Noisy voter model of the parliamentary attendance data

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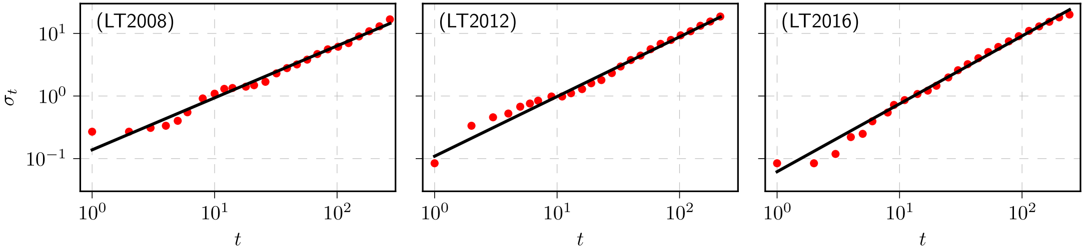
The question

What is the nature of the anomalous diffusion,

$$\sigma_t \sim t^{\alpha}, \qquad \text{with } \alpha \approx 0.9,$$

in the parliamentary attendance data?



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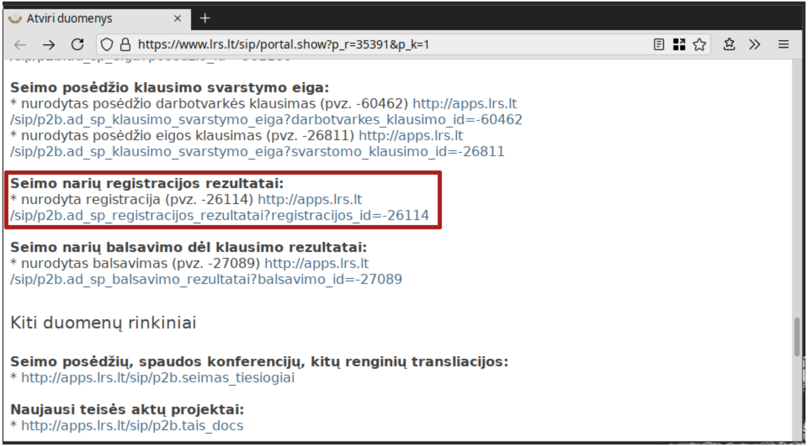
Talk based on: Kononovicius, J Stat Mech 2020: 063045 (2020)

Obtaining registration to vote data

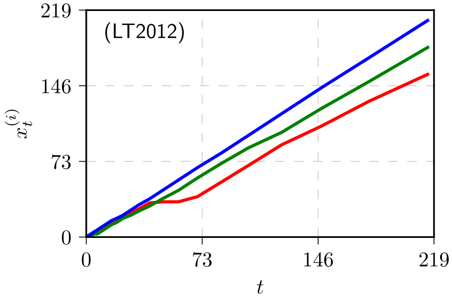


Raw data (available under CC-BY 4.0): https://www.lrs.lt/sip/portal.show?p_r=35391&p_k=1

Cumulative attendance series

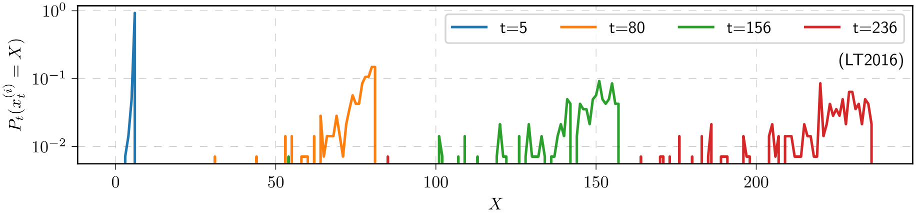
- Assume presence if $1+$ registrations in said a day.
- Representative attendance series: $\eta_t^{(i)} = 1$ if present, otherwise $\eta_t^{(i)} = 0$.
- Join attendance series of the "swapped" representatives.
- Cumulative attendance series:

$$x_t^{(i)} = \sum_{k=1}^{t} \eta_k^{(i)}.$$



Additional processing

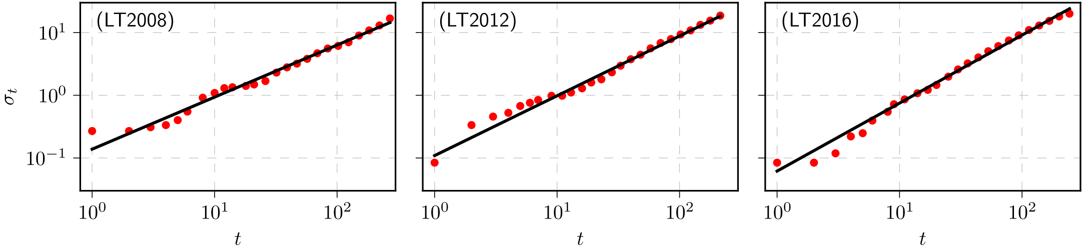
- Detect replacements and join the records.
- Replace troublesome records ($\sim 10\%$) with a random sample of valid records.
- Consider all cumulative attendance series during a single legislature (141 records) as a single statistical ensemble.



