Activities accompanying Precison Monte Carlos:

past, today, and future with FCC perspective

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(A) Life experience three ambiguity regimes **(i)** $\sim$ 1, 0.5% **(ii)** 0.3 % **(iii)** 0.043 % .

More thyan 43 years, Stanislaw Jadach effort started before 1981. Monumental projects of B. Lynn, R. Stuart, D. Bardin, W. Hollik, B.F.L. Ward. Important, indirect, insight/inspiration from lectures in Cracow: by W. Furmanski, A. Burzynski, H. Arodz., Z. Klimek. I realized importance of this background only years later ...

Bumpy path, often fog covered of unfulfilled expectations, non confirmed conjectures.

(B) Incomplete lists of needed methodology domains, and interwined projects:

1. (i) Phase space: symmetries (ii) matrix element preparation $\rightarrow$ factorizations (iii) program and development process design (iv) testing strategies (v) user interaction (vi) software tools (vii) partners and competitors

2. (i) FOWL (ii) GENRAP (iii) Mustraal (iv) Koralb (v) Lesko (vi) Tauola (vii) KoralZ, (viii) Lumlog (ix) Oldbab (x) Bhlumi (xi) Bhwide (xii) KKMC

(C) Challenges! I will use LEP 1 experience $\rightarrow$ there 0.1% precision threshold crossed.

Trivialities

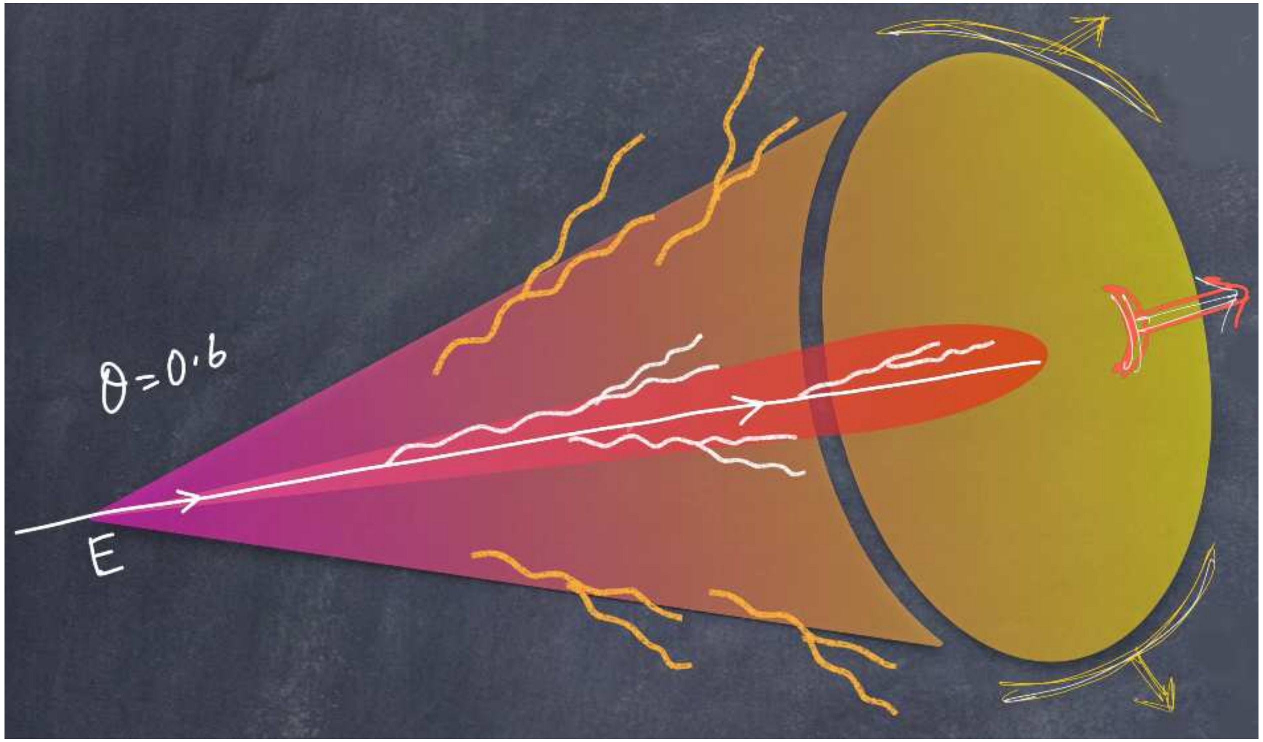
- We want to confront measurements with field theory predictions.
- Agreements mean precision tests of theory, measurements of coupling constants masses etc. Disagreement point to **NEW PHYSICS**
- What is needed to perform such program in case of accelerator measurements? Especially in cases when precision is essential?
- I want to comment on possible future work directions.
- Too many aspects too little time....
- I can see 3 precision regimes: (i) easy: 1% , (ii) intermediate (unfolding?): 0.3%, (ii) challenging: 0.1% and better.
- These precision regimes are defined by the size of α_{QED} and detector granularity.
- At level (iii) interplay of all aspects play the role. **Message for theoretical sophistications.**

Introduction: experimental side.

