PROBING GLUON TMD MODELS WITH DRELL–YAN STRUCTURE FUNCTIONS

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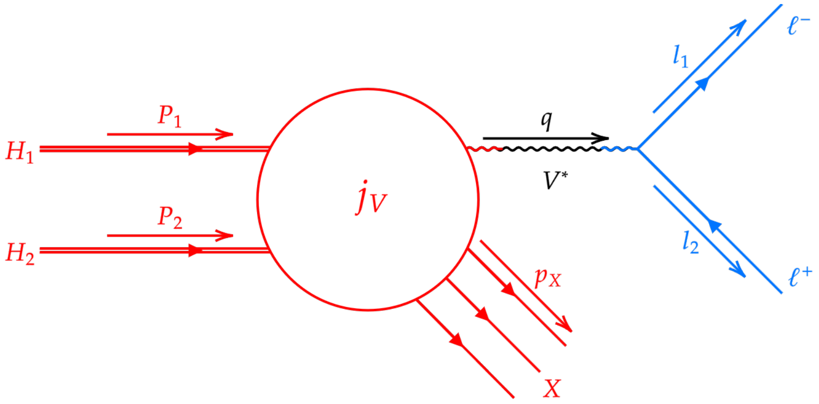
Outline

- The Drell–Yan process
- Structure Functions
- Factorization
- Lam–Tung relation
- Amplitudes from the spinor-helicity formalism
- TMD models overview
- Results

The Drell–Yan process as a probe of the hadrons structure

$$H_1(P_1) + H_2(P_2) \longrightarrow V^*(q) + X(p_X) \longrightarrow \ell^-(l_1) + \ell^+(l_2) + X(p_X)$$



$$\mathcal{A}\left(H_1 H_2 \to l^+ l^- X\right) = \overline{u}(l_1) ie\Gamma^\mu_V v(l_2) D_V(q)_{\mu\nu} \left\langle X | iej^\nu_V | H_1 H_2 \right\rangle$$

The differential cross section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}M^2 \mathrm{d}\Omega \mathrm{d}Y \mathrm{d}^2 q_T} = \frac{|\mathcal{A}|^2}{128\pi S} = \frac{\alpha_e^2}{8S}\left|D_V\left(M^2\right)\right|^2 W_{\mu\nu} L^{\mu\nu}$$

$L^{\mu\nu}$ - Standard QED calculation

$W_{\mu\nu}$ - Sensitive for an internal hadron structure

STRUCTURE FUNCTIONS ARE MEASURABLE SOURCE OF INFORMATION ABOUT HADRONS

The lepton angular decomposition is usually parameterized in the following way

$$\frac{d\sigma}{dM^2 dY d^2 q_T d\Omega} \propto \Big[1 + \cos^2\vartheta + \frac{1}{2}A_0\left(1 - 3\cos^2\vartheta\right) + A_1 \sin(2\vartheta)\cos\phi + \frac{1}{2}A_2 \sin^2\vartheta\cos(2\phi)$$
$$+ A_3 \sin\vartheta\cos\phi + A_4\cos\vartheta + A_5\sin^2\vartheta\sin(2\phi) + A_6\sin(2\vartheta)\sin\phi + A_7\sin\vartheta\sin\phi\Big],$$

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$$+\underbrace{A_3\sin\vartheta\cos\phi+A_4\cos\vartheta+A_5\sin^2\vartheta\sin(2\phi)+A_6\sin(2\vartheta)\sin\phi+A_7\sin\vartheta\sin\phi}_{\text{present for } Z^0 \text{ and } W^\pm}\Bigg],$$

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where the structure functions are given by the polarized cross sections

$$A_0 = \frac{2d\sigma^{(00)}}{d\sigma^{U+L}}, \qquad A_1 = \frac{d\sigma^{(+0)} + d\sigma^{(0+)} - d\sigma^{(-0)} - d\sigma^{(0-)}}{\sqrt{2}d\sigma^{U+L}}, \qquad A_2 = 2\frac{d\sigma^{(+-)} + d\sigma^{(-+)}}{d\sigma^{U+L}},$$

$$A_3 = \sqrt{2}c_l \frac{d\sigma^{(+0)} + d\sigma^{(0+)} + d\sigma^{(-0)} + d\sigma^{(0-)}}{d\sigma^{U+L}}, \qquad A_4 = 2c_l \frac{d\sigma^{(++)} - d\sigma^{(--)}}{d\sigma^{U+L}},$$

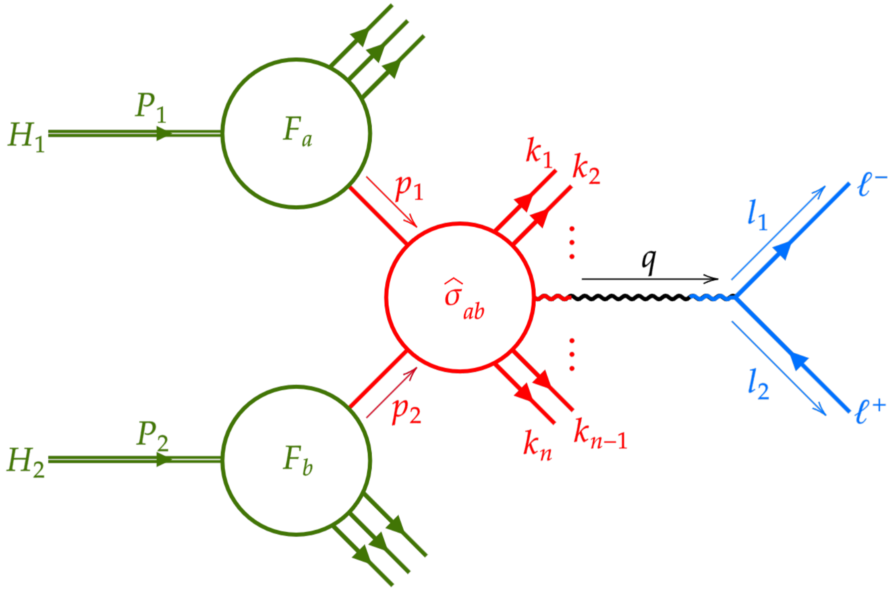
with

$$d\sigma^{U+L} = \sum_{r\in\{0,+,-\}} d\sigma^{(rr)}, \qquad c_l = \frac{2v_l a_l}{v_l^2 + a_l^2}.$$

THE STRUCTURE FUNCTIONS CAN BE CALCULATED IN QCD

Factorization:

$$\underbrace{\sigma\left(H_1 + H_2 \to \ell^- + \ell^+ + X\right)}_{\text{Hadronic cross section}} = \sum_{a,b} \underbrace{F_a \otimes F_b}_{\text{Parton Distributions}} \otimes \underbrace{\hat{\sigma}\left(a + b \to \ell^- + \ell^+ + X\right)}_{\text{Partonic cross section within the perturbative QCD}}$$



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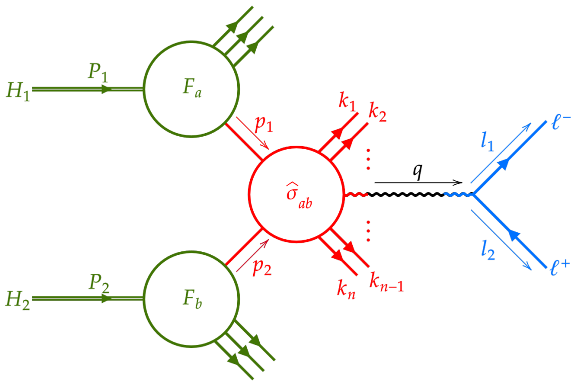

$$d\sigma \propto \begin{cases} L_{(rr')}\sum_{a,b}\int_0^1 dx_1 f_a\left(x_1,\mu_F^2\right)\int_0^1 dx_2 f_b\left(x_2,\mu_F^2\right)d\hat{\sigma}_{ab}^{(rr')} & \text{Collinear factorization} \\ L_{(rr')}\sum_{a,b}\int_0^1\frac{dx_1}{x_1}\int\frac{d^2p_{1T}}{\pi}\mathcal{F}_a\left(x_1,\mathbf{p}_{1T}^2,\mu_F^2\right)\int_0^1\frac{dx_2}{x_2}\int\frac{d^2p_{2T}}{\pi}\mathcal{F}_b\left(x_2,\mathbf{p}_{2T}^2,\mu_F^2\right)d\hat{\sigma}_{ab}^{*(rr')} & k_T\text{-factorization}\end{cases}$$

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Collinearly: $p_1 = x_1 P_1$, $p_2 = x_2 P_2$

High-energy: $p_1 = x_1 P_1 + p_{1T}$, $p_2 = x_2 P_2 + p_{2T}$ with $p_1^2 = -\mathbf{p}_{1T}^2$, $p_2^2 = -\mathbf{p}_{2T}^2$

Collinear — k_T-factorization relation:

$$d\hat{\sigma}^* \xrightarrow{\mathbf{p}_{1T}, \mathbf{p}_{2T} \to 0} d\hat{\sigma}.$$

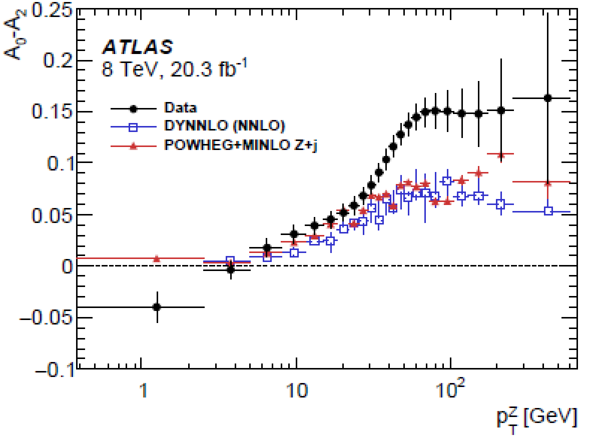
$$x f_a\left(x, \mu^2\right) = \int_0^{\mu^2} dk_T^2 \mathcal{F}_a\left(x, k_T^2, \mu^2\right).$$

