Discontinuous phase transitions in the q-voter model with generalized anticonformity on random graphs.

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Motivation

- In 2019, a generalization of q-voter model with anticonformity was proposed [1]. The novelty was decoupling of sizes of groups of influence needed for conformity and anticonformity. In result, a previously unseen discontinuous phase transition was observed.
- Discontinuous phase transitions are connected with interesting social phenomena such as tipping points and hysteresis.
- The q-voter model with generalized anticonformity has not been examined on random graphs, which are more realistic from a sociological point of view. Hence, one may ask the question, does the discontinuous phase transition survive the transition to random graphs?

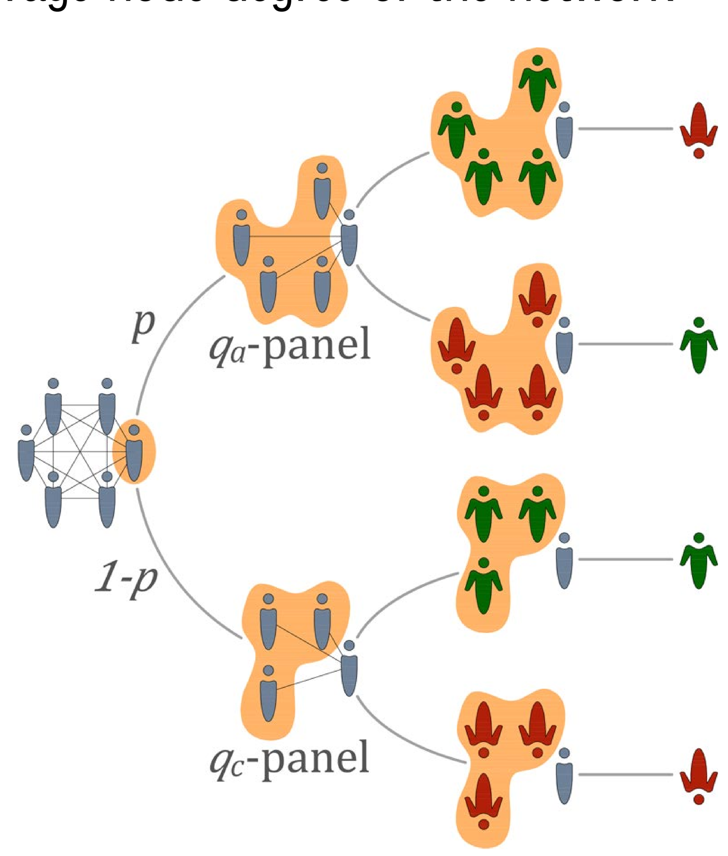
Model description

We consider a collection of N voters located in the vertices of a random graph. Each of them is characterized by a binary variable $S_i(t) = \pm 1$, $i = 1, \ldots N$, where t denotes time. They interact only with their neighbors, i.e., vertices with a direct link to them.

Dynamics

Model parameters:

- p - probability of anticonformity,
- q_a - necessary group size for anticonformity,
- q_c - necessary group size for anticonformity.
- k - average node degree of the network



