

Nuclear Parton Distributions

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Outline

Introduction

Global Analysis of PDFs

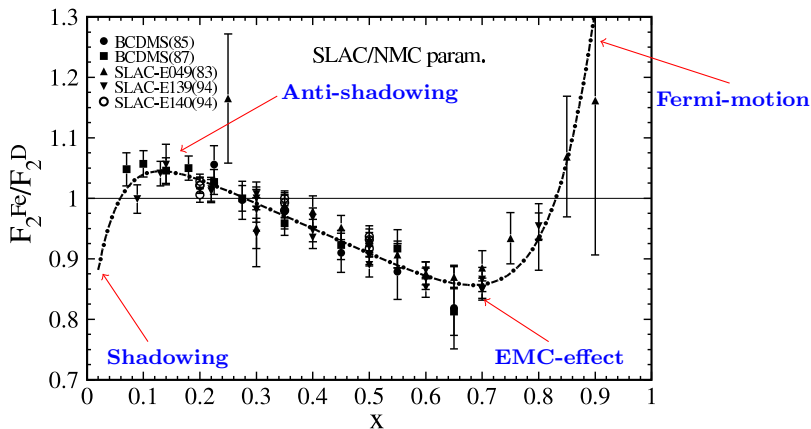
Summary of available nuclear PDFs

Selected results

Conclusions

Motivation

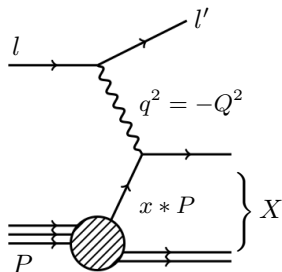
- Cross-sections in nuclear collisions are modified



- Can we translate this modifications into a universal quantities like nPDFs?

Motivation

- Factorization in case of deep inelastic scattering (DIS)

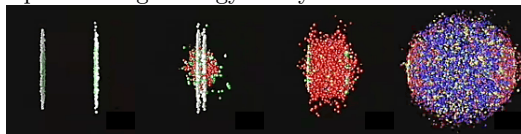


$$\frac{d^2\sigma}{dx dQ^2} = \sum_i f_i^A(x, Q^2) \otimes d\hat{\sigma}_{il \rightarrow l' X} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

- We assume that the nuclear effects can be absorbed into the universal **nPDFs** $[f_i^A(x, Q^2)]$.
- Do not consider any cold nuclear matter effects (e.g. energy loss).
- Include kinematic cuts to suppress non-leading terms, e.g.
 - nCTEQ: $\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$ EPS: $Q > 1.3 \text{ GeV}$

Motivations: Why do we need nuclear PDFs?

- ▶ Information on the structure of nucleus
 - ▶ small- x : onset of non-linear dynamics
 - ▶ large- x : QCD confinement, EMC effect, possible non-perturbative source of charm or beauty
- ▶ Description of high-energy heavy ion collisions at the **LHC** and **RHIC**



Key ingredient to use perturbative probes of QGP

- ▶ Computation of prompt atmospheric neutrino flux.
- ▶ Differentiate flavors in free-proton PDFs (e.g. strange)

charged lepton DIS

$$F_2^{l^\pm} \sim \left(\frac{1}{3}\right)^2 [d + s] + \left(\frac{2}{3}\right)^2 [u + c]$$

neutrino DIS

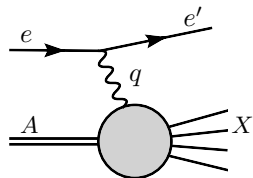
$$F_2^\nu \sim [d + s + \bar{u} + \bar{c}]$$

$$F_2^{\bar{\nu}} \sim [\bar{d} + \bar{s} + u + c]$$

$$F_3^\nu \sim 2[d + s - \bar{u} - \bar{c}]$$

$$F_3^{\bar{\nu}} \sim 2[u + c - \bar{d} - \bar{s}]$$

Variables: DIS of nuclear target $eA \rightarrow e'X$



- ▶ DIS variables in case on nucleons

$$\text{in nucleus } \begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2 p_A \cdot q} \end{cases}$$

- ▶ p^A – nucleus momentum
- ▶ $x_A \in (0, 1)$ – analog of Bjorken variable
(fraction of the nucleus momentum carried by a nucleon)
- ▶ Analogue variables for partons:
 - ▶ $p_N = \frac{p_A}{A}$ – average nucleon momentum
 - ▶ $x_N \equiv \frac{Q^2}{2 p_N \cdot q} = A x_A$ – parton momentum fraction with respect to the average nucleon momentum p_N
 - ▶ $x_N \in (0, A)$ – parton can carry more than the average nucleon momentum p_N .

Assumptions entering the nuclear PDF analysis

1. **Factorization** & DGLAP evolution

- ▶ allow for definition of **universal PDFs**
- ▶ make the formalism **predictive**
- ▶ needed even if it is broken

2. Isospin symmetry $\left\{ \begin{array}{l} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{array} \right. \quad f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$

3. The *bound proton* PDFs have the *same evolution equations* and sum rules as the free proton PDFs *provided we neglect any contributions from the region $x > 1$* (which is expected to have negligible contribution [PRC 73, 045206 (2006), [arXiv:hep-ph/0509241](#)])

Then observables $\mathcal{O}^A$ can be calculated as:

$$\mathcal{O}^A = Z \mathcal{O}^{p/A} + (A - Z) \mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

Schematics of Global Analysis

1. Choose experimental data (e.g. DIS, DY, inclusive jet prod., etc.)
2. Parametrize **bound proton PDFs** at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$$
$$f_i^{p/A}(x, Q_0) = f_i^{p/A}(x; a_0, a_1, \dots) = a_0 x^{a_1} (1-x)^{a_2} P(x; a_3, \dots)$$

with $a_j = a_j(A)$ depending on the nuclei.

3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\text{max}}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS} , σ_{DY} , etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$
$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}} \right)^2$$

(by default $w_n = 1$)

6. Minimize χ^2 function with respect to parameters $a_0, a_1, \dots$

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Uncertainties in global analysis

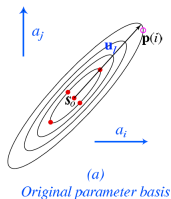
- ▶ **Experimental errors** (included in PDFs error analysis)
- ▶ **Theoretical uncertainties** (e.g. HF schemes; not included)
- ▶ **“Details” of Global Fits**
(e.g. parametrization; not included)

Propagating experimental errors to PDFs:

- ▶ Hessian Method
 - ▶ Eigenvector PDFs
 - ▶ Quadratic approximation
 - ▶ Simple computation of correlations
- ▶ Lagrange Multipliers
- ▶ Monte Carlo Methods
 - ▶ generate N data samples by varying data within errors;
 - ▶ perform N fits to the samples $\rightarrow$ PDF replicas
 - ▶ estimate uncertainty by calculating moments of PDF replicas

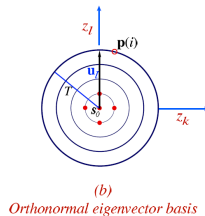
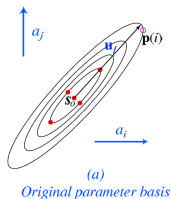
- **Expand** χ^2 function around minimum, $\{a_i^0\}$,

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2} (a_i - a_i^0)(a_j - a_j^0) \left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right)_0$$



- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

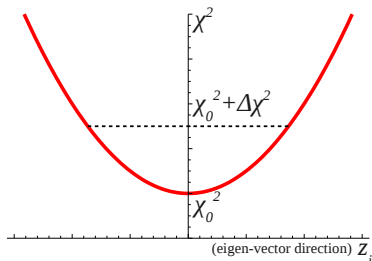
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- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1- σ uncertainty),
 - ideal case $\Delta\chi^2 = 1$
 - realistic global analysis $\Delta\chi^2 \sim 1 - 100$



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 - ▶ ideal case $\Delta\chi^2 = 1$
 - ▶ realistic global analysis $\Delta\chi^2 \sim 1 - 100$
- ▶ Construct error PDFs corresponding to each eigenvector direction:

$$f_i^\pm = f(\{z_i\}) = f(0, \dots, z_i = \pm\sqrt{\Delta\chi^2}, \dots, 0)$$

$$z_i = \pm\sqrt{\Delta\chi^2}$$

- ▶ Calculate errors of observable X :

$$\Delta X = \sqrt{\sum_i \left(\frac{\partial X}{\partial z_i} \times \delta z_i \right)^2} \simeq \frac{1}{2} \sqrt{\sum_i \left[X(f_i^+) - X(f_i^-) \right]^2}$$

Differences with the free-proton PDFs

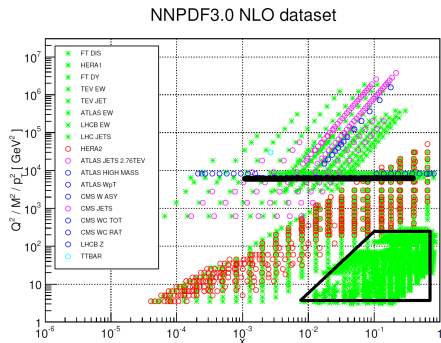
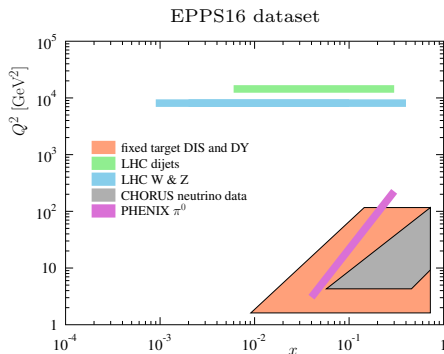
- ▶ Theoretical status of Factorization
- ▶ Parametrization – more parameters to model A -dependence
- ▶ Different data sets – much less data:
 - ▶ Less data $\rightarrow$ less constraining power $\rightarrow$ more assumptions (fixing) about a_i parameters
 - ▶ **Assumptions limit/replace uncertainties!**

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Available nuclear PDFs

► Multiplicative nuclear correction factors

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A) f_i^{\text{free proton}}(x_N, \mu_0)$$

- **HKN**: Hirai, Kumano, Nagai
[PRC 76, 065207 (2007), [arXiv:0709.3038](#)]
- **DSSZ**: de Florian, Sassot, Stratmann, Zurita
[PRD 85, 074028 (2012), [arXiv:1112.6324](#)]
- **EPPS16**: Eskola, Paakkinen, Paukkunen, Salgado
[EPJC 77 (2017) 163, [arXiv:1612.05741](#)]
- **KT16** H.Khanpour, S.A.Tehrani
[PRD 93, 014026 (2016), [arXiv:1601.00939](#)]

► Native nuclear PDFs

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0), \quad f_i(x_N, A=1, \mu_0) \equiv f_i^{\text{free proton}}(x_N, \mu_0)$$

- **nCTEQ15** [PRD 93, 085037 (2016), [arXiv:1509.00792](#)]
- **TUJU19** M.Walt, I.Helenius, W.Vogelsang
[PRD 100, 096015 (2019), [arXiv:1908.03355](#)]
- **nNNPDF2.0** Khalek, Ethier, Rojo, van Weelden
[JHEP 09 (2020) 183, [arXiv:2006.14629](#)]

► Parametrization

- PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- bound neutron PDFs, $f_i^{n/A}$, constructed assuming iso-spin symmetry
- bound proton PDFs parametrized:

nCTEQ15 [[arXiv:1509.00792](https://arxiv.org/abs/1509.00792)]

EPPS16 [[arXiv:1612.05741](https://arxiv.org/abs/1612.05741)]

$$x f_i^{p/A}(x, Q_0) = x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad f_i^{p/A}(x, Q) = R_i^A(x, Q) f_i^p(x, Q),$$

$$c_k \rightarrow c_k(\textcolor{red}{A}) \equiv c_{k,0} + c_{k,1} \left(1 - \textcolor{red}{A}^{-c_{k,2}}\right)$$

$$R_i^A(x, Q_0) = \begin{cases} a_0 + a_1(x - x_a)^2 & x \leq x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \leq x \leq x_b \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_b \leq x \leq 1 \end{cases}$$

$$d_i \rightarrow d_i(\textcolor{red}{A}) = d_i(A_{\text{ref}}) \left(\frac{\textcolor{red}{A}}{A_{\text{ref}}} \right)^{\gamma_i [d_i(A_{\text{ref}}) - 1]},$$

with $d_i = a_i, b_i, \dots$ and $A_{\text{ref}} = 12$

Comparison of available nPDFs

	EPPS16	TUJU19	nCTEQ15	nCTEQ15WZ	nNNPDF2.0
FT NC DIS	✓	✓	✓	✓	✓
FT CC DIS	✓	✓	✗ [#]	✗	✓
FT Drell-Yan	✓	✗	✓	✓	✗
RHIC π^0	✓	✗	✓	✓	✗
LHC W/Z	✓	✗	✗ [*]	✓	✓
LHC dijet	✓	✗	✗	✗	✗
QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
No data points	1811	2336	740	860	1467
No free param.	20	16	16	19	NN
χ^2/dof	1.00	0.89(0.86)	0.81	0.91	0.98
Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
Tolerance $\Delta\chi^2$	52	50	35	35	–
Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

[#] In a separate dedicated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]

^{*} See a reweighting study [EPJC77, 488 (2017), 1610.02925]

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RHIC π^0	✓	✗	✓	✓	✗
LHC W/Z	✓	✗	✗ [*]	✓	✓
LHC dijet	✓	✗	✗	✗	✗
QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
No data points	1811	2336	740	860	1467
No free param.	20	16	16	19	NN
χ^2/dof	1.00	0.89(0.86)	0.81	0.91	0.98
Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
Tolerance $\Delta\chi^2$	52	50	35	35	–
Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

[#] In a separate dedicated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]

^{*} See a reweighting study [EPJC77, 488 (2017), 1610.02925]

Comparison of available nPDFs

	EPPS16	TUJU19	nCTEQ15	nCTEQ15WZ	nNNPDF2.0
FT NC DIS	✓	✓	✓	✓	✓
FT CC DIS	✓	✓	✗ [#]	✗	✓
FT Drell-Yan	✓	✗	✓	✓	✗
RHIC π^0	✓	✗	✓	✓	✗
LHC W/Z	✓	✗	✗ [*]	✓	✓
LHC dijet	✓	✗	✗	✗	✗
QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
No data points	1811	2336	740	860	1467
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χ^2/dof	1.00	0.89(0.86)	0.81	0.91	0.98
Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
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Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

