Magnetic properties of Hadron Resonance Gas

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in collaboration with Wojciech Broniowski

[based on PRC 112 (2025) 4,045202 and arXiv:2511.19255]

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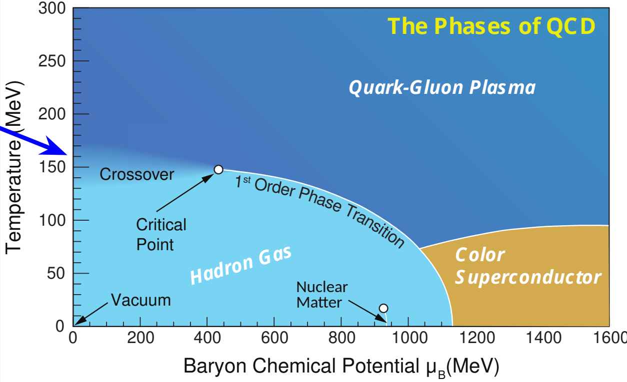
BIAŁASÓWKA HEP Seminar, 19.12.2025

Outline

- QCD phase diagram : Hadron Resonance Gas (HRG) and lattice QCD
- The HRG framework and its successes
- HRG in presence of B (static) : need for anomalous magnetic moment (κ)
- Failure of HRG for magnetic susceptibility $(\chi_B(T))$
- Possible solution through a non-interacting quark-meson model

QCD phase diagram: HRG and Lattice QCD

- At $\mu_B \approx 0$ (LHC and RHIC highest energy) $\longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV



Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

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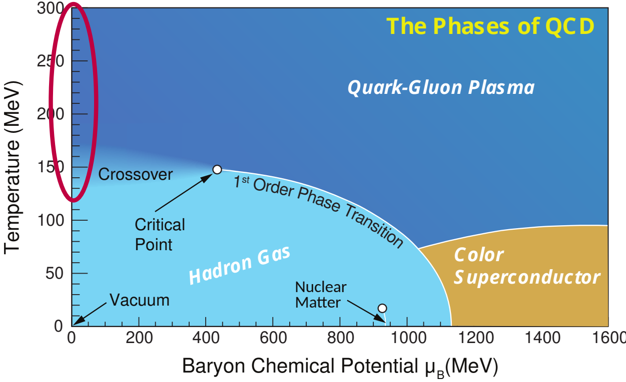

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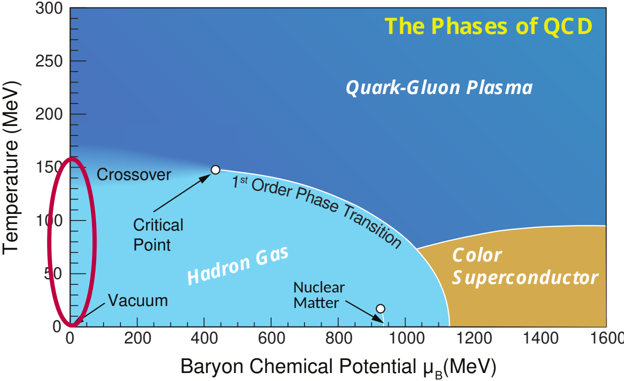
- **For $T \gtrsim 120$ MeV: lattice QCD (first principle description)**
- Below T_c: Hadron Resonance Gas (HRG) $\longrightarrow$ successfully describes numerous lattice data



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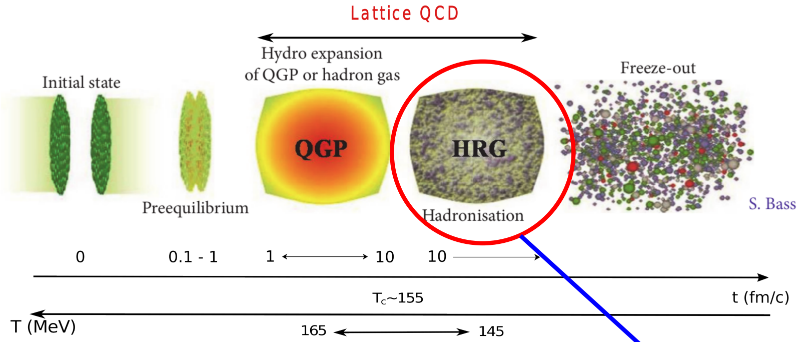
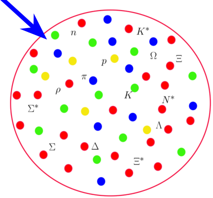
HRG in heavy-ion collision





- System of stable and resonance hadron states formed at **later stage of HI collision** [Braun-Munzinger *et al.*, PLB (1995), Andronic *et al.*, Nature (2018)]

  found in early-universe . . .

The HRG framework

- In non-interacting limit: thermal partial pressure (grand canonical system)

$$P^{th} = -\eta T(2s+1)\int \frac{d^3p}{(2\pi)^3}\log[1-\eta f(E,T,\mu)]$$

where, $f(E(p),T,\mu) = \frac{1}{\exp(\frac{E(p)-\mu}{T})+\eta}$, with $E_p = \sqrt{p^2+m^2}$, $\mu = \mu_{\mathcal{B}}\mathcal{B} + \mu_{\mathcal{S}}\mathcal{S} + \mu_{\mathcal{Q}}\mathcal{Q}$ and η=+ 1 (fermion) or -1 (boson)

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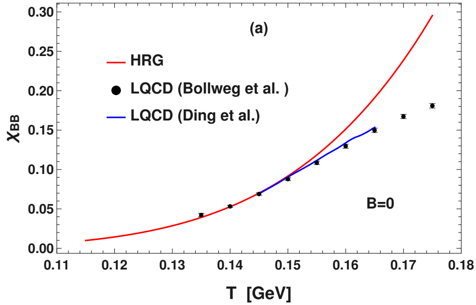
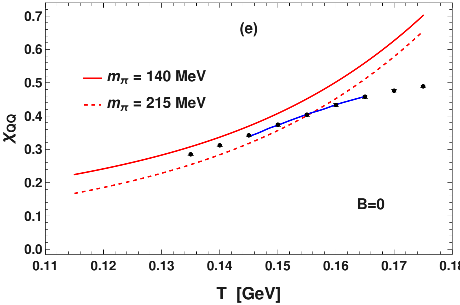
- Conserved charge susceptibility:

