universität freiburg

Probing optimal measurements of the electromagnetic dipole moments of the τ lepton

Based on [2410.23070]

Bialasowka seminar, Krakow

24 Jan 2025

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Introduction

Why electromagnetic dipole moments?

Particle spin is a crucial component of modern physics → historically studied using coupling of spin to EM fields / photons

At low energy these couplings manifest as the electromagnetic dipole moments

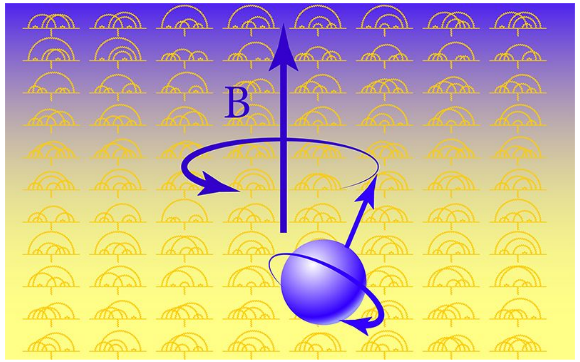

Figure from APS

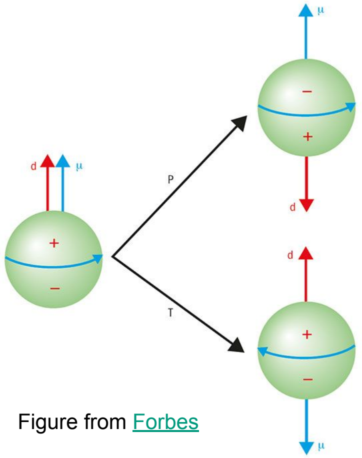

Figure from Forbes

$$\mathcal{H} \supset \boldsymbol{g}\, \mu_B\, \vec{S} \cdot \vec{B}$$

$$\mathcal{H} \supset \boldsymbol{d}\, \vec{S} \cdot \vec{E}$$

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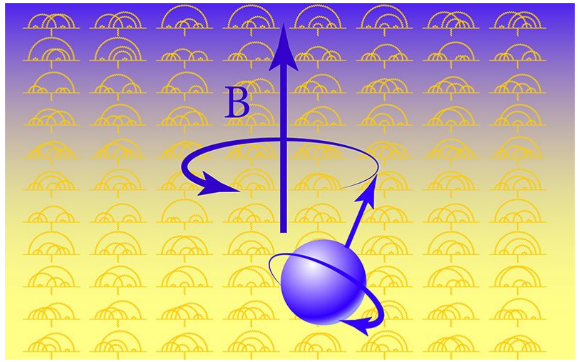

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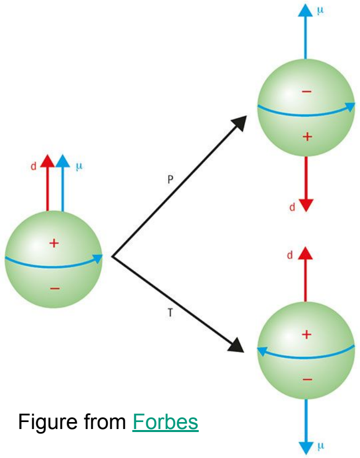

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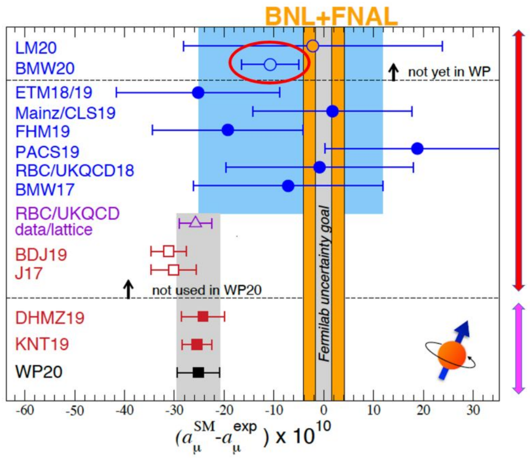
Sensitive to loop corrections from Standard Model (and maybe Beyond SM) particles and couplings → strong indirect probes of new physics



2023 muon *g*–2 result
Phys.Rev.Lett. 131 (2023) 16, 161802

Many BSM theories predict stronger coupling of new particles to heavier SM particles → in the lepton sector, tau lepton measurements are especially relevant

Past measurements of *g*–factor of tau lepton

DELPHI @ LEP (2003)
Eur.Phys.J.C 35 (2004) 159-170
$a_\tau \in [\,-0.052, +0.013\,]$ @ 95% CL
Total cross section as function of a_τ in e^+e^- collisions

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Visible $m_{\tau\tau}$ in peripheral pp collisions

CMS @ LHC (2024)
CMS-PAS-HIN-24-011
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Electron and muon p_T in ultraperipheral PbPb collisions

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Past measurements relied on cross-sections or kinematic distributions for extracting tau *g*–factor

Is it possible to define better observables?

ATL
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Towards optimal measurements

Cross section of a process of interest, differential in phase space variables φ

Parameters of interest α_i

$$\frac{d\sigma}{d\phi} = S_0 + \sum_i \alpha_i S_{1,i} + \sum_{i,j} \alpha_i \alpha_j S_{2,ij}$$

S_0, S_1, S_2 → Cross section terms

Estimators for α_i

Given integrated luminosity

$$V_{i,j}^{-1} = \mathrm{Cov}\left[\hat{\alpha}_i, \hat{\alpha}_j\right]^{-1} = \mathcal{L}_{\mathrm{int}} \int d\phi \frac{S_{1,i} S_{1,j}}{S_0}$$

This is the *minimum covariance bound*

Inverse covariance

Towards optimal measurements

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Inverse covariance

This is the *minimum covariance bound*

Estimators are, in general, based on functions of phase space variables

$$\hat{\alpha}_i = \hat{\alpha}_i(\mathcal{O}_i(\phi))$$

If these functions are chosen to calculate the estimators →

$$\mathcal{O}_i = S_{1,i}/S_0$$

Then the minimum variance bound is automatically achieved, i.e. the *smallest possible uncertainties* on measured α_i