[#] In a separate dedicated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]

^{*} See a reweighting study [EPJC77, 488 (2017), 1610.02925]

nCTEQ15 fit details

Fit properties:

- ▶ fit @NLO
- ▶ $Q_0 = 1.3\text{GeV}$
- ▶ using ACOT heavy quark scheme
- ▶ kinematic cuts:
 $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
 $p_T > 1.7\text{ GeV}$
- ▶ 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- ▶ 16+2 free parameters
 - ▶ 7 gluon
 - ▶ 7 valence
 - ▶ 2 sea
 - ▶ 2 pion data normalizations
- ▶ $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

- ▶ use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0)(a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- ▶ tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- ▶ eigenvalues span 10 orders of magnitude $\rightarrow$ require numerical precision
- ▶ use noise reducing derivatives

nCTEQ15 fit details

Kinematic cuts

nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3 \text{ GeV}$

HKN: $Q > 1 \text{ GeV}$

DSSZ: $Q > 1 \text{ GeV}$

Fit properties:

- ▶ fit @NLO
- ▶ $Q_0 = 1.3 \text{ GeV}$
- ▶ using ACOT
- ▶ kinematic cuts:
 - $Q > 2 \text{ GeV}$, $W > 3.5 \text{ GeV}$
 - $p_T > 1.7 \text{ GeV}$
- ▶ 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
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 - ▶ 7 gluon
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 - ▶ 2 pion data normalizations
- ▶ $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

$$H_{ij} = \frac{1}{\partial a_i \partial a_j}$$

- ▶ tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- ▶ eigenvalues span 10 orders of magnitude $\rightarrow$ require numerical precision
- ▶ use noise reducing derivatives

$$-a_i^0)(a_j - a_j^0)$$

nCTEQ15 fit details

Kinematic cuts

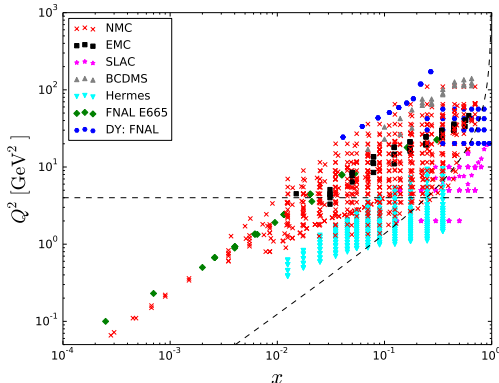
nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3 \text{ GeV}$

HKN: $Q > 1 \text{ GeV}$

DSSZ: $Q > 1 \text{ GeV}$



$$-a_i^0)(a_j - a_j^0)$$

$$\Pi_{ij} = \frac{\partial a_i}{\partial a_j}$$

$\chi^2 = 35$ (every nuclear
at 90% C.L.)

span 10 orders of
→ require numerical

nCTEQ: 740 data points
including derivatives
EPS09: 929 data points

nCTEQ15 fit details

Fit details

Hessian method

- ▶ choice of tolerance: $T = 35$
[PRD65 (2001) 014012,
arXiv:hep-ph/0101051]
- ▶ quadratic approximation

Error analysis:

- ▶ use Hessian method

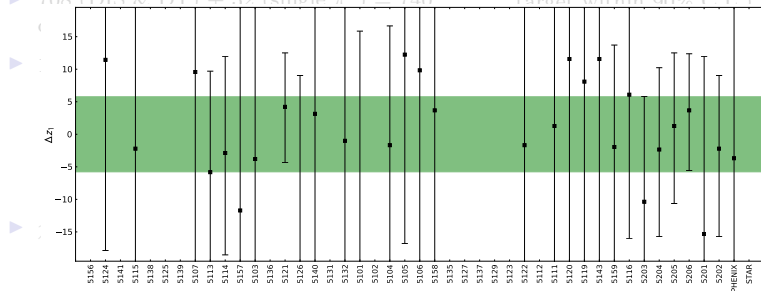
$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0) (a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- ▶ tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)

$p_T > 1.7$ GeV

- ▶ 708 (DIS & DY) + 32 (single π^0) = 740



s of
nuclear
ives

nCTEQ15 fit details

Fit details:

Hessian method

- ▶ choice of tolerance: $T = 35$
- ▶ quadratic approximation

▶ kinematic cuts:

$$Q > 2\text{GeV}, W > 3.5\text{GeV}$$

$$p_T > 1.7\text{ GeV}$$

▶ 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts

▶ 16+2 free parameters

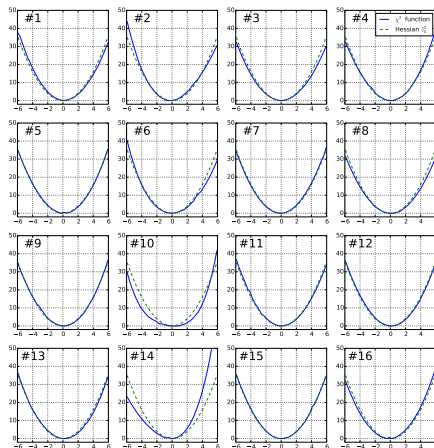
- ▶ 7 gluon
- ▶ 7 valence
- ▶ 2 sea

▶ 2 pion data normalizations

▶ $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

▶ use Hessian method

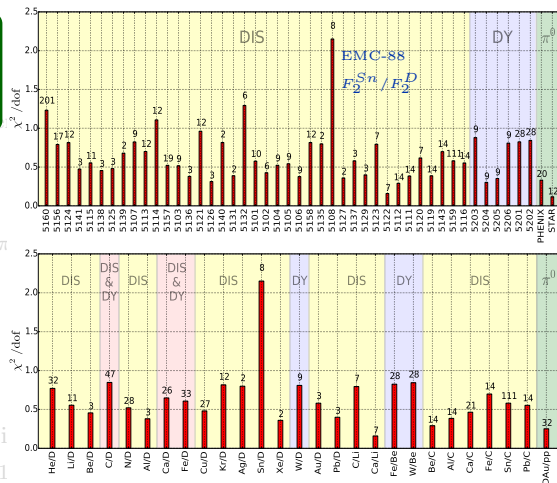


nCTEQ15 fit details

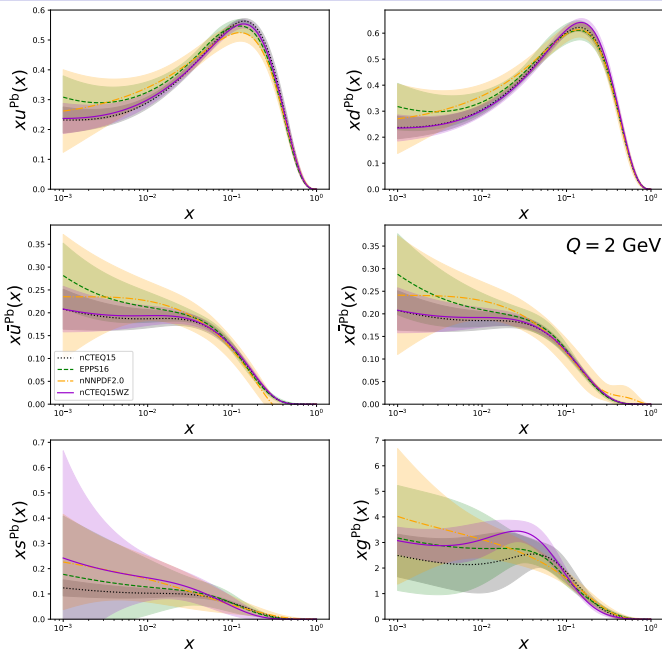
Fit quality

► $\chi^2/\text{dof} = 0.81$

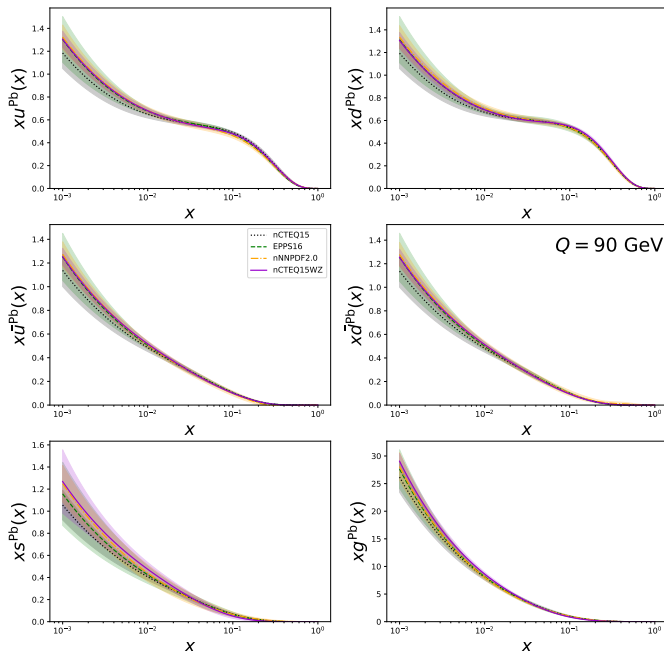
- kinematic cuts:
 $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
 $p_T > 1.7\text{ GeV}$
- 708 (DIS & DY) + 32 (single π) data points after cuts
- 16+2 free parameters
 - 7 gluon
 - 7 valence
 - 2 sea
 - 2 pion data normalization
- $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$



PDF comparison



PDF comparison



Current status

The picture coming from different nPDF analyses is rather *consistent* but we still don't know a lot of things:

- ▶ Compatibility of NC DIS and CC DIS → *universality*.
- ▶ Better information on *strange* and *gluon* nPDFs.
- ▶ *Small- x* (and onset of non-linear regime).
- ▶ *Large- x* (lack of good data, HT corrections, $x > 1$ region).
- ▶ *A-dependence*.

We can try to answer to some of these questions using current data but for some we need to wait for *new experiments*.

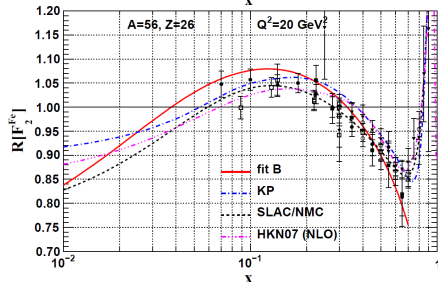
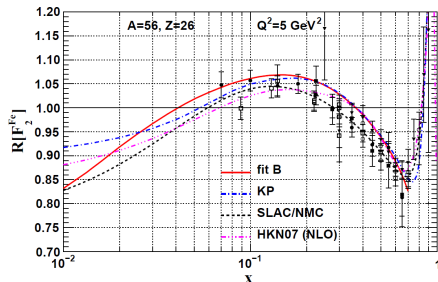
Selected results

- ▶ Compatibility of NC & CC DIS
- ▶ Strange nPDF
- ▶ Small- x gluon
- ▶ Large- x
- ▶ Future constraints

Compatibility of NC DIS and CC DIS [PRD 80 (2009) 094004]

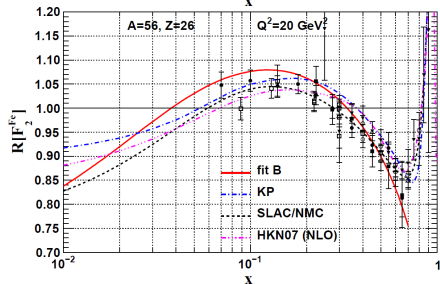
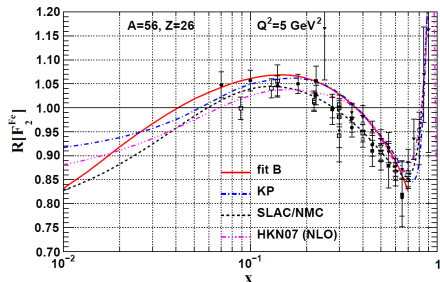
$$F_2^{\ell^\pm \text{Fe}} / F_2^{\ell^\pm \text{D}}$$

$$F_2^{\nu \text{Fe}} / F_2^{\nu \text{D}} \text{ [PRD 77 (2008) 054013]}$$

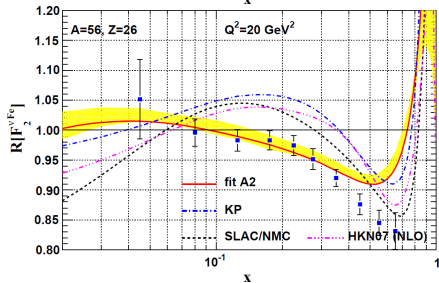
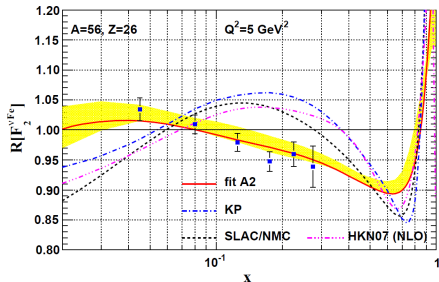


Compatibility of NC DIS and CC DIS [PRD 80 (2009) 094004]

$$F_2^{\ell^\pm \text{Fe}} / F_2^{\ell^\pm \text{D}}$$



$$F_2^{\nu \text{Fe}} / F_2^{\nu \text{D}}$$



Selected results

- ▶ Compatibility of NC & CC DIS
- ▶ Strange nPDF
- ▶ Small- x gluon
- ▶ Large- x
- ▶ Future constraints

Predominant information on strange used to come from difference of NC and CC DIS F_2 structure function (at LO neglecting charm)

$$\Delta F_2 = \frac{5}{18} F_2^{CC} - F_2^{NC} \sim \frac{x}{6} [s(x) + \bar{s}(x)]$$

