

Isospin-symmetry breaking by kaons in HIC

Francesco Giacosa

Jan Kochanowski U Kielce – Goethe U Frankfurt

In collaboration with

Wojciech Brylinski, Marek Gazdzicki,
Mark Gorenstein, Roman Poberezhnyuk,
Subhasis Samanta, Martin Rohrmoser
+NA61/SHINE

BIAŁASÓWKA - AGH

23/5/2025

Evidence of isospin-symmetry violation in high-energy collisions of atomic nuclei

Received: 6 March 2024

Accepted: 14 February 2025

Published online: 23 March 2025

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The NA61/SHINE Collaboration*, F. Giacosa ^{1,2}, M. Gorenstein ^{3,4},
R. Poberezhniuk ^{3,4,5} & S. Samanta ⁶

Strong interactions preserve an approximate isospin symmetry between up (u) and down (d) quarks, part of the more general flavor symmetry. In the case of K meson production, if this isospin symmetry were exact, it would result in equal numbers of charged (K^+ and K^-) and neutral (K^0 and \bar{K}^0) mesons produced in collisions of isospin-symmetric atomic nuclei. Here, we report results on the relative abundance of charged over neutral K meson production in argon and scandium nuclei collisions at a center-of-mass energy of 11.9 GeV per nucleon pair. We find that the production of K^+ and K^- mesons at mid-rapidity is $(18.4 \pm 6.1)\%$ higher than that of the neutral K mesons. Although with large uncertainties, earlier data on nucleus-nucleus collisions in the collision center-of-mass energy range $2.6 < \sqrt{s_{NN}} < 200$ GeV are consistent with the present result. Using well-established models for hadron production, we demonstrate that known isospin-symmetry breaking effects and the initial nuclei containing more neutrons than protons lead only to a small (few percent) deviation of the charged-to-neutral kaon ratio from unity at high energies. Thus, they cannot explain the measurements. The significance of the flavor-symmetry violation beyond the known effects is 4.7σ when the compilation of world data with uncertainties quoted by the experiments is used. New systematic, high-precision measurements and theoretical efforts are needed to establish the origin of the observed large isospin-symmetry breaking.

Outline

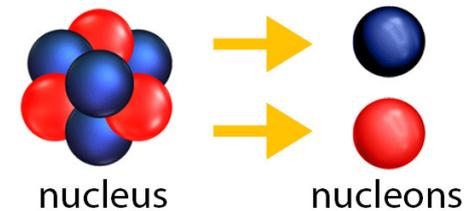


1. Isospin: brief recall
2. Kaon productions in heavy-ion collisions
3. Theory vs experiment (NA61/SHINE + other)
4. Quark coalescence model: post/predictions
5. Conclusions

Heisenberg (1932): the nucleon

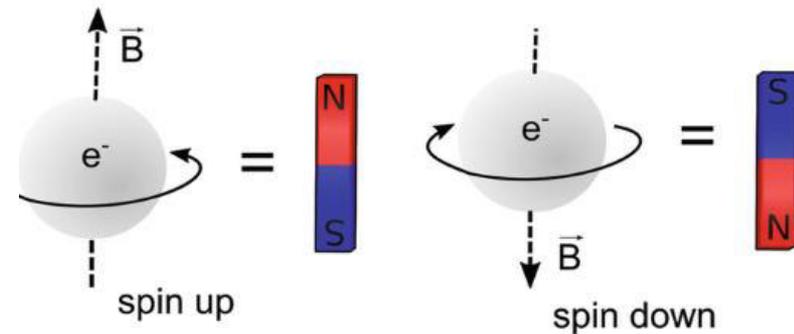
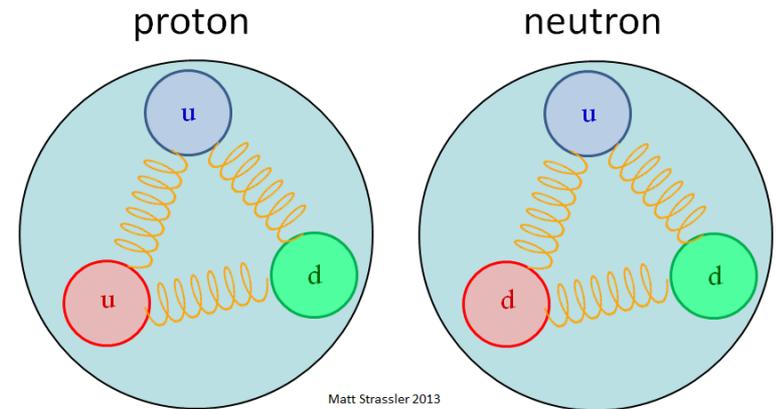
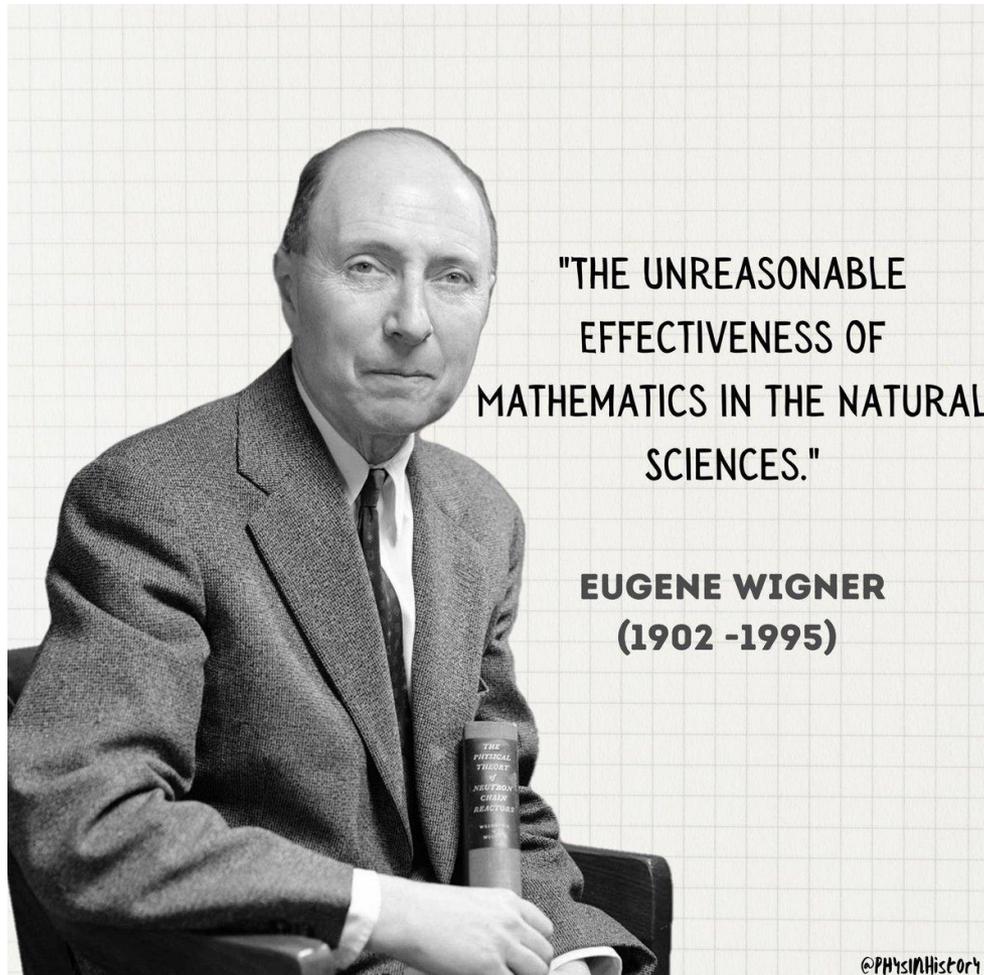


A nucleon is either a proton or a neutron as a component of an atomic nucleus



Proton and neutron merge into the nucleon
Masses very similar.

Wigner (1932): isotopic spin, thus isospin



Nucleon doublet: $I=1/2$

$$\begin{pmatrix} p \\ n \end{pmatrix} \rightarrow \hat{O} \begin{pmatrix} p \\ n \end{pmatrix}$$

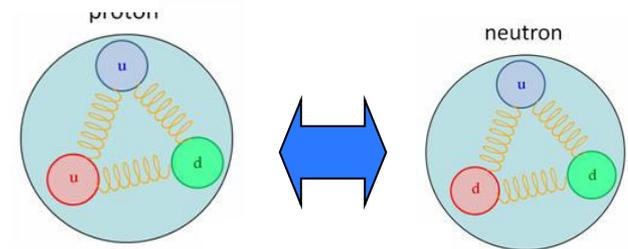
\hat{O} is a 2×2 unitary matrix. $\hat{O} = e^{i\theta_i \sigma_i / 2}$

A specific isospin transformation is the so-called
charge transformation

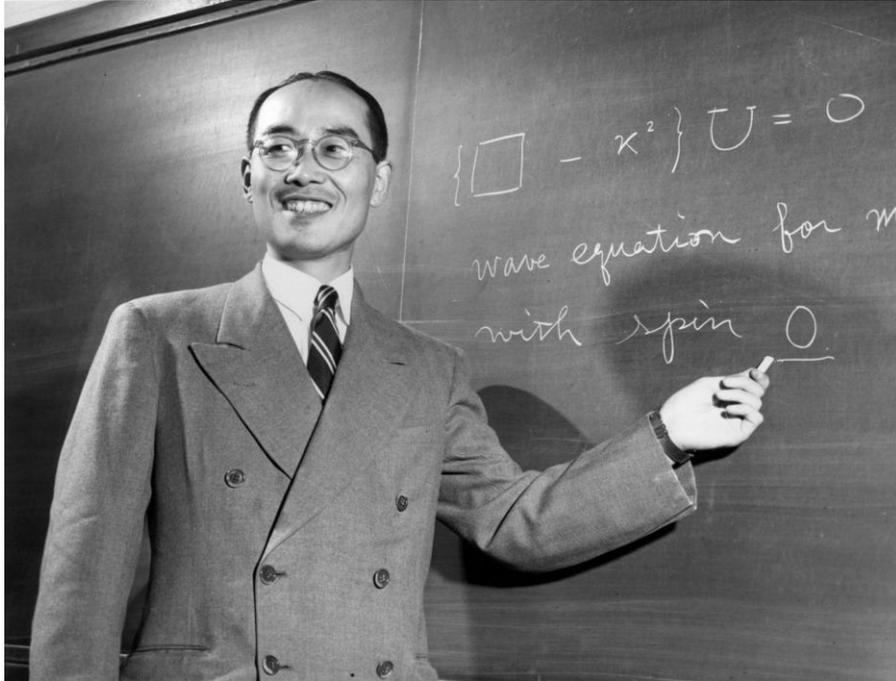
$$\hat{C} = e^{i\pi \sigma_2 / 2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Then under \hat{C} :

