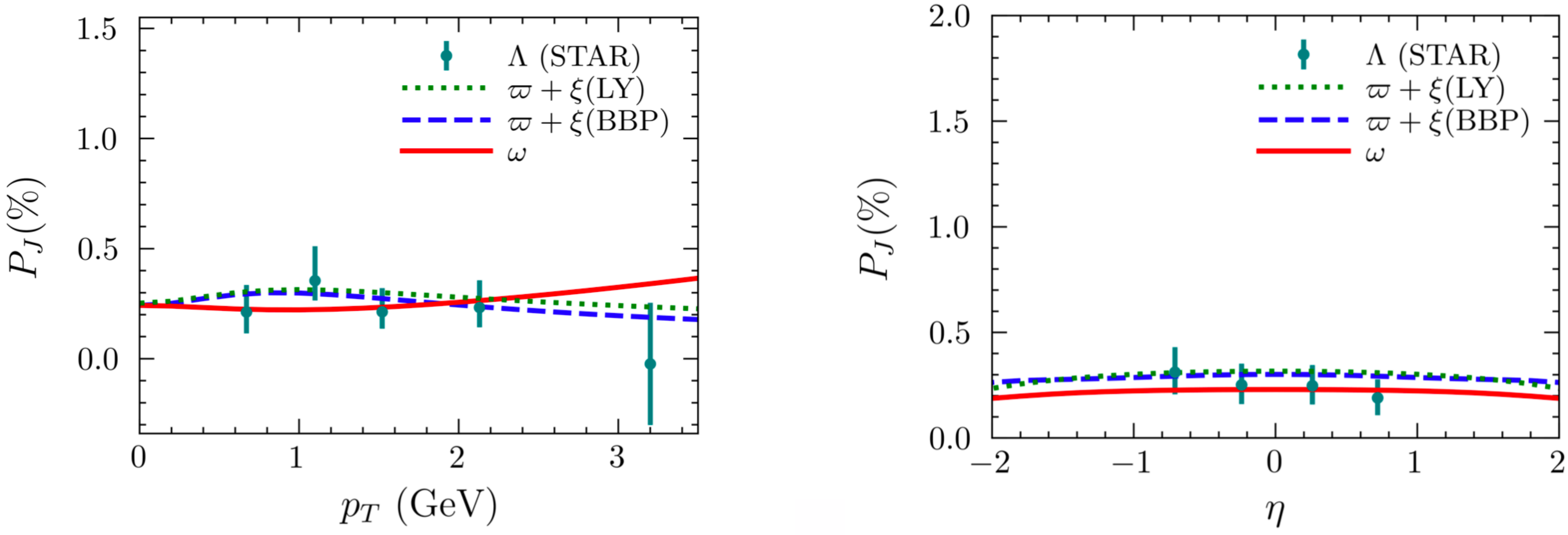
$\tau_0^s = 4$ fm and $m = m_\Lambda$



$\tau_0^s = 4$ fm and $m = 300$ MeV

$$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$$

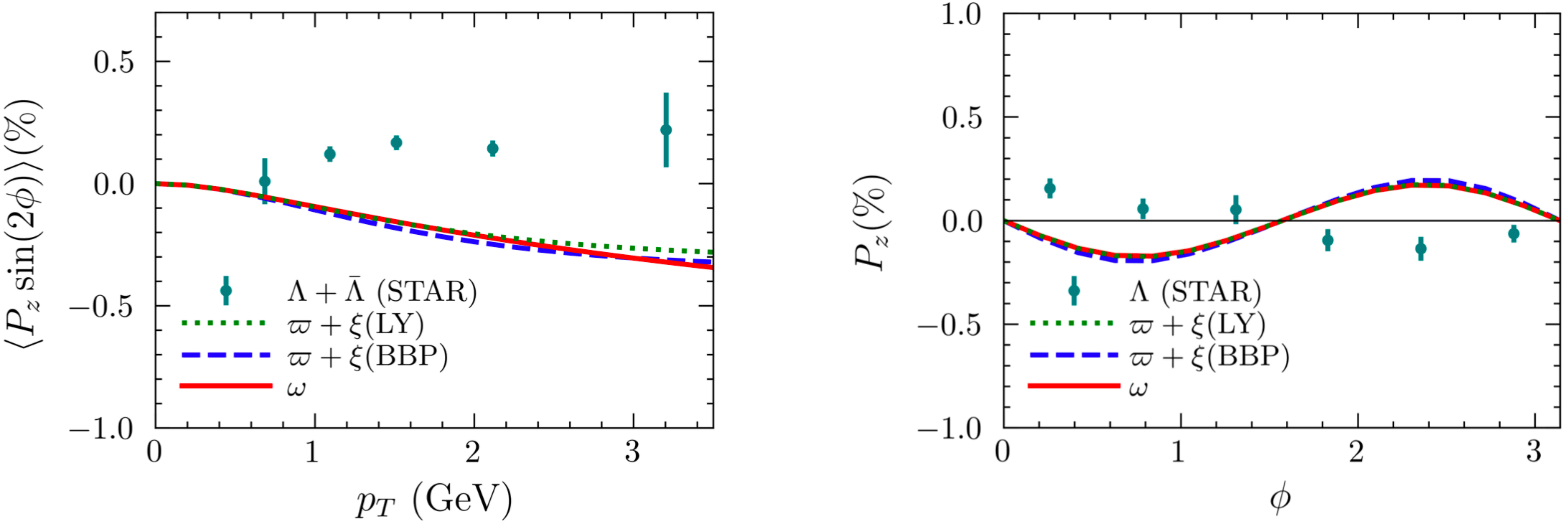
RESULTS FOR SPIN HYDRODYNAMICS



$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\text{iso}} + \varpi_{\mu\nu}^{\text{T}} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\text{iso}} + \xi_{\mu\nu}^{\text{T}}$$