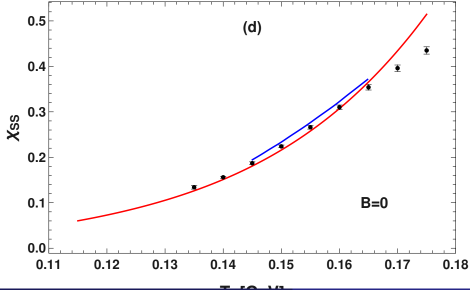
$$\chi_{Q_1Q_2} = \left.\frac{\partial^2(P/T^4)}{\partial(\mu_{Q_1}/T)\partial(\mu_{Q_2}/T)}\right|_{\mu_{Q_1},\mu_{Q_2}=0} \qquad Q_1,Q_2 \in \{\mathcal{B},\mathcal{S},\mathcal{Q}\}$$

$$\chi_{Q_1Q_2} = \frac{Q_1Q_2(2s+1)}{2\pi^2T^2}\int_0^\infty dp\,\left(m^2+2p^2\right)\frac{f(E_p)}{E_p}$$

HRG vs Lattice data







HRG successfully describes the lattice data upto $\sim T_c$

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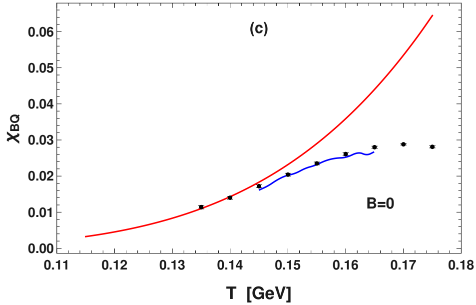

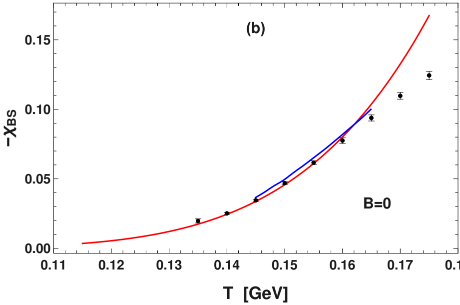

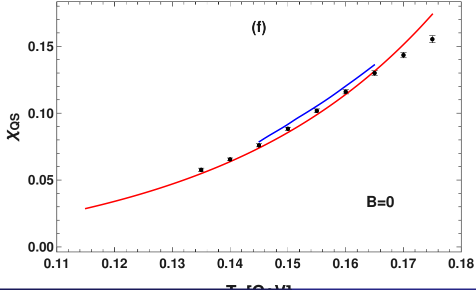

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Below T_c, HRG is a good model

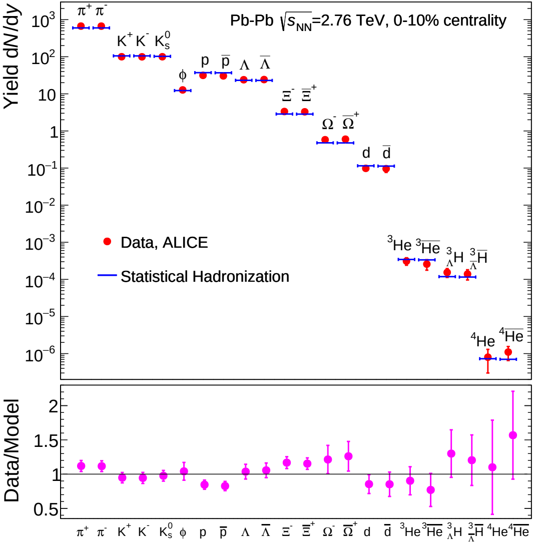
HRG vs particle yield

Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision

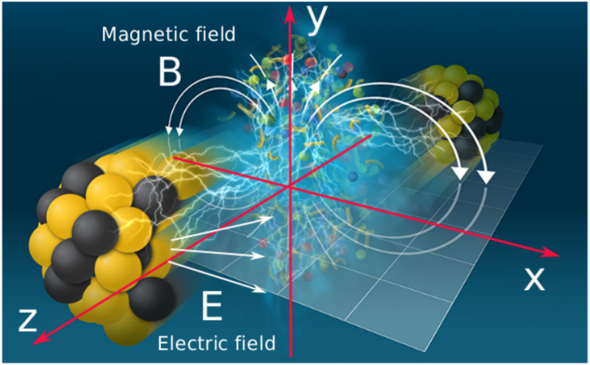
Andronic *et al.*, (2006) Nucl.Phys.A

Andronic *et al.* (2018), Nature

HRG has proved to be a very successful model at $B = 0$



HRG in magnetic field : HI collision



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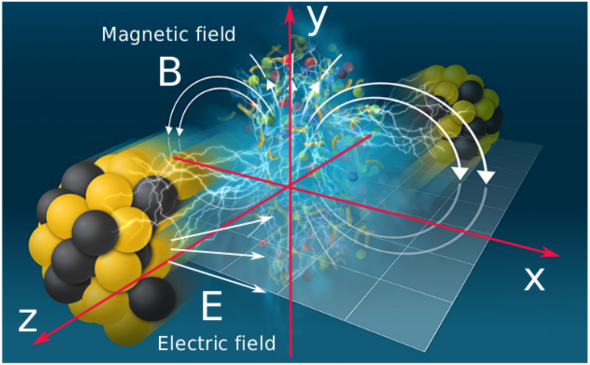

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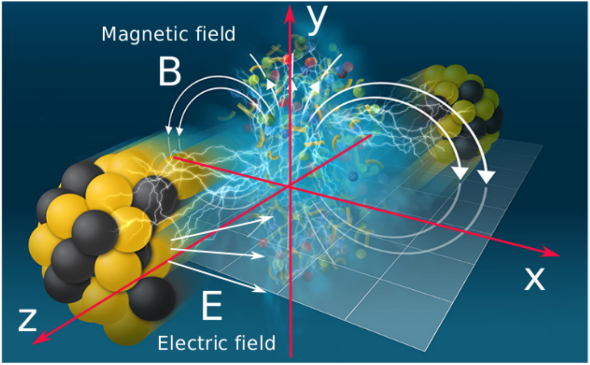