- Experiments (even pp) **measure best** directions of muons, **then** of electrons, **then** energies of muons, later of electrons. Jets are measured less precisely.
- Experiments, needs to adopt acceptance details, often irregular and direction dependent. Detectors consist of rectangular (nearly rectangular) cells, forming lattice or better say kind of 3 dimensional web. In this web there are dead zones (because of cables etc.). Detector response in barrel and forward regions differ.
- Sometimes, parts of detectors are faulty, and this makes even more irregular shapes of acceptance domain.
- **Picture of detector induced logarithms:**
  •: $\ln(1\ m/1\ mm \simeq 1000)$ (for collinear),
  •: $\ln(E_{max}/E_{min} \simeq 100)$ (for energy) logs.
- Points above define what is required for precision thresholds.
- The first two do not bring big complications, precision requirements are not very demanding.
- **Now** lessons from from the the past.

Precision regimes from experimental side

Precision regimes



(i) (i)Easy about 1% or even less strict.

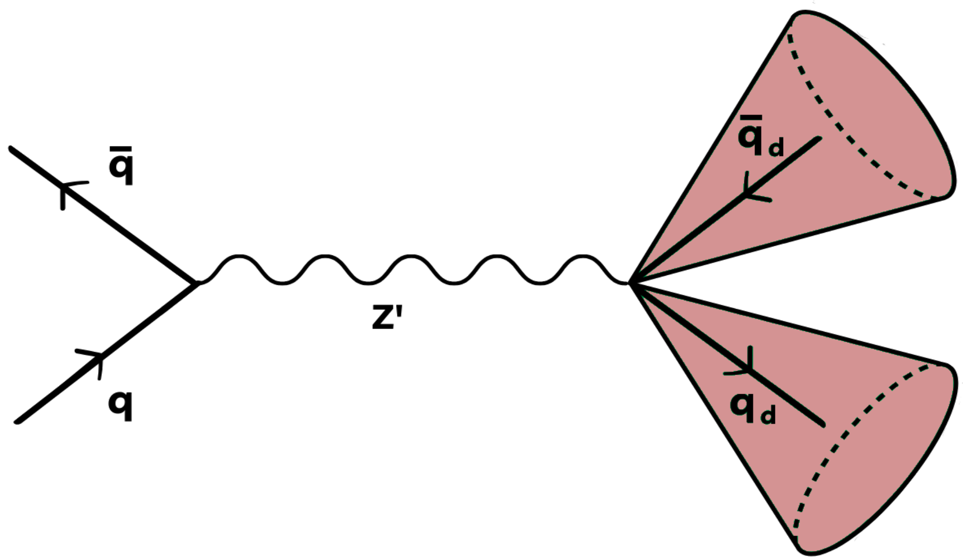
Cones are good for theory and experiments

For example, collinear photons do not need detailed treatment

Interferences between emissions around distinct sources are below ambiguity threshold

Precision regimes from experimental side

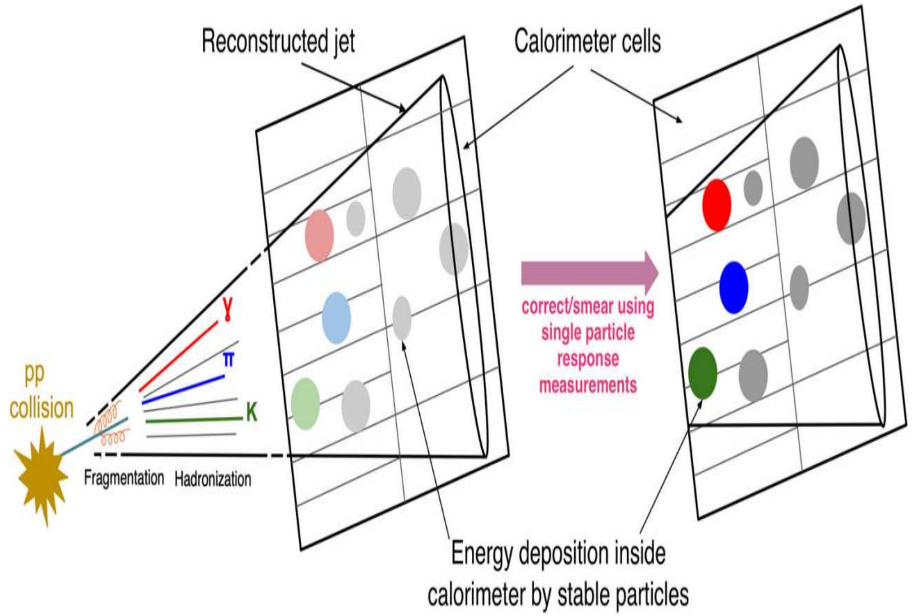
(ii) Intermediate level 0.3% **Theory:** interferences of emissions from distinct cones need to be checked and sometimes corrections need to be added, but they remain relatively small, no worry about interplay with other effects. E.g. only for optimal variables, used for fits. Similar treatment can be prepared for genuine weak corrections, etc.