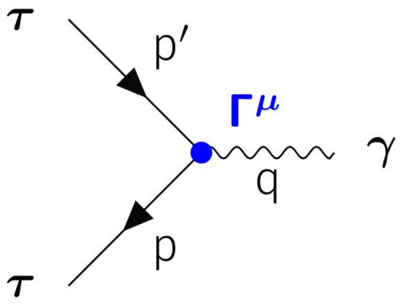
These are the ***optimal observables***

Our goal is to compare the sensitivity of optimal observables to kinematic observables in measurements of tau *g*–2 in ultraperipheral PbPb collisions @ LHC

Theoretical framework

Parameterizing the electromagnetic dipole moments

Photon–tau lepton vertex



Form factor parameterization

$$i\Gamma^{\mu}\left(p',p\right) = -ie\left[\gamma^{\mu}F_1\left(q^2\right) + \frac{i}{2m_\tau}\sigma^{\mu\nu}q_\nu F_2\left(q^2\right) + \frac{1}{2m_\tau}\sigma^{\mu\nu}\gamma^5 q_\nu F_3\left(q^2\right)\right]$$

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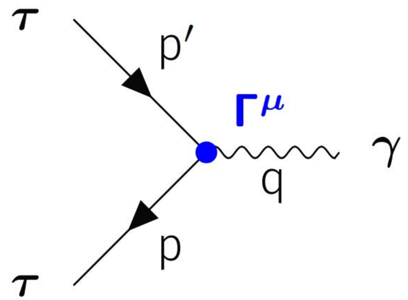


In the limit $q^2 \to 0$

$$F_1(0) = 1$$

$$F_2(0) = a_\tau = \frac{g_\tau - 2}{2}$$

$$F_3(0) = d_\tau \frac{2m_\tau}{e}$$

a, Anomalous magnetic dipole moment

d_τ → electric dipole moment

In this work we largely focus on a_τ

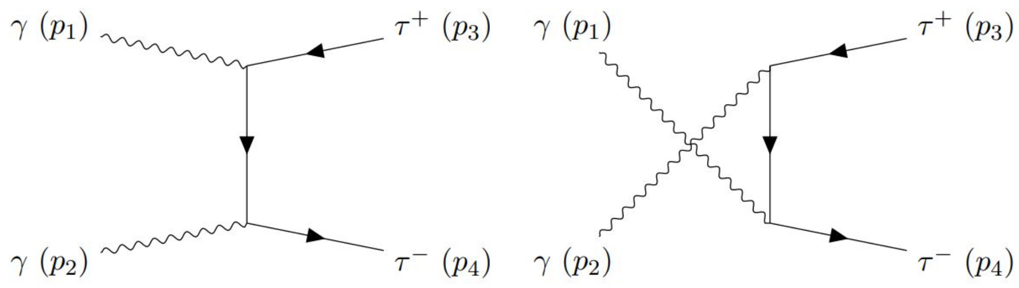
Matrix elements

Photon-induced di-tau production
in ultra-peripheral PbPb collisions

$$\mathrm{Pb} + \mathrm{Pb} \rightarrow \mathrm{Pb} + \mathrm{Pb} + \gamma\gamma \rightarrow \mathrm{Pb} + \mathrm{Pb} + \tau^{+}\tau^{-}$$
$$\gamma\,(p_1;\lambda_1) + \gamma\,(p_2;\lambda_2) \rightarrow \tau^{+}\,(p_3;\lambda_3) + \tau^{-}\,(p_4;\lambda_4)$$

$p_i \rightarrow$ particle 4-momenta
$\lambda_i \rightarrow$ particle helicity

Feynman diagrams at LO in QED



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$$\mathrm{Pb}+\mathrm{Pb}\to\mathrm{Pb}+\mathrm{Pb}+\gamma\gamma\to\mathrm{Pb}+\mathrm{Pb}+\tau^+\tau^-$$

$$\gamma\left(p_1;\lambda_1\right)+\gamma\left(p_2;\lambda_2\right)\to\tau^+\left(p_3;\lambda_3\right)+\tau^-\left(p_4;\lambda_4\right)$$

$p_i \to$ particle 4-momenta
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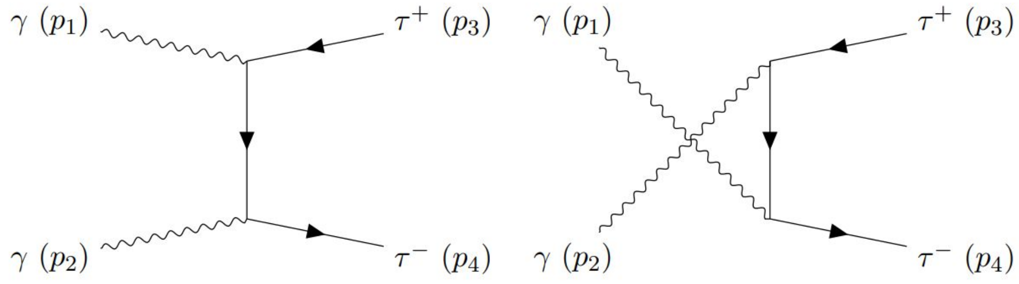

u, $v \to$ spinors
$\varepsilon \to$ photon polarization
$p_{t,u} \to$ exchange momenta

$$i\mathcal{M}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}=\epsilon_{1\mu}\left(\lambda_1\right)\epsilon_{2\nu}\left(\lambda_2\right)\bar{u}\left(p_3;\lambda_3\right)\left[i\Gamma^\mu\left(p_3,p_t\right)\frac{i\left(\not{p}_t+m_\ell\right)}{p_t^2-m_\ell^2}i\Gamma^\nu\left(p_t,-p_4\right)+i\Gamma^\nu\left(p_3,p_u\right)\frac{i\left(\not{p}_u+m_\ell\right)}{p_u^2-m_\ell^2}i\Gamma^\mu\left(p_u,-p_4\right)\right]v\left(p_4,\lambda_4\right)$$

Calculating the optimal observables for a_τ

Factorize the matrix element in powers of the dipole moments

$$\mathcal{M}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \sum_{i,j=0}^{2} (a_\tau)^i \, (d_\tau)^j \, \mathcal{M}_{ij}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}$$

M_{00} is the (tree-level) SM matrix element

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M_{00} is the (tree-level) SM matrix element

Take the square (summing / averaging over helicities if needed) and calculate the observables

$$\mathcal{OO}_{1,a_\tau}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \frac{2\mathrm{Re}\left\{\left(\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right)^* \mathcal{M}_{10}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right\}}{\left|\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right|^2}$$