$$p \iff n$$



Yukawa (1932) and Kemmer (1939): isospin triplet $I=1$



$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

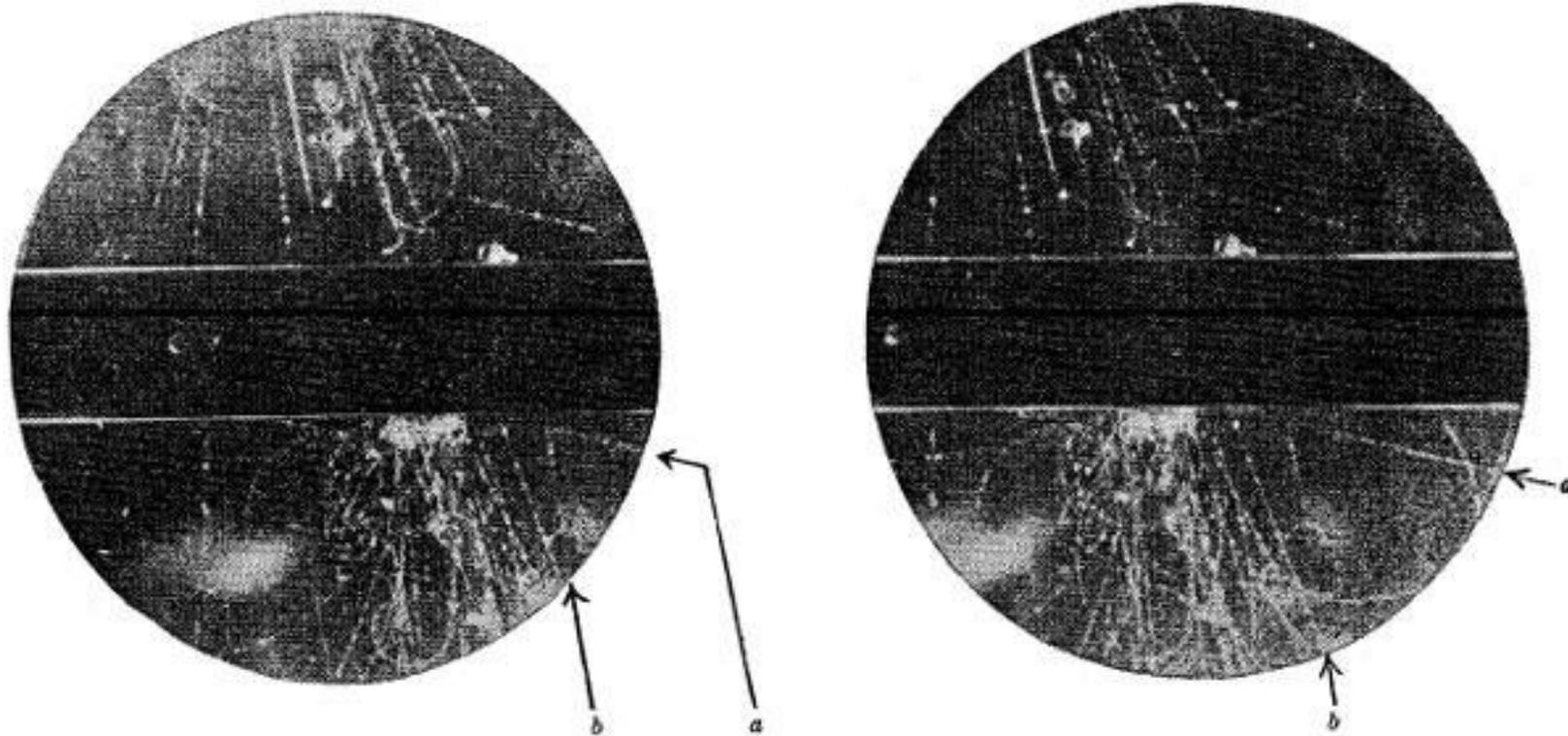
under \hat{C} :

$$\pi^+ \iff \pi^-$$

Kaons

20 DECEMBER 1947

Clifford Butler and George Rochester discover the kaon;
first strange particle



Kaons form isospin doublets, just as the nucleon

$$\begin{pmatrix} p \\ n \end{pmatrix} \quad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix} \quad \dots$$

under \hat{C} :

$$\begin{array}{ccc} p & \iff & n \\ K^+ & \iff & K^0 \\ \bar{K}^0 & \iff & K^- \end{array}$$

Quarks and QCD



up



charm



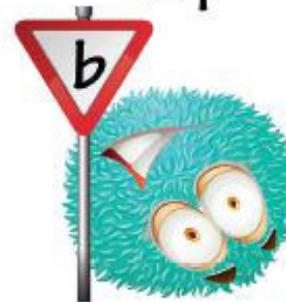
top



down

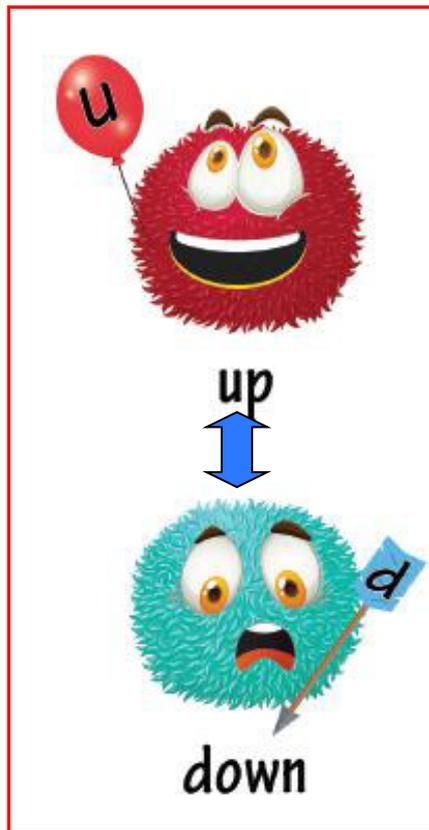


strange



bottom

Quarks and QCD, isospin:



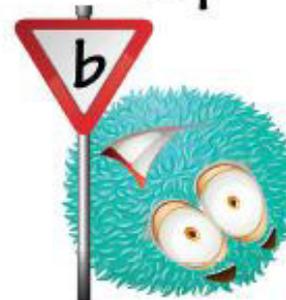
charm



strange



top



bottom

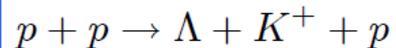
In terms of quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \hat{O} \begin{pmatrix} u \\ d \end{pmatrix}$

Then under \hat{C} : $u \longleftrightarrow d$

Isospin is an approximate symmetry of QCD

- Mesonic multiplets (nucleon doublet, pion triplet, kaon doublets).
- Reactions: if an initial state has a certain (I, I_z) , then the final state is also such. Indeed, pion-pion, pion-nucleon and nucleon-nucleon scattering conserve isospin (to a good level of accuracy).

Example: $(I=I_z=1)$



- Isospin symmetry is good, but not exact. Masses of u and d not equal (explicit symmetry breaking).
- Isospin transformations are a subset of flavor transformations.

Example of isospin breaking/1



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/84-27

March 8th, 1984

THE ISOSPIN-VIOLATING DECAY $\eta' \rightarrow 3\pi^0$

IHEP¹-IISN²-LAPP³ Collaboration

$$\text{BR}(\eta' \rightarrow 3\pi^0) = 5.2 \left(1 - \frac{m_u}{m_d} \right)^2 10^{-3}$$

Example of isospin breaking/2

$\phi(1020)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

$\phi(1020)$ MASS

| <u>VALUE (MeV)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|-------------------------|--------------------|--------------------|-------------|----------------|
| 1019.461 ± 0.016 | OUR AVERAGE | | | |

$\phi(1020)$ DECAY MODES

| Mode | Fraction (Γ_i/Γ) | Scale factor/ Confidence level |
|------------------------------|--------------------------------|-----------------------------------|
| $\Gamma_1 \quad K^+ K^-$ | (49.1 ± 0.5) % | S=1.3 |
| $\Gamma_2 \quad K_L^0 K_S^0$ | (33.9 ± 0.4) % | S=1.2 |

More on the resonance $\phi(1020)$

$\phi(1020)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

$\phi(1020)$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | COMMENT |
|-------------------------|------|-------------|------|--------------------|
| 1019.461 ± 0.016 | | | | OUR AVERAGE |

$\phi(1020)$ WIDTH

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | COMMENT |
|----------------------|------|-------------|------|--------------------|
| 4.249 ± 0.013 | | | | OUR AVERAGE |

Error includes scale factor of 1.1.

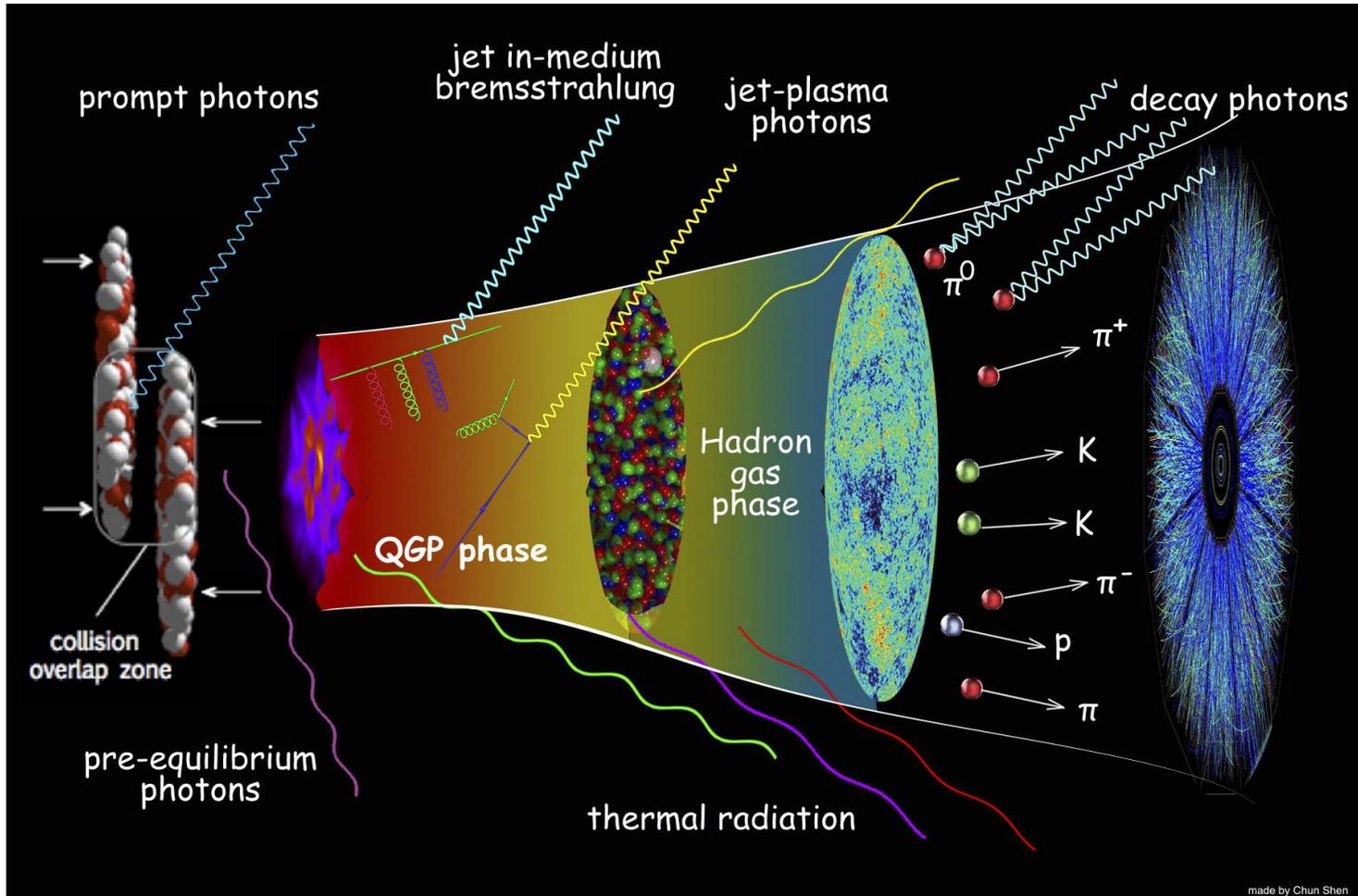
$\phi(1020)$ DECAY MODES

| Mode | Fraction (Γ_i/Γ) | Scale factor/ Confidence level |
|--|--------------------------------|-----------------------------------|
| Γ_1 K^+K^- | (49.1 ± 0.5)% | S=1.3 |
| Γ_2 $K_L^0 K_S^0$ | (33.9 ± 0.4)% | S=1.2 |
| Γ_3 $\rho\pi + \pi^+\pi^-\pi^0$ | (15.4 ± 0.4)% | S=1.2 |