Precision regimes from experimental side

ATLAS Collaboration, Eur. Phys. J. C (2025) 85:927

Fig. 1 Sketch of the particles associated with a jet and the corresponding energy depositions in the calorimeter. The colors represent the energy depositions of the various particle types. In the deconvolution method, the energy depositions are corrected and smeared to obtain the data-to-MC jet energy correction from calorimeter response measurements to single particles



(ii) Intermediate level 0.3% **Experimental side:** some attention needed too.

Around 0.3% level cones are still good for experimenal side, general refinements, interpolation to cones is enough.

Can this be named the unfolding regime?

But what changes if precison is even better, e.g. 0.043% like LEP 1 luminosity?

ALEPH Collaboration, Z. Phys. C 53 (1992) 375–390

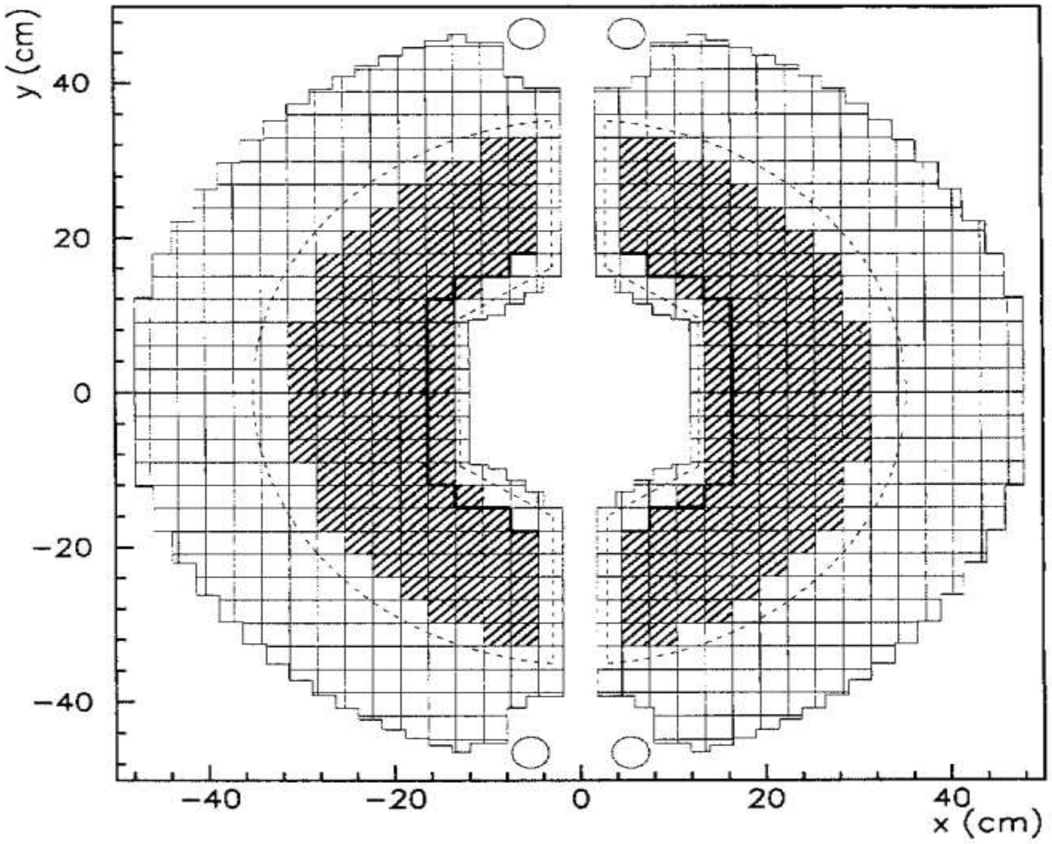
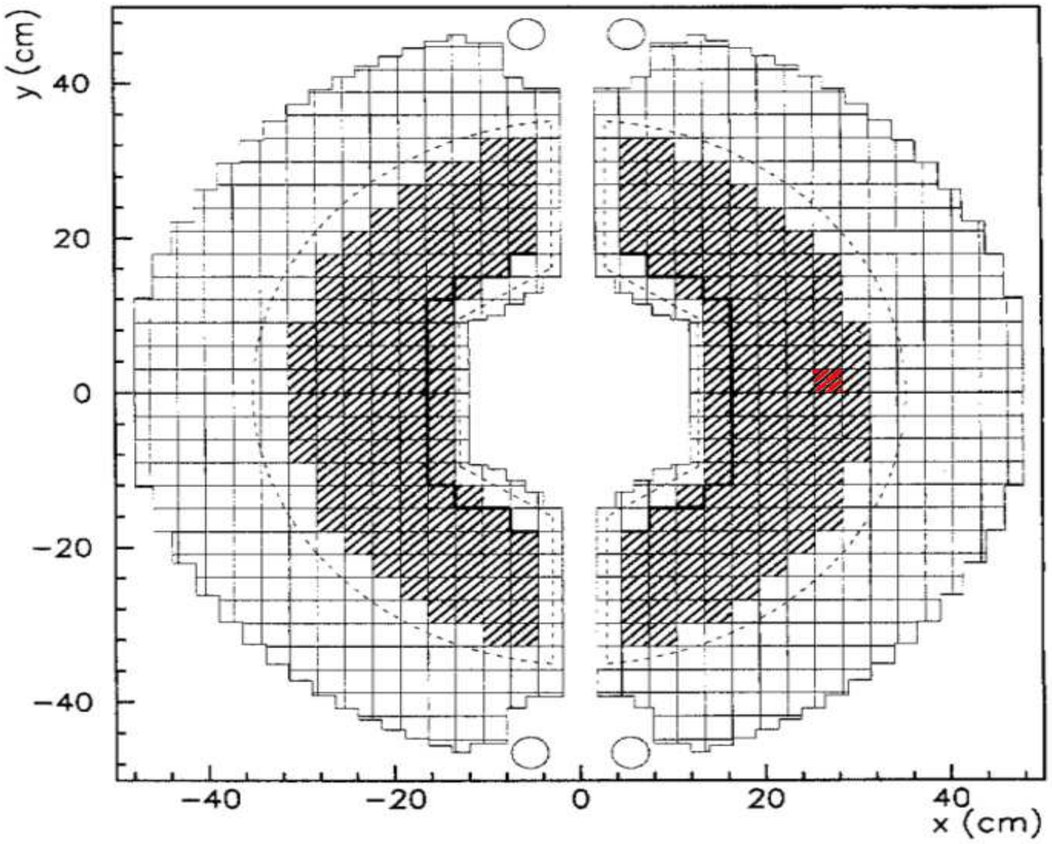


Fig. 2. The active area of an LCAL module showing the fiducial (hatched) and non-fiducial (dashed outline) regions. The squares represent the PCB pads; they are shown at $z = 280$ cm, the average position of maximum energy deposition. The bold line shows the inner boundary of a more restrictive fiducial area which is used for systematic studies. The support dowels are also shown