Fully polarized optimal observable

$$\mathcal{OO}_{1,a_\tau}^{\lambda_3,\lambda_4} = \frac{\sum_{\lambda_1,\lambda_2} 2\mathrm{Re}\left\{\left(\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right)^* \mathcal{M}_{10}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right\}}{\sum_{\lambda_1,\lambda_2}\left|\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right|^2}$$

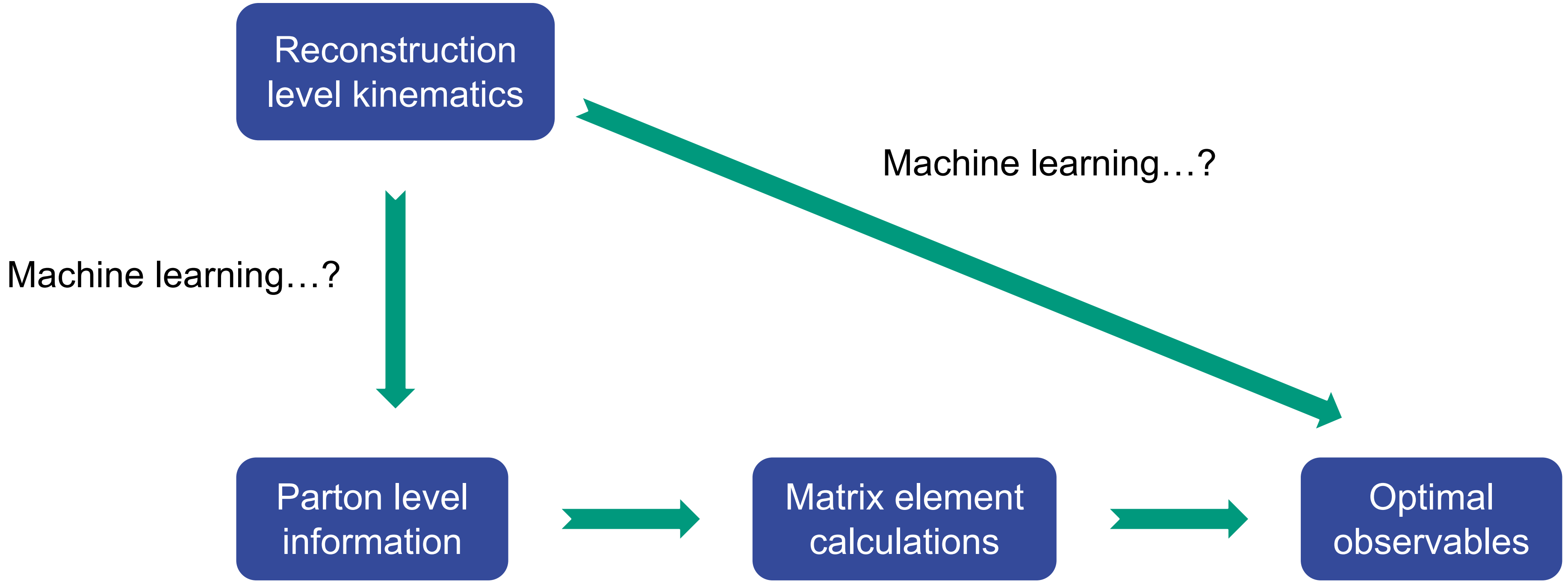
Photon-helicity averaged optimal observable

$$\mathcal{OO}_{1,a_\tau} = \frac{\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} 2\mathrm{Re}\left\{\left(\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right)^* \mathcal{M}_{10}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right\}}{\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\left|\mathcal{M}_{00}^{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right|^2}$$

Helicity averaged optimal observable

Less event information known

Calculating the optimal observables for a_τ



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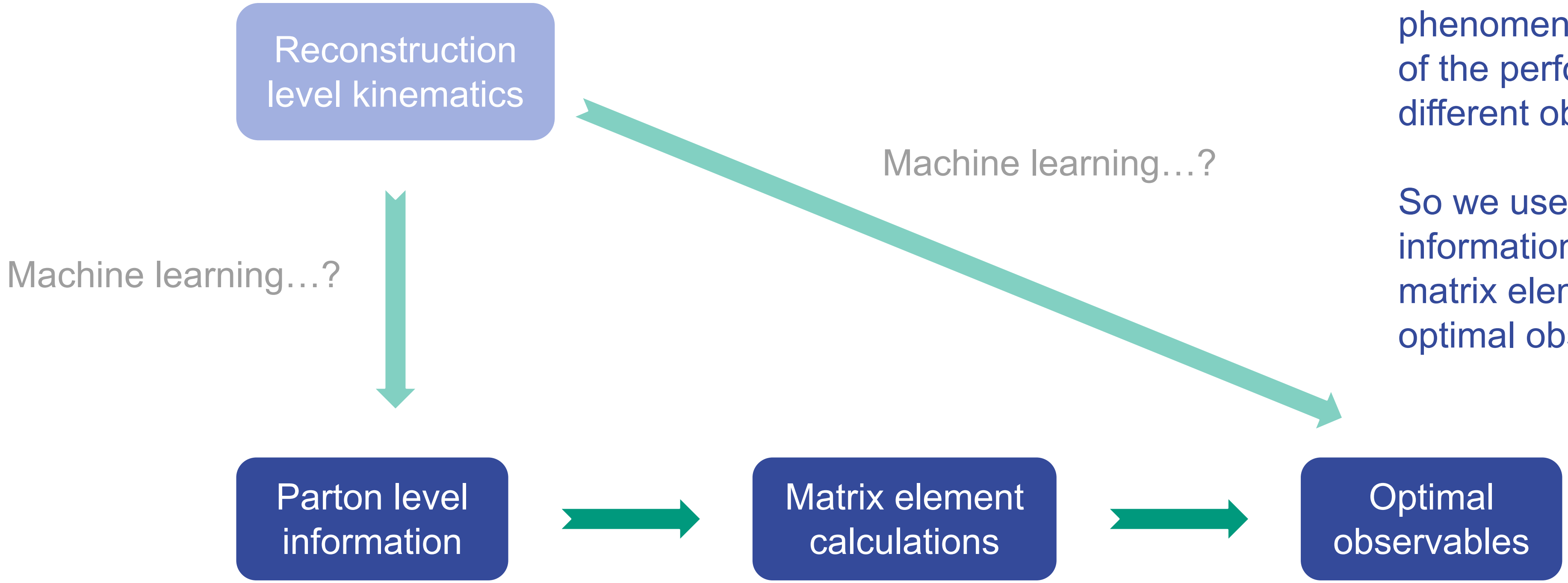