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- Lattice QCD: **data in the presence of uniform static B** [Bali et al., (2014, 2020) JHEP; Endrodi et al., (2022), JHEP; Ding et al., (2024, 2025) PRL, PRD]

Relativistic charged particle in magnetic field (Spin-0)

- For spin-0 particle in magnetic field one starts with Klein-Gordon equation :

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- Energy spectrum :

$$E_{ch} = \sqrt{m^2 + p_z^2 + B|Q|(2l+1)} \qquad l\text{: Landau levels}$$

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- Spin-1 (A. Proca, 1936) and spin-3/2 (Rarita-Schwinger state, Paoli *et al.*, 2013) : small $\kappa \rightarrow$ **non-relativistic treatment**

$$E_{ch} = \sqrt{m^2 + p_z^2 + B|Q|(2l+1) - 2QBs_z} - \mu_M B 2\kappa s_z$$

  Similar prescription for higher spin $s > 3/2$ ...

The pressure in magnetic field

- In presence of magnetic field:

$$(2s+1)\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{B|Q|}{2\pi^2}\sum_{l=0}^{\infty}\sum_{s_z=-s}^{s}\int_0^{\infty} dp_z$$

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- And so the charge susceptibilities:

$$\chi_{Q_1Q_2} = \frac{Q_1Q_2B|Q|}{2\pi^2T^3}\sum_{l=0}^{\infty}\sum_{s_z=-s}^{s}\int_0^{\infty} dp_z f(1-\eta f)$$

Effects of anomalous magnetic moment on χ_{BB}

- In HRG at **small** B, in **non-relativistic** and **Boltzmann limit**:

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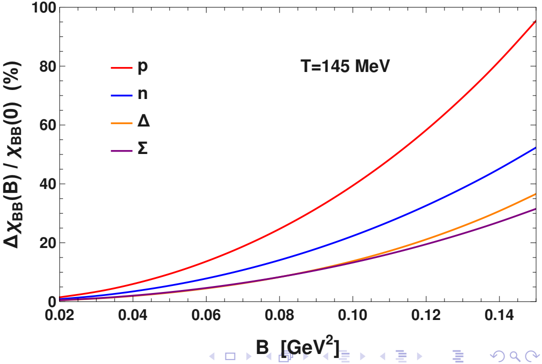

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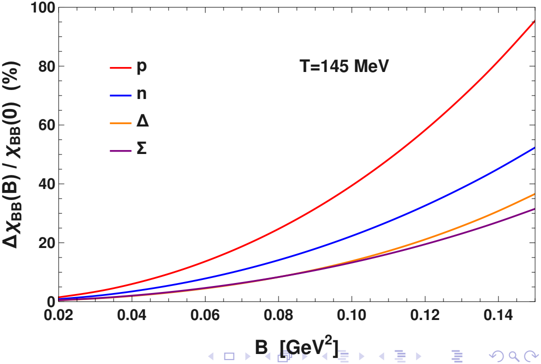
- Effect of κ is substantial and must be taken into account



Physical μ and g of hadrons

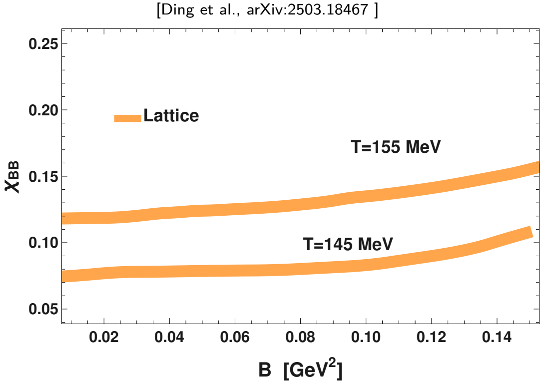
$$\mu_{exp} = g\ s\ \mu_M$$

$$\implies g = \frac{\mu_{exp}}{s\ \mu_N}\frac{m}{m_p}$$

Hadron species	μ/μ_N	g (Eq. 5)	Reference
$\rho^+(775)$	**1.94**(1)	1.60(1)	LQCD [42]
	2.21	1.82	χPT [43]
	2.37	1.96	QM [44]
$K^{*+}(892)$	**2.4**(2)	2.3(2)	LQCD [45]
	2.19	2.08	QM [44]
$K^{*0}(896)$	-0.183	-0.175	QM [44]
$p(938)$	2.793	5.586	PDG [46]
$n(939)$	-1.913	-3.831	PDG [46]
$\Lambda^0(1115)$	-0.613(4)	-1.458(9)	PDG [46]
$\Sigma^+(1189)$	2.458 (10)	6.232(25)	PDG [46]
$\Sigma^0(1192)$	**0.65**	1.65	χPT [47]
	0.791	2.011	QM [48]
$\Sigma^-(1197)$	-1.160(25)	-2.96(63)	PDG [46]
$a_1^+(1230)$	**1.7**(2)	2.2(2)	LQCD [49]
	1.44	1.89	QM [44]
$\Delta^{++}(1232)$	**3.7-7.5**	3.23-6.56	PDG [46]
	6.14(51)	5.37(45)	Lopez et.al [50]
	5.24(18)	4.58(16)	LQCD [51]
	4.97(89)	4.34(78)	χPT [52]
$\Delta^+(1232)$	$2.7^{+1.0}_{-1.3}\pm1.5\pm3$	$2.36^{+0.87}_{-1.14}$	PDG [46]
	2.6(5)	2.27(4)	χPT [52]
$\Delta^0(1232)$	0.02(12)	0.017(100)	χPT [52]
$\Delta^-(1232)$	-2.48(32)	-2.17(28)	χPT [52]
$\Xi^-(1321)$	-0.651(3)	-1.834(8)	PDG [46]
$\Xi^0(1321)$	-1.250(14)	-3.503(39)	PDG [46]
$\Sigma^{*+}(1383)$	**2.55**(26)	2.50(25)	LQCD [53]
	1.76(38)	1.73(37)	χPT [52]

HRG vs Lattice in B : $\chi_{\mathcal{BB}}$

- Lattice data **shows increase with** B, we focus on $B \lesssim 0.15$ GeV^2.



