Dynamics of spin polarization in the Gubser and Bjorken expanding background

Rajeev Singh

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

rajeev.singh@ifj.edu.pl

Collaborators:

Wojciech Florkowski (IF UJ), Radoslaw Ryblewski (IFJ PAN), Avdhesh Kumar (NISER), Gabriel Sophys (IFJ PAN), and Masoud Shokri (IPM)

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Outline:

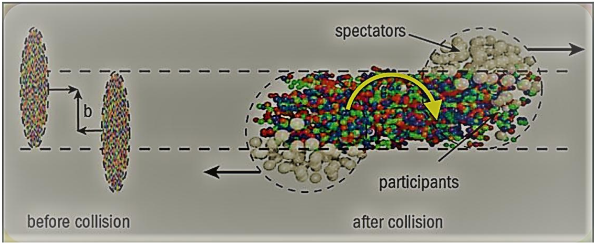
- Motivation.
- Methodology.
- Conservation laws.
- Connection to experiment.
- Boost-invariant and transversely homogeneous flow.
- Gubser flow.
- Bjorken-expanding resistive MHD background.
- Summary.

Motivation

Heavy-ion collisions:

- Non-central relativistic heavy-ion collisions creates global rotation of matter, which may induce spin polarization.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

nucl-th/0410079, nucl-th/0410089, arXiv:0708.0035.



Source: CERN Courier

Global polarization:

The first positive measurement of $\Lambda(\bar{\Lambda})$ global spin polarization by STAR.

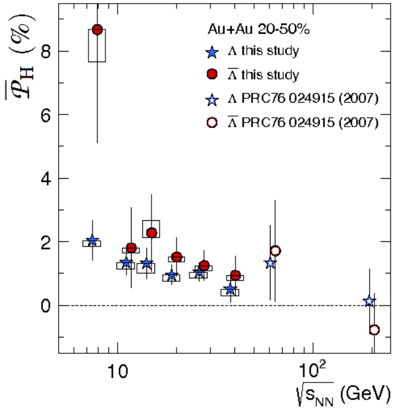
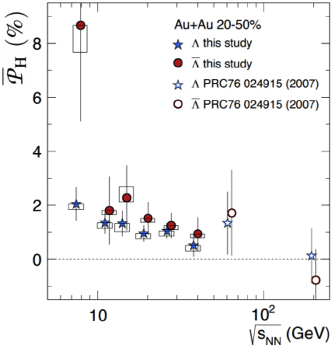


Figure: Average polarization $\bar{\mathcal{P}}_H$ (where $H = \Lambda$ or $\bar{\Lambda}$) versus collision energy in 20-50% central Au+Au collisions.

Source: L. Adamczyk et al.(STAR), Nature 548 (2017) 62-65

Global polarization:

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach $\longrightarrow$ $P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$ $\quad P_{\overline{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$

Becattini, F., Karpenko, I., Lisa, M., Upsal, I. , Voloshin, S. , PRC 95, 054902 (2017)

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = \left(P_\Lambda + P_{\overline{\Lambda}}\right) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \mathrm{s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Even larger than...

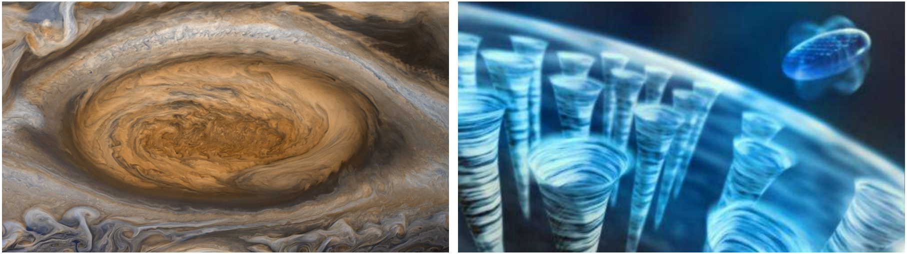


Figure: Jupiter great red spot ($10^{-4}s^{-1}$) & Nanodroplets of superfluid helium ($10^7 s^{-1}$).