In this work, we perform a phenomenological analysis of the performance of different observables

So we use parton-level information to calculate the matrix elements and optimal observables

Monte Carlo simulations

Event generators

Photon flux and matrix elements

gammaUPC + Madgraph5_aMC@NLO

Woods-Saxon potential for Pb ion charge distribution

FeynRules model to implement non-zero dipole moments

Tau decays

Pythia v8.245

Helicity from Madgraph matrix elements used in the decay

Final state photon radiation

Photos v3.61

Fiducial definitions

Closely follow the ATLAS 2022 signal region definitions

Object definitions

Leptons (electrons, muons)	Tracks (charged hadrons)
p_T > 4 GeV, $|\eta|$ < 2.5	p_T > 0.1 GeV, $|\eta|$ < 2.5

Fiducial region definitions

	1 μ + 1 track	1 μ + 3 tracks	1 μ + 1 e
N_{muons}, $N_{electrons}$, N_{tracks}	1, 0, 1	1, 0, 3	1, 0, 1
$\Delta R(\mu, trk(s)/e)$	> 0.1	> 0.1	> 0.1
Σcharge	0	0	0
$p_T(\mu+trk)$	> 1 GeV	–	–
m_{trks}	–	< 1.7 GeV	–
$A_\phi(\mu, trk(s))$	< 0.4	< 0.2	–

Matrix element based reweighting

To save compute resources, MC samples with many different a_τ are produced by modifying event weights of a reference sample, based on matrix element (squared) ratios

$$w^i_{\text{new}} = \frac{\left|\mathcal{M}^i_{\text{new}}\right|^2}{\left|\mathcal{M}^i_{\text{old}}\right|^2} \cdot w^i_{\text{old}} = R^i \cdot w^i_{\text{old}}$$

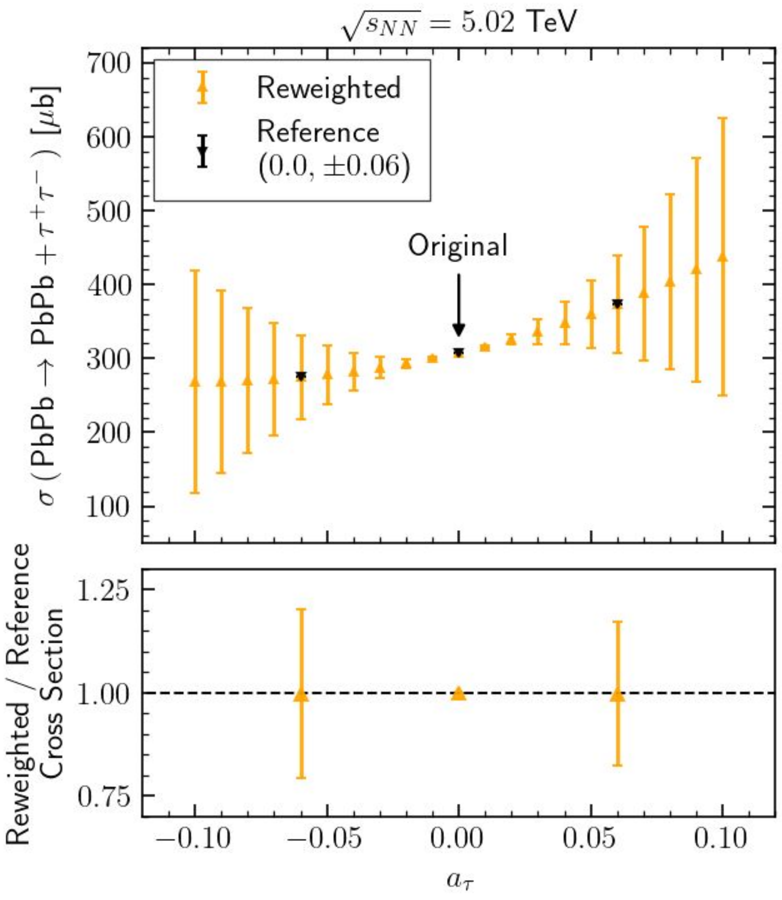
Uncertainties on observables $\mathcal{O}$ are propagated [Eur.Phys.J.C 76 (2016) 12, 674]

$$\mathcal{O}_{\text{new}} = \langle R \rangle \cdot \mathcal{O}_{\text{old}}$$
$$\Delta\mathcal{O}_{\text{new}} = \langle R \rangle \cdot \Delta\mathcal{O}_{\text{old}} + \Delta R \cdot \mathcal{O}_{\text{old}}$$

Average R^i (for $\langle R \rangle$)

RMS of R^i (for ΔR)

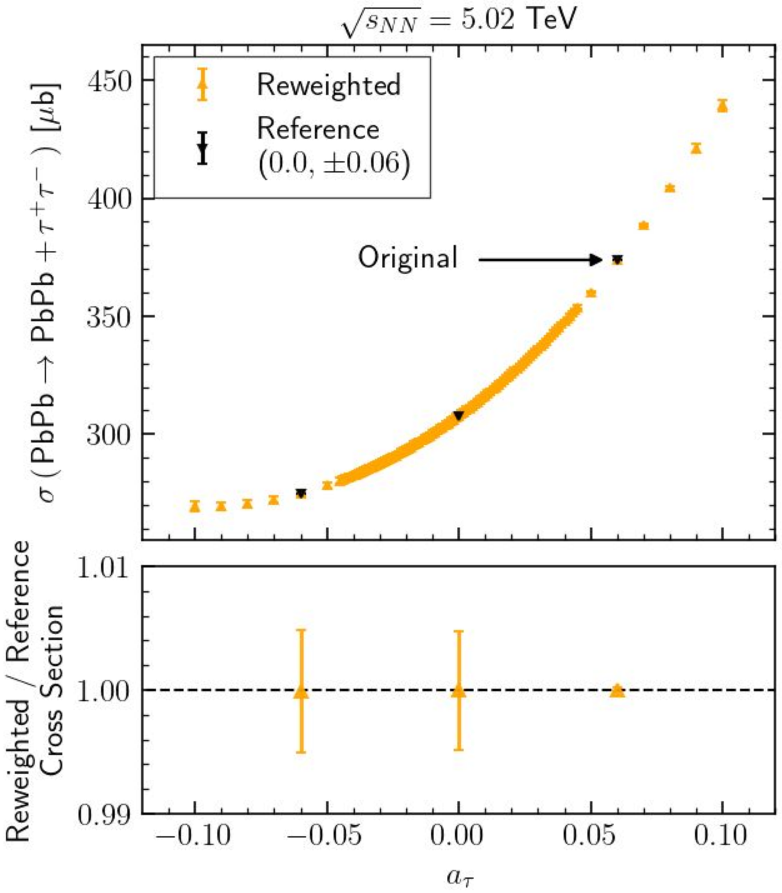
Validation of matrix element based reweighting



