Diffusion of innovation on networks: agent-based vs. analytical approach

M. Szurlej, A. Abramiuk-Szurlej, K. Sznajd-Weron

Introduction

We study an agent-based model of **innovation diffusion** on the **Watts–Strogatz random graphs**. This network topology allows us to control the **average number of nearest neighbors *k***. The model is based on the noisy *q*-voter model, which in its asymmetric version has been previously used to describe the **diffusion of green products and practices**. When the model is used to describe diffusion of innovation the up-down symmetry is not justified and therefore we introduce the asymmetry factor *f*. In this paper we provide the analytical approach to the model, via so called **pair approximation** (PA), as well as **Monte Carlo** results, and compare them with previous results obtained within **mean-field approximation** (MFA). We show in which cases the agent-based model can be reduced to the analytical one and when it cannot be done.

Conclusions

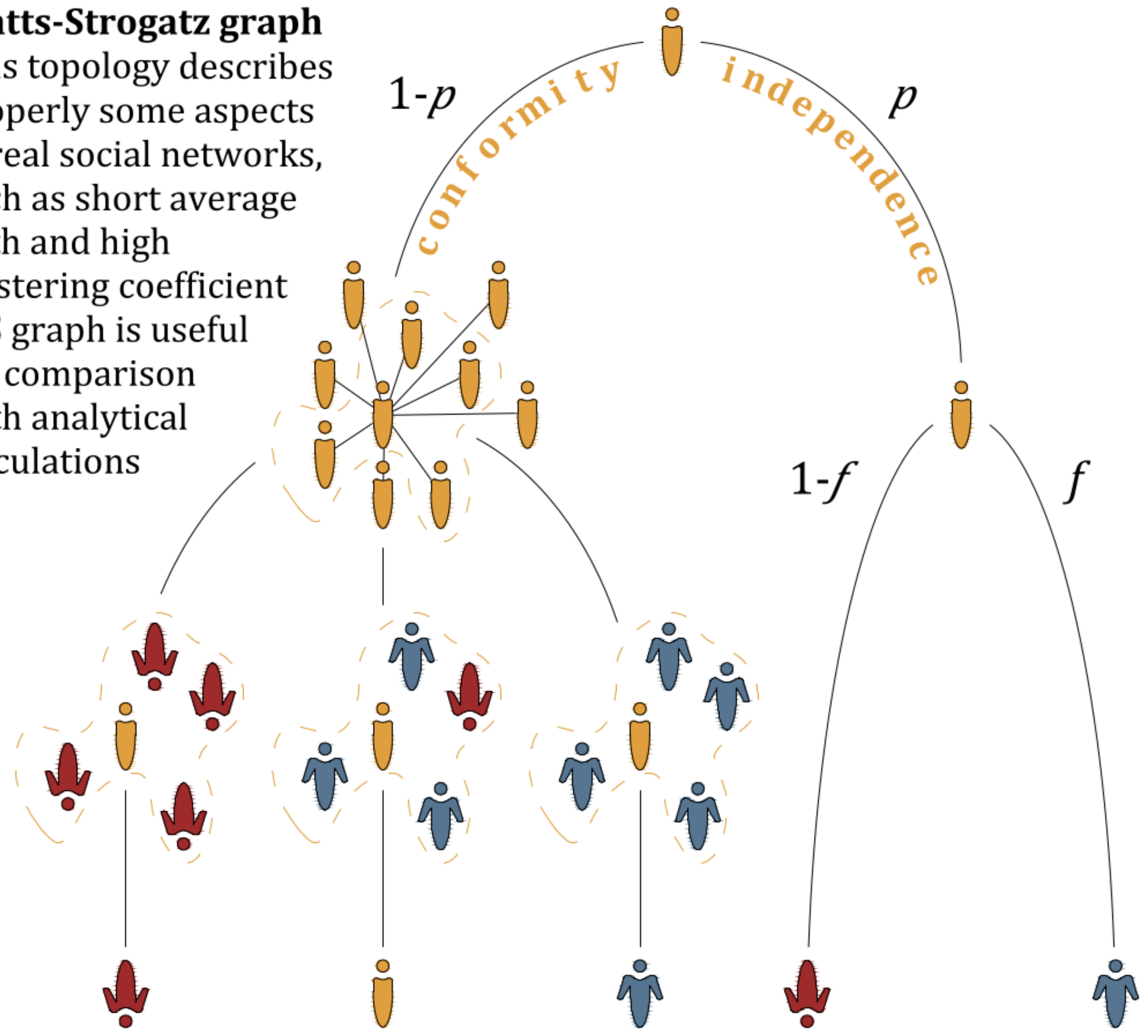
- Both the critical mass (i.e. the minimal number of initial adopters above which the innovation diffuse) and the stationary concentration of adopted agents depends on the values of parameters
- Phase diagrams show the agreement between Monte Carlo results and MFA for the complete graph
- For the Watts-Strogatz graph with $\beta = 1$ (small clustering coefficient) simulations overlap the PA results
- For the small world network (WS with $\beta = 0.05$) PA overestimates MC results
- The difference between PA and MFA is negligible for small and large values of p

