Partonic structure of the nucleon from Lattice QCD

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This project is supported by the National Science Center of Poland
SONATA BIS grant No 2016/22/E/ST2/00013 (2017-2022).

Outline of the talk

1. Introduction
2. PDFs: quasi and pseudo
3. Lattice impact on pheno?
4. Twist-3 PDFs
5. GPDs
6. Conclusions and prospects

Collaborators:

- C. Alexandrou (Cyprus)
- M. Bhat (Poznań)
- S. Bhattacharya (Temple)
- M. Constantinou (Temple)
- L. Del Debbio (Edinburgh)
- T. Giani (Edinburgh)
- K. Hadjiyiannakou (Cyprus)
- K. Jansen (DESY)
- A. Metz (Temple)
- A. Scapellato (Poznań)
- F. Steffens (Bonn)

Based on:

- M. Bhat, K. Cichy, M. Constantinou, A. Scapellato, "Parton distribution functions from lattice QCD at physical quark masses via the pseudo-distribution approach", arXiv:2005.02102
- S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, "One-loop matching for the twist-3 parton distribution $g_T(x)$", Phys. Rev. D102 (2020) 034005, "New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$", arXiv:2004.04130, "The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$", arXiv:2006.12347
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Unpolarized and helicity generalized parton distributions of the proton within lattice QCD", arXiv:2008.10573
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. D99 (2019) 114504
- K. Cichy, L. Del Debbio, T. Giani, "Parton distributions from lattice data: the nonsinglet case", JHEP 10 (2019) 137
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Light-Cone Parton Distribution Functions from Lattice QCD", Phys. Rev. Lett. 121 (2018) 112001,"Transversity parton distribution functions from lattice QCD", Phys. Rev. D98 (2018) 091503 (Rapid Communications)

Review of the field:

- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, AHEP 2019 (2019) 3036904, arXiv: 1811.07248

Nucleon structure

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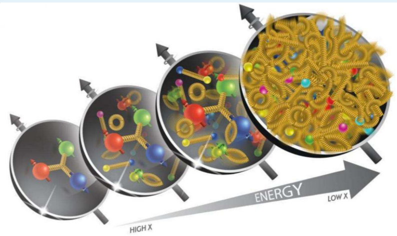

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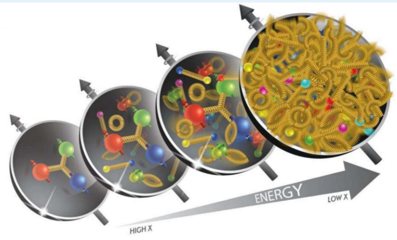

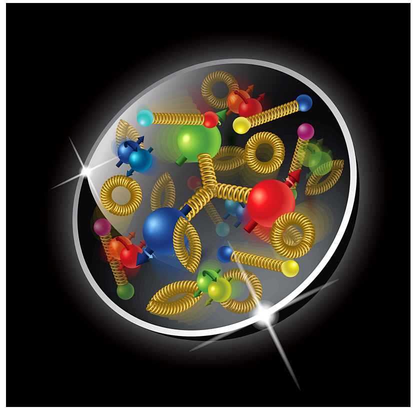

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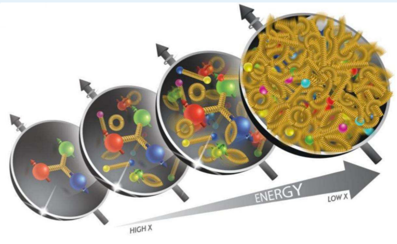

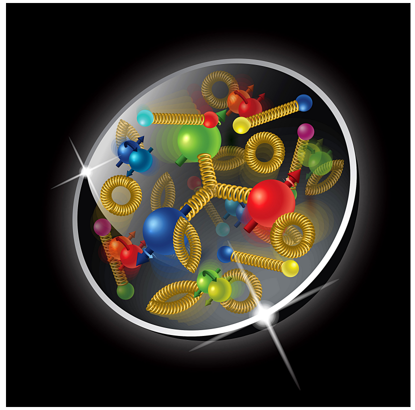

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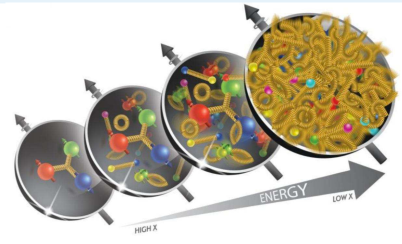

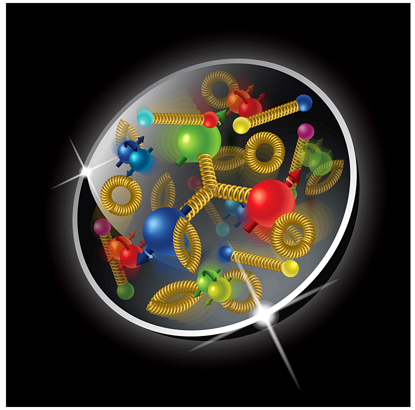

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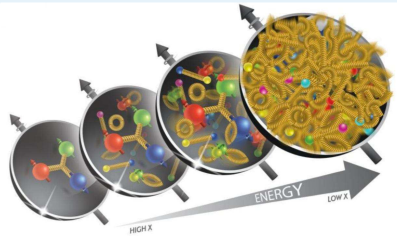

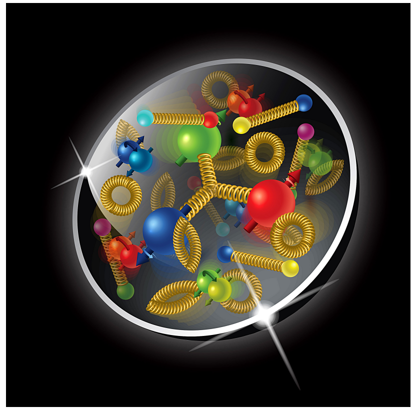

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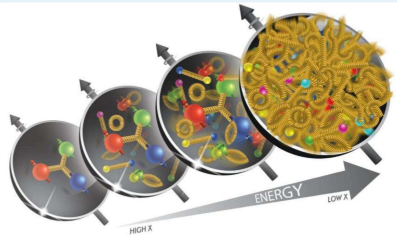

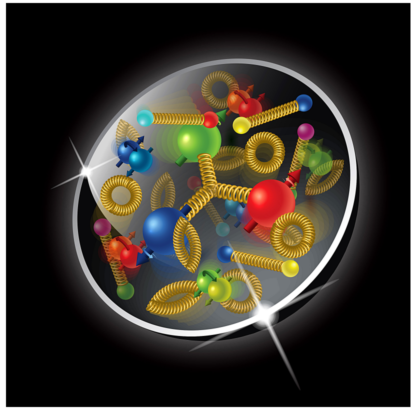

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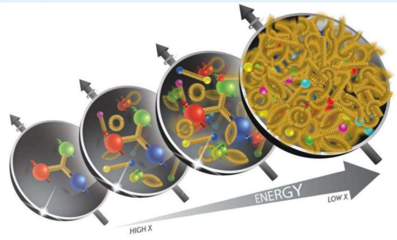

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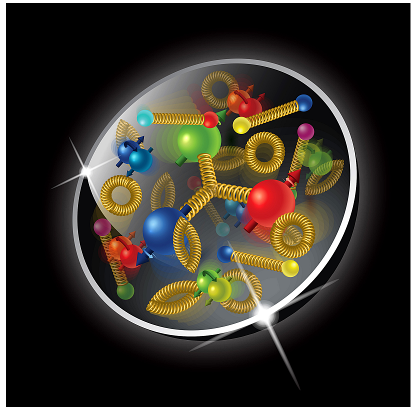

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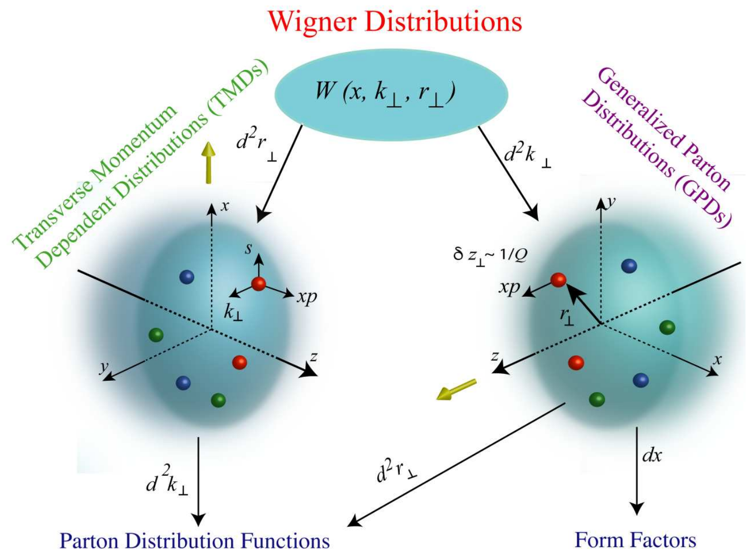

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Recently: new direct approaches to get x-dependence.

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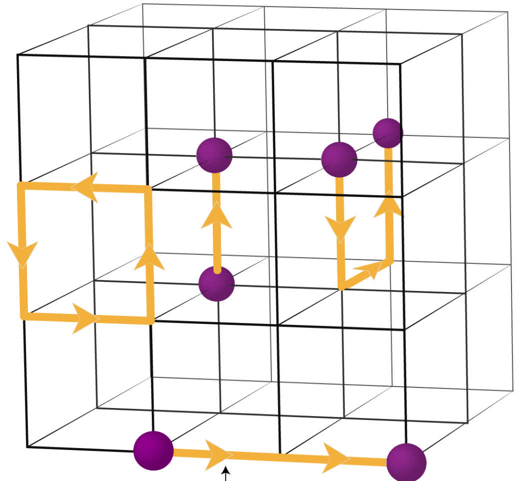

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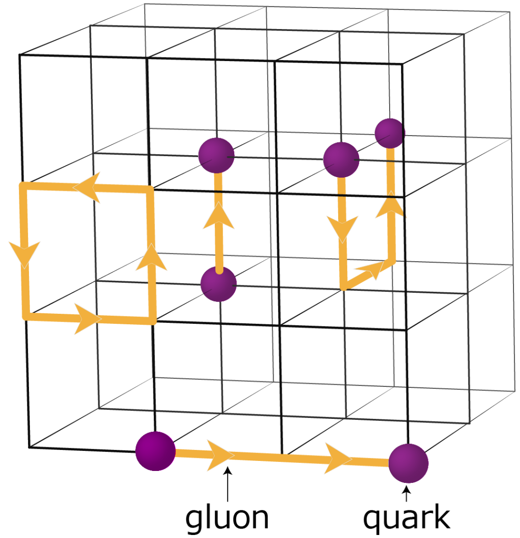

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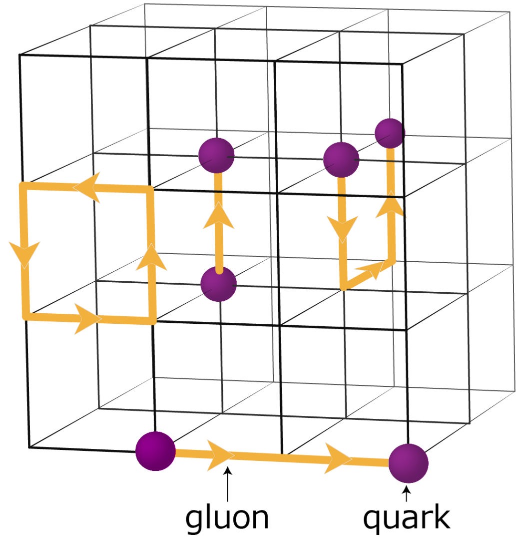

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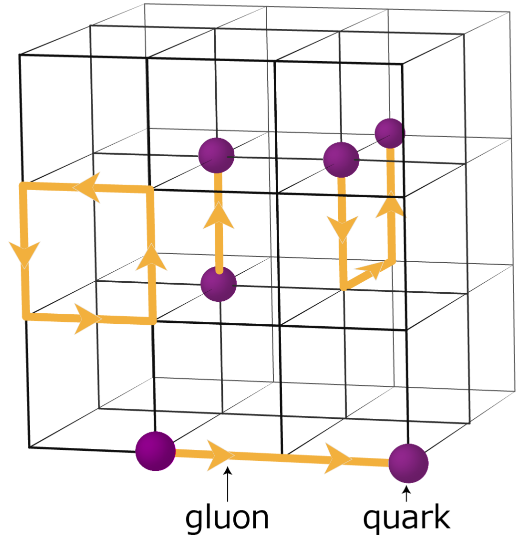

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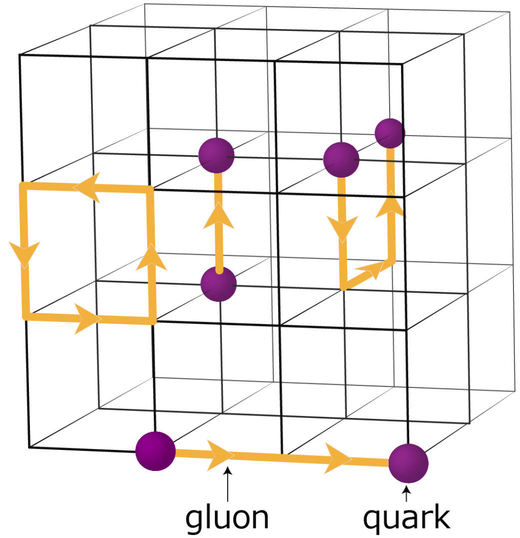

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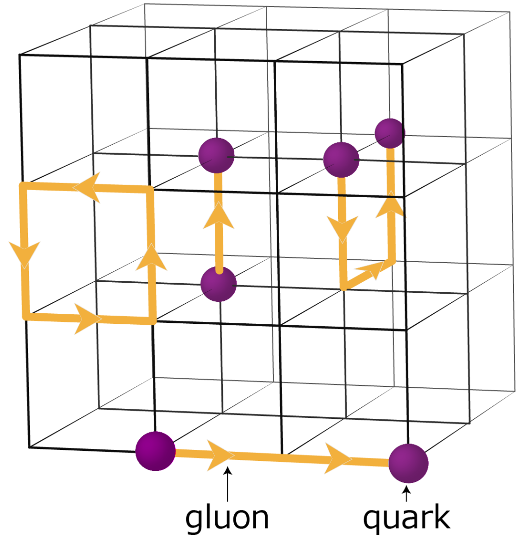

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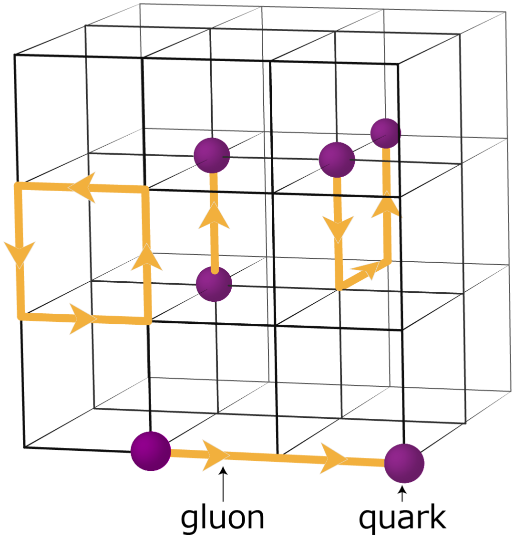

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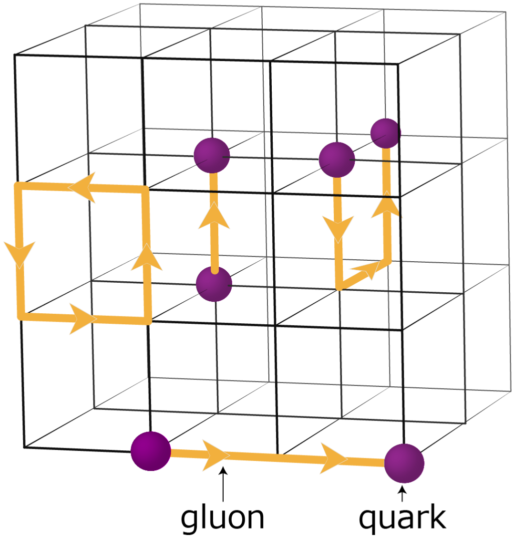

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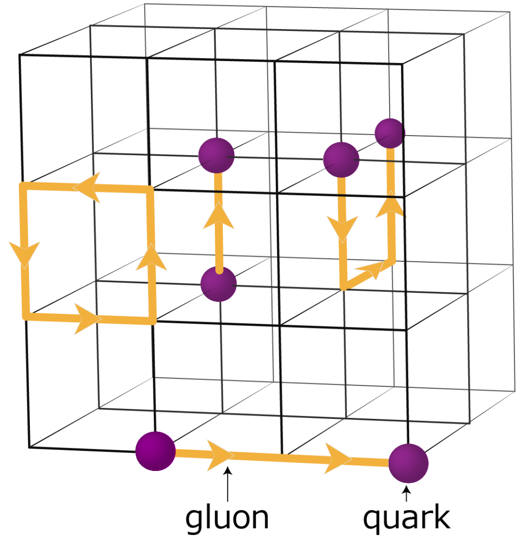

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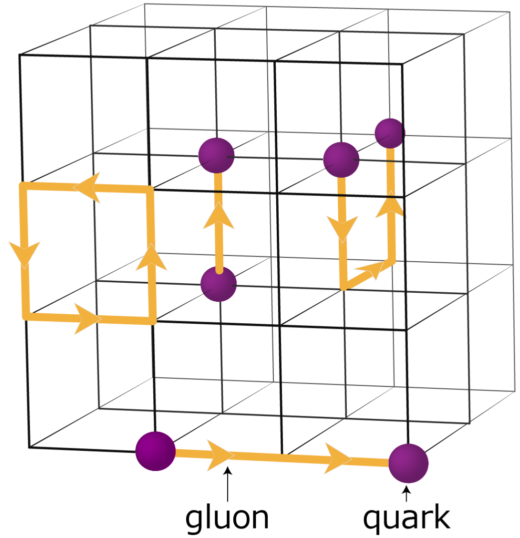
- **key aspect: control over various systematic effects**



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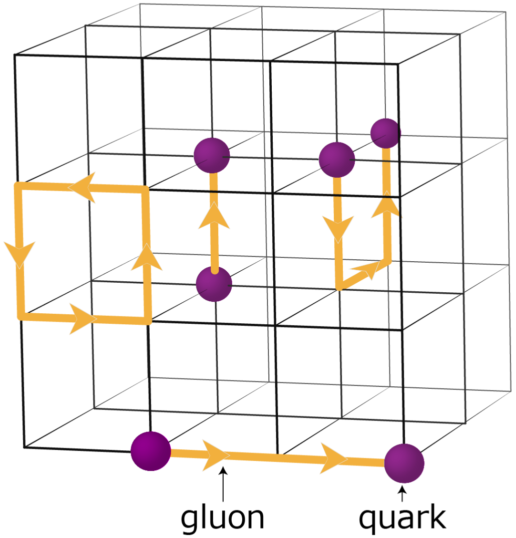
  - ⋆ ⇒ ∞-dim path integral → $10^8 - 10^9$-dim integral
- Monte Carlo simulations to evaluate the discretized path integral
- feasible, but still requires huge computational resources of $\mathcal{O}(1 - 1000)$ million core-hours, depending on the question asked
- formally, evaluation of a thermodynamic expectation value with respect to the Boltzmann factor $e^{-S_{\rm QCD}}$
- lattice regulates IR and UV divergences; the regulator needs to be removed ⇒ $L \to \infty$, $a \to 0$
- prior to regulator removal – (non-perturbative) renormalization
- **key aspect: control over various systematic effects**
- ⇒ exploratory studies vs. precision studies



Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}}=\int_{-1}^{1}\frac{dy}{y}\,C\left(\frac{x}{y},\mu_F,\mu_R\right)q(y,\mu_F),$$

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- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ, Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Overview of results from different approaches

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

- **Quasi-distributions**
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Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] **and Martha Constantinou**[2]

[1]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2]*Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

Adv. High Energy Phys. 2019 (2019) 3036904
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Recent update: M. Constantinou, proceedings of (would-be) plenary talk of LATTICE 2020, arXiv: 2010.02445
The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD

Quasi-PDFs

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X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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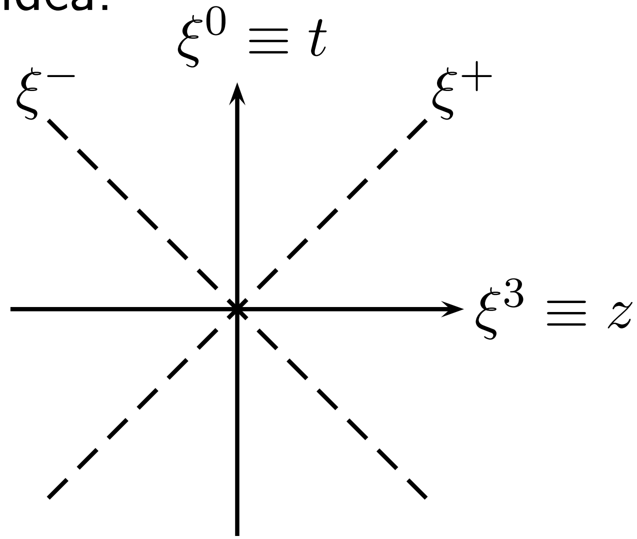

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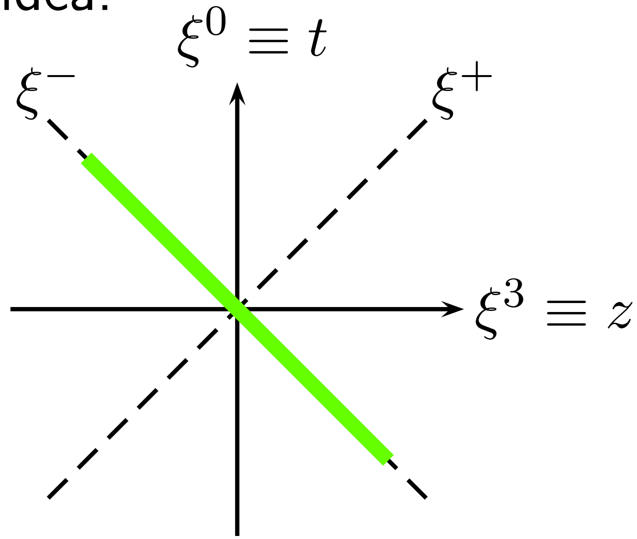

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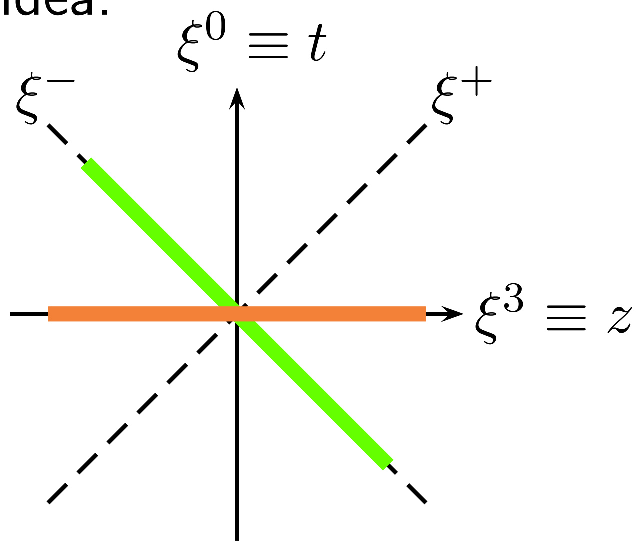

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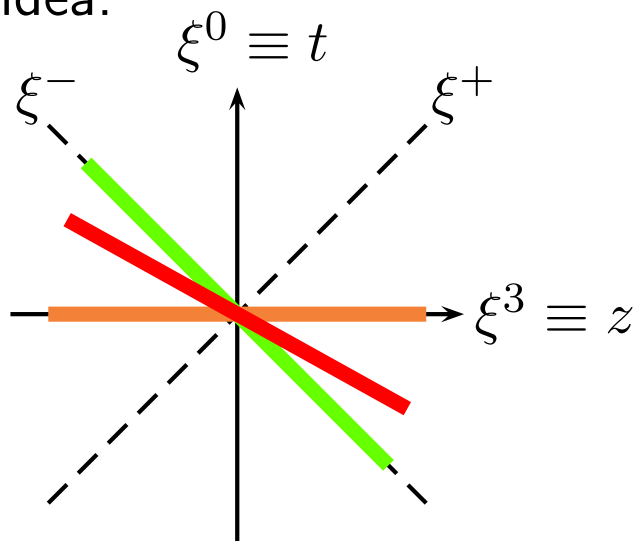

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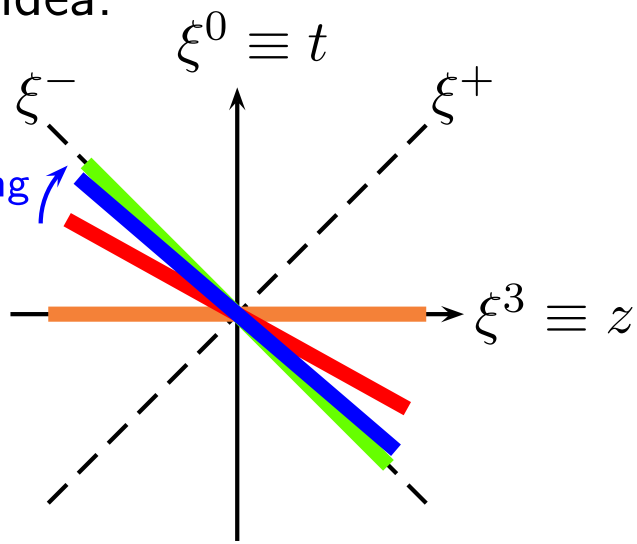

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X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

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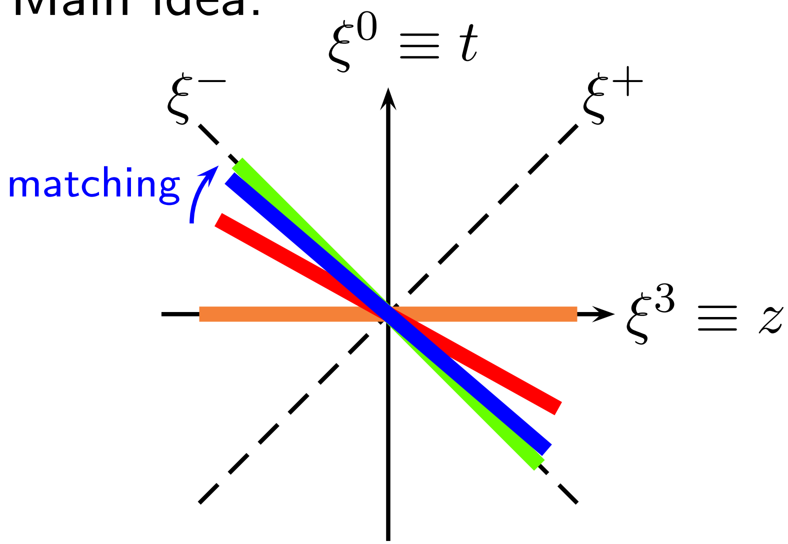

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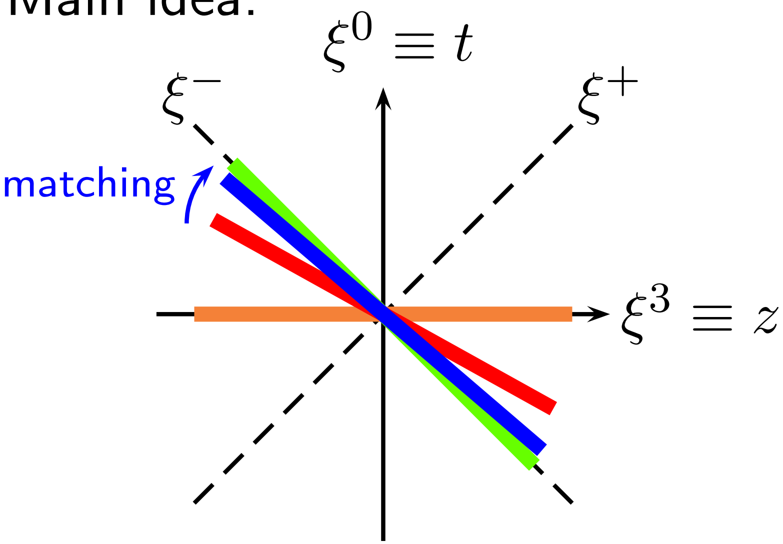