usually nice electron energy deposit

ALEPH Collaboration, Z. Phys. C 53 (1992) 375–390



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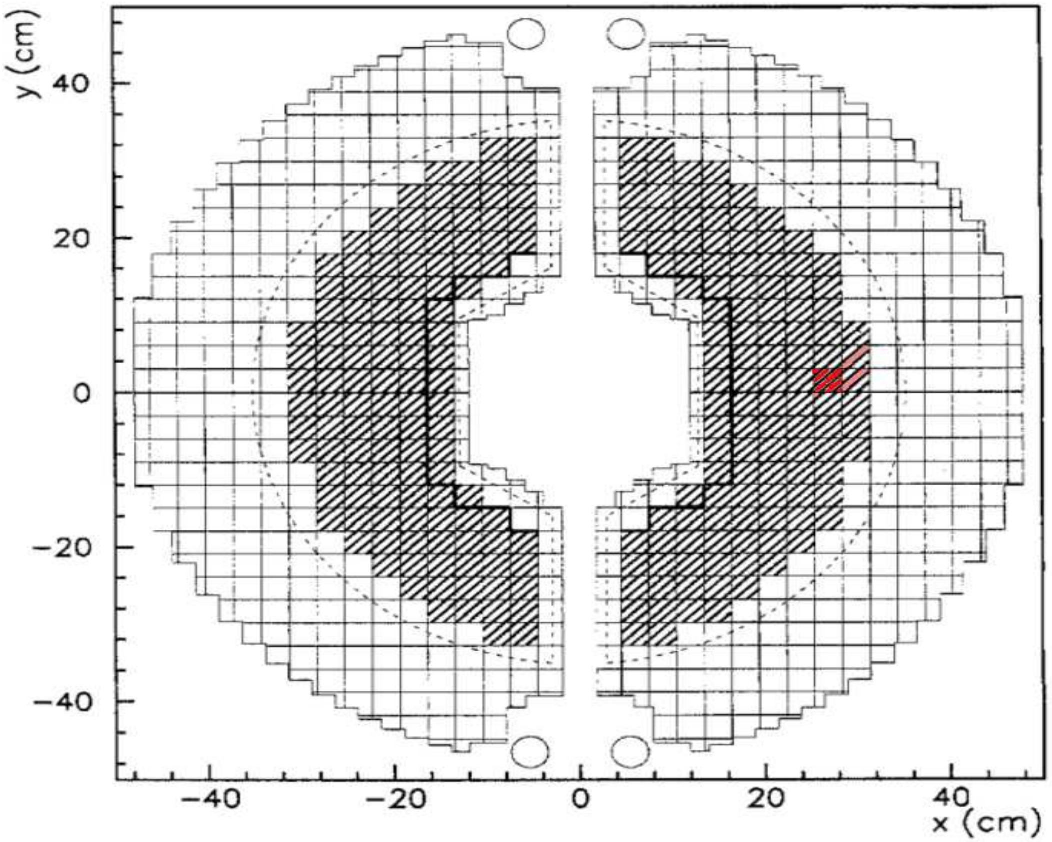
Deformaiton spotted! could be detector fault, but sometimes due to 3 nearby brem-photons.

ALEPH Collaboration, Z. Phys. C 53 (1992) 375–390



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Observation

- Level (iii) precision better than 0.3%:
  both theory and experiments side, have to take into account granularity.
- With improving precison level, more and more of the details need to be taken into account simultaneously.
- FCC expect precision better than 0.01% . All details will need to be taken into account simultaneously. That means in particular:
- detector details. Note that lepton with close extra photons (pairs) may be complicated object, looking like faulty detector cell or background.
- interferences between jets (hadronic or *electromagnetic*) “jets”.
- In short, all details, also of spotted dead detector cells.
- This, may be not so complicated for all observables, also not for New Physics.
  May be threshold, for (iii) level, will be bit lower than 0.3% ...
- **NOW** what may be/was needed from theory side for level (iii).

From the past...

- I will not cover details of the projects. Typically each one is involving many man-year efforts.
- I will rely on achievements, LEP observations. I will underline importance of efforts, not directly on pheno.-final programs, but of acitivities developed independently. More than often they needed laborous adaptations and complicated,interactions with users.
- Hopefully that experience will be useful for work on the future 0.01% precision regime of experiments such as at FCC. Precision at least one order of magnitude better than that at LEP.
- But for high precision or for rare processes where background tails play important role, **nothing is easy....**
- Precision predictions require managing **conflicting requirements**. Most important are those from experiments, because they involve massive human and financial efforts. That is why, I have mentioned that at the start. Can phenomenology work ease in these burdens? **Remove no-go-es**?

From the past...

- Theoretical calculations, because of ultraviolet infinities, tend/must be organized around renormalization schemes. That is understandable, that is the way. That implies that usually mass corrections (thus phase-space details too) are added later, sometimes at lower perturbation level.
- What does it mean, what are challenges and what are the solutions.
- Having theory and experimental things in one program is a target for level (iii)
- Nonetheless fitting functions (analytic or semi-analytic) with idealized acceptance, are essential, even now in ML world.
- Also intermediate detail level tools are helpful and I was advocating such solutions at the time.
- **Advantage:** For simplified acceptance with dressed leptons etc. mass corrections are less important and in general, higher order results are easier to get and incorporate.
- **Disadvantage:** detector acceptance details have to be calculated/simulated separately. Does it mean pushing difficulty on somebodys else desk?

From the past...

