Final State Interactions in Femtoscopy : chiral symmetry, off-shell ambiguities and all that

Enrique Ruiz Arriola [1]

[1] Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Granada, Spain.

Femtoscopy
Krakow, 2025

Based on recent work

- The role of chiral symmetry and the non-ordinary (700) nature in $\pi^+ K_S$ femtoscopic correlations
  Miguel Albaladejo, Alejandro Canoa, Juan Nieves, Jose Ramón Peláez, Enrique Ruiz-Arriola, Jacobo Ruiz de Elvira
  Phys.Lett.B 866 (2025) 139552
  e-Print: 2503.19746 [hep-ph]
- Description of femtoscopic correlations with realistic pion-kaon interactions: the κ(700) case
  PoS QNP2024 (2025) 091
  Miguel Albaladejo, Alejandro Canoa, Juan Nieves, Jose Ramón Peláez, Enrique Ruiz-Arriola,Jacobo Ruiz de Elvira
  e-Print: 2501.11408 [hep-ph]
- Femtoscopic signatures of the lightest S-wave scalar open-charm mesons
  Miguel Albaladejo, Juan Nieves, Enrique Ruiz-Arriola Phys.Rev.D 108 (2023) 1, 014020
  e-Print: 2304.03107 [hep-ph]

And older work

- Proton-proton hollowness at the LHC from inverse scattering
  Enrique Ruiz Arriola, Wojciech Broniowski Phys.Rev.D 95 (2017) 7, 074030
  e-Print: 1609.05597 [nucl-th]
- Coarse graining pipi scattering
  Jacobo Ruiz de Elvira, Enrique Ruiz Arriola Eur.Phys.J.C 78 (2018) 11, 878
  e-Print: 1807.10837 [hep-ph]
- Bethe-Salpeter approach for unitarized chiral perturbation theory
  Juan Nieves, Enrique Ruiz Arriola Nucl.Phys.A 679 (2000) 57-117
  e-Print: hep-ph/9907469 [hep-ph]

Outline

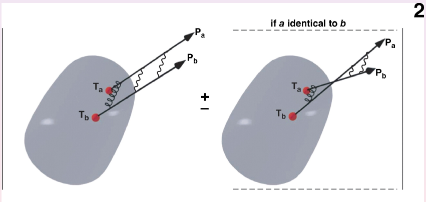
- Statement of the problem
- Femtoscopy and the inverse problem
- The Pratt-Koonin formula
- Pion-Kaon femto
- Chiral symmetry
- Conclusions

Statement of the problem

- Inclusive two particle production in High energy collisions

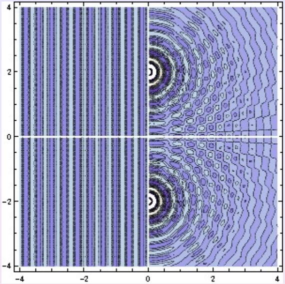
$$A + B \rightarrow X + p_1 + p_2$$

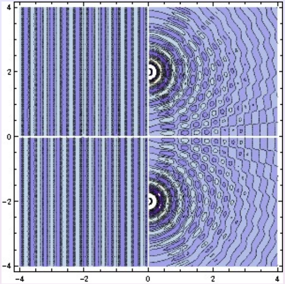
- If the detected particles interact we may in principle determine the phase-shifs
- It can be used in cases where no target exist
- Originally used to Hanbury Brown Twiss (HBT) bosonic or fermionic correlations

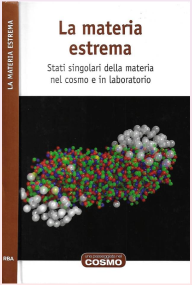


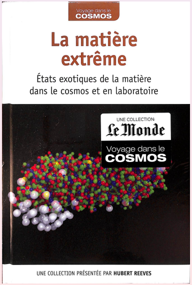
Lisa MA, et al. 2005.
Annu. Rev. Nucl. Part. Sci. 55:357–402

Water interference









Basic definitions

- Emision

$$\frac{d\sigma}{d^3p_1} = E_1 \langle a^\dagger_{p_1} a_{p_1} \rangle \quad (1)$$

$$\frac{d\sigma}{d^3p_1 d^3p_2} = E_1 E_2 \langle a^\dagger_{p_1} a^\dagger_{p_2} a_{p_1} a_{p_2} \rangle \quad (2)$$

- Correlation

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\sigma \frac{d\sigma}{d^3p_1 d^3p_2}}{\frac{d\sigma}{d^3p_1} \frac{d\sigma}{d^3p_2}} \quad (3)$$

The expectation value is over the density matrix ρ corresponding to the entire firball.