Reweighting from a_τ = 0

- Central value of cross section matches dedicated samples
- Errors are very large, dominated by spread in reweighting factors, *ΔR*
- Happens because some helicity combinations have zero matrix element at a_τ = 0 → incur large error due to extrapolation uncertainty

Validation of matrix element based reweighting



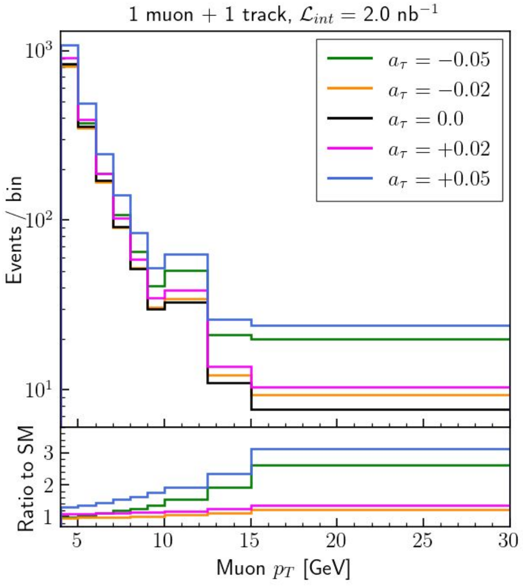
Reweighting from $a_\tau > 0$

- Central value of cross section matches dedicated samples
- Errors are in control, < 0.5%, because all helicity combinations have non-zero matrix elements when $a_\tau > 0$
- This is chosen as the preferred reweighting strategy

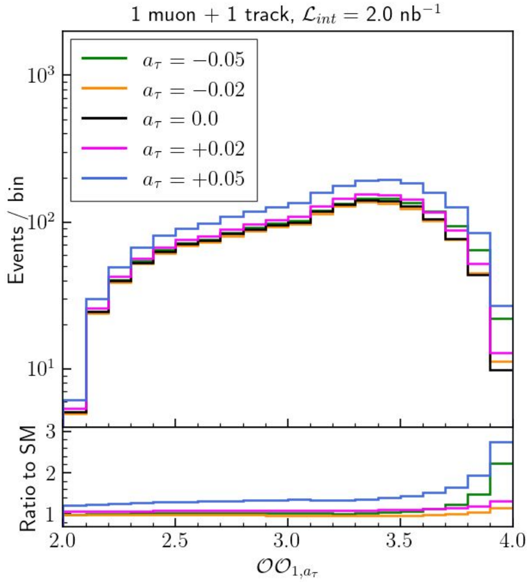
Statistical analysis

Distributions of observables (1μ+1trk, L=2.0 nb^{-1})

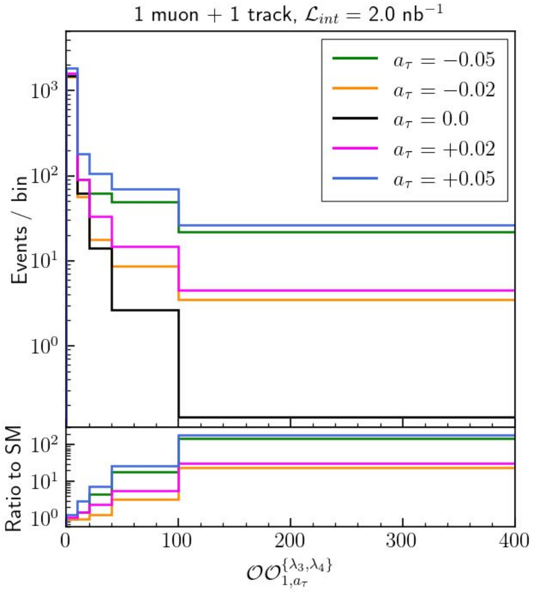
Muon p_T



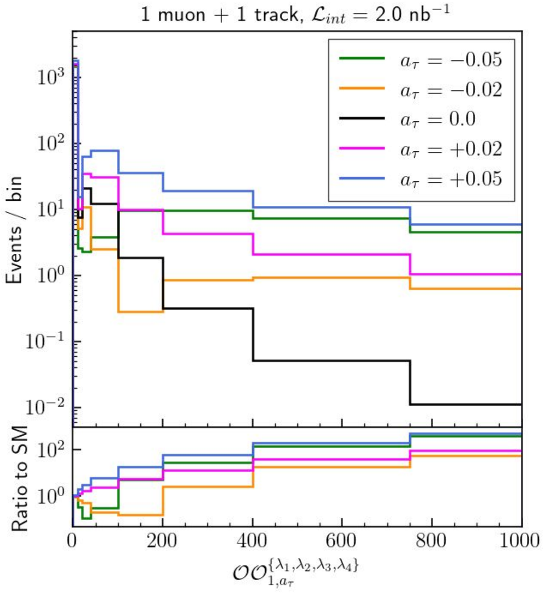
Helicity averaged optimal observable



Photon-helicity averaged optimal observable



Fully polarized optimal observable



More sensitivity with more information from the event

Methodology: binned ML fits

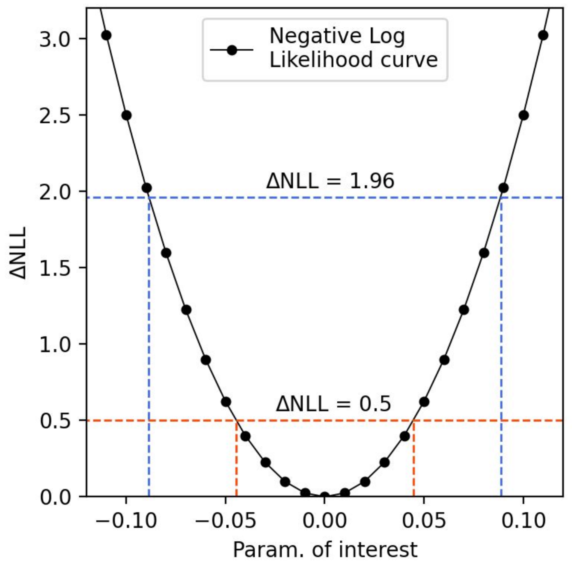
Likelihood assuming Poisson distributions

$$L(\alpha | \{n_i\}) = \prod_{i \in \text{bins}} \frac{(\mu_i(\alpha))^{n_i}}{n_i!} \exp(-\mu_i(\alpha))$$