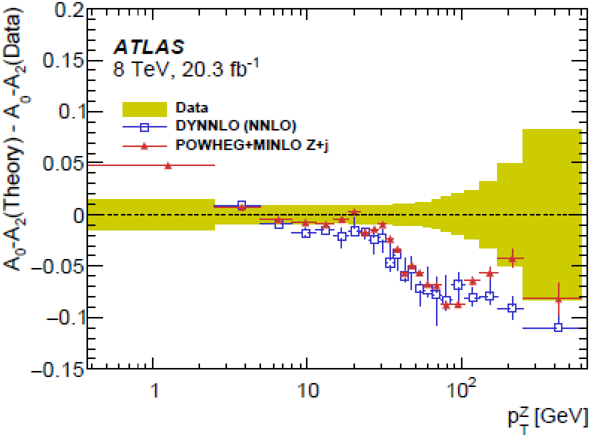
The Lam–Tung relation breaking is not completely explained

- The Lam–Tung relation is of the form:

$$\boxed{A_{LT} := A_0 - A_2 = 0.}$$

- Fulfilled up to the NLO (for V+jet) in collinear perturbative QCD
- Breaking of Lam–Tung relation may be traced back to a difference between partonic and hadronic collision planes $\longrightarrow$ sensitivity to partons' transverse momenta





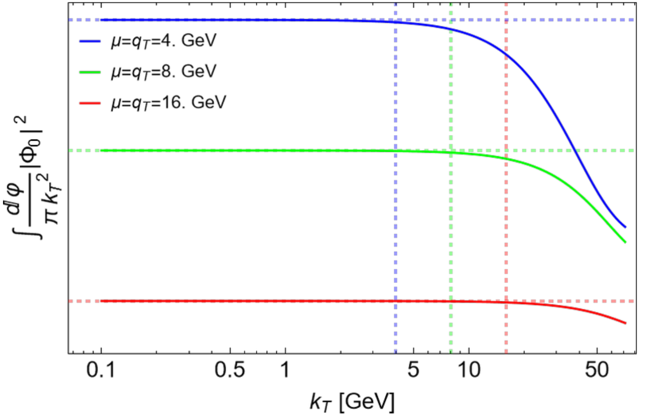
Motivation for including the partons' transverse momenta

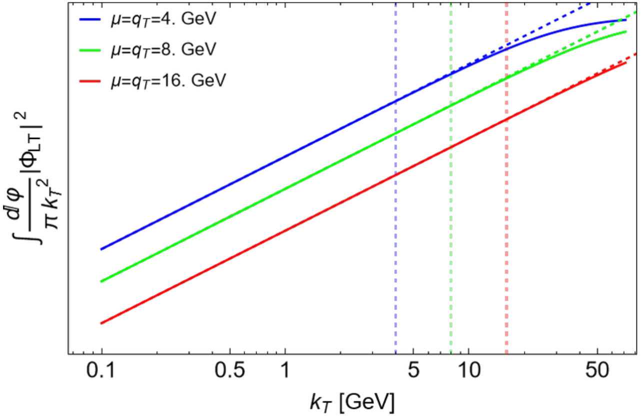
Consider the cross section with one off-shell parton

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 f\left(x_1, \mu^2\right) \int \mathrm{d}x_2 \int \frac{\mathrm{d}^2 k_T}{\pi k_T^2} \mathcal{F}\left(x_2, k_T^2, \mu^2\right) |\Phi|^2 \mathrm{d}PS.$$

The impact factor Φ is related to the collinear amplitude $\mathcal{A}_{\text{coll}}$ by

$$\int \frac{\mathrm{d}\varphi}{\pi} |\Phi|^2 \approx \theta\left(\mu^2 - k_T^2\right) |\mathcal{A}_{\text{coll}}|^2 k_T^2 + \mathcal{O}\left(\left(\frac{k_T}{\mu}\right)^4\right).$$





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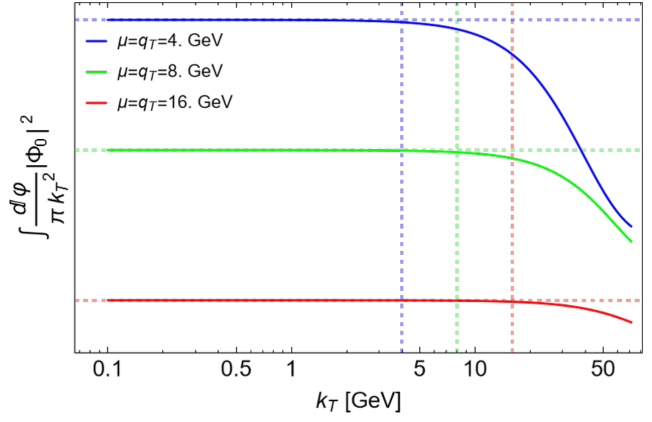
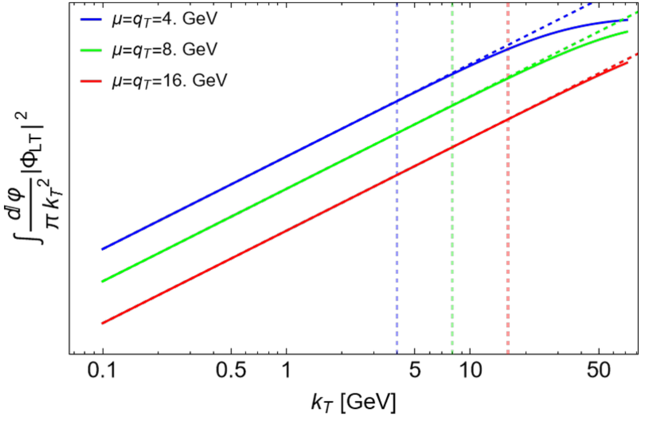


$$\int \mathrm{d}\varphi |\Phi_0|^2 \propto k_T^2$$



$$\int \mathrm{d}\varphi |\Phi_{LT}|^2 \propto k_T^4$$

k_T-FACTORIZATION APPROACH

- We want to take into account the effects of partons' transverse momenta $\Longrightarrow$ k_T**-factorization**

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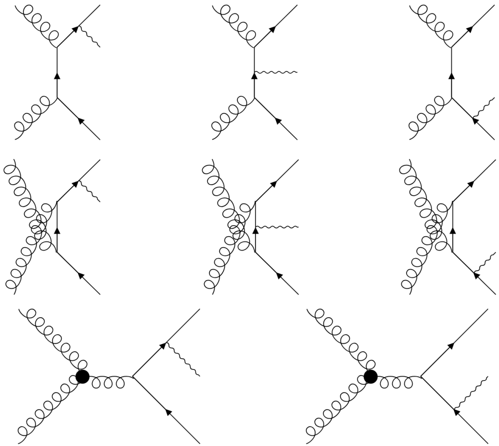
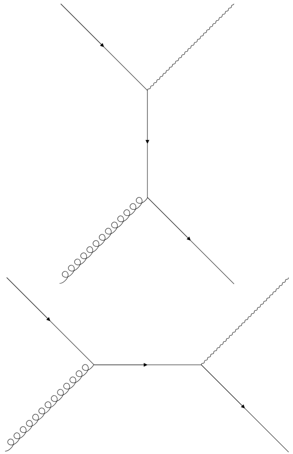
$$g^* g^* \longrightarrow q\bar{q}V^*$$



k_T-factorization

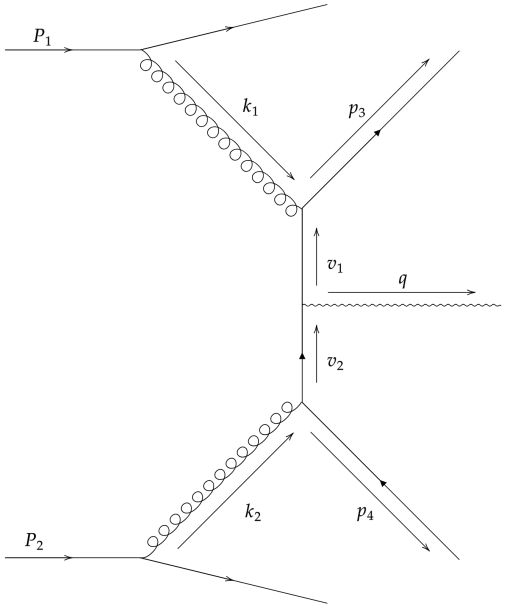
$$q_{\mathrm{val}}g^* \longrightarrow qV^*$$



Hybrid factorization

Scattering Amplitudes from the Spinor - Helicity Formalism

Example diagram:



From the expression for full diagram

$$\mathcal{A}^{ab,\mu}_{ij,\sigma_3\sigma_4} = -ig^2\left(T^aT^b\right)_{ij} A^{\mu}_{\sigma_3\sigma_4},$$

we can extract the spinorial part

$$A^{\mu}_{\sigma_3\sigma_4} = \overline{u}_{\sigma_3}(p_3)\not{P}_1\frac{\not{v}_1}{v_1^2}\Gamma^{\mu}_{V}\frac{\not{v}_2}{v_2^2}\not{P}_2 v_{\sigma_4}(p_4) = \boxed{?},$$

where

$$\Gamma^{\mu}_{V} = (v + a\gamma_5)\gamma^{\mu},$$

which can be divided into left and right chiral parts

$$A^{\mu}_{\sigma_3\sigma_4} = (v+a)R^{\mu}_{\sigma_3\sigma_4} + (v-a)L^{\mu}_{\sigma_3\sigma_4}$$