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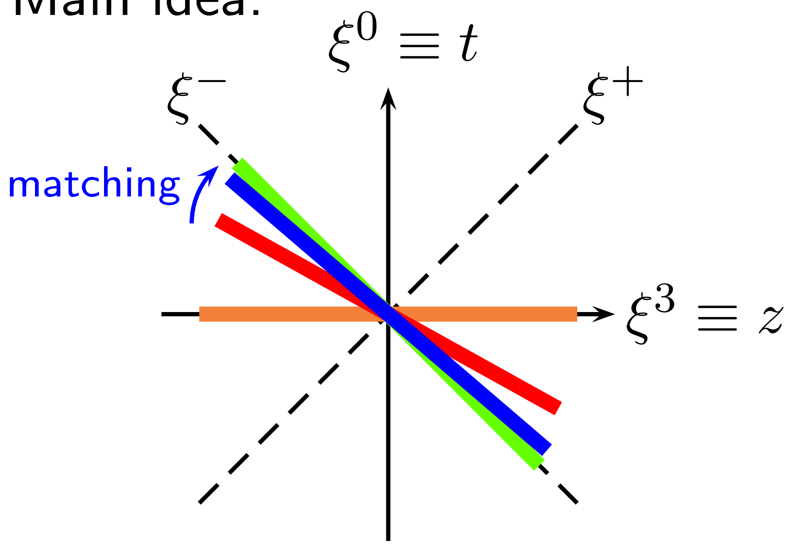

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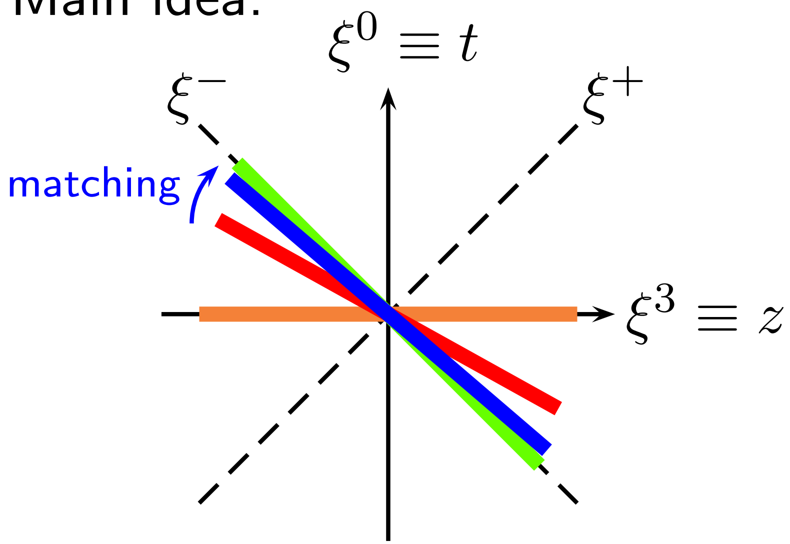

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A. Radyushkin, Phys. Rev. D96 (2017) 034025

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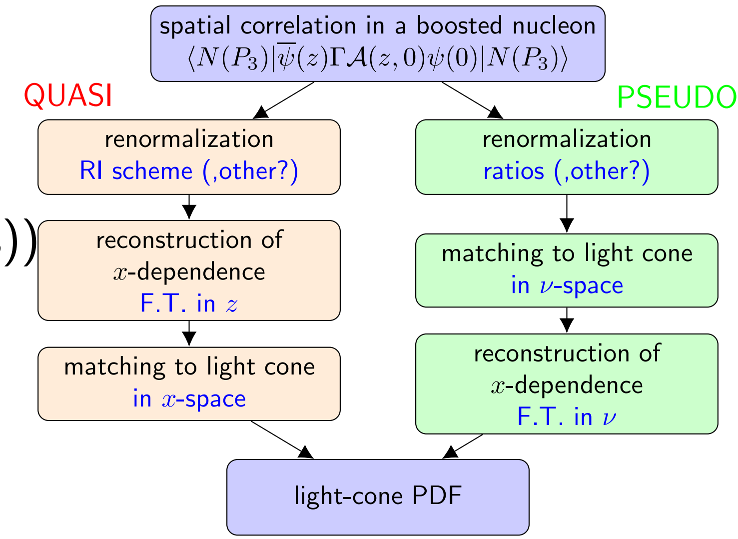
A. Radyushkin, Phys. Rev. D96 (2017) 034025



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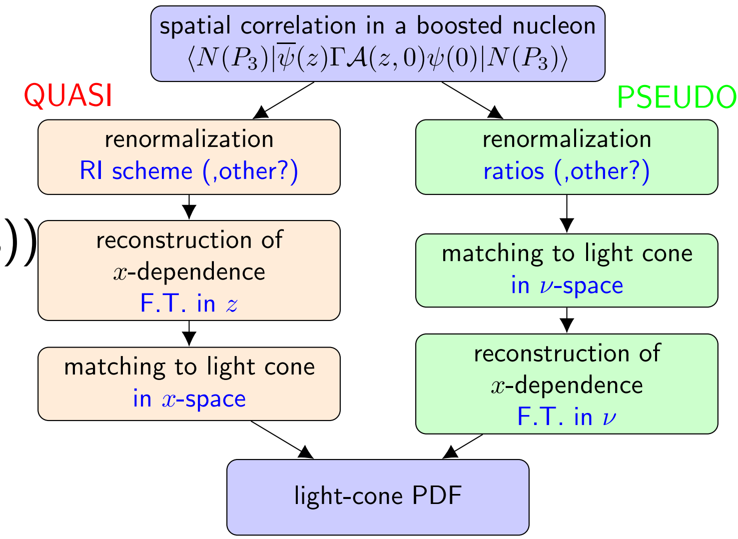

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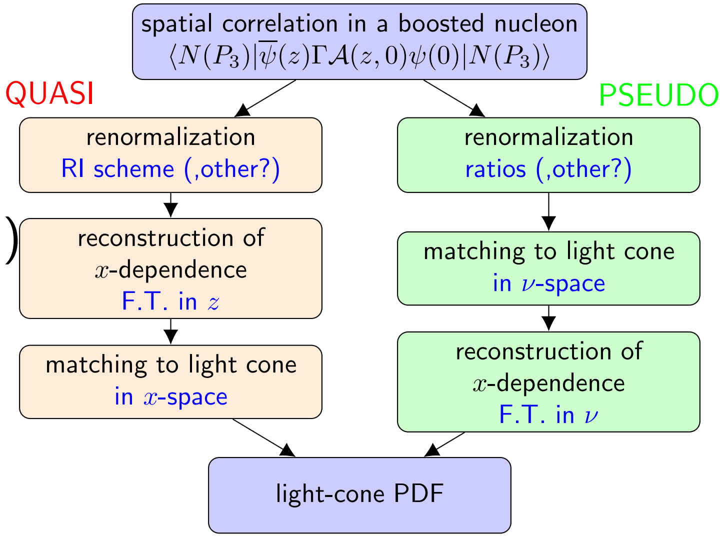

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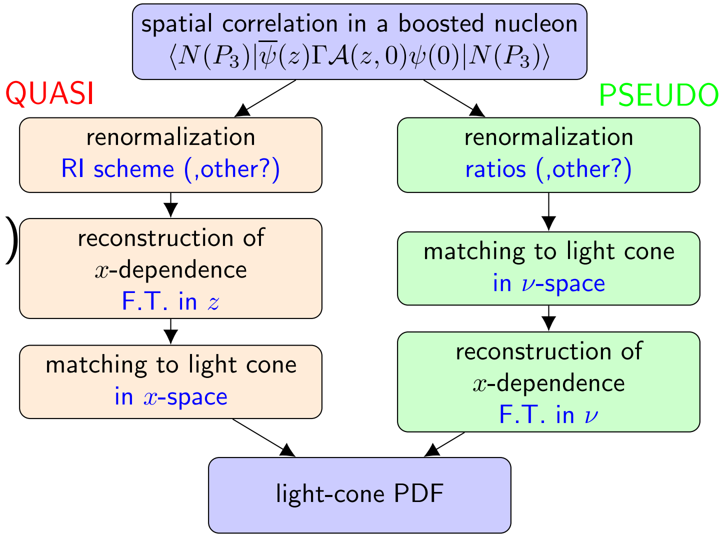

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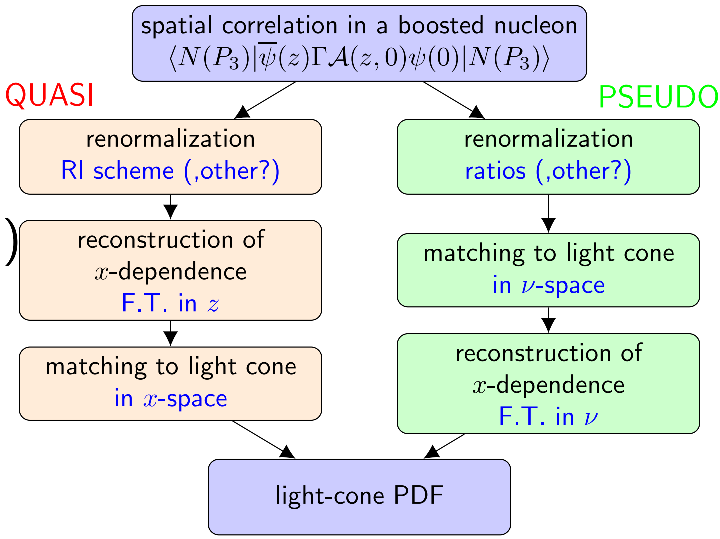

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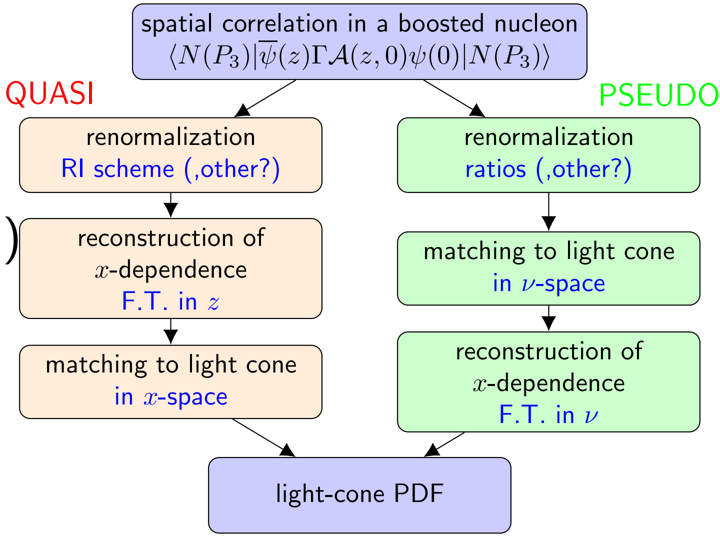

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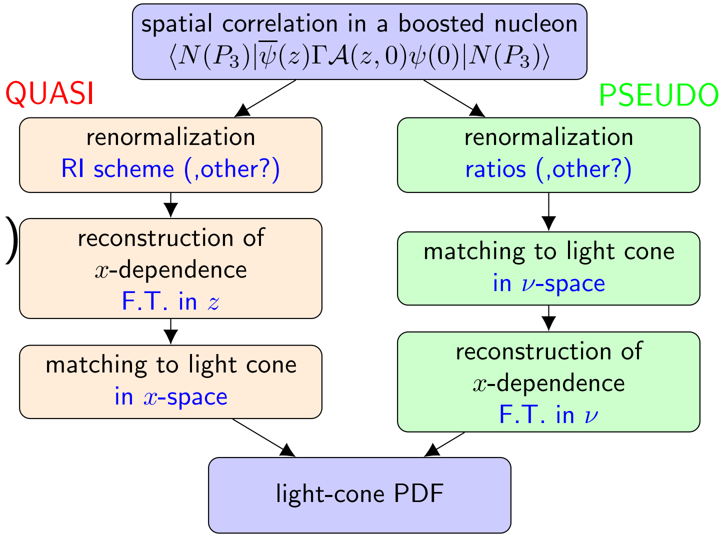

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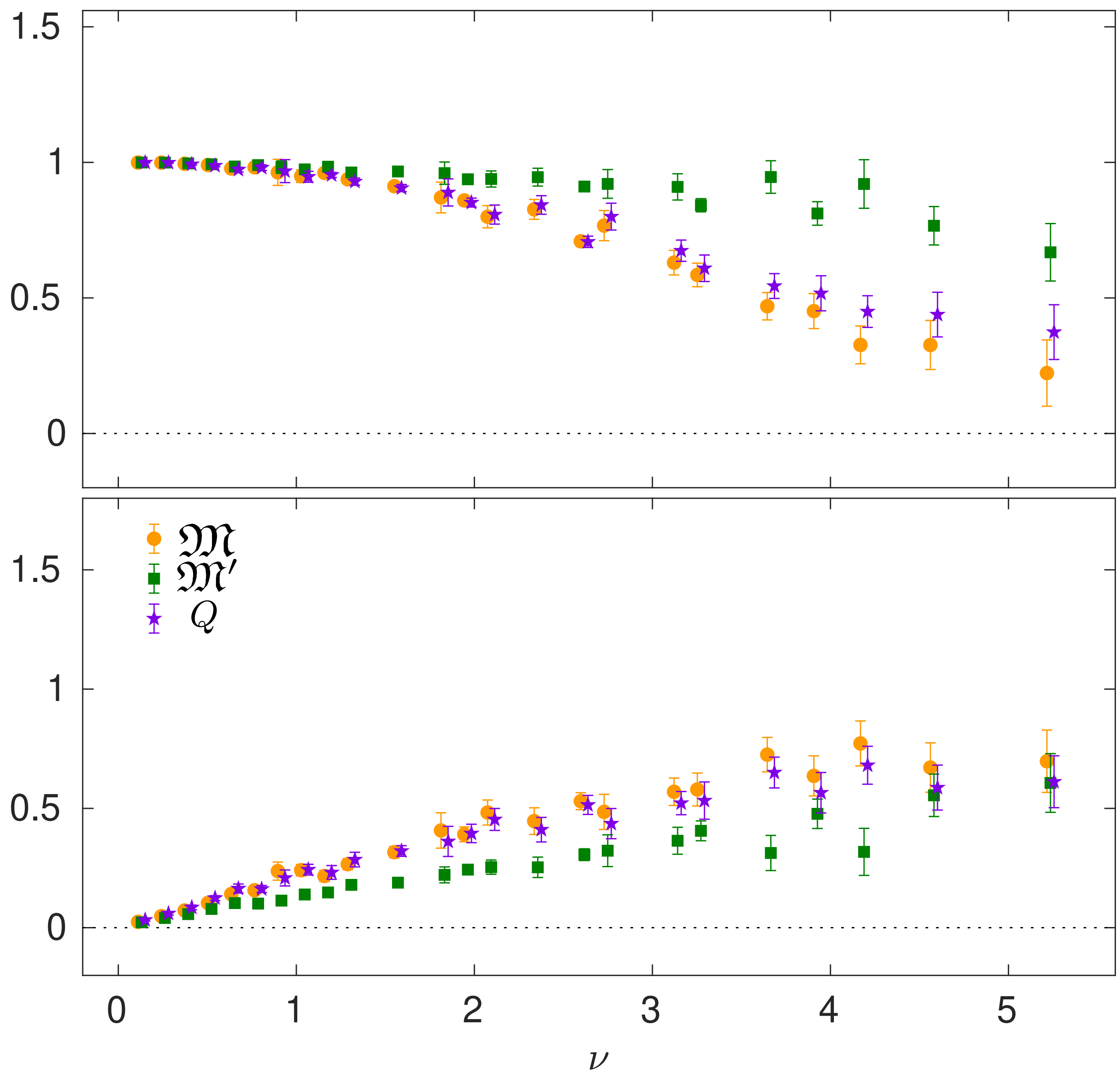
- Important: **inverse problem!** J. Karpie, K. Orginos, A. Rothkopf, S. Zafeiropoulos, JHEP 04 (2019) 057

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

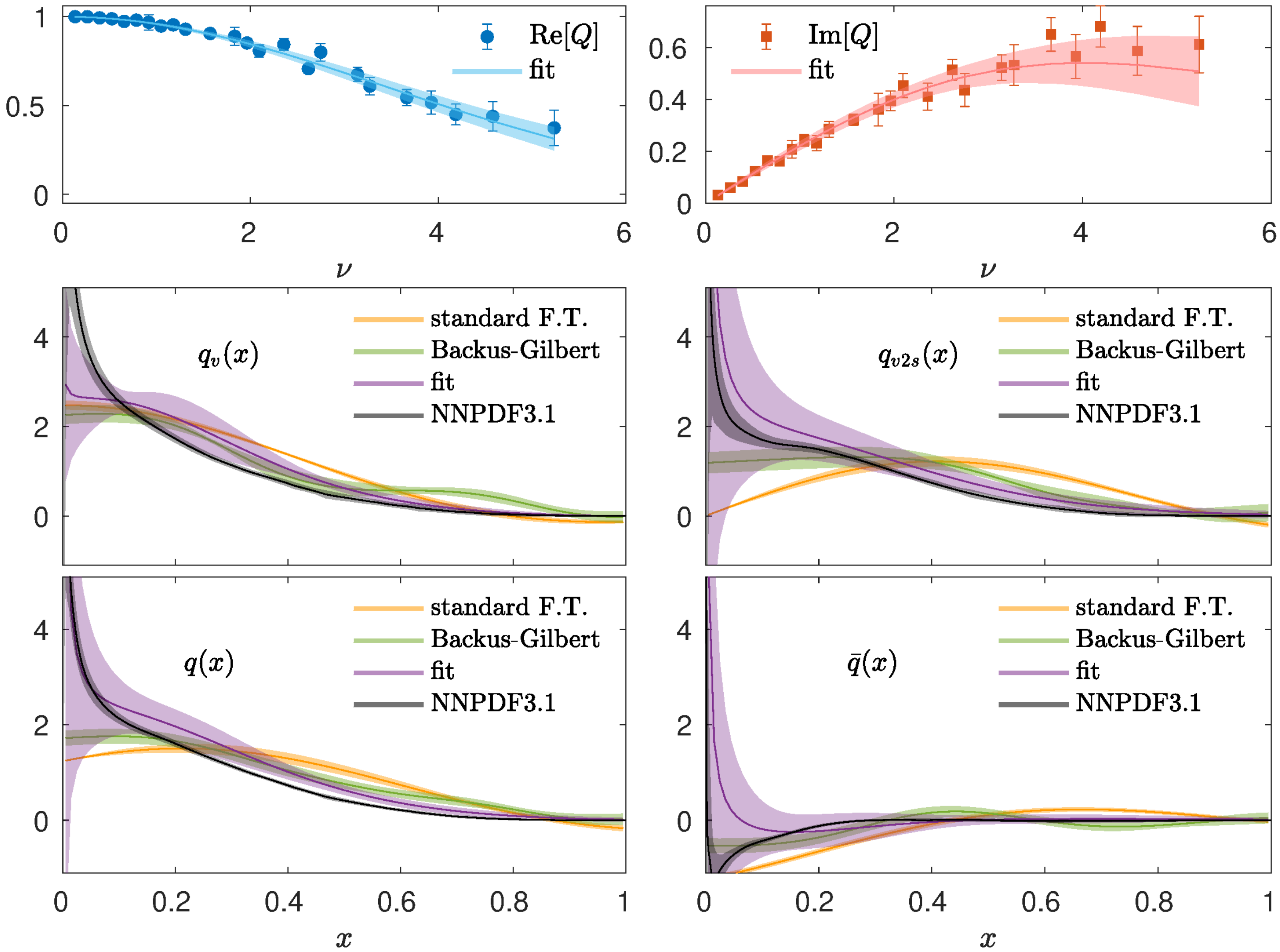
β=2.10, $c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm	
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

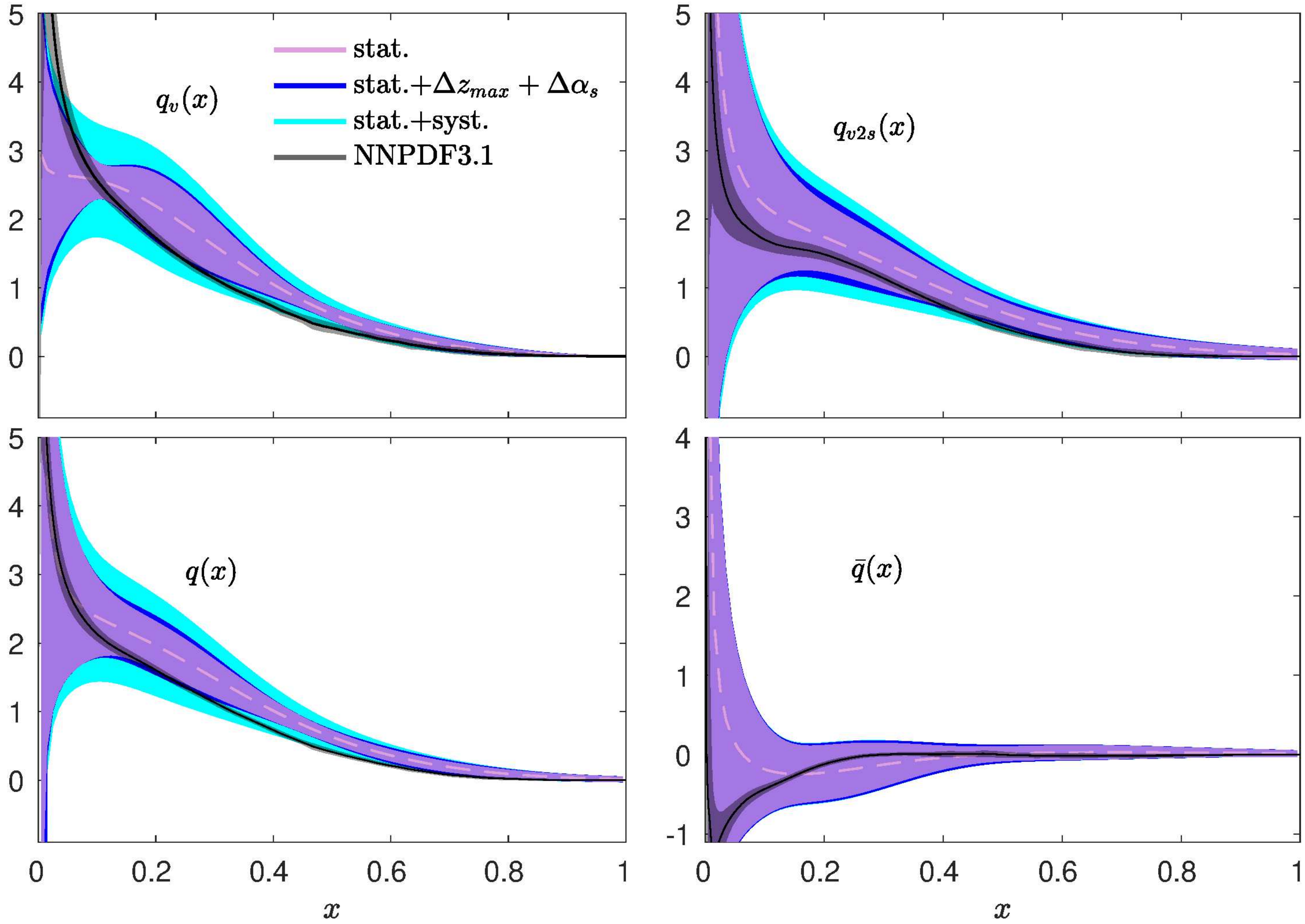
P_3	P_3 [GeV]	$N_{\rm confs}$	$N_{\rm meas}$
0	0	20	320
$2\pi/L$	0.28	19	1824
$4\pi/L$	0.55	18	1728
$6\pi/L$	0.83	50	4800
$8\pi/L$	1.11	425	38250
$10\pi/L$	1.38	811	72990

Reduced, evolved and matched ITDs

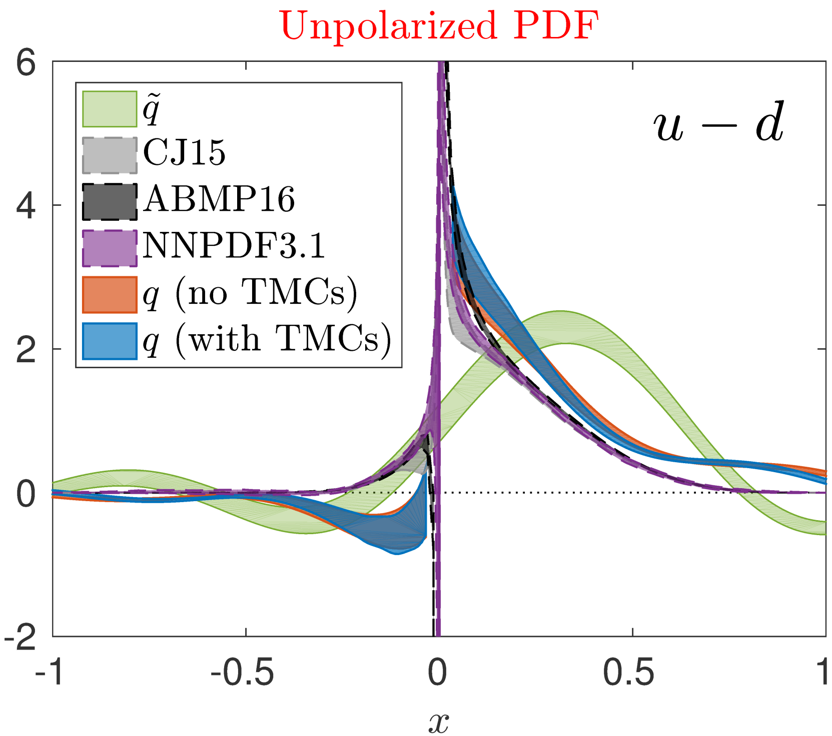


Reconstructed PDFs

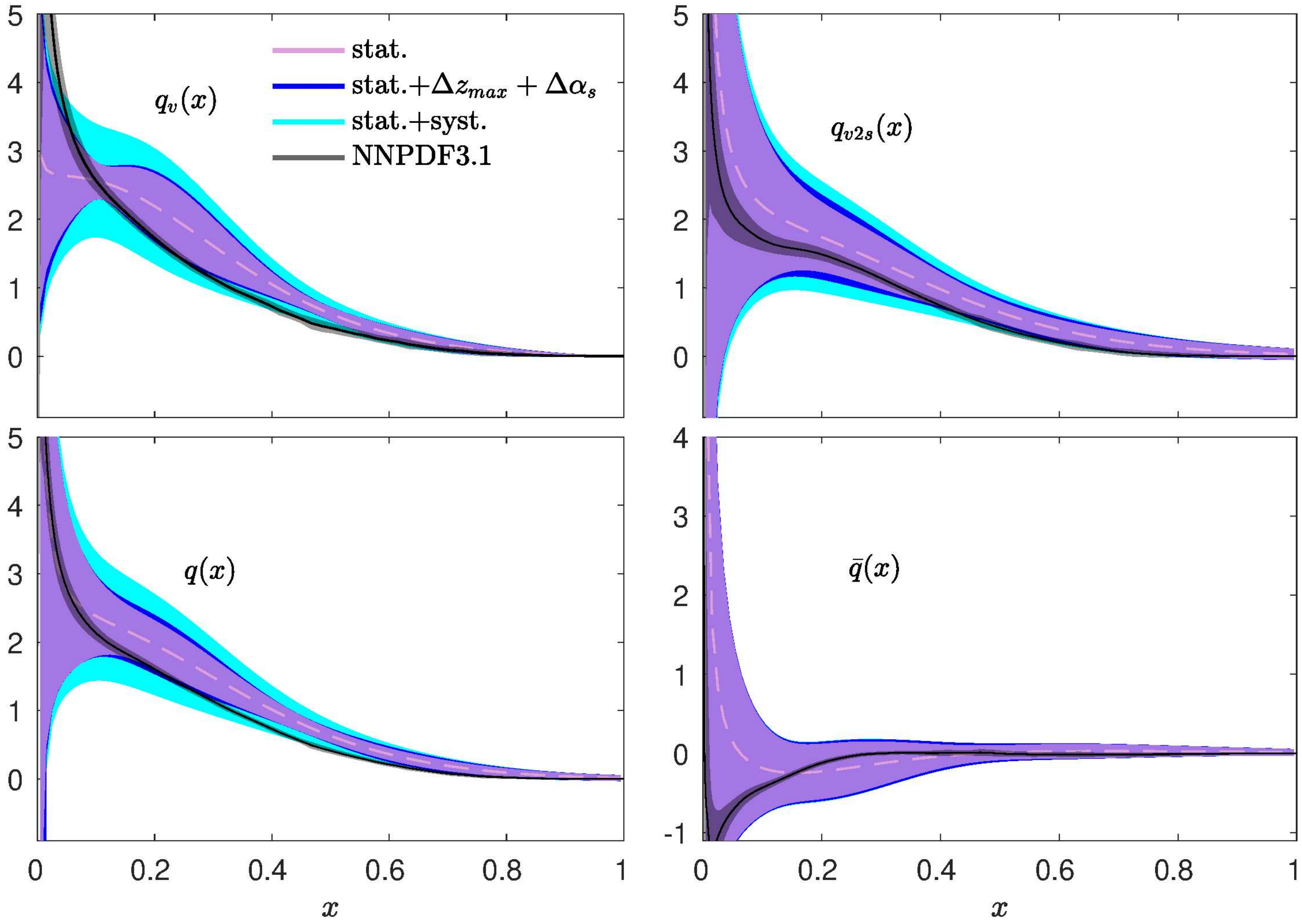



stat.
stat.+$\Delta z_{max} + \Delta\alpha_s$
stat.+syst.
NNPDF3.1
$q_v(x)$
$q_{v2s}(x)$
$q(x)$
$\bar{q}(x)$
x
x