- Defining the density matrix

$$\rho(p_1, p_2) = \sqrt{E_1 E_2} \langle a^\dagger_{p_1} a_{p_2} \rangle$$

- In the independent pair approximation one can use Wick's theorem and hence

$$C(p_1, p_2) = 1 + \frac{|\rho(p_1, p_2)|^2}{\rho(p_1, p_1)\rho(p_2, p_2)}$$

- The density matrix

$$\rho(p_1, p_2) = \rho(P + \frac{1}{2}k, P - \frac{1}{2}k) = \int d^4x S(x, P) e^{ikx}$$

- Correlation

$$C_{12}(k) = \int d^3P C(\frac{1}{2}P + k, \frac{1}{2}P - k)$$

The Koonin-Pratt formula

- For a local and static and completely random (incoherent) source

$$C(k) = \int d^3\vec{r}\, S(r) |\psi_k^{(+)}(\vec{r})|^2 \equiv \langle \psi_k^{(+)} | S | \psi_k^{(+)} \rangle$$

- For a local and static and partially coherent source

$$C(k) = 1 + \lambda \int d^3\vec{r}\, S(r) \left[|\psi_k^{(+)}(\vec{r})|^2 - 1 \right]$$

- Out going wave

$$(-\nabla^2 + U(x))\psi_k^{(+)}(x) = k^2 \psi_k^{(+)}(x) \implies \psi_k^{(+)}(x) \to e^{ik\cdot x} + \frac{e^{ikr}}{r} f(\hat{k},\hat{x})$$

- Differential cross section

$$\frac{d\sigma}{d\Omega} = |f(\hat{k},\hat{x})|^2$$

- Spherical source $\implies$ partial wave expansion

$$\Psi_k^{(+)}(x) = \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} P_l(\cos\theta) \frac{u_l(r)}{r} \implies f(\hat{k},\hat{x}) = \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \frac{\sin\delta_l}{k} P_l(\cos\theta)$$

- Radial reduced solution for spherical potential

$$-u_l''(r) + \left[\frac{l(l+1)}{r^2} + U(r) \right] u_l(r) = k^2 u_l(r) \qquad u_l(r) \to \sin(kr - \frac{l\pi}{2} + \delta_l)$$

-

$$C(k) = 1 + \lambda \sum_{l=0}^{\infty} (2l+1) \int 4\pi S(r) dr \left[u_l(r)^2 - \hat{j}_l(kr)^2 \right]$$

Lednicky approximation

- $$C(k) = 1 + \int d^3x S(x) \left[|\Psi_k^{(+)}(x)|^2 - 1 \right]$$
- Take the asymptotic wave function
$$\Psi_k^{(+)}(x) \underbrace{\to}_{r \gg a} \Phi_k^{(+)}(x) = e^{ikx} + f(\hat{k}, \hat{x}) \frac{e^{ikr}}{r}$$
- Assume spherical source function
$$C(k) = 1 + \int r^2 S(r) \int d\Omega \left[f(\hat{x}, \hat{k}) \frac{e^{ikr}}{r} + f(\hat{x}, \hat{k})^* \frac{e^{-ikr}}{r} + \frac{|f(\hat{x}, \hat{k})|^2}{r^2} \right]$$
- $$\int d\Omega f(\hat{x}, \hat{k}) = 4\pi e^{i\delta_0} \frac{\sin \delta_0}{k} \qquad , \int d\Omega |f(\hat{x}, \hat{k})|^2 = \sigma$$
- $$C(k) = 1 + \int_0^\infty 4\pi r^2 S(r) \left[\sin \delta_0 \frac{\cos(kr + \delta_0)}{kr} + \frac{\sigma}{4\pi r^2} \right] \to C(0) = 1 + \int_0^\infty 4\pi r^2 S(r) \left[-\frac{\alpha_0}{r} + \frac{\alpha_0^2}{r^2} \right]$$

Range correction

- Separate contributions

$$C(k) = 1 + \int d^3x S(x) \left[|\Phi_k^{(+)}(x)|^2 - 1 \right] + \int d^3x S(x) \left[|\Psi_k^{(+)}(x)|^2 - |\Phi_k^{(+)}(x)|^2 \right]$$

- Assume $S(r) \sim S(0)$ for $r \le a$. One can show that for large sources $R \gg a$

$$\Delta C = \int d^3x S(x) \left[|\Psi_k^{(+)}(x)|^2 - |\Phi_k^{(+)}(x)|^2 \right] = S(0) \sum_l (2l+1) \frac{d}{dk^2} k \cot \delta_l(k) \left[1 + \mathcal{O}(a^2/R^2) \right]$$

ONLY ON-SHELL INFORMATION NEEDED !!

