ENTROPY AND MULTIPLICITY OF HADRONS IN THE HIGH ENERGY LIMIT WITHIN DIPOLE CASCADE MODELS

Bialasowka Seminar

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INTRODUCTION

- “The confinement of coloured quarks inside hadrons provides perhaps the most dramatic example of quantum entanglement that exists in Nature.”

PRL 124 (2020) 6, 062001

- Proton: entangled partonic system → possibly could explain:
  - Thermal distributions in small systems
  - Snowballs in hell
  - Eigenstate thermalization hypothesis
- How to observe it?
- von Neumann entropy of IS → proxy: Shannon entropy of FS
- pQCD calculations with maximal entropy agrees with DIS data
- More data in proton-proton → What is the variable?

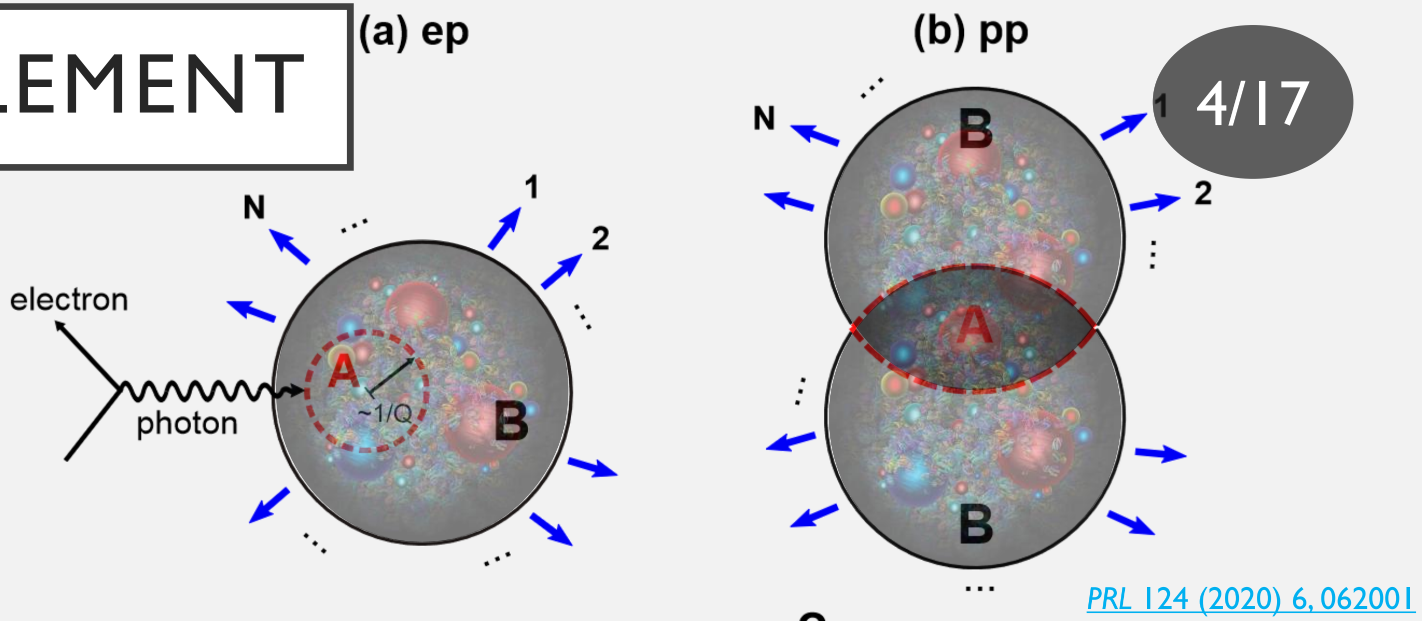
ENTANGLEMENT ENTROPY

Let’s consider on of the Bell states:  $= \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

- The full density matrix: $\rho_{AB} = |$  $\rangle\langle$  $|$
- Let’s trace-out B : $\rho_A = Tr_B(\rho_{AB}) = \cdots = \frac{1}{\sqrt{2}}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) = \frac{1}{2}\mathbb{I}_2 = \rho_B$
- Now, calculate the entanglement entropy: $S(\rho_A) = -Tr\big(\rho_A \ln(\rho_A)\big) = \cdots = \ln(2)$
- In general, for states in n-dimensional Hilbert space $\mathcal{H}$: $S_{max}(\rho) = \ln(\dim(\mathcal{H}))$.
- Equivalent to the Gibbs – Boltzmann entropy definition

INITIAL STATE ENTANGLEMENT

- Collision: sampling
- Observable: final state distribution of charged particles
- Maximally entangled: partonic microstates have equal probability

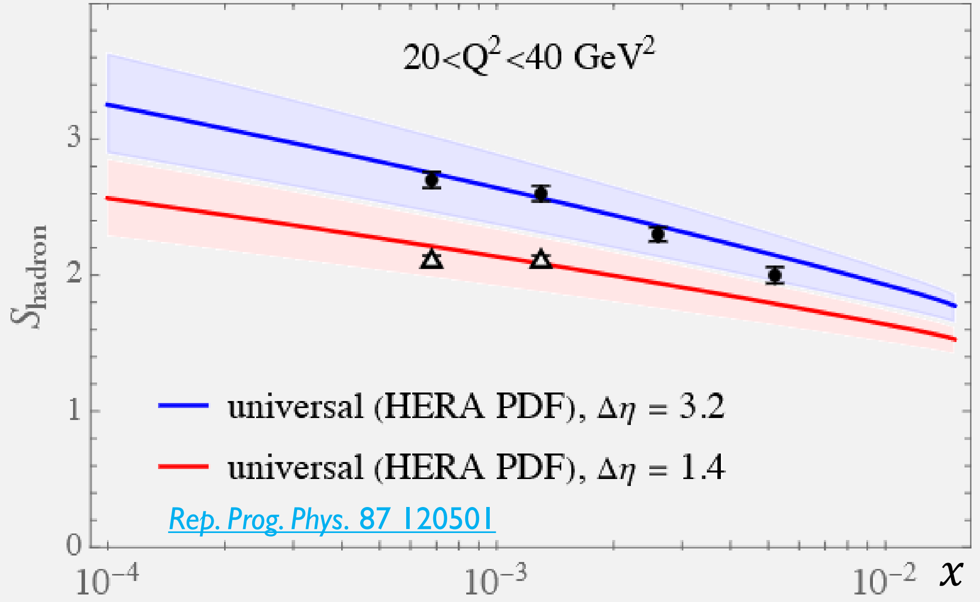


$$S_A = -tr[\hat{\rho}_A \ln \hat{\rho}_A] = S_B = -tr[\hat{\rho}_B \ln \hat{\rho}_B] \stackrel{?}{=} S_{hadron} = -\sum P(N) \ln P(N)$$