The Spinor - Helicity Formalism

We introduce the spinorial notation

$$p_{A\dot{A}} := p_\mu \sigma^\mu_{A\dot{A}}, \qquad p^{\dot{A}A} := p^\mu \overline{\sigma}_\mu^{\dot{A}A} \qquad \Longrightarrow \qquad \not{p} := p_\mu \gamma^\mu = \begin{bmatrix} 0 & p_{A\dot{A}} \\ p^{\dot{A}A} & 0 \end{bmatrix}$$

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Dirac bispinors u_σ and v_σ are defined as

$$\overline{u}_+(p) = [p| := \begin{bmatrix} \lambda(p)^A & 0 \end{bmatrix}, \qquad \overline{u}_-(p) = \langle p| := \begin{bmatrix} 0 & \overline{\lambda(p)}_{\dot{A}} \end{bmatrix},$$

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in order to satisfy the Dirac equation and such that they have specified helicity (chirality)

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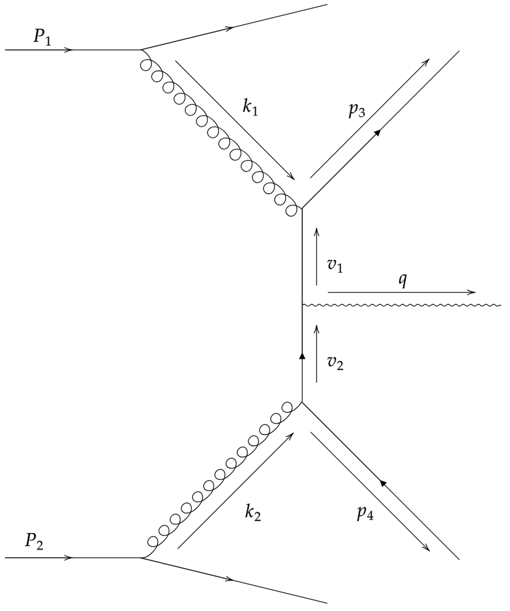
A lot of products vanish automatically

$$\langle p_1 | \gamma^{\mu_1} \dots \gamma^{\mu_n} | p_2] = [p_1 | \gamma^{\mu_1} \dots \gamma^{\mu_n} | p_2\rangle = 0 = \langle p_1|\gamma^{\mu_1} \dots \gamma^{\mu_{n+1}}|p_2\rangle = [p_1 | \gamma^{\mu_1} \dots \gamma^{\mu_{n+1}} | p_2], \tag{2}$$

if n is even.

Example diagram:



From the above identities

$$
\begin{aligned}
(1) \quad &\Longrightarrow \quad R^{\mu}_{+-} = 0 = L^{\mu}_{-+}, \\
(2) \quad &\Longrightarrow \quad R^{\mu}_{++} = R^{\mu}_{--} = 0 = L^{\mu}_{++} = L^{\mu}_{--}.
\end{aligned}
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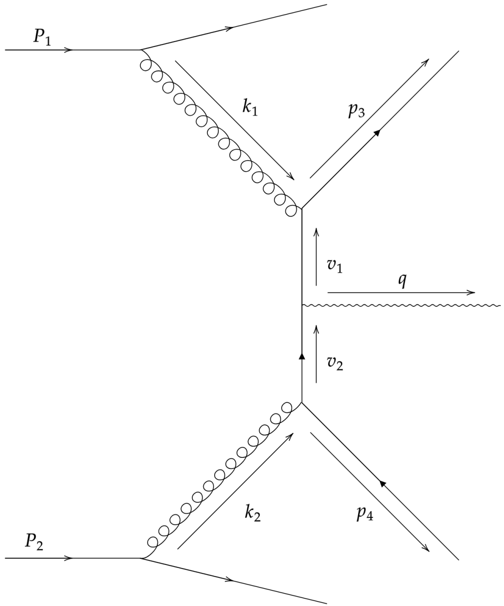
$\Longrightarrow$ We are left with two non-vanishing terms: R^{μ}_{-+} and L^{μ}_{+-}.

SCATTERING AMPLITUDES FROM THE SPINOR - HELICITY FORMALISM

Example diagram:



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$$
\begin{aligned}
R^{\dot{A}A}_{-+} =& \langle p_3 \,|\, \not{P}_1 \frac{\not{v}_1}{v_1^2} P_+ \gamma^{\dot{A}A} \frac{\not{v}_2}{v_2^2} \not{P}_2 \,|\, p_4] \\
=& \boxed{-\frac{2S}{v_3^2 v_2^2 \sqrt{p_3^+ p_4^+}} p_4^{\dot{0}0} p_3^{\dot{1}0} v_3^{\dot{A}1} v_2^{\dot{0}A}}
\end{aligned}
$$

Symmetries Simplify Calculation

Spinorial products satisfy the following identities

$$[p_1 \,|\, \Gamma_\pm \,|\, p_2\rangle = \overline{\langle p_1 \,|\, \Gamma_\mp \,|\, p_2]}, \qquad [p_1 \,|\, \Gamma_\pm \,|\, p_2] = -\overline{\langle p_1 | \Gamma_\mp | p_2\rangle}.$$

where $\Gamma_\pm$ – any composition of the gamma matrices and one of the projections $P_\pm$.

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This leads to the relations between left and right chiral amplitudes

$$R^\mu_{-+} = \overline{L}^\mu_{+-}, \quad L^\mu_{-+} = \overline{R}^\mu_{+-}, \quad R^\mu_{--} = -\overline{L}^\mu_{++}, \quad L^\mu_{--} = -\overline{R}^\mu_{++},$$

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$$\Longrightarrow \boxed{\text{1 independent expression for each diagram}}$$

THE CONSIDERED GLUON TMD MODELS

- The Gaussian model
- The Jung–Hautmann (JH) model
- The Kimber–Martin–Ryskin (KMR) model
- The "Weizsäcker–Williams" (WW) model (in analogy with photon flux) + modified "Weizsäcker–Williams" (WW') model

THE CONSIDERED GLUON TMD MODELS

- The Gaussian model — GBW saturation model

$$\mathcal{F}_G\left(x, k_T^2\right) = N_2(1-x)^7 \exp\left[-\left(\frac{x}{x_0}\right)^\lambda \frac{k_T^2}{k_0^2}\right],$$

  with parameters $k_0 = 1$ GeV, $N_2 = 28.44\ \text{GeV}^{-2}$, $\lambda = 0.29$ and $x_0 = 3 \cdot 10^{-4}$.

- The Jung–Hautmann model

- The Kimber–Martin–Ryskin model

- The "Weizsäcker–Williams" model + modified "Weizsäcker–Williams" model

The Considered Gluon TMD models

- The Gaussian model
- Jung–Hautmann (JH) model — Catani–Ciafaloni–Fiorani–Marchesini (CCFM) evolution equation - 3 versions from different fits (2013, 2018, 2023) from `TMDlib`. [`arXiv:1312.7875v2`, `arXiv:1804.11152v2`, `arXiv:2312.08655v3`]

  $$\mathcal{F}(x,k_T,\mu) = \mathcal{F}_0(x,k_T,\mu_0)\Delta_s(\mu,\mu_0) + \int \frac{\mathrm{d}z}{z}\int \frac{\mathrm{d}q^2}{q^2}\theta(\mu - zq)\Delta_s(\mu,zq)P(z,q,k_T)\mathcal{F}\left(\frac{x}{z}, k_T + (1-z)q, \mu\right),$$

  where the Sudakov form factor is of the form

  $$\Delta_s(\mu,\mu') = \exp\left[-\int_{\mu'^2}^{\mu^2}\frac{\mathrm{d}q^2}{q^2}\int_0^{1-\mu'/q}\mathrm{d}z\frac{C_A\alpha_s\left((1-z)^2q^2\right)}{\pi}\frac{1}{1-z}\right],$$

  and $P(z,Q,k_T)$ is the unintegrated splitting function.
- The Kimber–Martin–Ryskin model
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The Considered Gluon TMD models

- The Gaussian model
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$$\mathcal{F}_{\mathrm{KMR}}\left(x, k_T^2, \mu^2\right) = \frac{T_g(k_T, \mu)}{k_T^2} \frac{\alpha_s\left(k_T^2\right)}{2\pi} \int_x^{\mu/(k_T+\mu)} \mathrm{d}z \left[P_{gg}(z) \frac{x}{z} f_g\left(\frac{x}{z}, k_T^2\right) + \sum_q P_{gq}(z) \frac{x}{z} f_q\left(\frac{x}{z}, k_T^2\right) \right],$$

with the Sudakov factor of the form

$$T_g(Q, \mu) = \exp\left[- \int_{Q^2}^{\mu^2} \frac{\mathrm{d}p^2}{p^2} \frac{\alpha_s(p^2)}{2\pi} \int_0^{\mu/(p+\mu)} \mathrm{d}z z \left(P_{gg}(z) + \sum_q P_{qg}(z) \right) \right],$$

where $P_{ij}(z)$ are the DGLAP splitting functions and $f_i(x, \mu^2)$ are the collinear PDFs.