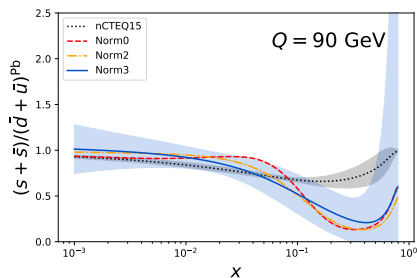
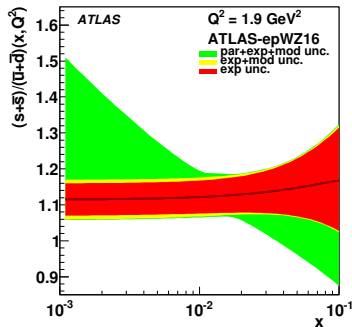
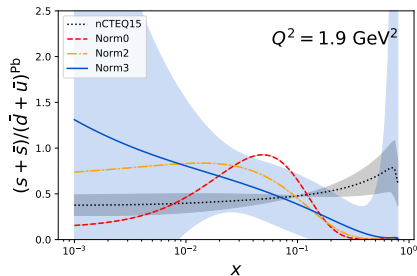
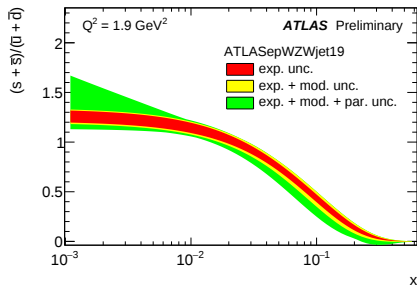
s is small compared to u and d PDFs $\rightarrow$ large uncertainties $\rightarrow$ it was assumed (CTEQ6.1, CTEQ6.5)

$$s(x) = \bar{s}(x) \sim \kappa \frac{\bar{u}(x) + \bar{d}(x)}{2}, \quad \kappa = \frac{1}{2}$$

$\rightarrow$ underestimation of s PDF uncertainty, as $\bar{u}$, $\bar{d}$ are much better constrained.

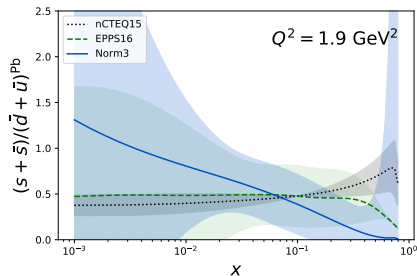
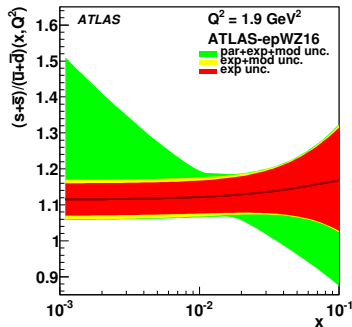
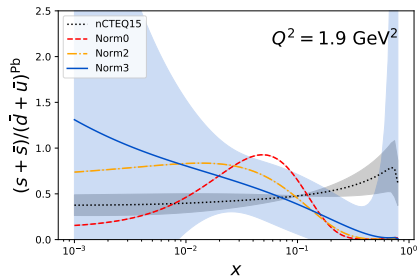
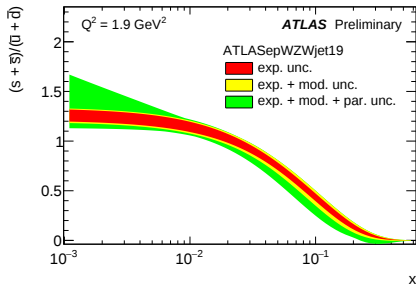
Strange nPDF from pPb W/Z LHC data

[EPJC 80 (2020) 10, 968]



Strange nPDF from pPb W/Z LHC data

[EPJC 80 (2020) 10, 968]



Strange nPDF from pPb W/Z LHC data

- ▶ pPb W/Z data from LHC also prefers large strange distribution similar to the pp data.
 - ▶ Run II CMS $W^\pm$ plays important role as the most precise one (not included in EPPS16)
- ▶ Need to do a combined analysis of pPb W/Z LHC data and neutrino data.

Possible future constraints of s -PDF

- ▶ **Lattice QCD** see e.g. [\[arXiv:2006.08636\]](#)
 - ▶ not too long future use **PDF-moments** calculated from Lattice;
 - ▶ less established more powerful: **quasi/pseudo-PDFs** (give full x -dependence);
 - ▶ especially viable for *helicity PDFs* which are less constraint.
- ▶ **EIC**
 - ▶ e.g. using charm jets [\[arXiv:2006.12520\]](#)
 - ▶ big advantage: will provide data for **protons** but also for a **range of nuclei**.

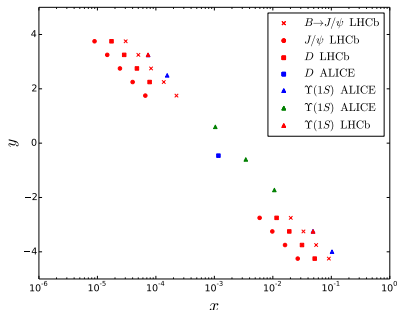
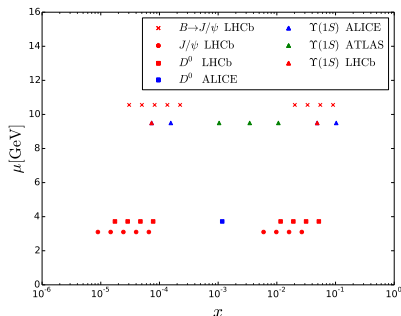
Selected results

- ▶ Compatibility of NC & CC DIS
- ▶ Strange nPDF
- ▶ Small- x gluon
- ▶ Large- x
- ▶ Future constraints

Small- x gluon from pPb LHC heavy-flavour data

[PRL 121, 052004 (2018)]

	D^0	J/ψ	$B \rightarrow J/\psi$	$\Upsilon(1S)$
μ_0	$\sqrt{4M_{D^0}^2 + P_{T,D^0}^2}$	$\sqrt{M_{J/\psi}^2 + P_{T,J/\psi}^2}$	$\sqrt{4M_B^2 + \left(\frac{M_B}{M_{J/\psi}} P_{T,J/\psi}\right)^2}$	$\sqrt{M_{\Upsilon(1S)}^2 + P_{T,\Upsilon(1S)}^2}$
$p+p$ data	LHCb [1]	LHCb [2,3]	LHCb [2,3]	ALICE [4], ATLAS [5], CMS [6], LHCb [7,8]
R_{pPb} data	ALICE [9], LHCb [15]	ALICE [10,11], LHCb [16,12]	LHCb [12]	ALICE [13], ATLAS [14], LHCb [17]

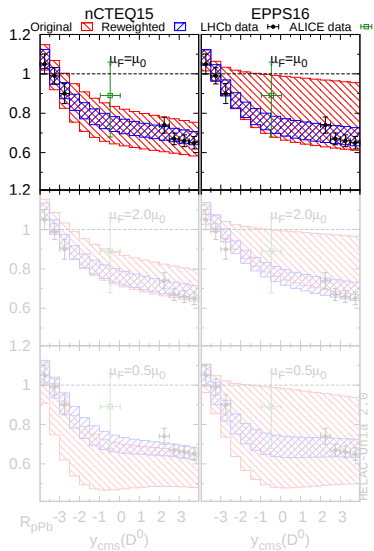


Expected nuclear effects on heavy quark(onium) production in pA collisions

- ▶ Nuclear modification of PDFs: initial-state effect
- ▶ Energy loss (w.r.t. pp collisions): initial-state or final-state effect
- ▶ Break up of the quarkonium in the nuclear matter: final-state effect
- ▶ Break up by comoving particles: final-state effect
- ▶ Colour filtering of intrinsic QQ pairs: initial-state effect
- ▶ ...

- ▶ We assume leading twist factorization is valid – ONLY modifications of PDFs are present → “shadowing-only” hypothesis.

