Transverse momentum broadening effects of dijets in QGP

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based on:

[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876] (entropy in jets)

[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910] (dijet production)

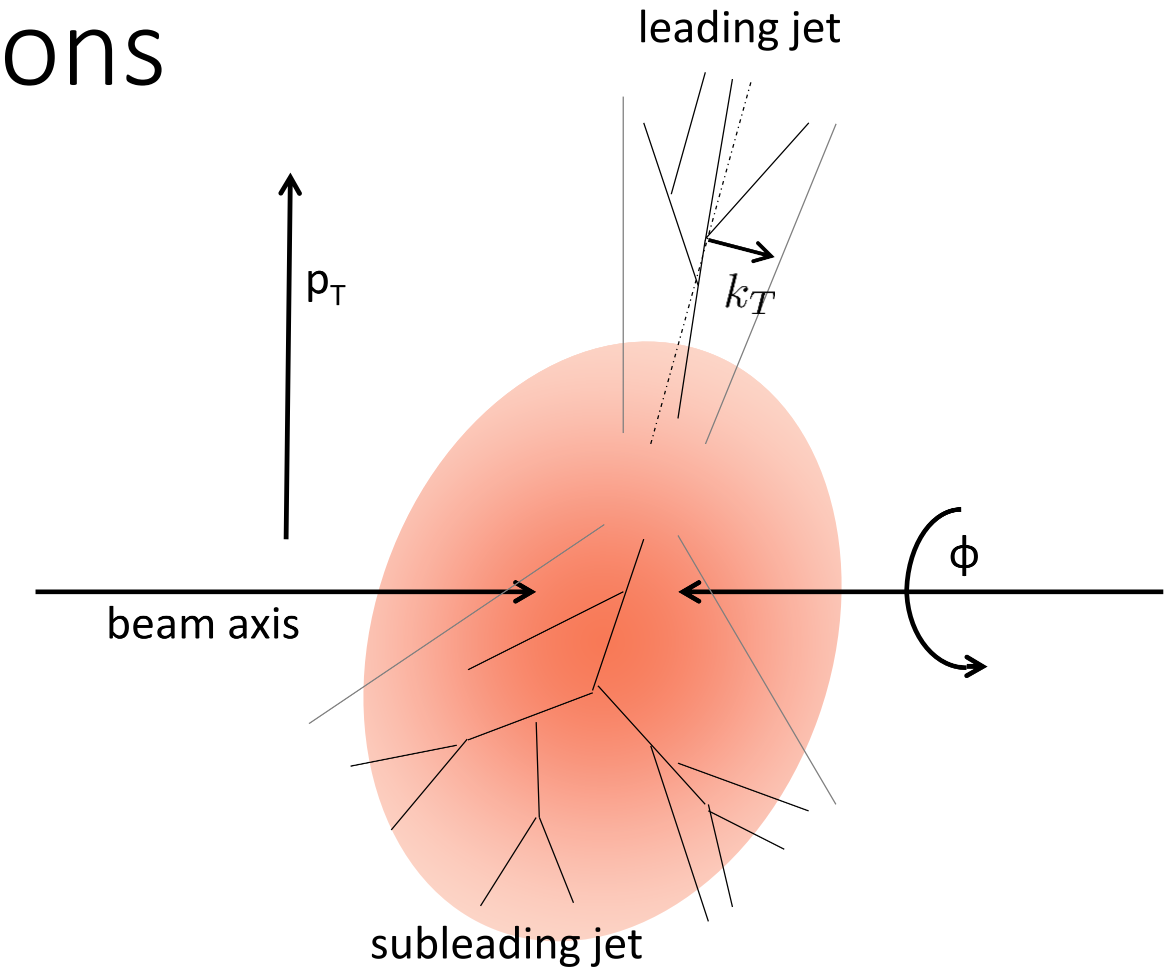
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (Monte Carlo for gluon fragmentation functions)

Jets in Heavy Ion collisions

Jets interact with medium → Jet Quenching!

↓

probe of the medium



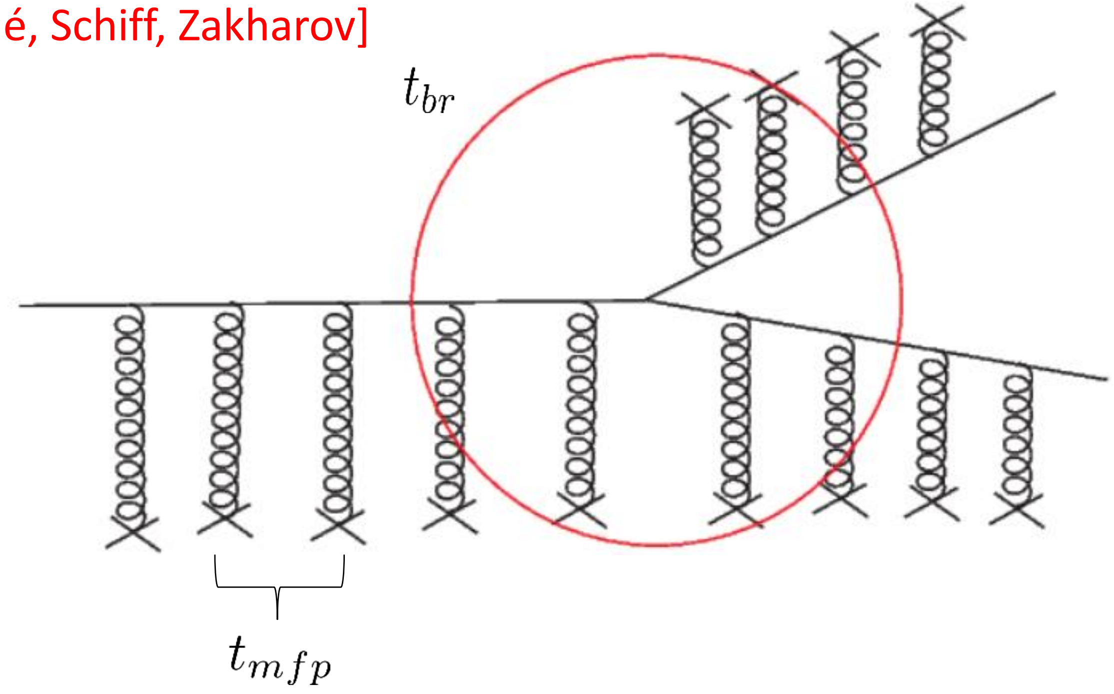
Outline of my Talk:

- Coherent emission in medium
- Monte-Carlo algorithm for jet evolution
- Single-jet results
  - k_T broadening
  - Entropy
- Hard collision
- Dijet results

Coherent emission

…à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$



$t_{br} \sim t_{mfp}$: one scattering + radiation
…Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

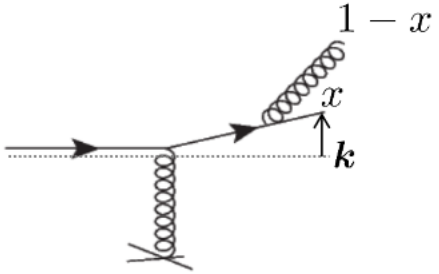
Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]



Momentum distribution:
$p \to xp$
Momentum transfer:
$p \to p + \mathbf{k}$

- Generalizes BDMPS-Z approach
- Includes transverse momentum broadening

For gluon-jets:

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t).$$

Induced Radiation:

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{2}{p_0^+} \frac{P_{gg}(z)}{z(1-z)} \sin\left[\frac{\mathbf{Q}^2}{2k_{br}^2}\right] \exp\left[-\frac{\mathbf{Q}^2}{2k_{br}^2}\right]$$

$$\omega = x p_0^+, \; k_{br}^2 = \sqrt{\omega_0 \hat{q}_0}, \; \mathbf{Q} = \mathbf{k} - z\mathbf{q}, \; \omega_0 = z(1-z) p_0^+$$

$$\hat{q}_0 = \hat{q} f(z), \; f(z) = 1 - z(1-z), \; P_{gg}(z) = N_c \frac{[1 - z(1-z)]^2}{z(1-z)}$$

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2 \boldsymbol{q}' w(\boldsymbol{q}')$$

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

→ Generalizes BDMPS-Z approach ?

k_T averaged Kernel:

$$\int_0^\infty d^2\mathbf{Q}\mathcal{K}(z,\mathbf{Q},p_0^+) = 2\pi\sqrt{\frac{\hat{q}}{p_0^+}}N_c\mathcal{K}(z) \longrightarrow \mathcal{K}(z) = \frac{(1-z+z^2)^{\frac{5}{2}}}{[z(1-z)]^{\frac{3}{2}}}$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{\omega}} \propto \frac{1}{t_{br}}$$

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$

Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

Different models

- Broadening in branching:

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2}\left[2\mathcal{K}(\mathbf{Q},z,\frac{x}{z}p_0^+)D\left(\frac{x}{z},\mathbf{q},t\right) - \mathcal{K}(\mathbf{q},z,xp_0^+)D(x,\mathbf{k},t)\right]$$
$$+\int \frac{d^2\mathbf{l}}{(2\pi)^2}C(\mathbf{l})\,D(x,\mathbf{k}-\mathbf{l},t).$$

  - No scattering
  - Scattering: $w(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

- No broadening in branching:

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$

  - Scattering: $w(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

- Gaussian broadening:

x given by $\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,t)\right]$

$\boldsymbol{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

All models yield the same k_T averaged splitting kernel $\mathcal{K}(z)$!

BDIM Equation as Integral Equation

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

$$\frac{\partial}{\partial t} D(x,t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x,t) \right]$$

↓

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)} D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy \delta(x - zy) \sqrt{\frac{z}{x}} z \mathcal{K}(z) e^{-\phi(x)(\tau-\tau')} D(y,\tau')$$

$$\tau = \frac{t}{t^*}$$

$$\phi(x) = \frac{1}{\sqrt{x}} \int_0^{1-\epsilon} dz z \mathcal{K}(z)$$

Monte-Carlo algorithm

Other codes implementing BDMPS-Z:

MARTINI, JEWEL, QPYTHIA, …

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)}D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy\delta(x-zy)\sqrt{\frac{z}{x}}z\mathcal{K}(z)e^{-\phi(x)(\tau-\tau')}D(y,\tau')$$

M
I
N
C
A
S

Set/Select x_0 at τ_0

1. Select τ_{i+1}:

   Probability density:

   $$\phi(x_i)e^{-\phi(x_i)(\tau_i-\tau_{i+1})}$$

2. Select $z = \frac{x_{i+1}}{x_i}$:

   Probability density:

   $$\frac{z\mathcal{K}(z)}{\int_0^{1-\epsilon} dz' z'\mathcal{K}(z')}$$

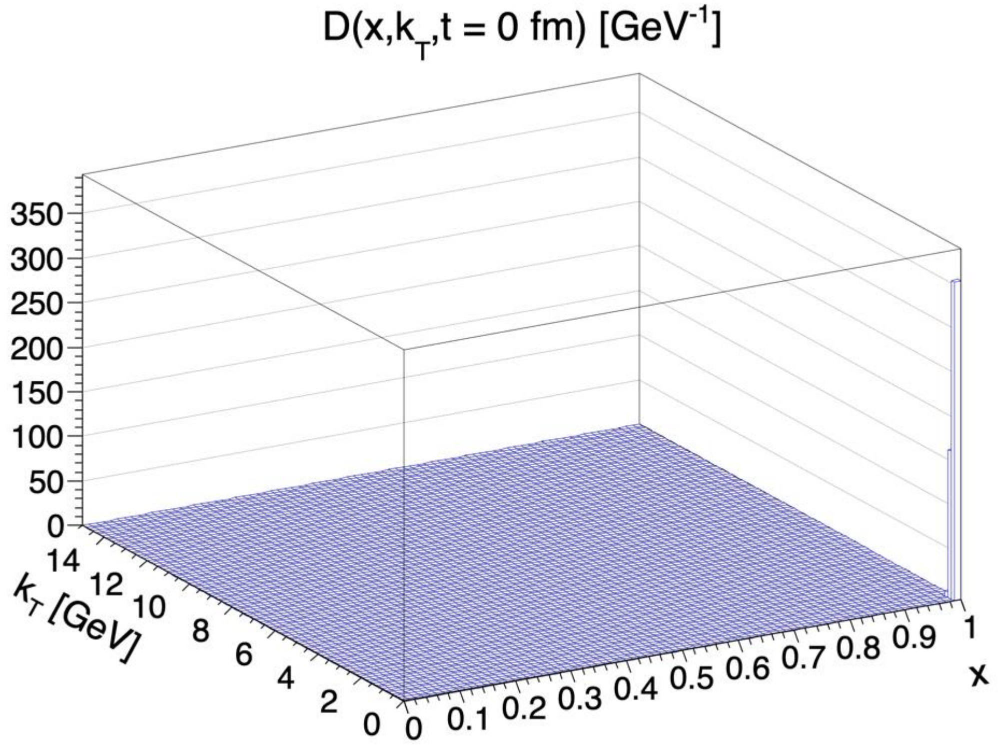
Repeat for next step in τ and x

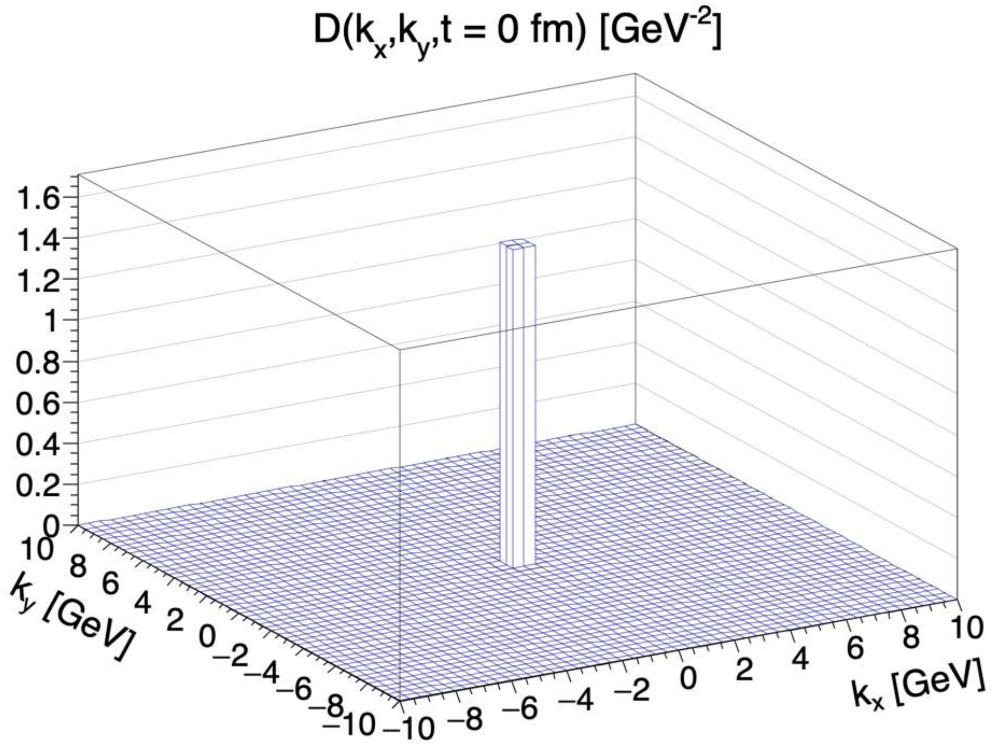
Stop once $\tau > \tau_L$

Analogous for the k_T dependent equation in x,k_T, and,τ!

Evolution of $D(x,k_T,t)$ (1/2)

$\mathcal{K}(z) \quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$

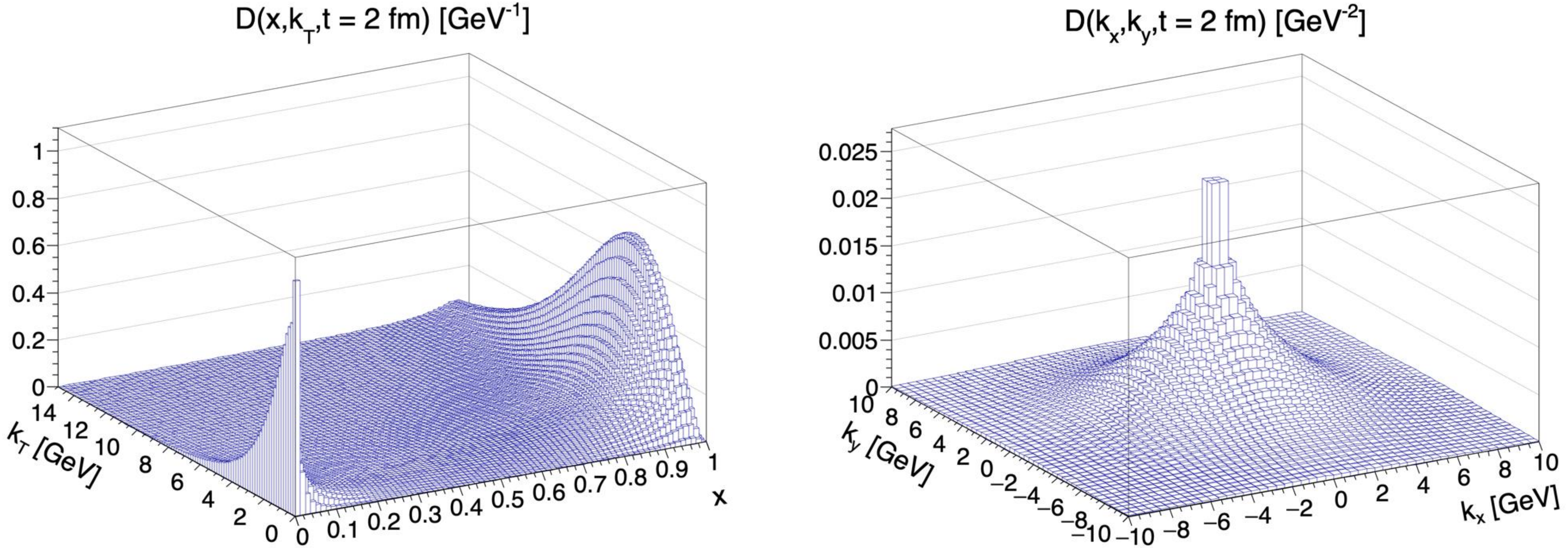




[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution of $D(x,k_T,t)$ (2/2)

$$\mathcal{K}(z) \qquad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Departure from Gaussian broadening

always same distribution for changes $p \to p + \boldsymbol{q}$
→central limit theorem

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \int \frac{d^2 \boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k} - \boldsymbol{q}, t)$$
$$+ \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z)$$
$$\left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \boldsymbol{k}, t) \right]$$

Splitting à la $p \to zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:

$$p \to z_1 p \to z_1 p + \boldsymbol{q}_1$$
$$\to z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$
$$\to z_2 (z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \to \ldots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

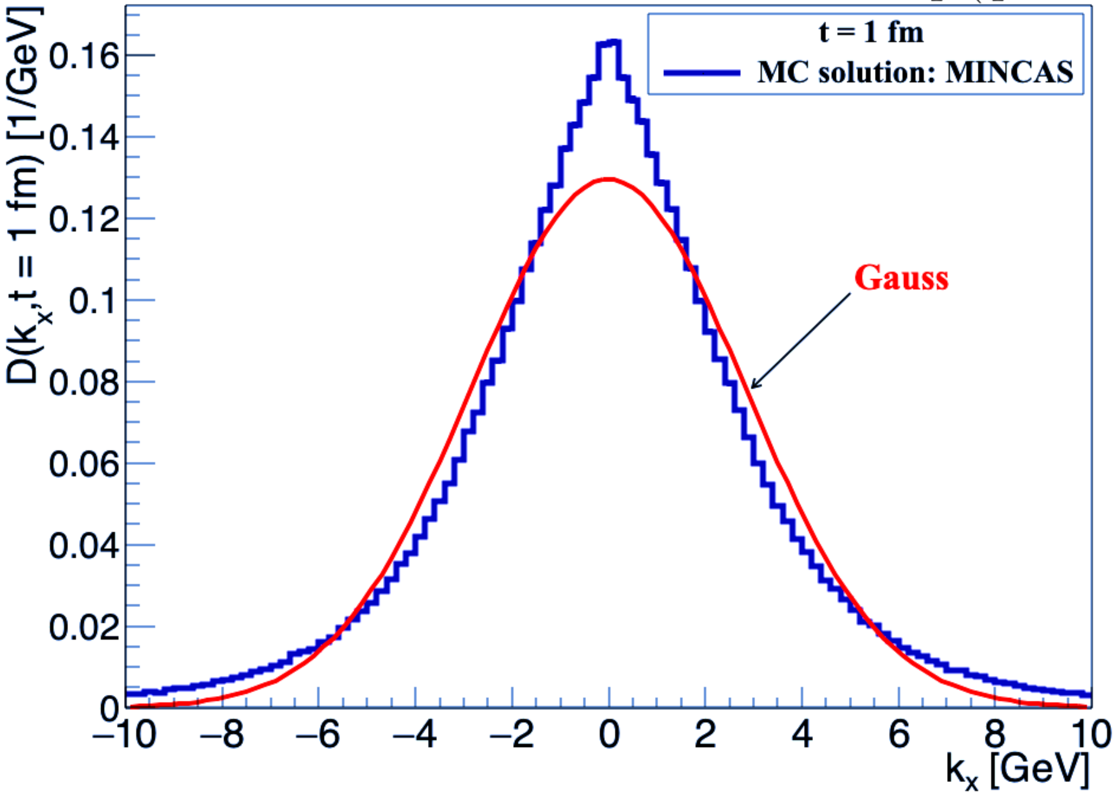
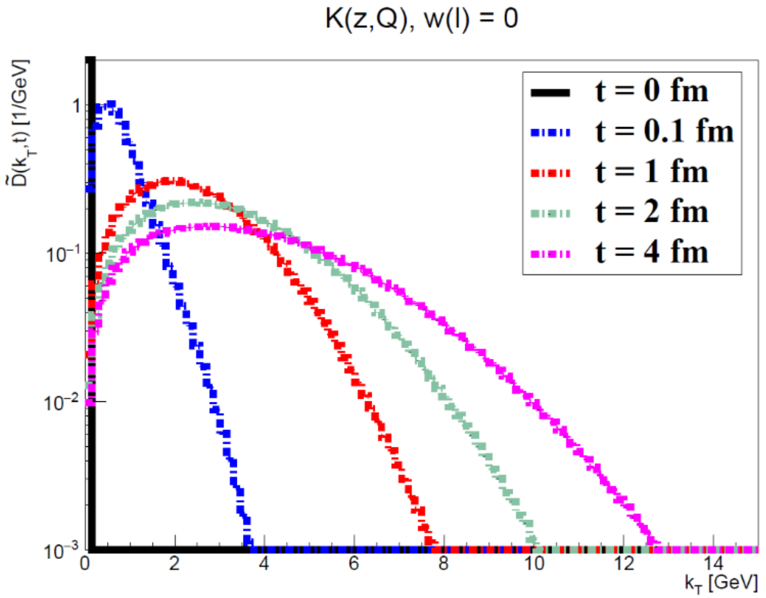
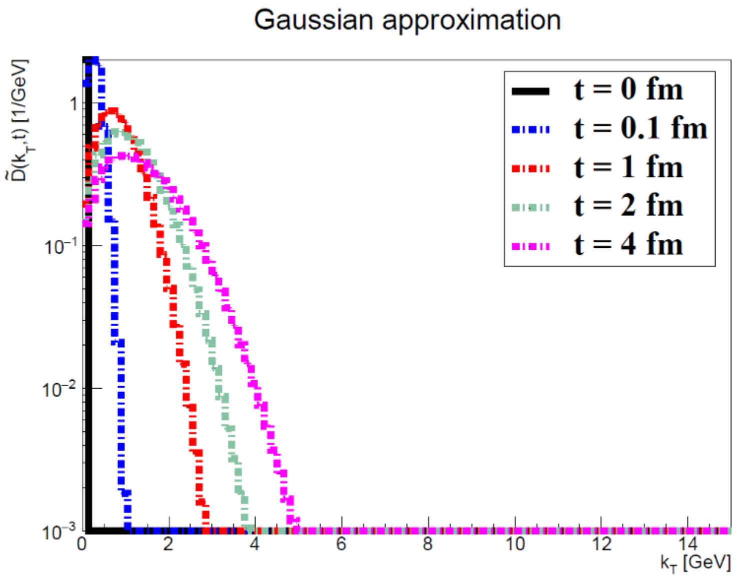


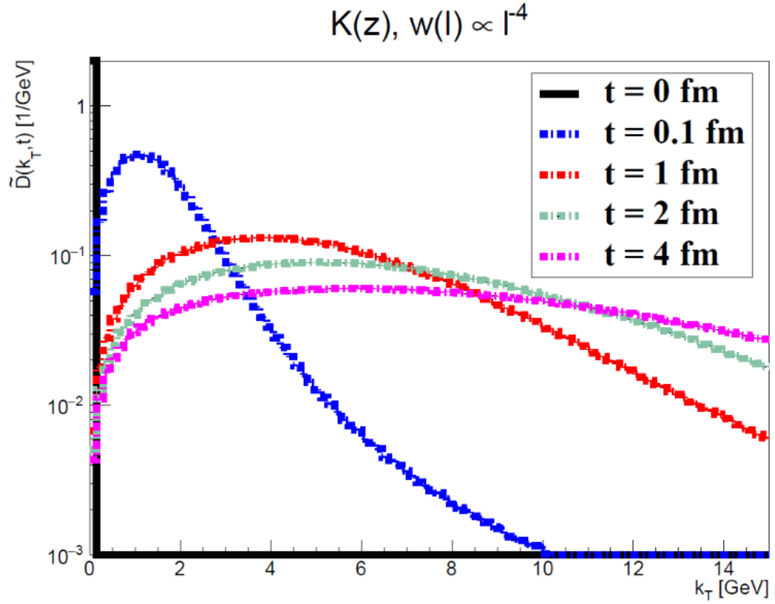
Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Broadening