- Small source $R \ll a$

$$C(k) = |\Psi_k^{(+)}(R)|^2$$

- PROBLEM : $C(k)$ Observable BUT $|\Psi_k(0)|^2$ is NOT.
- If the potential is UNKNOWN and only constrained by physical measurable information than the wave function inside the potential IS NOT.
- ¿ What is the range of the source ?
- ¿ What is the range of the interaction ?

Unitary transformations

- Correlation is physical BUT wave function is NOT

$$C(k) =\equiv \langle \psi_k^{(+)} | S | \psi_k^{(+)} \rangle$$

- Simplest case: Wave function in momentum space

$$\Psi_k(p) = \langle p | \Psi_k \rangle \to e^{i\alpha(p)} \Psi_k(p) \implies \Psi_k(x) \to \int dx' U(x,x') \Psi_k(x') , \qquad U(x,x') = \int dp e^{i\alpha(p) - p(x-x')}$$

- The correlation changes

$$C(k) = \int S(x) |\Psi(x)|^2 \to \int d^3x' d^3x'' \Psi(x')^* \Psi(x'') d^3x S(x) U(x,x') U(x,x'')^* \neq C(k)$$

- General Unitary transformation $|\psi_k^{(+)}\rangle \to U\psi_k^{(+)}\rangle$ implies source MUST change (Meissner,Epelbaum 2025)

$$C(k) \to C(k) \psi_k^{(+)} | U^\dagger USU^\dagger U | \psi_k^{(+)} \rangle$$

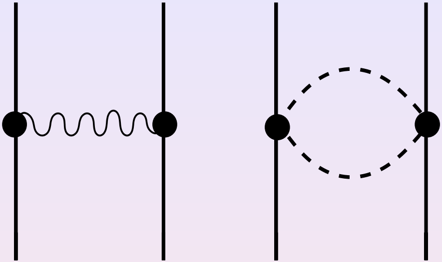
- $\implies$ SOURCE IS NOT UNIVERSAL

$$S \to U^\dagger SU$$

- What is the anatomy of source and interactions ?

Size of the interaction

- Interactions among mesons. The longest distance is $\pi\pi$ exchange



$$(x - x')^2 \lesssim \frac{1}{4m_\pi^2} \implies \Delta x \lesssim \frac{1}{2m_\pi} = 0.7\mathrm{fm}$$

- Chiral Potential (local and energy independent)

$$V_{\pi\pi}(x) = \frac{1}{f_\pi^4}\left[c_1 m^4 \Delta(x)^2 + c_2 m^2 (\partial_\mu \Delta(x))^2 + c_3 \left(\partial^\mu \partial^\nu \Delta(x)\right)^2\right] \sim e^{-2m_\pi r} + \mathcal{O}\left(\frac{m^6}{f^6}\right)$$

- Coulomb potential (non-local and energy dependent)

$$V_C(x) = \frac{q_1 q_2}{4\pi^2}\left[\frac{s}{x^2} + \partial^2 \frac{1}{x^2} + \frac{1}{x^2}\partial^2 + 2\partial^\mu \frac{1}{x^2}\partial_\mu\right]$$

Relativistic Equation

- Bethe Salpeter equation (with optical potential)

$$T_P(p,k) = V_P(p,k) + \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} V_P(p,q)\Delta(q_+)\Delta(q_-)T_P(q,k)$$

$\Delta(q_\pm) = 1/(q_\pm^2 - m_\pi^2 + i0^+)$

- $p = \frac{p_1 - p_2}{2}$, $k = \frac{k_1 - k_2}{2}$, $P = p_1 + p_2$ and $q_\pm = P/2 \pm q$.
- Perturbative matching

$$V_P(p,k) = V_P^{(2)}(p,k) + V_P^{(4)}(p,k) + \dots \qquad T_P(p,k) = T_P^{(2)}(p,k) + T_P^{(4)}(p,k) + \dots$$

- On-shell amplitude

$$T(s,t) = T_P(p,k), \quad p^2 = k^2 = \frac{s}{4} - m_\pi^2, \quad P\cdot p = P\cdot k = 0$$

- Off-shell ampiguities

[Nieves,RA (2000)]

Relativistic Equation II

- Mass invariant equation

[Allen,Payne,Polizou (2000)]

$$\mathcal{M}^2 = P^\mu P_\mu + W$$

- Non-relativistic matching

$$W = 4 m_\pi V$$

- CM system $\rightarrow$ Effective Schrödinger equation

$$(-\nabla^2 + m_\pi V)\Psi = (s/4 - m_\pi^2)\Psi \, .$$

- Inelasticity $\rightarrow$ Complex potential

$$V(\vec{r}, s) = \mathrm{Re}\, V(\vec{r}, s) + i\, \mathrm{Im}\, V(\vec{r}, s)$$