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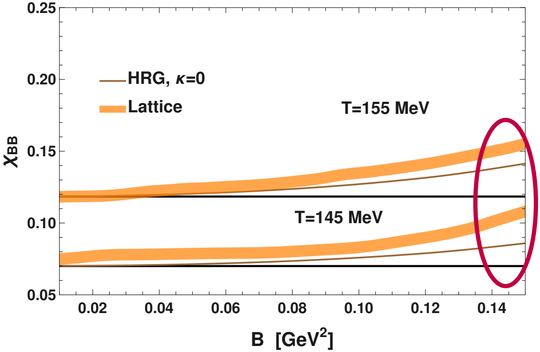

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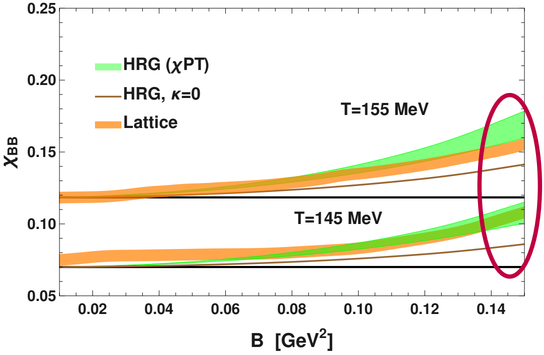

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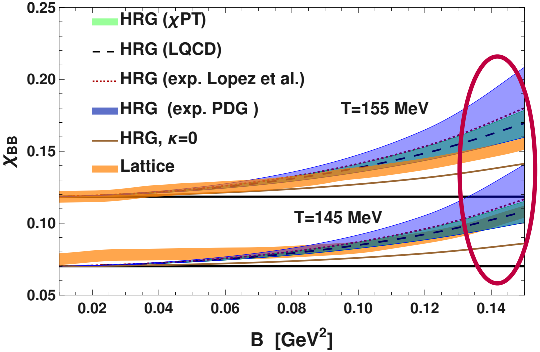
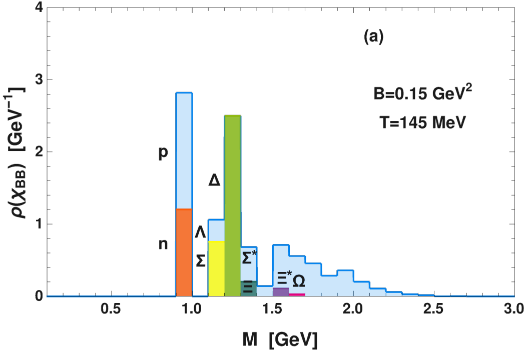
- **Large error bands** $\longrightarrow$ uncertainty in the estimate/measurement of $\mu_{\Delta^{++}}$



Anatomy of $\chi_{\mathcal{BB}}$

- $\rho(\chi_{BB}) \rightarrow$ **relative contributions of baryon states**



Individual contributions from baryons

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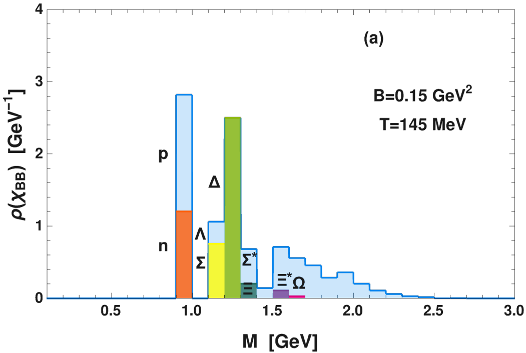
- $T = 145$ MeV: **dominant contribution from nucleons, followed by Δ**

  $(p+n) + \Delta = 50\%$
  $\Sigma + \Lambda = 10\%$



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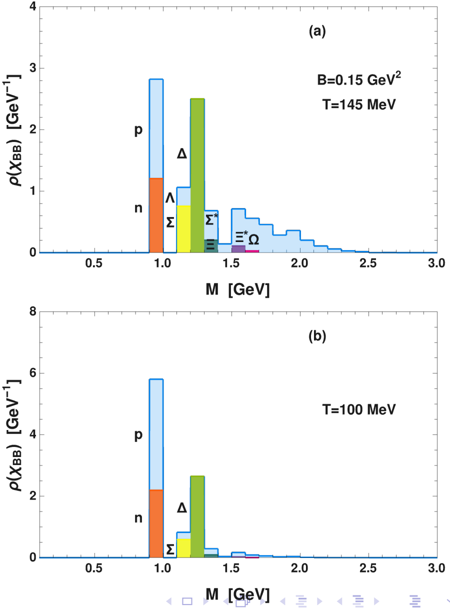
- T decreases $\rightarrow$ **thermal suppression of heavy baryons**

$T = 100$ MeV: $(p+n) \sim 60\%$



HRG vs Lattice in B : χ_{QQ} and χ_{SS}

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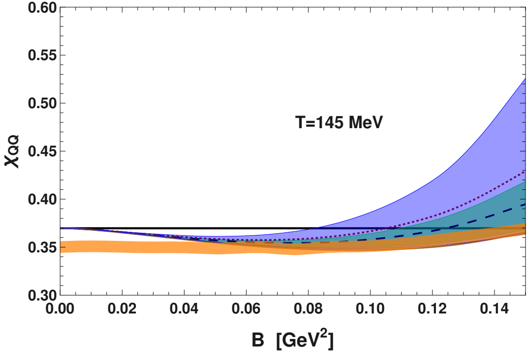

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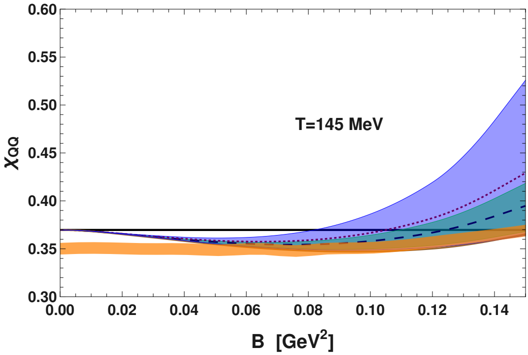