$$\frac{\Gamma_{K^+K^-}}{\Gamma_{K^0\bar{K}^0}} = \frac{g_{K^+K^-}^2}{g_{K^0\bar{K}^0}^2} \frac{\left(\frac{m_\phi^2}{4} - m_{K^+}^2\right)^{3/2}}{\left(\frac{m_\phi^2}{4} - m_{K^0}^2\right)^{3/2}} = \frac{g_{K^+K^-}^2}{g_{K^0\bar{K}^0}^2} 1.52 \stackrel{\text{PDG}}{=} 1.45 \pm 0.03$$

$$\frac{g_{K^+K^-}}{g_{K^0\bar{K}^0}} = 0.98 \pm 0.01$$

Heavy-ion collisions



C. Shen, U. Heinz,
Nucl. Phys. News 25
(2015) 2, 6-11

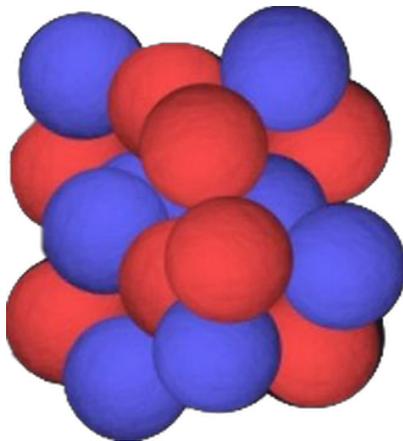
At the freeze-out, the emission of hadrons is well described by e.g. thermal models.

- Kaon production: unexpected large violation of isospin in charged to neutral kaon ratio
- Adhikary et al. [NA61/SHINE], Excess of Charged Over Neutral K Meson Production in High-Energy Collisions of Atomic Nuclei, [arXiv:2312.06572 [nucl-ex]]
(Nat. Comm.)
- ...as well as to a compilation of other experiments
- Previous theoretical considerations:
Brylinski et al., Large isospin symmetry breaking in kaon production at high energies, [arXiv:2312.07176 [nucl-th]].

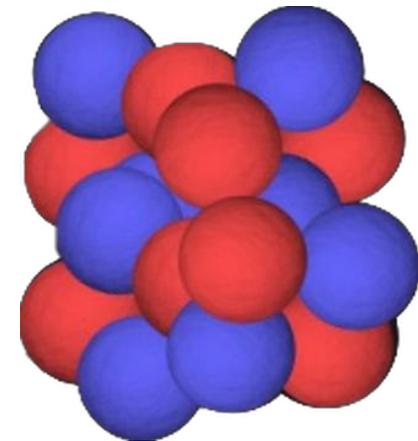
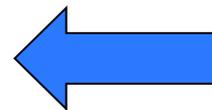
Nucleus-nucleus collision with equal numbers of protons and neutrons

$$Z = N = A/2, \quad Q/B = 1/2$$

$$|A + A\rangle$$



Oxygen-16



Oxygen-16

$I_z = 0$ (typically also $I = 0$ for each nucleus, thus total isospin also vanishing)

Expected kaon multiplicities

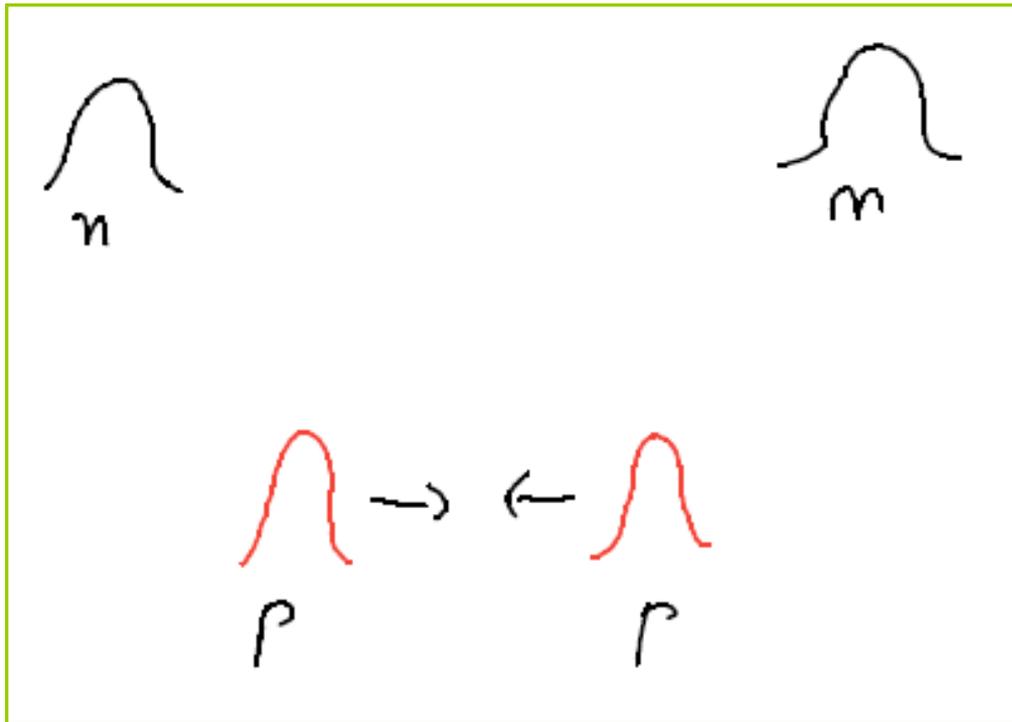
Charge symmetry means that strong interactions are invariant under the inversion of the third component of the isospin of hadron of the initial and final states.

Then:

$$\langle K^+ \rangle = \langle K^0 \rangle$$

$$\langle K^- \rangle = \langle \bar{K}^0 \rangle$$

ppmm \mapsto ?



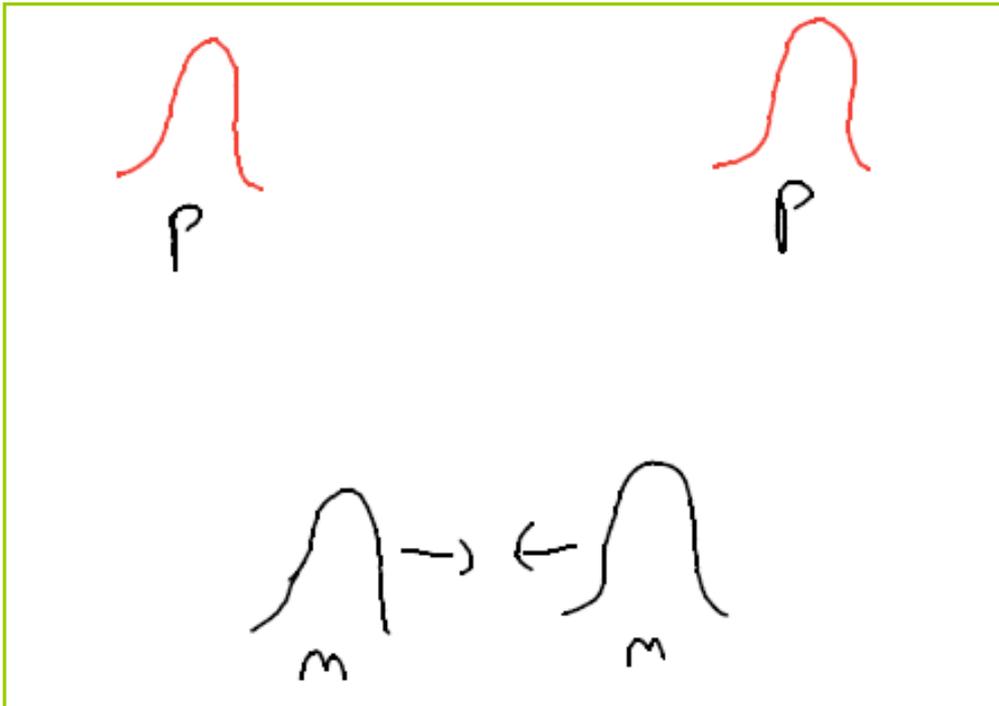
Just as pp!

More K^+ than K^0

Is then the previous argumentation wrong?

No.
One needs to average.

But ... \hat{C} transform



This is the C-transformed version for the previous reaction.

Here, the protons are spectators and the neutrons interact.

Just as mm scattering!

More K^0 than K^+

Averaging leads to...

If both initial states
are equally probable



$$\langle K^+ \rangle = \langle K^0 \rangle$$

holds!

This is a general result!

Formally:

$$\hat{\rho} = \sum_n p_n |\Psi_n\rangle \langle \Psi_n|$$

$$\hat{C} \hat{\rho} \hat{C}^\dagger = \hat{\rho}$$

Neutral kaons and the ratio R_K

$$\begin{pmatrix} |K_S^0\rangle \\ |K_L^0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$\langle K_S^0 | = \frac{1}{2} \langle K^0 | + \frac{1}{2} \langle \bar{K}^0 | = \langle K_L^0 | \qquad \langle K^+ | + \langle K^- | = 2 \langle K_S^0 |$$

$$Q/B = 1/2$$

$$R_K \equiv \frac{\langle K^+ | + \langle K^- |}{\langle K^0 | + \langle \bar{K}^0 |} = \frac{\langle K^+ | + \langle K^- |}{2 \langle K_S^0 |} = 1$$

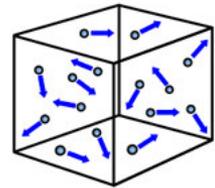
+ isospin exact...