- Causality $\rightarrow$ Fixed-r dspersion relation

$$\mathrm{Re}\, V(r, s) = V(r) + \frac{1}{\pi} \int_{s_0}^{\infty} s' \, \frac{\mathrm{Im}\, V(r, s')}{s' - s - i\epsilon} \, ,$$

- Mandelstam representation for scattering amplitude

[Cornwall,Ruderman, PR (1965)]

Relativistic Equation III

(J. Ruiz de Elvira, ERA , 2017)

- $\pi\pi$ scattering
- Isospin and exchange potential

Au,Lommon,Feschbach (1962)]

$$V(r) = \left[V_A(r) + V_B(r)\,\vec{I}_1 \cdot \vec{I}_2 + V_C(r)\,(\vec{I}_1 \cdot \vec{I}_2)^2 \right] (1 + \mathcal{P}_{12}) = V_{\rm D}(r) + V_{\rm X}(r), o$$

- Exchange operator $\mathcal{P}_{12} = (-1)^{I+J}$
- Partial waves $\Psi(\vec{x}) = \frac{u_l(r)}{r} Y_{lm_l}(\hat{x})$,

$$-u_l''(r) + \left[U(r) + \frac{l(l+1)}{r^2} \right] u_l(r) = p^2 u_l(r),$$

- Scattering conditions

$$u_l(r) \to \sin\left(pr - \frac{l\pi}{2} + \delta_l \right).$$

Chiral potentials

- Isospin and exchange potential

$$V(r) = \left[V_A(r) + V_B(r)\,\vec{I}_1 \cdot \vec{I}_2 + V_C(r)\,(\vec{I}_1 \cdot \vec{I}_2)^2 \right] (1 + \mathcal{P}_{12}) = V_{\rm D}(r) + V_{\rm X}(r),$$

- Exchange operator $\mathcal{P}_{12} = (-1)^{l+J}$
- Partial waves $\Psi(\vec{x}) = \frac{u_l(r)}{r} Y_{lm_l}(\hat{x})$,

$$-u_l''(r) + \left[U(r) + \frac{l(l+1)}{r^2} \right] u_l(r) = p^2 u_l(r),$$

- Scattering conditions

$$u_l(r) \to \sin\left(pr - \frac{l\pi}{2} + \delta_l \right).$$

Chiral potentials I

- Perturbation theory

$$f(p,\cos\theta) = -\frac{1}{4\pi}\int {}^3\vec{r}\, U(r) e^{-i\vec{q}\cdot\vec{r}} - \int^3 \vec{r}_1 d^3\vec{r}_2 e^{i(\vec{p}'\cdot\vec{r}_2 - \vec{p}\cdot\vec{r}_1)} \frac{e^{ipr_{12}}}{r_{12}} U(r_1)U(r_2) + \dots$$

$\equiv T_2 + T_4 + \dots$

- $$U_l(r,s) = U_l^{(2)}(r,s) + U_l^{(4)}(r,s) + \dots ,$$

- Leading order

$$\begin{aligned} U_0^{(2)}(r,s) &= \frac{-1}{4\sqrt{s}} \frac{m_\pi^2 - 2s}{2f^2} \delta^{(3)}(\vec{r}) , \\ U_1^{(2)}(r,s) &= \frac{-1}{4\sqrt{s}} \frac{4m_\pi^2 - 2\nabla^2 - s}{2f^2} \delta^{(3)}(\vec{r}) , \\ U_2^{(2)}(r,s) &= \frac{-1}{4\sqrt{s}} \frac{s - 2m_\pi^2}{2f^2} \delta^{(3)}(\vec{r}) . \end{aligned} \tag{6}$$

Chiral potentials II

- Next to leading order

$$U_0 = \frac{\left(-23m_\pi^5 r^2 - 200m_\pi^3\right) K_1(2m_\pi r)}{128\pi^3 f^4 r^4 \sqrt{s}} + \frac{\left(-24m_\pi^4 r^2 - m_\pi^2 r^2 s - 100m_\pi^2\right) K_2(2m_\pi r)}{32\pi^3 f^4 r^5 \sqrt{s}},$$

$$U_1 = \frac{\left(-13m_\pi^5 r^2 - 40m_\pi^3\right) K_1(2m_\pi r)}{128\pi^3 f^4 r^4 \sqrt{s}} + \frac{\left(-18m_\pi^4 r^2 - m_\pi^2 r^2 s - 40m_\pi^2\right) K_2(2m_\pi r)}{64\pi^3 f^4 r^5 \sqrt{s}},$$

$$U_2 = \frac{\left(-17m_\pi^5 r^2 - 80m_\pi^3\right) K_1(2m_\pi r)}{128\pi^3 f^4 r^4 \sqrt{s}} + \frac{\left(-30m_\pi^4 r^2 + m_\pi^2 r^2 s - 80m_\pi^2\right) K_2(2m_\pi r)}{64\pi^3 f^4 r^5 \sqrt{s}}.$$