- From (ii) to (iii) We have used solutions of incomplete phase space details, and it worked well down to 0.5 % precision level.
- First-order simulations with correlated leading logs simulations of collinear multi-photons OLDBAB+LUMLOG
- **But** at 0.3 %, 0.1%, 0.043% levels they had to give way to more refined tools (remained as essential tests Phys.Lett.B 253 (1991) 469):
- Simulation had to take care of configurations with explicit 3 photons even at LEP. Long years of work for LEP to make it field theory consistent. FCC Challenge: 5 photons, EW second order, QED third order, eikonal QED 5th order.
- Such configurations are essential to understand effects due to detector cell structure, but their inclusion complicates installation of other theory effects. That points to important work domain. E.g. separate EW loops into QED and rest.
- – Simulations with all details of phase space and higher order induced kinematical configurations included, on one side, – fitting functions on other.

From the past...

- At somewhat lower (ii) precision level, simplified phase space tools, useful.
- Can the limit to (iii) be pushed below 0.1% ? At LEP 1, I tried longer than my colleagues and finally failed. Exclusive exponentiation supported with work on electroweak corrections to separate out QED part took the lead.
- I will not be able to demonstrate theoretical proofs. But this was solid work.
- How did we worked, before solutions of exponentialtion and separating out QED from EW become available?
- Later, many of these solutions became source of valuable tests.
- Some, turned out to be sufficient.

From the past. Two programs full phase space.

- An idea: $(1+A+B)^2 \rightarrow (1+A)^2 + (1+B)^2 - 1$ Define A and B well!

  A- effects of all tree-level diagrams with respect to double resonant ones (initial state exclusive exponentiation of QED only)

  B- effects of exclusive exponentiation for initial and final state QED effects but for hard interaction of double resonant diagrams only.

- **KandY** hep-ph/0104049: correlated simulations of 4-fermion final states with initial state exponentiation KORALW **all tree diagrams**, YFSWW for initial **and** final state exponentiation. Hard interaction **double resonant diagrams only**.

- That was essential step of work on Monte Carlos, and in many cases enough, even if it brings substantial difficulties for the experimental users.

- **Help for future:** step for new processes like for 4 fermion, 6 fermion (or jets) final states (WW, ZZ ZH intermediate states). Solutions will be working at 0.3% level and will open gate to new applications as benchmarks.

- No compromise on phase space details. **KandY was a step** forward with respect to earlier correlated samle solutions like OLDBAB+LUMLOG, even earlier Mustraal+YFS2.

- Useful to match with fixed order field theory calculations.

From the past. Second order QED

- Look at level (iii) and 2 fermion production $e^+e^- \to l\bar{l}$ also Bhabha scattering.

  Two fundamentals:

A Separation of SM into QED and hard interaction corrections: genuine weak, vacuum polarization, some strong interaction (also New Phyisics).

B Eikonal parts of amplitudes $\beta_0 \times \int \ldots$ plus corrections β_1, β_2 available perturbatively. YFS way of reordering of QED perturbation expansion:

- At zero-th order kinematical configurations with arbitrary large number of photons and all over the phase space.

- Gauge invariant, of internal singularity cancellation, sectors: initial/final state and interference (Bhabha upper/lower line and interference), β_1, β_2. Fundamental for matching with non-QED weak.

- Interference (real and virtual can be added at later steps by re-weighting (with bounded from above weights)) For interferences beware of β_1, β_2 and some virtual- real- corrections cancellations.

- Now to list of real topics.

From the past. Second order QED

- QED eikonal amplitude parts for initial and final state interaction can be solved to all orders; results represented as **functional** exponent. • Basis for algorithm of complete phase-space (number of photons is generation variable too). • Crude level: with truncated, but known exactly and analytically amplitudes and phase-space.
- **Possible:** conformal symmetry of eikonal factors and phase space for photons.
- That created basis of algorithm where: • initial state photons, • invariant mass of intermediate state • and final state radiation could be generated independently, • at start, independent photons and phase space limits adjusted with scale (conformal) symmetry.
- Minor technicality: to match virtual corrections an algorithm to deal with photon candidates of energies, after rescaling, below lower generation limit. The lower limit of photons energy can be arbitrarily low, but need be the same for all photons. Fortunately for that regime eikonal part of amplitudes is enough.
- In PHOTOS MC Kinoshita Lee Nauenberg theorem is used; difficulties for second order matrix element implementation.

From the past. Second order QED

- Second step, re-weighting to introduce matrix elements:
- First add effect of QED: β_1, β_2.
- Looks trivial, but is not always so. For ν_e final state one has to add charged Higgs ghost contribution to QED $\mathcal{O}(\alpha^2)$. Is it QED? Emissions from $W^\pm$'s...
- The cancellation of singularities require contributions to be organized first. Only then add hard interactions through *Form Factors*, to avoid ruined gauge invariance. That worked fabulously clear at 0.1% precision level and CMS energies below 205 GeV. but at 2012 GeV things became less intuitive.
- Huge effort on genuine weak corrections was needed, even at one loop level. First star couplings, to get electroweak form factors, finally formal proofs.

