Escaping polarization

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Introduction

In our model we employ two theories from social sciences: homophily theory and structural balance theory. **Homophily theory** assumes thatsimilar agents tend to create friendly relations and dissimilar agents tend to create unfriendly relations. **Structural balance theory** claims that balanced structures tend to last longer than unbalanced ones. A simple desciption of balanced structures is given by following four sentences [1]:

- 'friend of my friend is my friend',
- 'friend of my enemy is my enemy',
- 'enemy of my friend is my enemy' and
- 'enemy of my enemy is my friend'.

In a signed undirected complete network to analyse a structure it is enough to analyse triads. There are two versions of structural balance theory:

- a traditional version where all above sentences (S1-S4) need to be fulfilled and a balanced triad contains an even (that is 0 or 2) number of negative links. This is also called a **strong structural balance** definition. See Fig. 1a.
- **weak structural balance**, where the sentence S4 does not have to be fulfilled and the triad with 3 negative edges is also considered to be balanced. See Fig. 1b.

The structure of the balanced network may be significantly different depending whether it is balanced in the strong or the weak sense. If the network is balanced in the strong sense then either all the links are friendly and we have a so-called paradise state (Fig. 1c) or the nodes may be divided into two enemy groups (Fig. 1a) with all links within the groups positive and all links connecting the groups negative. In the network that is balanced in the weak sense it is also possible that there are multiple enemy groups (Fig. 1b).

Which state is polarized?

- A paradise, i.e. the state with all links positive is NOT a polarized state.
- A balanced structure (in the strong sense) with two enemy groups is a polarized state.
- A structure with all links negative is a polarized state. Such a state is balanced in the weak sense (number of groups is equal to number of nodes). Therefore:
- **Polarized state is the state that is balanced in the weak sense with at least two enemy groups**.

In terms of local properties of the network a polarized triad is the triad with two or three negative links and we define the **local polarization density** P_{LP} as the density of such triads in the network.