- Singular potentials at short distances (like van der Waals)

$$U_l(r,s) = -\frac{c_l}{16\pi^3 f^4 r^7 \sqrt{s}} + \ldots,$$

- Long distances

$$U_l(r,s) = -\frac{d_l m_\pi^{9/2} e^{-2m_\pi r}}{256\pi^{5/2} f^4 r^{5/2} \sqrt{s}} + \ldots$$

Chiral potentials III

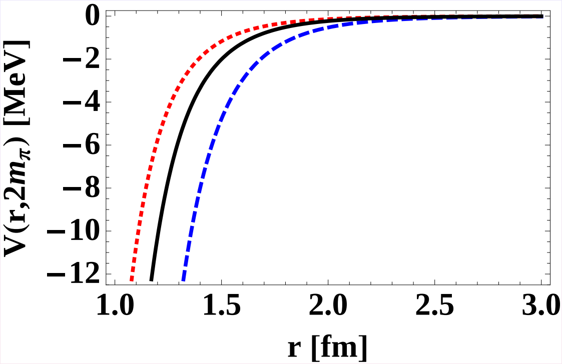


Figure: hiral 2π exchange potentials V_{IJ} at threshold $\sqrt{s} = 2m_\pi$ as a function of the distance for $IJ = 00$ (dashed), $IJ = 11$ (full) and $IJ = 20$ (dotted) channels.

Effective elementarity

- Electromagnetic Pion form factors

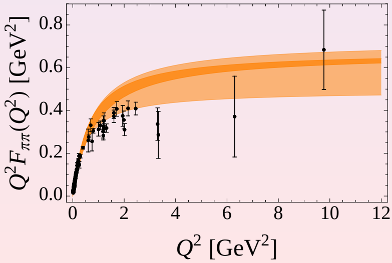
$$\langle \pi^+(p')|J^\mu(0)|\pi^+(p)\rangle = (p'^\mu + p^\mu)F_{\rm em}(q)$$

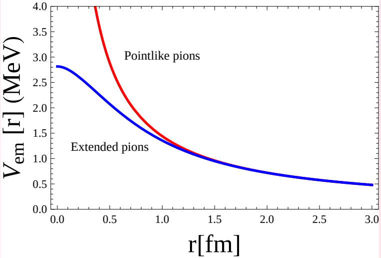
- Vector meson dominance (VMD)

$$F_V(q) = \sum_{V=\rho,\rho',\dots} c_V \frac{M_V^2}{M_V^2+q^2} \approx \frac{m_\rho^2}{m_\rho^2+q^2}$$

- Electromagnetic Coulomb interaction $\Delta m_\pi|_{\rm em} = V_{\rm em}(0) = 3{\rm MeV}$

$$V_{\rm em}(r) = \int \frac{d^3q}{(2\pi)^3}|F_{\rm em}(q^2)|^2 e^{i\vec{q}\cdot\vec{r}} = \frac{1}{r} - e^{-m_\rho r}\left[\frac{1}{2}m_\rho + \frac{1}{r}\right] \sim \frac{1}{r} \qquad r > r_e \sim 1 - 1.5{\rm fm}$$





Effective elementarity

- Gravitational Pion form factor

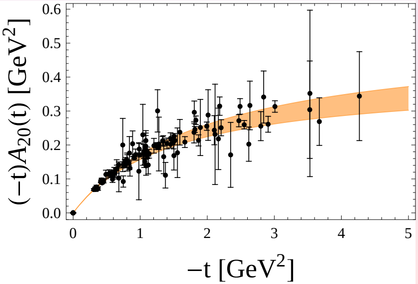
$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p)\rangle = \frac{1}{2}\delta^{ab}\left[(g^{\mu\nu}q^2 - q^\mu q^\nu)\Theta_1(q^2) + 4P^\mu P^\nu \Theta_2(q^2)\right]'^{\mu})$$

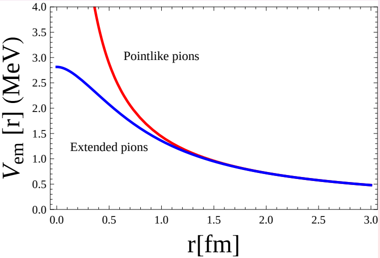
- Tensor meson dominance (TMD)

$$\Theta_1(q) = \sum_{T=f_2,f_2',\dots} c_T \frac{M_T^2}{M_T^2+q^2} \approx \frac{m_f^2}{m_f^2+q^2}$$