1301.6119, Science 345, 906–909 (2014)

Longitudinal polarization:

- Good agreement between experiment and models on global polarization.
  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770
- But...

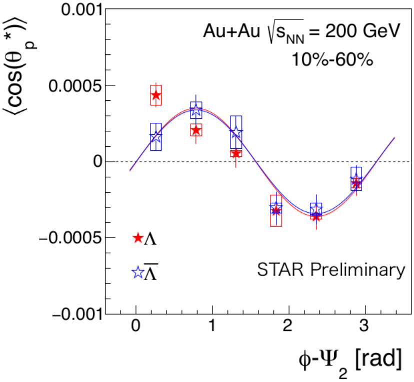


Figure: Longitudinal polarization of Λ-$\bar{\Lambda}$ (1905.11917)

Bigger picture:

- This study will help us to know the formation and characteristics of the QGP, a state of matter believed to exist at sufficiently high energy densities.

- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

Methodology

Our approach:

- Include spin degrees of freedom into the ideal standard hydrodynamics to form spin hydrodynamics formalism.
- $J^{\mu,\alpha\beta}(x) = x^{\alpha} T^{\mu\beta}(x) - x^{\beta} T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_{\lambda} J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_{\lambda} S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T^{\nu\mu}_{\rm GLW}(x) = T^{\mu\nu}_{\rm GLW}(x)$, we have $\partial_{\lambda} S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in the de Groot-van Leeuwen-van Weert (GLW) formulation.

1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot *et. al.*, Relativistic Kinetic Theory: Principles and Applications (1980).

Our spin hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin.
- Determination of the spin evolution in the hydrodynamic background.
- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Calculation of the spin polarization of particles in their rest frame. The spin polarization obtained is a function of the three-momenta of particles and can be directly compared with the experiment.
  1901.09655

Conservation laws

Conservation of net baryon number:

$$d_\alpha N^\alpha(x) = 0$$

where,

$$N^\alpha = 4\sinh\left(\frac{\mu}{T}\right) \mathcal{N}_{(0)} U^\alpha$$

Here, μ is baryon chemical potential, T is temperature and U^μ is 4-vector fluid flow.
$\mathcal{N}_{(0)}$ is number density for the case of ideal relativistic gas of classical massive particles (and antiparticles).

1811.04409

Conservation of energy and linear momentum:

$$d_\alpha T^{\alpha\beta}(x) = 0$$

where for perfect-fluid,

$$T^{\alpha\beta} = 4\cosh\left(\frac{\mu}{T}\right)\left[\left(\mathcal{E}_{(0)} + \mathcal{P}_{(0)}\right) U^\alpha U^\beta + \mathcal{P}_{(0)} g^{\alpha\beta}\right]$$

$\mathcal{E}_{(0)}$ and $\mathcal{P}_{(0)}$ are the energy density and pressure for the case of ideal relativistic gas of classical massive particles (and antiparticles), respectively.

1811.04409

Above conservation laws (charge and energy-linear momentum) provide closed system of five equations for five unknown functions: μ, T, and three independent components of U^μ (hydrodynamic flow vector) which needs to be solved to get the hydrodynamic background.

Conservation of spin:

$$d_\alpha S^{\alpha,\beta\gamma} = 0$$

where spin tensor is defined as

$$S^{\alpha,\beta\gamma} \;=\; \mathcal{C}\, U^\alpha \omega^{\beta\gamma} + S_\Delta^{\alpha,\beta\gamma}$$

and,

$$S_\Delta^{\alpha,\beta\gamma} \;=\; \mathcal{A} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}{}_\delta + \mathcal{B}\Big(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}{}_\delta + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}{}_\delta + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]}{}_\delta \Big)$$

with $\omega^{\alpha\beta}(x)$ being the spin polarization tensor and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are the thermodynamic quantities obtained from perfect-fluid dynamics

$$\mathcal{A} = 2\mathcal{C} - 3\mathcal{B}, \quad \mathcal{B} = -2\cosh\left(\frac{\mu}{T}\right)\frac{\mathcal{E}_{(0)} + \mathcal{P}_{(0)}}{(m^2/T)}, \quad \mathcal{C} = \cosh\left(\frac{\mu}{T}\right)\frac{\mathcal{P}_{(0)}}{T}$$