Light-cone PDFs from pseudo and quasi

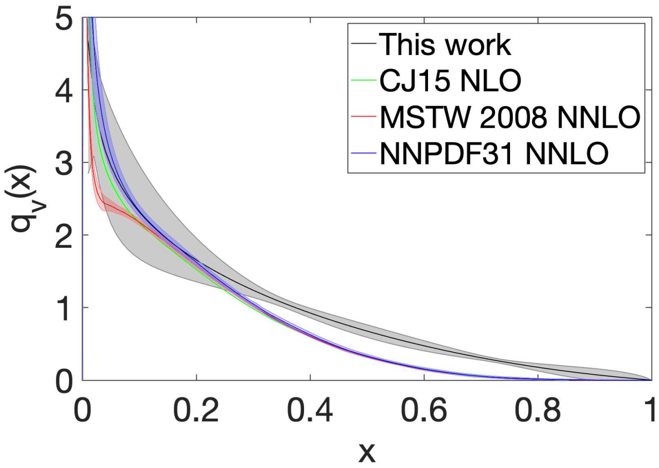


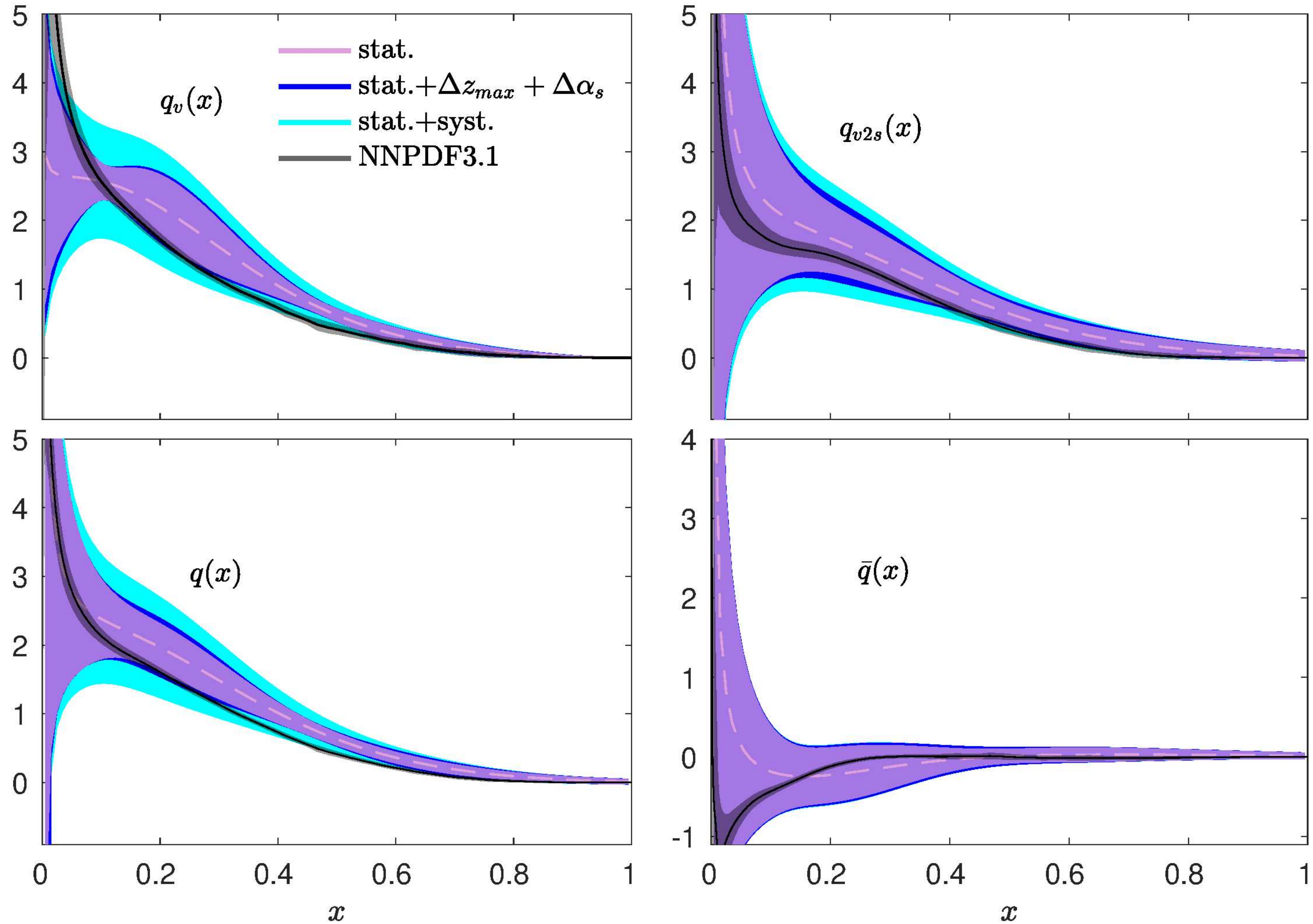
ETMC, Phys. Rev. Lett. 121 (2018) 112001
ETMC, Phys. Rev. D 99 (2019) 114504



ETMC, arXiv:2005.02102

Comparison with JLab





B. Joó et al., arXiv:2004.01687

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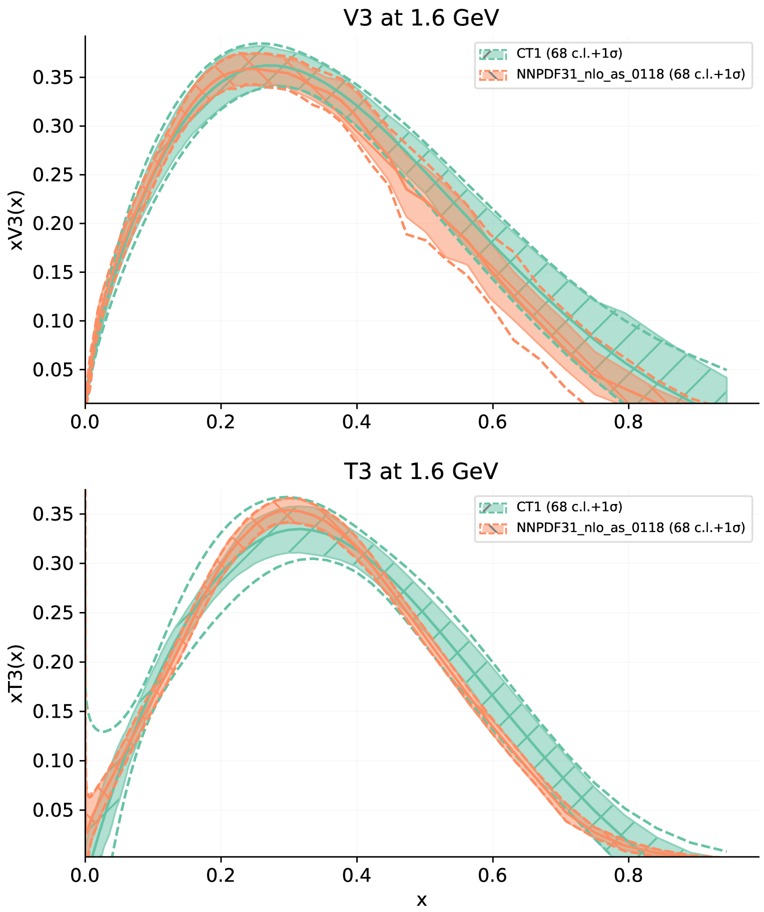

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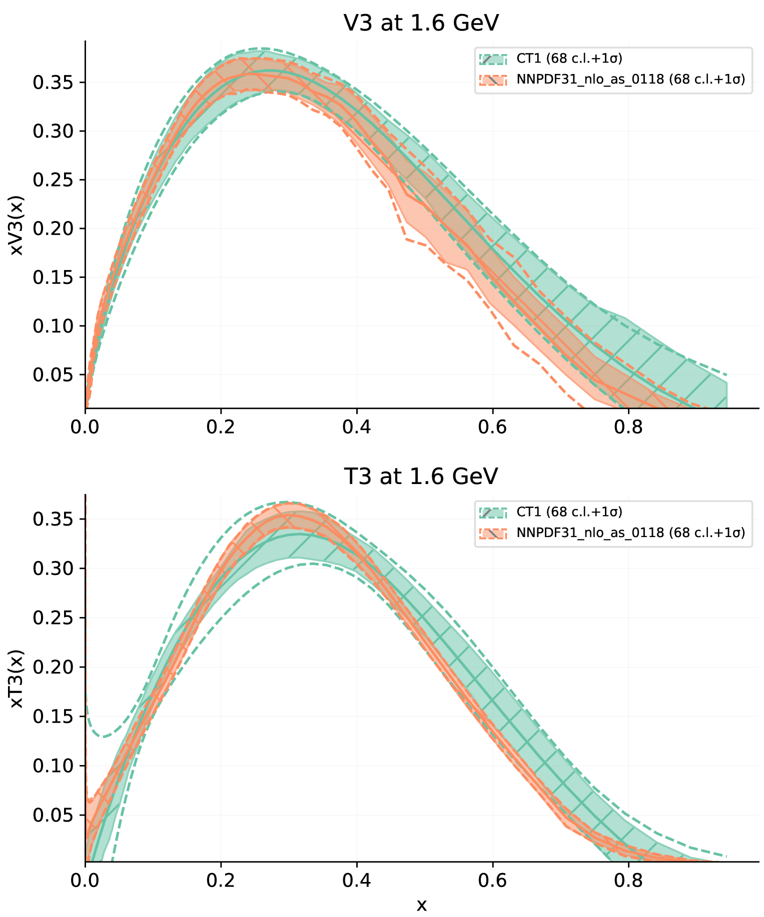

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Very robust result!

pseudo data:
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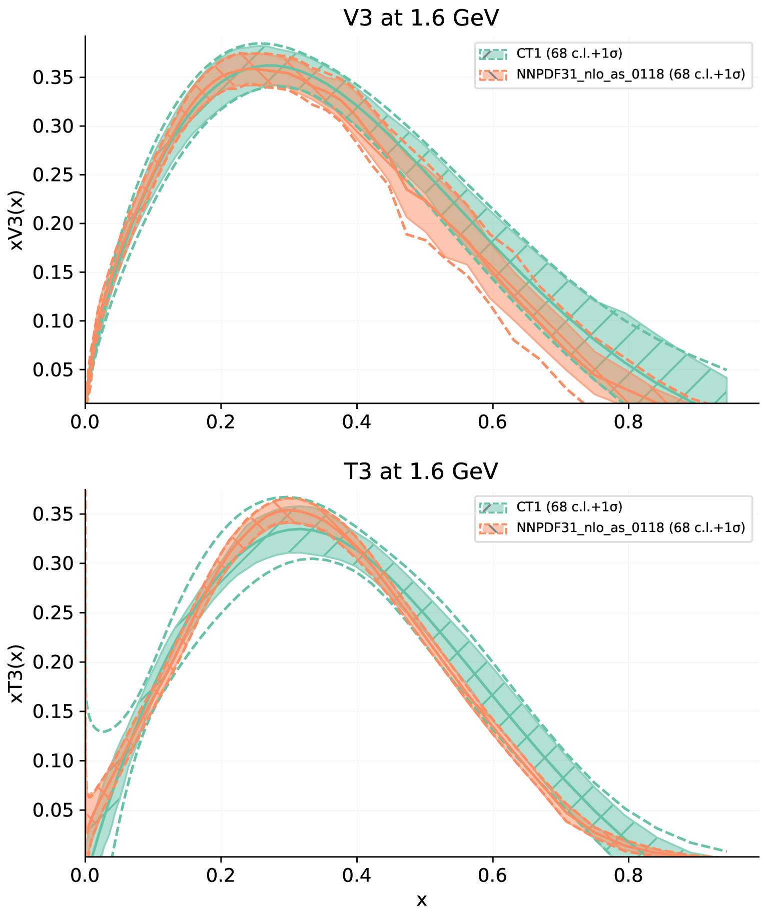

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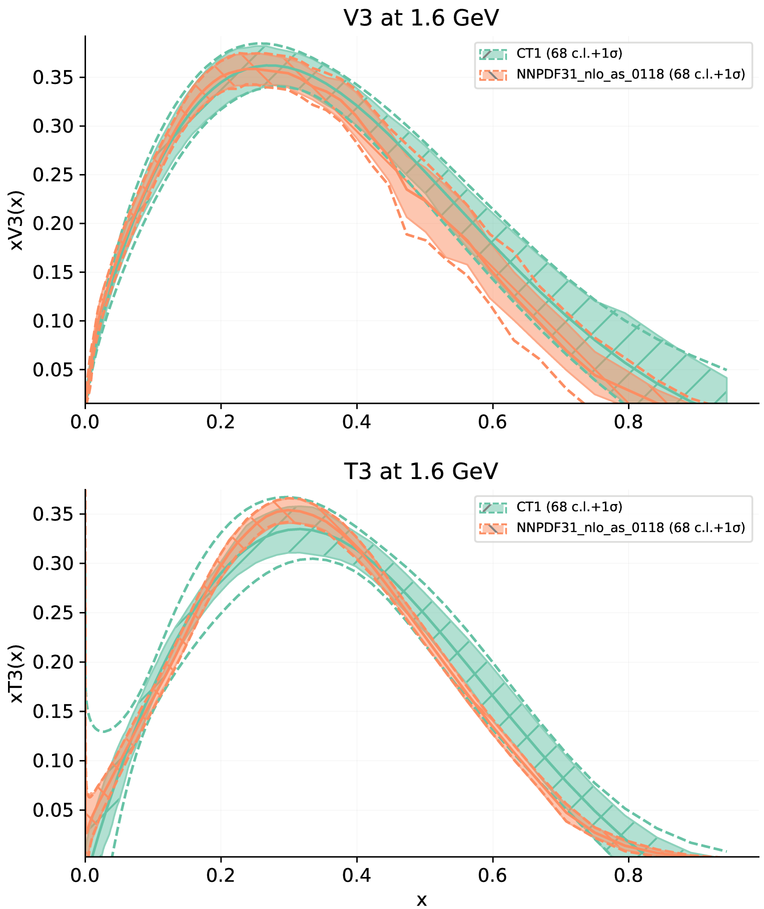

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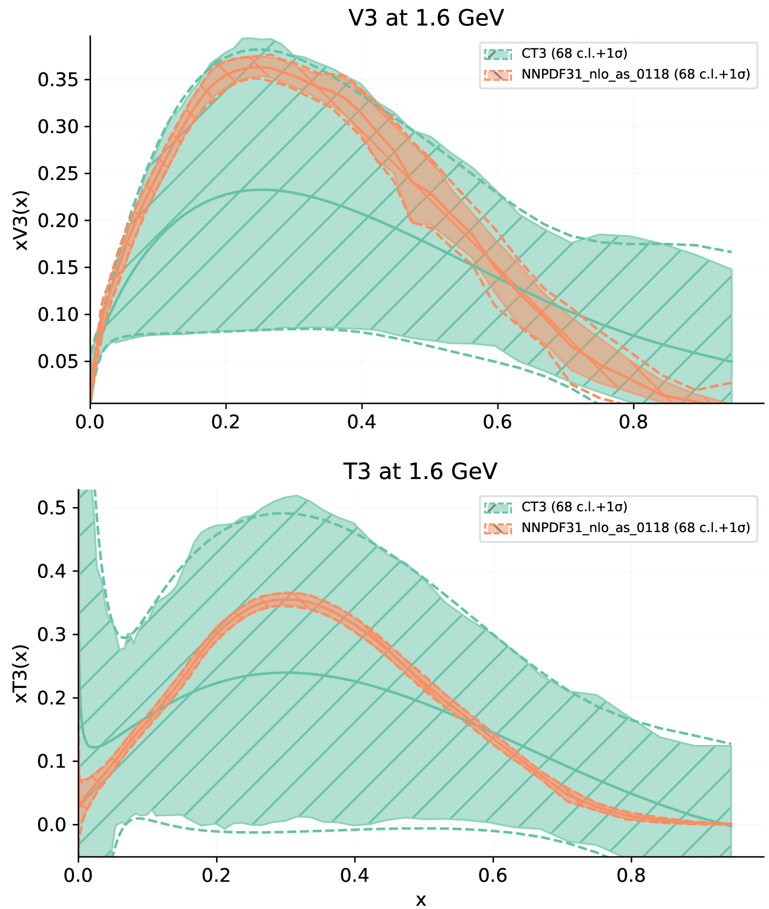

stat.error of ETMC lattice data + a scenario for systematics

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S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

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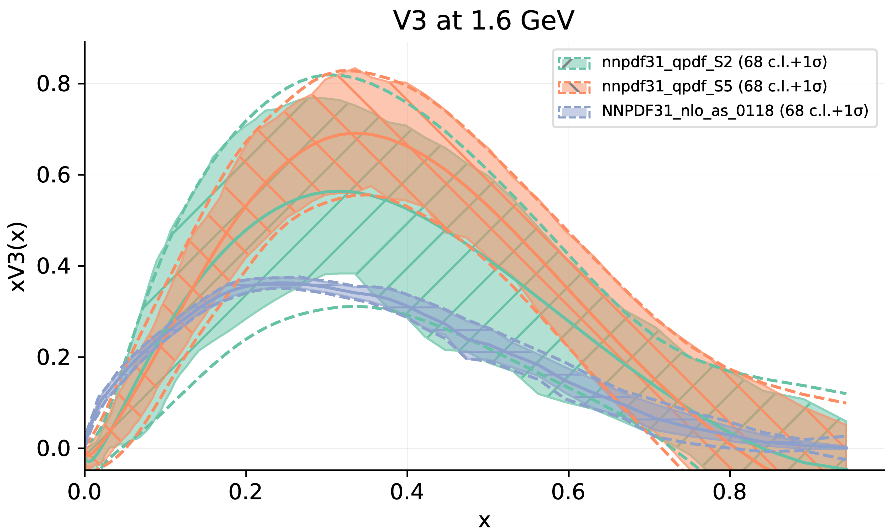
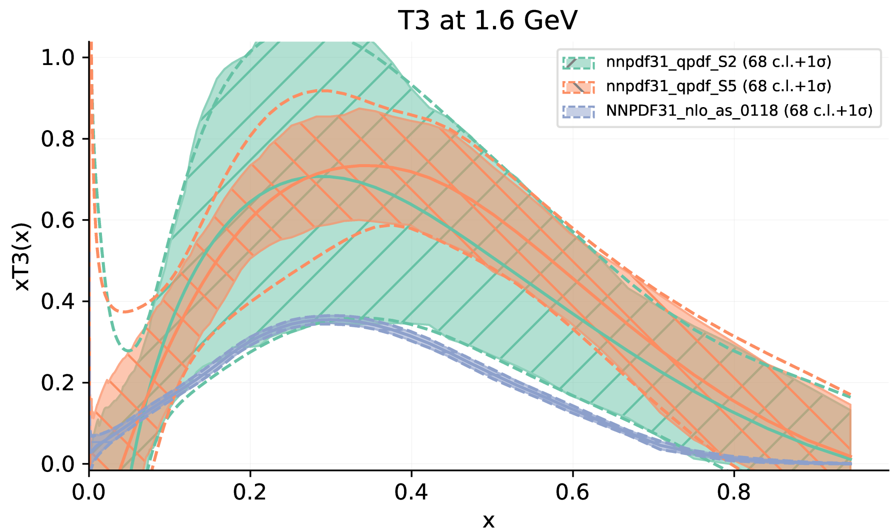
Results from scenarios S2 and S5 (“realistic”):

K.C., L. Del Debbio, T. Giani
JHEP 10 (2019) 137





Fitting actual lattice data

- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
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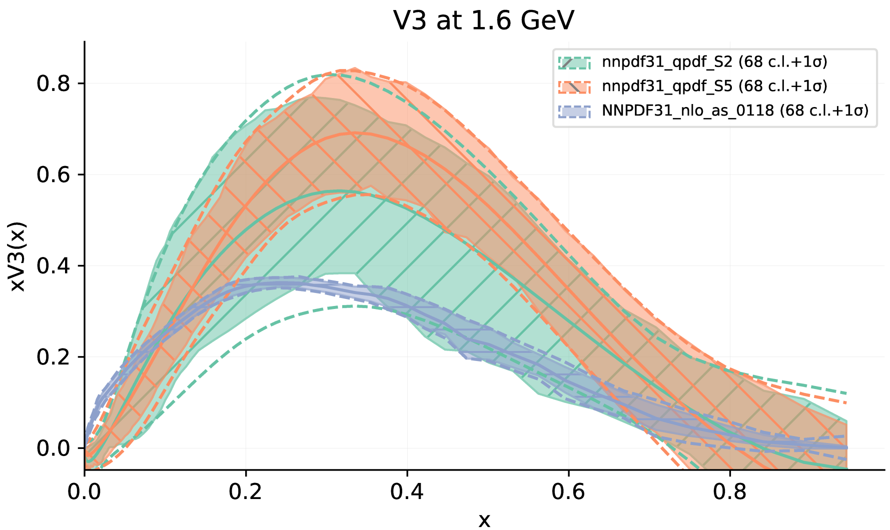

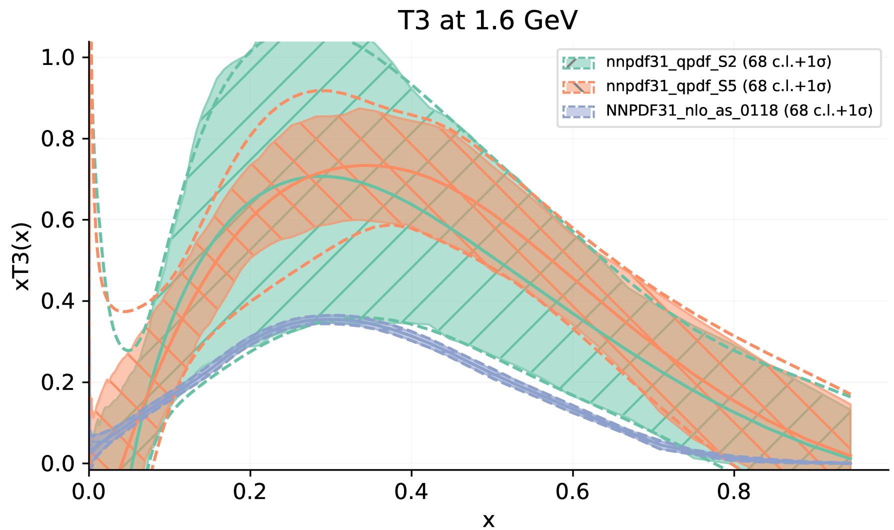

Reasonable agreement, but a lot of work for the lattice to reduce uncertainties!

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PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

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- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements difficult due to their suppressed $\mathcal{O}(1/Q)$ kinematical behavior.

Our work:

Our work:

- matching for twist-3 helicity $g_T(x)$
  proved factorization at 1-loop
  extracted the matching coefficient between quasi and light-cone
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- role of zero-mode contributions for twist-3 transversity $h_L(x)$ and scalar $e(x)$
  light-cone and quasi do not fully agree in the infrared
  breakdown of matching?