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$







k_T Broadening

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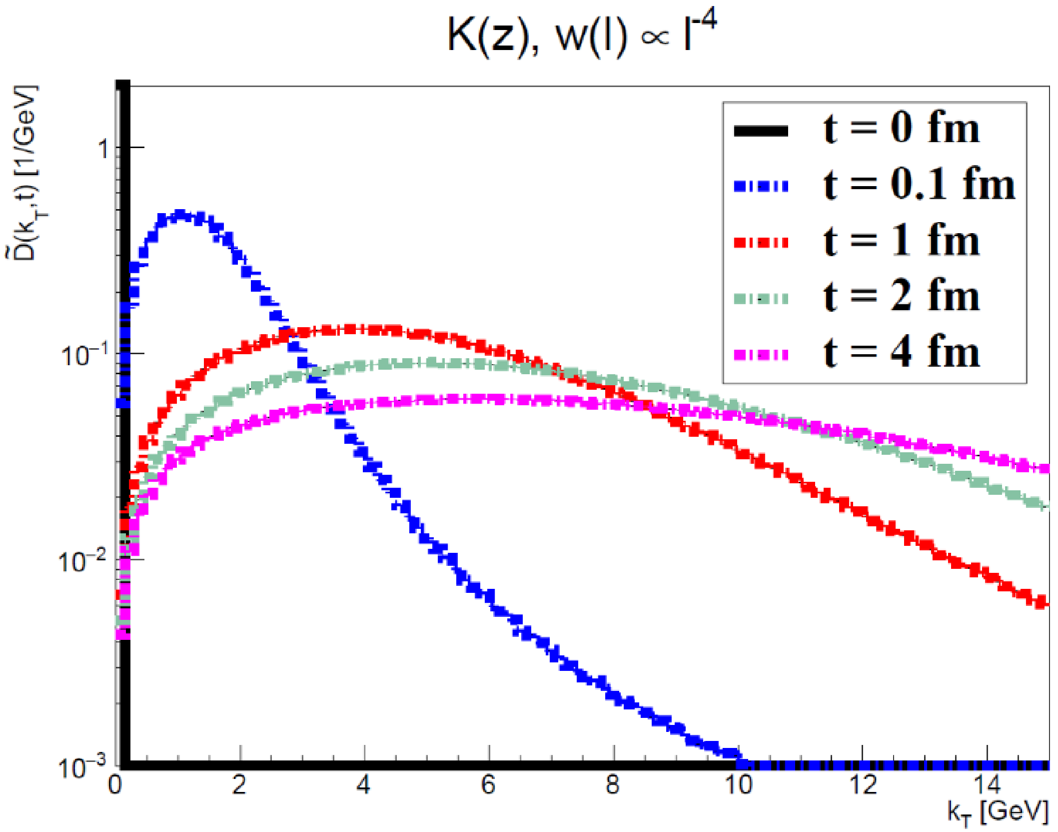
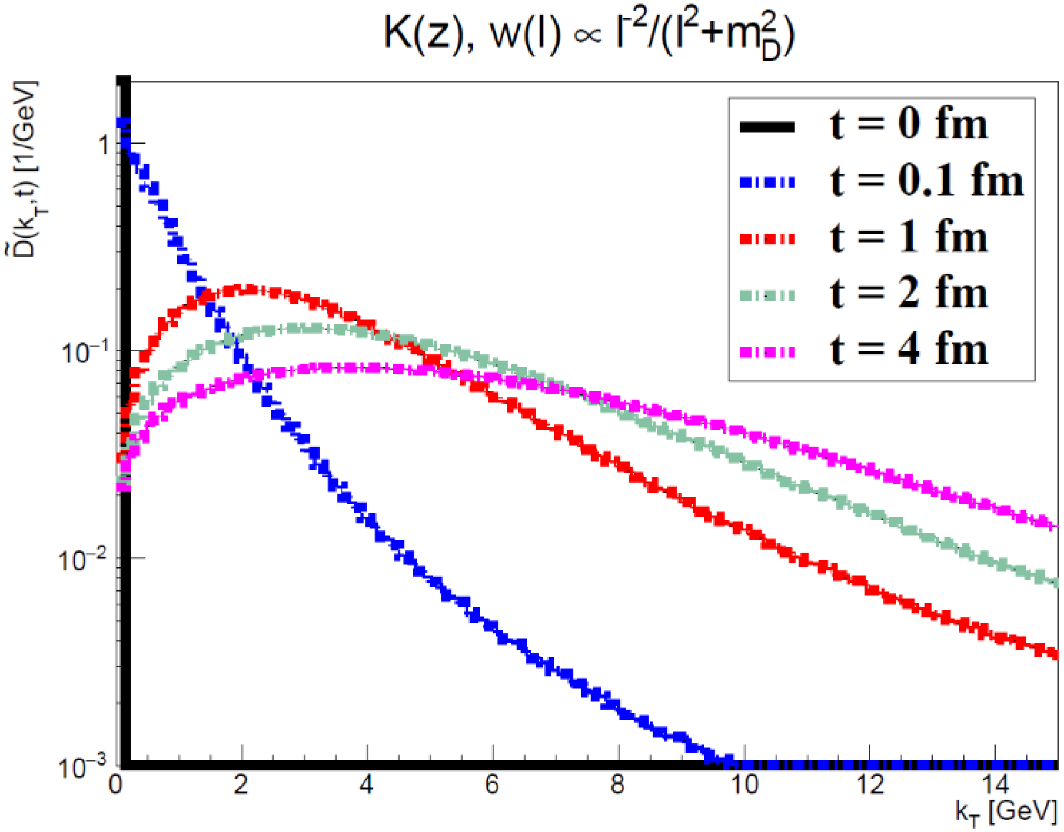
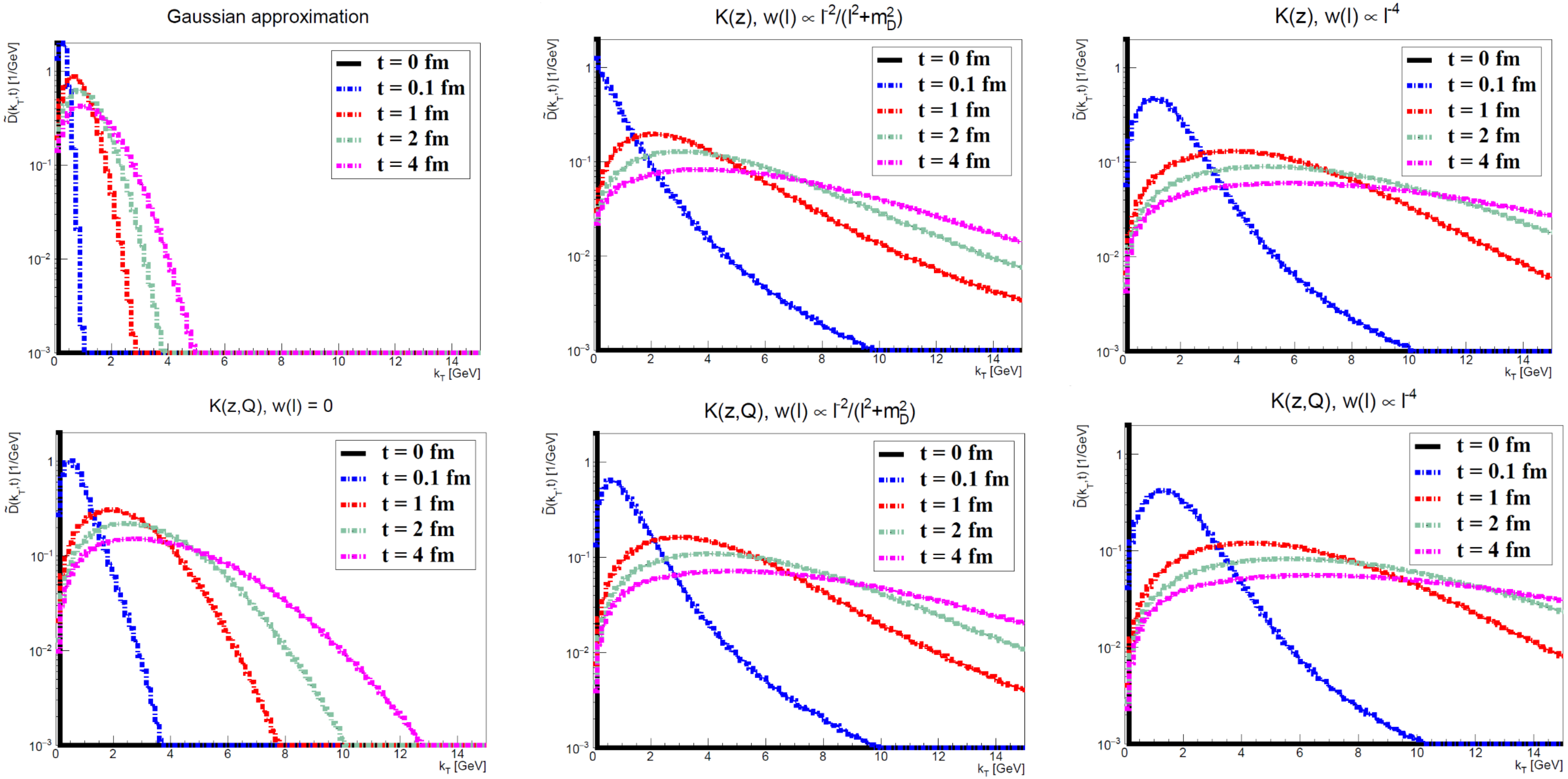




k_T Broadening

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Entropy for leading particles

Delta-Entropy: [B. Chen, Y. Zhu, J. Hu and J. C. Principe,System Parameter Identification. InformationCriteria and Algorithms. Elsevier, 2013, https://doi.org/10.1016/C2012-0-01233-1]

$$S_\Delta(t) = -\sum_{i=1}^{N} p_i(x,k_T,t)\,\ln p_i(x,k_T,t) + P(t)\ln\left[\Delta x\,\Delta k_T\right]$$

$$\Delta x \to 0,\ \Delta k_T \to 0$$

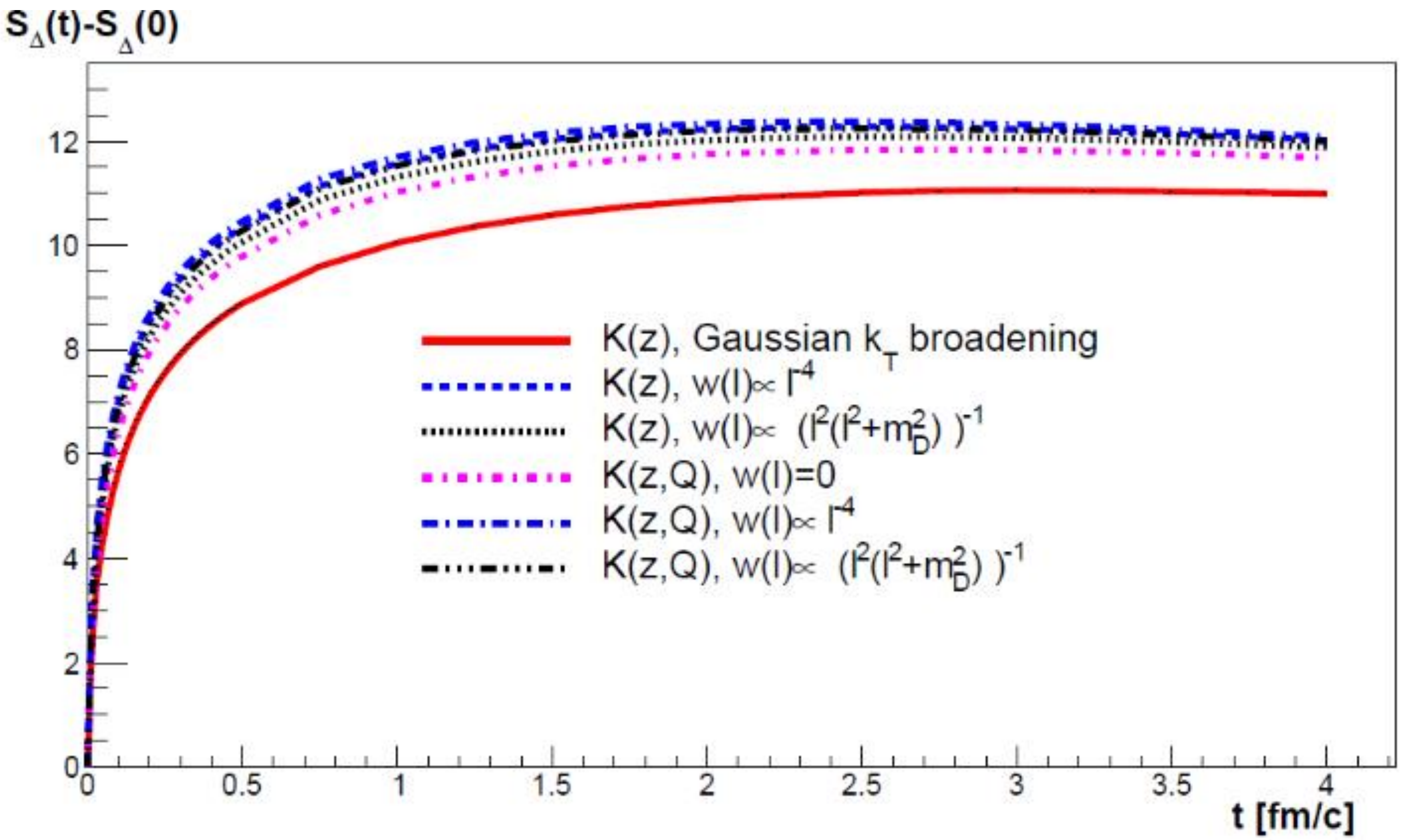
$$S_\Delta(t) \to -\int dx dk_T \tilde{D}(x,k_T,t)\ln\left(\tilde{D}(x,k_T,t)\right)$$