Processed data: https://github.com/akononovicius/lithuanian-parliamentary-presence-data

Statistical moments of the ensembles

Mean, μ_t, shows a boring linear dependence.



While σ_t exhibits sublinear ($\alpha \approx 0.9$) growth.

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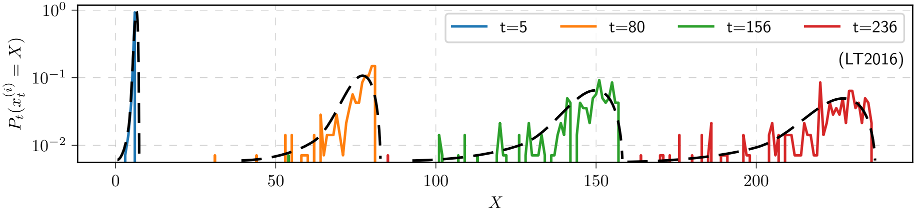
Phenomenological model

[VRJM2019] have observed that $P_t(x_t^{(i)} = X)$ is well approximated by a scaled Beta distribution.



It was shown that the following diffusion equation could be an appropriate model:

$$\frac{\partial P_t}{\partial t} = D t^{\beta} \frac{\partial}{\partial X} \left[X^{-\theta} \frac{\partial}{\partial X} \left(X^{\gamma} P_t^{\nu} \right) \right].$$

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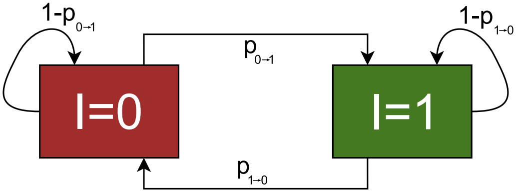
[VRJM2019]: Vieira et al., PRE 99: 042141 (2019).

Noisy voter model

Let the agents switch between the attending ($I = 1$) and the absent ($I = 0$) states with probabilities:

$$p^{(i)}_{1\to 0} = h \left[\varepsilon_0 + \frac{X_0}{N} \right], \qquad p^{(i)}_{0\to 1} = h \left[\varepsilon_1 + \frac{X_1}{N} \right] = h \left[\varepsilon_1 + \left(1 - \frac{X_0}{N} \right) \right].$$

Unlike in the original NVM many agents may switch at the same time.



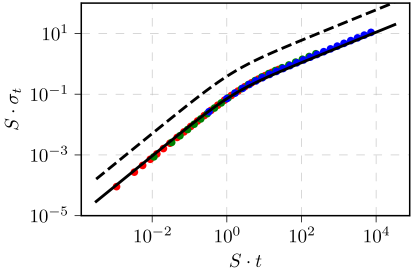
Anomalous diffusion modeled by NVM

Numerical exploration of the model reveals that:

$$\sigma_t \approx \frac{\theta_0 t}{\sqrt{\theta_1 + S t}}.$$

Here:

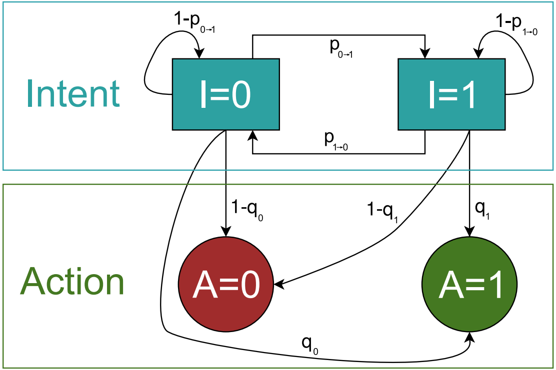
- θ_0 depends on asymmetry.
- $\theta_1 \approx 1.4$.
- $S = h\left(1 + \varepsilon_0 + \varepsilon_1\right)$.



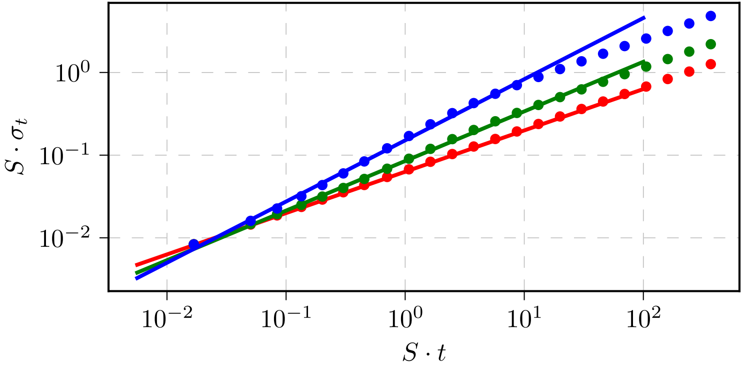
Parameters: $\varepsilon_0 = 0.1$, $\varepsilon_1 = 10$ and $N = 141$ (all cases), $h = 10^{-4}$ (red dots), 10^{-3} (green dots), $3 \cdot 10^{-2}$ (blue dots).