1811.04409

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be defined by the four-vectors κ^μ and ω^μ,

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta,$$

where,

$$\kappa^\alpha = a_X X^\alpha + a_Y Y^\alpha + a_Z Z^\alpha, \quad \omega^\alpha = b_X X^\alpha + b_Y Y^\alpha + b_Z Z^\alpha$$

U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions:

$$U \cdot U = 1$$

$$\begin{aligned} X \cdot X &= Y \cdot Y = Z \cdot Z = -1, \\ X \cdot U &= Y \cdot U = Z \cdot U = 0, \\ X \cdot Y &= Y \cdot Z = Z \cdot X = 0. \end{aligned}$$

Assumption: Restricted to leading order terms in $\omega^{\mu\nu}$.

Connection to experiment

Mean spin polarization per particle:

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

The above equation is the ratio of the invariant momentum distribution of the total Pauli-Lubański vector and the momentum density of particles and antiparticles expressed as

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = \frac{\cosh(\frac{\mu}{T})}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p} \, \tilde{\omega}_{\beta\mu} \, p^\beta$$

and

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\frac{\mu}{T})}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p}$$

respectively, where $\tilde{\omega}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}$ is the dual polarization tensor and $\Delta\Sigma_\lambda$ is the infinitesimal element of the freeze-out hypersurface.

- Polarization vector $\langle \pi^{\star}_{\mu} \rangle$ in the local rest frame of the particle can be obtained by using the canonical boost.

- Components of $\langle \pi^{\star}_{\mu} \rangle$ are then obtained as functions of transverse momentum components p_x and p_y in mid-rapidity, which can be compared with the experiment.

Boost-invariant and transversely homogeneous flow

Perfect-fluid background dynamics:

- Conservation law of charge can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

  Therefore, for Bjorken type of flow we can write,

$$\partial_\tau n + \frac{n}{\tau} = 0$$

- Conservation law of energy-momentum can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

  Hence for the Bjorken flow,

$$\partial_\tau \varepsilon + \frac{(\varepsilon + P)}{\tau} = 0$$

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Initial baryon chemical potential $\mu_0 = 800$ MeV
Initial temperature $T_0 = 155$ MeV
Particle (Lambda hyperon) mass $m = 1116$ MeV
Initial and final proper time is $\tau_0 = 1$ fm and $\tau_f = 10$ fm, respectively.

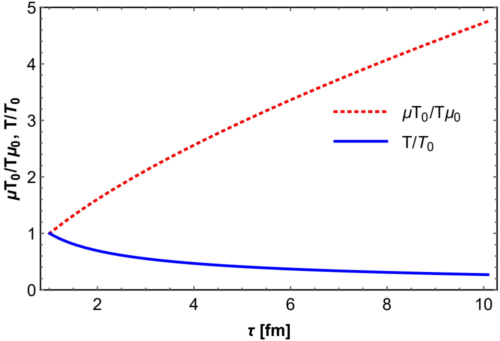

Figure: Proper-time dependence of T divided by its initial value T_0 (solid line) and the ratio of baryon chemical potential μ and temperature T re-scaled by the initial ratio μ_0/T_0 (dotted line) for a boost-invariant one-dimensional expansion.