Reweighting with D^0 data

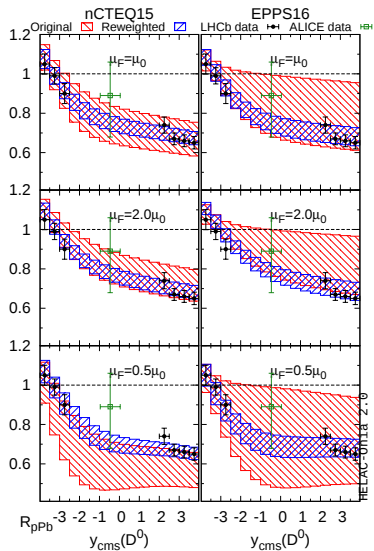


LHCb [JHEP 1710 (2017) 090, 1707.02750]

ALICE [PRL113, 232301 (2014), 1405.3452]

- ▶ Initial description of data is good for both nCTEQ15 and EPPS16.
- ▶ Substantial reduction of uncertainty especially for EPPS16.

Reweighting with D^0 data

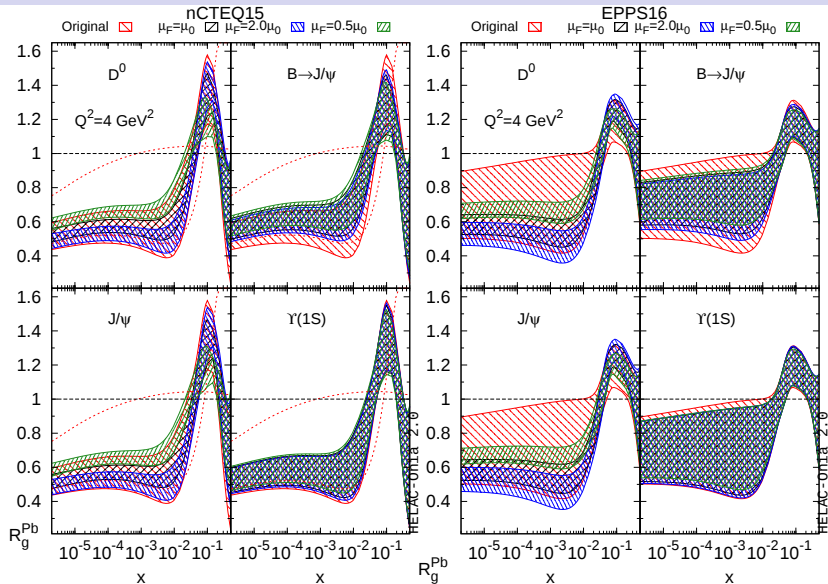


LHCb [JHEP 1710 (2017) 090, 1707.02750]

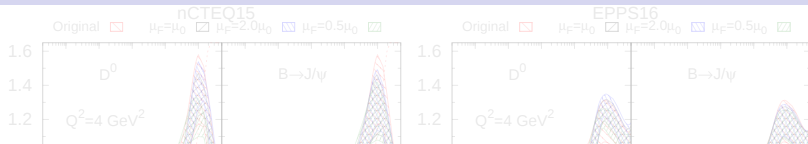
ALICE [PRL113, 232301 (2014), 1405.3452]

- ▶ Initial description of data is good for both nCTEQ15 and EPPS16.
- ▶ Substantial reduction of uncertainty especially for EPPS16.
- ▶ If we include factorization scale uncertainty errors increase and it can become the dominant uncertainty.

Reweighting results: $R_g^{\text{Pb}} = f_g^{\text{Pb}} / f_g^p$



Reweighting results: $R_g^{\text{Pb}} = f_g^{\text{Pb}} / f_g^p$



Comments

- ▶ We observe global coherence of the data constraints: necessary condition to assume a shadowing-only approach.
- ▶ First clear experimental observation on gluon SHADOWING at low x ; visible reduction of the EPPS16 uncertainties; confirmation of the extrapolation done in nCTEQ15.
- ▶ Confirmation of the existence of a gluon anti-shadowing: $R_g(0.05 \lesssim x \lesssim 0.1) > 1$.
- ▶ The scale ambiguity for D^0 and J/ψ production is now the dominant uncertainty.
- ▶ Non-prompt J/ψ are really promising if improved data can be obtained.

Consistency with other data

We checked the consistency of the reweighted (nCTEQ15) nPDFs with other data sets entering global analysis:

- ▶ DIS data (the most precise set NMC Sn/C [NPB 481 (1996) 23]).
- ▶ LHC W/Z boson production data [EPJC 77, (2017) 488].
- ▶ PHENIX J/ψ R_{dAu} data [PRL 107 (2011) 142301; PRC 87, (2013) 034904].

This is very non-trivial and further confirms the “shadowing-only” hypothesis of leading twist factorization is valid within the current data precision!

Consistency with other data

- The results of the [PRL 121 (2018), 052004] study were successfully used e.g. to describe data at RHIC.

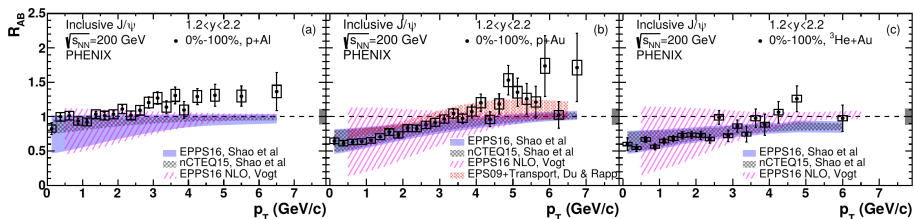


FIG. 10. Nuclear modification factor of inclusive J/ψ as a function of p_T at forward rapidity ($p/{}^3\text{He}$ -going direction) for 0%–100% $p+Al$, $p+Au$, and ${}^3\text{He}+Au$ collisions. Bars (boxes) around data points represent point-to-point uncorrelated (correlated) uncertainties. The theory bands are discussed in the text.

[arXiv:1910.14487](https://arxiv.org/abs/1910.14487)

see also: K. Smith, Quark Matter 2019

Selected results

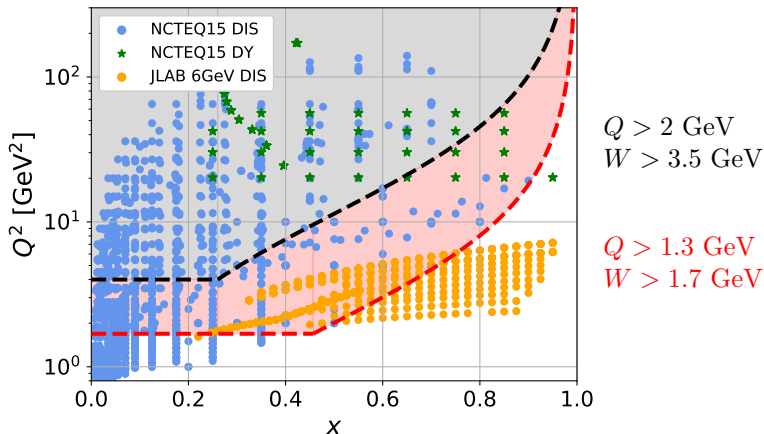
- ▶ Compatibility of NC & CC DIS
- ▶ Strange nPDF
- ▶ Small- x gluon
- ▶ Large- x
- ▶ Future constraints

Large- x data from JLAB

In (n)PDF analyses we use kinematic cuts to exclude data that are

- ▶ in non-perturbative region
- ▶ have significant higher-twist corrections.