Predictions for $a_\tau = 0$

Predictions for $a_\tau \neq 0$

Take logarithm, subtract by minimum

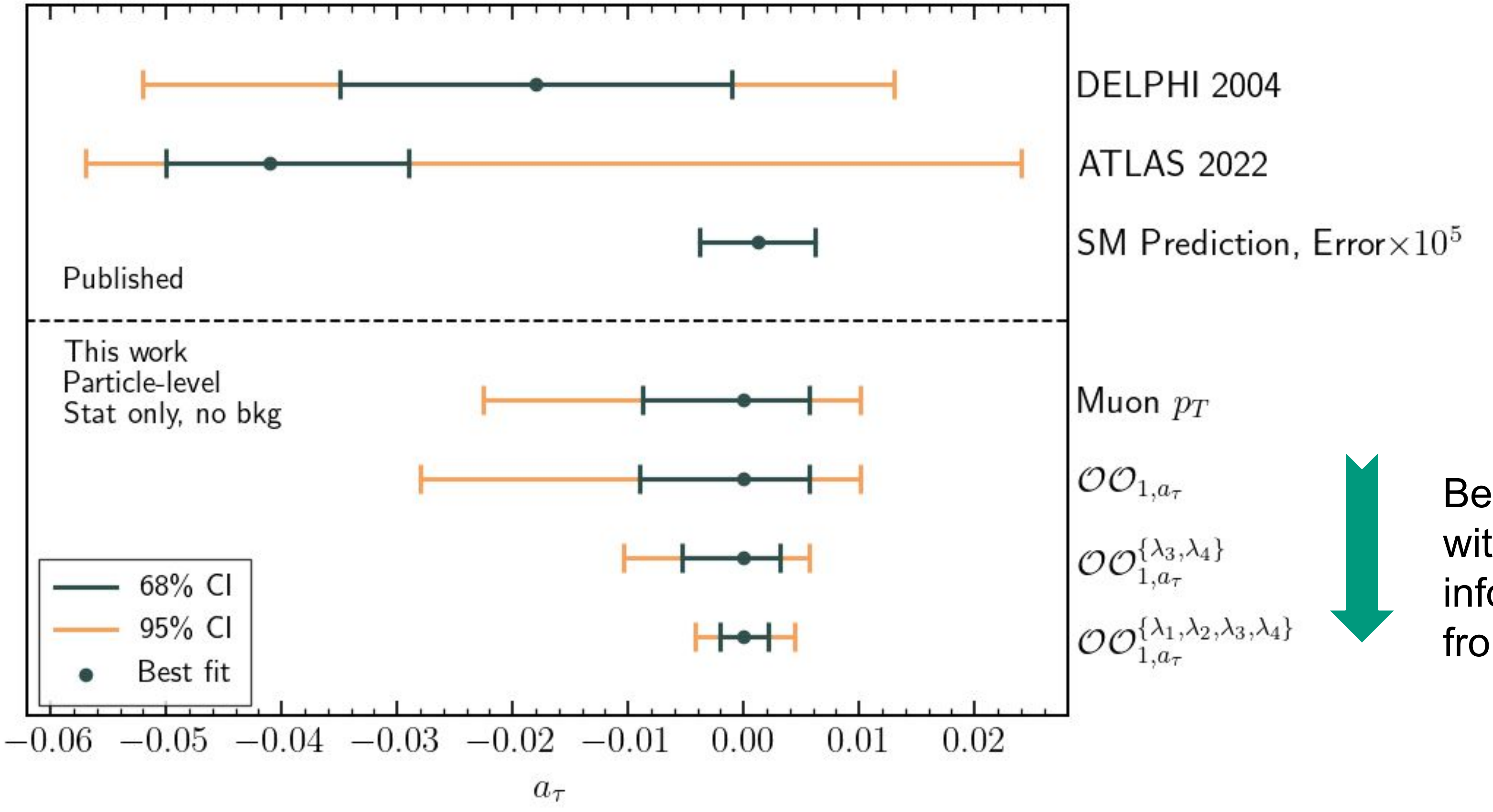


68% CI at threshold of 0.5
95% CI at threshold of 1.96

Analysis is performed *background-free without systematics*, since we are interested in comparing performance of observables

