From relation to interactions: a case study in Reddit website

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Introduction
Elucidating what factors are salient in emerging interactions in social networks is still an open question. Hence, we develop an agent-based models (ABMs) for generating interactions in signed networks. The ABMs, based on the Activity Driven Network model, use signed relations between agents to reproduce their interaction frequencies and crucial network distributions. They also consider the agents’ preferences and information about neighbours of neighbours in the past. The calibration and validation step is performed on the Reddit Hyperlink Network, where agents are represented by subreddits, and links by hyperlinks between communities. We devise a profound methodology using Anderson-Darling statistic to assess the performance of the models. The proposed ABM successfully reproduces basic node-link such as a degree distribution or a link-weight distribution of the empirical dataset. Furthermore, ABMs recover a high clustering coefficient which is visible in Reddit Hyperlink Network.

Activity Driven Network model (ADN) was proposed to study highly dynamical networks [2]. It uses the activity potential, which describes the willingness of agents to create links. The algorithm proceeds with generating an empty graph of size \(N\) and assigning an activity potential \(x_i\) to each agent. The potential activity is usually drawn from a power-law distribution \(F(x)\). At each discrete time step \(t\) with probability \(a_i \Delta t\), where \(a_i\) is a vector of vertices that becomes active and generates \(m_i\) links to randomly selected nodes. Then, at the next time step \(t+\Delta t\) all the edges in the network are deleted and the process runs once again. We propose a few extensions: (1) use the input signed network to generate both the activity and preference distributions. (2) we use signed network as agent’s preference (see Fig. 3).

We also incorporate the copying mechanism which accounts for neighbours of neighbours. It takes into account paths of length 2 between agents \(i\) and \(j\):

\[
\text{copying}_{ij}(t) = \frac{1}{\min(k_i, k_j)} A_{ij}(t),
\]

and copying\(_{ij}(t) = 0\), where \(A(t)\) is the accumulated undirected adjacency matrix at time \(t\). One can define the total rate that the node \(j\) obtains a link from the active node \(s\) as follows:

\[
\text{rate}_{ij}(t) = \alpha \cdot \text{pref}_i \cdot f(\text{sign}_{ij}) + \gamma \cdot \text{copying}_{ij}(t-1),
\]

where \(\alpha, \beta, \gamma\) and \(\text{sign}_{ij}\) are constants which correspond to the strength of preferential term, sign term and copying mechanism, respectively. All constants are in range \([0,1]\).

Fig. 3: A figure presents our approach to leverage signed network as an input to the ADN model. The procedure yields both activity and preference distributions from the signed input network for modified ADN model. We begin with (1) finding exponent for \(P(k)\) distribution of signed network, which is used in the next step (2) to generate activity distribution. Analogously, the exponent in preference distribution is obtained from \(P(k)\) (3). Then we generate agents’ preferences (4) which go into rate equation (5). The step in orange is optional (2*). It correlates the activity with the out-degree of the input signed network, i.e. the agent with the highest out-degree will generate more links. The last step (6) constitutes the final rate calculation, where we also utilize the input signed network.

Model and methods

Reddit is the social news aggregation website. Users can post content while others can rate and discuss it (see Fig. 1). We can distinguish communities, so-called subreddits, which gather people with similar opinions and beliefs. An example post is presented in Fig. 1. User submitted content on one subreddit and included hyperlink to another. We denote such a hyperlink as an interaction between two subreddits. To study such interactions among subreddits we use dataset from SNAP repository [1]. The Reddit Hyperlink Network (RHN) represents the directed connections between subreddits. A link (hyperlink) originates from a post in the source community and links to a post in the target community. The network is directed, signed and temporal.

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Function \(f\) is defined as:

\[
f(\text{sign}_{ij}) = \begin{cases} 1, & \text{if the link } (i,j) \text{ is positive}. \\ 0, & \text{if the link } (i,j) \text{ is negative}. \end{cases}
\]

Here we focus on the parts marked by green circles (see Fig. 3), i.e. whether to correlate activity and test the impact of the input signed network.

Impact of the input signed network

For the model without input signed network \((\beta = 0)\) and with activity correlation (blue line) the maximum link weight matches the corresponding value from RHN (Fig. 4).

![Fig. 4: Link-weight distribution for ADN models with and without input signed network.](image)

The model with the input signed network \((\beta = 1)\) and activity correlation (in violet) in 6 out of 7 cases the highest number of motifs that are closer to empirical data than the rest of the models (Fig. 5).

![Fig. 5: Triad census for models with and without input signed network.](image)

Conclusions
- We reveal the possibility of developing the ABM to generate interactions from agents’ relations, their preference and their memory about the neighbours of neighbours.
- The results lead to the conclusion of only a slight impact of the input signed network on the \(P(k)\) distribution.
- We successfully restore a similar number of motifs’ types by analyzing the triad census.

References

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