This is typically done by cuts on Q^2 and $W^2 = Q^2(1-x)/x + M_N^2$.

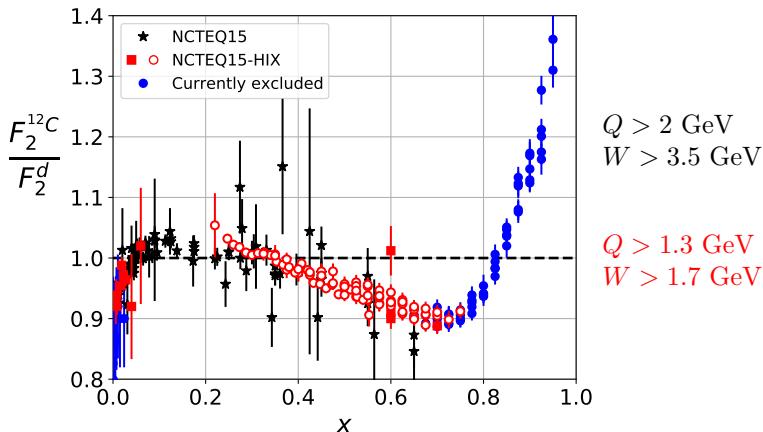


Large- x data from JLAB

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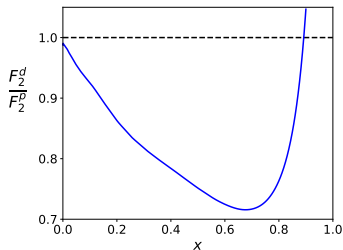
Large- x data from JLAB

Target	Experiment	ID	Ref.	# data	#data after cuts
$^{208}\text{Pb}/\text{D}$	CLAS	9976	[11]	25	24
$^{56}\text{Fe}/\text{D}$	CLAS	9977	[11]	25	24
$^{27}\text{Al}/\text{D}$	CLAS	9978	[11]	25	24
$^{12}\text{C}/\text{D}$	CLAS	9979	[11]	25	24
$^4\text{He}/\text{D}$	Hall C	9980	[12]	25	17
		9981	[12]	26	16
$^3\text{He}/\text{D}$	Hall C	9982	[12]	25	17
		9983	[12]	26	16
$^{64}\text{Cu}/\text{D}$	Hall C	9984	[12]	25	17
		9985	[12]	26	16
$^9\text{Be}/\text{D}$	Hall C	9986	[12]	25	17
		9987	[12]	26	16
$^{197}\text{Au}/\text{D}$	Hall C	9988	[12]	24	17
		9989	[12]	26	16
$^{12}\text{C}/\text{D}$	Hall C	9990	[12]	25	17
		9991	[12]	17	7
		9992	[12]	26	16
		9993	[12]	18	6
		9994	[12]	17	7
		9995	[12]	15	2
		9996	[12]	19	7
		9997	[12]	16	2
		9998	[12]	21	8
		9999	[12]	18	3
Total				546	428

Large- x data from JLAB

Effects we include:

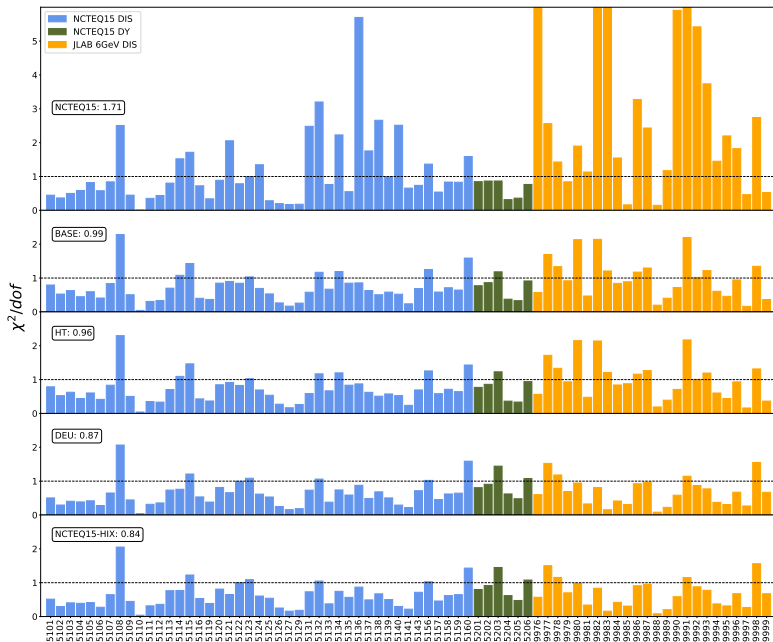
- ▶ *Target-mass corrections* (OPP) & *dynamic higher-twist* effects
→ to good extent cancel in ratio.
- ▶ *Deuteron corrections*



Effects that would need to be included when allowing for even higher- x (lower W):

- ▶ Non-vanishing structure functions at $x > 1$ and corresponding extension of DGLAP evolution.
- ▶ Threshold resummation.

Large- x data from JLAB



Selected results

- ▶ Compatibility of NC & CC DIS
- ▶ Strange nPDF
- ▶ Small- x gluon
- ▶ Large- x
- ▶ Future constraints

A-dependence

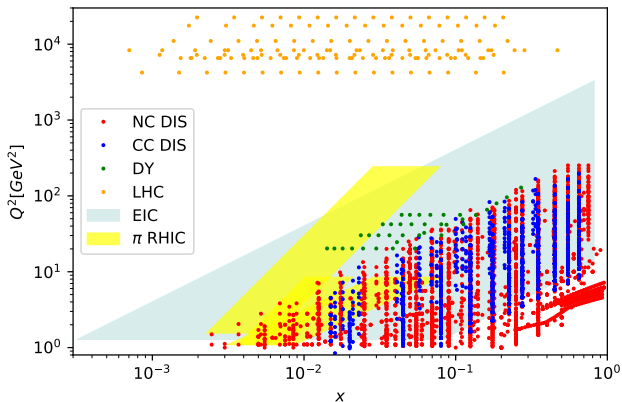
- ▶ At the moment the A -dependence is predominantly assumed in certain form, e.g.

$$c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}) \quad \text{or} \quad d_i(A) = d_i(A_{\text{ref}}) \left(\frac{A}{A_{\text{ref}}} \right)^{\gamma_i [d_i(A_{\text{ref}}) - 1]}$$

- ▶ There is not enough data to really constrain the A -dependence and check if such assumption is justified.
- ▶ Hopefully data from EIC will allow to answer this.