$$P(N) \sim Uniform(N)$$

$$S = \ln(N(x, Q^2)) = \ln\left(\frac{2}{3}(xq(x, Q^2) + xg(x, Q^2))\right)$$

Eur.Phys.J.C 82 (2022) 2, 111

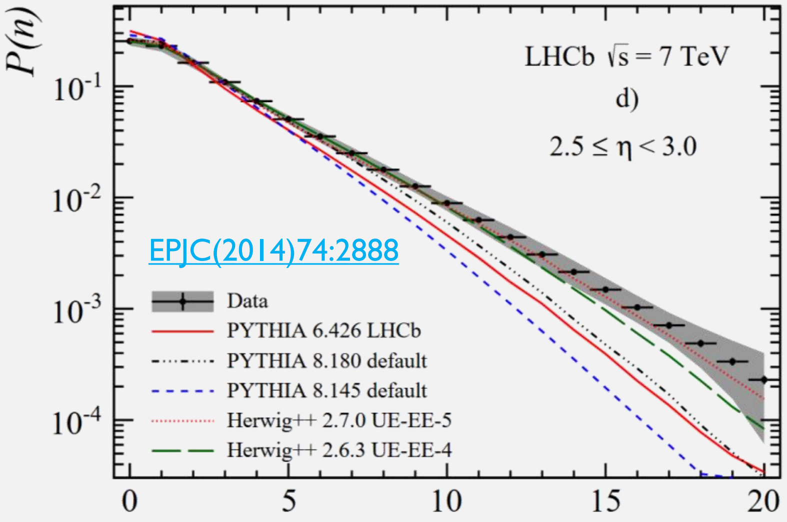


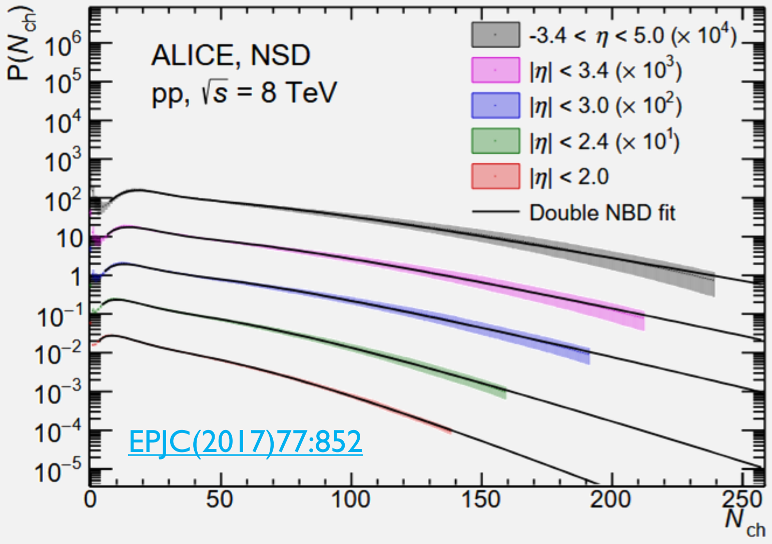
- pQCD agrees with HERA DIS data

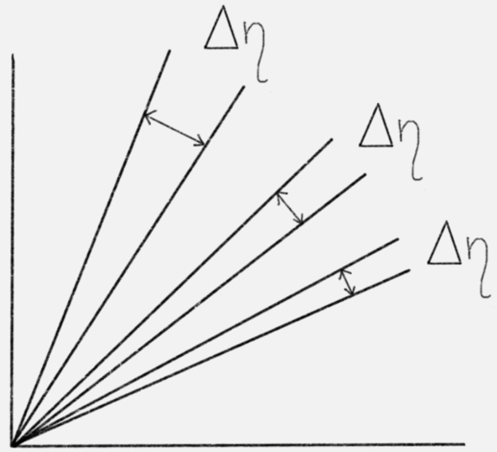
conjecture: $\underbrace{S_{parton}}_{initial\ state} = \underbrace{S_{hadron}}_{final\ state}$

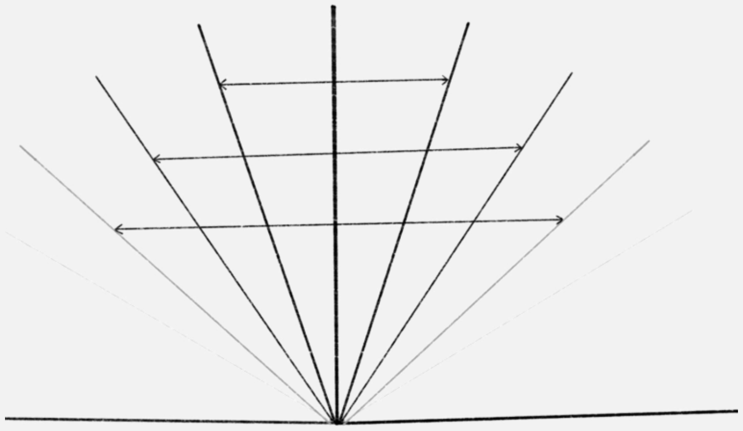
FINAL STATE DISTRIBUTION

- Principle of maximal entropy can be applied (later)
- Technical: how the rapidity defined in experiment?
- $P(n)$ measured
  - moving rapidity window (LHCb, H1)
  - opening rapidity window (ALICE, CMS, STAR)
- Cross-experimental comparison $\boldsymbol{S(\ln\langle n\rangle)}$
- Calculable directly from data
  - $\ln\langle n\rangle = \ln \sum_n nP(n)$
  - $S = -\sum_n P(n) \ln P(n)$