- Analytic and anti analytic constraints of field theory need to be preserved. Proven to work well at at 0.1% precision level, no compromises. Anti-analytic features, optical theorem, Cutkosky rules, broken only at $\mathcal{O}(\alpha^2) \sim 10^{-4}$.
- Massive effort of revisiting all perturbation expansion details for its re-ordering, before Monte Carlo and other pheno applications.

From the past, summary...

- Even at LEP precision level, eikonal part of QED was needed to at least third order. Thanks to re-ordering of perturbation expansion one loop genuine weak effects were sufficient.
- For FCC that will probably mean two loop genuine weak plus third order QED and may me 5-th order for its eikonal, real emissions part?
- We have some hints how to start work for 0.01% precision level, see eg 2505.00272 but question is how to keep projects on-going, expertise to be passed to new contributors and goals to be reached.
- Old style solutions OLDBAB+LUMLOG or KORALZ(YFS2) +Mustraal offered gate to the framework of NLL NNLL picture in ambiguities evaluation.
- KandY ; same phase space, Taylor expansion for ME refinements.
- At the end, observable dependent evaluation of ambiguities in % has to be provided. Only that is meaningful for measurement-theory comparisons.

Now for the future...

- Surely, for precision level of 0.01 % two loop electroweak corrections will be needed. Third order QED and possibly up to 5-th order of dominant (eikonal parts) for detector response to configurations with 5 explicit photons.
- Two loops electroweak corrections, separate into parts: QED and genuine weak. This to match with (QED only) higher orders.
- Two loop electroweak corrections will impose new pressures on how necessary higher order QCD/QED corrections, be prepared. This is also true for the part of non perturbative QCD effects taken from low energy $e^+e^- \to hadrons$ data.
- At the level of two electroweak loops, such h.o. QED QCD need to be added for s-channel and for t-channel exchanges in vertex corrections simultaneously.
- Then, what about complex masses schemes? Solutions proposed by R. Stuart and later by A. Denner, to preserve constraints of optical theorem Cutkosky rules need then to remain valid with the first offending terms at $\mathcal{O}(\alpha^3)$ level. And all that with full granularity.

Now for the future...

- At 0.01% precision level corrections due to 4-fermion processes will be needed in full, that complicates patterns for bremsstrahlung, interference's and crude level generation. Language of tangent spaces and CW-complexes may be helpful to systematize matching of collinear/soft sub-spaces. That was the case in PHOTOS Monte Carlo for radiative corrections in decays
- For s-channel t-channel processes contributing simultaneously, use of contact interaction and expansion with respect to contact interaction useful. It was used (hep-ph/0406045) for ν_e and hopefully will be helpful in future too.
- Question of tests. It is so easy to make the code working 'nearly correctly'. Difficult, we had to continue work with respect to orthodox perturbative calculations, automatic algebraic manipulations formulae could not be used without adaptations $\longrightarrow$ laborous steps needed.
- What should be understood as QED subset of SM, what should be understood as eikonal subset of QED?
- OK, it is really needed, but is it possible? What it actually mean?

Now for the future...

- QED may mean those parts of amplitudes which are necessary to control collinearly enhanced regions of phase space.
- eikonal those parts of amplitudes which are enough to conrol soft photon enhanced regions of phase space.
- genuine weak the rest.
- once summed these three parts must give complete/exact electroweak amplitudes.
- Not only! The eikonal (also QED parts) of the given perturbation order must be helpful to construct dominant parts of higher order amplitudes.
- Sounds impossible? At LEP it was obtained after years of work, for one loop electroweak, two loop QED level.

Now for the future...

- Most of the calculations can not be used as off shelf segments, there is quite broad spectrum of necessary adaptations/tests (luckily these sometimes coincide with phenomenological projects).
- Pressure: need to identify the QED part of EW and detector acceptance details.
- Lots of new tests techniques: theoretical calculations to provide benchmarks, numerical results to evaluate reliability of physics parts.
- Long time projects. Software effort, physics input, algorithms development, tests. For computer software engineer perspective see PhD thesis of Tomasz Przedzinski 2203.11650 and Comput.Sci.Eng. 22 (2020) 4, 86-98
- **Man power and expertise of all sub domains. Survive? Or not?**
- How to identify requested parts of amplitudes? Lorentz group sub-groups and layers? Extened theories and their symmetries? Use of topology/differential geomenty, triangulation theory. Long way to go...
- Subtracting at one loop clean, but later? Negative weights? Beware of detector granularities. $\rightarrow$ So many things.