Theoretical approaches

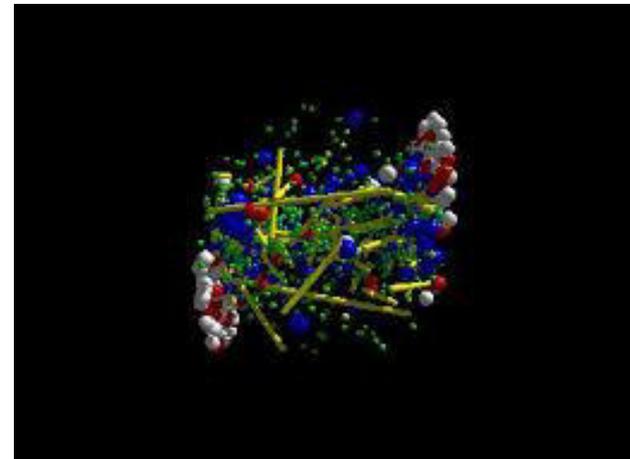
- HRG (hadron resonance gas approach)

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}$$

$$\ln Z_k^{\text{stable},} = f_k V \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T} \right]^{\pm 1}$$

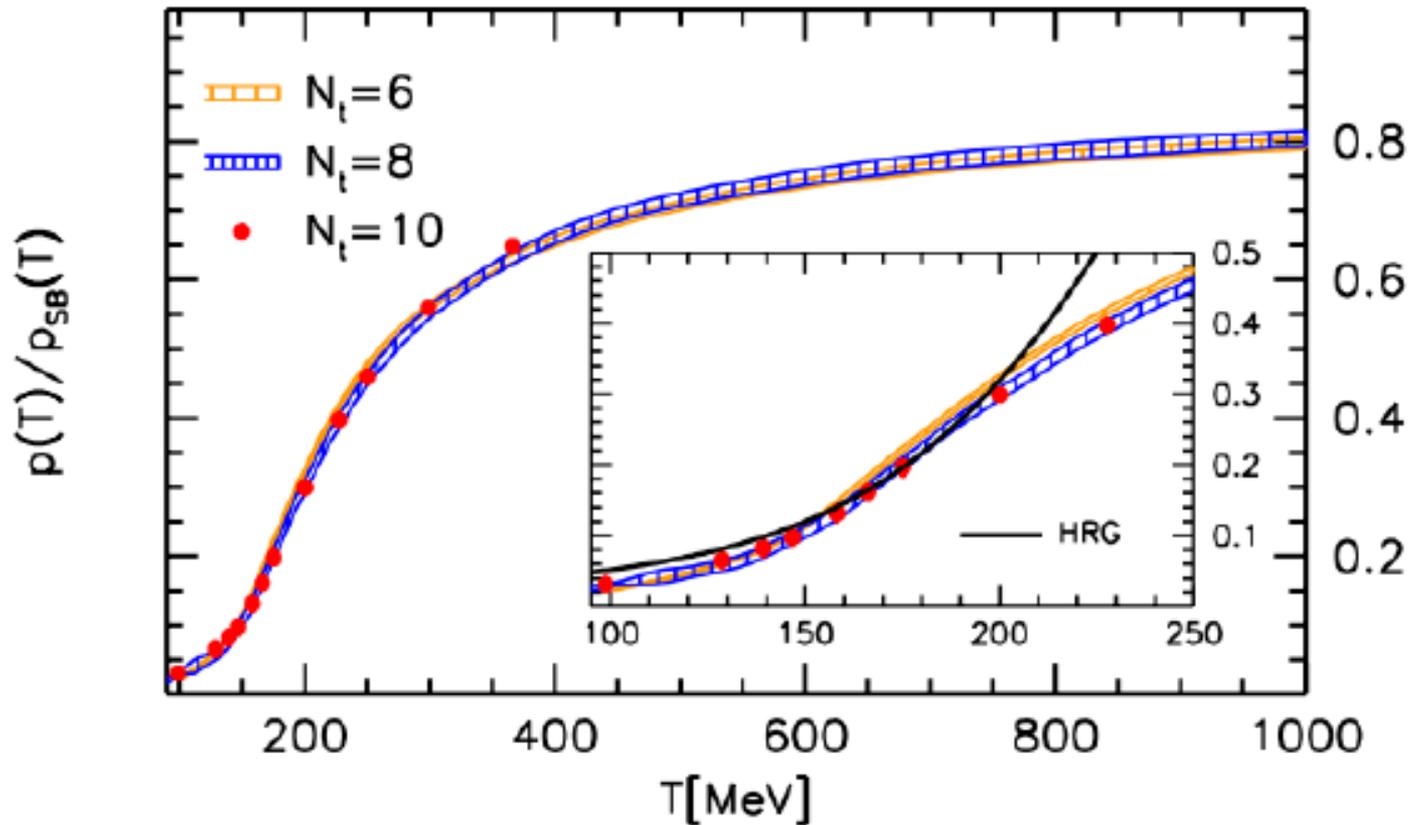


- UrQMD (Hadron-String transport model, fully integrated Monte Carlo simulation of nucleus-nucleus simulations)

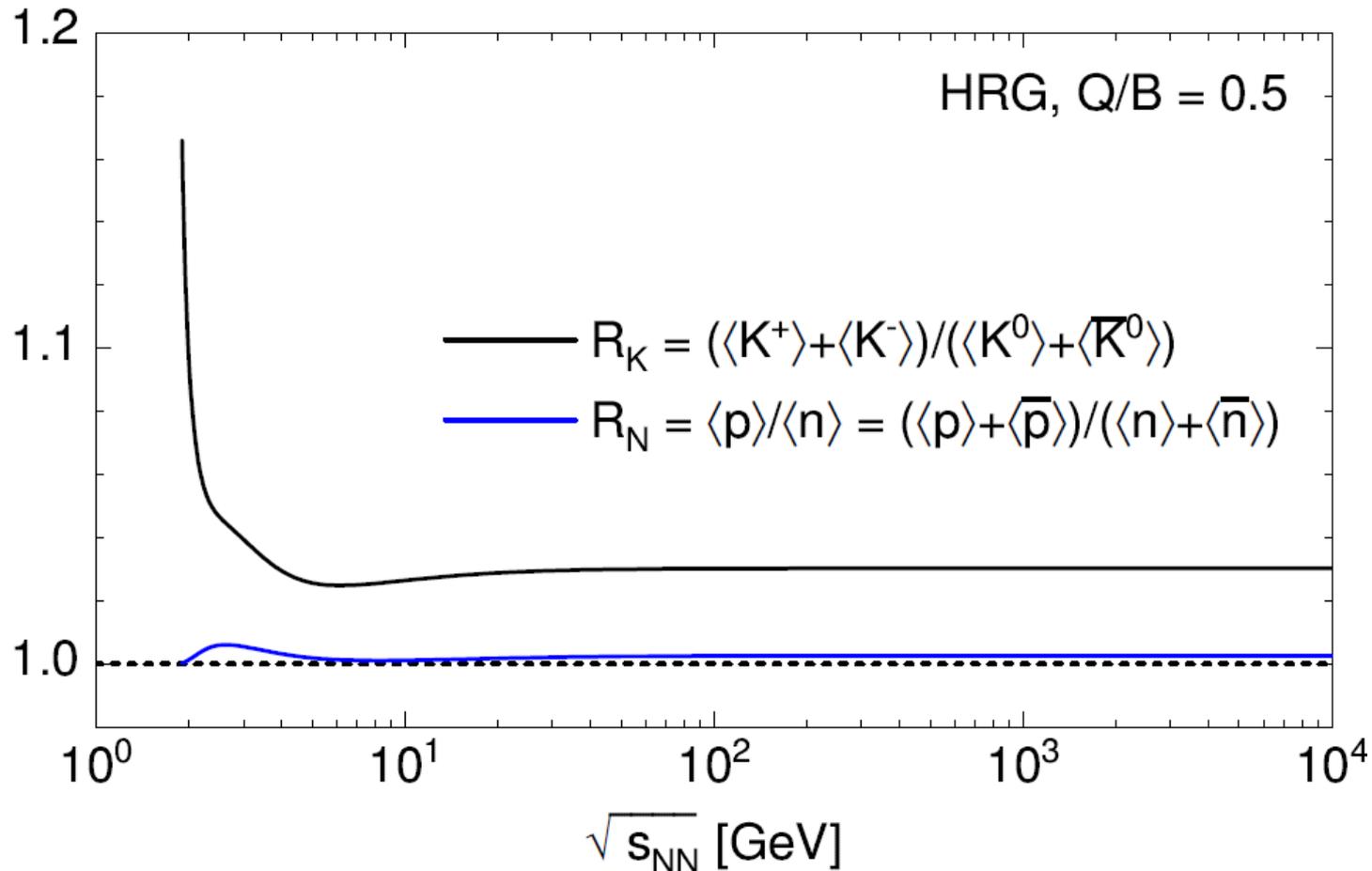


Hadron resonance gas vs lattice results

- All baryons and mesons ($m < 2.5$ GeV) from PDG [Borsnayi et al. JHEP11(2010)077]



HRG for $Q/B=1/2$



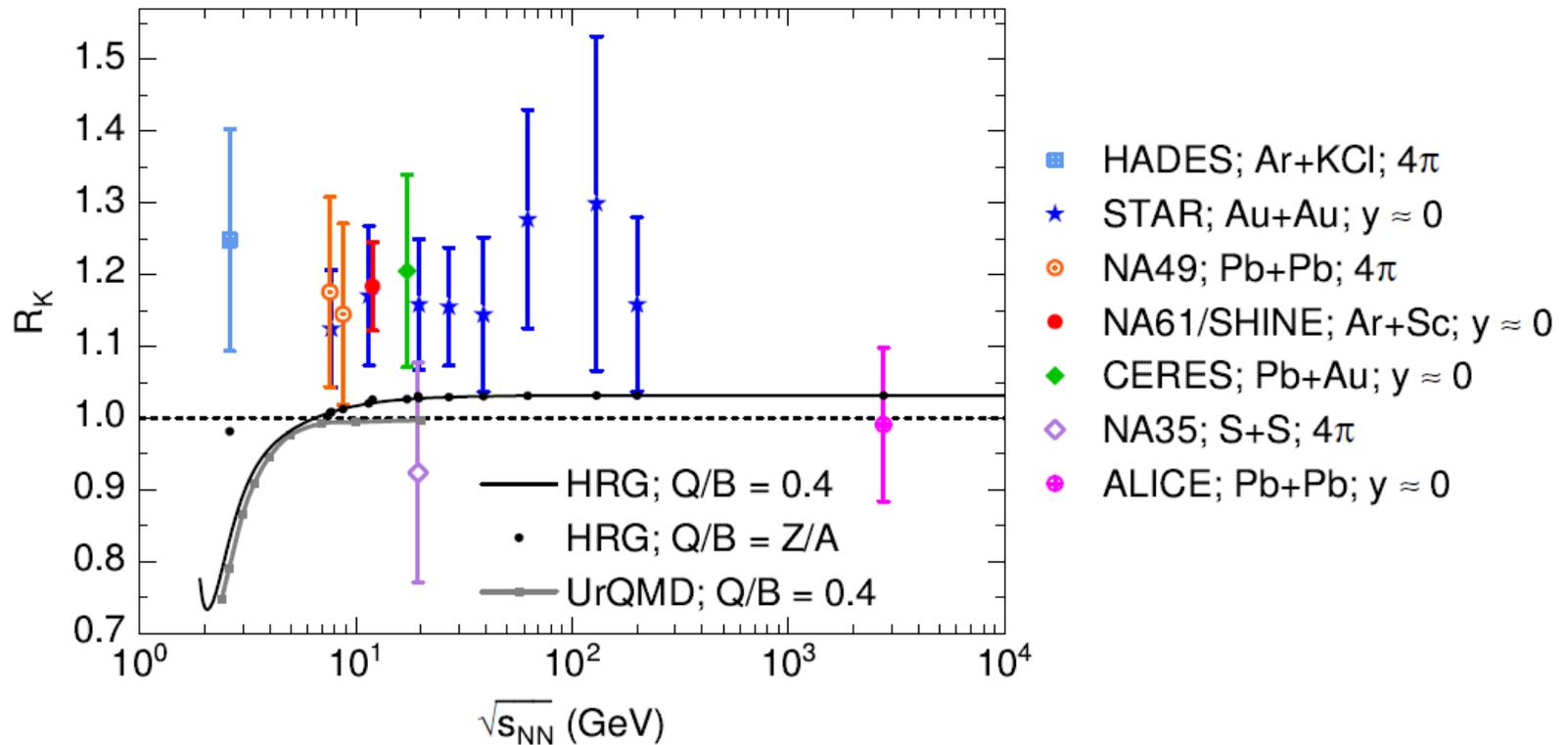
If we enforce isospin symmetry to be exact, $R_K = 1$ for any energy. 26