- Gravitational Coulomb interaction

$$V_{\rm grav}(r) = \int \frac{d^3q}{(2\pi)^3}|\Theta(q^2)|^2 e^{i\vec{q}\cdot\vec{r}} = \frac{1}{r} - e^{-m_f r}\left[\frac{1}{2}m_f + \frac{1}{r}\right] \sim \frac{1}{r} \qquad r > r_e \sim 1 - 1.2\text{fm}$$





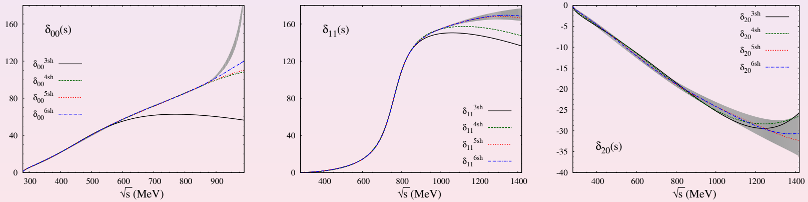
$\pi\pi$ Phase shifts

- Coarse graining

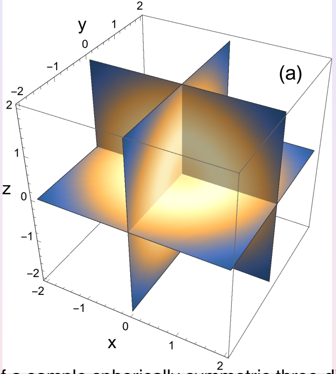
$$V_{\pi\pi}(r) = \left\{ \sum_i V(r_i)\delta(r - r_i) \right\} \theta[r_e - r] + V_{QFT}(r)\theta[r - r_e]$$

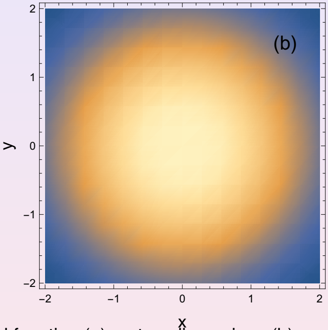
- Energy independent fits with $N = 4, 5, 6$ delta-shells

$$\Delta r = 0.3\text{fm}$$



Size of the source: pp Inelastic Collisions



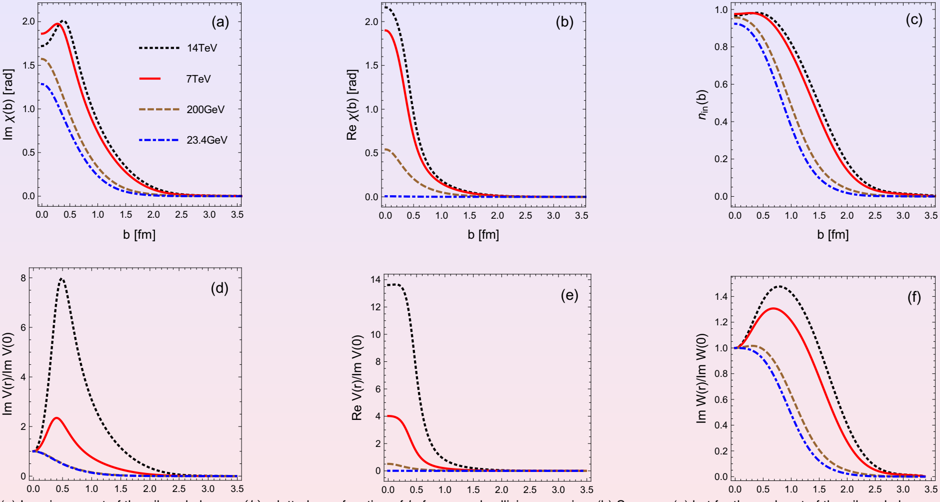


Projection of a sample spherically symmetric three-dimensional function (a) on two dimensions (b), as in Eq. (**??**). A shallow hollow present in (a) disappears in (b), where it is only reflected with a flatness on the central region.

- 3 D reconstruction of inelasticity

Hollowness ?

(W. Broniowski, ERA)



(a) Imaginary part of the eikonal phase $\chi(b)$, plotted as a function of b, for several collision energies. (b) Same as (a) but for the real part of the eikonal phase. (c) Same as (a) but for the inelasticity profile $n_{\rm in}(b)$. (d) Imaginary part of the optical potential $V(r)$ divided with $\mathrm{Im}\,V(0)$, plotted as a function of the radius r, for collision energies as in (a). (e) Same as (d) but for the real part of the optical potential. (e) Same as (d) but for the imaginary part of the on-shell optical potential $W(r)$ divided with $\mathrm{Im}\,W(0)$. The plots in the lower row are obtained, correspondingly, from the plots in the upper row via the transformations of Eqs. (**??**) or (**??**).