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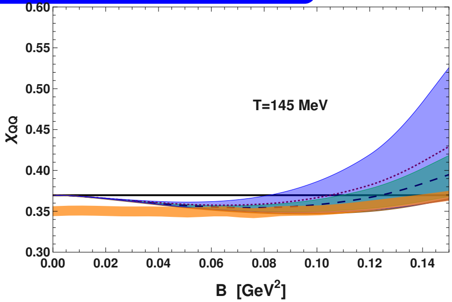
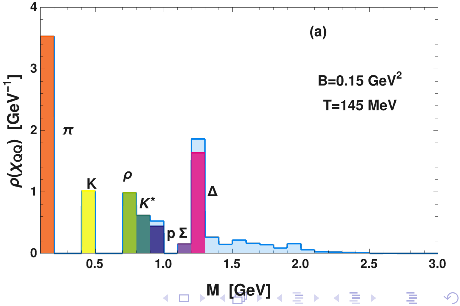
- Large error band $\longrightarrow \mu_{\Delta}^{++}$ (for χ_{QQ} effect is multiplied by 4)
- $T = 145$ MeV : **dominant contribution from π (35 %) followed by $\Delta(\sim$ 20 %)**





HRG vs Lattice in B : χ_{QQ} and χ_{SS}

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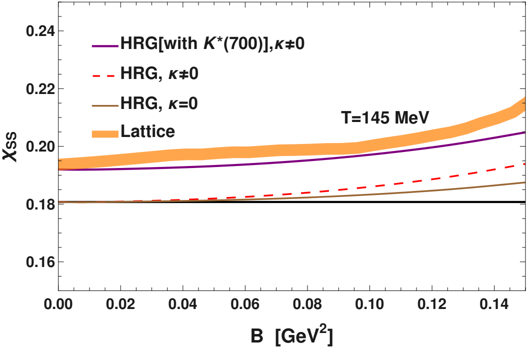

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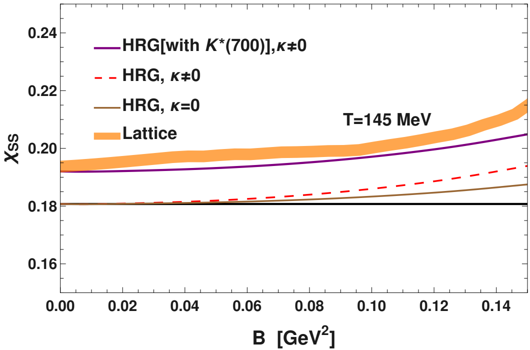

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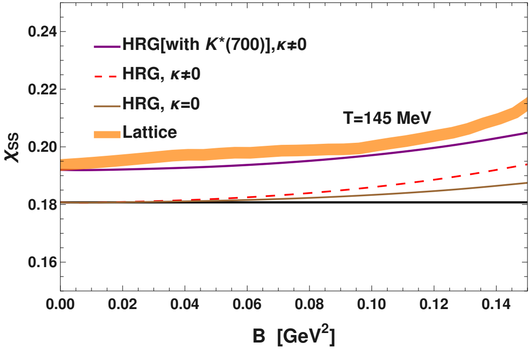
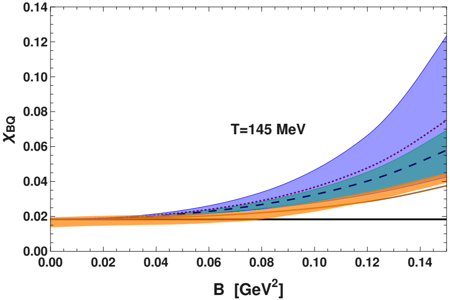
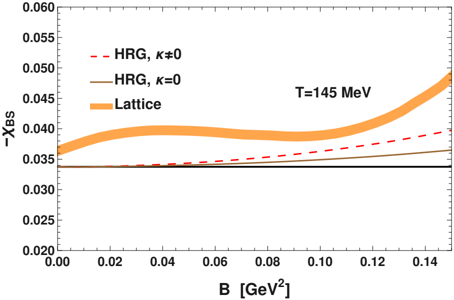
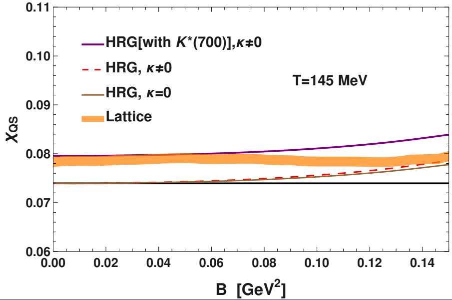


Inclusion of $\kappa(K^*_{700})$ makes up the gap

Non-diagonal susceptibilities (χ_{BQ}, χ_{BS} and χ_{QS})







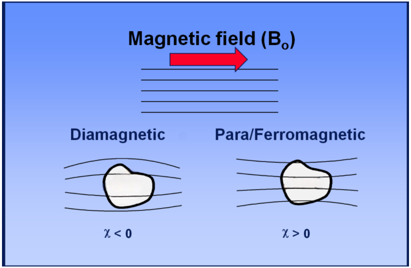
- χ_{BQ} : **not affected by κ of neutral baryons.**
- χ_{QS} : requires $K^*(700)$ state to reproduce the lattice data.

Magnetic Susceptibility

- **Magnetic response of a medium to the applied magnetic field,**

$$\chi_B = \frac{\partial \mathcal{M}}{\partial B}\Big|_{B=0}$$

$\mathcal{M} \rightarrow$ Magnetization of the system



Thermodynamic system under magnetic field

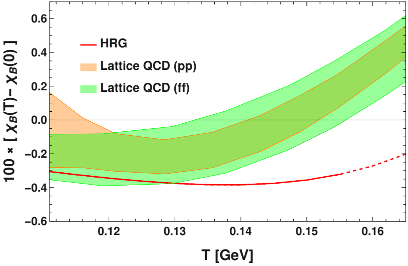
- Magnetic susceptibility:

$$\chi_B^{\rm th} = \frac{\partial^2 P^{\rm th}}{\partial B^2}\Big|_{B=0} = \begin{cases} + \quad \text{if } s \neq 0 : \text{ Paramagnetic} & \text{e.g } p, \rho, \Delta \\ - \quad \text{if } s = 0 : \text{ Diamagnetic} & \text{e.g. } \pi, K \end{cases}$$