- The "Weizsäcker–Williams" model + modified "Weizsäcker–Williams" model

THE CONSIDERED GLUON TMD MODELS

- The Gaussian model
- The Jung–Hautmann model
- The Kimber–Martin–Ryskin model
- The "Weizsäcker–Williams" model (in analogy with photon flux)

$$\mathcal{F}_{\mathrm{WW}}\left(x, k_T^2\right) = \frac{N_1}{k_0^2}(1-x)^7 x^{-\lambda} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \frac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases} .$$

The modified "Weizsäcker–Williams" model

$$\mathcal{F}'_{\mathrm{WW}}\left(x, k_T^2, \mu^2\right) = \frac{x f_g\left(x, \mu^2\right)}{k_0^2\left[1+\log\left(\frac{\mu^2}{k_0^2}\right)\right]} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \frac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases} .$$

where $f_g\left(x, \mu^2\right)$ is gluon PDF and the parameters have values $N_1 = 0.6111$, $k_0 = 1$ GeV and $\lambda = 0.29$.

THE CONSIDERED GLUON TMD MODELS

- The Gaussian model

$$\mathcal{F}_G\left(x, k_T^2\right) = N_2(1-x)^7 \exp\left[-\left(\frac{x}{x_0}\right)^{\lambda} \frac{k_T^2}{k_0^2}\right] \qquad - \qquad \text{Narrow in } k_T$$

- The Jung–Hautmann model

- The Kimber–Martin–Ryskin model

- The “Weizsäcker–Williams” model

$$\mathcal{F}_{WW}\left(x, k_T^2\right) = \frac{N_1}{k_0^2}(1-x)^7 x^{-\lambda} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \dfrac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases} \qquad - \qquad \text{Wide in } k_T$$

+ modified “Weizsäcker–Williams” model.

Normalization to the total Drell–Yan cross section

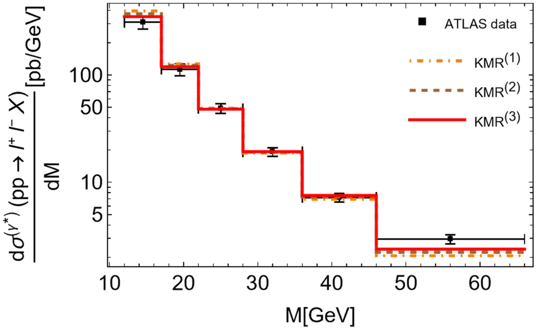
Problem: Some models do not describe the Drell–Yan cross-section data well.

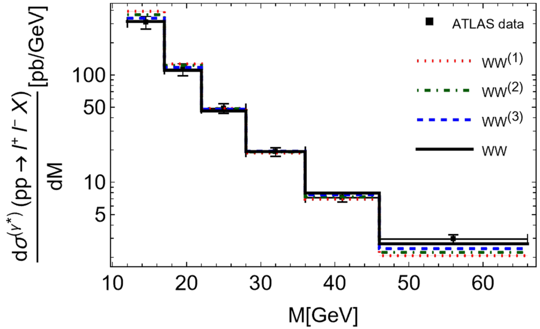
Phenomenological solution: Normalization + x rescaling (Motivated by different kinematics in the collinear and non-zero k_T approaches)

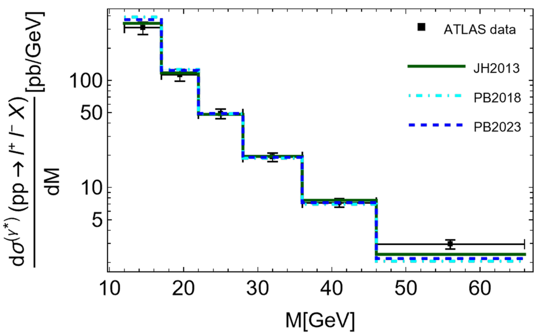
We considered three modifications of KMR and WW′ models of the form

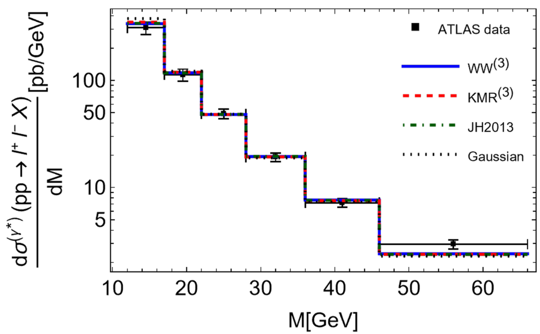
$$\mathcal{F}^{(1)}\left(x, k_T^2, \mu^2\right) = N^{(1)}\mathcal{F}\left(x, k_T^2, \mu^2\right),$$
$$\mathcal{F}^{(2)}\left(x, k_T^2, \mu^2\right) = N^{(2)}\mathcal{F}\left(x/2, k_T^2, \mu^2\right),$$
$$\mathcal{F}^{(3)}\left(x, k_T^2, \mu^2\right) = N^{(3)}\mathcal{F}\left(x/4, k_T^2, \mu^2\right).$$