$\pi^{\pm} K_S$ femtoscopy from ALICE

- ALICE

$$C'(k^*) = \kappa \left[C_S(k^*) + \epsilon \frac{dN_{BW}}{dm} \frac{dm}{dk^*} \right] . \qquad (7)$$

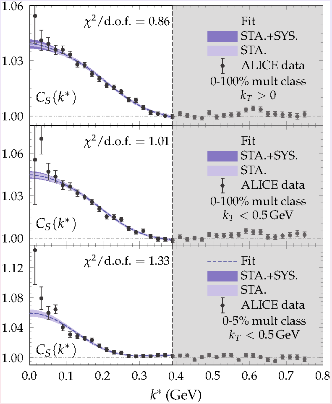


Figure: ALICE data on femtoscopic S-wave correlations for three different datasets. The blue lines and bands result from fits using our formalism with realistic πK interactions and relativistic corrections. The formalism presented here is valid in the elastic πK regime, but we show data beyond the ηK threshold for completeness.

$\pi^{\pm}K_S$ femtoscopy from Theory

- Koonin-Pratt

$$C(k^*) = \int d^3\vec{r}\, S(r) |\psi^*(\vec{k^*}, \vec{r})|^2$$

S(r)=exp$(-r^2/4R^2)/(4\pi R^2)^{3/2}$

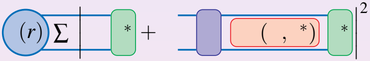


Figure: Schematically, the final state f is produced directly from the source S or intermediate states i (including also f) with virtual momentum p, through the half off-shell f^{if} amplitude. In factorization schemes, the $f^{if}(p, k^*)$ amplitudes factorize on-shell out of the p integral, i.e., $f^{if}(k^*)$, leaving just the pair of propagators inside the p integration.

$$C_{\rm S}(k^*) = 1 + \frac{\lambda}{2}\left(\frac{1}{3}\delta C^{(1/2)}(k^*) + \frac{2}{3}\delta C^{(3/2)}(k^*)\right), \quad (8)$$

$$\delta C^{(I)}(k^*) = \frac{f_0^{(I)}(k^*)}{R} I_1(s,R) - \frac{f_0^{(I)}(k^*)}{R} I_2(s,R) + \left|\frac{f_0^{(I)}(k^*)}{R}\right|^2 I_3(s,R) + \Delta C^{(I)}, \quad (9)$$

$$\Delta C^{(I)}(k^*) = -\frac{|f_0^{(I)}(k^*)|^2}{2R^3\sqrt{\pi}} \frac{d\left(1/f_0^{(I)}(k^*)\right)}{d(k^{*2})}. \quad (10)$$

Here, $s = ((m_\pi^2 + k^{*2})^{1/2} + (m_K^2 + k^{*2})^{1/2})^2$ is the usual Mandelstam variable. The λ parameter, called correlation strength, is included to measure the purity of genuine pairs
Namely, $I_1 = J_1$, $I_2 = J_1$ and

$$J_1(s,R) = -16\pi R\sqrt{s}\int d^3\vec{r}\, S(r)\, j_0(k^* r) g(s,r),$$

$$I_3(s,R) = 64\pi^2 R^2 s \int d^3\vec{r}\, S(r)\, |g(s,r)|^2 ,$$

where j_0 is a Bessel function (from the spherical *S*-wave) and

$$g(s,r) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{E_K + E_\pi}{2E_K E_\pi} \frac{j_0(pr)}{s - (E_K + E_\pi)^2 + i\epsilon}.$$

Note the use of relativistic energies $E_a = \sqrt{m_a^2 + p^2}$.