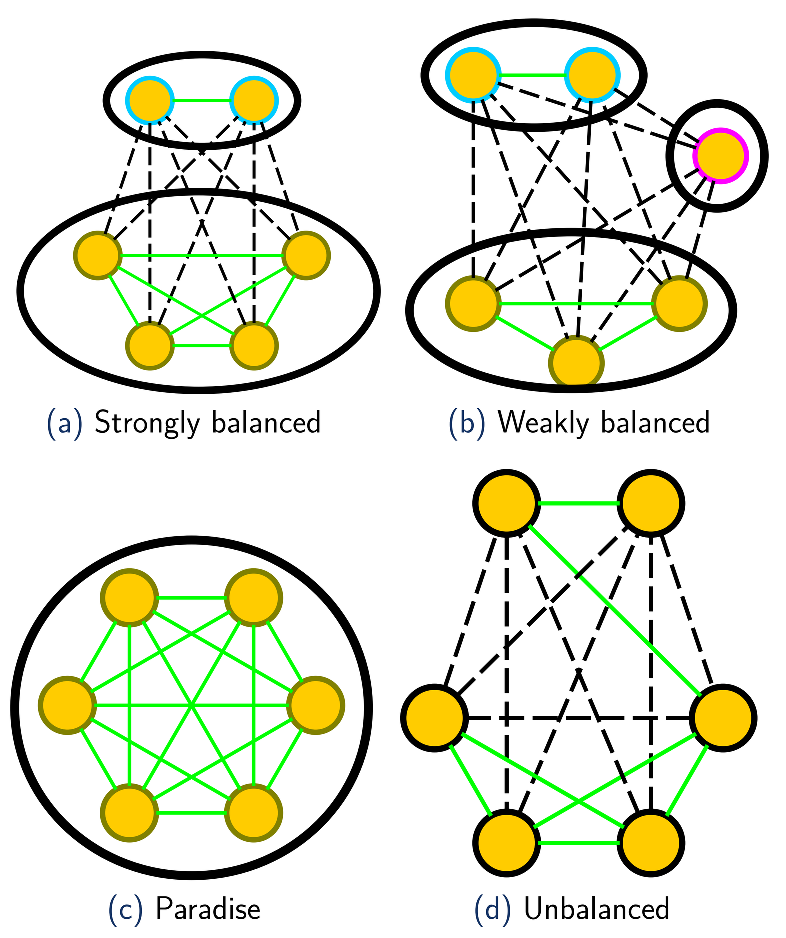

Fig. 1: A balanced state in the strong sense (a) or in the weak sense (b) can be polarized.
A balanced, paradise state (c) is not polarized. An unbalanced state (d) is not polarized.

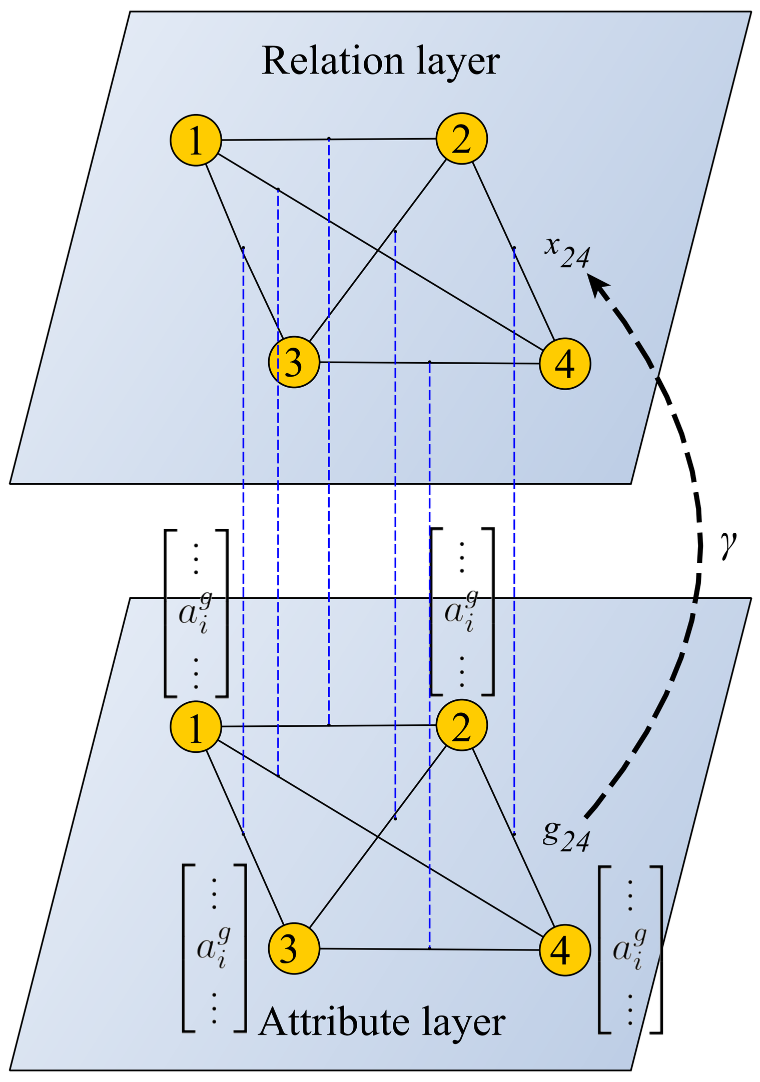

Fig. 2: The influence of the attribute layer (AL) on the relation layer (RL). Each agent has a set of attributes $\{a_i^g\}$ that allows to define the weights g_{ij} in the AL. AL influences RL with the coupling strength γ.