Normalization to the total Drell–Yan cross section



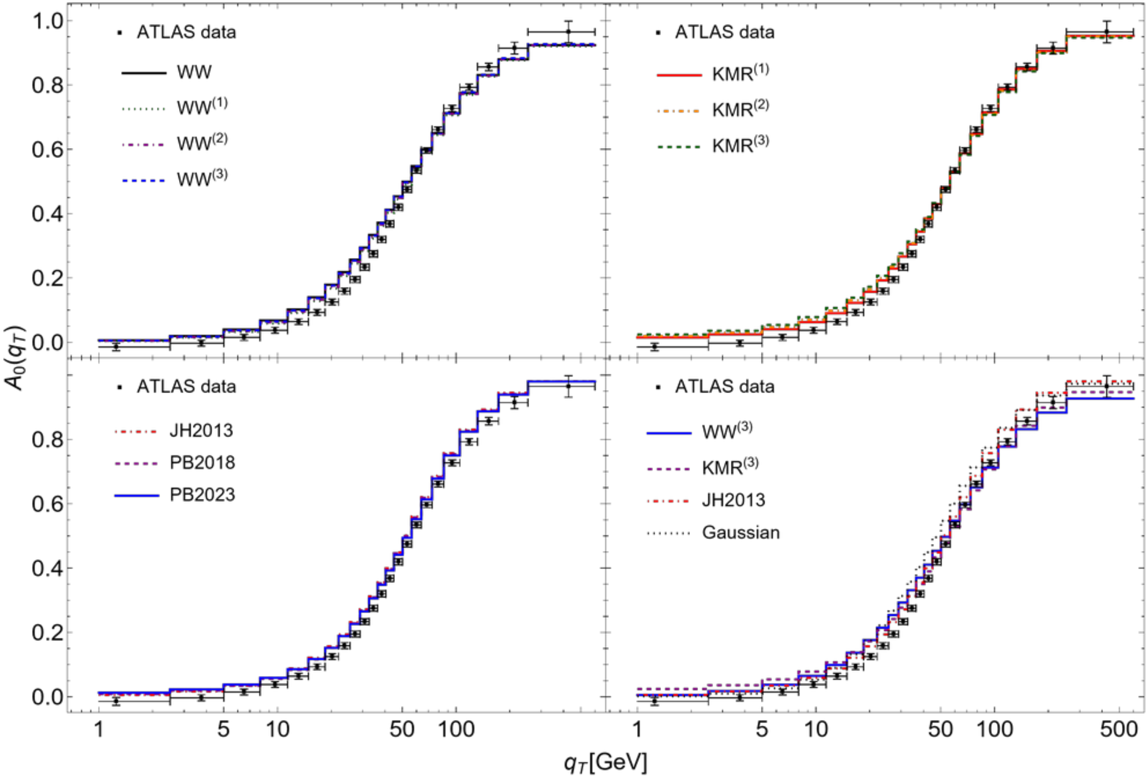




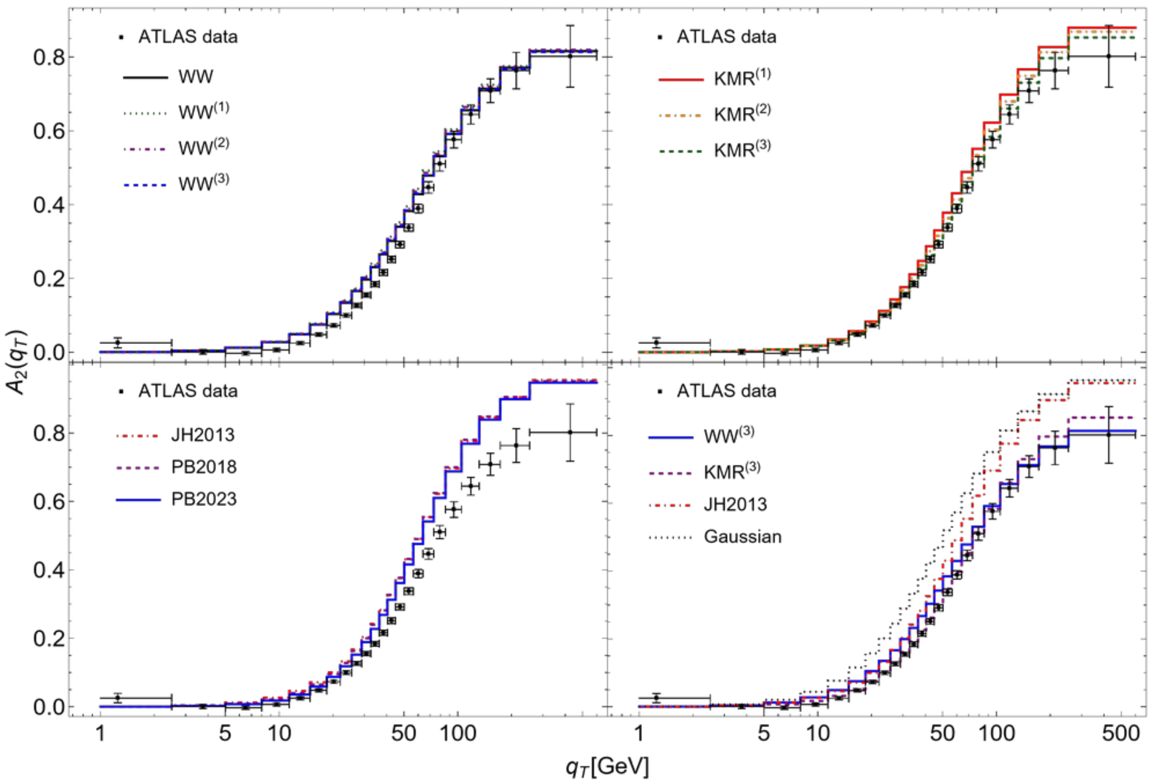


Model	χ^2/d.o.f
WW	0.53
$\mathrm{WW}^{(1)}$	3.02
$\mathrm{WW}^{(2)}$	1.82
$\mathrm{WW}^{(3)}$	0.98
$\mathrm{KMR}^{(1)}$	3.08
$\mathrm{KMR}^{(2)}$	1.90
$\mathrm{KMR}^{(3)}$	1.13
JH2013	1.07
PB2018	3.04
PB2023	2.10
Gauss.	1.74

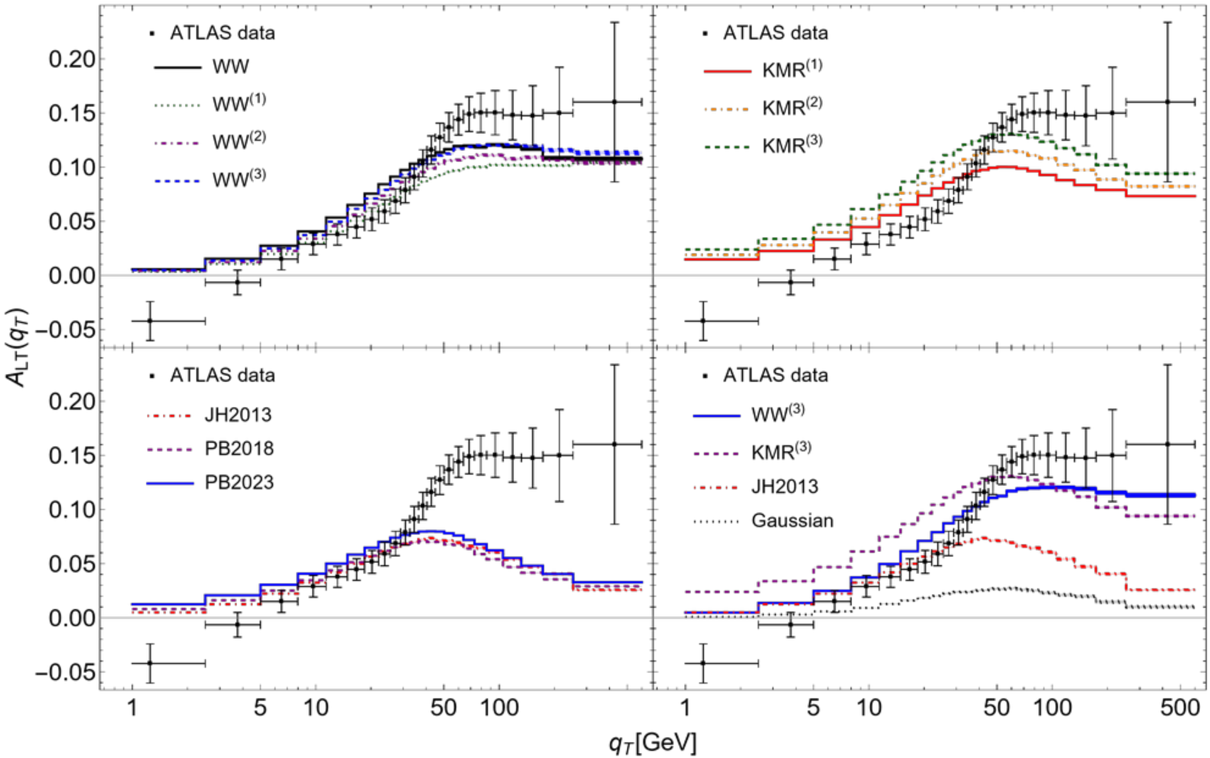
Results for the Parity Conserving Structure Functions: A_0



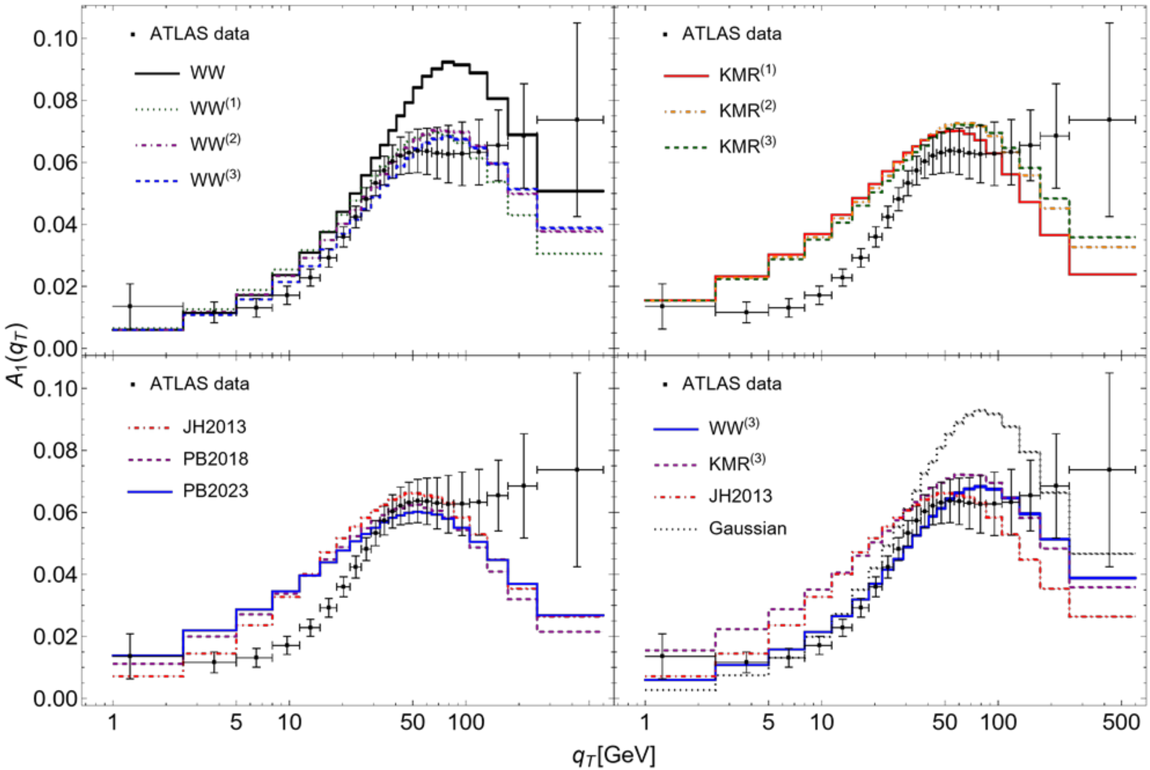
RESULTS FOR THE PARITY CONSERVING STRUCTURE FUNCTIONS: A_2



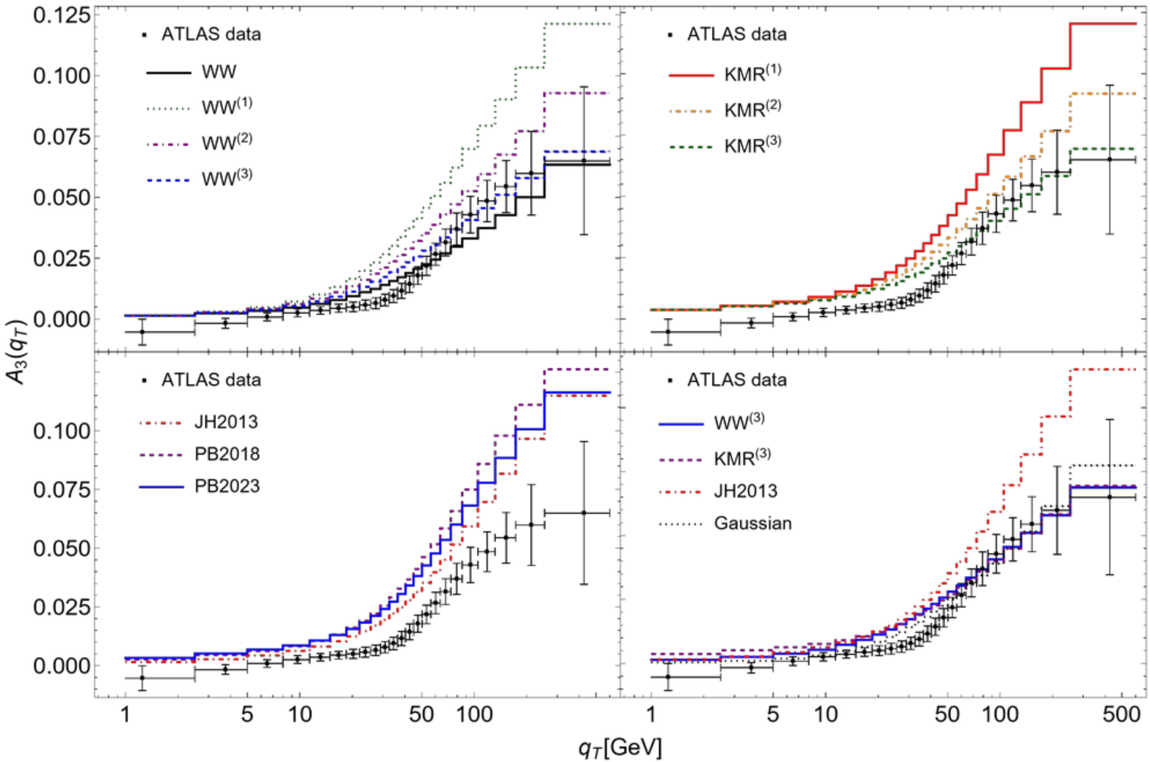
Results for the Parity Conserving Structure Functions: A_{LT}