  S. Bhattacharya et al., arXiv:2006.12347

Lattice matrix element:

$$\mathcal{M}_{g_T}(P,z) = \langle P|\,\overline{\psi}(0,z)\,\gamma^j\,\gamma^5\,W(z)\,\psi(0,0)\,|P\rangle\,.$$

$\gamma^j = \gamma^x$ or γ^y

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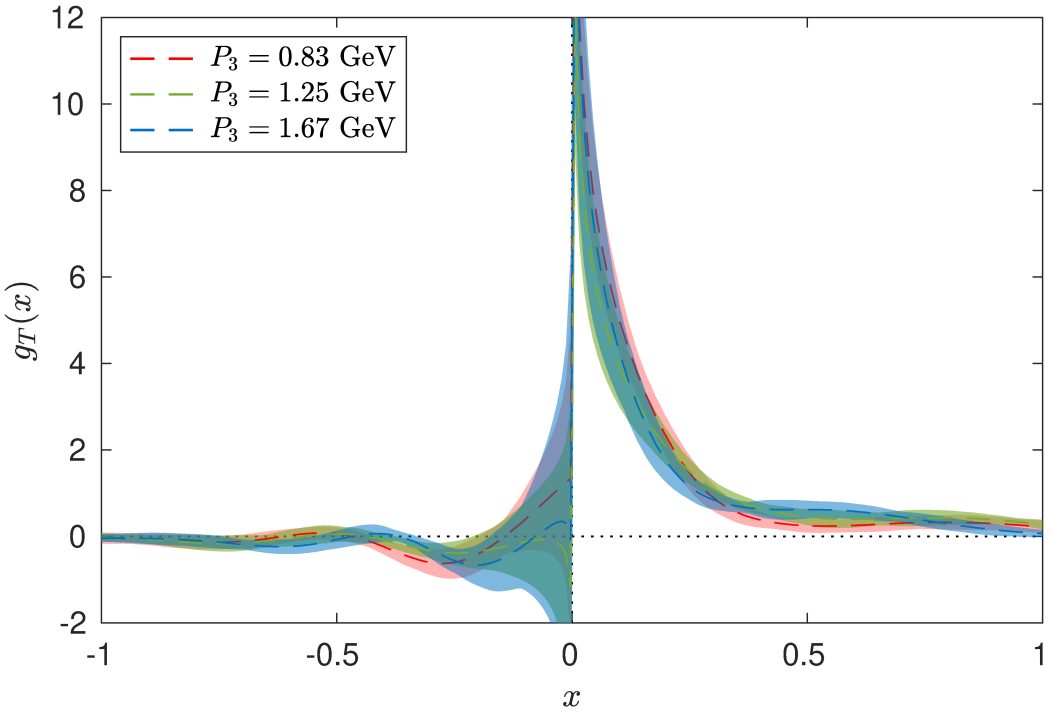
Nucleon boost dependence
(after matching)
(quasi-g_T reconstructed with BG)



Results for g_T and g_1

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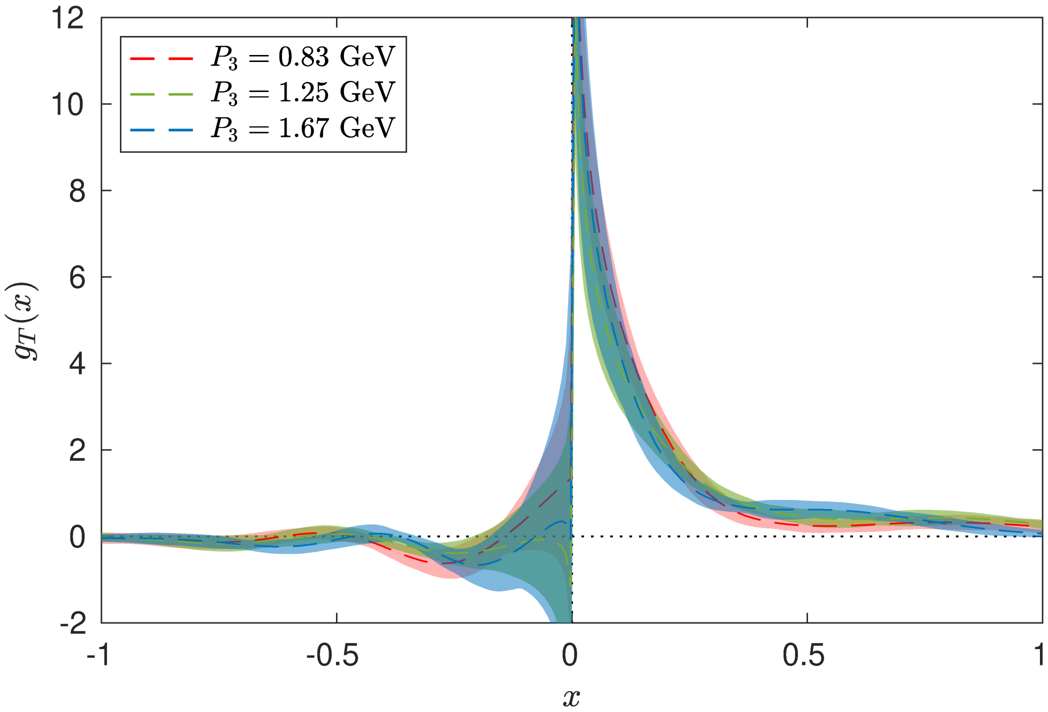

Twist-2 g_1 vs. twist-3 g_T
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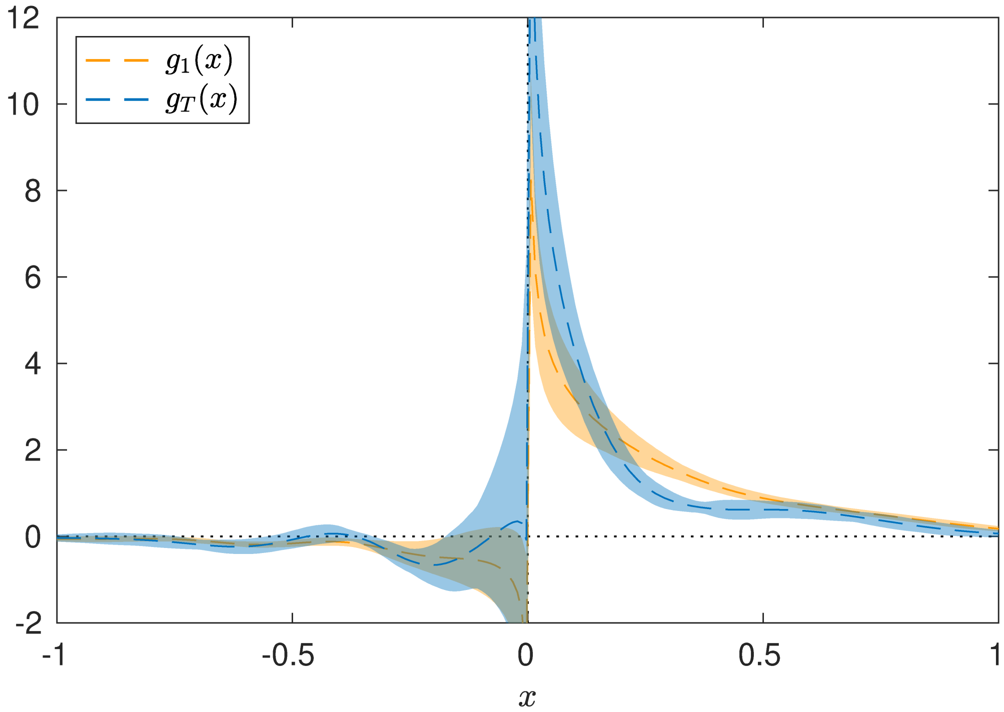

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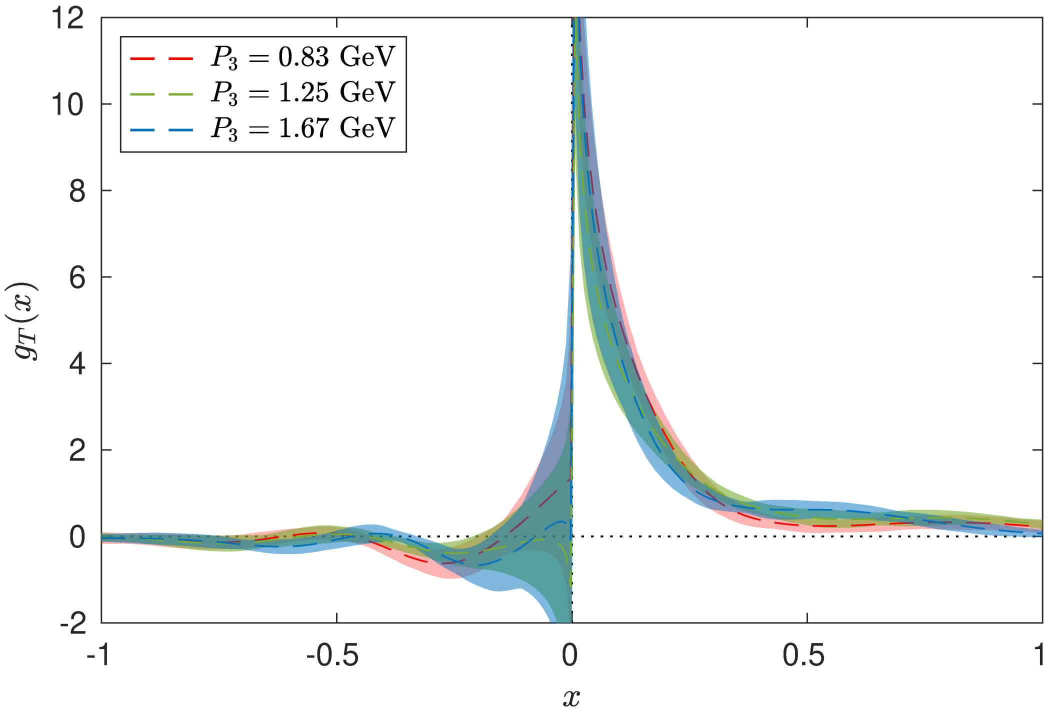

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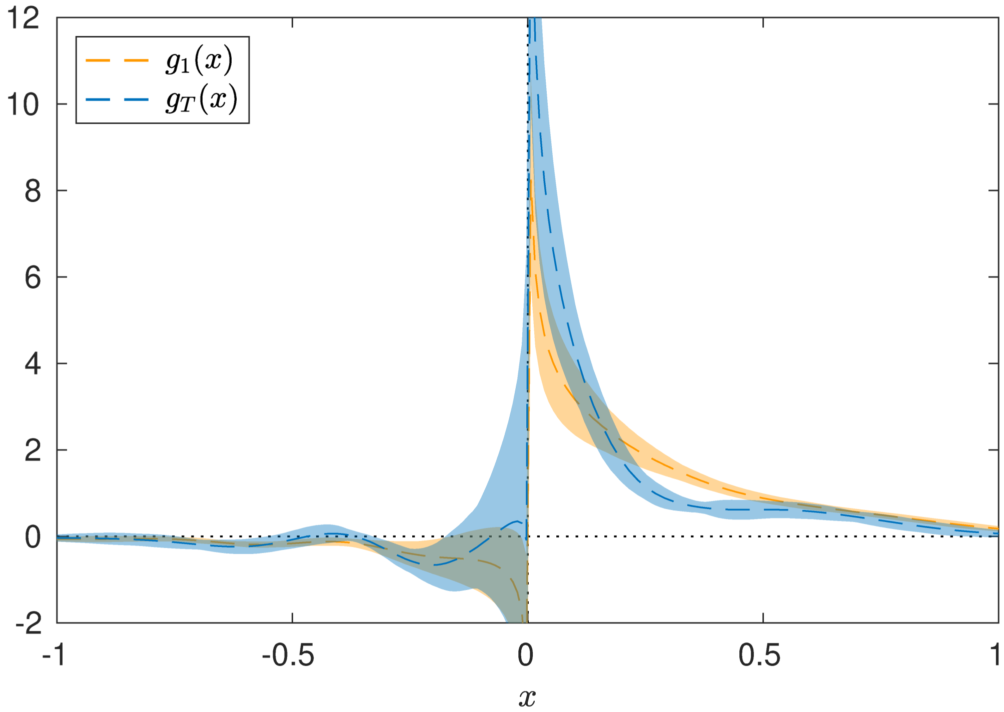

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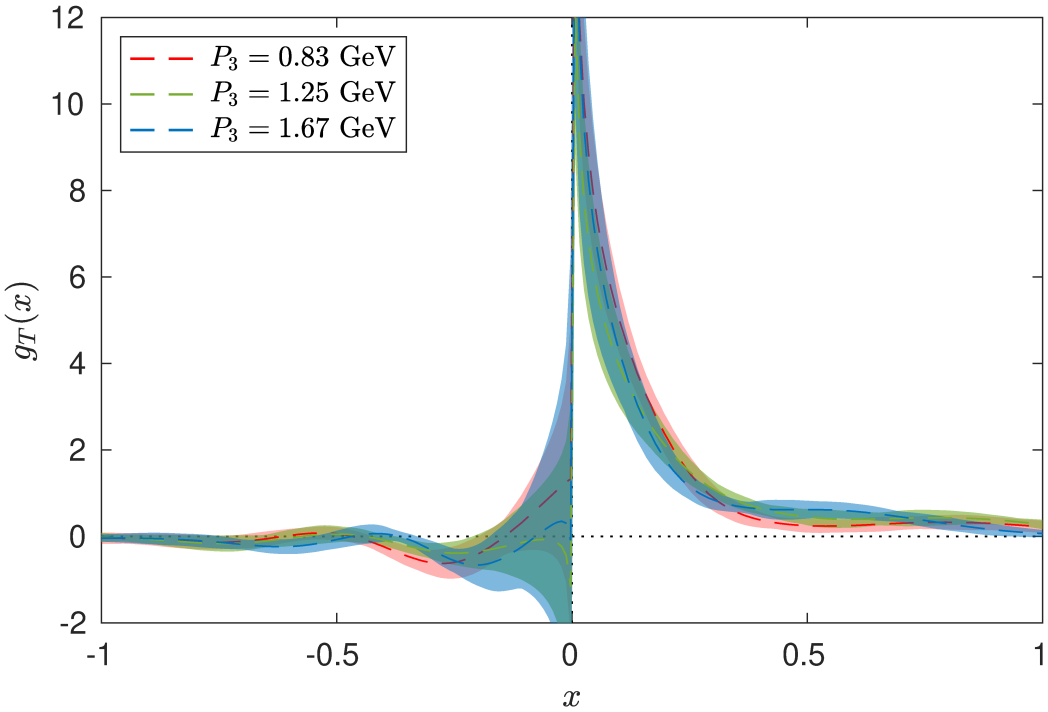

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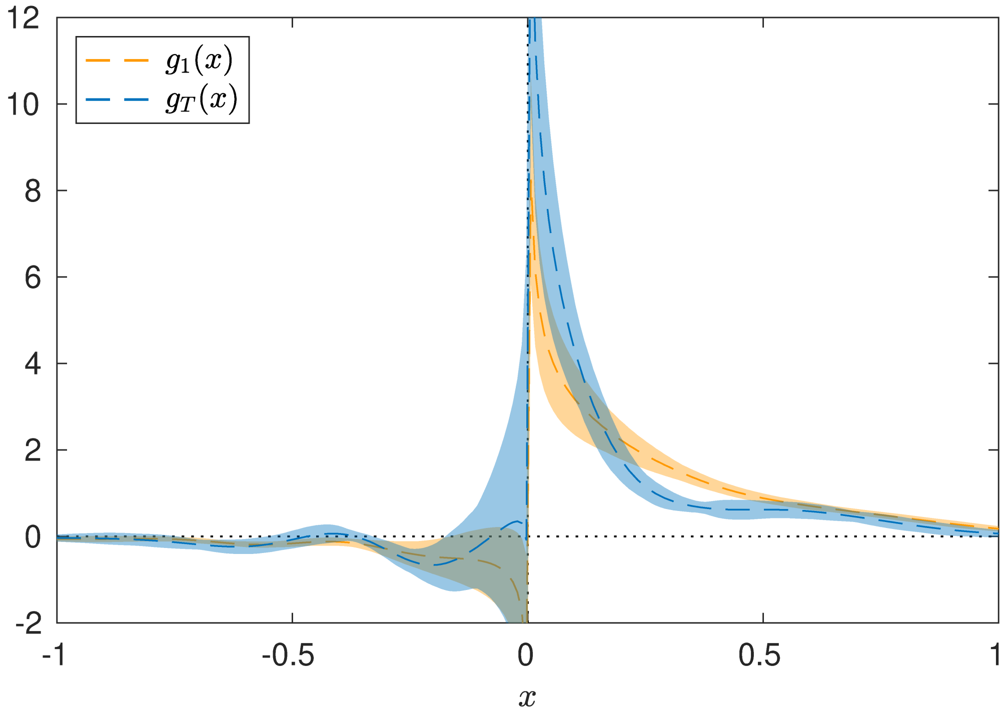

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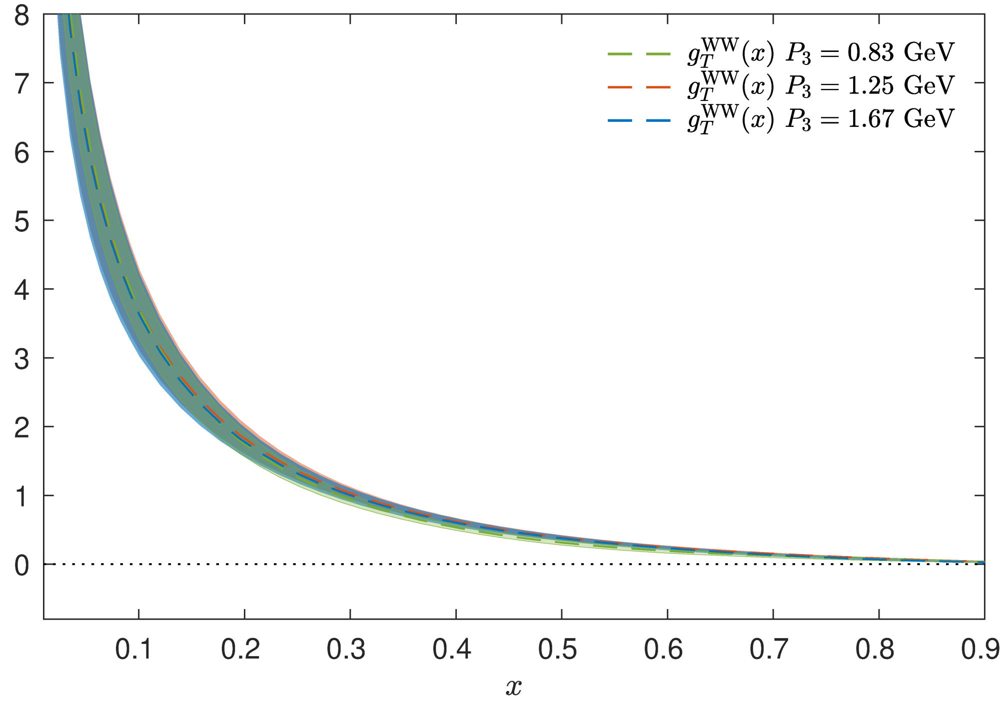

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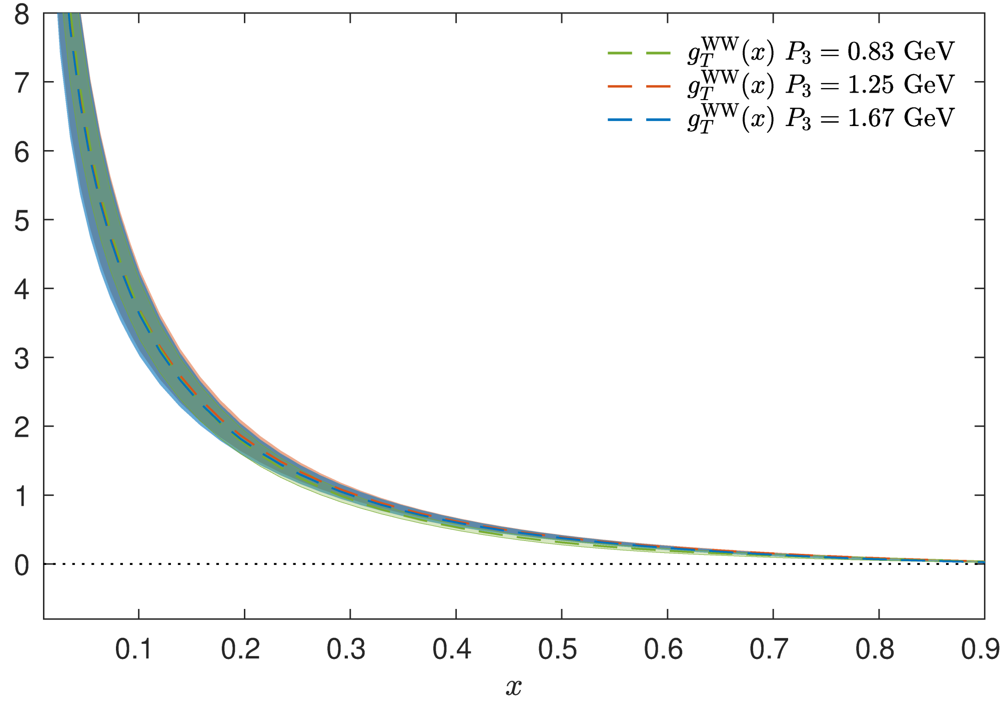

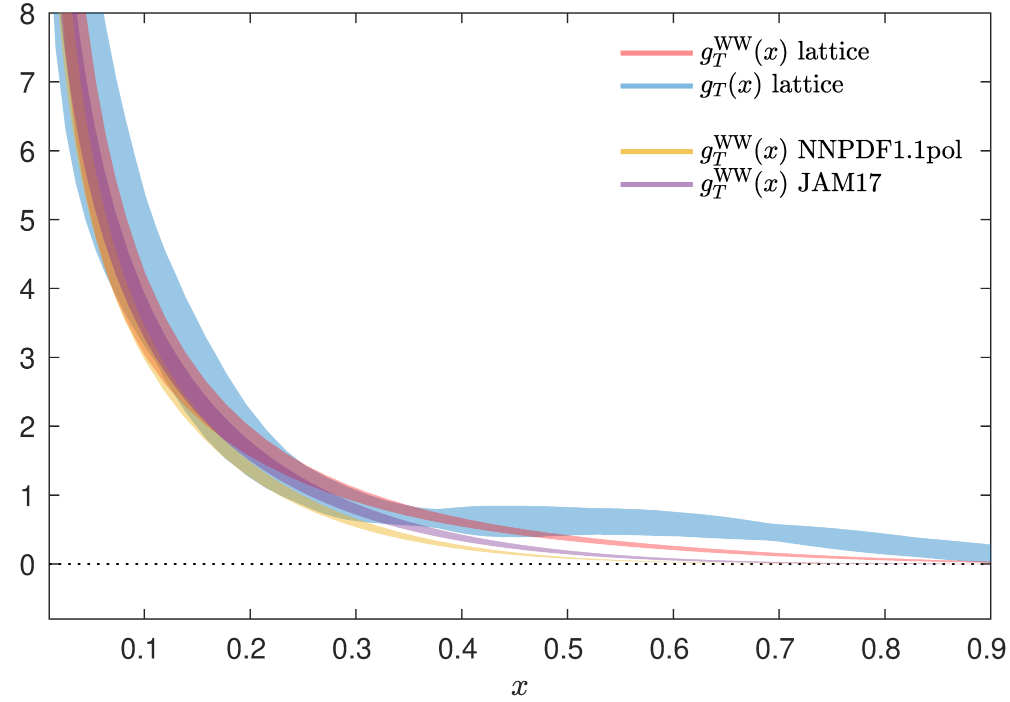

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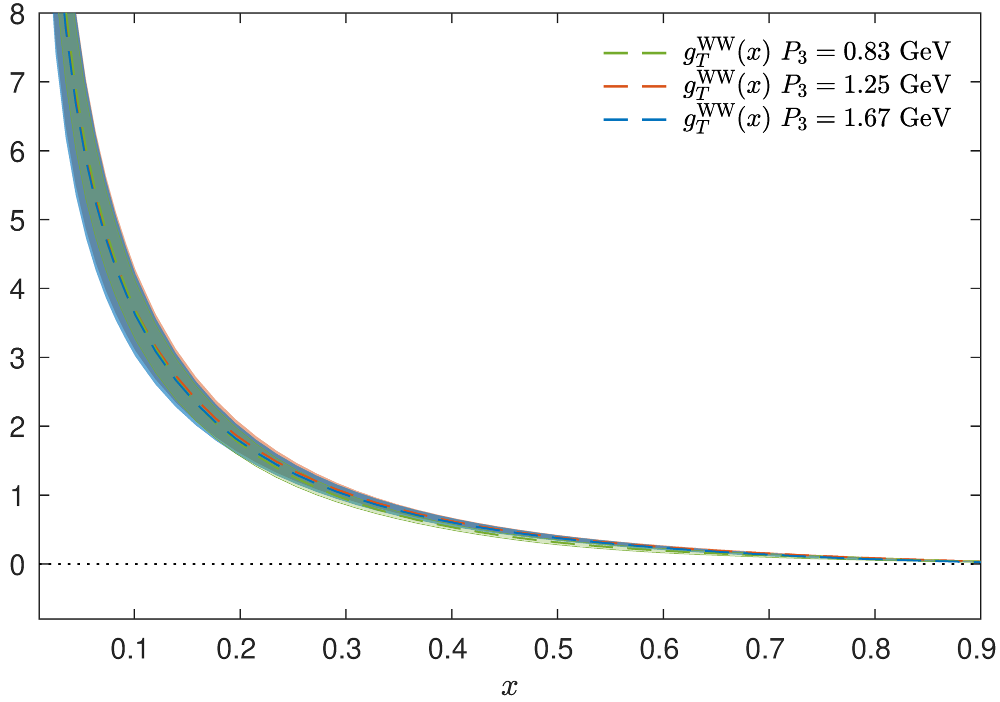

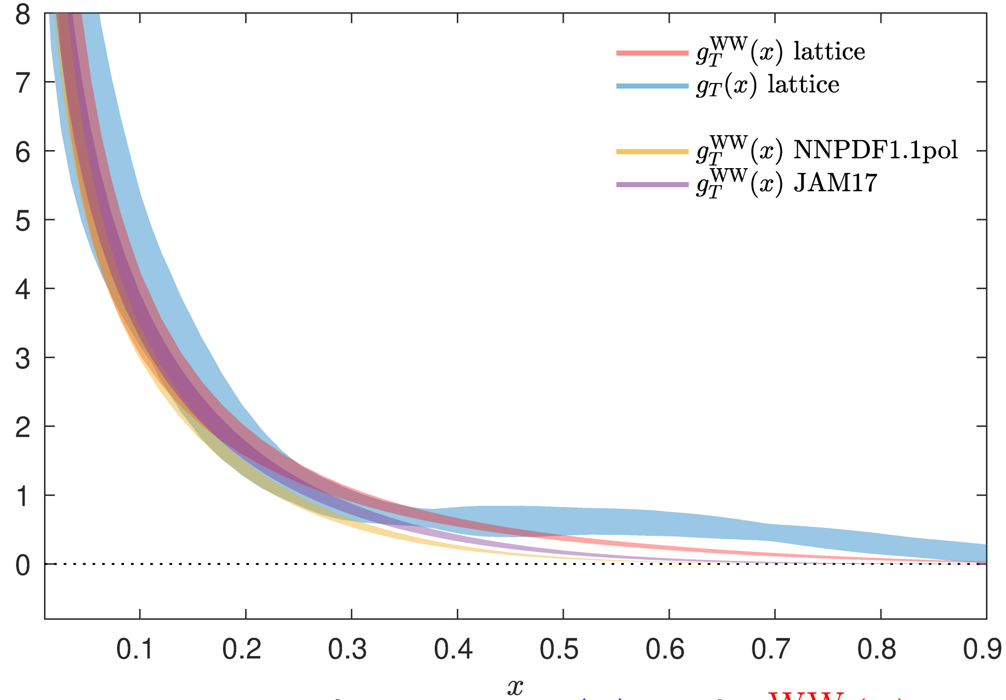

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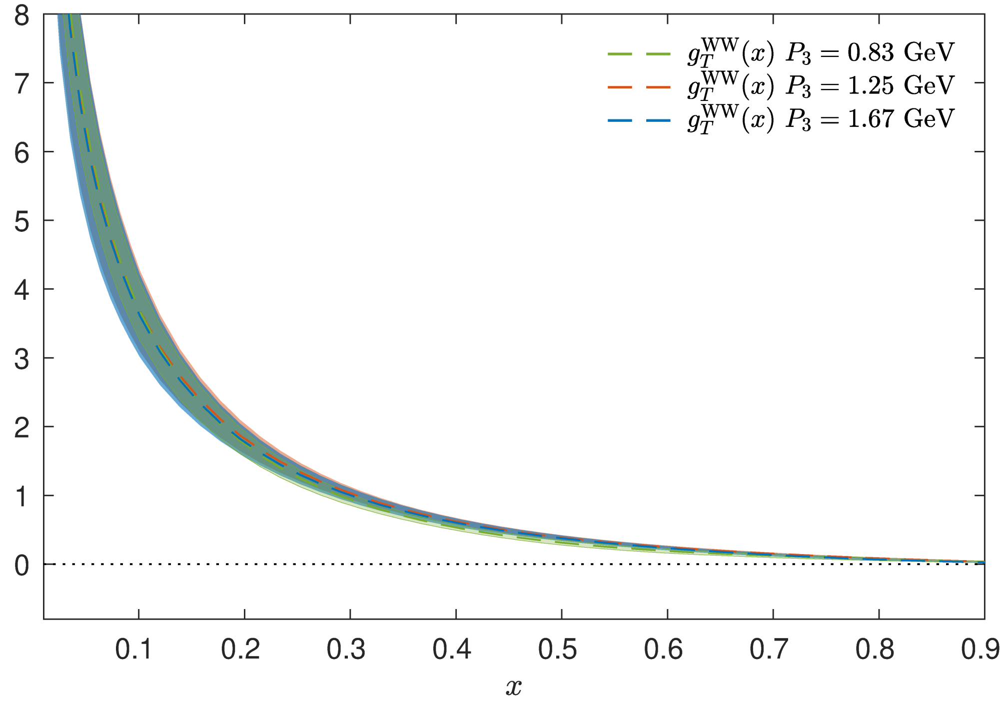

