Weighted Axelrod model

JAGIELLONIAN UNIVERSITY IN KRAKÓW

Zuzanna Kalinowska | Bartłomiej Dybiec

Faculty of Physics, Astronomy and Applied Computer Science | Jagiellonian University

Introduction

The Axelrod model is a well-known model of culture development and dissemination describing a possible mechanism for the emergence of cultural domains. It is based on two sociological phenomena: **homophily** and the **theory of social influence**. Technically, it assumes that every culture is represented by a vector of F cultural traits (features), each taking any of the q allowed opinions (values). The model assumes that an individual can interact with local neighbours if and only if they share some common traits. The agents are conservative in the sense that they are more likely to interact with other agents who are similar to them.

On the one hand, at every successful interaction, one of the interacting agents accepts the agent's point of view on a topic on which both agents differ. Consequently, interactions increase the similarity between agents and make them even more likely to interact in the future. On the other hand, acceptance of opinion can result in differentiation of noninteracting neighbours. The Axelrod model allows for **coexistence of multiple cultural domains** where neighbouring cultures are completely different, as agents belonging to adjacent clusters do not share any common traits.

Motivation

The Axelrod model does not take into account the fact that cultural attributes may have **different significance for a given individual**. This is a limitation in the context of how the model reflects the mechanisms driving the evolution of real societies. The study aims to modify the Axelrod model by giving individual features different weights that have a decisive impact on the possibility of changing the opinion and in turn on interactions between two individuals.

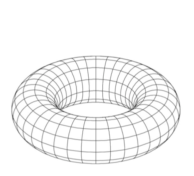
Agent-based modelling

The model consists of a set of agents that take on a finite collection of states. This is known as agent-based modelling, a technique well suited for modelling many different types of systems. The state of an agent depends on the system's previous state and is determined through a set of rules. Those rules describe the mechanics of the agent's interaction with other agents in the neighbourhood.

Axelrod's model is based on a concept of **cellular automata**. They are agent-based models that are local in their interactions, discrete in space/time, and homogeneous in space/time (same update rule at all cells at all times). Cellular automaton typically consist of:

1. A D-dimensional space of $\{i\}$ cells.
2. A k sized set of allowed states for each cell, usually the same for all of them: $\{s_1, s_2, ..., s_k\}$.
3. $O(s_i)$ – a neighbourhood which defines which cells are considered to pass information to a given cell s_i.
4. F – a transition rule which specifies how given a cell and the states of its neighbours, a new state is produced in the $t+1$ moment: $s_i(t+1) = F(\{s_j(t)\})$ where $j \in O(s_i)$.

Periodic boundary conditions



To approximate a large-scale system – society – the agents placed on the 2D space borders were "glued" together by using periodic boundary conditions. In topological terms, the space made by two-dimensional PBC can be thought of as being mapped onto a torus.

Original Axelrod model

In the original model each agent x is characterised by a vector $X(x)$ consisting of F cultural attributes that can take any of the allowed q values:

$$X(x) = (X^1(x), ..., X^F(x))$$
where $X^i(x) \in \{1, 2, ..., q\}$ for $i = 1, 2, ..., F$.

The model dynamics can be described in a few steps:

1. Choose a random agent.
2. Choose a random neighbour of this agent.
3. Choose a random cultural attribute that will be the subject of interaction between the above agents. Note that the chosen attribute needs to have different values for both entities.
4. Perform an interaction between the chosen agents based on their similarity – agent takes over the value of the cultural attribute of its neighbour with the probability equal to the ratio of common values to all possible cultural attributes F.
5. Repeat steps 1-4 (one repetition = one iteration) until one of the final conditions is met:
   - Homogeneity – all agents have the same values, i.e. their similarity is equal to 1 – no more further interactions will introduce any changes to the system,
   - Polarization – agents are split into subgroups (*cultural clusters*) and the similarity between neighbouring entities is equal to 0 – no further interactions are possible.
   - Time constraint – simulation has exceeded the set limit of iterations.

Weighted Axelrod model

In the modified Axelrod model each agent x is characterised by a vector $X(x)$ consisting of F cultural attributes that can take any of the allowed q values as well as a vector $W(x)$ representing the weights of respective attributes:

$$X(x) = (X^1(x), ..., X^F(x))$$
$$W(x) = (w_1, w_2, ..., w_F)$$
where $X^i(x) \in \{1, 2, ..., q\}$ and $0 < w_i < 1$ for $i = 1, 2, ..., F$.