Model

We propose a link-multiplex model (Fig. 2) consisting of N agents belonging to two layers: relation layer (RL) and attribute layer (AL). Each agent possesses G attributes of the same type and along the homophily theory positive or negative links are created in the AL. We assume AL does not change in time and influences the RL through the coupling with strength γ. The links in the RL have continuous weights: $x_{ij} \in [-1, +1]$. The links are dependent on time and change according to the following differential equation [2]:

$$\dot{x}_{ij} = \left(1 - (x_{ij})^2\right)\left(\frac{1}{N-2}\sum_{k=1}^{k=N} x_{ik}x_{kj} + \gamma g_{ij}\right) \qquad (1)$$

The sum in this equation fulfills assumptions of strong version of the structural balance theory.

Attributes

The weight in the AL is the function depending on all attributes. We assume the attributes are uncorrelated and uniformly distributed.

$$g_{ij} \equiv g_{ij}(a_i^1, \ldots, a_i^G; a_j^1, \ldots, a_j^G) = \frac{1}{G}\sum_g h_{ij}(a_i^g, a_j^g), \qquad (2)$$

where h_{ij} is the similarity between distinct attributes. We consider following attribute types (Fig. 3):

- Binary attributes (BA), where an attribute's value is a choice between two values.
- Ordered attributes (OA), where an attribute can have one of v ordered values (e.g. salary, age).
- Unordered negative attributes (UA), where an attribute can have one of the v unordered values (e.g. race).
- Unordered positive attributes (UPA) are like UA, but dissimilar values of this attribute does not influence the relation between agents (e.g. having a passion for fishing)

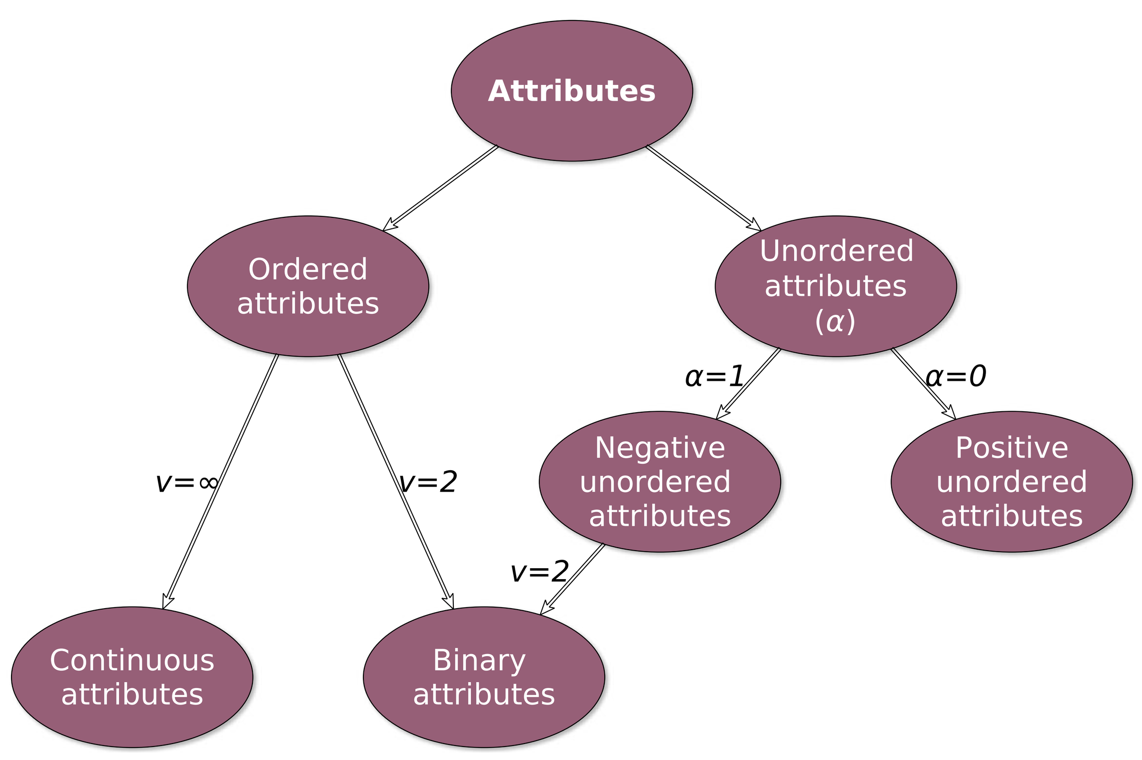

Fig. 3: Classification of considered attributes. Binary attribute is a special case of both ordered and unordered attributes.