Illustrations: Anita Czinder

THEORY COMPARISON: DIPOLE MODELS

- 1 D Mueller model, branching partons, no recombination

$$\partial_y P_n(y) = -\alpha n P_n(y) + \alpha(n-1)P_{n-1}(y)$$

- Solution: $P_n(y) = \frac{1}{C}e^{-\alpha y}\left(1 - \frac{1}{C}e^{-\alpha y}\right)^{n-1}$ ← **geometric distr.**
- Single fluctuating source produces particles in multiplicative cascade

- 1D generalized dipole model (PRD110(2024)8,085011)

$$\partial_y P_n(y) = -\alpha(n+2h)P_n(y) + \alpha(n-1+2h)P_{n-1}(y)$$

- Solution: $P_n(y) = \frac{\Gamma(2h+n)}{n!\Gamma(2h)}\left(\frac{1}{C}e^{-\alpha y}\right)^{2h}\left(1 - \frac{1}{C}e^{-\alpha y}\right)^{n}$ ← **NBD**

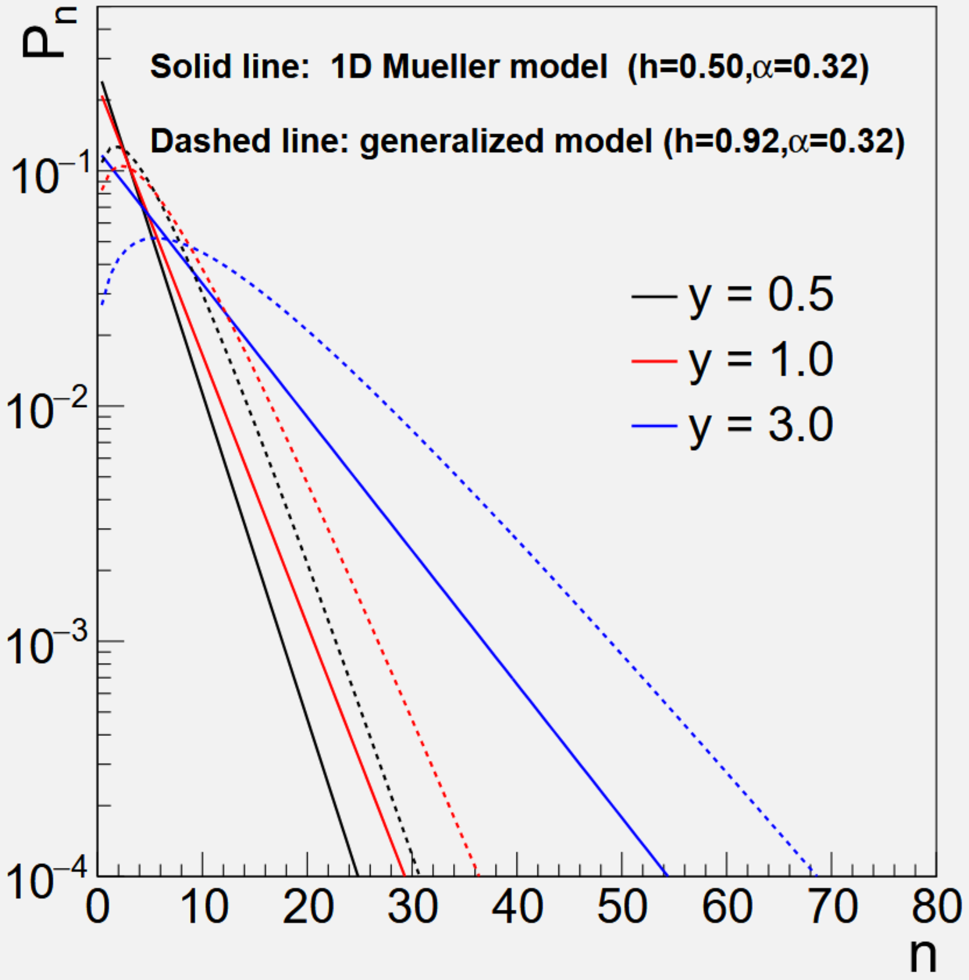
- Superposition of many fluctuating particle sources, semi-independent cascades

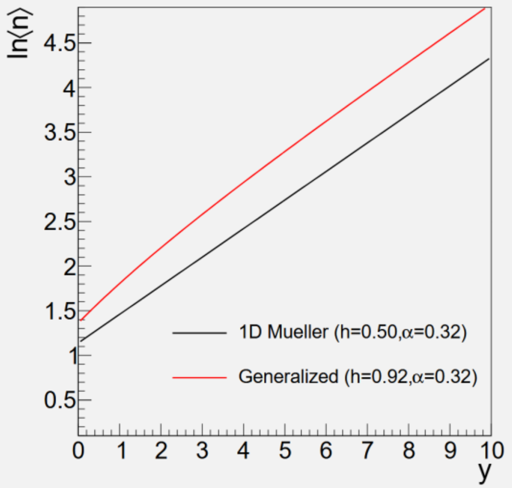
- New parameter h conformal weight of SL(2,C) (see the details in K. Kutak @ Bialasowka)

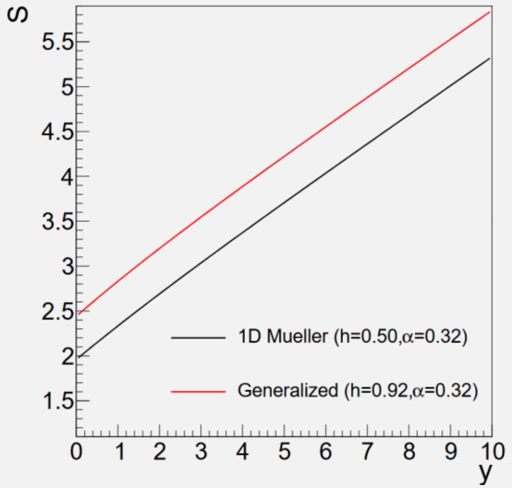
OBSERVABLES FROM THE MODELS

- h introduce NBD shape
- More general than the Mueller model
- $S(\ln\langle n\rangle)$ can be calculated and compared to data