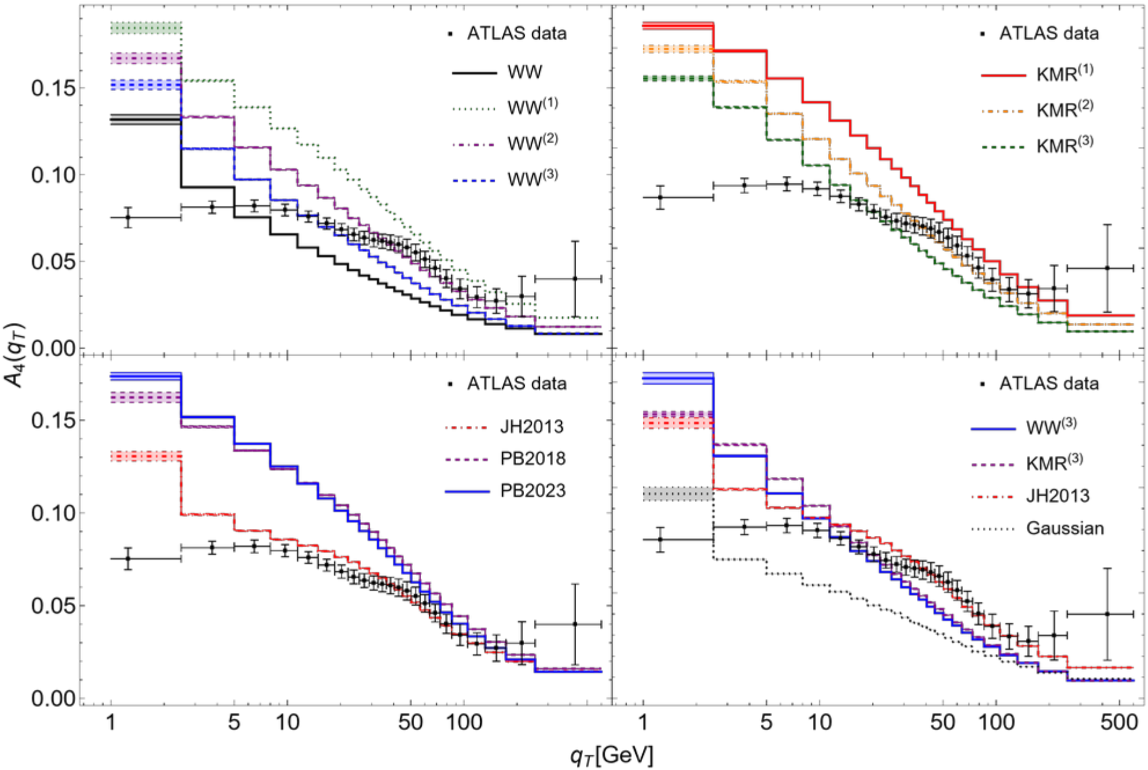
Results for the Parity Conserving Structure Functions: A_1



Results for the Parity Violating Structure Functions: A_3



Results for the Parity Violating Structure Functions: A_4



Comparison of the Results for Different TMD Models

Reduced χ^2 for the structure function data

$$\chi^2_n/\text{d.o.f} = \frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \frac{\left(A^{\text{th}}_{n,i} - A^{\text{data}}_{n,i}\right)^2}{\sigma^2_{i,\text{data}} + \sigma^2_{i,\text{th}}} \qquad \text{where} \qquad \sigma_{i,\text{th}} = 0.12 \cdot A^{\text{th}}_{n,i}.$$

$$\chi^2_{\min}/\text{d.o.f} = \min\left\{\chi^2/\text{d.o.f}\right\} \approx \boxed{\boxed{2.50}} \qquad \text{where} \qquad \chi^2/\text{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi^2_n/\text{d.o.f}.$$

Comparison of the Results for Different TMD Models

Reduced χ^2 for the structure function data

$$\chi_n^2/\text{d.o.f} = \frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \frac{\left(A_{n,i}^{\text{th}} - A_{n,i}^{\text{data}}\right)^2}{\sigma_{i,\text{data}}^2 + \sigma_{i,\text{th}}^2} \qquad \text{where} \qquad \sigma_{i,\text{th}} = 0.12 \cdot A_{n,i}^{\text{th}}.$$

$$\chi_{\min}^2/\text{d.o.f} = \min\left\{\chi^2/\text{d.o.f}\right\} \approx \boxed{\boxed{2.50}} \qquad \text{where} \qquad \chi^2/\text{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi_n^2/\text{d.o.f.}$$

χ^2/d.o.f for the structure function data:

Model	WW	WW$^{(1)}$	WW$^{(2)}$	**WW$^{(3)}$**	KMR$^{(1)}$	KMR$^{(2)}$	**KMR$^{(3)}$**	**JH2013**	PB2018	PB2023	**Gauss.**
χ^2/d.o.f	3.83	4.50	2.84	2.50	4.34	3.02	2.78	2.91	4.77	4.28	5.56

Comparison of the Results for Different TMD Models

Reduced χ^2 for the structure function data

$$\chi^2_n/\text{d.o.f} = \frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \frac{\left(A^{\text{th}}_{n,i} - A^{\text{data}}_{n,i}\right)^2}{\sigma^2_{i,\text{data}} + \sigma^2_{i,\text{th}}} \qquad \text{where} \qquad \sigma_{i,\text{th}} = 0.12 \cdot A^{\text{th}}_{n,i}.$$

$$\chi^2_{\text{min}}/\text{d.o.f} = \min\left\{\chi^2/\text{d.o.f}\right\} \approx \boxed{\boxed{2.50}} \qquad \text{where} \qquad \chi^2/\text{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi^2_n/\text{d.o.f.}$$

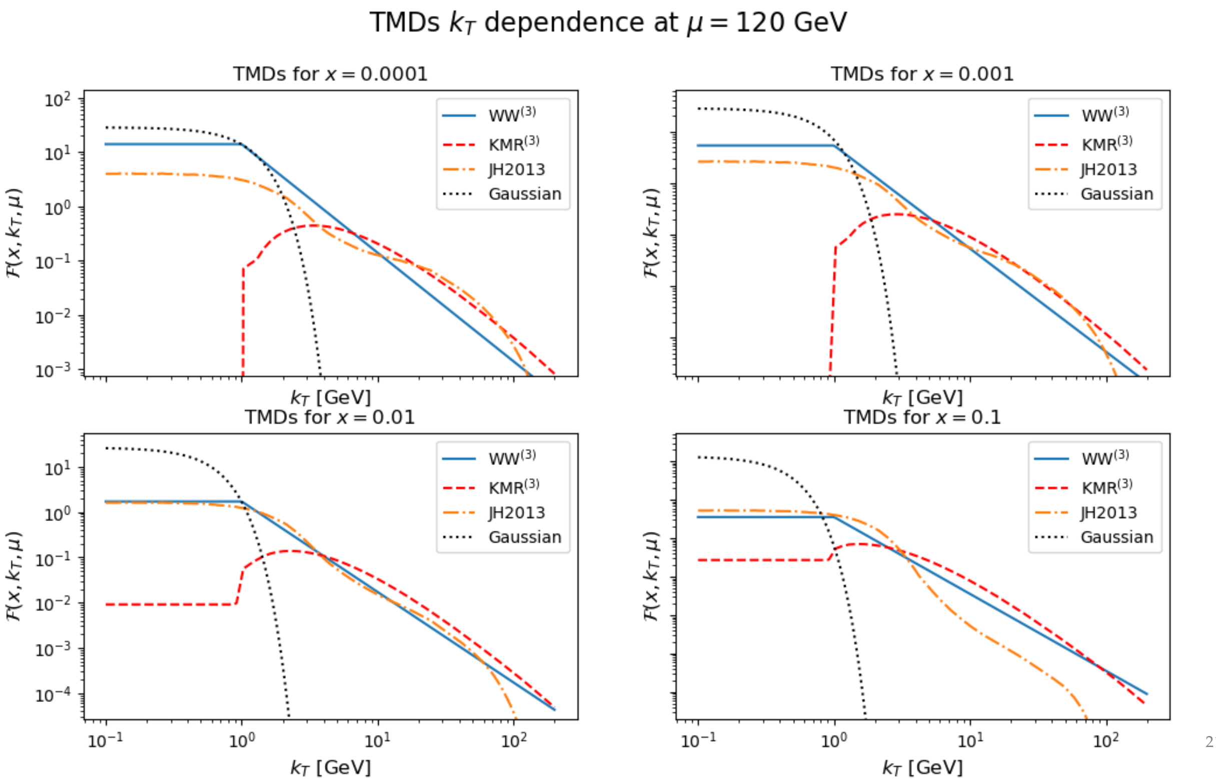
χ^2/d.o.f for the structure function data:

Model	WW	WW$^{(1)}$	WW$^{(2)}$	**WW$^{(3)}$**	KMR$^{(1)}$	KMR$^{(2)}$	**KMR$^{(3)}$**	**JH2013**	PB2018	PB2023	**Gauss.**
χ^2/d.o.f	3.83	4.50	2.84	2.50	4.34	3.02	2.78	2.91	4.77	4.28	5.56

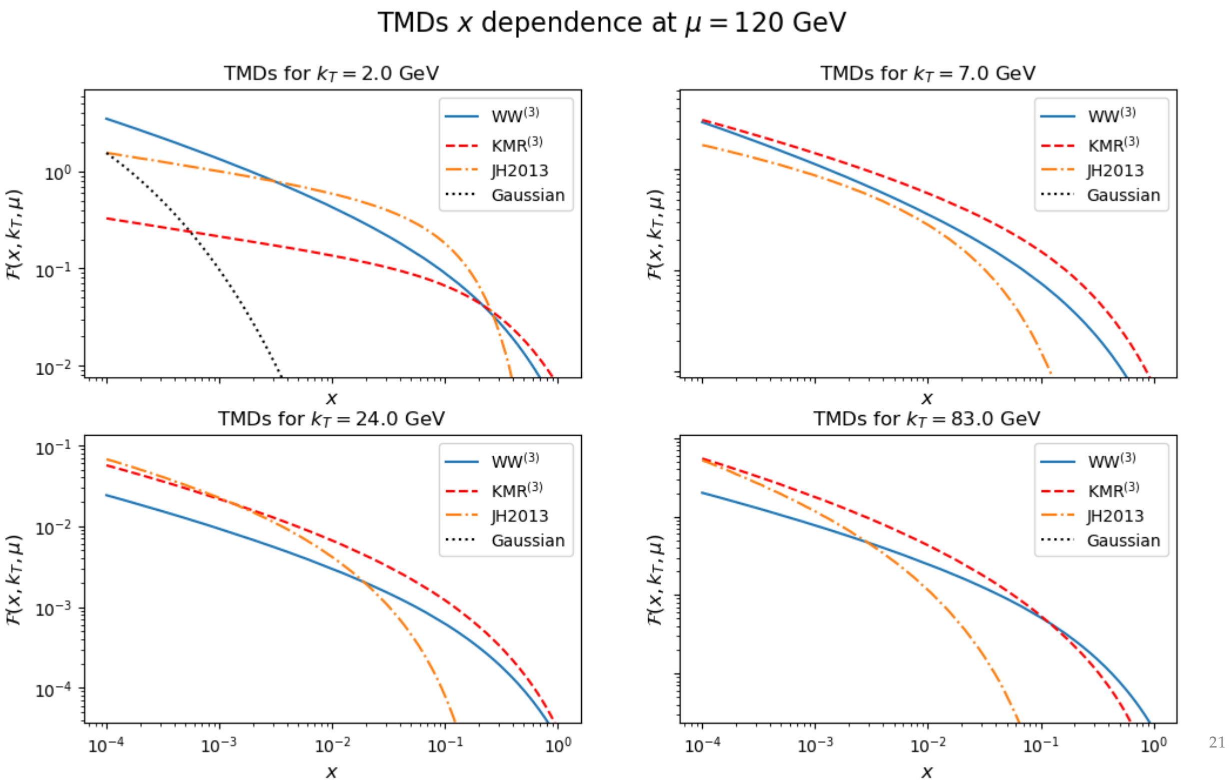
$$\chi^2_{LT,\text{min}}/\text{d.o.f} = \min\left\{\chi^2_{LT}/\text{d.o.f}\right\} \approx \boxed{\boxed{1.43}}$$

Model	WW$^{(3)}$	KMR$^{(3)}$	JH2013	Gauss.
χ^2_{LT}/d.o.f	1.43	4.02	7.37	25.33

Comparison of the TMD Models



Comparison of the TMD Models



Summary

- The Drell–Yan process is an excellent probe of internal hadron structure: five non-vanishing structure functions, three kinematical variables dependence (q_T, Y, M^2).
- Wide in k_T Weizsäcker-Williams TMD model, among the models we considered, was found to give the best overall description of the data within the presented formalism.
- In particular, an even better description was found for the Lam–Tung combination.
- We continue the program of constraining gluon TMD with Drell–Yan observables by:
  - fitting TMD to a wider set of data,
  - extending the formalism to include off-shell quarks,
  - A future goal: Inclusion of the NLO corrections in k_T-factorization

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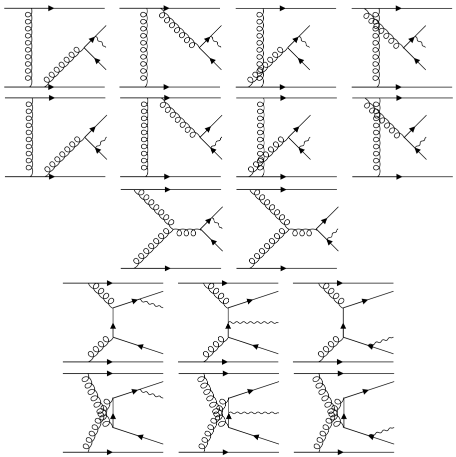
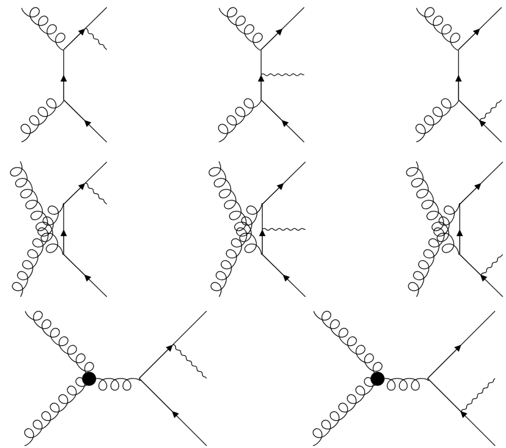
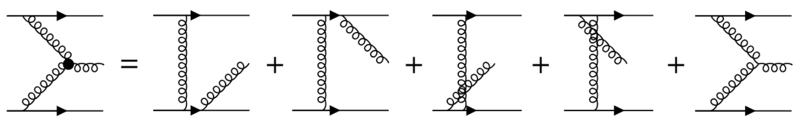
ADDITIONAL MATERIAL

Leading Diagrams

Lipatov Effective Vertex







$$\Gamma^{\mu\nu\rho} = \frac{2}{S}\left[\left(x_1 + \frac{2\mathbf{k}_{1T}^2}{x_2 S}\right)P_1^\mu + \left(x_2 + \frac{2\mathbf{k}_{2T}^2}{x_1 S}\right)P_2^\mu - (k_{1T} - k_{2T})^\mu\right]P_1^\nu P_2^\rho$$

Relations between collinear and high energy channels

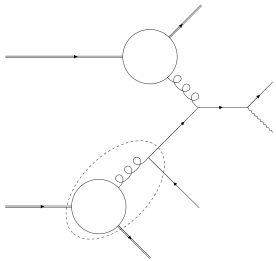
The $q_{\mathrm{val}} g^* \to qV$ channel

- contains $g \to \bar{q}$ LO DGLAP splitting for $q_{\mathrm{val}}\bar{q} \to V$ matrix element and the NLO $qg \to qV$ contribution.
- neglects the NLO loop corrections for $q\bar{q} \to V$.

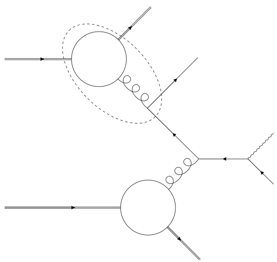
The $g^* g^* \to q\bar{q}V$ channel

- contains $g \to q_{\mathrm{sea}}, g \to \bar{q}$ NLO DGLAP splittings for the $q_{\mathrm{sea}}\bar{q} \to V$ matrix element
- and $g \to q_{\mathrm{sea}}, g \to \bar{q}$ LO DGLAP splittings for $q_{\mathrm{sea}}g \to qV$ and $\bar{q}g \to \bar{q}V$ respectively.
- contains the leading contribution to the NNLO collinear $gg \to q\bar{q}V$ matrix element.
- neglects the loop corrections for $q_{\mathrm{sea}}g \to qV$, $\bar{q}g \to \bar{q}V$ and $q\bar{q} \to V$.