Experimental data: NA61/SHINE and previous exp

| Experiment | Collision system | $\sqrt{s_{NN}}$ (GeV) | R_K | σ_{stat} | σ_{total} |
|--------------|------------------|-----------------------|--------|-----------------|------------------|
| NA61/SHINE | Ar+Sc | 11.9 | 1.1839 | 0.0138 | 0.0615 |
| HADES | Ar+KCl | 2.6 | 1.2483 | 0.1027 | 0.1545 |
| STAR (BES I) | Au+Au | 7.7 | 1.1247 | - | 0.0819 |
| STAR (BES I) | Au+Au | 11.5 | 1.1707 | - | 0.0973 |
| STAR (BES I) | Au+Au | 19.6 | 1.1584 | - | 0.0910 |
| STAR (BES I) | Au+Au | 27 | 1.1553 | - | 0.0819 |
| STAR (BES I) | Au+Au | 39 | 1.1446 | - | 0.1079 |

| | | | | | |
|-------|-------|------|--------|--------|--------|
| NA49 | Pb+Pb | 7.6 | 1.1758 | 0.0198 | 0.1325 |
| NA49 | Pb+Pb | 8.7 | 1.1447 | 0.0295 | 0.1263 |
| CERES | Pb+Au | 17.3 | 1.2052 | 0.0539 | 0.1340 |
| NA35 | S+S | 19.4 | 0.9238 | - | 0.1533 |
| STAR | Au+Au | 62.4 | 1.2774 | - | 0.1525 |
| STAR | Au+Au | 130 | 1.2994 | - | 0.2331 |
| STAR | Au+Au | 200 | 1.1586 | - | 0.1214 |
| ALICE | Pb+Pb | 2760 | 0.9909 | - | 0.1071 |

Experimental results (NA61/SHINE plus others)



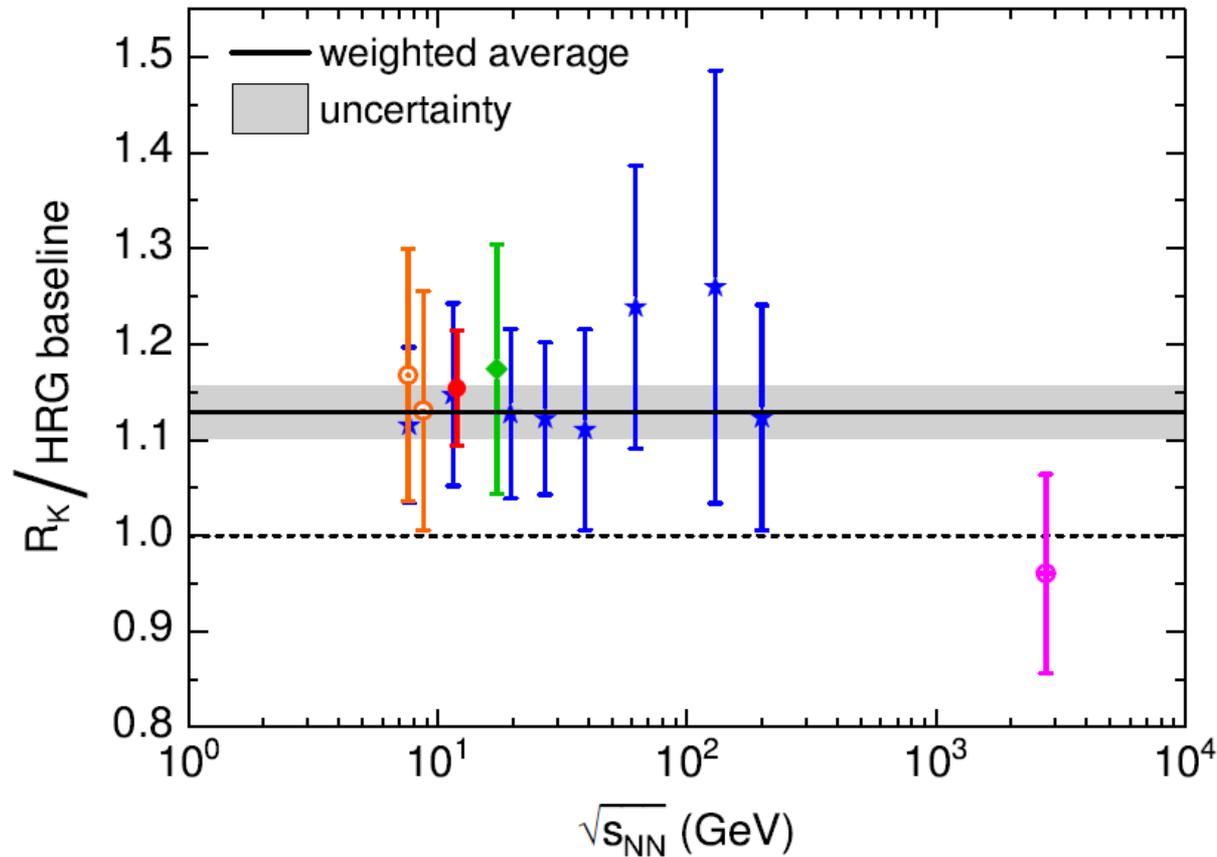
Latest NA61/SHINE result: $R_K = 1.184 \pm 0.061$

Note, however, most experiments have $Q/B < 0.5$

Experiment vs theory (HRG): ratio

$1.129 \pm 0.027.$

$\chi_{min}^2/\text{dof} \approx 0.3$



The exp/th mismatch is 4.7σ .

Considerations

- HRG and UrQMD agree with each other
- $Q/B < 1/2$ favors neutral kaons
- charged kaons are lighter than neutral ones:
this favors charged kaons

- Non-QCD effects: weak processes are negligible
- Non-QCD effects: electromagnetic processes are small, of the order of α^2 . However, nonperturbative effects possible for soft charged kaons?
- Decays of $\phi(1020)$ meson generates quite small effects.
- Role of $a_0(980)$ and $f_0(980)$ is also small.

Toward a simple 'quark counting' model

- Provided the large isospin-symmetry breaking is true, two questions can be asked: why and which are its consequences.
- 'Why' is, as usual, a difficult question. Can electromagnetic interaction enhance $K+K^-$? We argued that this is not the case. But...
- What about a sum over many small effects? All ϕ - f_0 - a_0 etc effects would lead to the measured results.
- Eventually a combination of both QED and many small contributions...

Quark recombination model: references

Joanna Stepaniak and Damian Pszczel. On the relation between K_s^0 and charged kaon yields in proton–proton collisions. *Eur. Phys. J. C*, 83(10):928, 2023.

M. Bonesini, A. Marchionni, F. Pietropaolo, and T. Tabarelli de Fatis. On Particle production for high-energy neutrino beams. *Eur. Phys. J. C*, 20:13–27, 2001. As reported in Ref. [25] the model was developed by N. Doble, L. Gatignon, P. Grafstrom, NA31 Internal note 83 (1990). According to the authors, the formula and its derivation are due to Horst Wachsmuth.

Valence and sea quarks

$$n_u = n_u^{val}$$

$$n_d = n_d^{val}$$

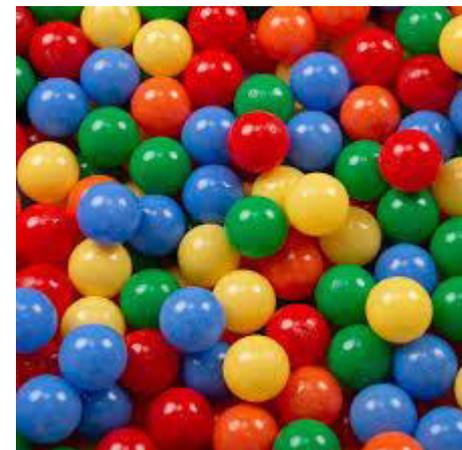
$$\alpha = n_u^{sea} = n_{\bar{u}}^{sea}$$

$$\beta = n_d^{sea} = n_{\bar{d}}^{sea}$$

$$\gamma = n_s^{sea} = n_{\bar{s}}^{sea}$$

$$n_{tot} = n_u + n_d + 2\alpha + 2\beta + 2\gamma$$

$$p(u) = \frac{n_u + \alpha}{n_{tot}}$$



Kaon probabilities

$$p(K^+) \propto n_u \gamma + \alpha \gamma$$

$$p(K^0) \propto n_d \gamma + \beta \gamma$$

$$p(K^-) \propto \alpha \gamma$$

$$p(\bar{K}^0) \propto \beta \gamma$$

'Grundschulmathematik' leads to:

$$R_K = \frac{\langle K^+ \rangle + \langle K^- \rangle}{\langle 2K_S^0 \rangle} = \frac{n_u + 2\alpha}{n_d + 2\beta}$$

isospin-symmetric limit ($\alpha = \beta$)

$$R_K = 1 \quad \text{if } n_u = n_d$$

$$Q/A = 1/2$$

From R_K to \tilde{R}_K

$$\tilde{R}_K = R_K + \left(\frac{1 - 2\frac{Q}{A}}{1 + \frac{Q}{A}} \right) \frac{\langle K^+ \rangle - \langle K^- \rangle}{2 \langle K_S^0 \rangle} = \frac{n_d + 2\alpha}{n_d + 2\beta}$$

2504.02113

isospin-conserved

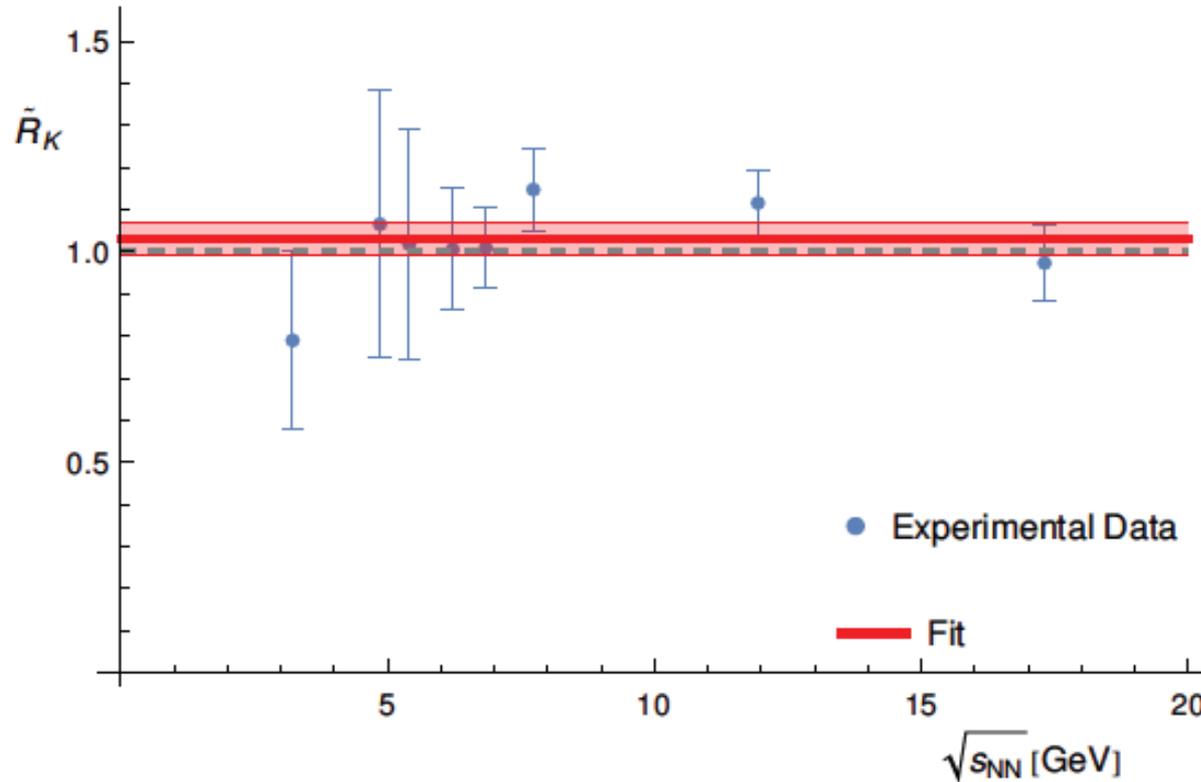
$$\alpha = \beta \quad \rightarrow \quad \tilde{R}_K = 1$$