- For now, it's pp from 200 GeV – 13 TeV



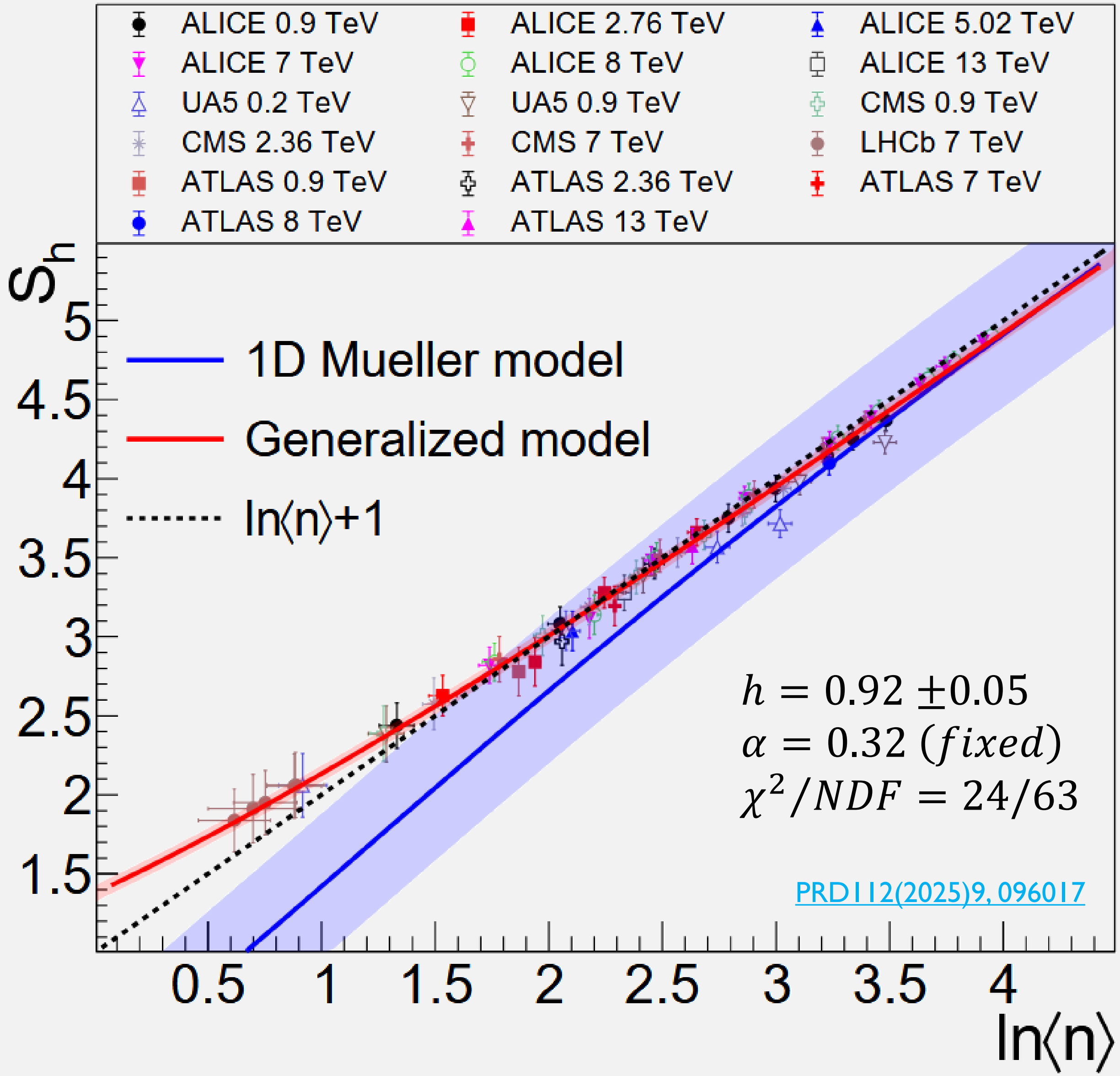




PRD112(2025)9, 096017

DATA COMPARISON

- Data from 200 GeV to 13 TeV
- With different
  - η definitions
  - distribution support (i.e. statistics)
- Observable: $S(\ln\langle n\rangle)$
- Deviation from the Mueller case at low multiplicities
- General model describes the data (no recombination!)
- Both converge to the maximal entropy case



ORIGIN OF NBD?

See also: K. Golec-Biernat @ Bialasowka

- Apply maximal entropy principle:
  - Probability distr. with maximum entropy and fixed mean: **geometric distribution**
  - Indistinguishable particles, combinatorial structure: **Poisson distribution**
- Event activity fluctuations: mean Poisson multiplicity is not constant but $g(\mu)$
- Apply MaxEnt on $g(\mu)$ with fixed logarithmic moment: **Gamma distribution**
- Poisson–Gamma mixture:

$$P(n) = \int_0^\infty P(n|\mu) g(\mu) = \int_0^\infty \frac{\mu^n e^{-\mu}}{n!} \cdot \frac{\mu^{k-1} e^{-\frac{\mu}{\theta}}}{\Gamma(k)\theta^k} = \frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)} \left(\frac{k}{k+\overline{n}}\right)^k \left(\frac{\overline{n}}{k+\overline{n}}\right)^n$$

- Event activity fluctuations → NBD and $k = \frac{\overline{n}^2}{Var(\mu)}$ measures the relative strength
- How to disentangle the effects?

LIMITS OF NBD

- $k \to 0^+$: extreme broad distribution $Var(n) \to \infty$
- $k = 1$:
  - reduces to the geometric distribution
  - single fluctuating particle cascade
- $1 < k < \infty$: most general case, $Var(n) = \overline{n} + \frac{\overline{n}^2}{\mathrm{k}}$
  - superposition of particle clusters (clan model)
- $k \to \infty$: limiting case when $g(\mu) \to \delta(\mu - \overline{n})$, $Var(n) = \overline{n}$
  - Independent particle emission, no clusters or correlations
- General equations with saturation have solutions in NBD form EPJC 85 (2025) 1215
- How's about it in smaller systems like DIS?