Imperfect behavior: Intents and actions

- Let NVM describe how agents' intent, I, evolves.
- Let the agents behave imperfectly: randomness in actions.
- Let q_I be the probability that agent with intent I attends the session (takes action $A = 1$).

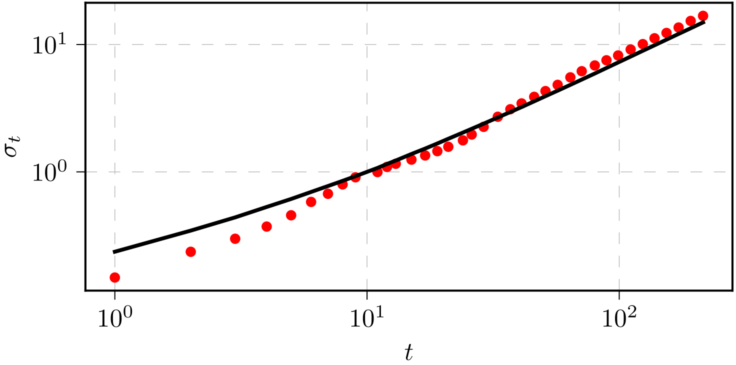


Anomalous diffusion modeled by the imperfect NVM



Parameters: $\varepsilon_0 = \varepsilon_1 = 0.06$, $h = 1.5 \cdot 10^{-2}$, $N = 141$ (all cases), $q_0 = q_1 = 0.5$ (red dots), $q_0 = 0.43$ and $q_1 = 0.57$ (green dots), $q_0 = 0.32$ and $q_1 = 0.68$ (blue dots). Fits: $\alpha = 0.5$ (red line), 0.6 (green line) and 0.75 (blue line).

Reproducing empirical anomalous diffusion



Averaged empirical σ_t (red dots) vs model σ_t (black line). Parameters: $\varepsilon_0 = 0.21$, $\varepsilon_1 = 0.43$, $h = 7.1 \cdot 10^{-4}$, $N = 141$, $q_0 = 0.8$ and $q_1 = 0.98$.

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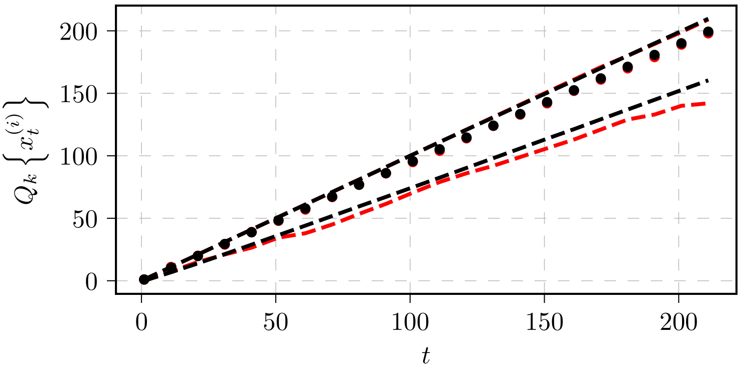
Reproducing empirical distribution



95% of $x_t^{(i)}$ are within the dashed lines (empirical data – red, model – black). Median $x_t^{(i)}$ is represented by a dot. Parameters same as before.

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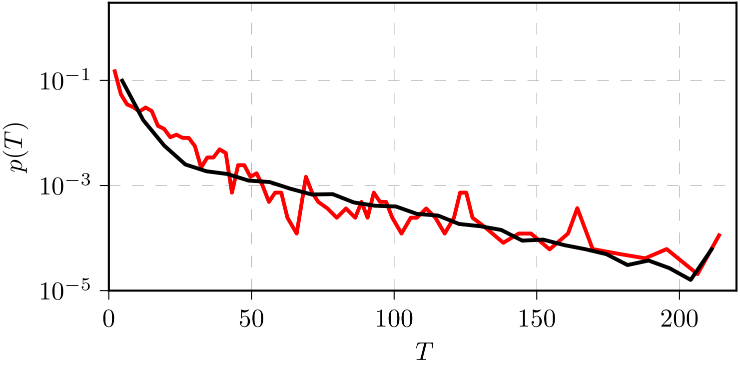
Reproducing attendance streaks



Probability density function of the attendance streak length (empirical data - red, model - black). Parameters same as before.

Summary

- Here the ensemble is multiple agents within the same NVM.
- Perfect NVM has two regimes: balistic and normal diffusion.
- Imperfect NVM enables reproduction of superdiffusion.
- Superdiffusion may emerge due to coordination between the individuals.
- Superdiffusion may emerge due to relatively slow dynamics.

- Taking ensemble of NVMs leads to balistic regime [KK2021].
- Transformation of observable and time may lead to other regimes [KK2021].

[KK2021]: Kazakevičius & Kononovicius, PRE 103: 0321541 (2021)

Thank you for your attention!

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