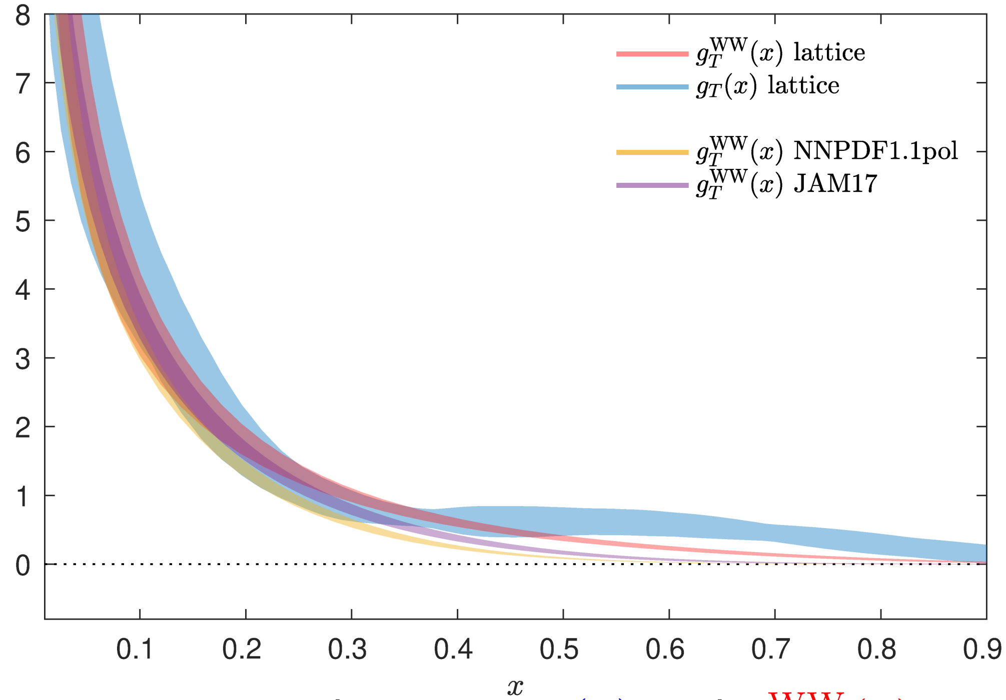

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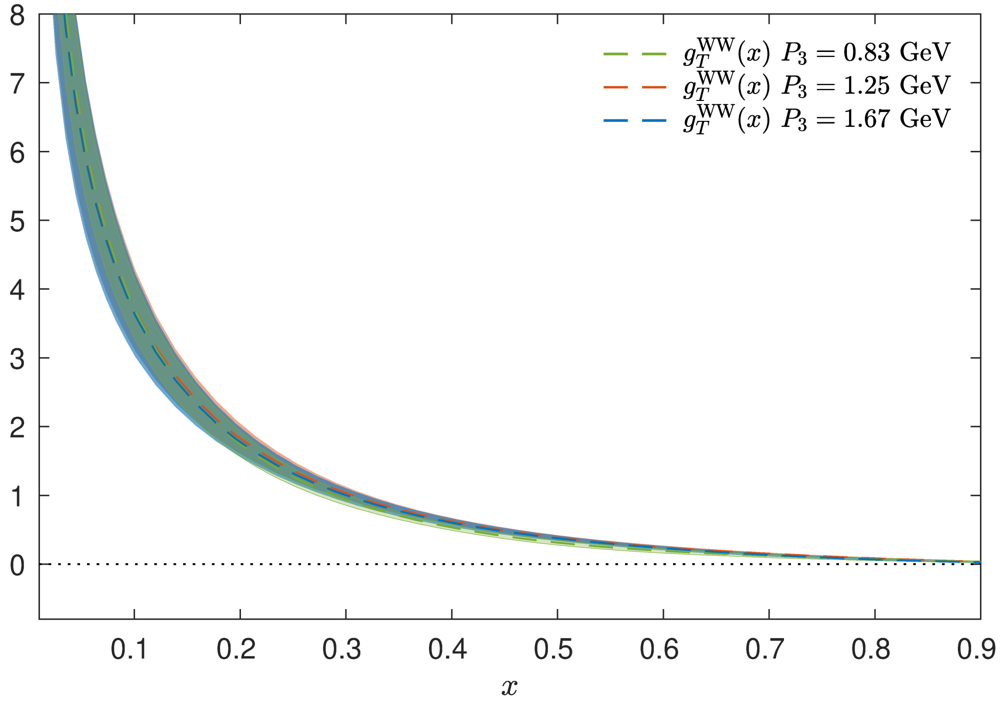

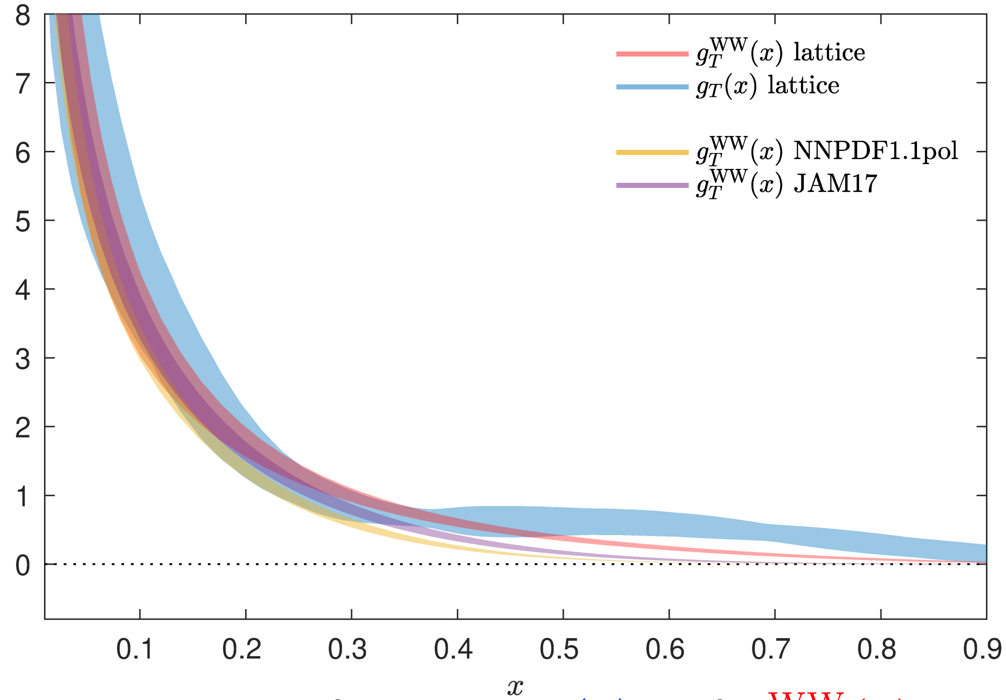

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interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:
A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093

Generalized parton distributions (GPDs)

- Parton distribution functions (PDFs) – formal definition:
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  The only difference: **momentum transfer**
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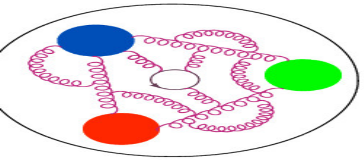

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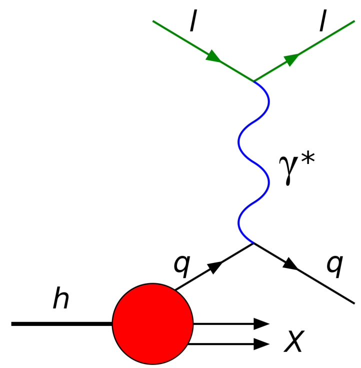

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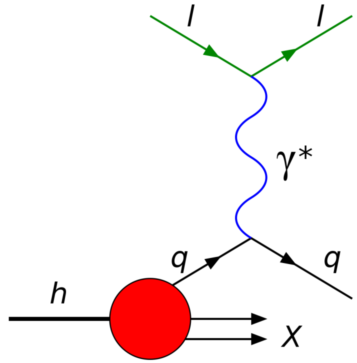

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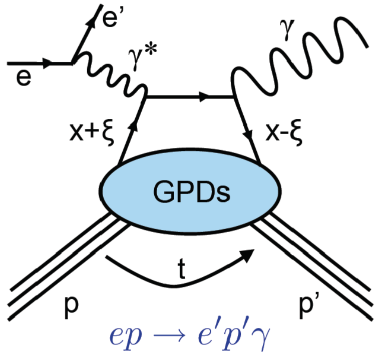

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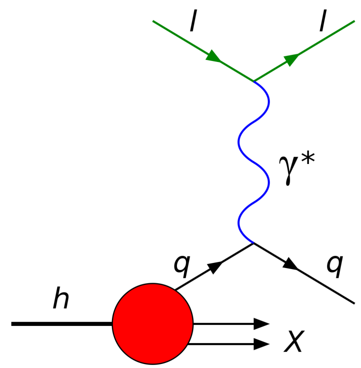

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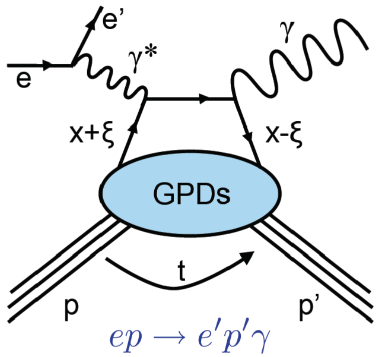

Quasi-GPDs: similar procedure to quasi-PDFs
Important new aspect: 2 or 4 GPDs need to be disentangled, e.g. H and E:

$$\mathcal{M}(z,t,\xi;\,\mu_R;\,\Gamma,\overline{\Gamma}) = \mathcal{K}_H(\Gamma,\overline{\Gamma})H(z,t,\xi;\mu_R) + \mathcal{K}_E(\Gamma,\overline{\Gamma})E(z,t,\xi;\mu_R).$$

Bare matrix elements

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$P_3 = 0.83, 1.25, 1.67$ GeV
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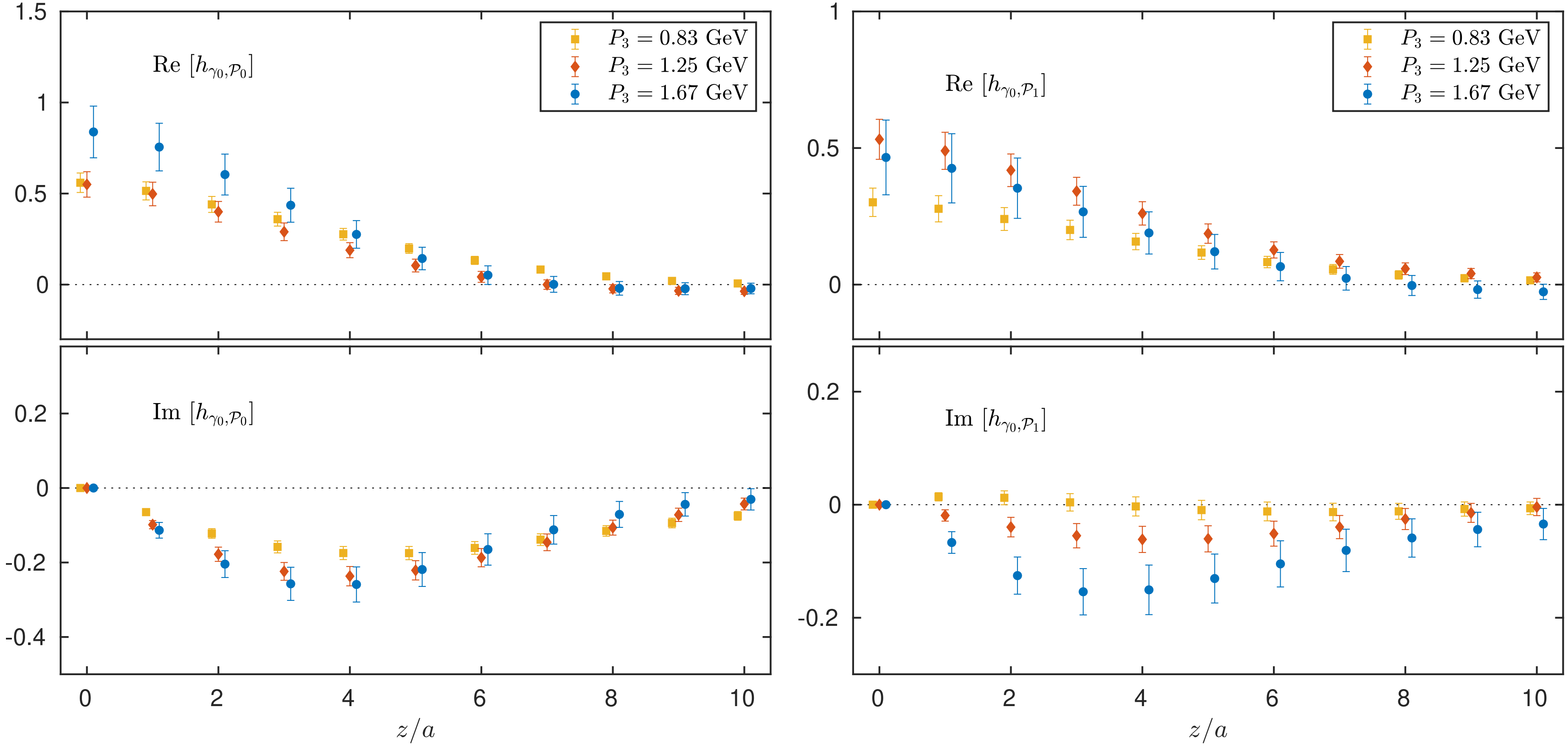
unpolarized projector | polarized projector



Disentangled renormalized matrix elements

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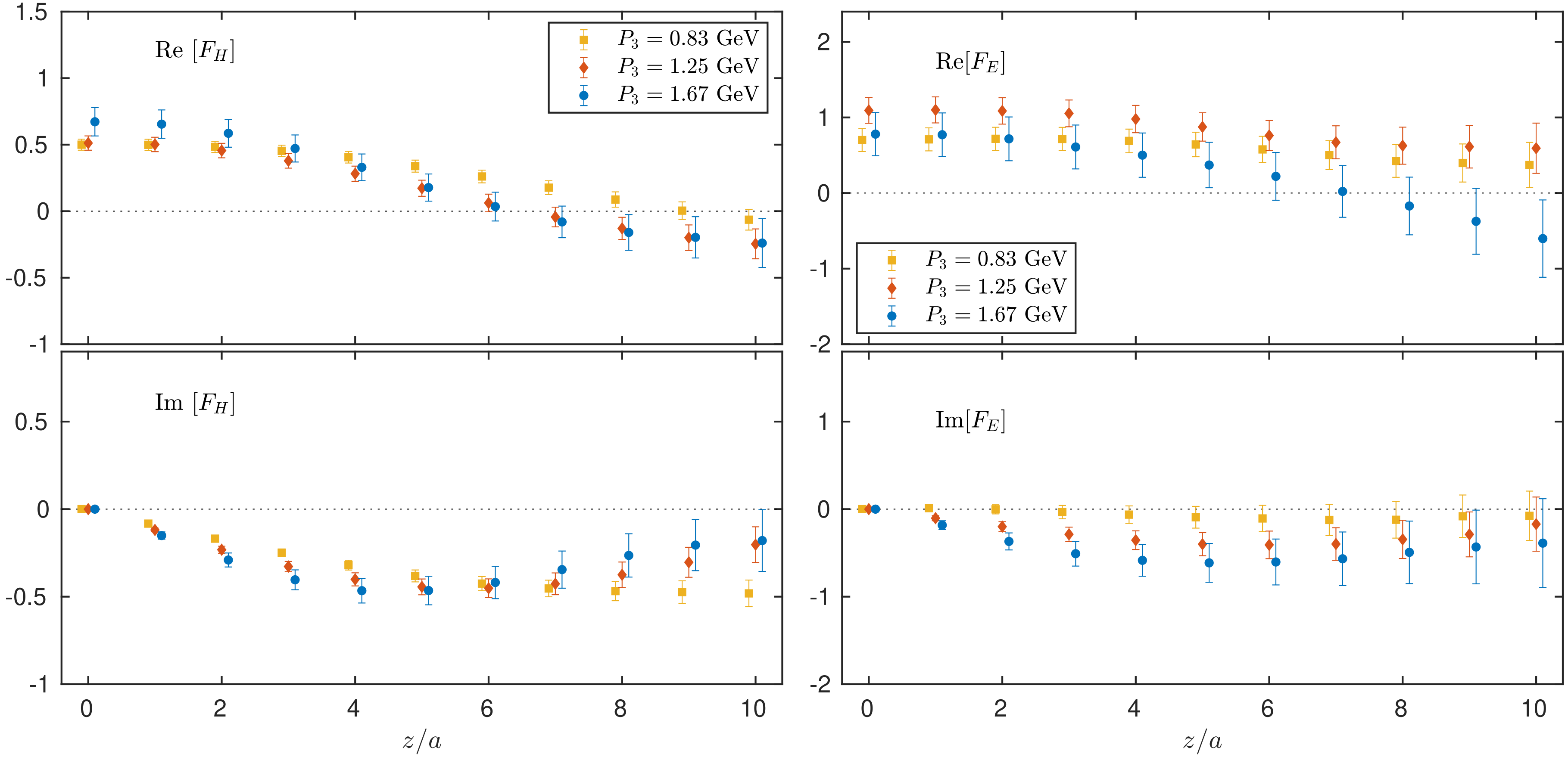
ME of H-function   ME of E-function



After matching: H and E functions

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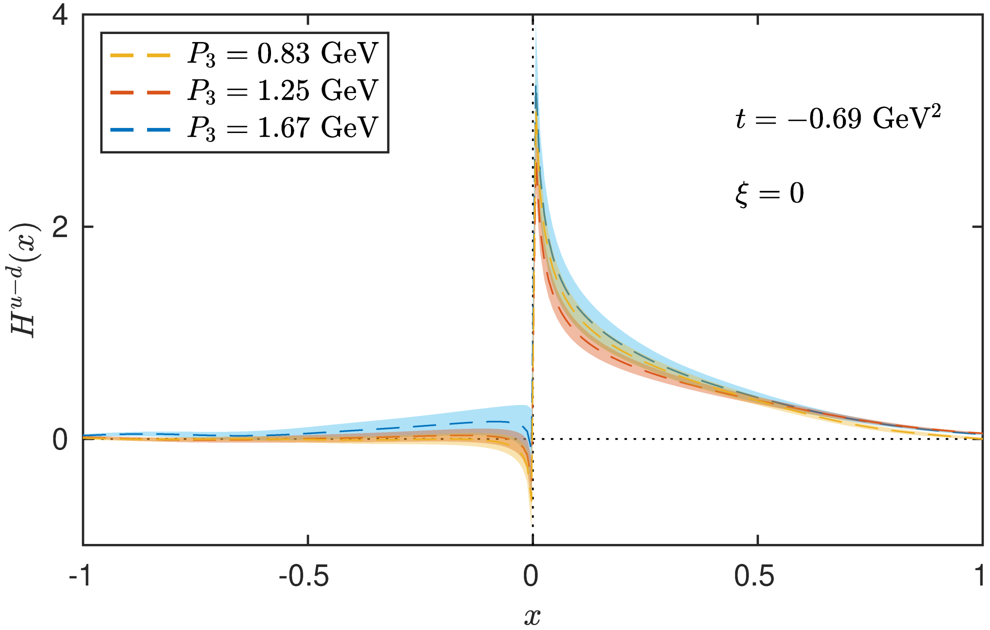
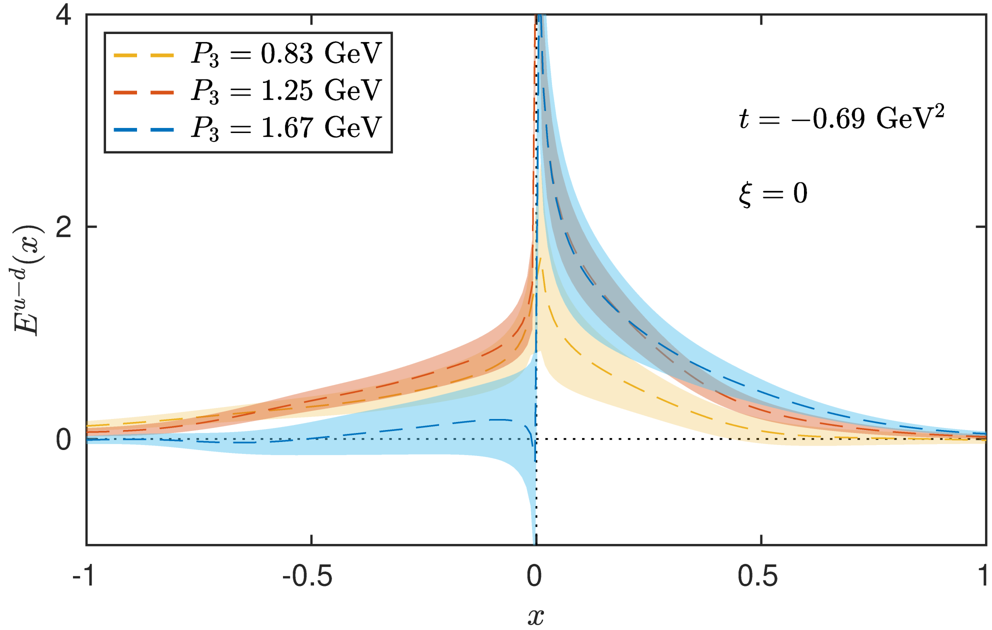
$$\xi = 0$$

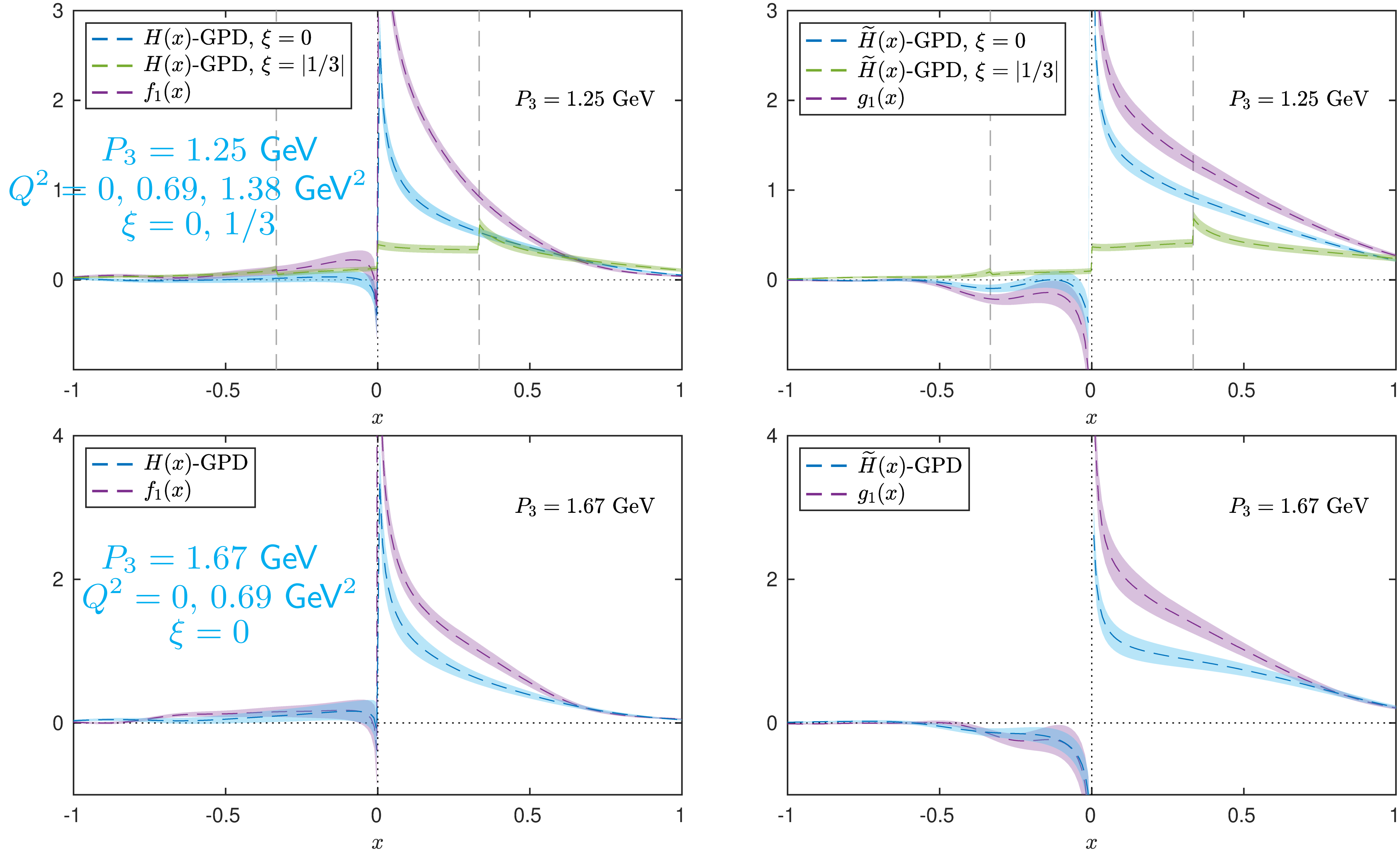
H-function



E-function



Comparison of PDFs and H-GPDs



unpolarized

helicity

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**

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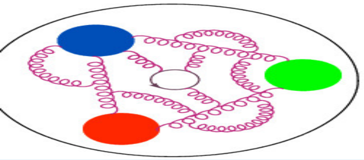

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Thank you for your attention!

Backup slides

Renormalization from a double ratio

The matrix element $\langle N(P_3)|\overline{\psi}(z)\gamma_0\mathcal{A}(z,0)\psi(0)|N(P_3)\rangle$ exhibits two kinds of divergences:

- standard logarithmic divergence,
- power divergence related to the Wilson line.

Shown to be multiplicatively renormalizable to all orders in PT

T. Ishikawa et al., PRD96(2017)094019, X. Ji et al., PRL120(2017)112001

Both divergences can be canceled by forming a double ratio with zero-momentum and local $(z=0)$ matrix elements:

(also removes part of HTE (generically $\mathcal{O}(z^2\Lambda^2_{\rm QCD})$))

$$\mathfrak{M}(\nu,z^2)=\frac{\mathcal{M}(\nu,z^2)\,/\,\mathcal{M}(\nu,0)}{\mathcal{M}(0,z^2)\,/\,\mathcal{M}(0,0)}.$$

$\mathfrak{M}(\nu,z^2)$ – “reduced” matrix elements (or pseudo-ITDs).

The double ratio defines a renormalization scheme with renormalization scale proportional to $1/z$.

Matching to light-cone ITDs

The reduced matrix elements, $\mathfrak{M}(\nu, z^2)$, defined at different scales $1/z$, need to be:

- evolved to a common scale,
- scheme-converted to the $\overline{\mathrm{MS}}$ scheme $\longrightarrow Q(\nu, \mu^2)$.

The full 1-loop matching equation: A. Radyushkin, PLB781(2018)433, PRD98(2018)014019; J.-H. Zhang et al., PRD97(2018)074508; T. Izubuchi et al., PRD98(2018)056004

$$\mathfrak{M}(\nu, z^2) = Q(\nu, \mu^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4} \right) B(u) + L(u) \right] Q(u\nu, \mu^2)$$

with:

$$B(u) = \left[\frac{1+u^2}{1-u} \right]_+ , \qquad L(u) = \left[4 \frac{\ln(1-u)}{1-u} - 2(1-u) \right]_+ ,$$

$$\int_0^1 [f(u)]_+ \, Q(u\nu) = \int_0^1 f(u) \left(Q(u\nu) - Q(\nu) \right).$$

Matching to light-cone ITDs

We invert the matching equation and look separately into the effect of evolution and scheme conversion:

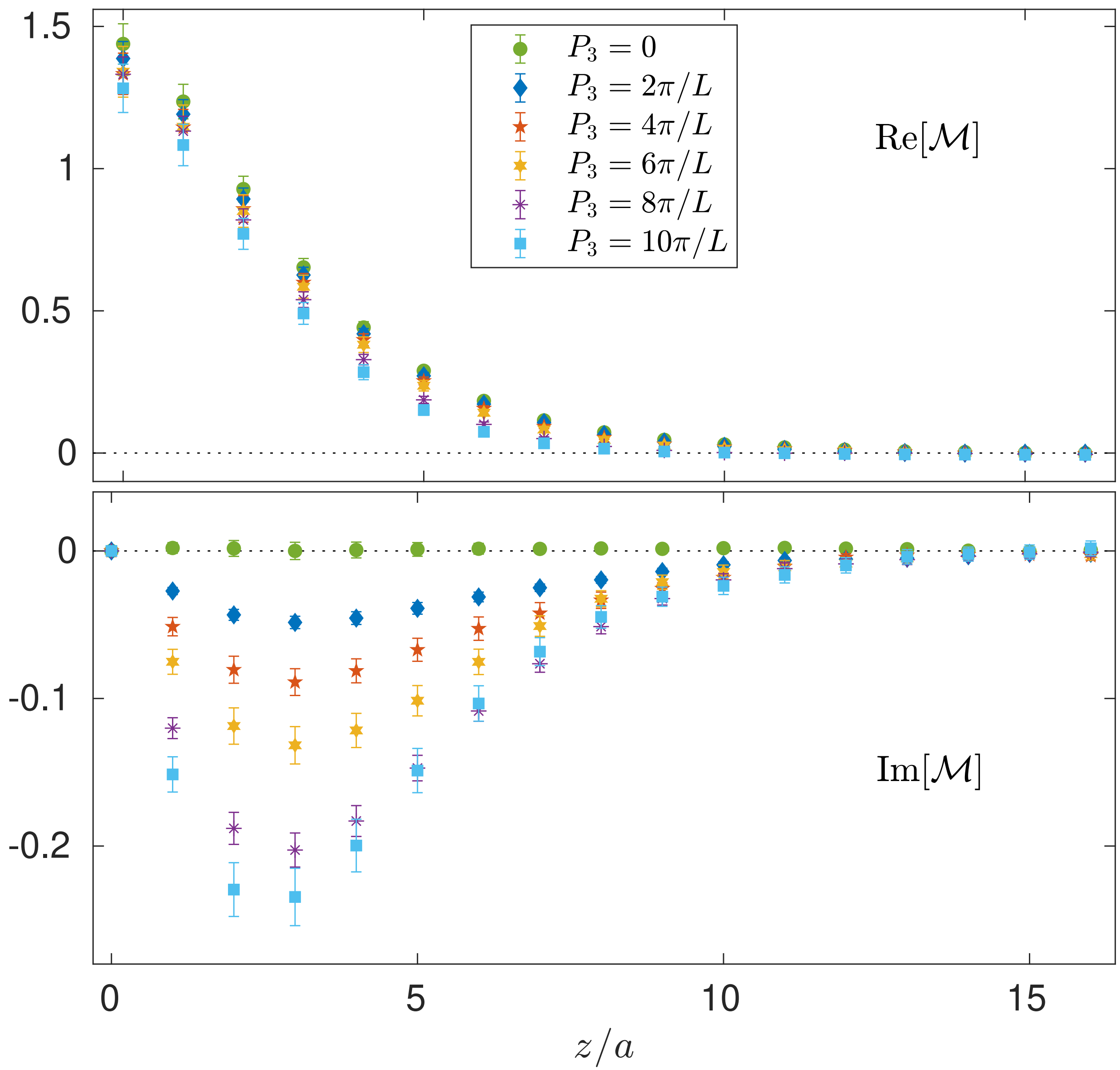
- evolution:
  $\mathfrak{M}'(\nu, z^2, \mu^2) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u)\mathfrak{M}(u\nu, z^2)$,
  The evolved ITD $\mathfrak{M}'$ has 3 arguments:
  the Ioffe time ν, the common scale μ, the initial scale z.
  In principle, values should be independent of the initial scale $\longrightarrow$ test this.