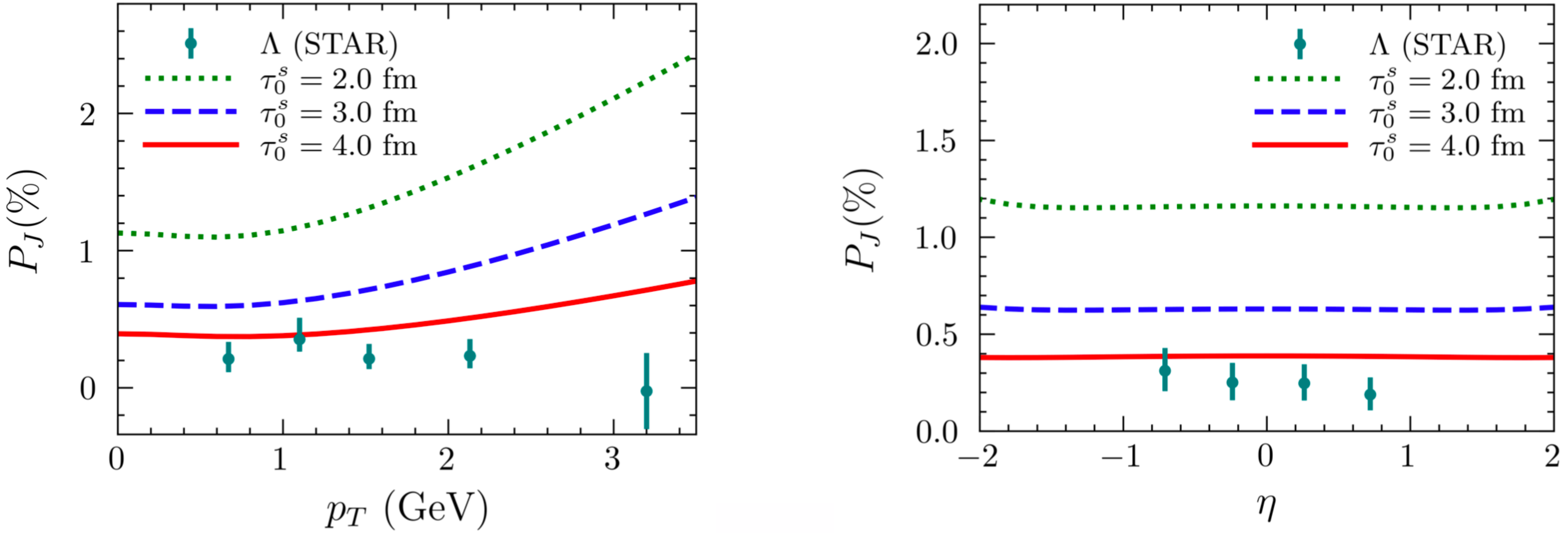
Results for spin hydrodynamics



$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi^{\text{iso}}_{\mu\nu} + \varpi^{\text{T}}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\text{iso}}_{\mu\nu} + \xi^{\text{T}}_{\mu\nu}$$

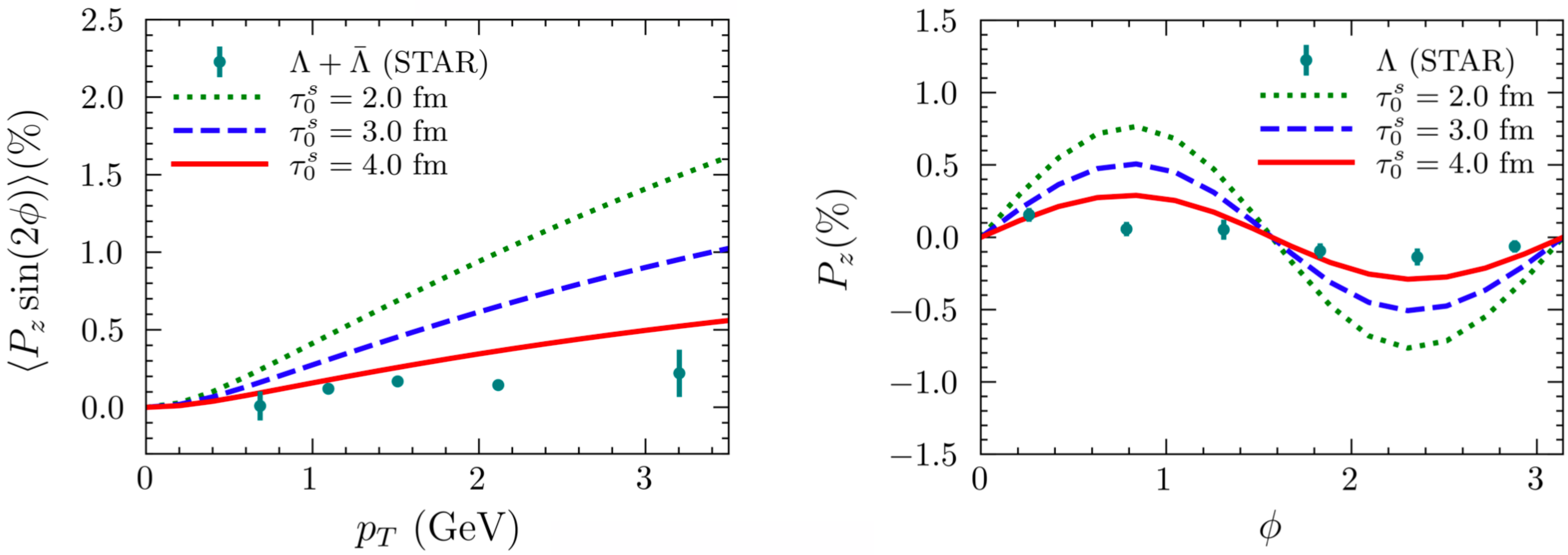
Results for spin hydrodynamics



$$m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

RESULTS FOR SPIN HYDRODYNAMICS



$$m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

Summary and outlook

We developed a complete numerical framework for perfect spin hydrodynamics

We solved perfect spin hydrodynamics in realistic 3+1 dimensional case

We tuned the background to describe basic hadronic observables

We determined polarization vector for Lambda hyperons and compared with data and other frameworks

Acceptable agreement is obtained with delayed initialization time for spin evolution

Thank you for your attention