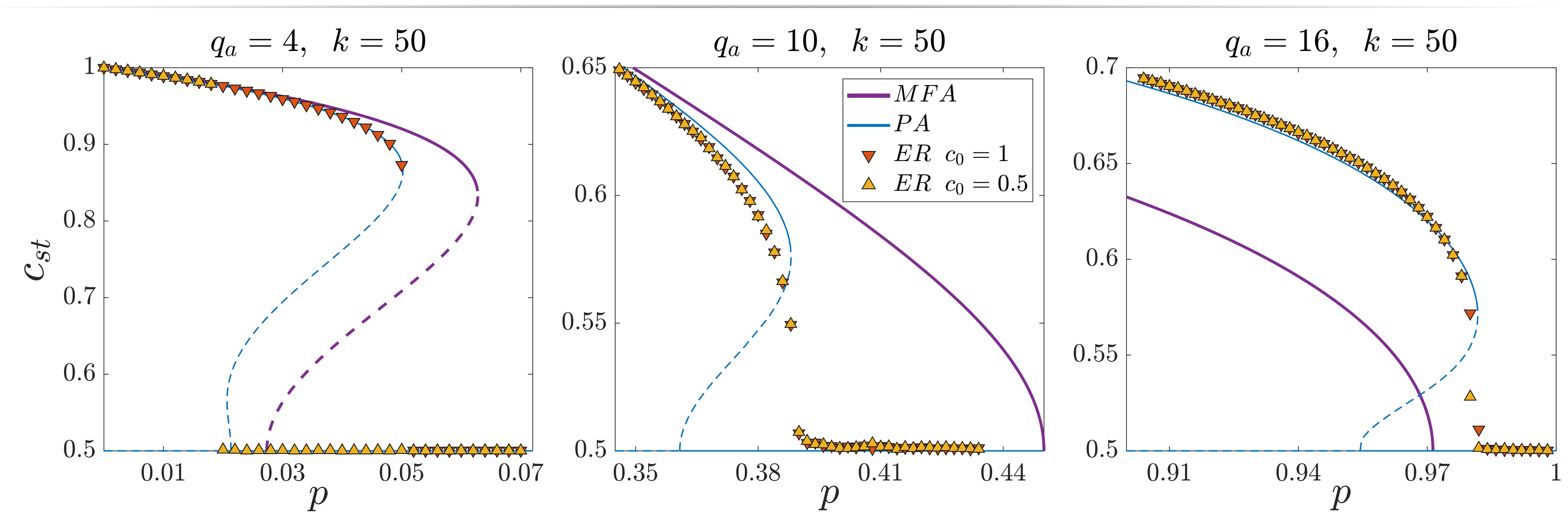
Pair approximation

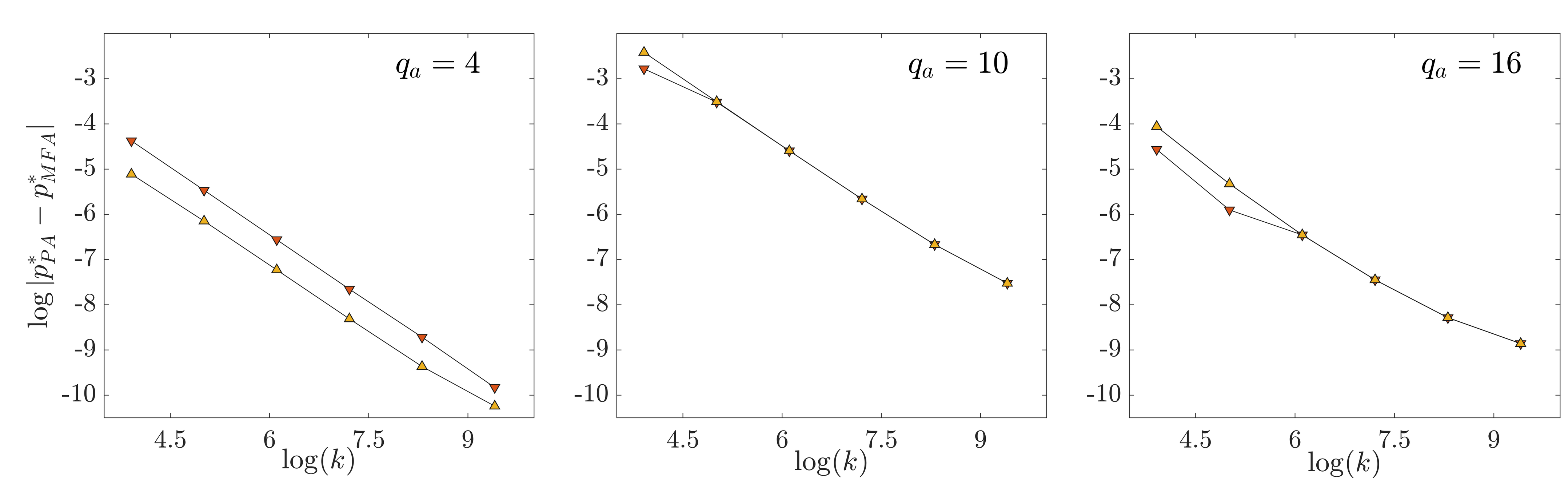
In our work, we consider the network topology different from the complete graph, hence the usually employed mean field approximation is insufficient. We need to take into account the correlations between the nearest neighbors. Here, the model is described by two variables, the concentration of positive opinion $c(t) \equiv \frac{N_{\uparrow}(t)}{N}$, and the concentration of active bonds $b(t) = \frac{1}{2Nk}\sum_{i=1}^{N}\sum_{j=1}^{N}(1 - S_i(t)S_j(t))\mathbb{1}_{(i,j)\in E}$ i.e. bonds between opposite spins. Such a description is possible because of two assumptions:

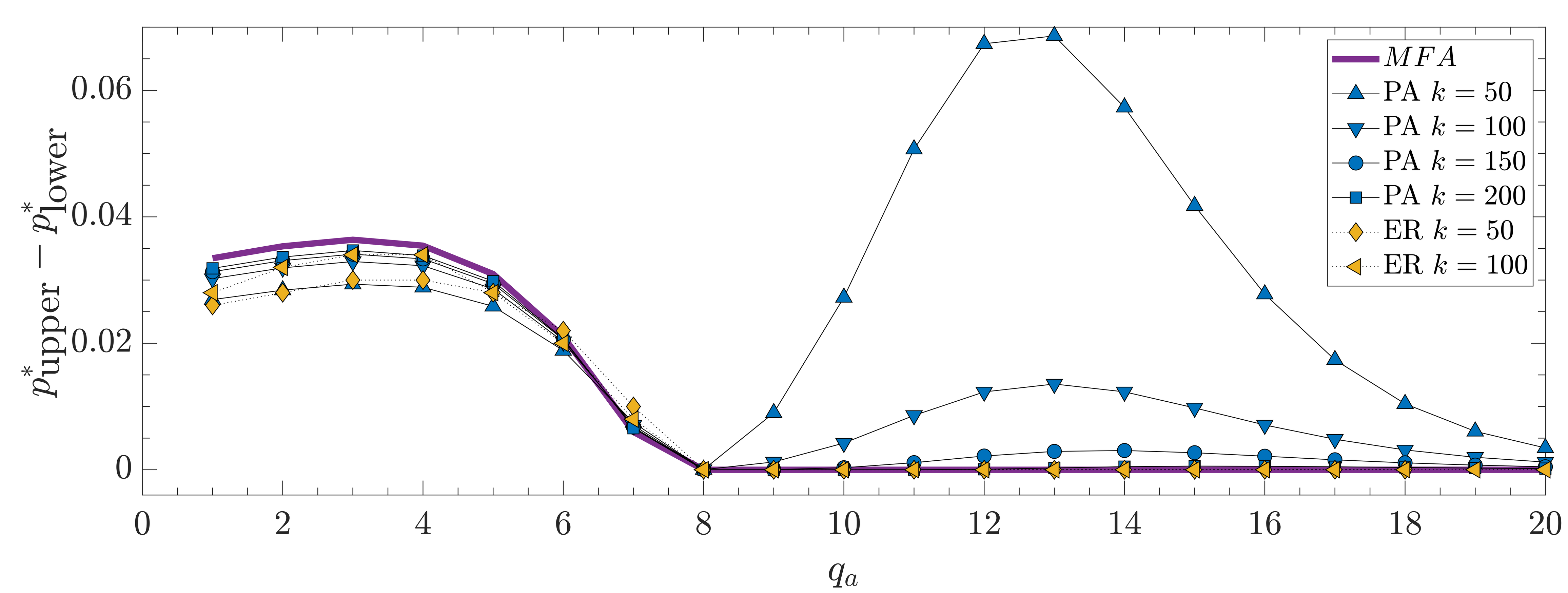
- the states of agent's neighbors are independent of each other,
- there is no correlation between vertex degree and finding an agent with a given opinion or a bond in a given state

This constitutes the Pair approximation [2], resulting in two coupled differential equations to be solved by numerical methods - see [3].

Results for $q_c = 10$







Conclusion

- We implemented the q-voter model with generalized anticonformity on random E-R graphs and conducted Monte Carlo simulations.
- It occurred that the theoretical predictions obtained via Pair Approximation do not match the simulation results for values of q_a fulfilling: $q_a \geq q_c - 1$.
- In these cases, PA yields a discontinuous phase transition and nonzero width of the hysteresis, whereas Monte Carlo simulations produce a continuous phase transition, the same as MFA.
- To conclude, PA should be used with caution, and to ensure the validity of the results, it is best to complement it with simulation results.

References

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