For pp collisions $Q/A = 1$

$$\langle K^+ \rangle + 3 \langle K^- \rangle = 4 \langle K_S^0 \rangle$$

See J. Stepaniak and D. Pszczel, EPJC 83 2023

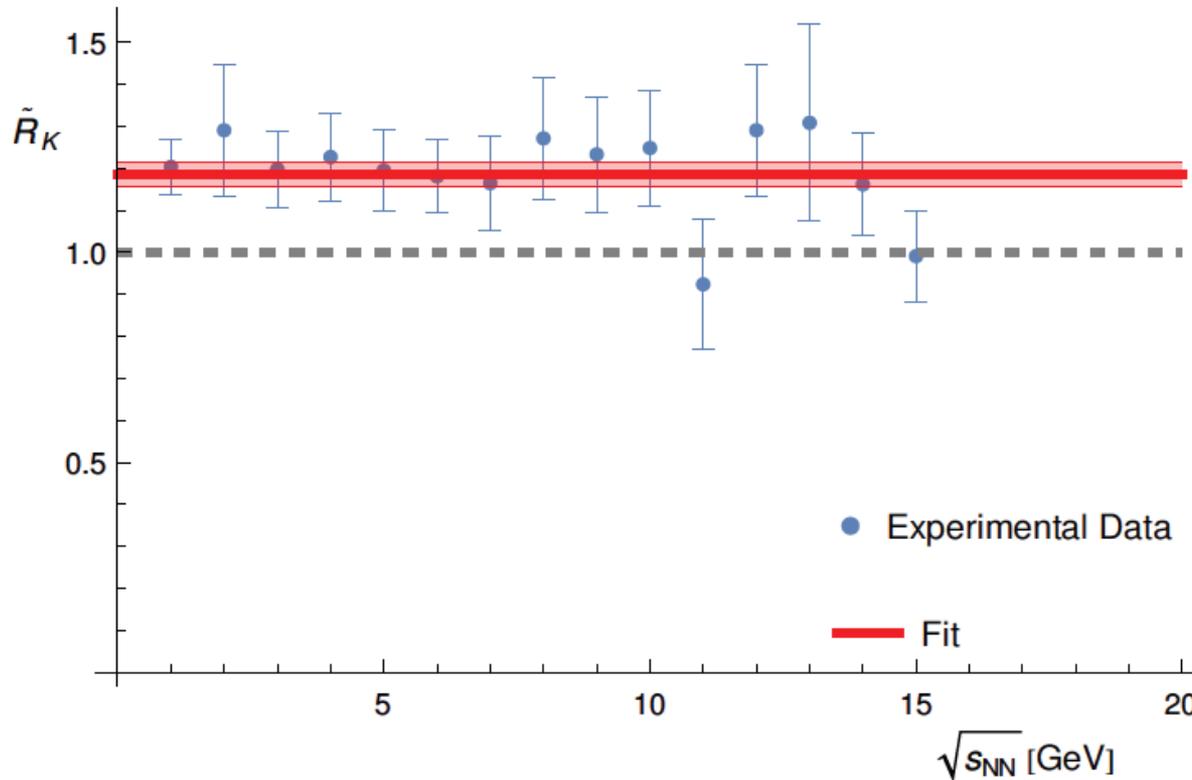
Proton-proton results: isospin ok



2504.02113

$$\tilde{R}_K = 1.030 \pm 0.038.$$

Nucleus-nucleus results for \tilde{R}_K : constant but not 1

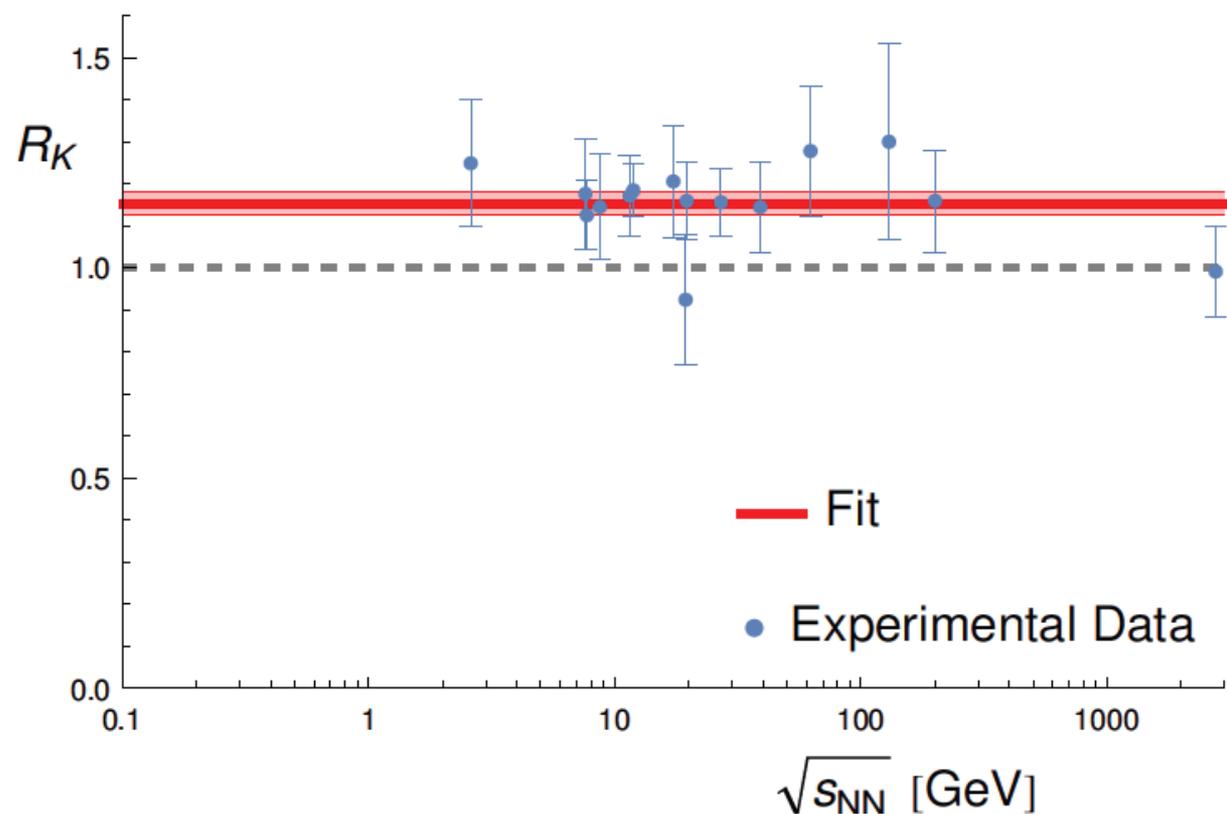


2504.02113

$$\tilde{R}_K = 1.185 \pm 0.029$$

This is 6.4σ away from 1.

Nucleus-nucleus results for R_K : constant, not 1, and compatible with $R_{\text{tilde}K}$



2504.02113

$$R_K = 1.152 \pm 0.027$$

Predictions

| Ratio | Estimated value |
|---|--------------------------------------|
| $R_K = \frac{K^+ + K^-}{K^0 + \bar{K}^0}$ | $r = 1.185 \pm 0.029$ |
| p/n | $r = 1.185 \pm 0.029$ |
| π^+ / π^0 | $\frac{2r}{1+r^2} = 0.986 \pm 0.004$ |
| Σ^+ / Σ^0 | $r = 1.185 \pm 0.029$ |
| Σ^+ / Σ^- | $r^2 = 1.404 \pm 0.068$ |

Predictions

| Ratio | Estimated value |
|----------------------------|-------------------------|
| Δ^{++} / Δ^{+} | $r = 1.185 \pm 0.029$ |
| Δ^{++} / Δ^{0} | $r^2 = 1.404 \pm 0.069$ |
| Δ^{++} / Δ^{-} | $r^3 = 1.67 \pm 0.12$ |

Pion-nucleus scattering antiquarks in the initial state

$$R_K = \frac{\langle K^+ \rangle + \langle K^- \rangle}{\langle 2K_S^0 \rangle} = \frac{n_u + n_{\bar{u}} + 2\alpha}{n_d + n_{\bar{d}} + 2\beta}$$

$R_K = 1$ in the isospin limit ($\alpha = \beta$)
for $n_u + n_{\bar{u}} = n_d + n_{\bar{d}}$.

This is the case for pion-carbon.

(In fact for π^+C : $n_u = 18+1$, $n_{\bar{u}} = 0$, $n_d = 18$, $n_{\bar{d}} = 1$)

But isospin-symmetry is broken.

Hence our prediction for pion-carbon:

$$R_K^{\pi^+C} = R_K^{\pi^-C} \simeq 1.185 \pm 0.029$$

See NA61/SHINE
PRD 107 (2023) 062004
Where R_K is about 1.2

\tilde{R}_K for (anti)quarks u and d

$$\begin{aligned}\tilde{R}_K &= R_K + \frac{n_d + n_{\bar{d}} - n_u - n_{\bar{u}}}{n_u - n_{\bar{u}}} \frac{\langle K^+ \rangle - \langle K^- \rangle}{\langle 2K_S^0 \rangle} \\ &= \frac{n_d + n_{\bar{d}} + 2\alpha}{n_d + n_{\bar{d}} + 2\beta}\end{aligned}$$

$\tilde{R}_K = 1$ in the isospin-symmetric limit

valid also for initial states with $n_s = n_{\bar{s}}$

$\eta, \eta',$ and $\phi, \quad K^+ \Lambda$

Most general case

In the most general case with arbitrary $n_{u,d,s}$ and $n_{\bar{u},\bar{d},\bar{s}}$ the quantity \tilde{R}_K reads

$$\tilde{R}_K = \frac{(n_d + \alpha)(n_{\bar{s}} + \gamma) + (n_{\bar{d}} + \alpha)(n_s + \gamma)}{(n_d + \beta)(n_{\bar{s}} + \gamma) + (n_{\bar{d}} + \beta)(n_s + \gamma)}.$$