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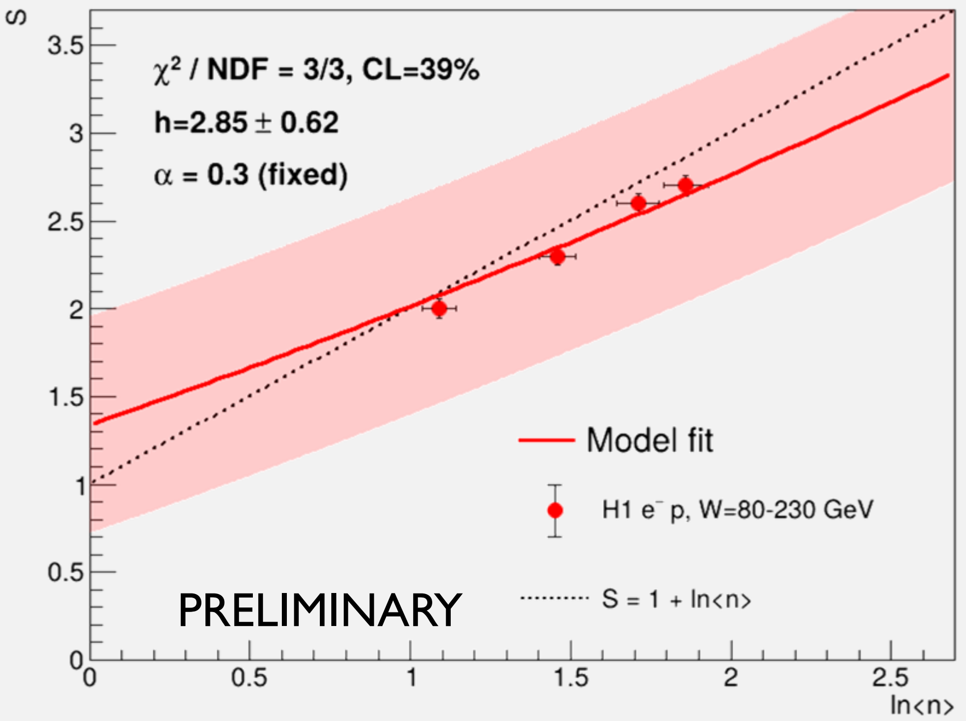

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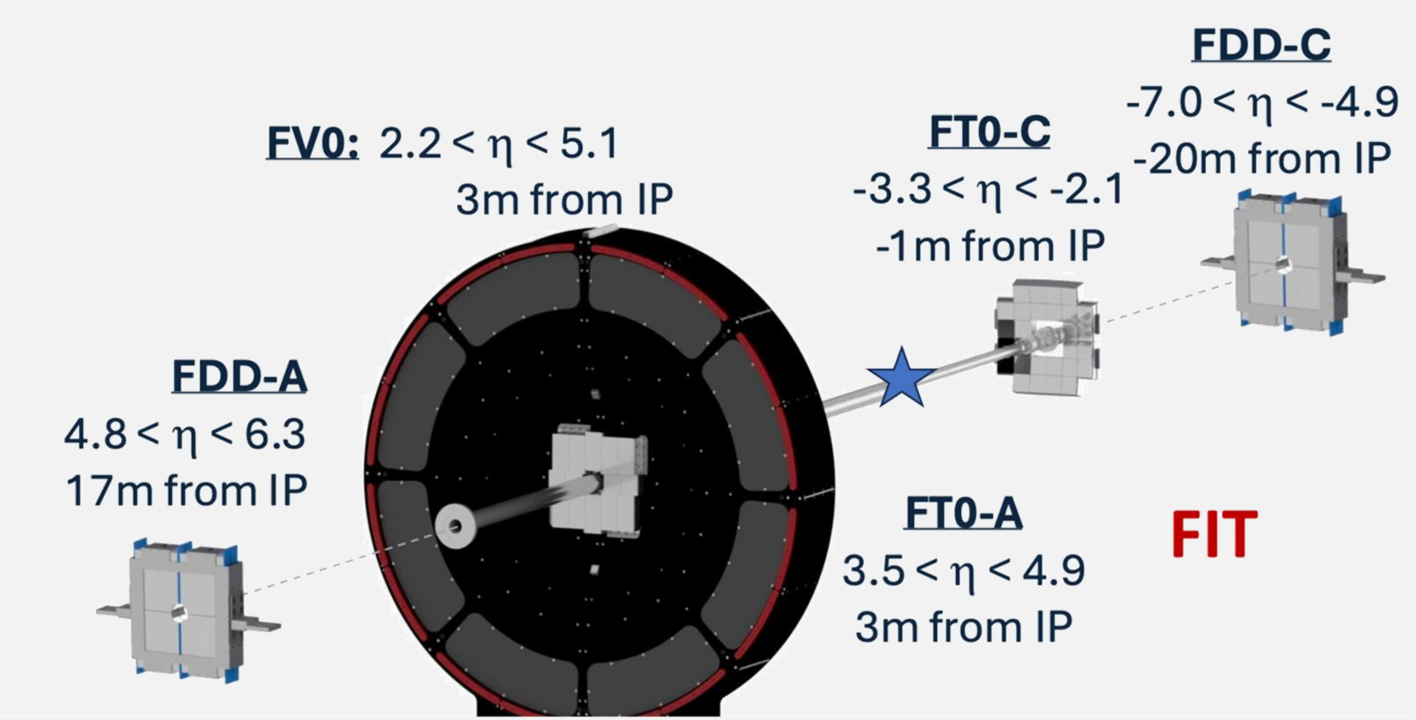
  - Independent particle emission, no clusters or correlations
- General equations with saturation have solutions in NBD form EPJC 85 (2025) 1215
- How's about it in smaller systems like DIS? More data?