The only difference in the algorithm for the weighted version of the model is in the interaction step. In the original model, the interaction was solely driven by the similarity between two chosen entities. In the modified version of the Axelrod model, step number 4 in the dynamics outlined in the above section can be defined as:

Perform an interaction between the chosen agents based on their similarity – agent takes over the value of the cultural attribute of its neighbour with the probability equal to the ratio of common values to all possible cultural attributes F **if the agent's weight for the chosen cultural attribute is smaller than the similarity**.

Simulations

The weighted model was examined with a focus on the influence of the introduced modification on the evolution of the system and the final states. One of the simulation parameters was W_{max}, a maximum value which the randomly chosen weight for the agent's cultural attribute could not exceed. 100 agents placed within the von Neumann neighbourhoods were used in the simulations – periodic boundary conditions allowed to approximate a bigger system without using too many resources. Each experiment was repeated 100 times so that it was possible to average the numbers and obtain results that are less dependent on random initial conditions. The maximum number of iterations was set to 10 000.

Experiments

Five core experiments were performed:

1. No. of attributes $F = 8$, values $q \in \{1, ..., 5\}$, weights $W_{max} \in \{0.125, 0.3, 0.7, 0.875\}$.
2. No. of attributes $F = 8$, values $q \in \{1, 2\}$, weights $W_{max} \in \{0.125, 0.3, 0.7, 0.875\}$.
3. No. of attributes $F = 8$, values $q \in \{1, ..., 8\}$, weights $W_{max} \in \{0.125, 0.3, 0.7, 0.875\}$.
4. No. of attributes $F = 4$, values $q \in \{1, ..., 5\}$, weights $W_{max} \in \{0.25, 0.3, 0.7, 0.75\}$.
5. No. of attributes $F = 16$, values $q \in \{1, ..., 5\}$, weights $W_{max} \in \{0.0625, 0.3, 0.7, 0.9375\}$.

For $W_{max} = 1/F$ the modified model is equivalent to the standard Axelrod model.

Results

Table 1. Simulation duration for Original ($W_{max} = 1/F$) vs Modified ($W_{max} = 1 - 1/F$) model

Exp. No.	Avg. Duration - Original	Avg. Duration - Weighted
1	579	1 311
2	343	431
3	649	2 972
4	436	605
5	980	4 259

For the modified version of the model with a high maximum weight a significant increase in time required to reach the final state was observed. This effect is less visible in experiments that have a relatively small number of cultural attributes or their values. The more complicated the system, the stronger the effect that the weights have on the results.

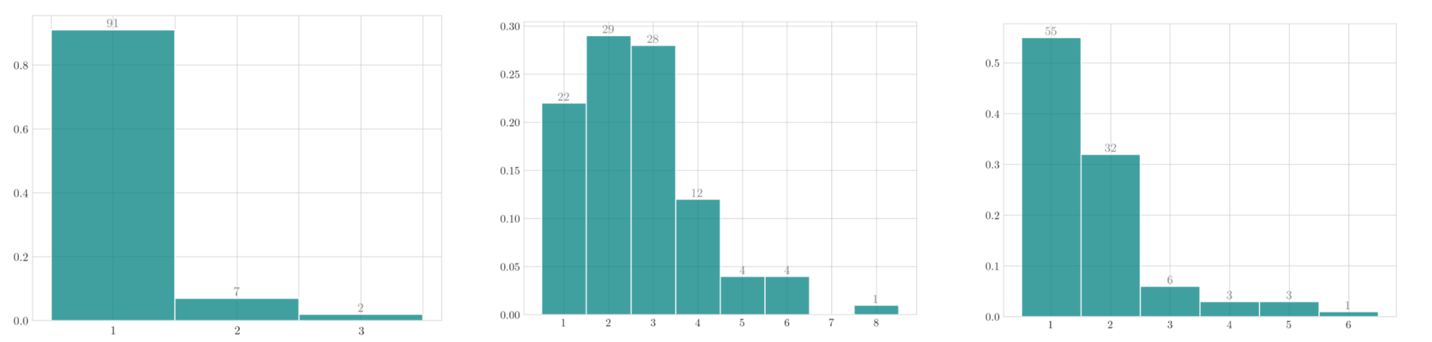


Figure 1. Probability histograms of culture clusters number in the final states
From left: Exp. 1 where $W_{max} = 0.875$, Exp. 3 where $W_{max} = 0.875$, Exp. 4 where $W_{max} = 0.75$

For all the above experiments, the maximum weight input corresponding with the original Axelrod model ended in full system homogeneity in almost all simulations.

Summary

The comparison of the results obtained for the classic Axelrod model and its modified version shows that the introduced weights have **a significant impact** on the system evolution, in particular, they **increase the polarization** of the system in the final state.

References

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zuzanna.kalinowska@student.uj.edu.pl