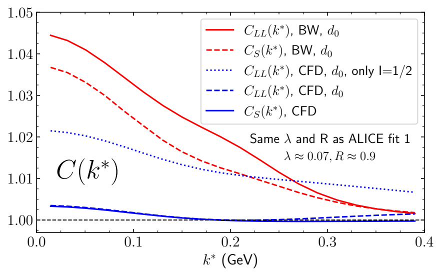
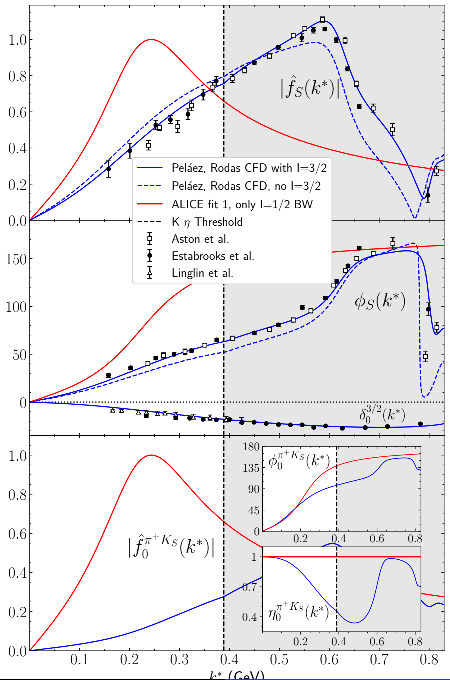
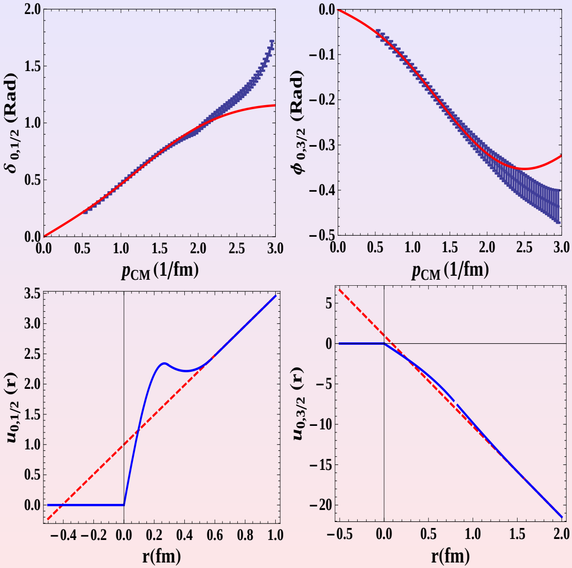


Figure: The red curve is ALICE fit 1 with the non-relativistic LL model and I=1/2 elastic BW $\pi^{\pm} K_S$ interaction only. In the red-dashed line, the LL formula is replaced by our relativistic equation. The dotted blue line is the LL model with realistic I=1/2 πK interactions (CFD, [**?**]) only. The continuous blue line is our relativistic model with realistic interactions for both I=1/2 and 3/2 but without the derivative in $\delta C^{(I)}$ set to $d_0^{(I)}/2$ (as done for other lines).



Peláez, Rodas CFD with I=3/2
Peláez, Rodas CFD, no I=3/2
ALICE fit 1, only I=1/2 BW
K η Threshold
Aston et al.
Estabrooks et al.
Linglin et al.
$|\hat{f}_S(k^*)|$
$\phi_S(k^*)$
$\delta_0^{3/2}(k^*)$
$|\hat{f}_0^{\pi^+K_S}(k^*)|$
$\phi_0^{\pi^+K_S}(k^*)$
$\eta_0^{\pi^+K_S}(k^*)$
k^* (GeV)

Square wells πK_S



Chiral lagrangians: field ambiguities

- Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{4} \mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \frac{f^2 m^2}{4} \mathrm{tr}\left(U + U^\dagger - 2\right) \qquad U = e^{i\vec{\phi}\cdot\vec{\tau}/f_\pi} \tag{11}$$

- Three equivalent representations

$$U(x) = e^{i\vec{\phi}\cdot\vec{\tau}/f_\pi} \tag{12}$$

$$U(x) = \sqrt{1 - \vec{\phi}^{\,2}/f^2} + \mathrm{i}\vec{\phi}\cdot\vec{\tau}/f \tag{13}$$

$$U(x) = (1 + \mathrm{i}\vec{\phi}\cdot\vec{\tau}/2f)/(1 - \mathrm{i}\vec{\phi}\cdot\vec{\tau}/2f) \tag{14}$$

$$\vec{\phi} \to \vec{\phi}(1 + \alpha\vec{\phi}^2 + \dots) \tag{15}$$

- Fourth order in $1/f_\pi$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\vec{\phi})^2 - \frac{m^2}{2}\vec{\phi}^{\,2} - \frac{\alpha}{4f^2}(\partial_\mu\vec{\phi})^2\vec{\phi}^{\,2} + \frac{1-\alpha}{2f^2}(\vec{\phi}\,\partial_\mu\vec{\phi})^2 + (\alpha - \frac{1}{2})\frac{m^2}{4f^2}\vec{\phi}^{\,4} \tag{16}$$

- Scattering amplitudes are INDEPENDENT of α but

$$C(k) = c0(\alpha) + c2(\alpha)k^2 + \frac{m_\pi^2}{f_\pi^2}F(k/m_\pi) + \mathcal{O}\left(\frac{m_\pi^4}{f_\pi^4}\right)$$

- This is a low energy theorem for $\pi\pi$ femtoscopy

Conclusions

- The femtoscopy program has achieved many goals and high precision data are available BUT it may need some refinements in order to be predictive:
- What aspects of the source are COMMON ?
- What aspects of the interaction are COMMON ?
- How can we describe the combined ambiguities of SOURCE+FINAL STATE INTERACTION in a MODEL INDEPENDENT WAY ?