$$p_i(x,k_T,t) = \left[\tilde{D}(x,k_T,t)\,\Delta x\,\Delta k_T\right]_i = \left[2\pi k_T\, D(x,k_T,t)\,\Delta x\,\Delta k_T\right]_i$$

$$P(t) = \sum_{i=1}^{N} p_i(x,k_T,t)$$

Entropy for leading particles

$\hat{q} = 2\ \mathrm{GeV}^2/\mathrm{fm}$



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Entropy for leading particles

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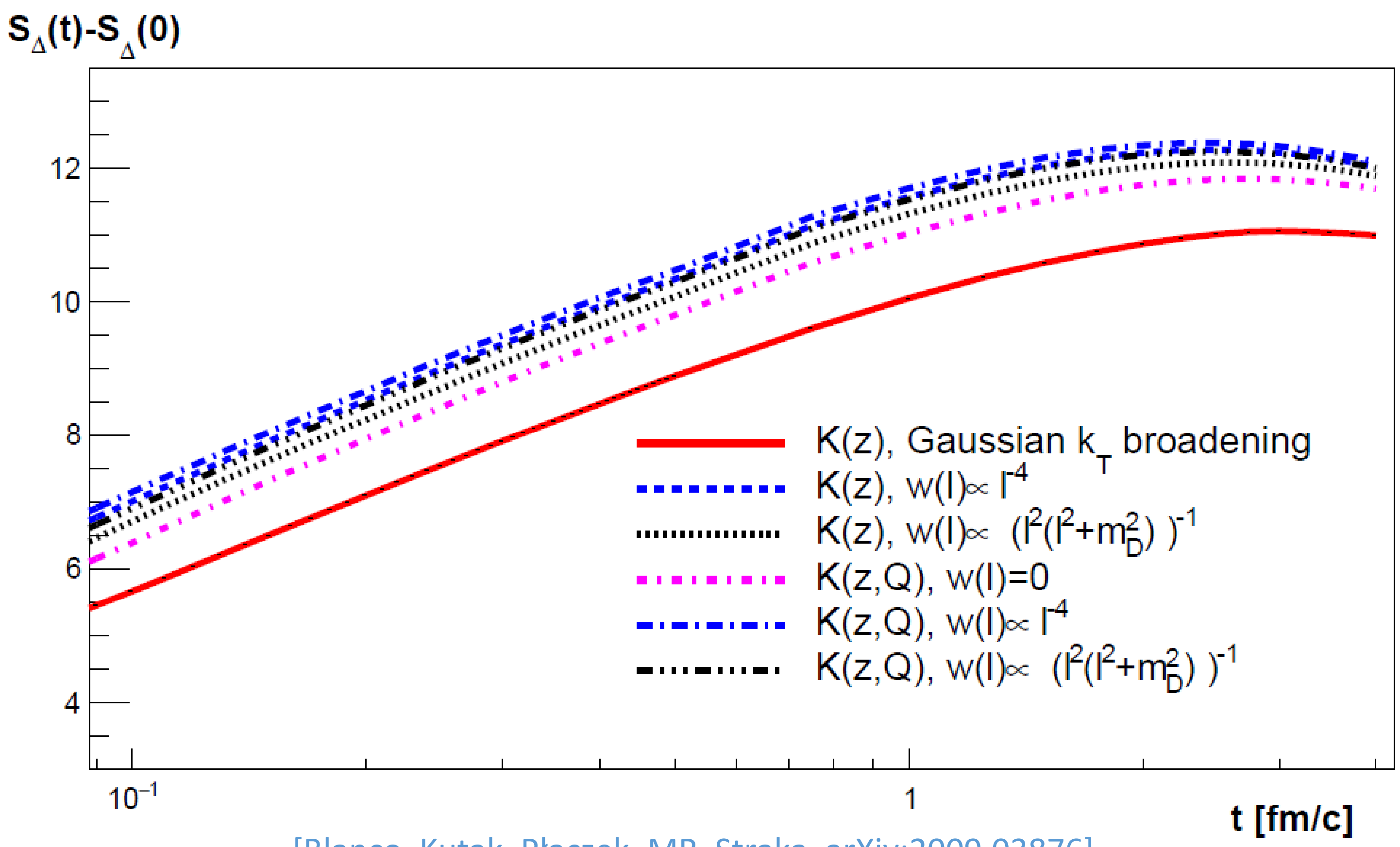
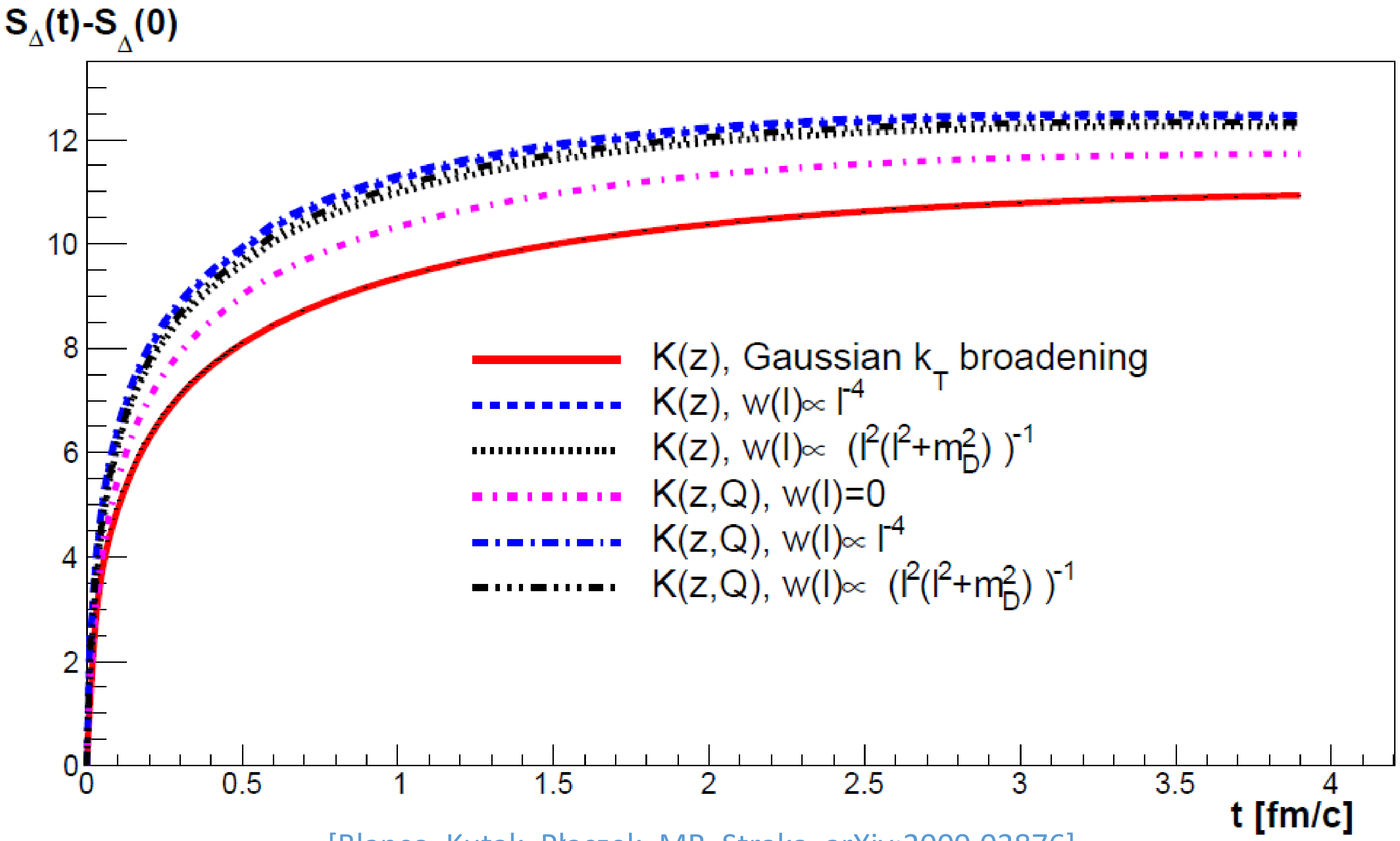


[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

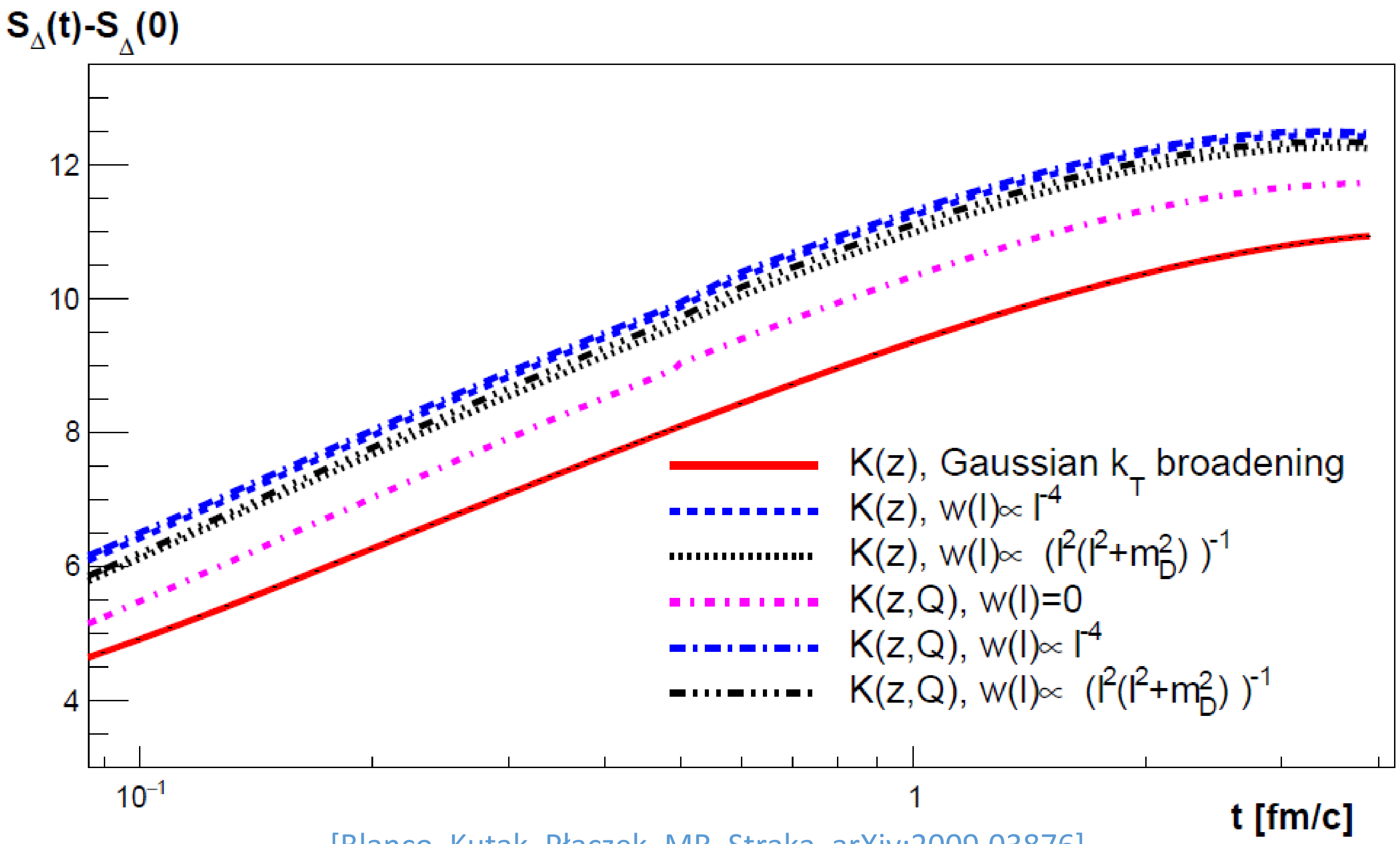
Entropy for leading particles



$\hat{q} = 1\ \mathrm{GeV}^2/\mathrm{fm}$

[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Entropy for leading particles