Now for the future...

Final message

- Later years, it was not so important. From LEP 1 we could see that complications start for precision of about 0.1%
- At LEP 1 this required theoretical efffort: separation at amplitude level, of EW effects into QED and genuine weak.
- Possible because of semi-separating interaction of massive bosons.
- Then reordering of QED perturbative expansion.
- Will it extend to two loops EW easily? How much effort will be needed?
- Fortunately not all observables require precision of 0.1% or better, even at FCC.

Thanks for listening.

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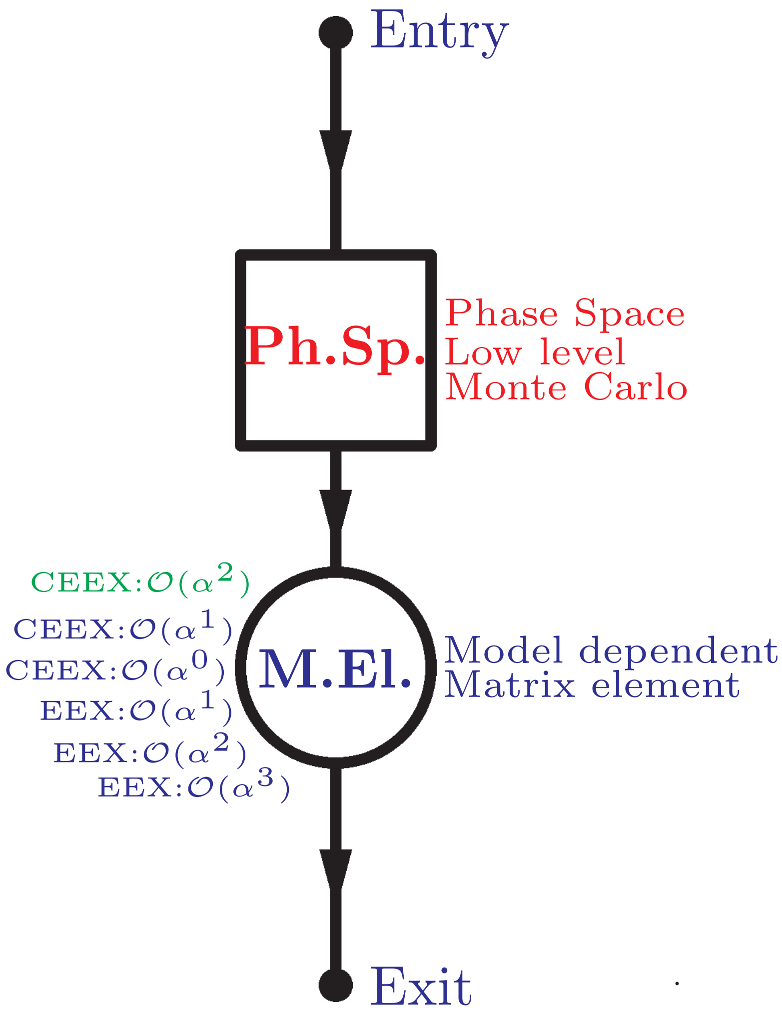
KKMC rigorous “matrix element × full phase space” implementation



- Phase-space Monte Carlo simulator is a module producing “raw events” (including importance sampling for possible intermediate resonances/singularities)
- Library of Matrix Elements; input for “model weight”; independent module
- This was used extensively for LEP precision Monte Carlos.
- Earlier difficulties with the k_0 boundary. Consequence of defined perturbation order paradigm. Soft photons were integrated out and resulting function summed with the virtual corrections to cancel infrared singularities. The k_0 not too small or negative weight events, bad for the detector response.

- KKMC works not only at spin amplitudes to reduce formulae lenghth. Interferences can be calculated directly from squares of the sum of complex contributions (for real emissions).
- As there are no sector phase space decompositions interferences cancellatios (resonance lifetimes $\rightarrow$ position time representation) are easier to control. That will be essential for processes mediated by boson pairs, like ZH.
- Vector objects residing in Feynman diagram (like $\not{k}$) are represented as external product of spinors. That is because of Kleiss-Stirling spin amplitude language.
- Enabled a lot of tests for partial results too.
- Finally: From spin amplitudes density matrices for τ-pairs are calculated. Safely, accurately, from first principles, with unlimited number of real photons configurations.

Heuristic CW complexes

We define our crude distribution over yellow space (surface=1) (reprersented by sum of: red point, green lines and flat yellow square), then we do projections using transformation T and matrix elements.