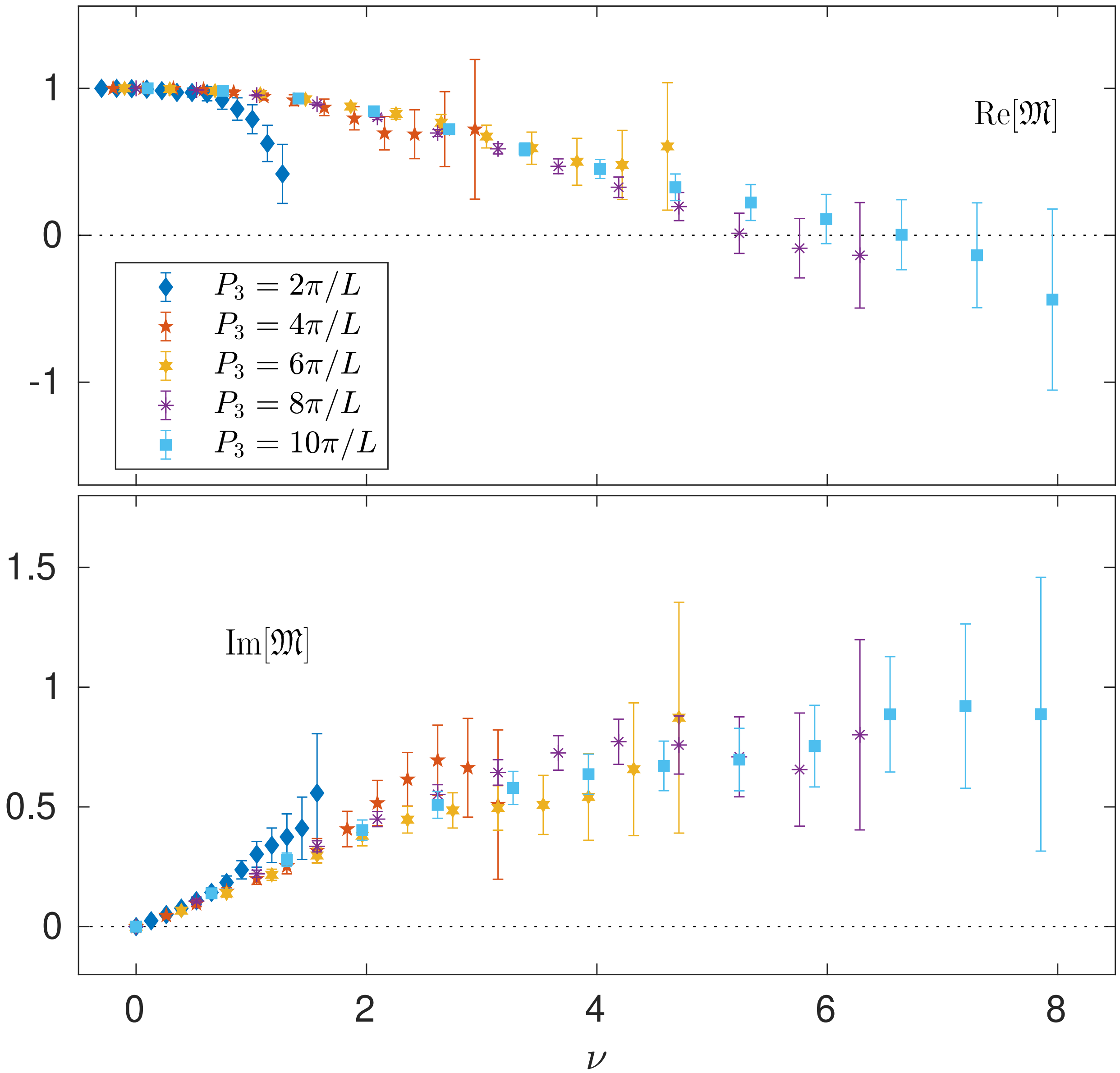
- scheme conversion:
  $Q(\nu, z^2, \mu^2) = \mathfrak{M}'(\nu, z^2, \mu^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du L(u)\mathfrak{M}(u\nu, z^2)$.

  Again 3 arguments and test of independence on the initial scale.

For the reconstruction of the final PDF
$\longrightarrow$ average the matched ITDs $Q(\nu, z^2, \mu^2)$ for cases where a given Ioffe time is achieved by different combinations of (P_3, z), denote such average by $Q(\nu, \mu^2)$.

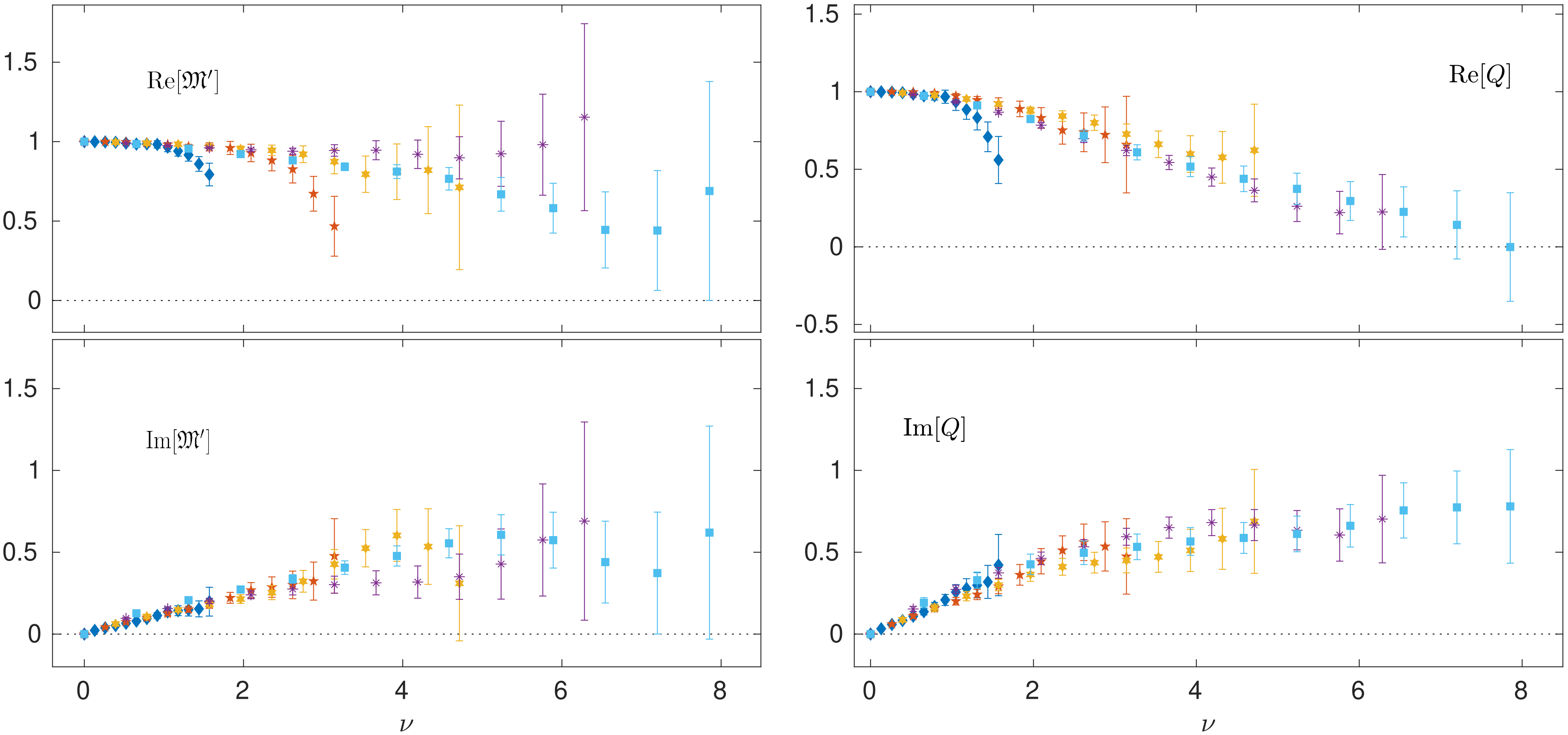
Pseudo-PDFs – bare ME

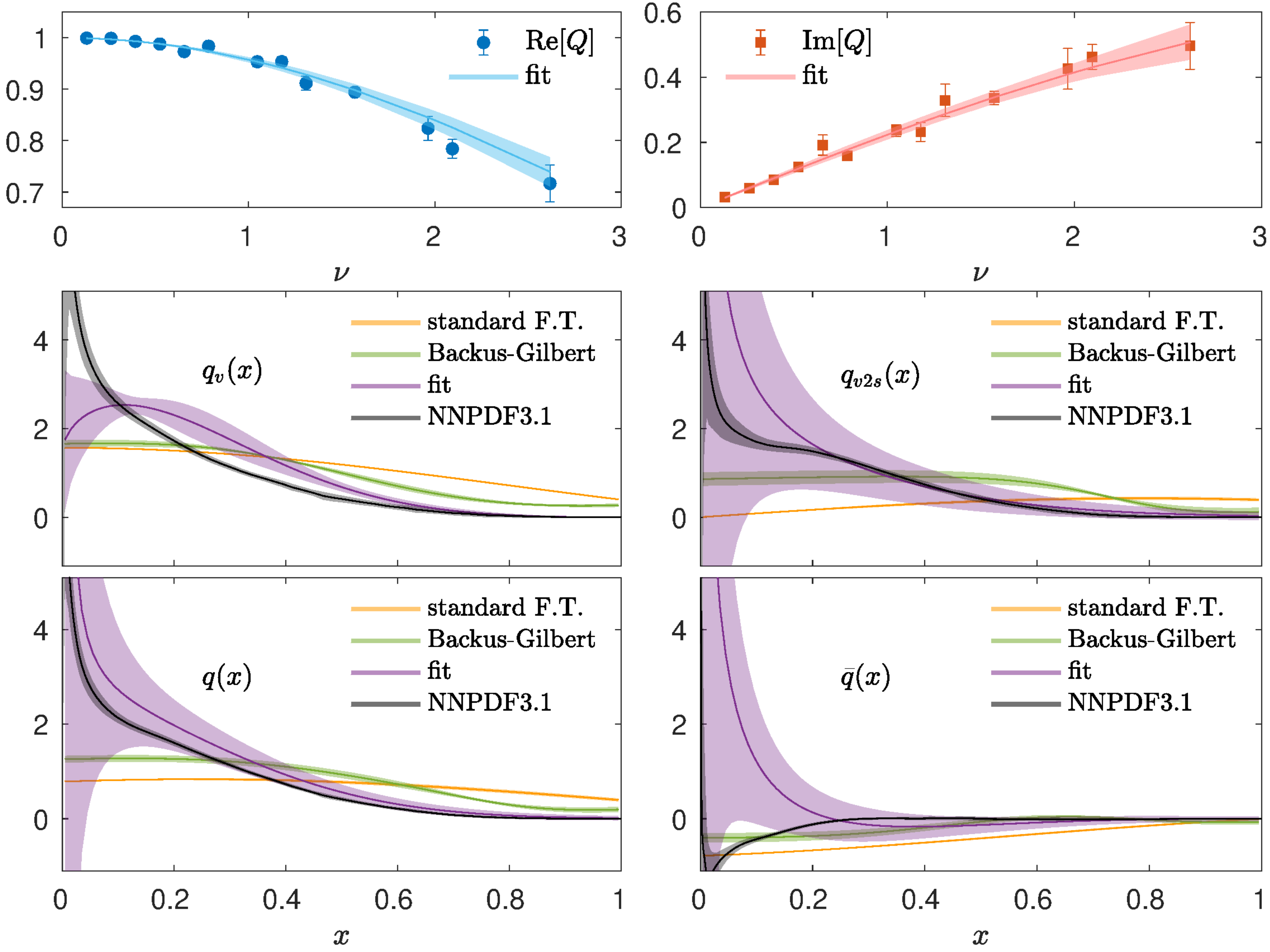




Re[$\mathfrak{M}$]
$P_3 = 2\pi/L$
$P_3 = 4\pi/L$
$P_3 = 6\pi/L$
$P_3 = 8\pi/L$
$P_3 = 10\pi/L$
Im[$\mathfrak{M}$]
1
0
-1
1.5
0.5
2
4
6
8
ν

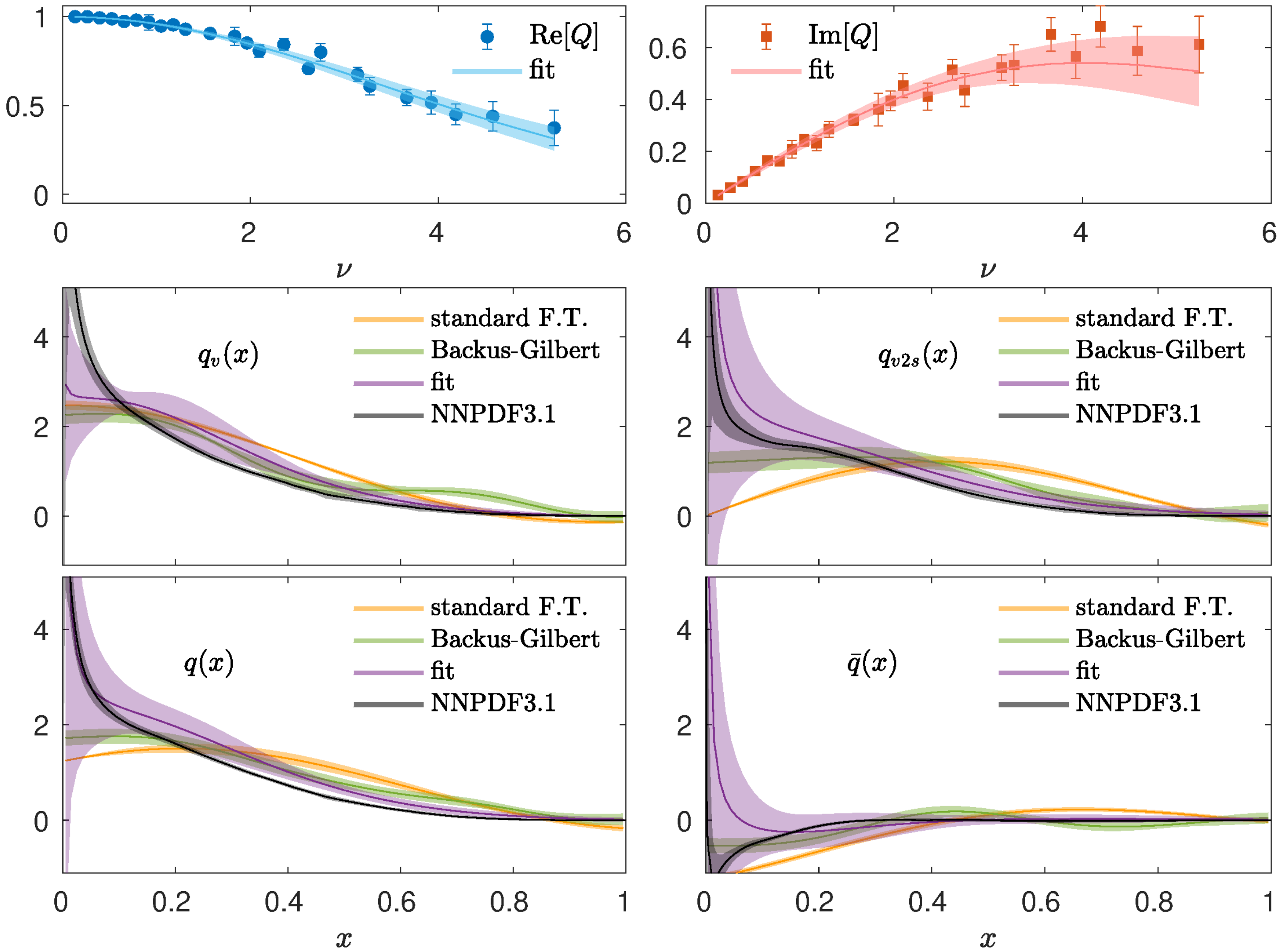
Pseudo-PDFs – evolved and $\overline{\mathrm{MS}}$-converted matrix elements



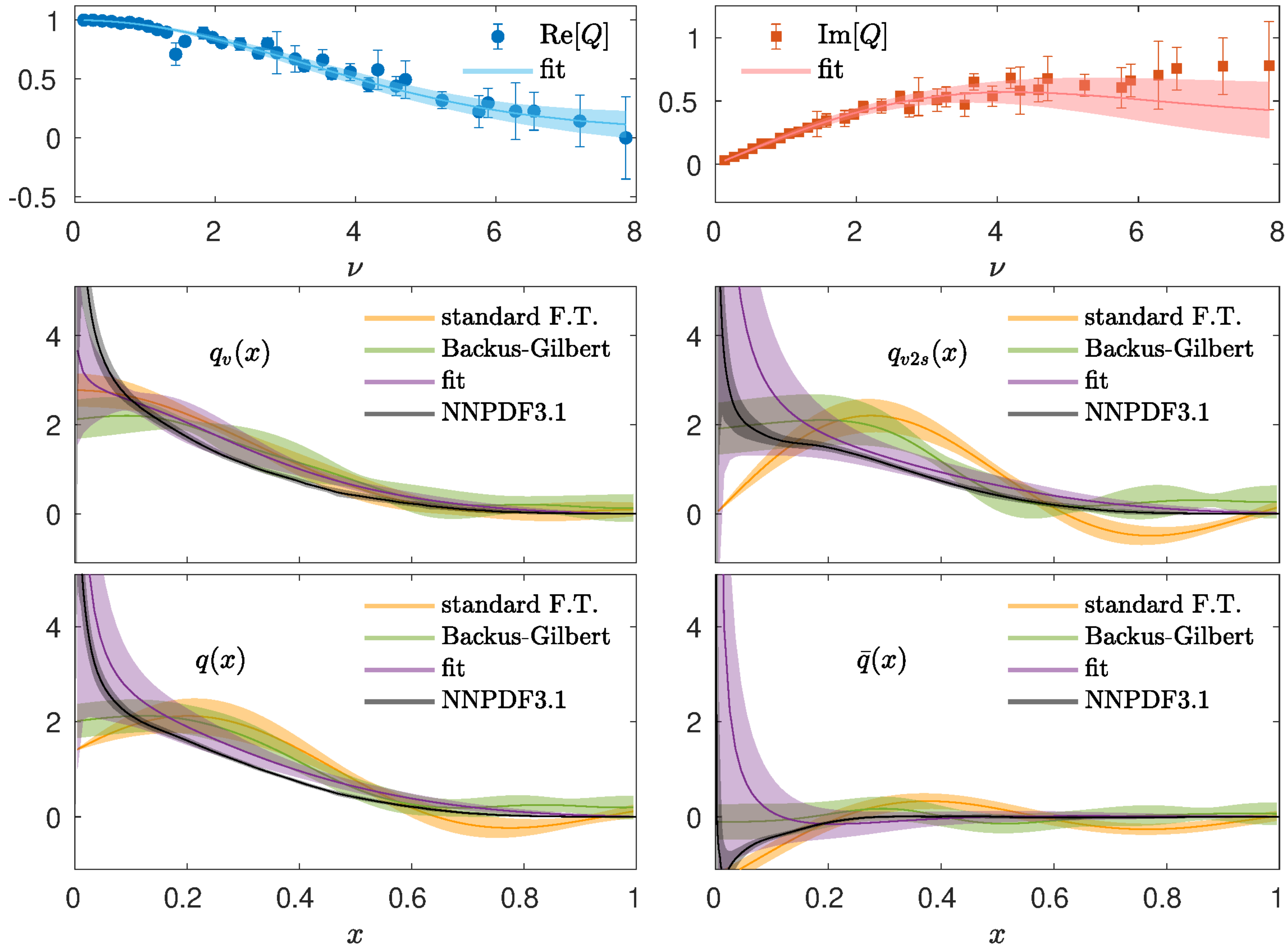


Re[Q]
fit
Im[Q]
fit
ν
ν
$q_v(x)$
$q_{v2s}(x)$
$q(x)$
$\bar{q}(x)$
standard F.T.
Backus-Gilbert
fit
NNPDF3.1
x
x