$\hat{q} = 1 \text{ GeV}^2/\text{fm}$

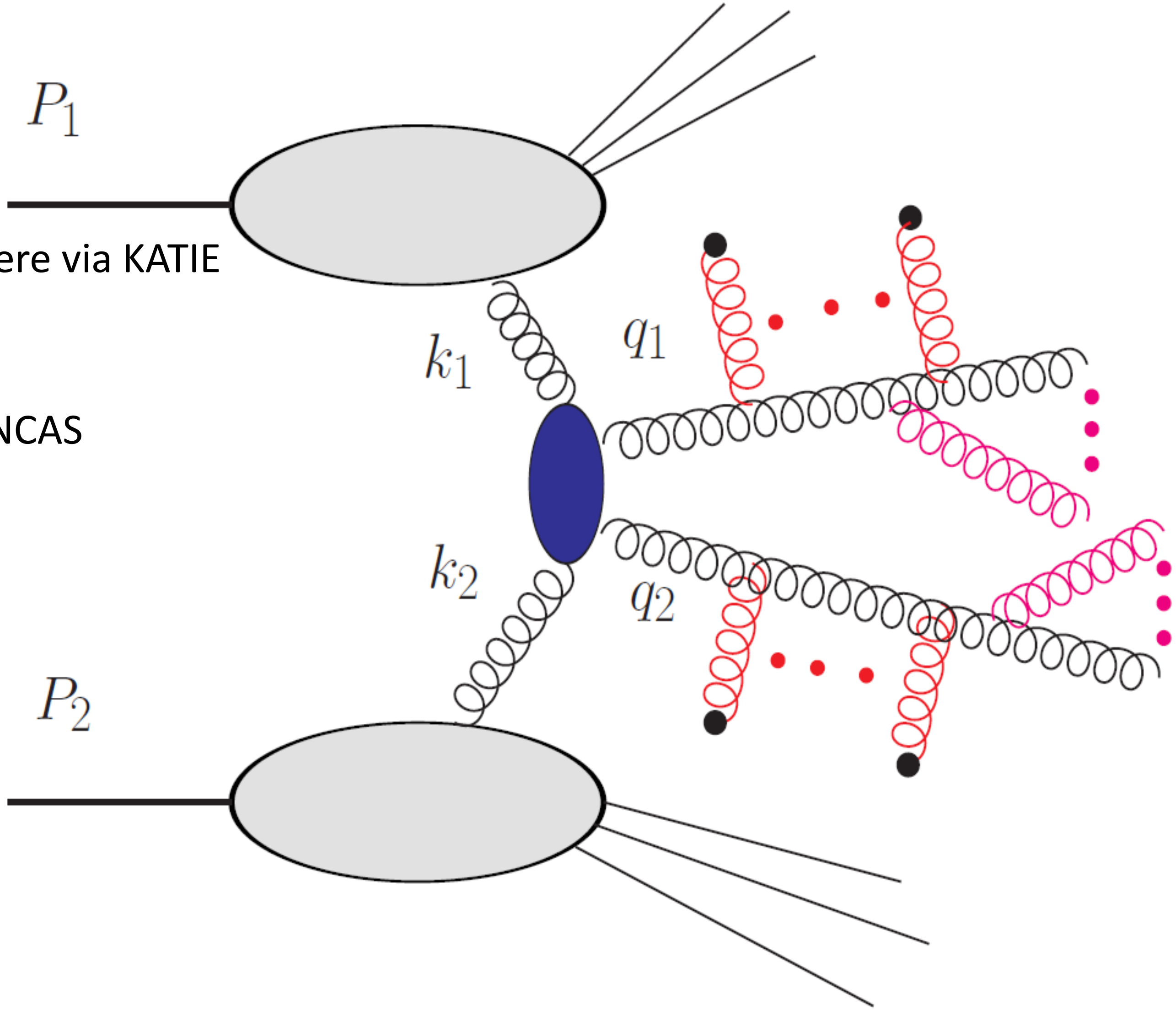
[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Jet Production(1/3)

Cross section =
PDF1(or TMD1)*PDF2(TMD2)
*hard cross section
*fragmentation of jet1
*fragmentation of jet2

Here via KATIE

Here via MINCAS



Jet Production (2/3)

k_T factorization:

$$\frac{d\sigma_{pp}}{dy_1 dy_2 d^2 q_{1T} d^2 q_{2T}} = \int \frac{d^2 k_{1T}}{\pi} \frac{d^2 k_{2T}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}^{\text{off-shell}}_{g^* g^* \to gg}|^2}$$
$$\times \delta^{(2)} \left(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_{1T} - \vec{q}_{2T} \right) \mathcal{F}_g(x_1, k_{1T}^2, \mu_F^2) \mathcal{F}_g(x_2, k_{2T}^2, \mu_F^2)$$

$\mathcal{F}_g(x, k_T^2, \mu_F^2)$ …transverse momentum distribution (TMD)

→ full phase space access at LO
particularly relevant at low x

Jet Production (3/3)

Factorization for AA collisions:

$$\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2\boldsymbol{l} \int_0^1 \frac{d\tilde{x}}{\tilde{x}} \delta(p^+ - \tilde{x}q^+)\delta^{(2)}(\boldsymbol{p} - \boldsymbol{l} - \boldsymbol{q}) D(\tilde{x}, \boldsymbol{l}, \tau(q^+)) \frac{d\sigma_{pp}}{d\Omega_q}$$

$$d\Omega_q = dq^+ d^2\boldsymbol{q}$$

$$\tau(q^+) = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{q^+}}L$$

↓

$$\frac{d^2\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} = \int d\Omega_{q_1} \int d\Omega_{q_2} \int d^2\boldsymbol{l}_1 \int d^2\boldsymbol{l}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1}\delta(p_1^+ - \tilde{x}_1 q_1^+) \int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2}\delta(p_2^+ - \tilde{x}_2 q_2^+)$$
$$\delta^{(2)}(\boldsymbol{p}_1 - \boldsymbol{l}_1 - \boldsymbol{q}_1)\delta^{(2)}(\boldsymbol{p}_2 - \boldsymbol{l}_2 - \boldsymbol{q}_2) D(\tilde{x}_1, \boldsymbol{l}_1, \tau(q_1^+)) D(\tilde{x}_2, \boldsymbol{l}_2, \tau(q_2^+)) \frac{d^2\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}}$$

Program: KATIE+MINCAS

- Use KATIE for hard initial collisions:
  - PDFs/TMDs for colliding nucleons
  - Hard collision cross-section (Monte-Carlo simulation)
  - Resulting particles→initial particles of jets

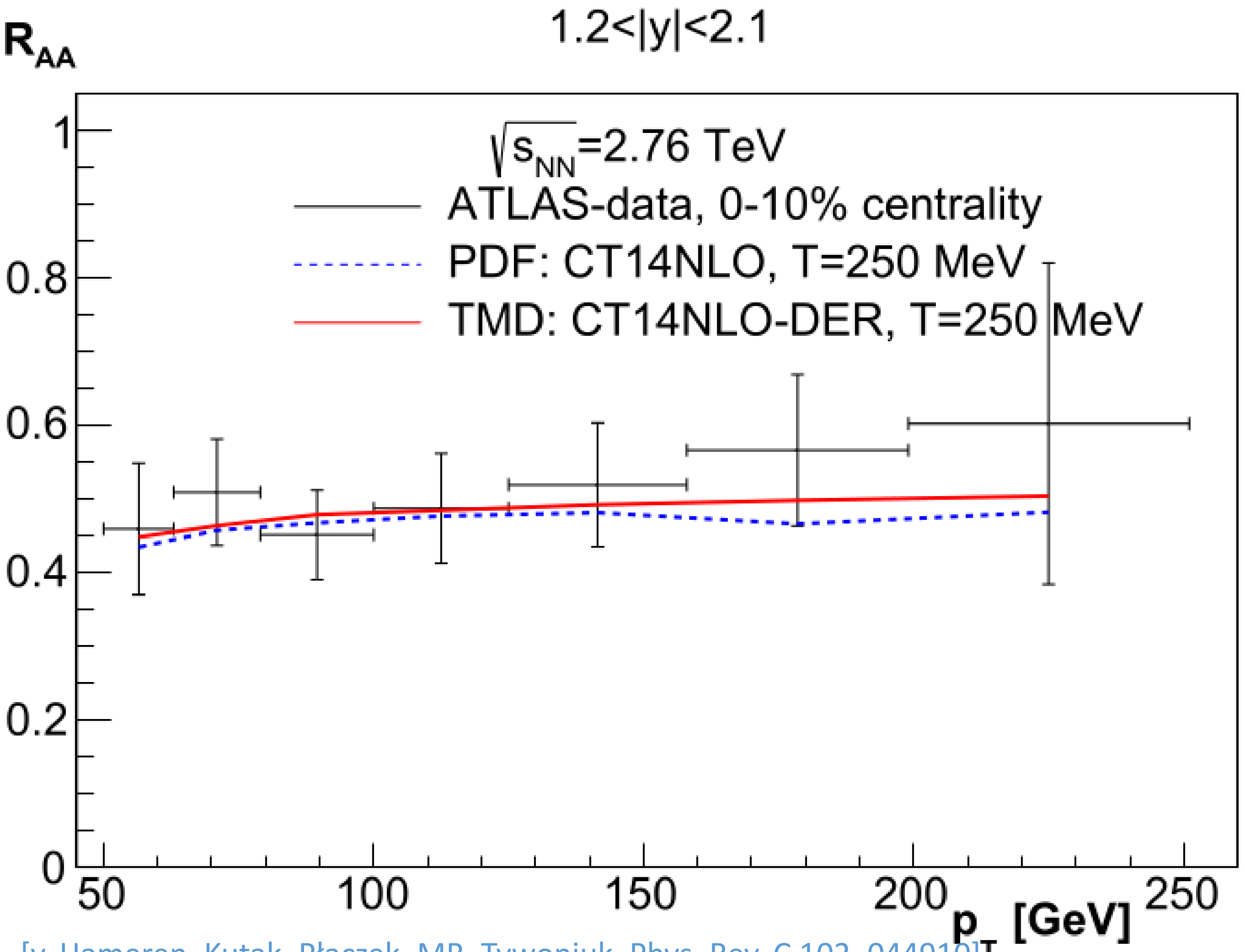
[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380]

- Jets: by MINCAS
  - Monte-Carlo simulation of BDIM equation
  - Time-evolution of jets in medium