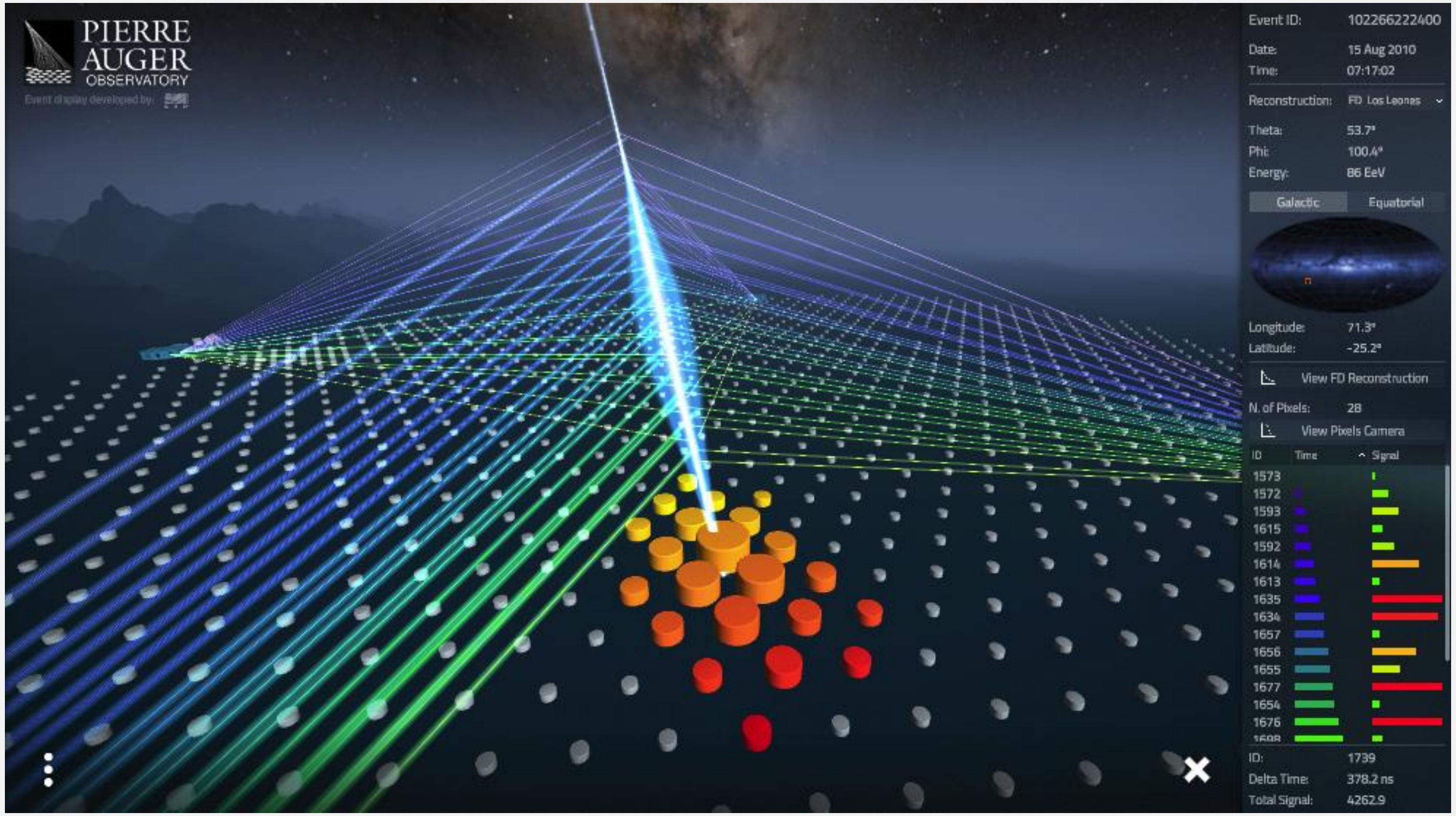
PROSPECTS FOR ALICE

- pp is not the best system, DIS is not available in LHC
- UPC is the best candidate in ALICE, a good job for FIT
- Analysis is ongoing, stay tune for the results