Magnetic Susceptibility (χ_B): HRG vs lattice QCD

- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$



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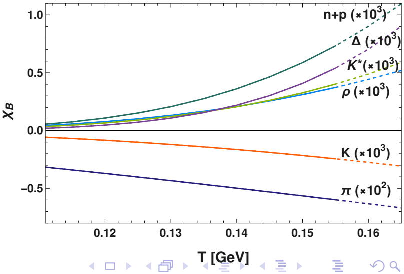

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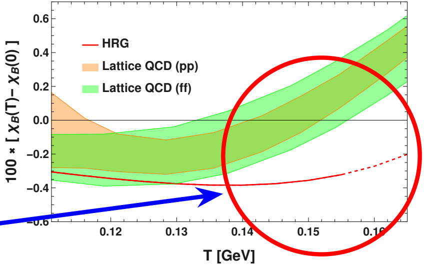

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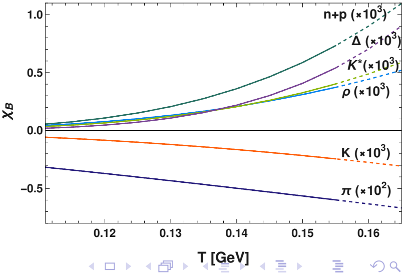

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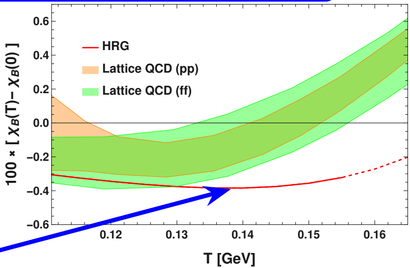

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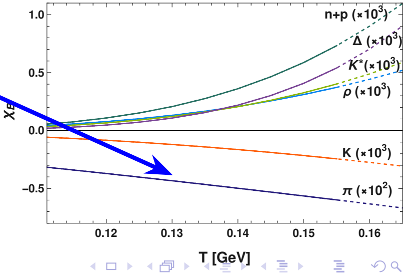

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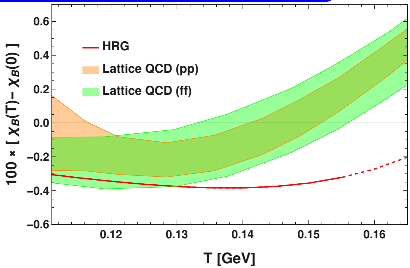

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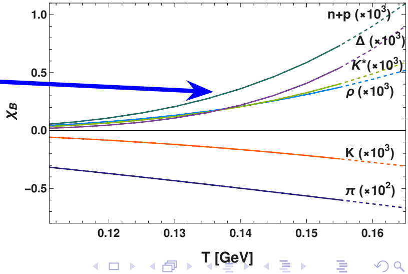

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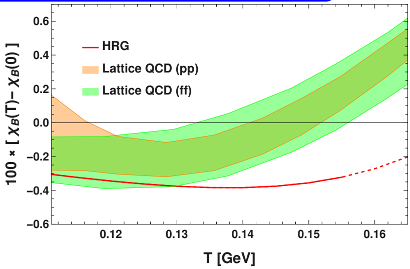
- **Genuine need for light paramagnetic source (e.g. quarks?)**



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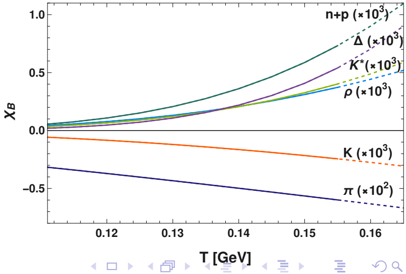
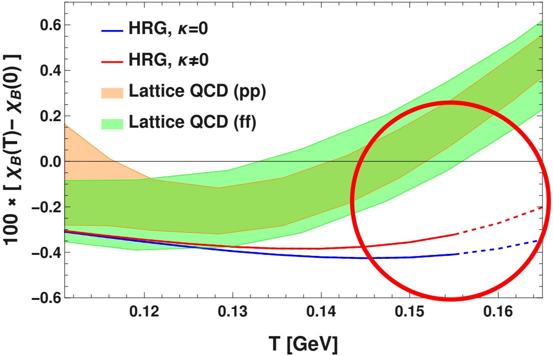


κ does not help for χ_B

- Non-zero κ of hadrons are not sufficient for χ_B



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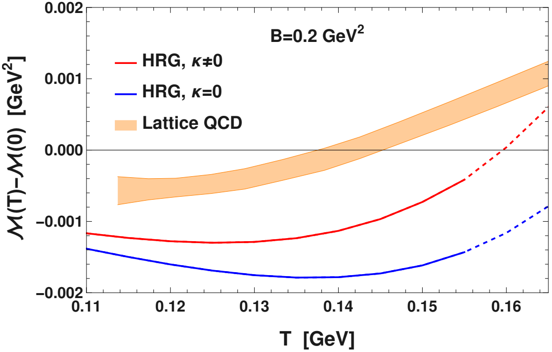
$\mathcal{M} = \frac{\partial P}{\partial B}$



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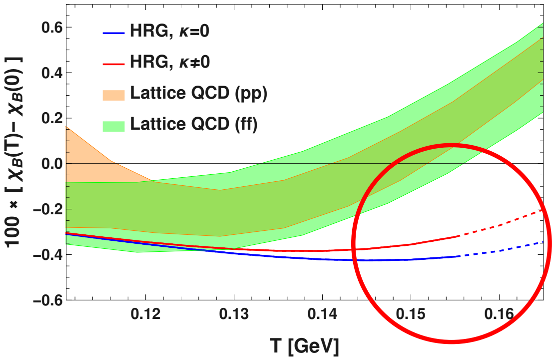

We address this failure as a genuine problem

Importance of vacuum pressure

- The total pressure can be decomposed:

$$P = \underbrace{P^{\rm vac}(B;m)}_{\text{Vacuum part}} + \underbrace{P^{\rm th}(T,\mu,B;m)}_{\text{Thermal part}}$$

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$$\frac{\partial P}{\partial m} = 0$$