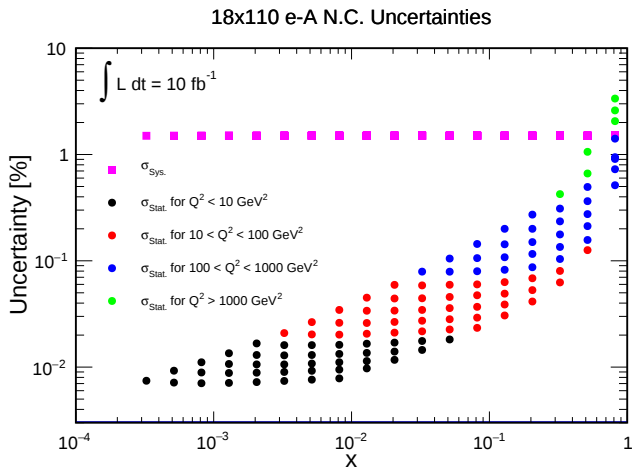
Electron-Ion Collider opportunities for nPDFs

- ▶ Different nuclei: Au, Cu, Fe, C, He, ...
- ▶ Wide kinematic coverage



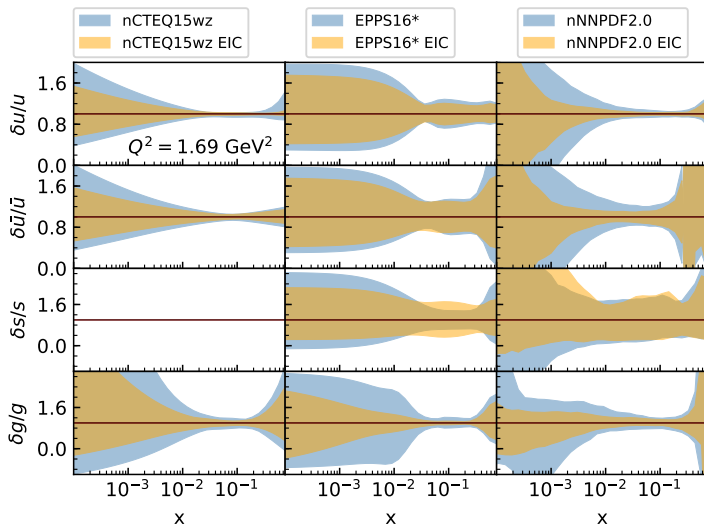
Electron-Ion Collider opportunities for nPDFs

► Small uncertainties

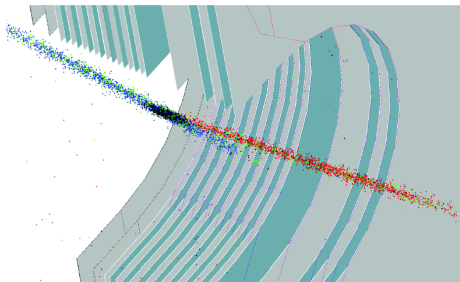
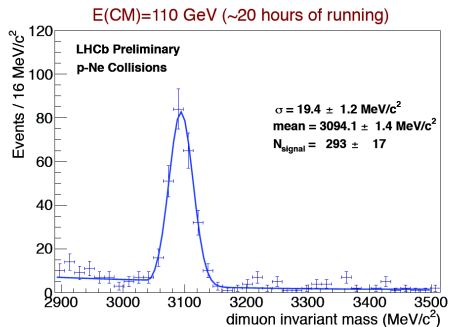


Electron-Ion Collider opportunities for nPDFs

- ▶ Great prospects for understanding nuclear structure in particular nPDFs



LHCb developed the **S**ystem for **M**easuring the **O**verlap with **G**as to obtain a high-precision (1%) luminosity measurement by injecting a noble gas into the VELO to profile the beams -- but also permits running in fixed-target mode!



In fixed-target mode, LHCb is a central-backward detector that probes energy densities between that of the SPS and RHIC. Data collected: p-He, p-Ne, p-Ar and Pb-Ne, Pb-Ar.

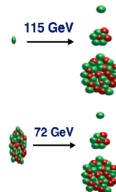
Energy range

7 TeV proton beam on a fixed target

c.m.s. energy: $\sqrt{s} = \sqrt{2m_N E_p} \approx 115 \text{ GeV}$	Rapidity shift: $y_{c.m.s.} = 0 \rightarrow y_{lab} = 4.8$
Boost: $\gamma = \sqrt{s} / (2m_N) \approx 60$	

2.76 TeV Pb beam on a fixed target

c.m.s. energy: $\sqrt{s_{NN}} = \sqrt{2m_N E_{pb}} \approx 72 \text{ GeV}$	Rapidity shift: $y_{c.m.s.} = 0 \rightarrow y_{lab} = 4.3$
Boost: $\gamma \approx 40$	



Such $\sqrt{s}$ allow, for the first time, for systematic studies of W boson, bottomonia, p_T spectra, associated production, ..., in the fixed target mode

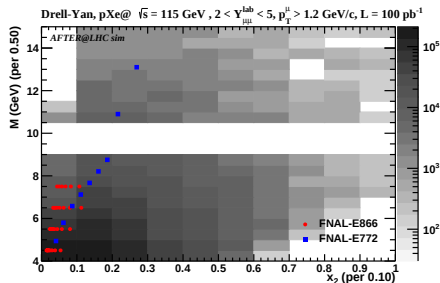
Effect of boost :

[particularly relevant for high energy beams]

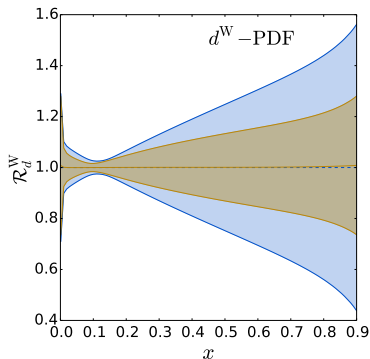
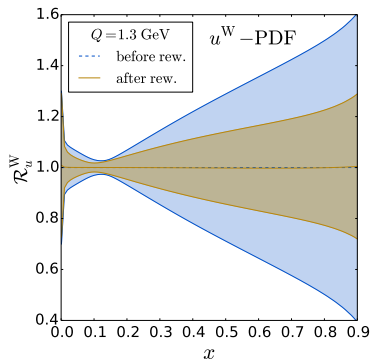
- LHCb and the ALICE muon arm become **backward detectors** [$y_{c.m.s.} < 0$]
- With the reduced $\sqrt{s}$, their acceptance for physics grows and nearly covers half of the backward region for most probes [$-1 < x_F < 0$]
- Allows for backward physics up to high $x_{\text{target}} (\equiv x_2)$
[uncharted for proton-nucleus; most relevant for $p\text{-}p^\uparrow$ with large $x^\uparrow$]



- ▶ Would be certainly very useful for PDF/nPDF determination
 - ▶ allow to collect data on different targets: Pb, W, Xe,...
 - ▶ e.g. Drell-Yan lepton pair production



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Summary

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- ▶ I presented the schematics of global nPDF analysis and reviewed available nPDFs as well as some related recent results.
- ▶ The current nPDFs are generally compatible with each other but we still don't have satisfactory knowledge of all flavours especially at small and large x .
- ▶ The currently available data (LHC, JLAB) can be utilize to further constrain nPDFs, however, only to a certain extent.
- ▶ On the other hand the planed EIC will bring the nPDFs to a precise era, and will allow to studay not only the collinear distributions but also more complicted objects like TMDs, GPDs.

nCTEQ

nuclear parton distribution functions

- Home
- PDF grids & code
 - nCTEQ15
 - previous PDF grids
- Papers & Talks
- Subversion
- Tracker
- Wiki

nCTEQ project is an extension of the **CTEQ** collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). All details on the framework and the first complete results can be found in [arXiv:1507.07444 \[hep-ph\]](#). The effects of the nuclear environment on the parton densities can be shown as modified parton densities or nuclear correction factors (for example for lead as shown below)