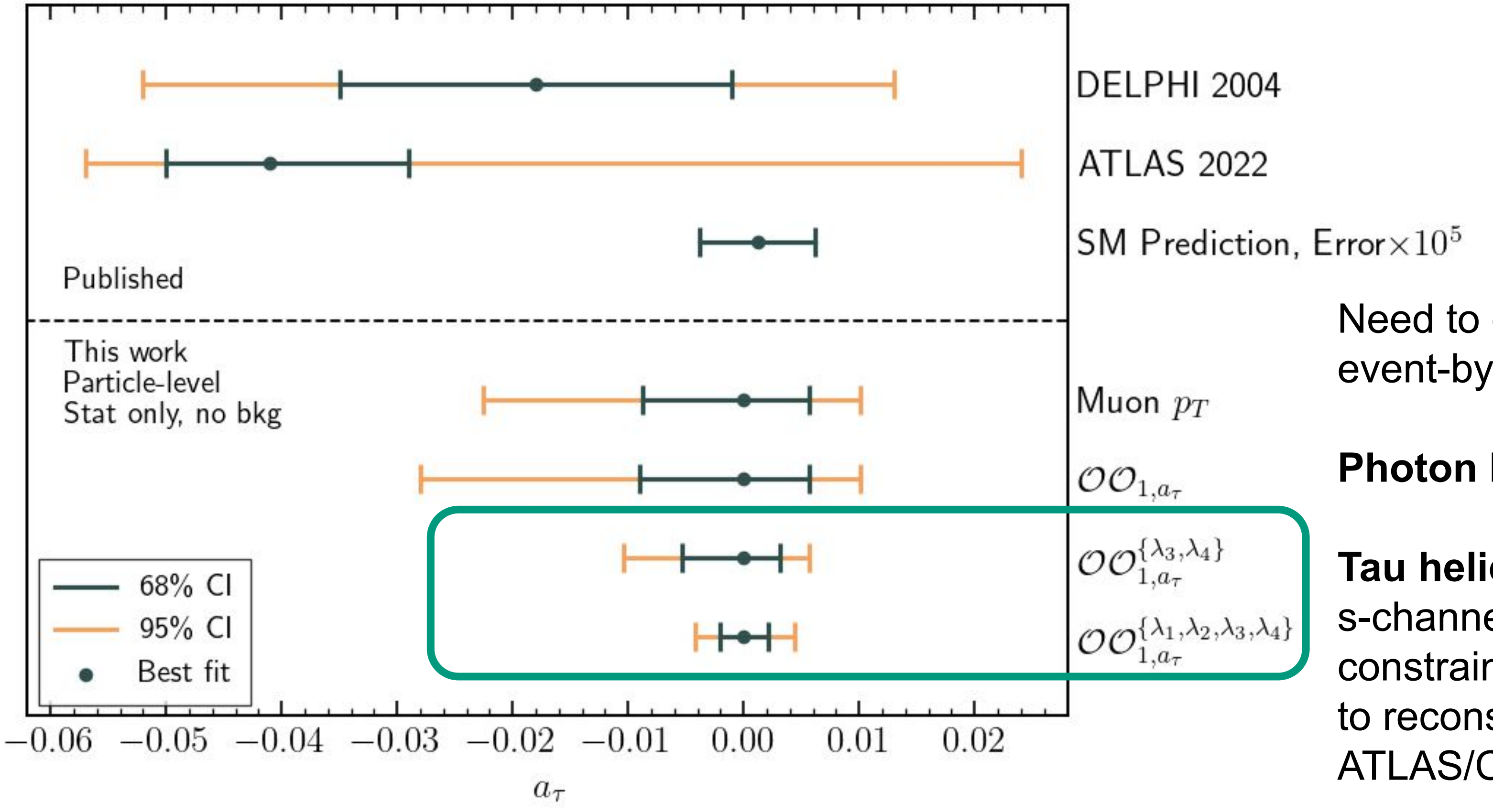
Note: real analyses (ATLAS, CMS) have ~10% backgrounds, and are statistically dominated

Results



Better limits with more information from the event

Results



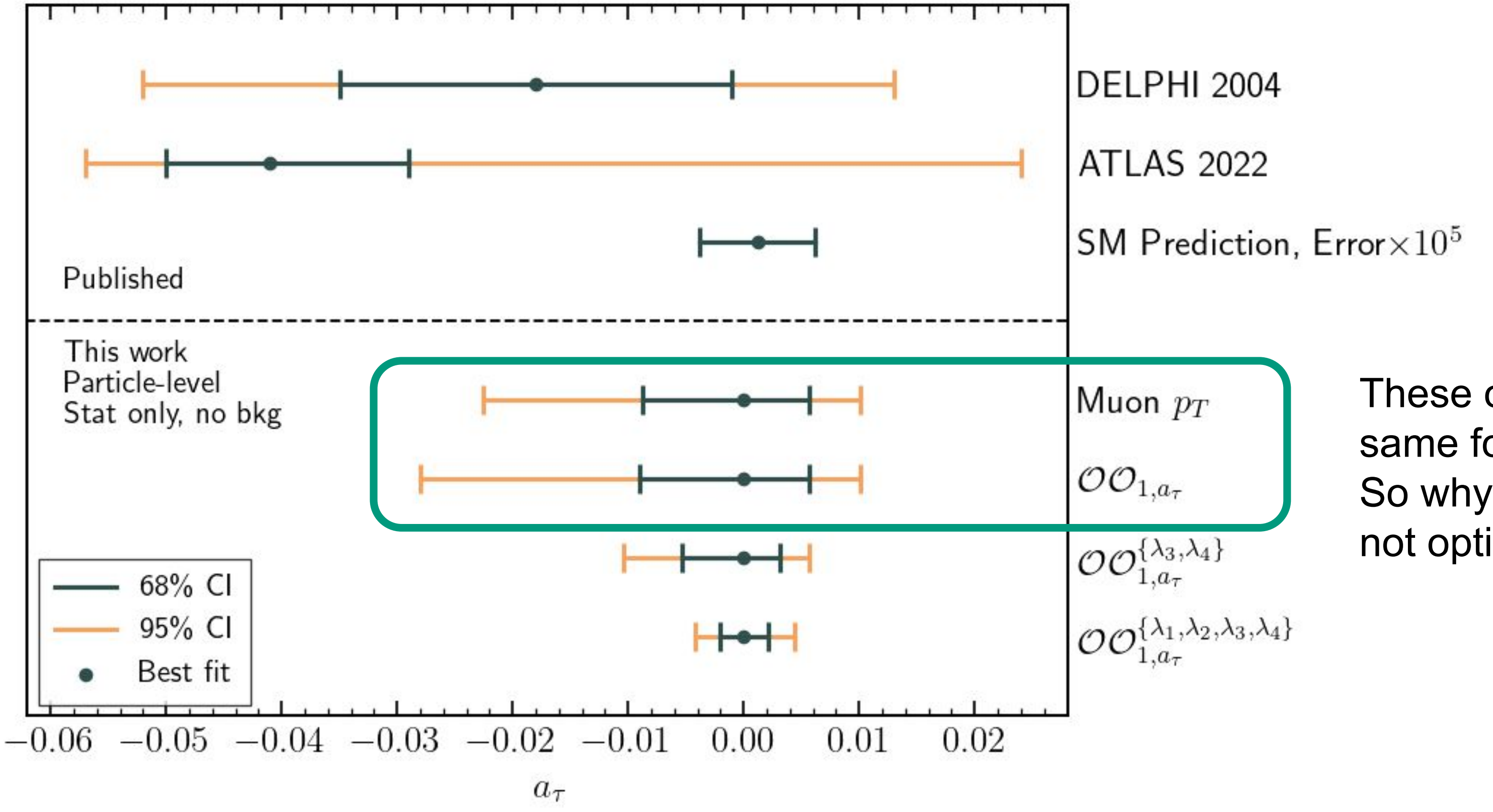
Need to determine particle helicities event-by-event to use these in experiment

Photon helicity → never thought about

Tau helicity → done before at LEP in s-channel e^+e^- process using kinematic constraints, not possible here. Also need to reconstruct π^0, almost hopeless in ATLAS/CMS

Sadly, these observables are not practical

Results



These observables are on the same footing concerning helicity. So why is the optimal observable not optimal?