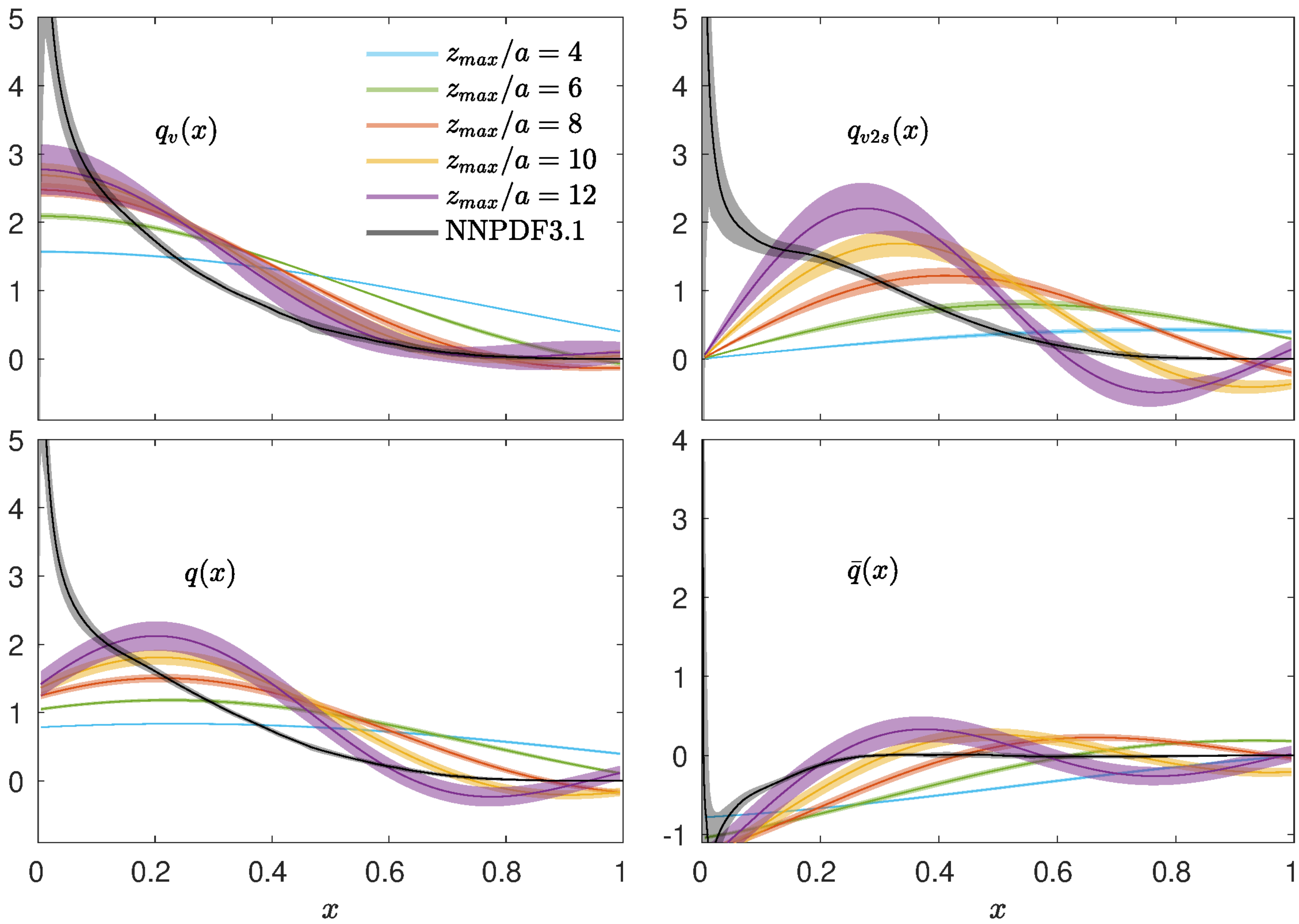
PDFs using ITDs with $z_{\max} = 8a$



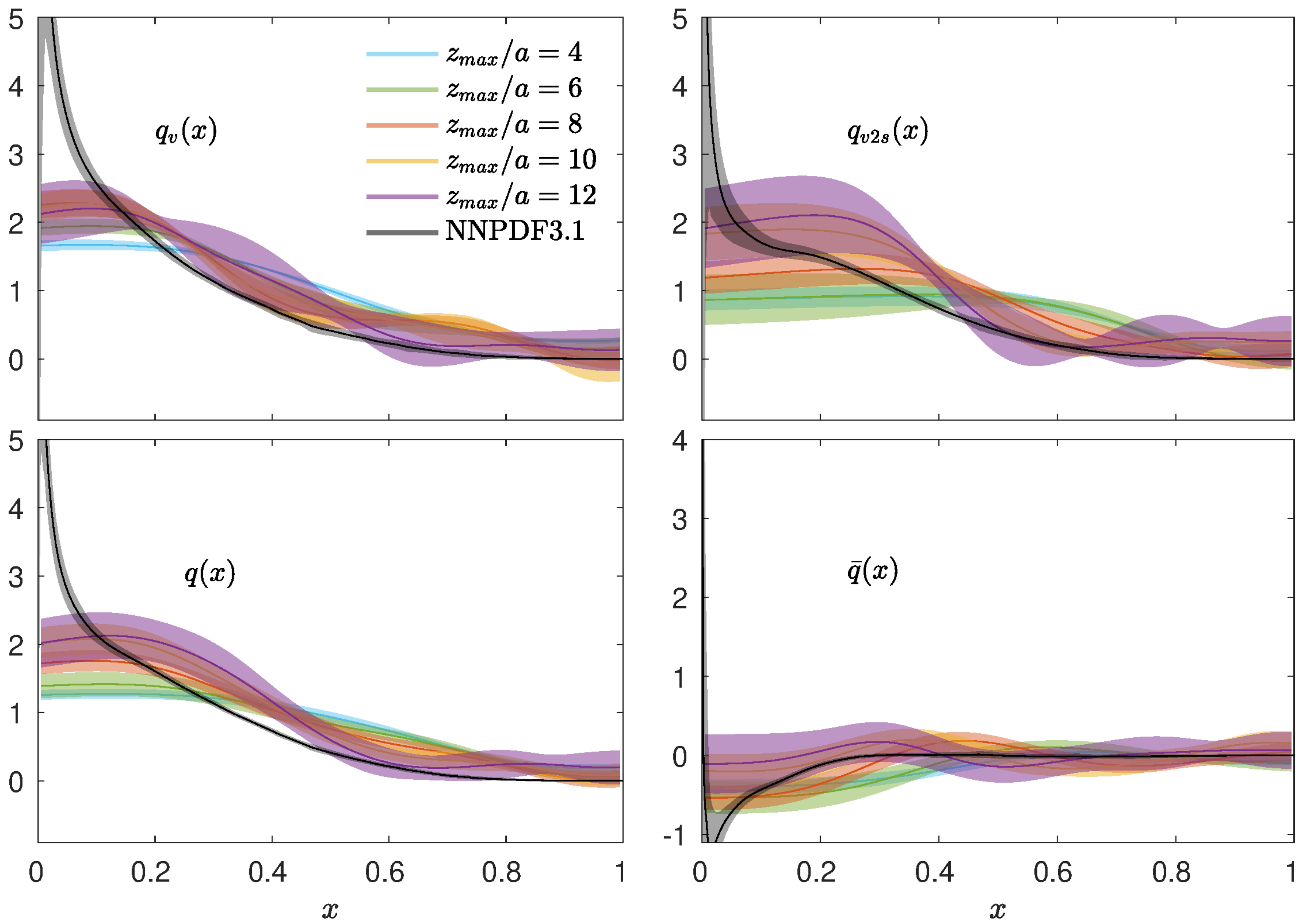
PDFs using ITDs with $z_{\max} = 12a$



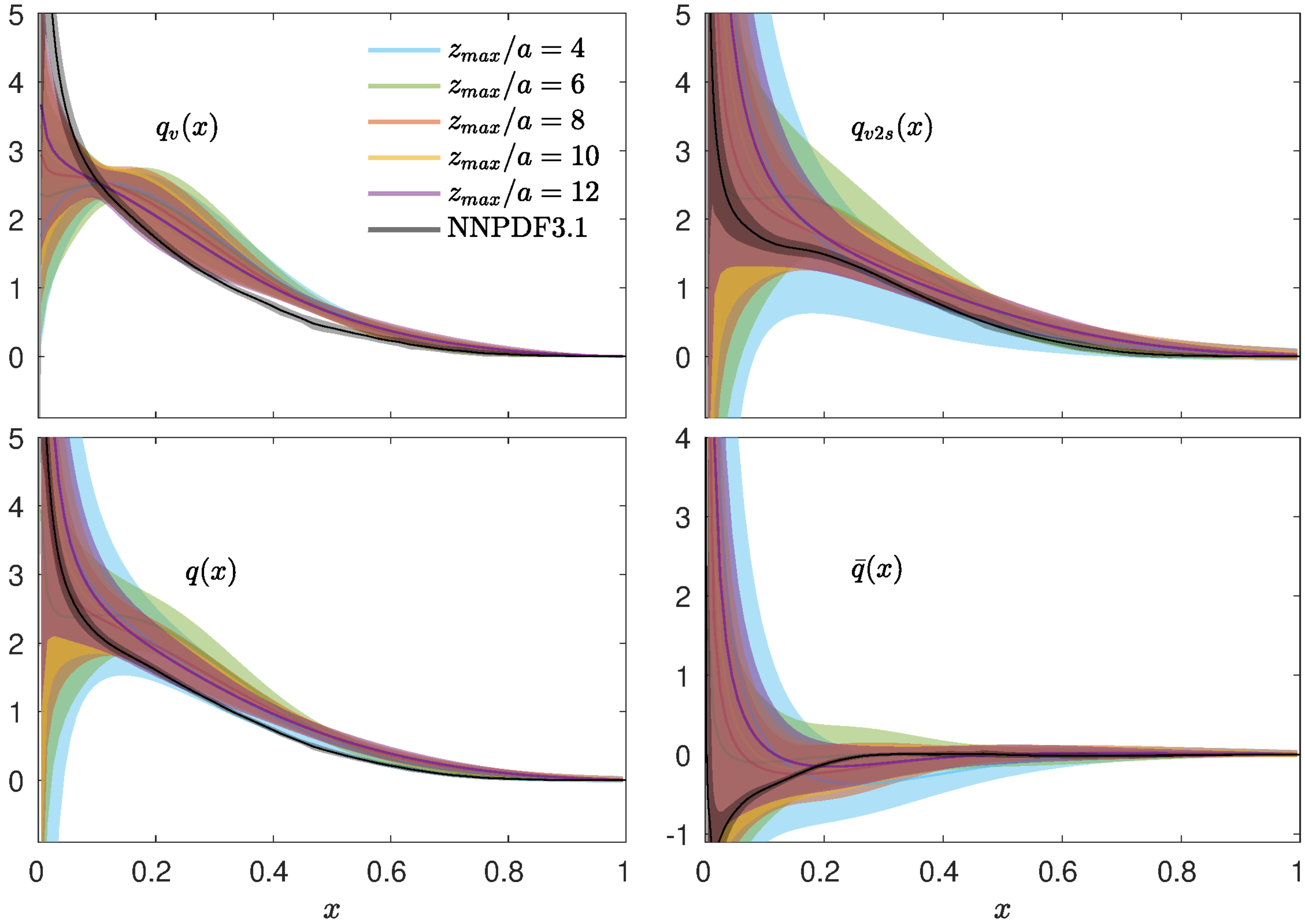
PDFs from naive FT – $z_{\max}$-dependence



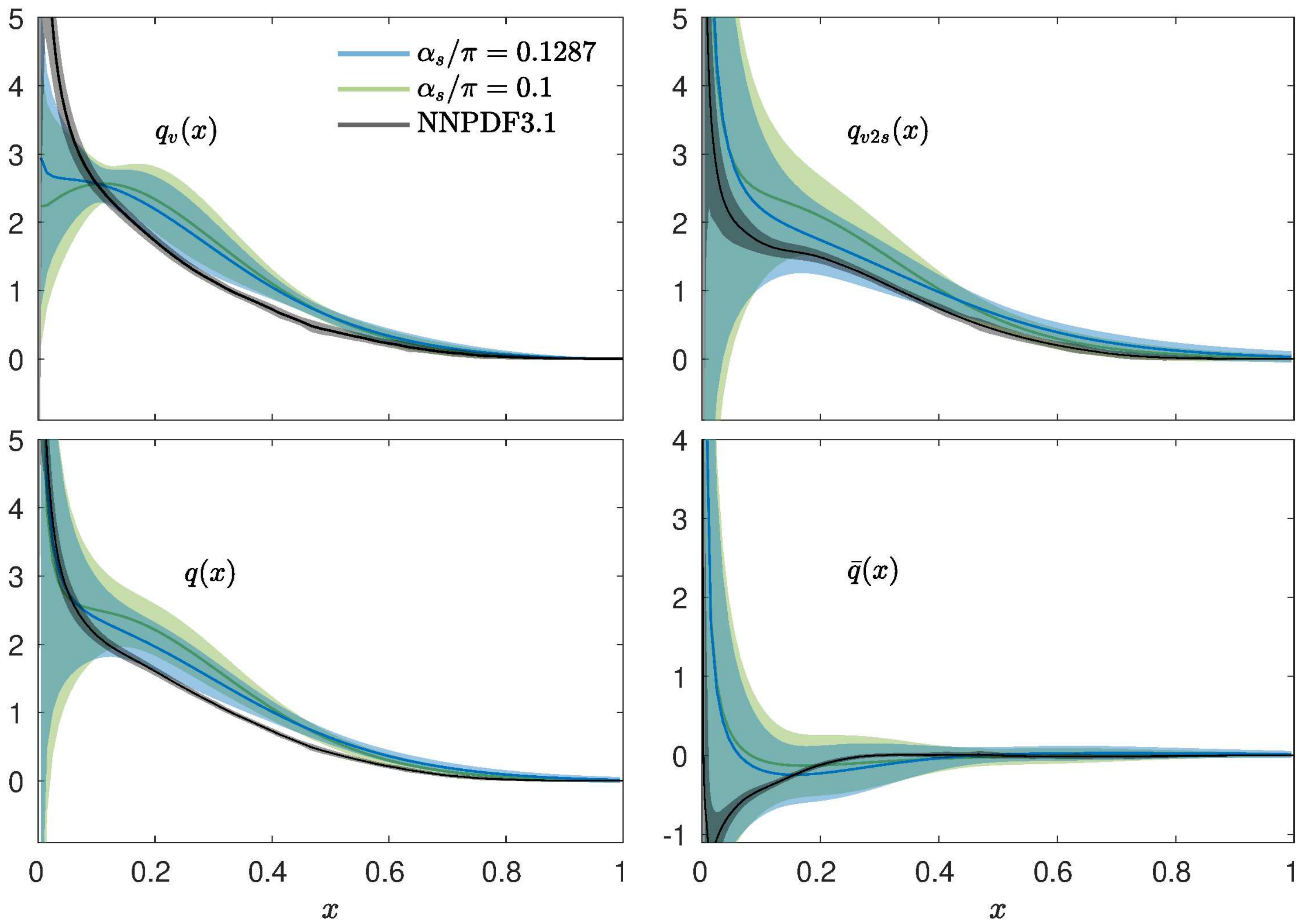
PDFs from BG – $z_{\max}$-dependence



PDFs from fits – z_{max}-dependence



PDFs from fits – α_s-dependence



Systematics

Quantified systematics:

- $z_{\rm max}$: $\Delta z_{\textsf{max}}(x) = \frac{|q_{z_{\textsf{max}}/a=12}(x) - q_{z_{\textsf{max}}/a=4}(x)|}{2}$,
- α_s: $\Delta\alpha_s(x) = |q_{\alpha_s/\pi=0.129}(x) - q_{\alpha_s/\pi=0.1}(x)|$.

Estimated systematics:

- Discretization effects: assume 20%
  indirect support: no violation of continuum dispersion relation, $E^2 = P_3^2 + m_N^2$,
  computations of moments of unpolarized PDFs by different groups: deviations of $\mathcal{O}(5-15\%)$
  from continuum at similar lattice spacings.
- FVE: assume 5%
  indirect support: $\exp(-m_\pi L) \approx 0.05$ for our setup,
  enhanced FVE? R. Briceño et al., Phys. Rev. D 98 (2018) 014511
  toy scalar model, relevant parameter for FVE: $m_N(L-z) \longrightarrow$ tiny,
  worst case: relevant parameter for FVE in QCD: $m_\pi(L-z) \longrightarrow$ still rather small for small z/a,
  also indirectly no indication for such effects in Z-factors for quasi-PDFs.
- Excited states: assume 10%
  evidence: ETMC, Phys. Rev. D 99 (2019) 114504 – suppressed below stat. precision
- Matching (truncation effects and HTE): assume 20%
  indirect support: little dependence on α_s and on $z_{\rm max}$
  needed: 2-loop matching, explicit computation of HTE?

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

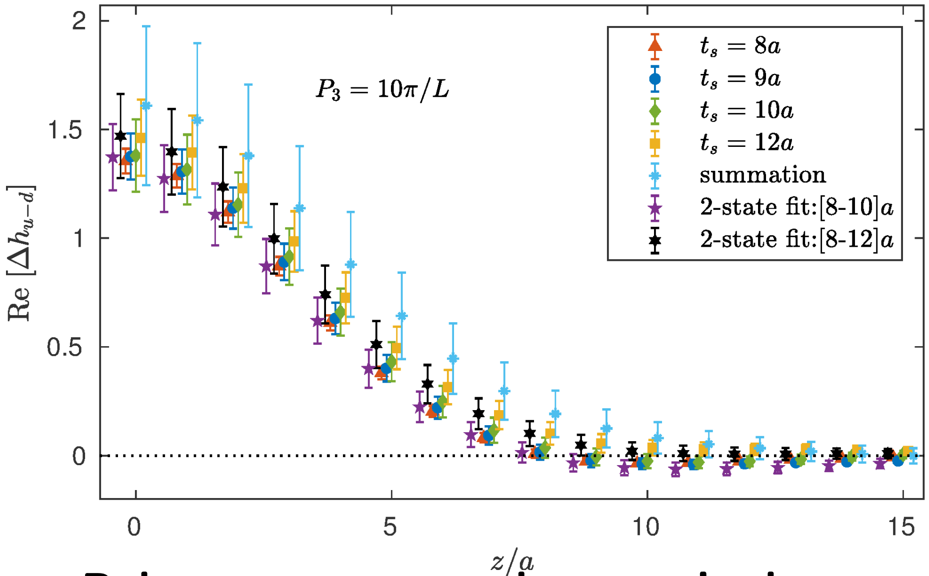
1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \rightarrow Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \rightarrow q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

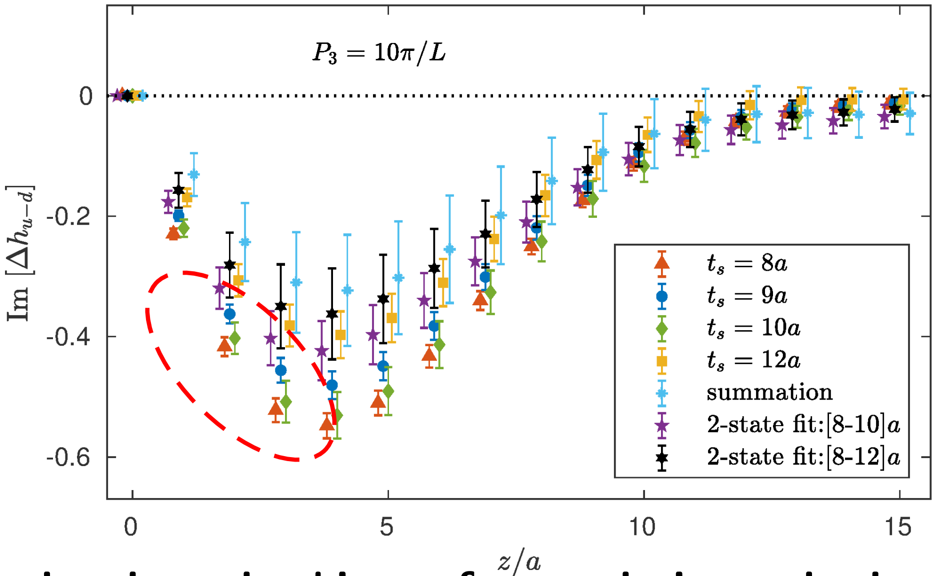
Choice of nucleon momentum

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing → UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state → excessive excited states contamination → one needs to go to large enough source-sink separation t_s ⇒ COSTLY!





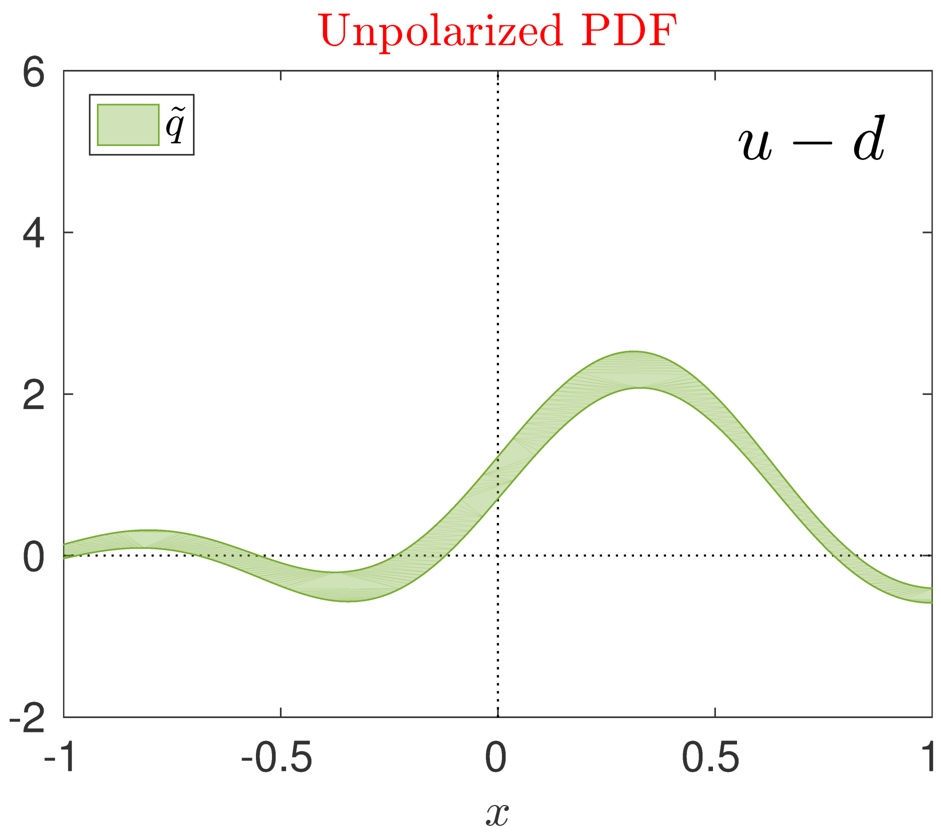
- **Robust statements about excited states only when checking a few analysis methods.**
  here: 2-state fit with $t_s/a = 8, 9, 10, 12$ shows full consistency with the 1-state fit at $t_s = 12a$.

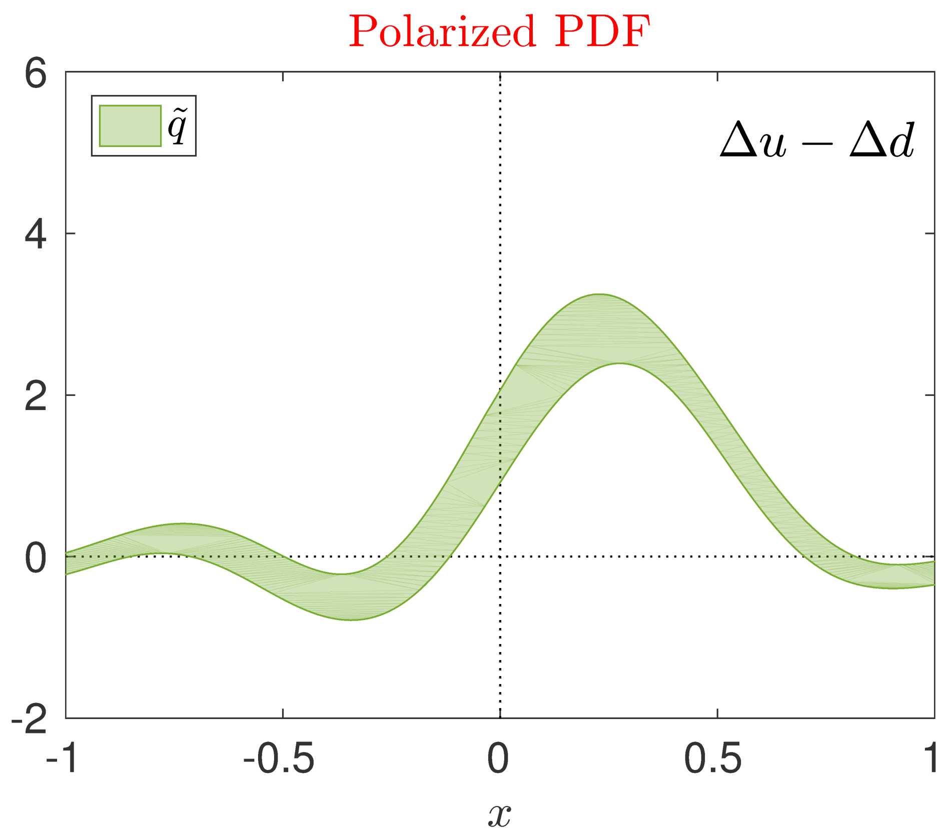
Our largest momentum: ≈ 1.4 GeV

- safely below UV cut-off,
- excited states contamination shown to be smaller than statistical errors.

Fourier transform

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

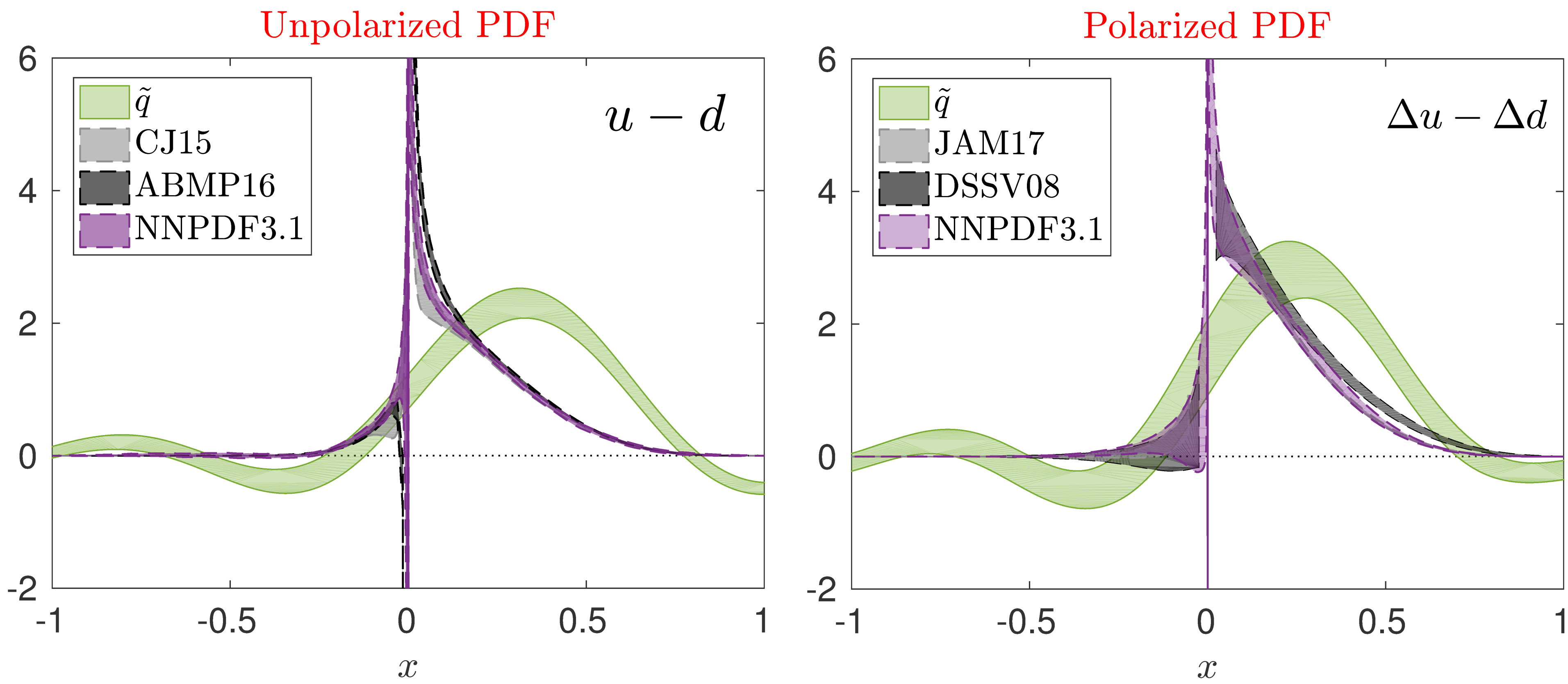




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Quasi-PDFs + pheno

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

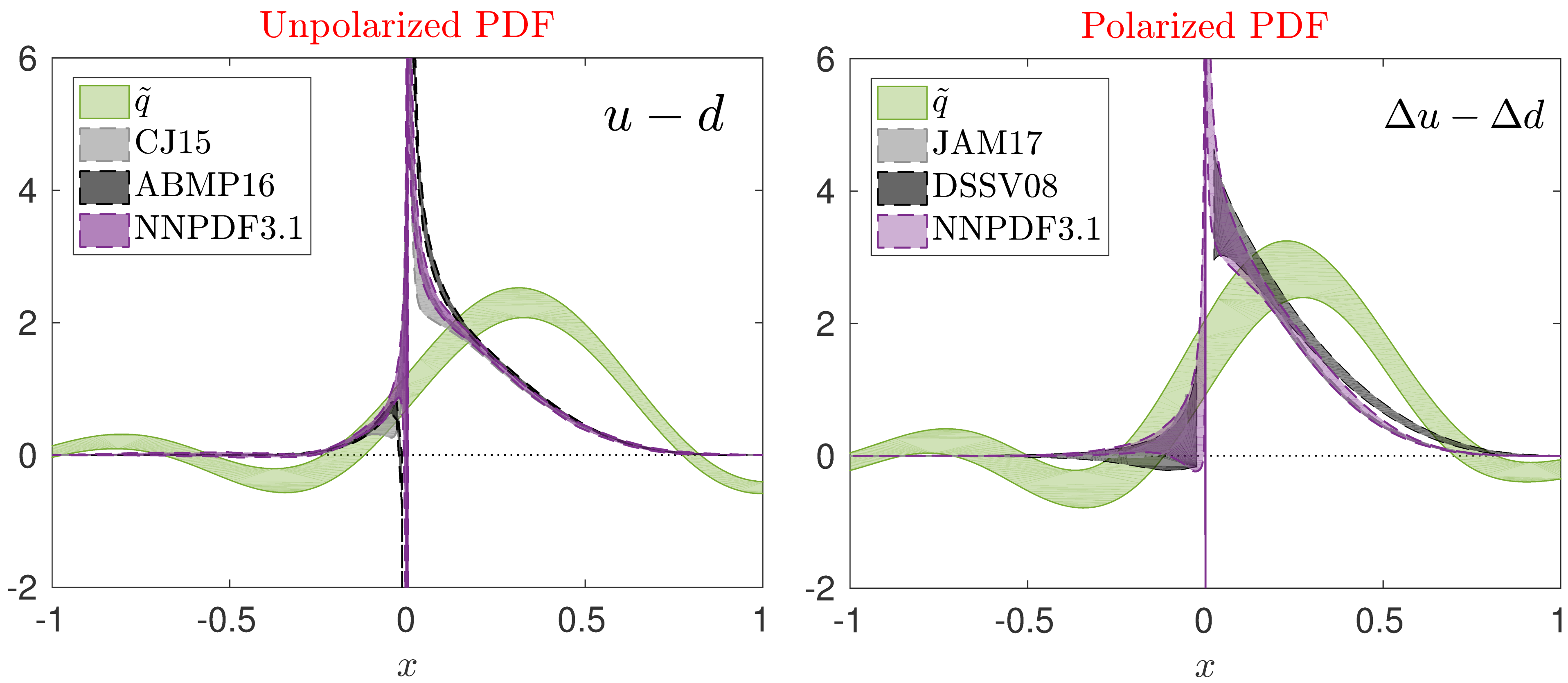
$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{x^2P_3^2}{\xi^2\mu^2}(4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF $\to$ $\mathrm{M}\overline{\mathrm{MS}}$ scheme.
- In this procedure, vector current is **conserved**.

Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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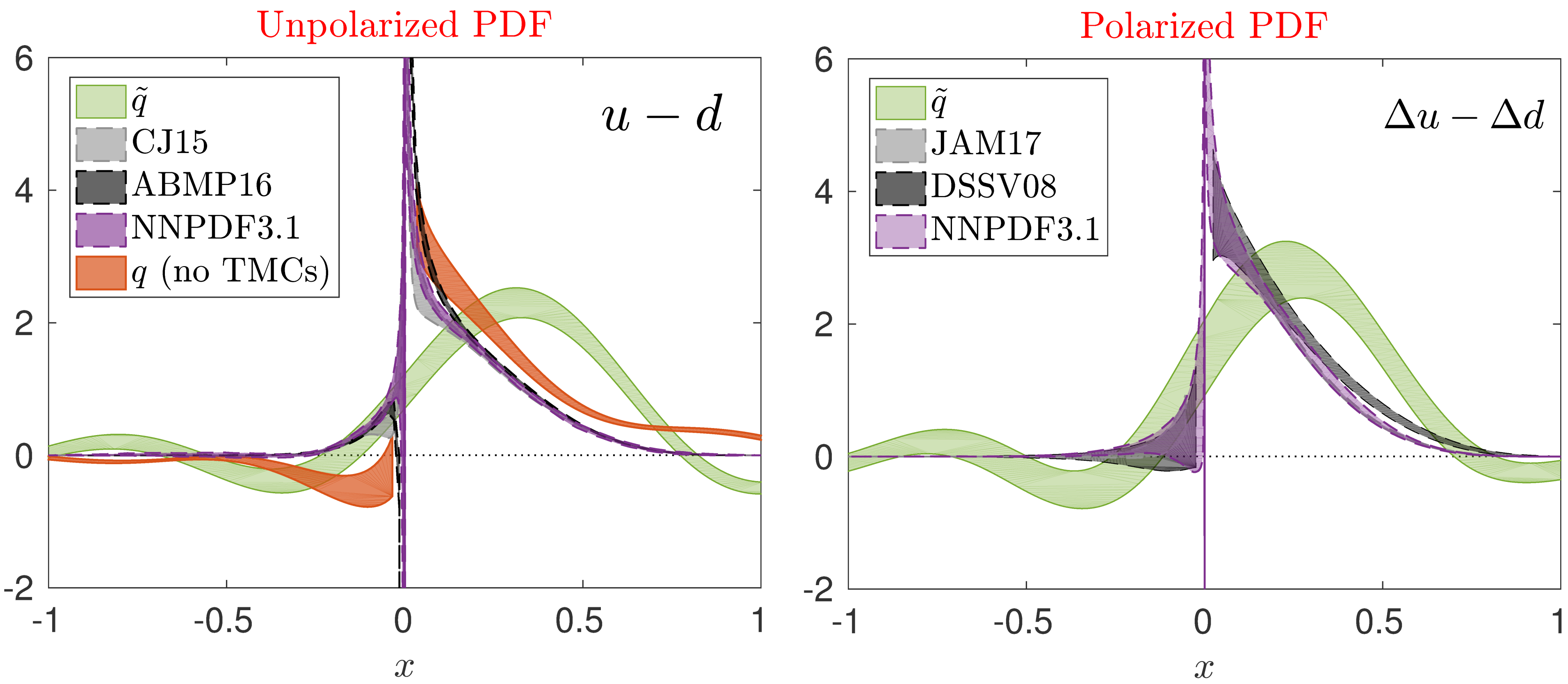

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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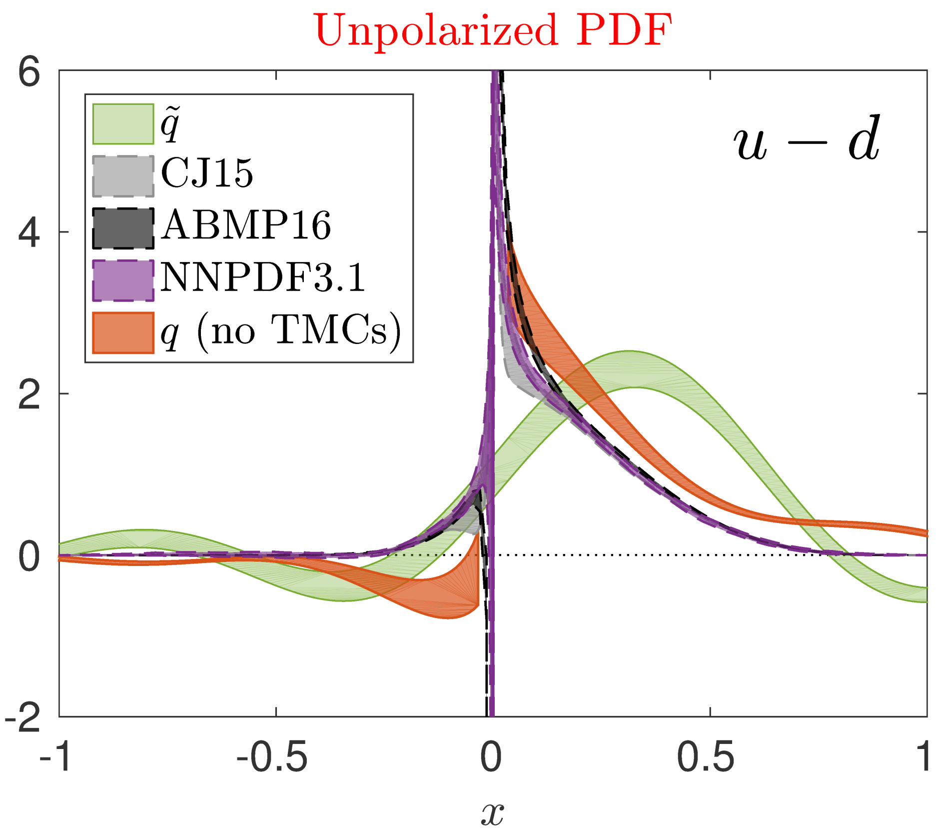
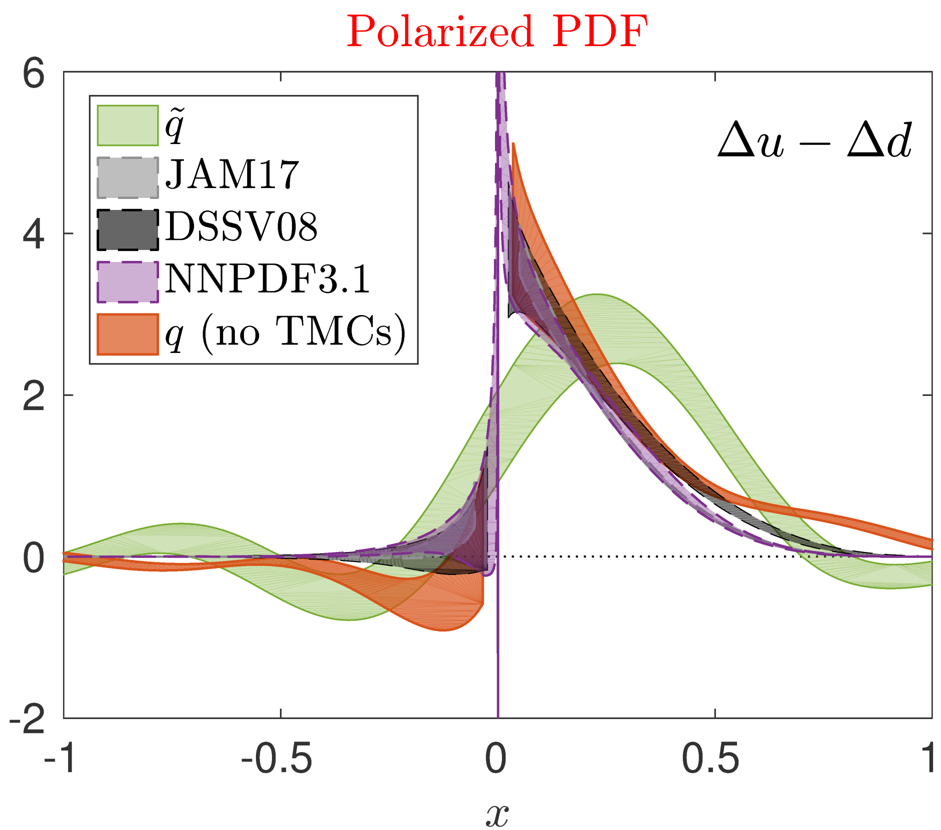
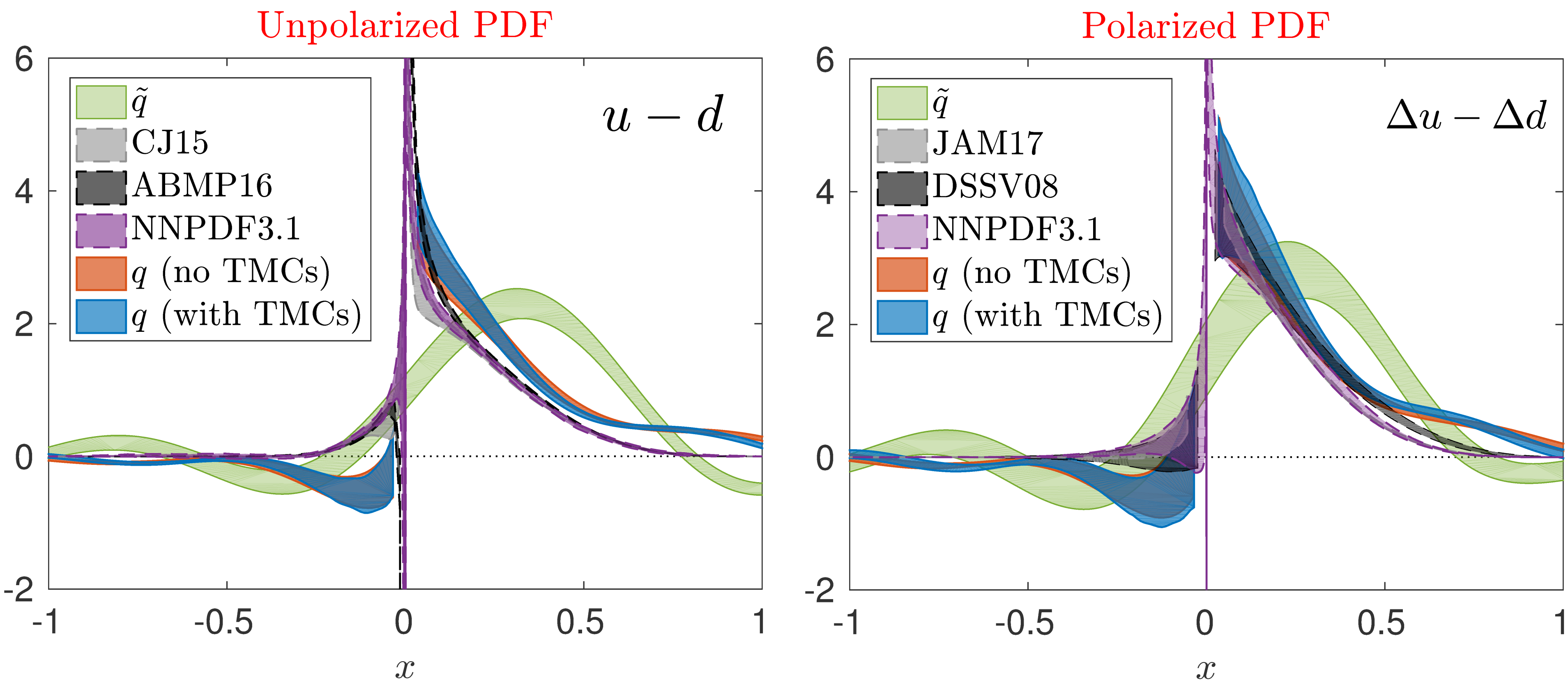




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

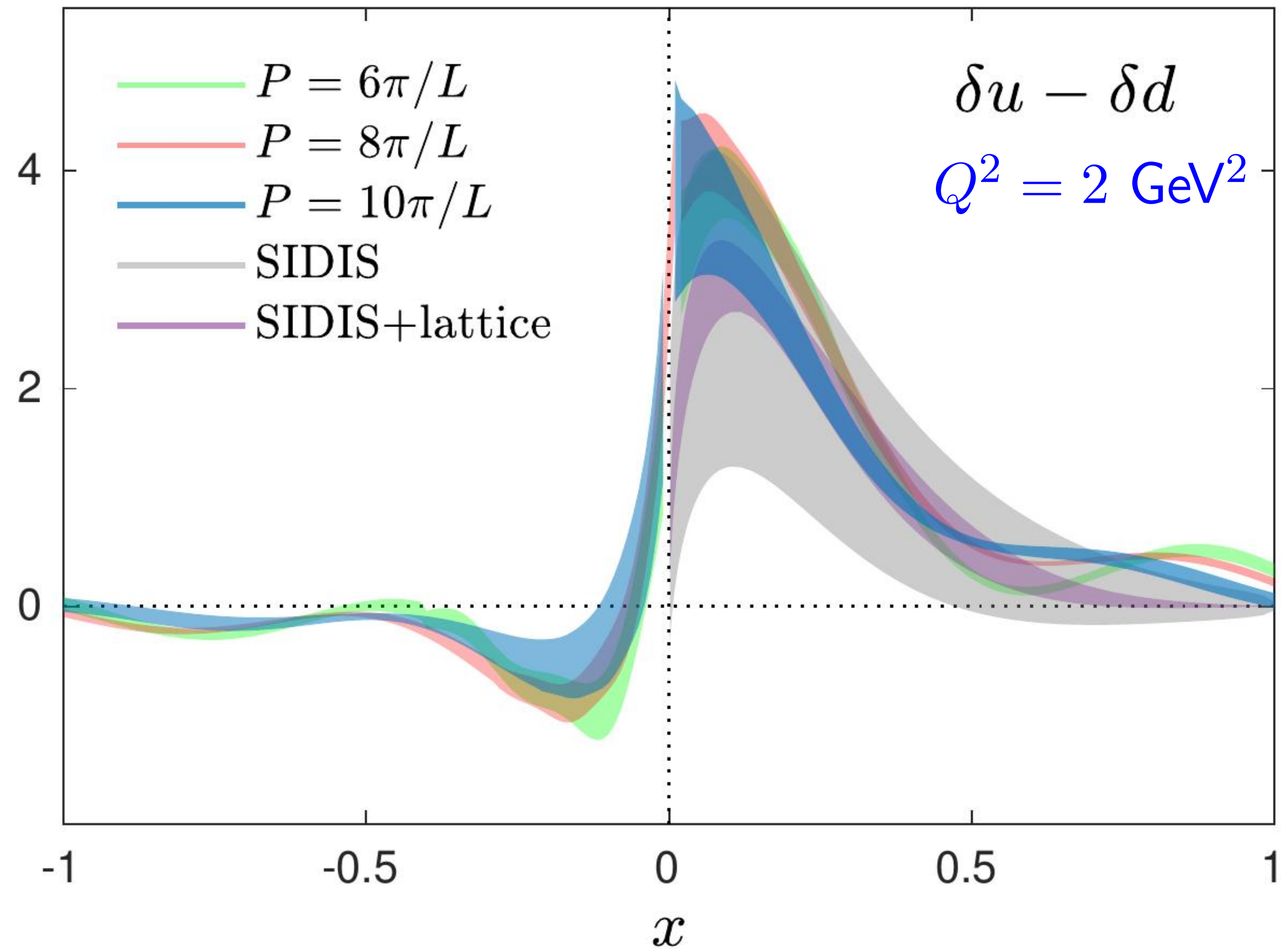
Matched PDF + TMCs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



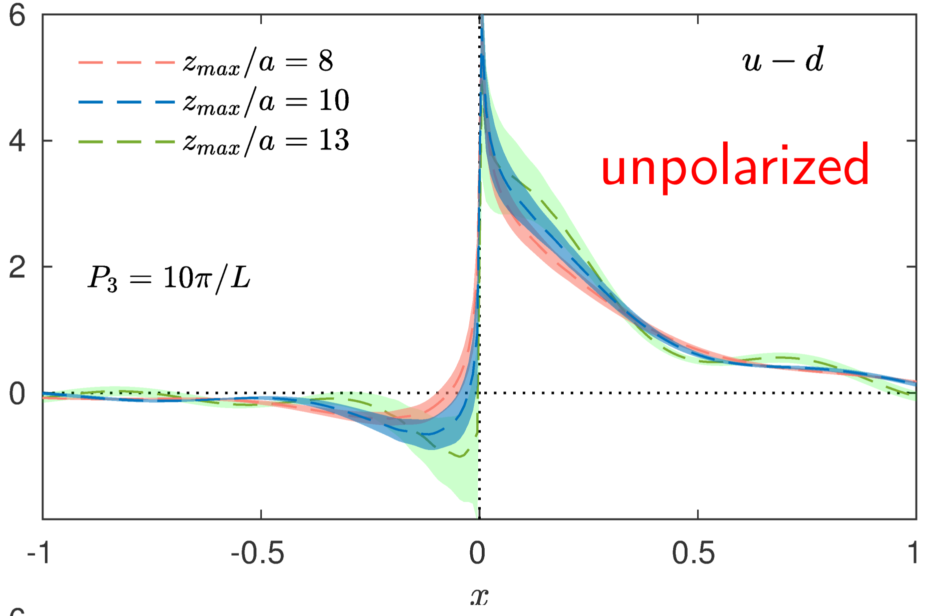
C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

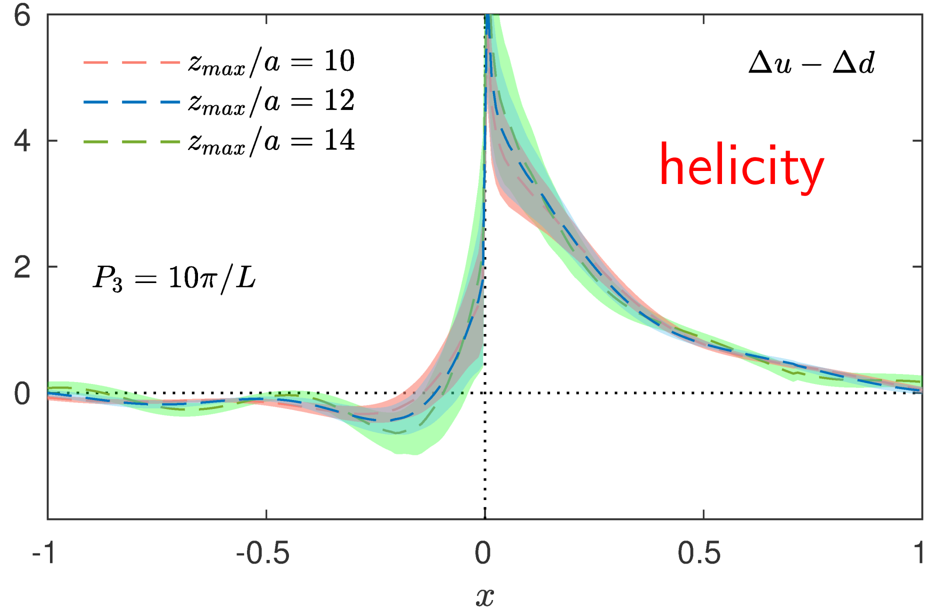
C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)

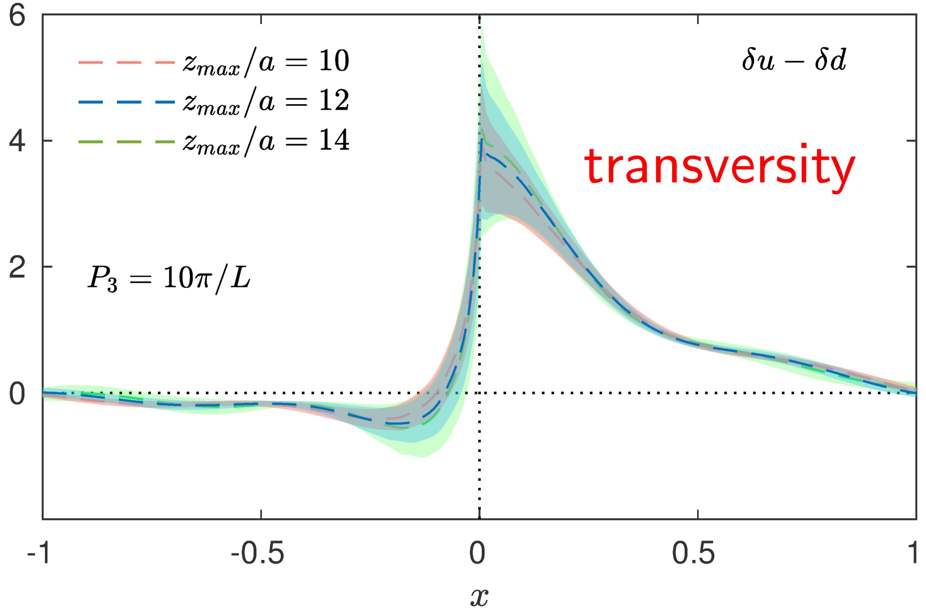


Statistical precision already much better than the precision of phenomenological fits from SIDIS: JAM Collaboration, Phys. Rev. Lett. 120 (2018) 152502

Truncation of Fourier transform





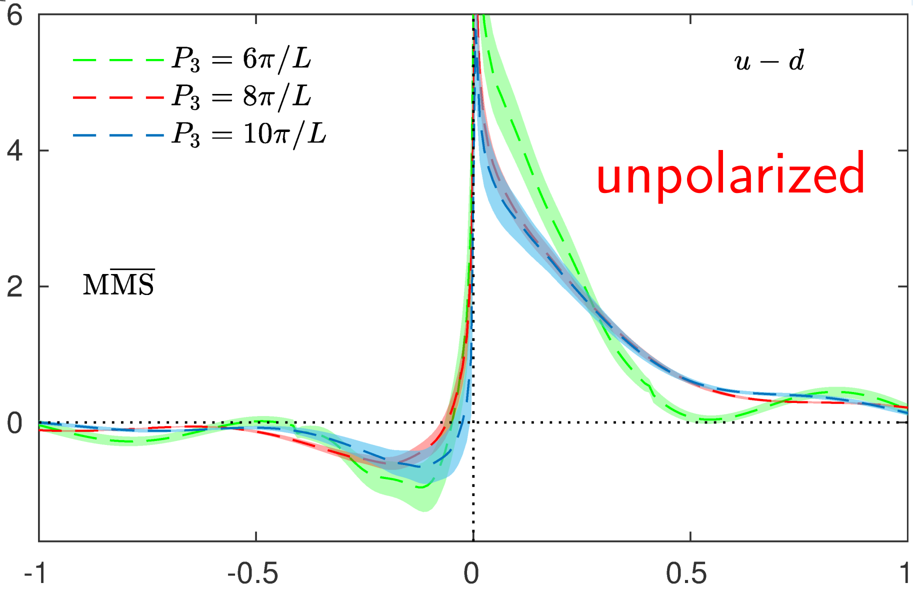


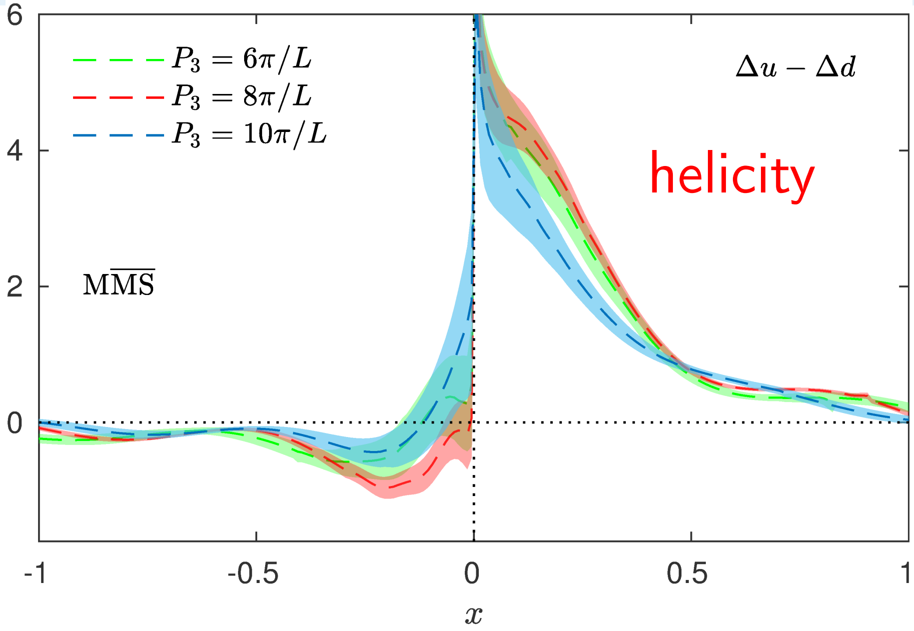
Nucleon momentum $\frac{10\pi}{48}$

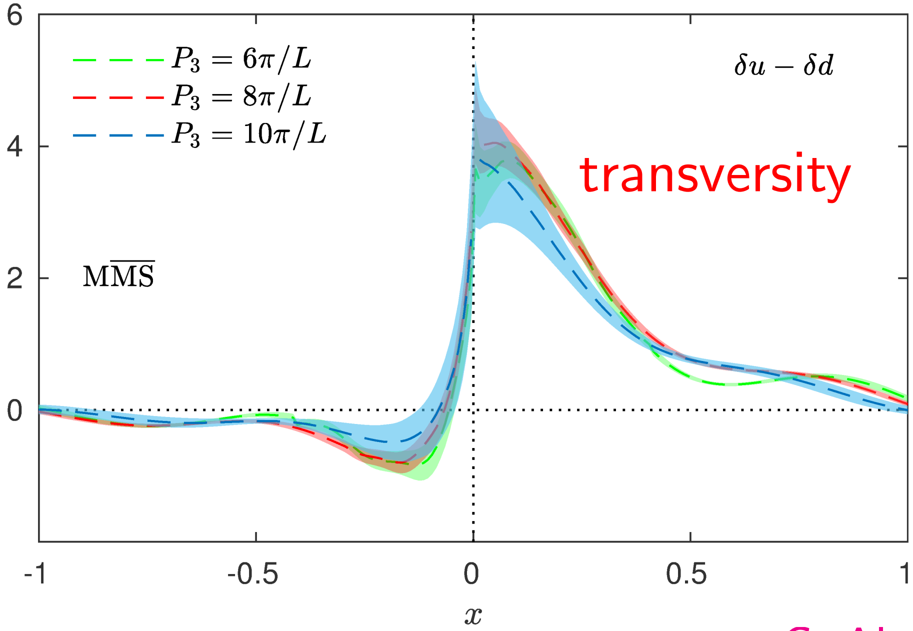
Needs the use of advanced reconstruction techniques
J. Karpie et al., JHEP 1904 (2019) 057

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Momentum dependence of final PDFs







Nucleon momenta $\frac{6\pi}{48}$, $\frac{8\pi}{48}$, $\frac{10\pi}{48}$

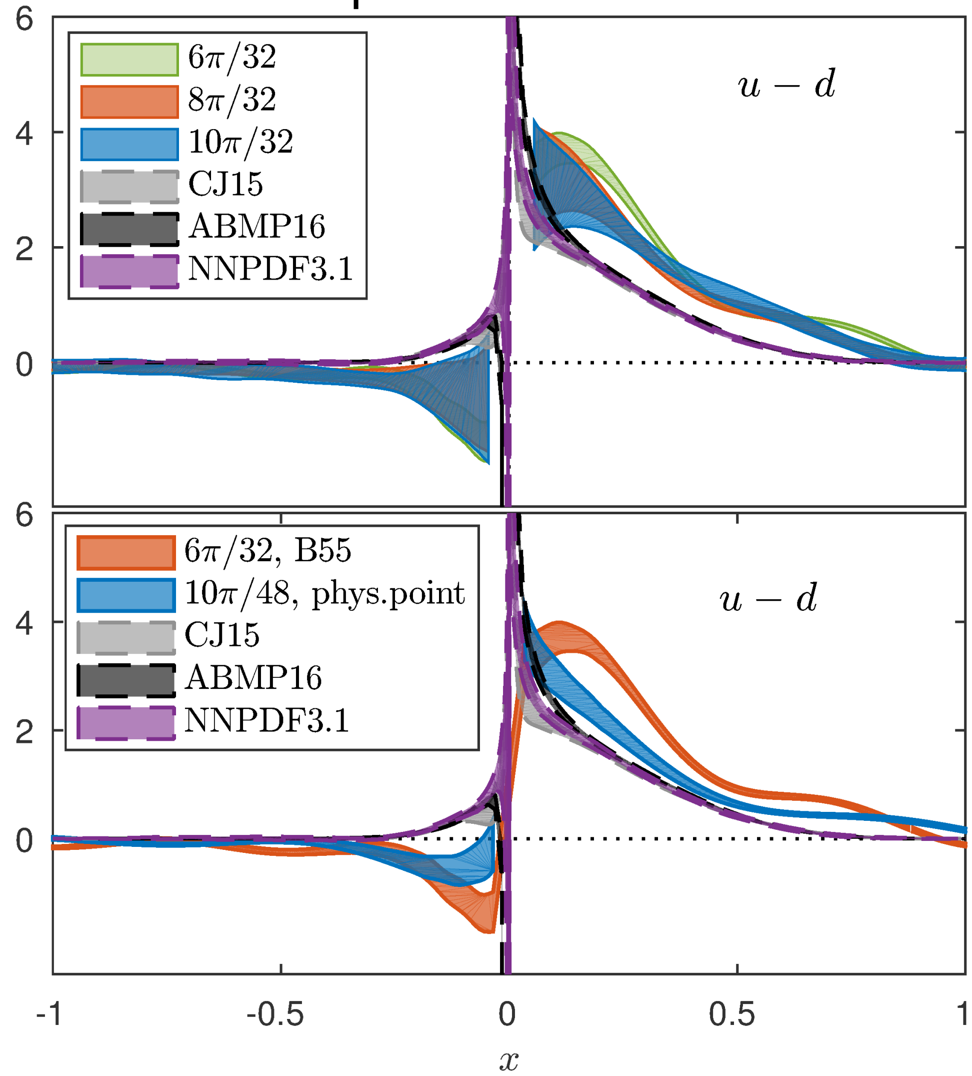
Results seem to indicate convergence in nucleon boost

Expected HTE:
$\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Comparison with non-physical pion mass

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001