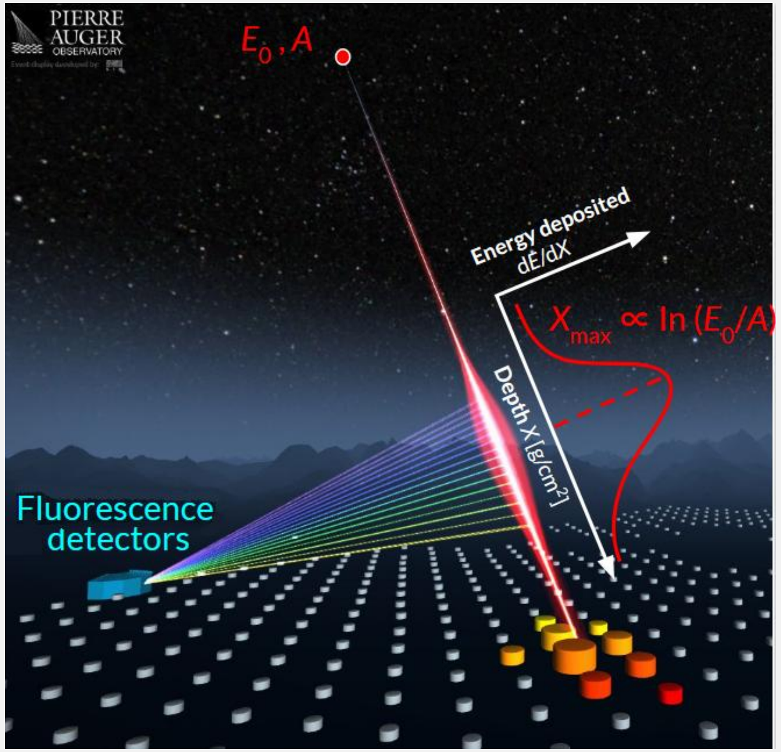
- But until then…

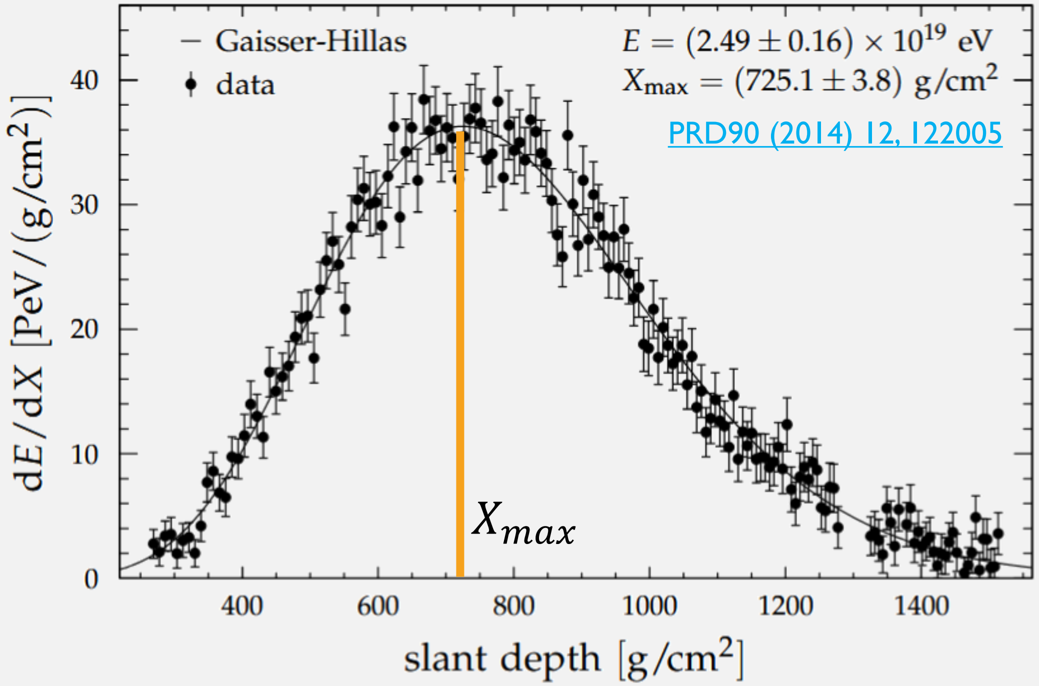
HIGHER ENERGIES? PIERRE AUGER OBSERVATORY!



- Ultra high energy cosmic rays in the hundreds of Eev range
- $\sqrt{s_{NN}} \geq 200$ TeV fix target
- Systems: pp, pHe, pN, pFe
- Shower (cascade) develops in the atmosphere

PRELIMINARY STUDIES AT 200 TEV



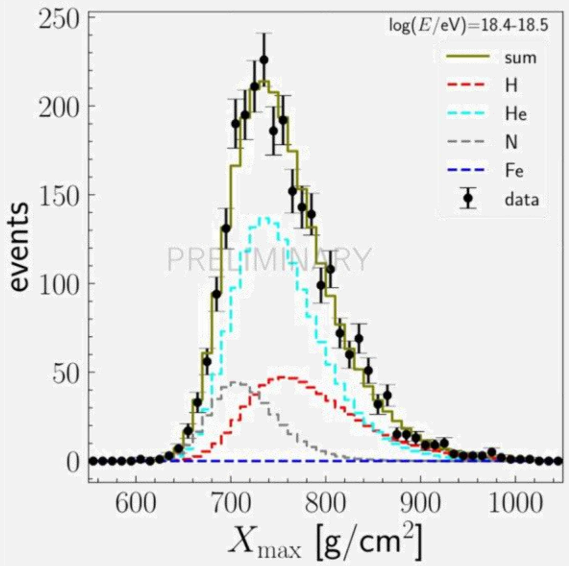


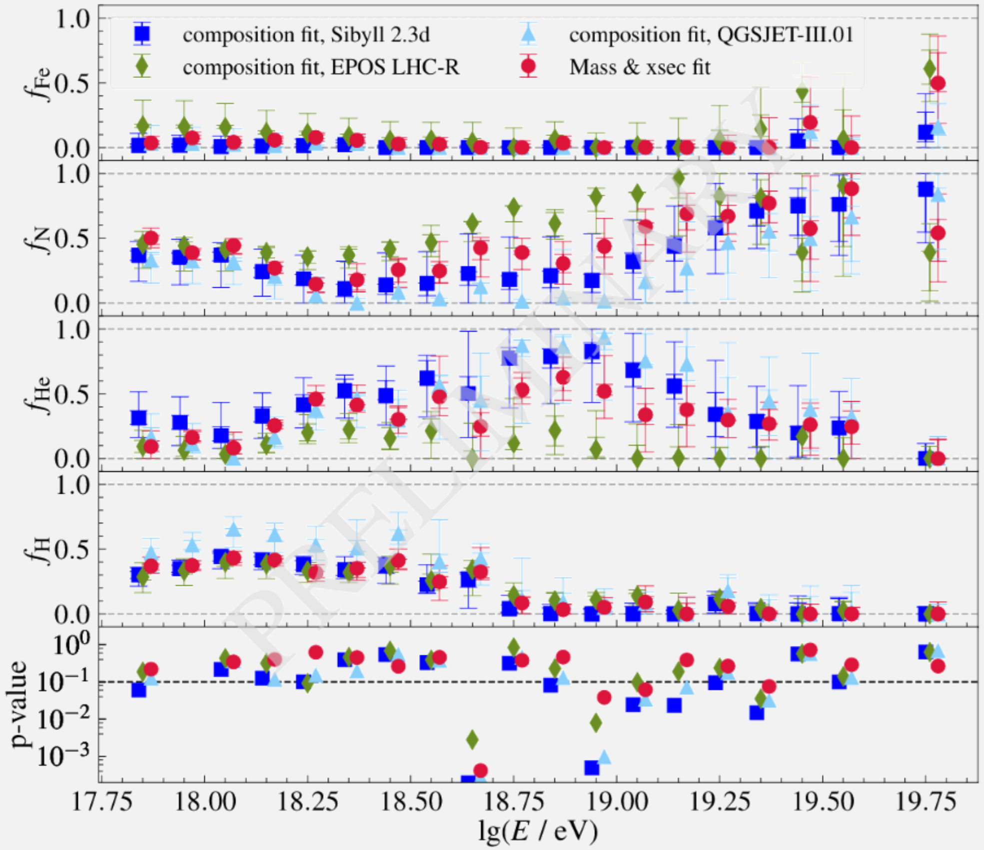
$$f_{GH}(X; X_{max}, X_0, \lambda) = \left(\frac{dE}{dX}\right)_{max} \left(\frac{X - X_0}{X_{max} - X}\right)^{\frac{X_{max}-X_0}{\lambda}} e^{\frac{X_{max}-X}{\lambda}}$$