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

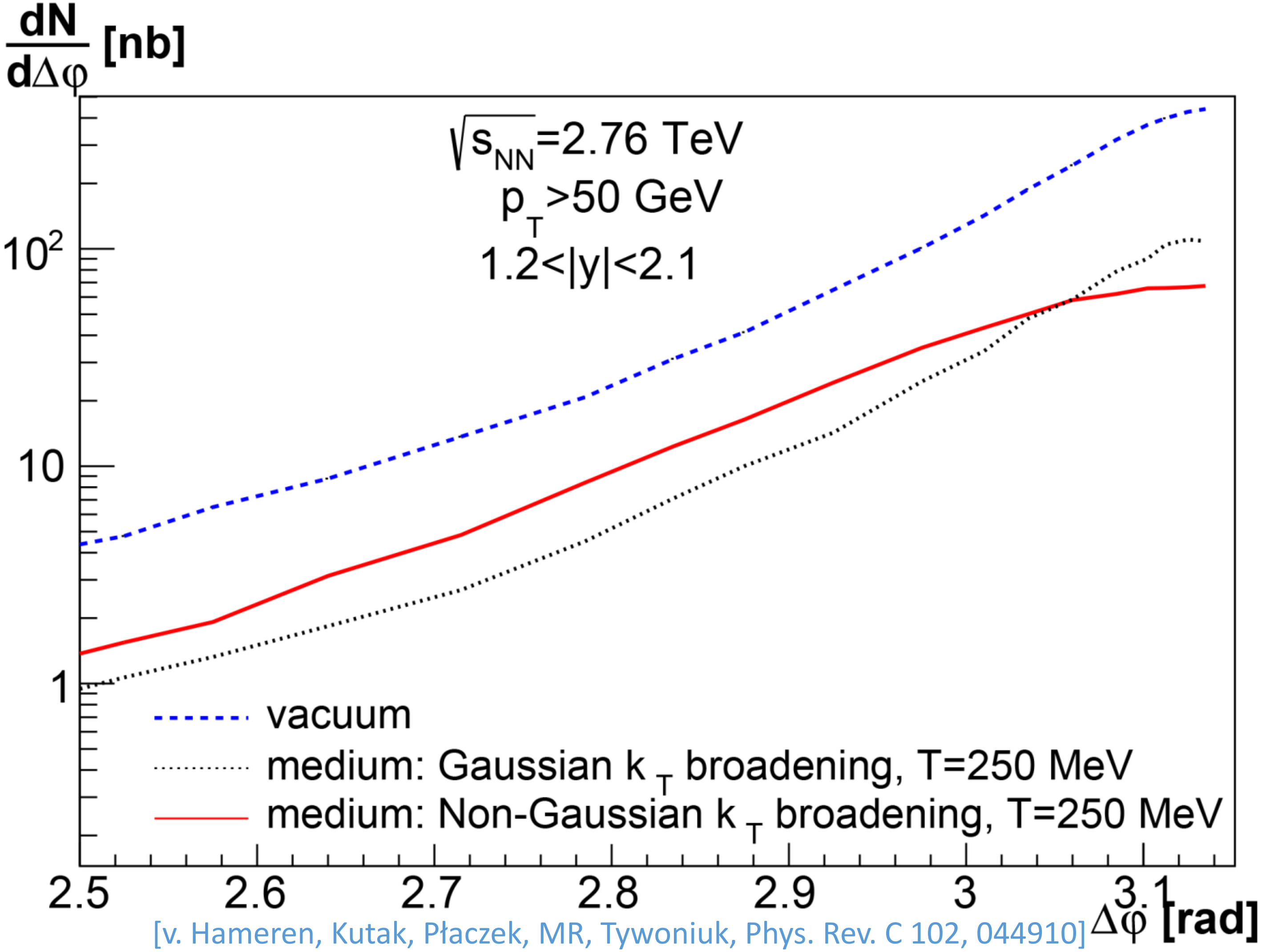
R_{AA}

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA}\rangle \frac{d\sigma_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$



[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910]

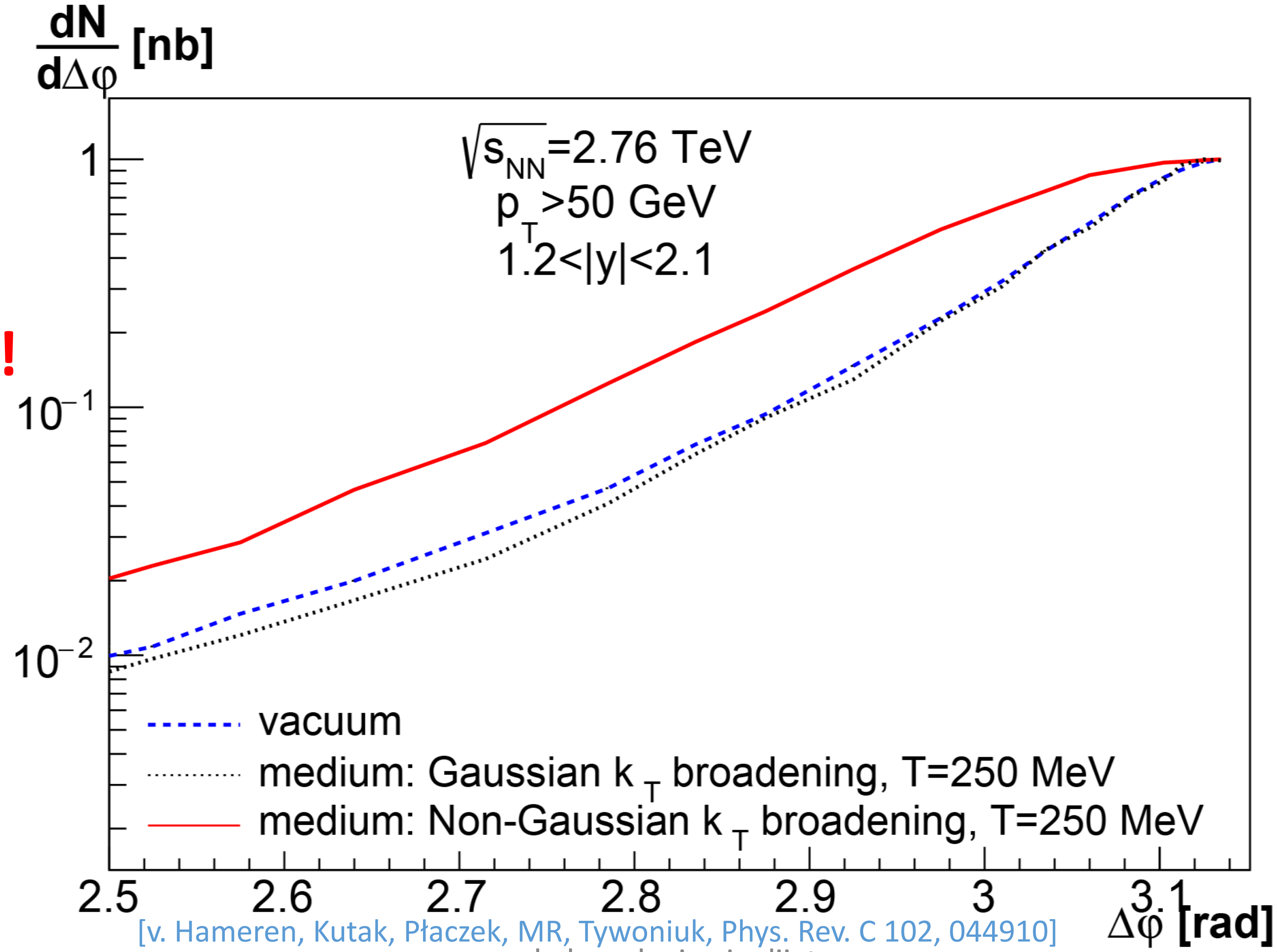
Azimuthal Decorrelations



[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910]

Azimuthal Decorrelations

Normalized to maximum!



[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910]

Summary

- MINCAS: jet evolution based on coherent emission and scattering
- Combination with KATIE: allows for calculation of jet-observables

- Transverse momentum broadening differs from Gaussian distribution
- …leads to broadening in angular dijet decorrelations
- Entropy of leading particle increases with k_T broadening.

Outlook

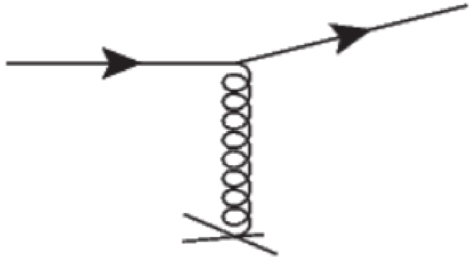
- to account for quarks
- to study more forward processes

Thank you for your attention!

Back-up slides

Processes in Jets

scattering…



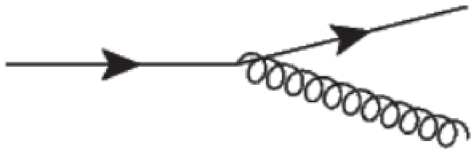
Transverse momentum transfer!

$$p \to p + k_T$$

Scattering Kernel: $C(k_T)$

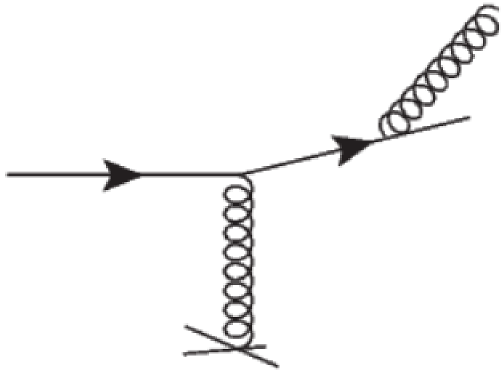
Average transfer: $\hat{q}$

…splitting…



Bremsstrahlung as in vacuum.

…induced radiation



Momentum distribution:

$$p \to zp$$

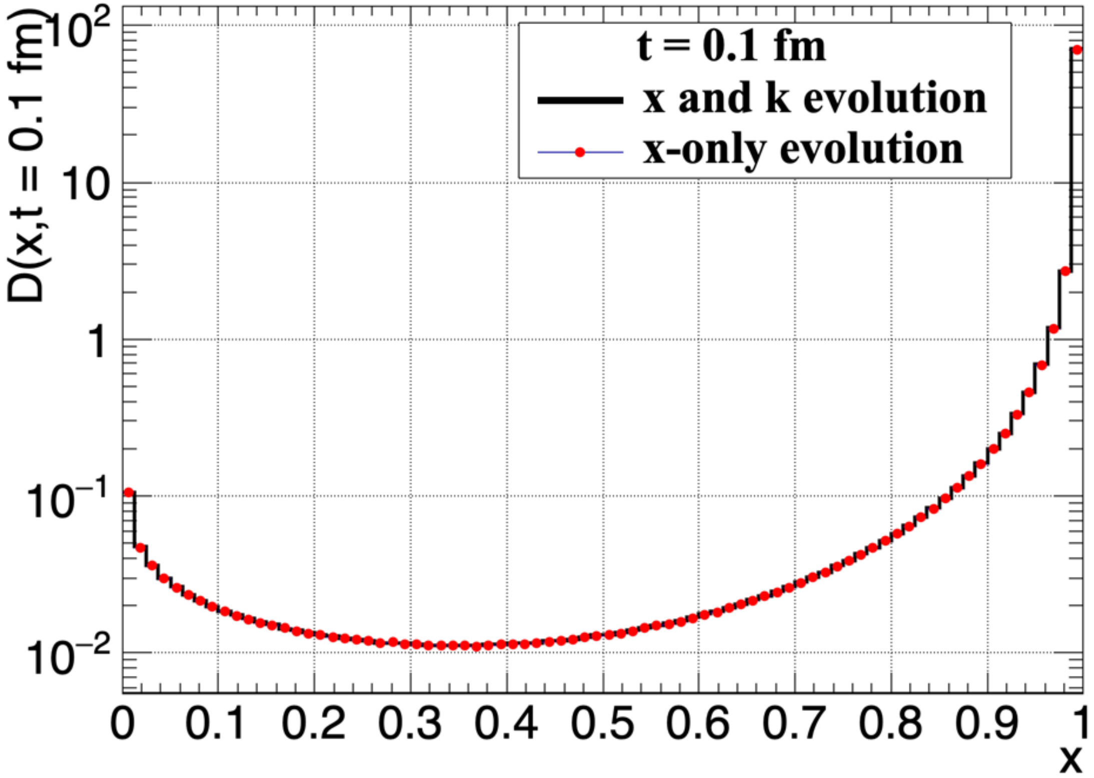
+Momentum transfer:

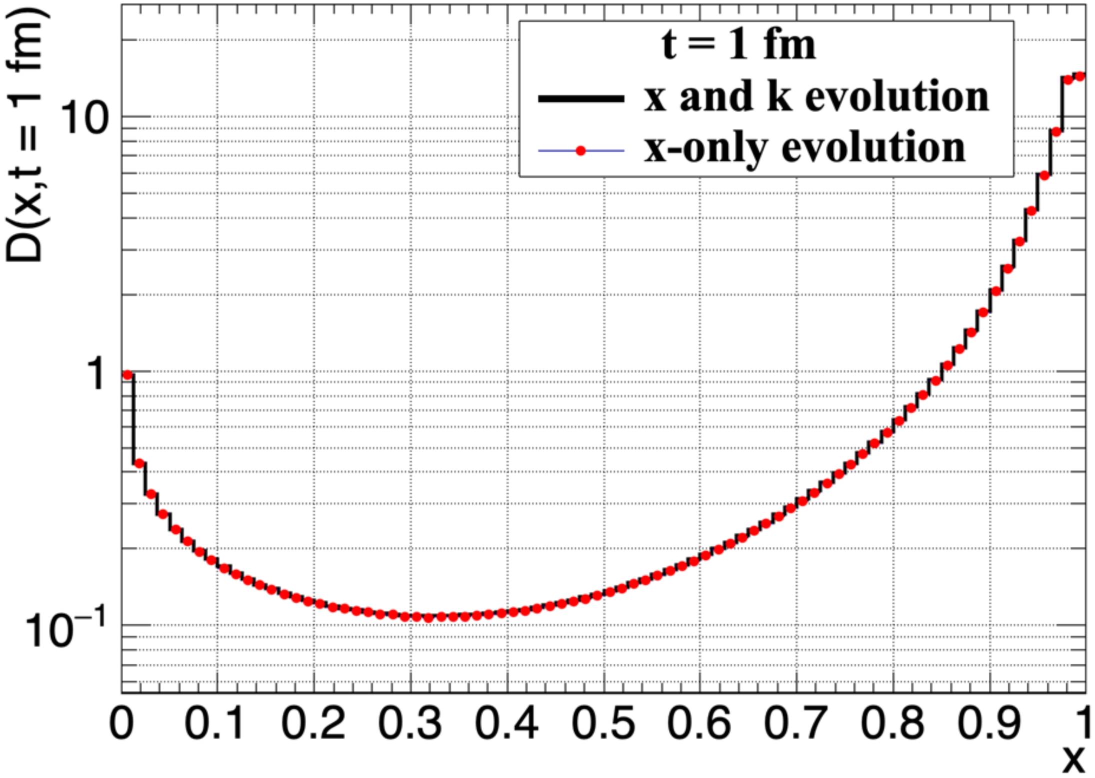
$$p \to zp + k_T$$

Kernel: $\mathcal{K}(z, k_T)$

Our results: combination of scattering and induced radiation processes!