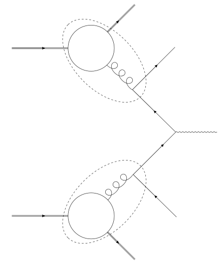
Relations between collinear and high energy channels



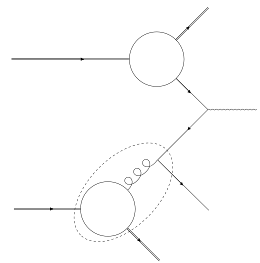
(a)



(b)



(c)



(d)

(a) $g \to q_{\text{sea}}$ splitting

(b) $g \to \bar{q}$ splitting

(c) $g \to q_{\text{sea}}$ and $g \to \bar{q}$ splittings

(d) $g \to \bar{q}$ splitting

Proof of the Bispisnor Products Identy

The charge conjugation acts in the following way

$$\mathcal{C}\psi = \mathcal{C}\begin{bmatrix}\eta_A \\ \overline{\chi}^{\dot{A}}\end{bmatrix} = \begin{bmatrix}\chi_A \\ \overline{\eta}^{\dot{A}}\end{bmatrix} = \overline{\psi}^T, \qquad \text{satisfies:} \qquad \mathcal{C}^2 = \mathbb{1}, \quad \mathcal{C}\gamma^\mu\mathcal{C} = \gamma^\mu, \quad \mathcal{C}\gamma_5\mathcal{C} = \gamma_5,$$

and is anti-linear. Now we will define another anti-linear operator

$$\mathcal{Q} := \gamma_5\mathcal{C}, \qquad \text{which satisfies} \qquad \mathcal{Q}^2 = -\mathbb{1}, \quad \mathcal{Q}\gamma^\mu\mathcal{Q}^{-1} = \gamma^\mu, \quad \mathcal{Q}\gamma_5\mathcal{Q}^{-1} = -\gamma_5.$$

For any two bispinors we have that

$$\overline{\mathcal{Q}\psi_1}\mathcal{Q}\psi_2 = \overline{\overline{\psi_1}\psi_2}.$$

It can be directly shown that $\mathcal{Q}|p] = |p\rangle$, $\mathcal{Q}\,|p\rangle = -|p]$, and in consequence

$$[p_3\,|\,\Gamma\,|\,p_4\rangle = \overline{|p_3\rangle}\Gamma\,|p_4\rangle = \overline{\mathcal{Q}|p_3]}\Gamma\mathcal{Q}|p_4] = \overline{\mathcal{Q}|p_3]}\mathcal{Q}\Gamma|p_4] = \overline{\overline{|p_3]}\Gamma|p_4]} = \overline{\langle p_3\,|\,\Gamma\,|\,p_4]}, \qquad [p_3\,|\,\Gamma\,|\,p_4] = -\overline{\langle p_3|\Gamma|p_4\rangle}.$$

$\mathcal{Q}$ exchanges the projections $\mathcal{Q}P_\pm\mathcal{Q}^{-1} = P_\mp$, so

$$[p_3\,|\,\Gamma_\pm\,|\,p_4\rangle = \overline{\mathcal{Q}|p_3]}\Gamma_\pm\mathcal{Q}|p_4] = \overline{\mathcal{Q}|p_3]}\mathcal{Q}\Gamma_\mp|p_4] = \overline{\langle p_3\,|\,\Gamma_\mp\,|\,p_4]}, \qquad [p_3\,|\,\Gamma_\pm\,|\,p_4] = -\overline{\langle p_3|\Gamma_\mp|p_4\rangle}.$$

The Amplitudes Squared

Full amplitude $\mathcal{A}^\mu$ is given by the sum of all diagrams.

- For the $q_{\text{val}} g^* \to qV^*$, there are two diagrams with the same color structure, which gives the amplitude

$$\mathcal{A}^\mu = \mathcal{A}_1^\mu + \mathcal{A}_2^\mu = -igT_{ij}^a \left(A_1^\mu + A_2^\mu\right),$$

so the amplitude square is

$$\mathcal{M}^{\mu\nu} = \frac{1}{N\left(N^2-1\right)} \sum_{i,j,a} \mathcal{A}^\mu \overline{\mathcal{A}}^\nu = \frac{g^2}{2N} A^\mu \overline{A}^\nu.$$

- For the $g^* g^* \to q\bar{q}V^*$, the amplitude can be decomposed into symmetric (S) and antisymmetric (A) parts in the adjoint color indices

$$\mathcal{A}^\mu := \sum_{n=1}^{8} \mathcal{A}_n^\mu = \mathcal{A}_S^\mu + \mathcal{A}_A^\mu, \qquad \text{where} \qquad \mathcal{A}_S^\mu := -ig^2 \left(\frac{1}{N}\delta^{ab}\delta_{ij} + d^{abc}T_{ij}^c\right) A_S^\mu, \quad \mathcal{A}_A^\mu := g^2 f^{abc} T_{ij}^c A_A^\mu.$$

In the amplitude squared, the symmetric and antisymmetric parts in the adjoint color indices do not interfere:

$$\mathcal{M}^{\mu\nu} = \frac{1}{\left(N^2-1\right)^2} \sum_{i,j,a,b} \mathcal{A}^\mu \overline{\mathcal{A}}^\nu = \frac{g^4\left(N^2-2\right)}{2N\left(N^2-1\right)} A_S^\mu \overline{A}_S^\nu + \frac{g^4 N}{2\left(N^2-1\right)} A_A^\mu \overline{A}_A^\nu = \mathcal{M}_S^{\mu\nu} + \mathcal{M}_A^{\mu\nu}.$$

The Amplitudes Squared

Comparison with The Trace Method

The spinorial matrix $\mathfrak{A}_n^\mu$ appearing in the amplitude A_n^μ is defined by:

$$A_n^\mu = \overline{u}_{\sigma_3}(p_3)\mathfrak{A}_n^\mu v_{\sigma_4}(p_4).$$

In an analogous way we define the matrices $\mathfrak{A}_S^\mu, \mathfrak{A}_A^\mu$.
The amplitudes squared can be computed as traces

$$\begin{aligned}\sum_{\sigma_3,\sigma_4} \mathcal{M}_S^{\mu\nu} =& \frac{g^4\left(N^2-2\right)}{2N\left(N^2-1\right)}\operatorname{Tr}\left[\left(\not{p}_3+m_3\right)\mathfrak{A}_S^\mu\left(\not{p}_4-m_4\right)\mathfrak{A}_S^{\dagger\nu}\right],\\ \sum_{\sigma_3,\sigma_4} \mathcal{M}_A^{\mu\nu} =& \frac{g^4 N}{2\left(N^2-1\right)}\operatorname{Tr}\left[\left(\not{p}_3+m_3\right)\mathfrak{A}_A^\mu\left(\not{p}_4-m_4\right)\mathfrak{A}_A^{\dagger\nu}\right].\end{aligned}$$

Using the trace formulas one can check numerically the helicity structure functions of the form

$$\mathcal{M}_{rr'} = \varepsilon_\mu^{(r)}\mathcal{M}^{\mu\nu}\overline{\varepsilon}_\nu^{(r')},$$

where $r, r' \in \{+,-,0\}$ are basis polarizations of V^*.

The Cutted Cross Sections

From the obtained squared amplitudes one can derive the polarized cross sections by integrating over the phase space with the parton distributions.

- $q_{\rm val} g^* \to q V^*$

$$\frac{\mathrm{d}\sigma^{(q_{\rm val} g^*)}_{r_1 r_2}}{\mathrm{d}M^2 \mathrm{d}Y \mathrm{d}^2 q_T} = \sum_f \int \mathrm{d}x_q \wp_{f,\rm val}\left(x_q, \mu_F^2\right) \int \frac{\mathrm{d}^2 k_T}{\pi \mathbf{k}_T^2} \mathcal{F}\left(x_g, \mathbf{k}_T^2, \mu_F^2\right) \frac{2}{(8\pi)^2 x_q (1 - x_F) S^2} \mathcal{M}^{(q_{\rm val} g^*), f}_{r_1 r_2},$$

- $g^* g^* \to q \bar{q} V^*$

$$\frac{\mathrm{d}\sigma^{(g^* g^*)}_{r_1 r_2}}{\mathrm{d}M^2 \mathrm{d}Y \mathrm{d}^2 q_T} = \int \mathrm{d}x_1 \int \frac{\mathrm{d}^2 k_{1T}}{\pi \mathbf{k}_{1T}^2} \mathcal{F}\left(x_1, \mathbf{k}_{1T}^2, \mu_F^2\right) \int \mathrm{d}x_2 \int \frac{\mathrm{d}^2 k_{2T}}{\pi \mathbf{k}_{2T}^2} \mathcal{F}\left(x_2, \mathbf{k}_{2T}^2, \mu_F^2\right) \frac{(2\pi)^4}{2S} \mathcal{M}^{(g^* g^*)}_{r_1 r_2} \frac{\mathrm{d}z \mathrm{d}\phi_\kappa}{8(2\pi)^9}.$$

Reduced χ^2 for the resulting structure functions

$$\chi_n^2/\mathrm{d.o.f} = \frac{1}{N_{\mathrm{points}}} \sum_{i=1}^{N_{\mathrm{points}}} \frac{\left(A_{n,i}^{\mathrm{th}} - A_{n,i}^{\mathrm{data}}\right)^2}{\sigma_{i,\mathrm{data}}^2 + \sigma_{i,\mathrm{th}}^2} \qquad \text{where} \qquad \sigma_{i,\mathrm{th}} = 0.12 \cdot A_{n,i}^{\mathrm{th}}.$$

$$\chi_{\mathrm{min}}^2/\mathrm{d.o.f} = \min\left\{\chi^2/\mathrm{d.o.f}\right\} \approx 2.50 \qquad \text{where} \qquad \chi^2/\mathrm{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi_n^2/\mathrm{d.o.f}.$$

Model	WW$^{(3)}$	KMR$^{(3)}$	JH2013	Gauss.
$\chi_0^2/\chi_{\mathrm{min}}^2$	0.84	1.26	0.52	0.82
$\chi_1^2/\chi_{\mathrm{min}}^2$	0.15	1.10	0.98	0.70
$\chi_2^2/\chi_{\mathrm{min}}^2$	1.29	0.27	1.56	4.35
$\chi_3^2/\chi_{\mathrm{min}}^2$	1.34	1.78	2.40	0.55
$\chi_4^2/\chi_{\mathrm{min}}^2$	1.39	1.15	0.35	4.70

The ratios of χ^2 per degree of freedom of each structure function (χ_n^2) for each TMD model to the minimal global reduced χ^2 — χ_{min}^2.