Model description

We consider a set of N voters described by a dynamical binary variable $S_i = \pm 1$ that represents state (adopted/unadopted)
The state of the whole system is described by a fraction of adopted agents c

Q-voter model with asymmetric independence

Watts-Strogatz graph
This topology describes properly some aspects of real social networks, such as short average path and high clustering coefficient
WS graph is useful for comparison with analytical calculations



References

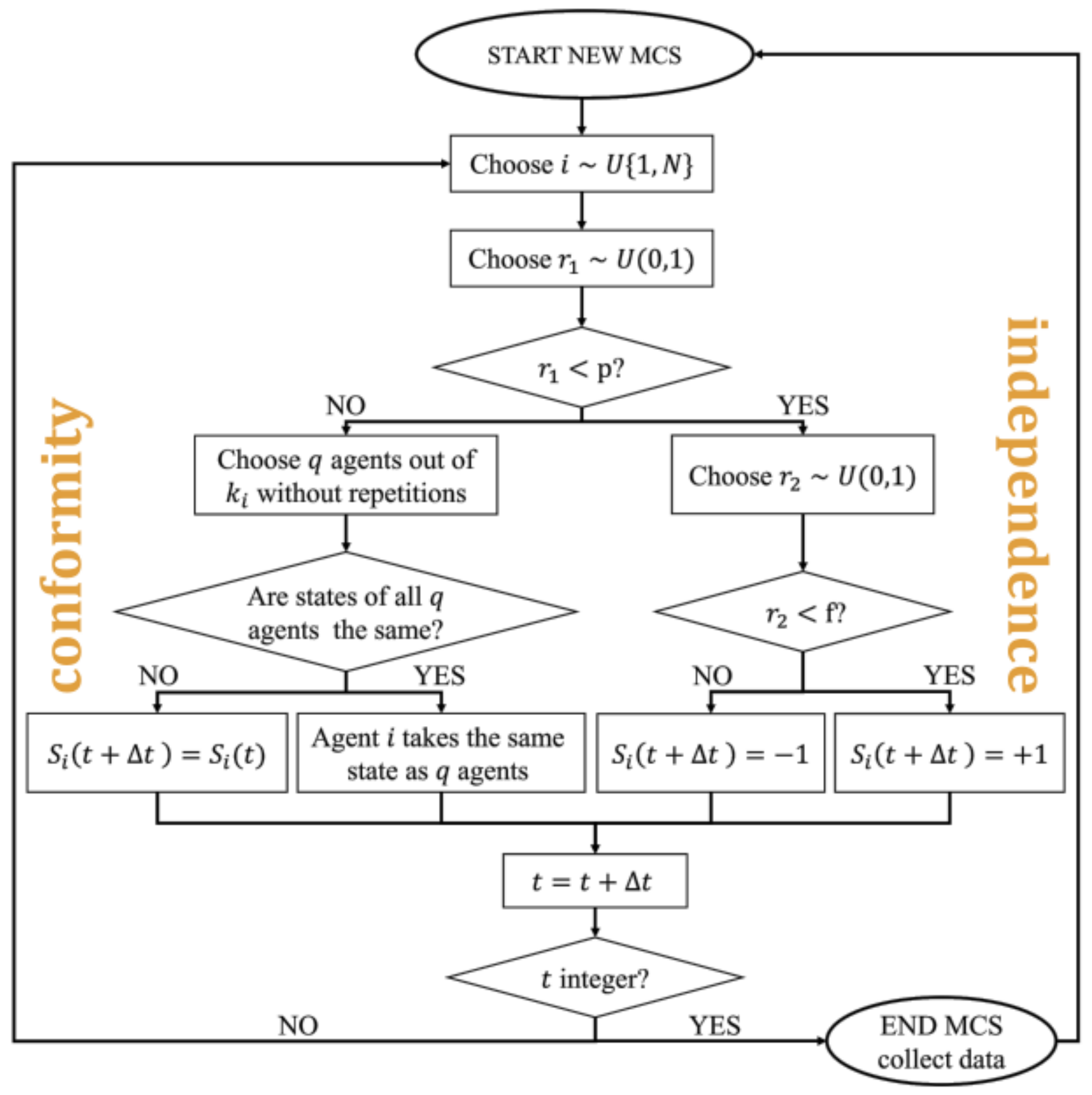
K. Byrka, A. J˛edrzejewski, K. Sznajd-Weron, and R. Weron, *Difficulty is critical: The importance of social factors in modeling diffusion of green products and practices.* Renewable and Sustainable Energy Reviews 2016, 62, 723–735