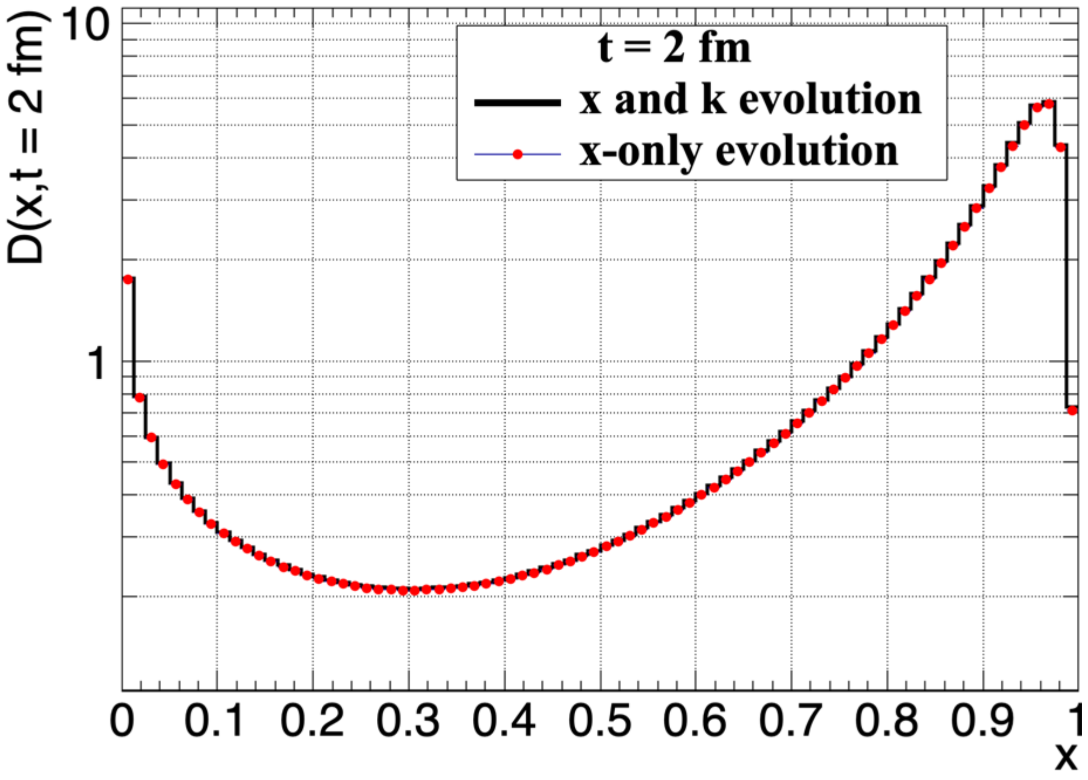
Turbulent behavior (1/2)

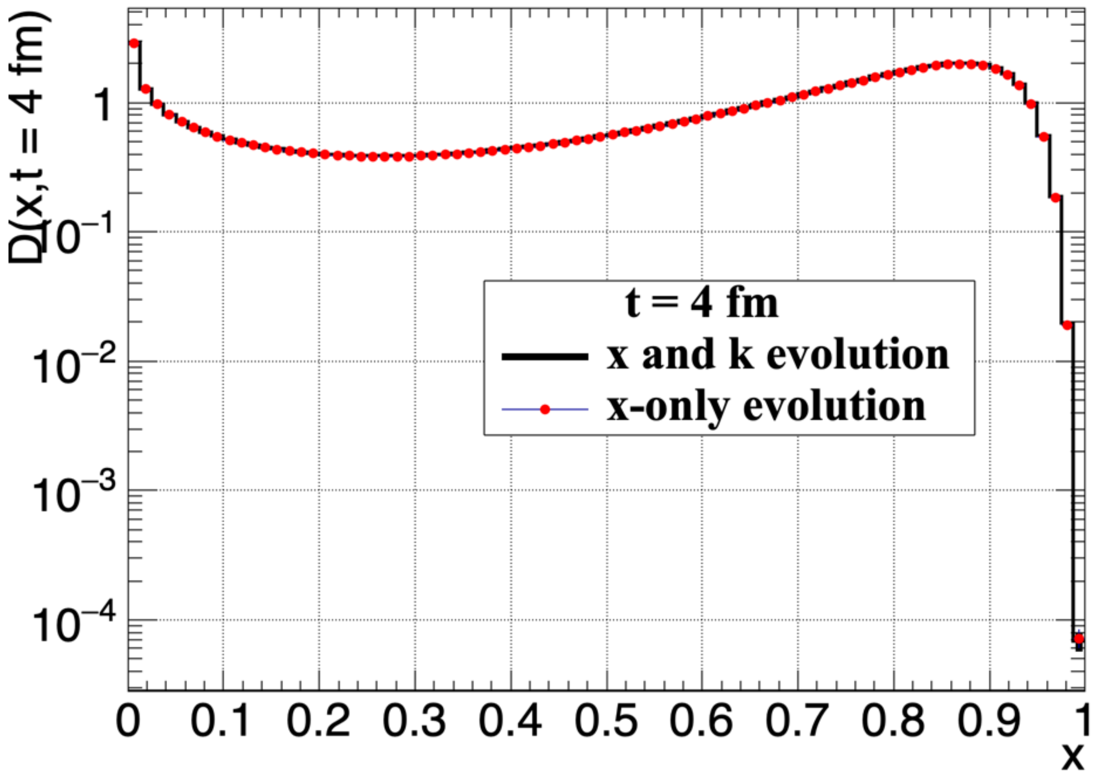




[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Turbulent behavior (2/2)

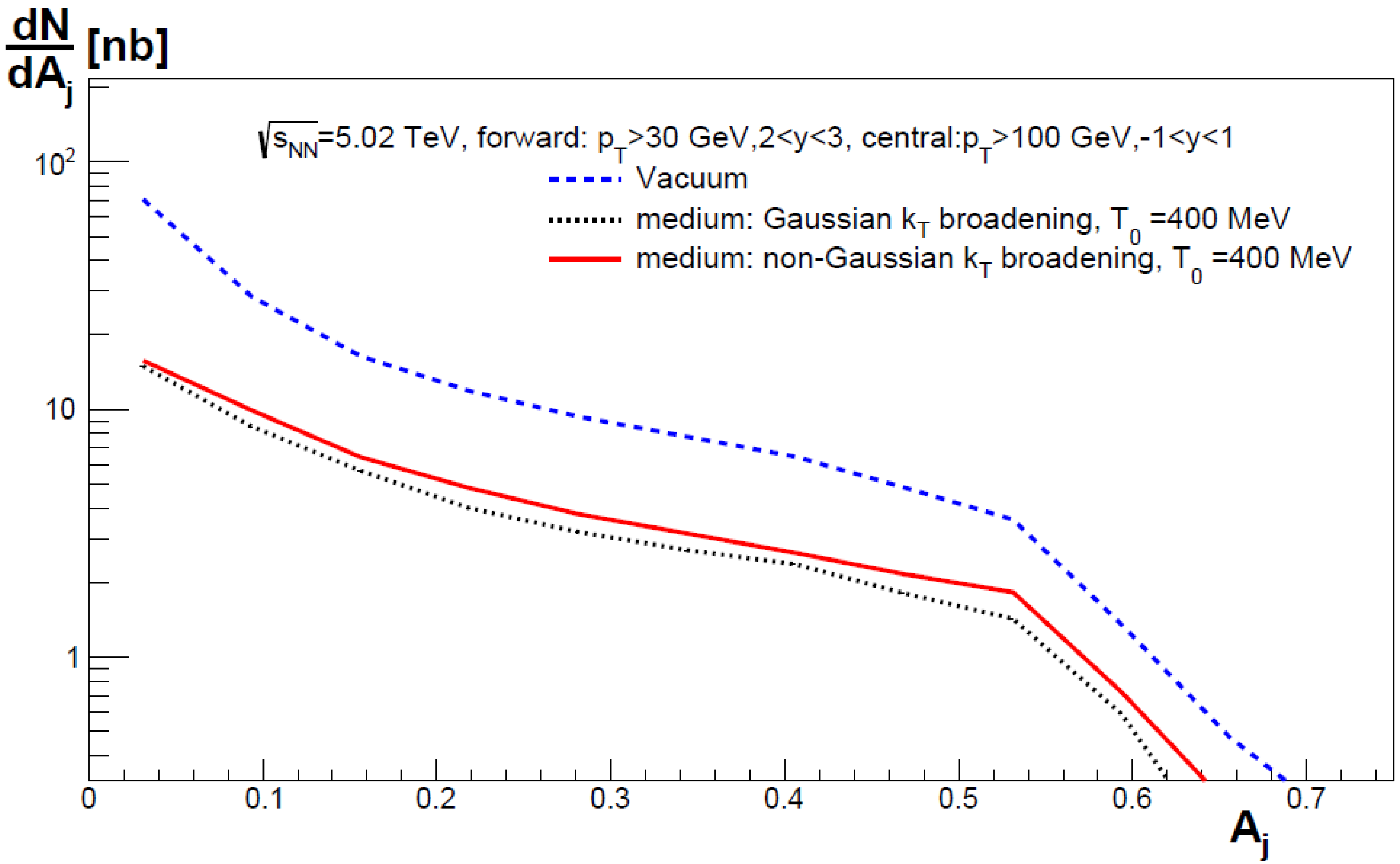




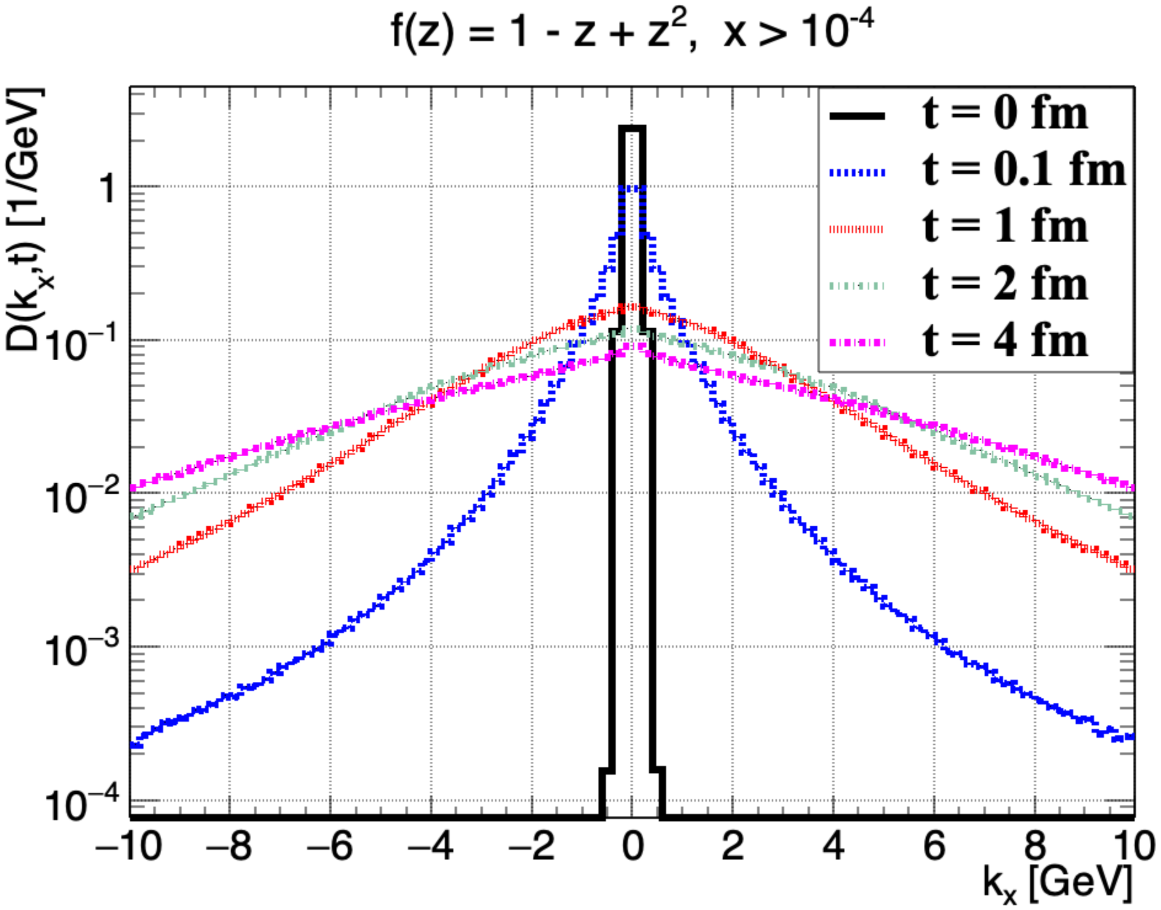
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