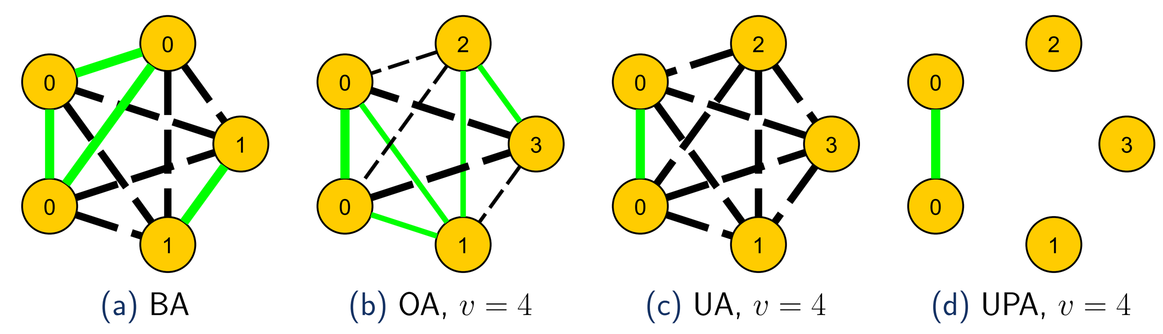

Fig. 4: Weights $g_{ij} = h_{ij}$ in the AL for considered types of attributes.

Simulations and Results

We perform the simulations in the following scenario. The system is initially in the strongly balanced state (paradise or two enemy groups) and then the attributes start to influence the relation layer. Our question is whether the attributes can destabilize the balanced state. Such a question for two enemy groups is equivalent to the following one:

Can the attributes destabilize the polarized state?

Here, we present results of two simulation analyses:

- What is the effect of the increase of the number of attributes G?
- What is the effect of the increase of the network size N?