Dependence on P^{vac}

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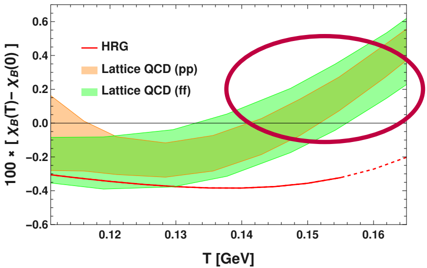
- **This makes the modeling of vacuum extremely difficult $\longrightarrow$ possible room for other degrees of freedom**

Model

- We need paramagnetism (but also light): constituent quarks



Temp-dependent quark masses

Quark degrees of freedom: Quark-Meson model

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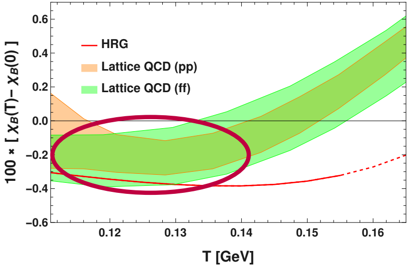

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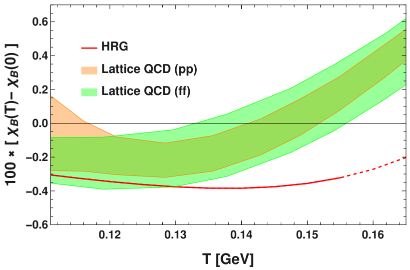

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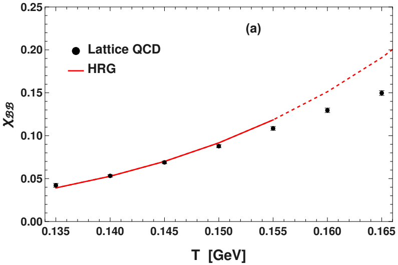

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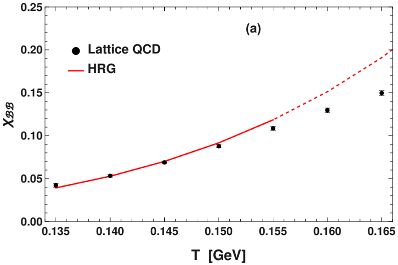

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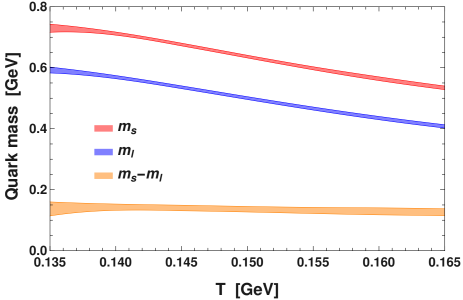

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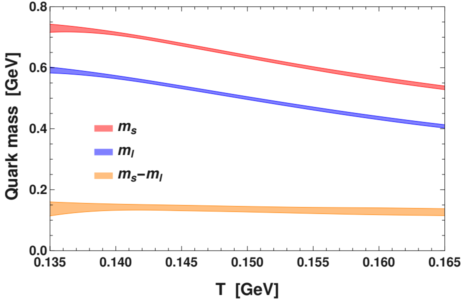

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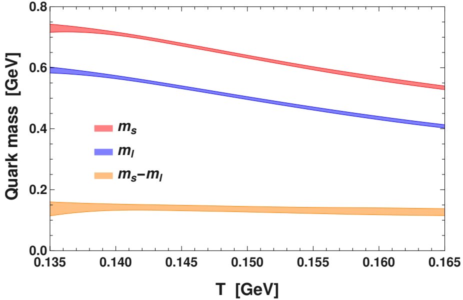

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- $m_s - m_l$ is roughly constant ~ 120 MeV

The vacuum contributions of quarks and mesons

Endrodi *et al.* (2013), JHEP; Kamikado *et al.* (2015), JHEP

- Can be calculated from photon vacuum polarization involving a quark/pion loop ($\Pi^{\mu\nu}$).

$$\chi_B^{\text{vac}} = \frac{1}{2} \lim_{\vec{q}\to 0} \lim_{q_0\to 0} \frac{\partial^2 \text{Re}[\Pi_s^{\text{vac}}]}{\partial q^2}$$

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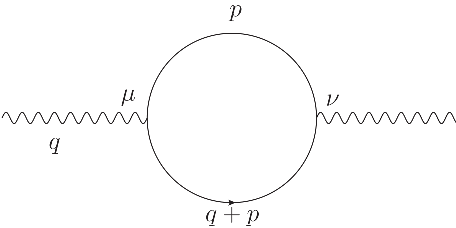
Coupling with anomalous magnetic moment (κ)

$$\Gamma^\mu = \gamma^\mu + \frac{\kappa}{2m}\sigma^{\mu\nu}q^\nu$$



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- After regularizing the divergence:

$$\chi_B^{\text{vac}}(T) - \chi_B^{\text{vac}}(0) = f(m(T), m(0), \kappa, \Lambda)$$

$\Lambda \sim 800$ MeV $\longrightarrow$ regularization scale

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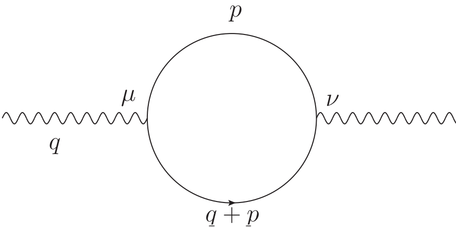

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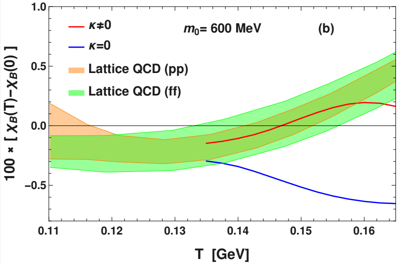
Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP