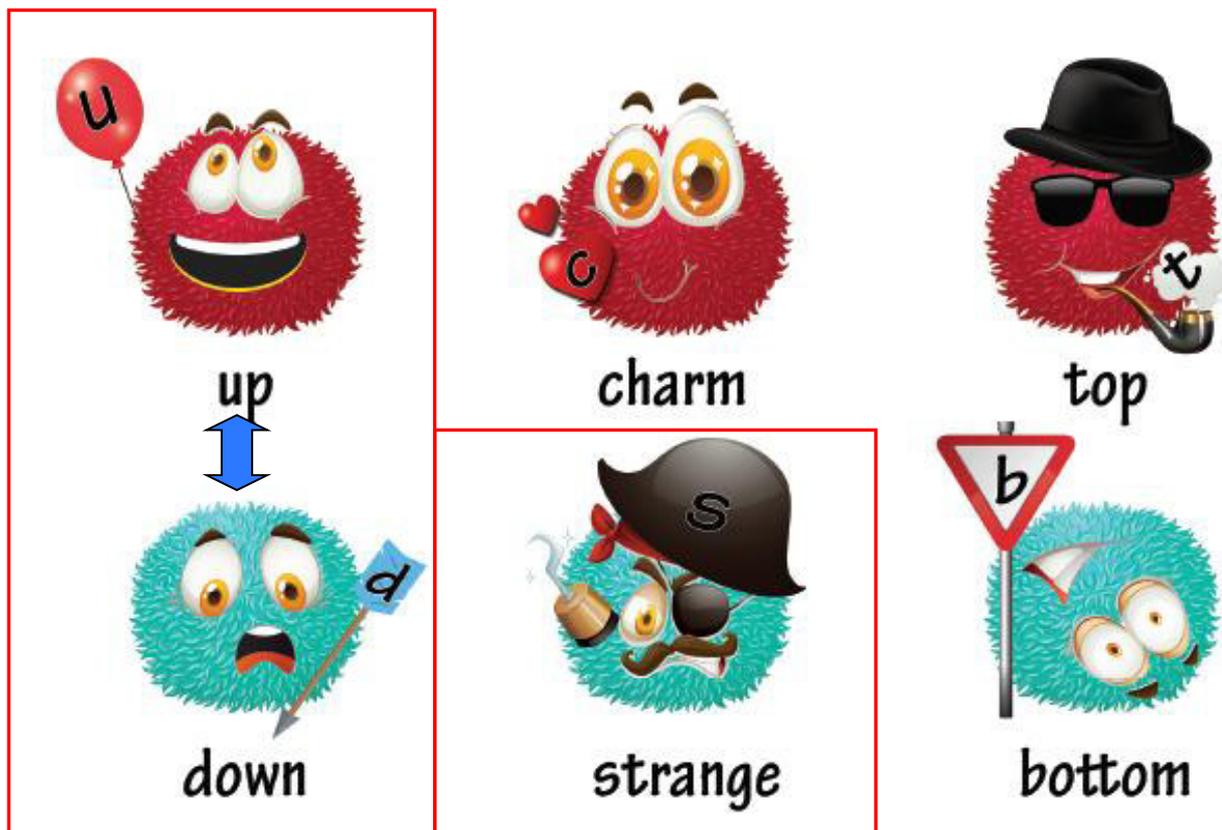
However, it cannot be expressed as a function of the three multiplicities $\langle K^+ \rangle$, $\langle K^- \rangle$, and $\langle K_S^0 \rangle$, but it involves separately $\langle K_0 \rangle$ and $\langle \bar{K}_0 \rangle$ [38]. This fact is not convenient because only K_S^0 is usually detected. Moreover, even measuring K_L^0 would not help, since (neglecting a very small CP -breaking) $\langle K_L^0 \rangle = \langle K_S^0 \rangle$, implying that the multiplicities $\langle K_0 \rangle$ and $\langle \bar{K}_0 \rangle$ cannot be obtained.

Summary and conclusions

- Theory (HRG) cannot explain experiment(s) on charged-vs-neutral kaons
- UrQMD: new paper 2503.10493 increased ratio via a novel parameter.
- A simple quark-counting scheme valid for any Q/A shows: proton-proton data agree with isospin symmetry, nucleus nucleus do not.
- This model reproduces data for a large isospin breaking (about 20% more u than d quarks from QCD vacuum)
- $\pi^- + C$ and $\pi^+ + C$ of nuclei with $Z = N = A/2$ highly desired.
- Study ratios of other isospin multiplets (nucleons, hyperons)

Thanks!

Quarks and QCD, flavor symmetry:



Flavor transformation is a rotation in the (u,d,s) space.
Isospin is a subgroup of flavor.

Historical recall: „Shmushkevich” rule

An initial ‘uniform’ ensemble of hadronic state (that is, one with an equal mean number of each member of any isospin multiplet, such as the scattering of two isosinglet nuclei) evolves into a uniform final-state ensemble.

Uniform stays uniform

Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **103**, 235 (1955)

Dushin, N., Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **106**, 801 (1956)

MacFarlane, A.J., Pinski, G., Sudarshan, G.: Shmushkevich’s method for a charge independent theory. Phys. Rev. **140**, 1045 (1965) <https://doi.org/10.1103/PhysRev.140.B1045>

Wohl, C.G.: Isospin relations by counting. American Journal of Physics **50**(8), 748–753 (1982) <https://doi.org/10.1119/1.12743>

Pal, P.: An Introductory Course of Particle Physics -CRC Press, (2014)

Important remark:

Initial ensemble C-invariant: probabilities of having initial states related by this transformation are equal.

This is the case of nucleus-nucleus collisions where each nucleus has an equal number of protons and neutrons (thus, $I_z = 0$). Then, the invariance under C-transformation holds also for the final state ensemble.

$$\langle K^+ \rangle = \langle K^0 \rangle$$

$$\langle K^- \rangle = \langle \bar{K}^0 \rangle$$

Chat-GPT and e.m. interaction

1. Strong Interaction with Isospin Breaking:

- Quark mass differences m_u and m_d break isospin symmetry, leading to slightly different couplings for u -quark and d -quark production rates.
- The effective strong interaction rates now include a dependence on quark masses:

$$\alpha_s^u = \alpha_s(1 - \delta), \quad \alpha_s^d = \alpha_s(1 + \delta),$$

where δ is a small parameter quantifying the isospin-breaking effect due to $m_d > m_u$.

2. Electromagnetic Contribution:

- The electromagnetic terms remain as before:

$$\alpha_{\text{em}} Q_u^2 \quad \text{and} \quad \alpha_{\text{em}} Q_d^2.$$

3. Total Rates:

- The total rates now include both effects:

$$\text{Rate}(u\bar{u}) = \alpha_s^u + \alpha_{\text{em}} Q_u^2,$$

$$\text{Rate}(d\bar{d}) = \alpha_s^d + \alpha_{\text{em}} Q_d^2.$$

4. Ratio of Effective Rates:

- Incorporating isospin breaking, the ratio α/β becomes:

$$\frac{\alpha}{\beta} = \frac{\text{Rate}(u\bar{u})}{\text{Rate}(d\bar{d})} = \frac{\alpha_s(1 - \delta) + \alpha_{\text{em}} Q_u^2}{\alpha_s(1 + \delta) + \alpha_{\text{em}} Q_d^2}.$$

Chat-GPT and e.m. interaction /2

Numerical Calculation:

Using the same parameters as before:

- $\alpha_s = 0.1,$
- $\alpha_{\text{em}} = 1/137,$
- $Q_u^2 = 4/9, Q_d^2 = 1/9,$
- For isospin breaking: $\delta \approx 0.003$ (a typical estimate reflecting $m_d - m_u \sim 2 - 3$ MeV).

We compute:

$$\text{Numerator: } \alpha_s(1 - \delta) + \alpha_{\text{em}}Q_u^2 = 0.1 \cdot (1 - 0.003) + \frac{1}{137} \cdot \frac{4}{9} \approx 0.10075.$$

$$\text{Denominator: } \alpha_s(1 + \delta) + \alpha_{\text{em}}Q_d^2 = 0.1 \cdot (1 + 0.003) + \frac{1}{137} \cdot \frac{1}{9} \approx 0.10055.$$

The ratio α/β becomes:

$$\frac{\alpha}{\beta} = \frac{0.10075}{0.10055} \approx 1.002.$$

RK as function of Energy

$$R_K = \frac{N_u^{initial} + 2\alpha}{N_d^{initial} + 2\beta} \quad r = \frac{\alpha}{\beta} \sim 1.2$$

$$R_K = \frac{1 + 2\lambda \left(\sqrt{s_{NN}}\right)^k}{\frac{2-Q/A}{1+Q/A} + \frac{2\lambda}{r} \left(\sqrt{s_{NN}}\right)^k}$$

a₀(980) and f₀(980) data

Radiative phi decays with derivative interactions

Francesco Giacosa (Frankfurt U.), Giuseppe Pagliara (Frankfurt U.)

Apr, 2008

11 pages

Published in: *Nucl.Phys.A* 812 (2008) 125-139

e-Print: [0804.1572](https://arxiv.org/abs/0804.1572) [hep-ph]

DOI: [10.1016/j.nuclphysa.2008.08.011](https://doi.org/10.1016/j.nuclphysa.2008.08.011)

$$A_{f_0\pi\pi} = 2.88 \pm 0.22 \text{ GeV}, \quad A_{f_0KK} = 5.91 \pm 0.77 \text{ GeV}.$$

$$A_{a_0\pi\eta} = 3.33 \pm 0.15 \text{ GeV}, \quad A_{a_0KK} = 3.59 \pm 0.44 \text{ GeV},$$

[24] M. Ablikim *et al.* [BES Collaboration], *Phys. Lett. B* **607** (2005) 243 [arXiv:hep-ex/0411001].

[25] D. V. Bugg, V. V. Anisovich, A. Sarantsev and B. S. Zou, *Phys. Rev. D* **50** (1994) 4412.

[26] D. V. Bugg, *Eur. Phys. J. C* **47** (2006) 57 [arXiv:hep-ph/0603089]. D. V. Bugg, arXiv:hep-ex/0510014

| NA61/SHINE experiment | | | | | |
|--|---|------------------|------------|------------------------|---------------|
| Ar+Sc collisions at $\sqrt{s_{NN}} = 11.9$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $3.732 \pm 0.016 \pm 0.148$ | 0.15 | 0–10% | $0.0 < y < 0.2$ | [18] |
| K^- | $2.029 \pm 0.012 \pm 0.069$ | 0.070 | 0–10% | $0.0 < y < 0.2$ | [18] |
| K_S^0 | $2.433 \pm 0.027 \pm 0.102$ | 0.11 | 0–10% | $y = 0$ | this analysis |
| HADES experiment | | | | | |
| Ar+KCl collisions at $\sqrt{s_{NN}} = 2.6$ GeV | | | | | |
| hadron | Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $0.028 \pm 0.002 \pm 0.0014^{(*)}$ | 0.0024 | 0–35% | extrapolated to 4π | [43] |
| K^- | $0.00071 \pm 0.00015 \pm 0.000032^{(*)}$ | 0.00015 | 0–35% | extrapolated to 4π | [43] |
| K_S^0 | $0.0115 \pm 0.0005 \pm 0.0009$ | 0.0010 | 0–35% | extrapolated to 4π | [44] |
| STAR (BES I) experiment | | | | | |
| Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | 20.8 | 1.7 | 0–5% | $-0.1 < y < 0.1$ | [30] |
| K^- | 7.7 | 0.6 | 0–5% | $-0.1 < y < 0.1$ | [30] |
| K_S^0 | $12.67 \pm 0.12 \pm 0.44$ | 0.46 | 0–5% | $-0.5 < y < 0.5$ | [31] |