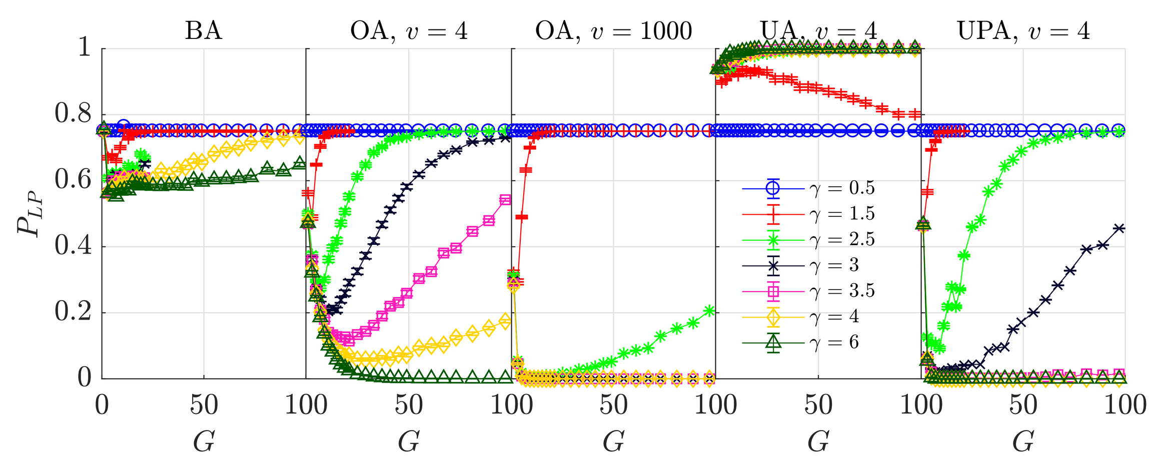

Fig. 5: Threshold values of coupling strength γ_{th} can be observed having sufficiently large numbers of attributes G. The plot shows density of local polarization $P_{LP}(G)$ for different attribute types for the network of size $N = 9$. The plot confirms predictions coming from analytics. We have expected the change of P_{LP} towards $P_{LP}(\gamma = 0.5)$ for $\gamma < \gamma_{th}$. For $\gamma > \gamma_{th}$ the system will stay unpolarized no matter G. Calculated threshold values are related to the expectation values of given similarity measures h_{ij}. These calculated values agree with simulation results.

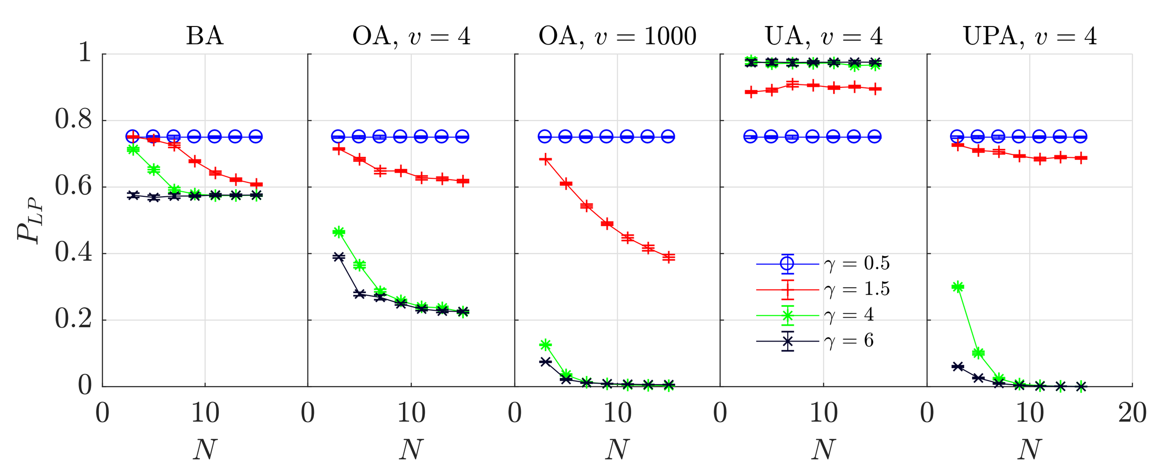

Fig. 6: For all attribute types for which the expected value of similarity h_{ij} is not negative an increase of the system size N makes a polarized state easier to be destroyed. The plot shows density of local polarization $P_{LP}(G)$ for different attribute types for systems with nodes possessing $G = 5$ attributes each. Attribute layer coupling strengths $\gamma < 1$ do not destabilize the system. Large coupling strengths ($\gamma = 4, 6$) destroy the polarization completely for UPAs and continuous attributes (OA with large v) and to certain extent for OAs and BAs. Interesting result is obtained for intermediate strength ($\gamma = 1.5$), where UPAs are less efficient than OAs and BAs. This is the contrary for what one would expect, that an attribute type without negative impact (UPA) will be always close to be most efficient in destroying the balanced state.

Conclusions

- Positive expected value of similarity function h is very important for destabilizing the polarized state.
- To destabilize the polarized state one should apply the attributes in following order: positive unordered attributes (UPA) ?>? continuous attributes (OA, large v) > ordered attributes (OA) > binary attributes (BA)
- A structurally balanced state and paradise state are not equivalent. One needs to remember that by destabilized a balanced state a different balanced state may be achieved.
- Destroying the structurally balanced state in the strong sense is the most efficient with unordered attributes (UA). However, such attributes lead to the state with plenty of negative links. Therefore, they do not decrease the polarization in the system.

References

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