- Observable: the place of the maximum development X_{max}
- Rapidity is not reconstructable

PRELIMINARY STUDIES AT 200 TEV

- $X_{max} \sim Gumbel(\mu, \beta)$
- Composite
- Understood based on MC
- Distr. pars energy dependent





- Modelling initial multiplicity with full MC or simple model

Let's assume:

- $\frac{dE}{dX}(X) \sim N(X)$ and $X \ll X_{max}$
- Ignore the compositeness of the measured distribution
- From $f_{GH}(X; X_{max}, X_0, \lambda)$ approximate N_0

PoS(ICRC2025)416

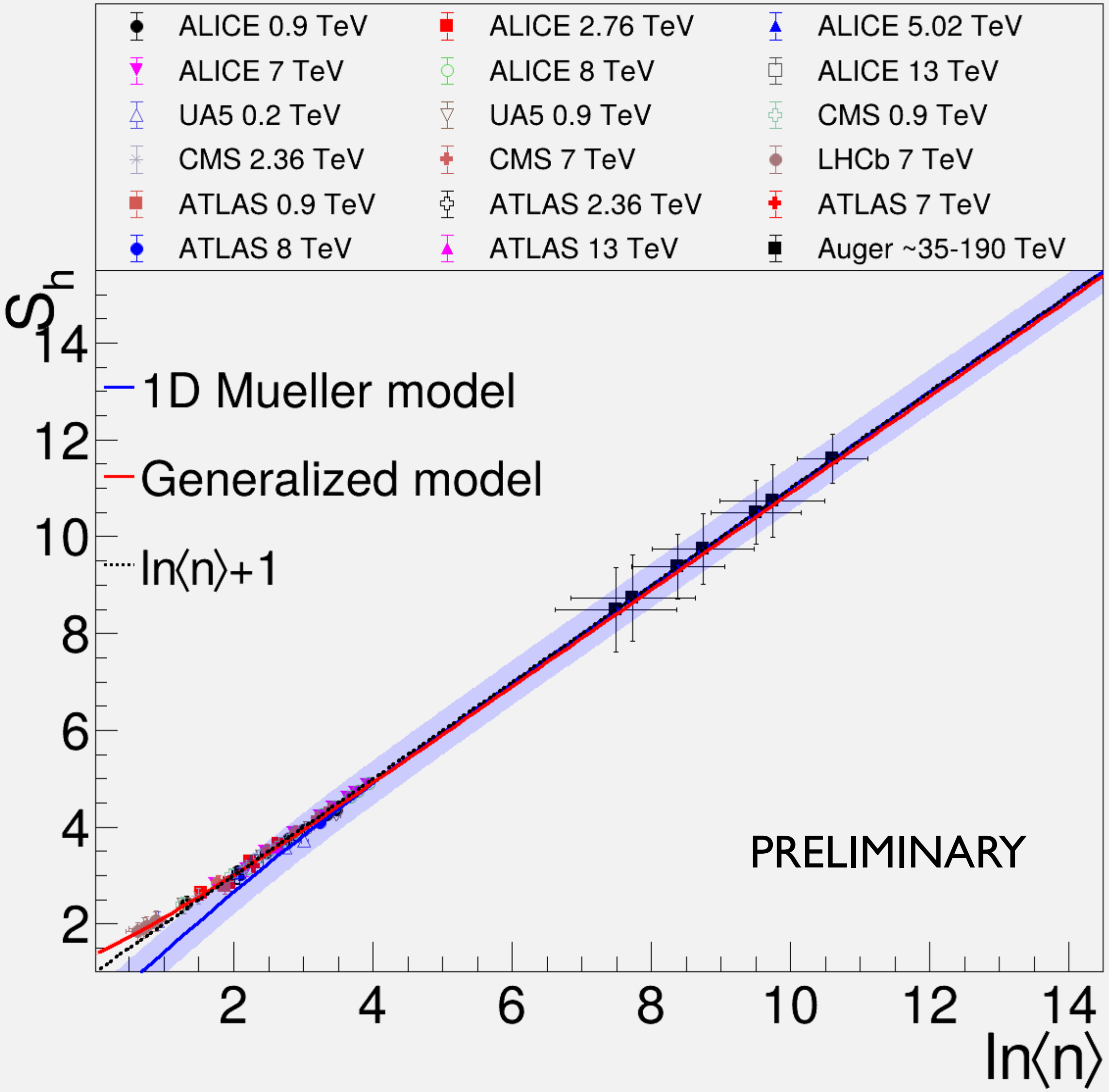
Thanks to Kevin Almeida Cheminant

HIGHER ENERGIES? (PRELIM!)

- Reconstruct the initial multiplicities from Gumbel-X_0 distributions as

$$N_0 \approx N_{max} e^{-X_{max}}$$

- Handy observable: $S(ln\langle n \rangle)$
- Next: more careful modelling with dedicated MC



SUMMARY AND OUTLOOK

- Final state entropy might be related to initial state entropy
- Models can describe data from ep and pp based on entanglement picture
- New observable: $S(\ln\langle n\rangle)$ described in pp by a general dipole model
- Plan to extend the studies
  - Available DIS data (H1 and even earlier), clarify the meaning of the h
  - Include recombination effects into the general model
  - Compare the model to ultra high energy cosmic ray data
- Open question: heavy ion, more complex systems? ETH?

THANK YOU FOR YOUR ATTENTION!