| STAR (BES I) experiment | | | | | |
|--|---|------------------|------------|------------------|------|
| Au+Au collisions at $\sqrt{s_{NN}} = 11.5$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | 25.0 | 2.5 | 0–5% | $-0.1 < y < 0.1$ | [30] |
| K^- | 12.3 | 1.2 | 0–5% | $-0.1 < y < 0.1$ | [30] |
| K_S^0 | $15.93 \pm 0.12 \pm 0.58$ | 0.50 | 0–5% | $-0.5 < y < 0.5$ | [31] |

| STAR (BES I) experiment | | | | | |
|--|---|------------------|------------|------------------------|----------|
| Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | 29.6 | 2.9 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K^- | 18.8 | 1.9 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K_S^0 | $20.89 \pm 0.08 \pm 0.67$ | 0.67 | 0-5% | $-0.5 < y < 0.5$ | [31] |
| STAR (BES I) experiment | | | | | |
| Au+Au collisions at $\sqrt{s_{NN}} = 27$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | 31.1 | 2.8 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K^- | 22.6 | 2.0 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K_S^0 | $23.24 \pm 0.09 \pm 0.70$ | 0.71 | 0-5% | $-0.5 < y < 0.5$ | [31] |
| STAR (BES I) experiment | | | | | |
| Au+Au collisions at $\sqrt{s_{NN}} = 39$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | 32.0 | 2.9 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K^- | 25.0 | 2.3 | 0-5% | $-0.1 < y < 0.1$ | [30] |
| K_S^0 | $24.9 \pm 0.1 \pm 1.7$ | 1.7 | 0-5% | $-0.5 < y < 0.5$ | [31] |
| NA49 experiment | | | | | |
| Pb+Pb collisions at $\sqrt{s_{NN}} = 7.6$ GeV | | | | | |
| hadron | Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $52.9 \pm 0.9 \pm 3.5^{(*)}$ | 3.6 | 0-7.2% | extrapolated to 4π | [40] |
| K^- | $16.0 \pm 0.2 \pm 0.4$ | 0.45 | 0-7.2% | extrapolated to 4π | [40] |
| K_S^0 | $29.3 \pm 0.3 \pm 2.9$ | 2.9 | 0-7.2% | extrapolated to 4π | [42] |
| NA49 experiment | | | | | |
| Pb+Pb collisions at $\sqrt{s_{NN}} = 8.7$ GeV | | | | | |
| hadron | Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $59.1 \pm 1.9 \pm 3$ | 3.6 | 0-7.2% | extrapolated to 4π | [41] |
| K^- | $19.2 \pm 0.5 \pm 1.0$ | 1.1 | 0-7.2% | extrapolated to 4π | [41] |
| K_S^0 | $34.2 \pm 0.2 \pm 3.4$ | 3.4 | 0-7.2% | extrapolated to 4π | [42] |
| CERES experiment | | | | | |
| Pb+Au collisions at $\sqrt{s_{NN}} = 17.3$ GeV | | | | | |
| hadron | Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $31.8 \pm 0.6 \pm 2.5$ | 2.6 | 0-7% | $y = 0$ | [27] |
| K^- | $19.3 \pm 0.4 \pm 2.0$ | 2.0 | 0-7% | $y = 0$ | [27] |
| K_S^0 | $21.2 \pm 0.9 \pm 1.7$ | 1.9 | 0-7% | $y = 0$ | [28, 29] |
| NA35 experiment | | | | | |
| S+S collisions at $\sqrt{s_{NN}} = 19.4$ GeV | | | | | |
| hadron | Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$ | σ_{total} | Centrality | y ranges | Ref. |
| K^+ | $12.5 \pm 0.4 \pm 0.375^{(*)}$ | 0.55 | 0-2% | extrapolated to 4π | [38] |
| K^- | $6.9 \pm 0.4 \pm 0.207^{(*)}$ | 0.45 | 0-2% | extrapolated to 4π | [38] |
| K_S^0 | 10.5 | 1.7 | 0-2% | extrapolated to 4π | [39] |

Example of isospin breaking/3

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

$D^*(2007)^0$

$$I(J^P) = \frac{1}{2}(1^-)$$

I, J, P need confirmation.

J consistent with 1, value 0 ruled out (NGUYEN 77).

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

$D^*(2010)^\pm$

$$I(J^P) = \frac{1}{2}(1^-)$$

I, J, P need confirmation.

$D^*(2007)^0$ DECAY MODES

$\bar{D}^*(2007)^0$ modes are charge conjugates of modes below.

| Mode | Fraction (Γ_i/Γ) |
|--------------------------|----------------------------------|
| Γ_1 $D^0 \pi^0$ | $(64.7 \pm 0.9) \%$ |
| Γ_2 $D^0 \gamma$ | $(35.3 \pm 0.9) \%$ |
| Γ_3 $D^0 e^+ e^-$ | $(3.91 \pm 0.33) \times 10^{-3}$ |

$D^*(2010)^\pm$ DECAY MODES

$D^*(2010)^\pm$ modes are charge conjugates of the modes below.

| Mode | Fraction (Γ_i/Γ) |
|-------------------------|--------------------------------|
| Γ_1 $D^0 \pi^+$ | $(67.7 \pm 0.5) \%$ |
| Γ_2 $D^+ \pi^0$ | $(30.7 \pm 0.5) \%$ |
| Γ_3 $D^+ \gamma$ | $(1.6 \pm 0.4) \%$ |

More on the resonances $f_0(980)$

$f_0(980)$

$$I^G(J^{PC}) = 0^+(0^{++})$$

See the related review(s):
 Scalar Mesons below 1 GeV

$f_0(980)$ DECAY MODES

$f_0(980)$ T-MATRIX POLE \sqrt{s}

Note that $\Gamma = -2 \operatorname{Im}(\sqrt{s})$.

| Mode | Fraction (Γ_i/Γ) |
|-----------------------|--------------------------------|
| Γ_1 $\pi\pi$ | seen |
| Γ_2 $K\bar{K}$ | seen |

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|-------------------------------------|-------------------------------|------|---------|
| (980-1010) - i (20-35) OUR ESTIMATE | (see Fig. 64.4 in the review) | | |

$$\Gamma(\pi\pi) / [\Gamma(\pi\pi) + \Gamma(K\bar{K})]$$

| VALUE | EVTS |
|-------|------|
|-------|------|

• • • We do not use the followin

| | |
|-----------------|------|
| 0.52 ± 0.12 | 9.9k |
|-----------------|------|

| | |
|------------------------|--|
| $0.75^{+0.11}_{-0.13}$ | |
|------------------------|--|

| | |
|-----------------|--|
| 0.84 ± 0.02 | |
|-----------------|--|

~ 0.68

| | |
|-----------------|--|
| 0.67 ± 0.09 | |
|-----------------|--|

| | |
|------------------------|--|
| $0.81^{+0.09}_{-0.04}$ | |
|------------------------|--|

| | |
|-----------------|--|
| 0.78 ± 0.03 | |
|-----------------|--|

The $\pi\pi$ mode dominates.

Similar consideration as for the $a_0(980)$ mesons.

Even including threshold effects,
no significant change of RK.

A simple 'quark counting' model

$$\alpha = N_u^{vac} = N_{\bar{u}}^{vac}$$

$$\beta = N_d^{vac} = N_{\bar{d}}^{vac}$$

$$\gamma = N_s^{vac} = N_{\bar{s}}^{vac}$$

$$r = \frac{\alpha}{\beta} \sim 1.2$$

Preliminary!!!

| Ratio | large $\sqrt{s_{NN}}$ result |
|---|------------------------------|
| $R_K = \frac{K^+ + K^-}{K^0 + \bar{K}^0}$ | $r \sim 1.2$ |
| $\frac{p}{n}$ | $r \sim 1.2$ |
| $\frac{\pi^+}{\pi^0}$ | $\frac{2r}{1+r^2} \sim 0.98$ |
| $\frac{\Sigma^+}{\Sigma^0}$ | $r \sim 1.2$ |
| $\frac{\Sigma^+}{\Sigma^-}$ | $r^2 \sim 1.4$ |

Pion-Carbon

$$R_K = \frac{N_u^{initial} + N_{\bar{u}}^{initial} + 2\alpha}{N_d^{initial} + N_{\bar{d}}^{initial} + 2\beta}$$

For π^+C and π^-C we have:

$$R_K^{\pi^+C} = \frac{19 + 2\alpha}{19 + 2\beta} = R_K^{\pi^-C}$$

and in the isospin-symmetric limit

$$R_K^{\pi^+C} = R_K^{\pi^-C} = 1$$

More on the resonance $a_0(980)$

$a_0(980)$

$$I^G(J^{PC}) = 1^-(0^{++})$$

$a_0(980)$ DECAY MODES

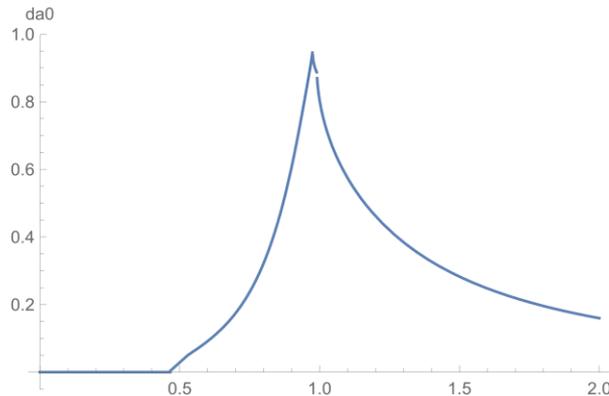
See the related review(s):
Scalar Mesons below 1 GeV

$a_0(980)$ T-MATRIX POLE \sqrt{s}

Note that $\Gamma = -2 \operatorname{Im}(\sqrt{s})$.

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---------------------------------------|-------------|------|-------------------------------|
| (970–1020) – i (30–70) OUR ESTIMATE | | | (see Fig. 64.2 in the review) |

| | Mode | Fraction (Γ_i/Γ) |
|------------|------------|--------------------------------|
| Γ_1 | $\eta\pi$ | seen |
| Γ_2 | $K\bar{K}$ | seen |
| Γ_3 | $\eta'\pi$ | seen |



Using the PDG average $\bar{K}K/\pi\eta = 0.172 \pm 0.019$ amounts to the following $\bar{K}K$ overall branching ratio :

$$\begin{aligned} \frac{\Gamma_{\bar{K}K}}{\Gamma_{tot}} &= \frac{\Gamma_{\bar{K}K}}{\Gamma_{\bar{K}K} + \Gamma_{\pi\eta} + \Gamma_{\pi\eta'}} \simeq \frac{\Gamma_{\bar{K}K}}{\Gamma_{\bar{K}K} + \Gamma_{\pi\eta}} \\ &= \frac{1}{1 + \frac{\Gamma_{\pi\eta}}{\Gamma_{\bar{K}K}}} = \frac{1}{1 + \frac{1}{\Gamma_{\pi\eta}/\Gamma_{\bar{K}K}}} = 0.15 \pm 0.01 . \end{aligned}$$

HRG: at first, equal amount for charged and neutral kaons.

Including threshold effects, leads to the branching ratio $K^+K^-/K^0\bar{K}^0 \simeq 1.1$

No significant effect on RK

Toward the general initial state

- For total initial $I = 0$ it is easy to show that $\langle K^+ \rangle = \langle K^0 \rangle$
- The result can be easily extended to any **fixed** total initial isospin $I=I_0$.
- It can be even generalized to initial states that are not isospin eigenstates, provided that an appropriate average is performed.