Phase space and luminosity scan

Going back to the definition of optimal observables…

Optimal observables sit at the minimum variance bound:

$$V_{i,j}^{-1} = \mathrm{Cov}\left[\hat{\alpha}_i, \hat{\alpha}_j\right]^{-1} = \mathcal{L}_{\mathrm{int}} \int d\phi \frac{S_{1,i} S_{1,j}}{S_0}$$

If one has phase space cuts:

➢ Domain of integration over *dφ* becomes smaller
➢ V_{ij} becomes larger

I.e. the measured uncertainty on the parameters of interest becomes larger

"*Optimal-ness*" can be recovered by increasing the integrated luminosity, i.e. recording more data

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Behavior tested in our work by scanning over:

- ➢ Minimum muon p_T in object definition:
  - ○ > 3 GeV, > 4 GeV (nominal) , > 5 GeV
- ➢ Scanning over assumed integrated luminosity
  - ○ 0.5 nb^{-1} to 14.5 nb^{-1}

The figure of merit is the *interval value difference*

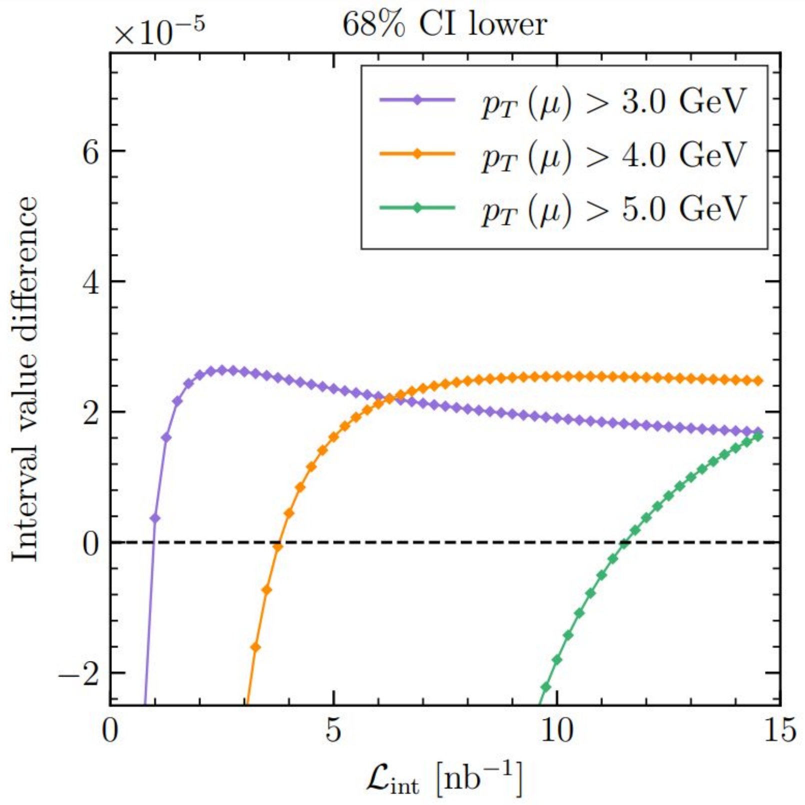
$$\begin{aligned} &68\%\,\mathrm{CI\,upper}\,(\mathrm{muon}\,p_T) - 68\%\,\mathrm{CI\,upper}\,(\mathcal{OO}) \\ &- \{68\%\,\mathrm{CI\,lower}\,(\mathrm{muon}\,p_T) - 68\%\,\mathrm{CI\,lower}\,(\mathcal{OO})\} \end{aligned}$$

Sign for lower limit inverted for interpretation:

A **positive** interval value difference means the **optimal observable is better** than the muon p_T

(same definitions also for the 95% CI)

Phase space and luminosity scan



Optimal observable is better

Muon p_T is better

For fixed integrated luminosity, optimal measurements can be achieved by setting looser phase space cuts

For fixed phase space cuts, optimal measurements can be achieved by recording more data

→ **Optimal observables are not always optimal!**

What about the electric dipole moment?

What about the electric dipole moment?

In the helicity amplitude framework, terms in the matrix element linear in the electric dipole moment are vanishingly small

$$\mathcal{M} = \mathcal{M}_{00} + d_\tau \mathcal{M}_{01} + \ldots$$

Non-zero only for very rare helicity combinations

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Recently, $\gamma\gamma\rightarrow\tau^+\tau^-$ matrix elements elements were calculated with full tau spin vectors [Phys.Rev.D 109 (2024) 1, 013002]

Found that terms linear in electric dipole moment exist only with transverse tau spin correlations

$$|\mathcal{M}|^2_{\mathrm{EDM}} = \frac{e^4}{4D^2}\beta B[(s_1^- s_2^+ - s_2^- s_1^+)(\beta^2\cos(4\theta) + 4\cos(2\theta) + 15\beta^2 - 20) + 2(s_2^- s_3^+ - s_3^- s_2^+)\gamma(\beta^2\cos(2\theta) - 3\beta^2 + 2)\sin(2\theta)].$$

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These terms vanish in the helicity amplitude approach → no optimal observables

The calculations are present as an additional reweighting in TauSpinner to account for spin correlations

But for a phenomenological / experimental realization of optimal observables, we need event-by-event tau spin information!

Current MC generators and experimental techniques don't meet these requirements

Summary / Outlook

Summary / Outlook

- Electromagnetic dipole moments of fundamental particles are powerful (indirect) probes of BSM physics
  - Precision measurements of the **tau lepton** have a lot to say!

- In this work we investigated **optimal observables** for optimal measurements in PbPb UPC @ LHC

- Found that current phase space definitions limit what can be achieved for the tau lepton *g–2* with optimal observables
  - **Looser phase space** required → lower trigger thresholds (challenging), but introduces more background
  - Counter by collecting **more data**

Summary / Outlook

- ➢ Better limits on tau *g–2* if **particle helicities** known, but difficult to determine in data
- ➢ Proper measurements of the EDM need more work
  - Theory: Event generators should write particle spins, not just helicity
  - Experiment: Techniques to reconstruct tau lepton spin
- ➢ Overall, an **exciting time** to think about these measurements!

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A summer sunset in Freiburg