P. Nyczka, K. Sznajd-Weron, J. Cisło, *Phase transitions in the q-voter model with two types of stochastic driving.* Phys. Rev. E 2012, 86, 011105

Model dynamics

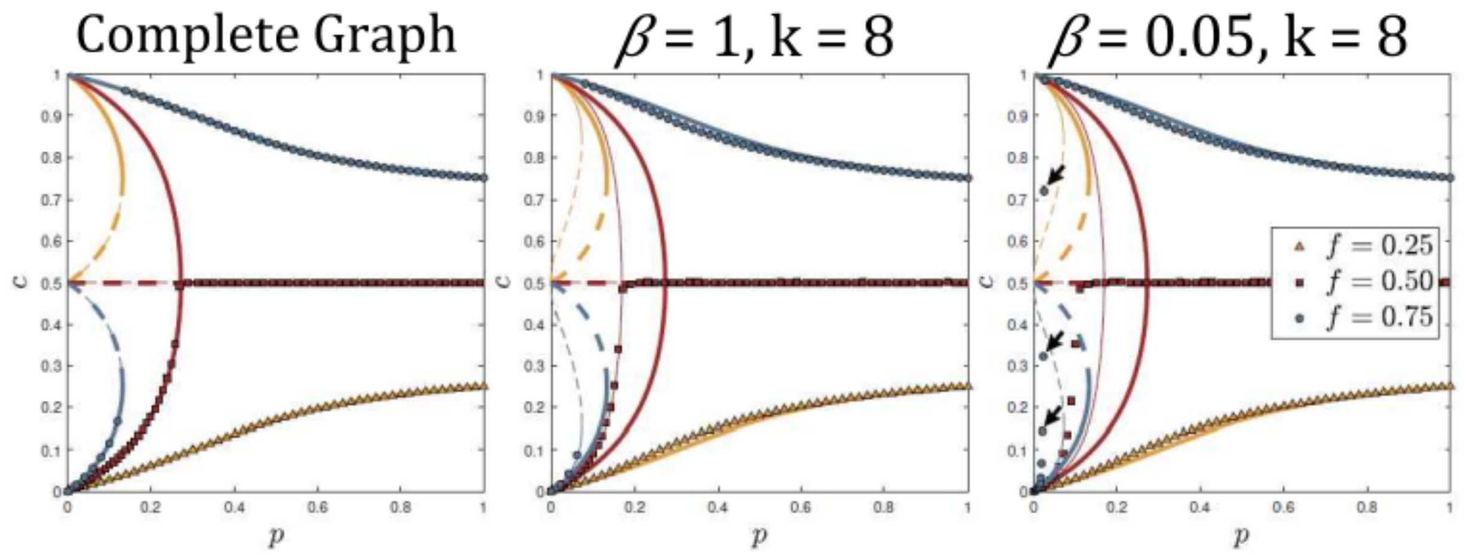
Three parameters of the model:

p — level of independence
f — the asymmetry factor, probability of being adopted
q — size of a group of influence