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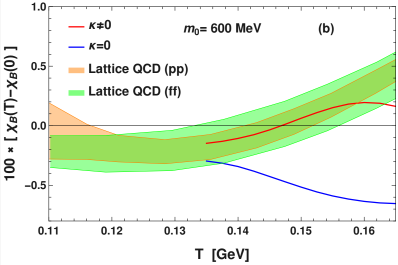
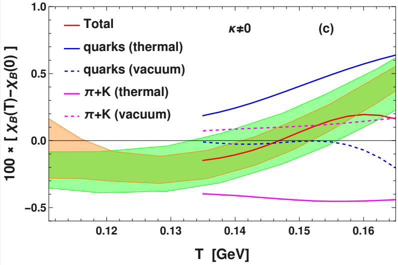


Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
- Quark anomalous magnetic moment (κ) is essential
- χ_B^{vac}(mesons): paramagnetic (temperature dependence of mass flattens the thermal part)



Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP
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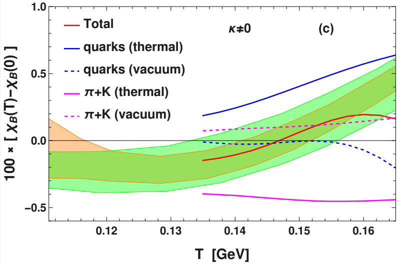

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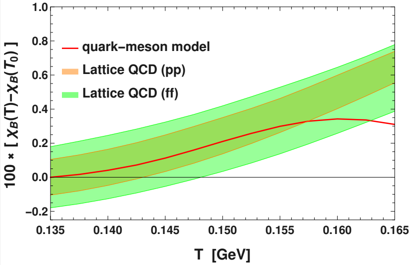
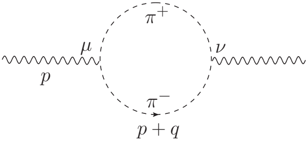


Contribution of π-vector meson loop

- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.



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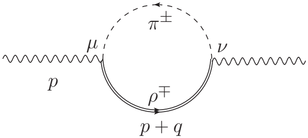

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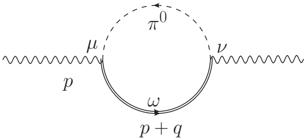

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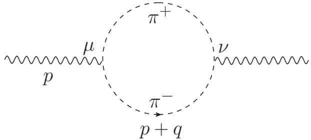

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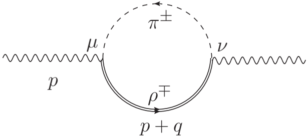

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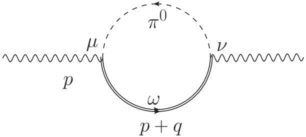

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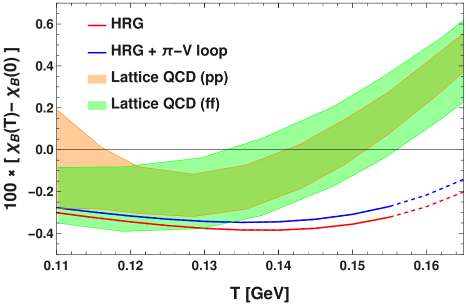

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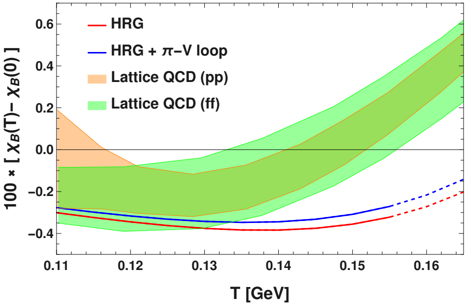

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- **The quark-meson model can successfully describe lattice data for χ_B while simultaneously reproducing $\chi_{\mathcal{BB}}$ and $\chi_{\mathcal{BS}}$**

Thank you !

Backup

Pauli-Villars Regularization

- Vacuum contribution to quark:

$$\chi_B^{\rm vac} = R_0 + R_1\kappa + R_2\kappa^2$$

where , e.g. $R_0 = \frac{4}{3}Q^2\pi^2\left[(q^2+2m^2)B_0(q^2,m^2,m^2) - 2A_0(m^2)\right]$

- A_0 and B_0 are the one-loop Passarino-Veltman (PaVe) functions

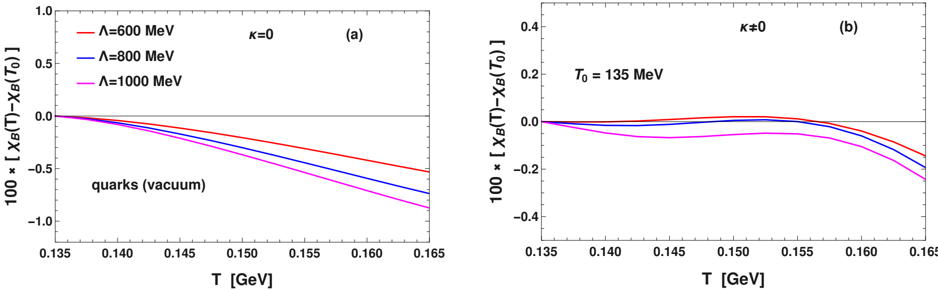
$$i\pi^2 A_0(m^2) = \int d^4p\, \frac{1}{p^2 - m^2 + i\epsilon}$$

$$i\pi^2 B_0(q^2, m_1^2, m_2^2) = \int d^4p \left[\frac{1}{(p+q)^2 - m_1^2 + i\epsilon}\,\frac{1}{p^2 - m_2^2 + i\epsilon}\right]$$

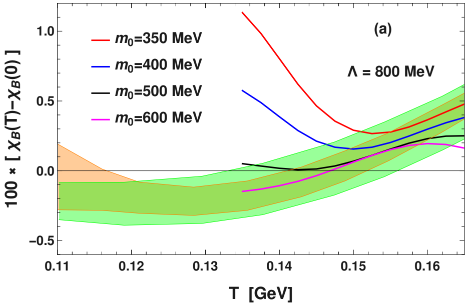
- Replace $F(\{m^2\})$ with

$$F^{\Lambda}(\{m^2\}) = F(\{m^2\}) - F(\{m^2+\Lambda^2\}) + \Lambda^2\frac{dF(\{M^2+\Lambda^2\})}{d\Lambda^2}$$

Effect of Λ



Fixing $m_q(T=0)$



We try several values of $m_l(0)$ ($m_s(0) = m_l(0) + 120$ MeV) and fix it to $m_l(0) = 600$ MeV $\longrightarrow$ best agreement with the data