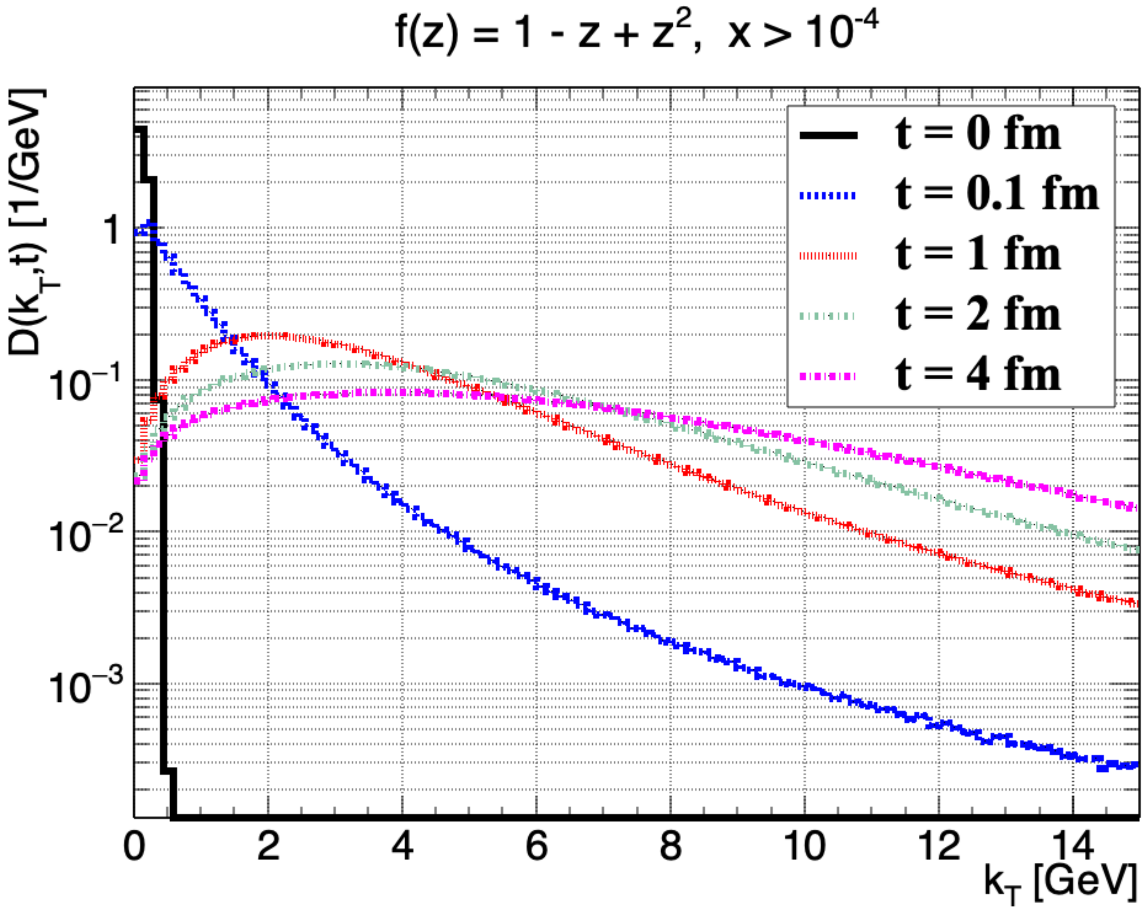
Asymmetry A_j

$$A_j = \frac{p_{Tc} - p_{Tf}}{p_{Tc} + p_{Tf}}$$



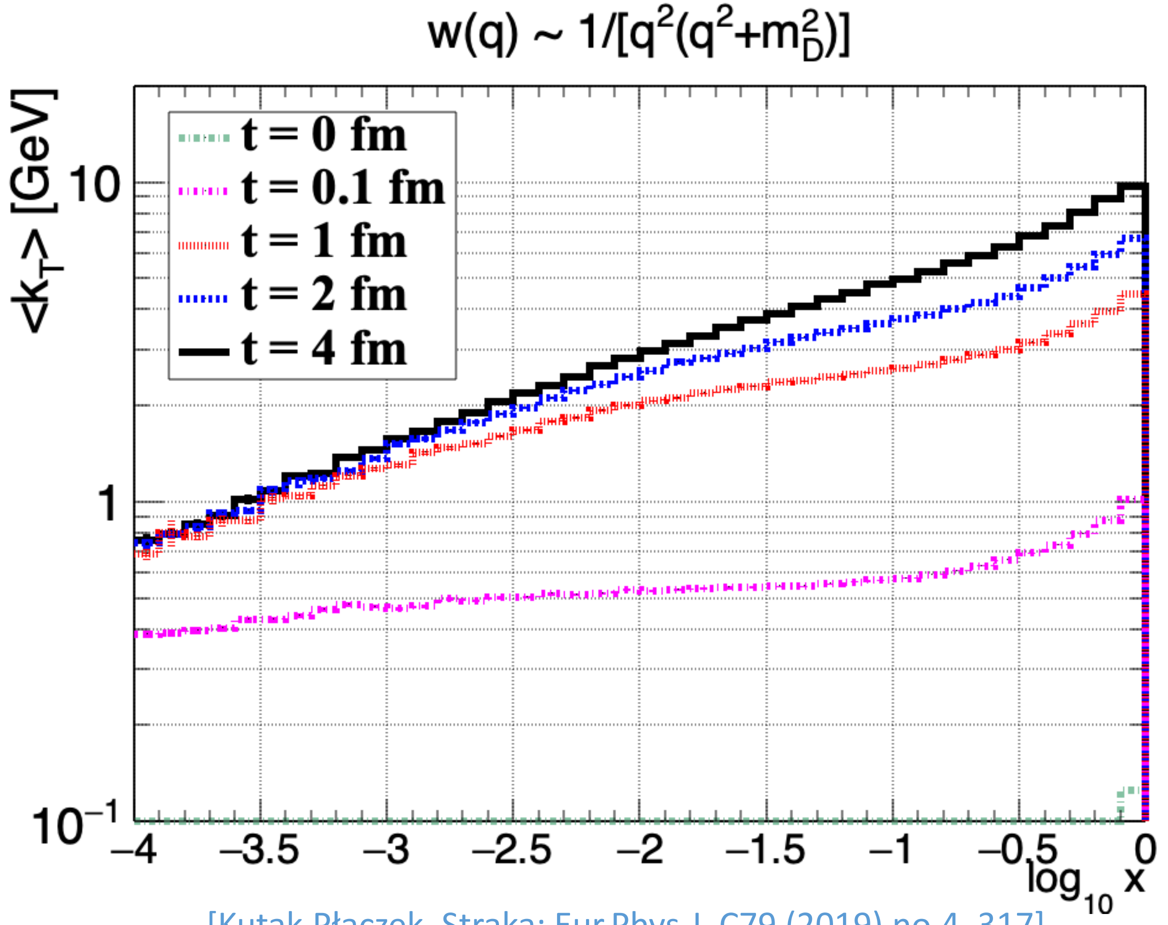
k_T Distribution(2/3)





[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Distribution(3/3)



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Gaussian k_T broadening

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$

↓ Integrate over $d^2\boldsymbol{k}$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

↓

For comparison with full equation: add $\boldsymbol{k}$ selected from Gaussian! width: $\sigma^2 \sim \hat{q}L$

Medium Model

Bjorken Model:

$$T(t) = T_0 \left(\frac{t_0}{t}\right)^{\frac{1}{3}}$$

T ...temperature at time t

T_0...temperature at time t_0

fixed		free		resulting	
c_q	3.7	t_0	0.6 fm/c	$\langle \hat{q} \rangle$	0.54 $\mathrm{GeV}^2/\mathrm{fm}$
c_n	5.228	t_L	5 fm/c	$\langle n \rangle$	0.154 GeV^3
		T_0	0.4 GeV	$\langle m_D \rangle$	0.684 GeV

From Phenomenology (the JET-Collaboration):

$$\hat{q}(T) = c_q T^3$$

[JET Collaboration, Burke et al.: Phys. Rev.C90(2014) 014909]

From HTL:

$$m_D^2 = \left(\frac{N_C}{3} + \frac{N_F}{6}\right) g^2 T^2$$

cf. [Laine, Vuorinen: Lect. Notes Phys.925(2016)pp.1–281, 1701.01554]

Bose-Einstein/Fermi-Dirac Distribution

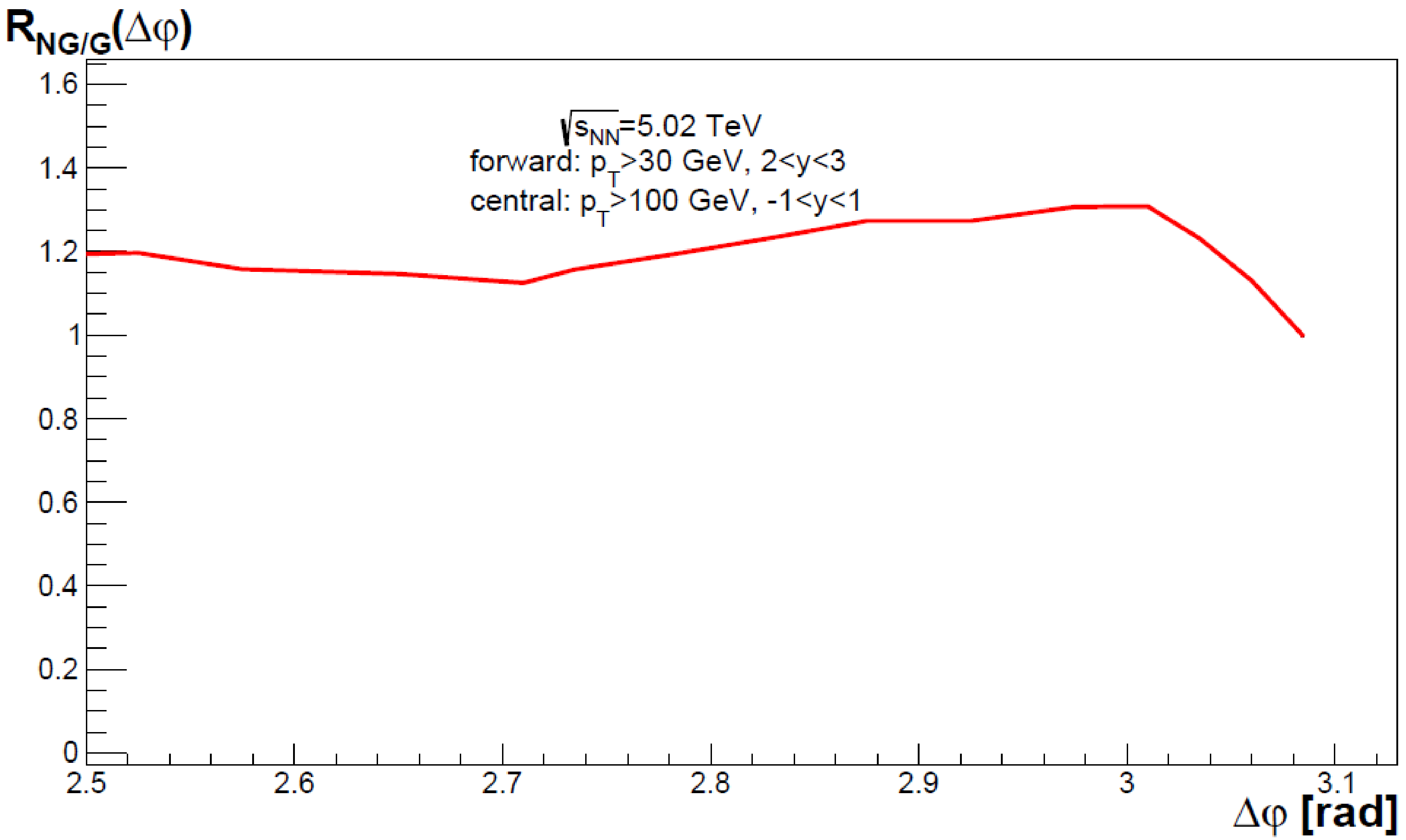
↓

Taylor expansion, lowest order:

$$n(T) = n_q + n_{\bar{q}} + n_g = c_n T^3$$

cf.:[K.C.Zapp, PhD-Thesis, Heidelberg U., 2008]

Azimuthal Decorrelations (ratio)



R_{AA}

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA}\rangle \frac{d\sigma_{pp}}{dp_T}}$$

$$\approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$