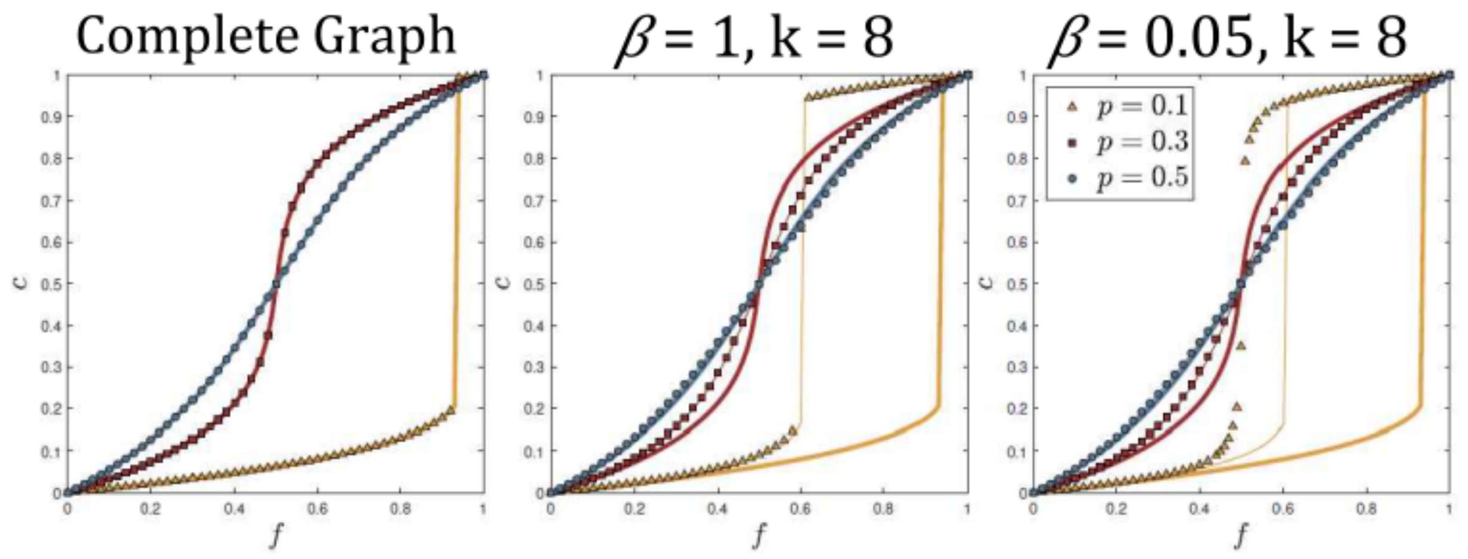


Results

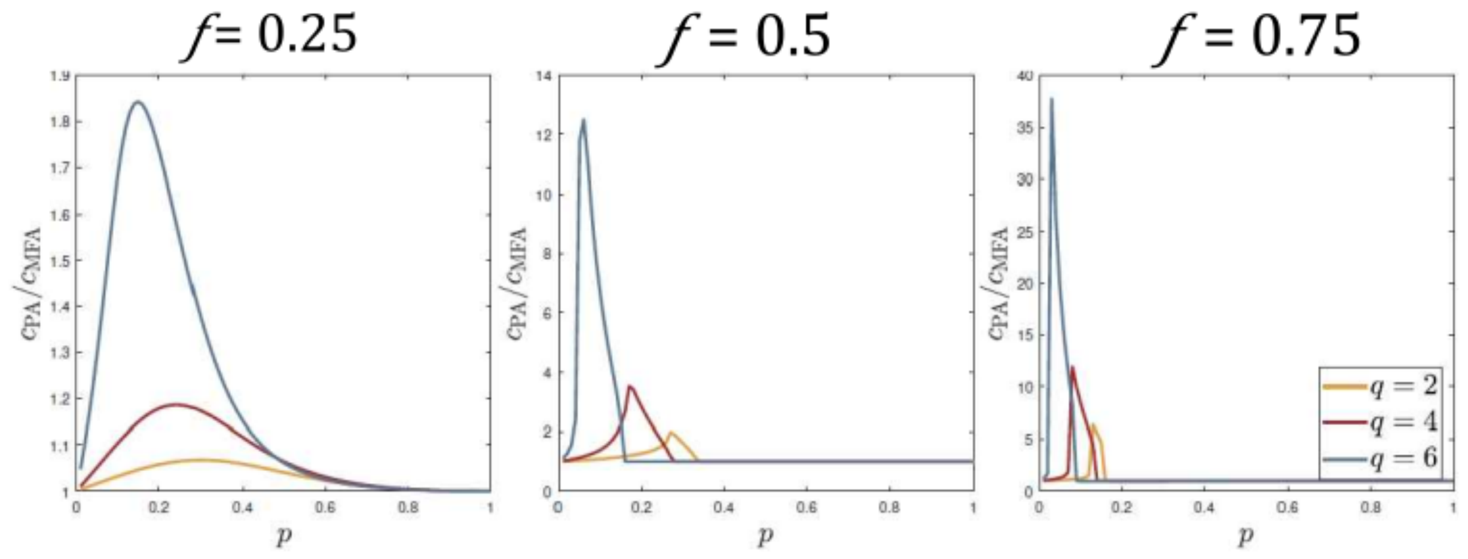
Stationary fraction of adopted agents c as a function of probability of independence p for $q = 4$



Stationary fraction of adopted agents c as a function of asymmetry factor f for $q = 4$



Ratio between stationary fractions of adopted agents obtained analytically for PA and MFA



Wrocław University of Science and Technology

Acknowledgements: This work was partially supported by funds from Polish Ministry of Science and Higher Education through project "Diamentowy Grant" no.DI2019 0150 49