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Spin polarization coefficients evolution:

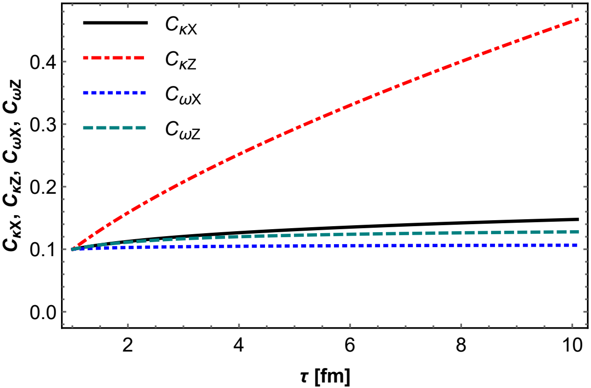

Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{\kappa Y}$ and $C_{\omega Y}$ satisfy the same differential equations as the coefficients $C_{\kappa X}$ and $C_{\omega X}$.

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Momentum dependence of polarization:

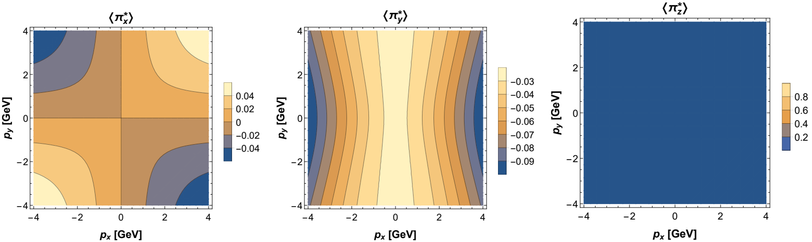


Figure: Components of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0, 0, 0)$, and $\boldsymbol{C}_{\omega,0} = (0, 0.1, 0)$ for $y_p = 0$.

1901.09655

Gubser flow

- Solve the perfect-fluid hydrodynamical equations using the Gubser flow.
- Obtain analytical solutions for T and μ.
- Derive the equations of motion for spin polarization components in de Sitter coordinates.
- The background solutions are not spoiled by the breaking of the symmetry at the level of angular momentum conservation.
- The coupling between the spin polarization coefficients emerge due to the conformal symmetry breaking.

  2011.14907

Space-time evolution of Temperature:

$T(\tau_0 = 1\,\mathrm{fm}, r = 0) = 1.2\,\mathrm{fm}^{-1}$

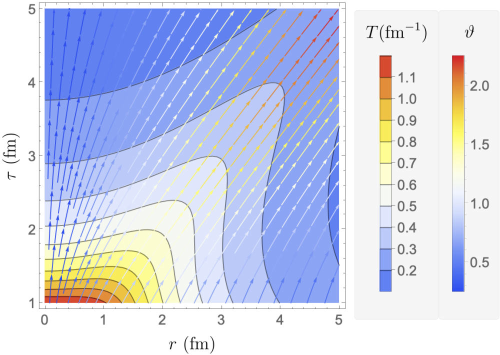


Figure: The space-time dependence of temperature (contours) and flow-vector components $(U^\tau, U^r)/\sqrt{(U^\tau)^2 + (U^r)^2}$ (stream lines – the coloring of arrows is given by the rapidity ϑ).

2011.14907

Spin polarization coefficients:

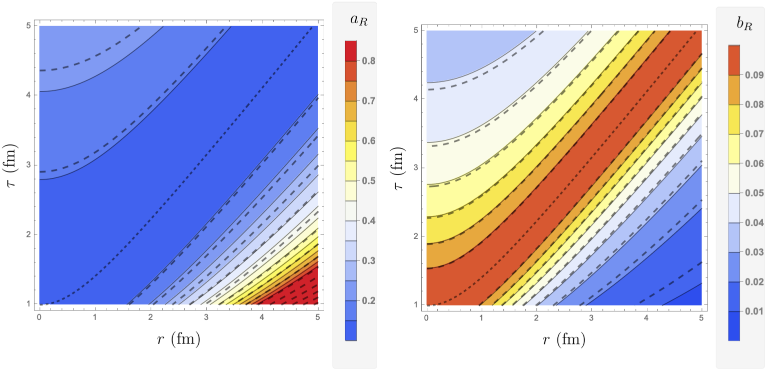


Figure: Numerical solutions for a_R and b_R components of the spin polarization tensor as functions of proper time τ and radial distance r.

2011.14907

Bjorken-expanding resistive MHD background

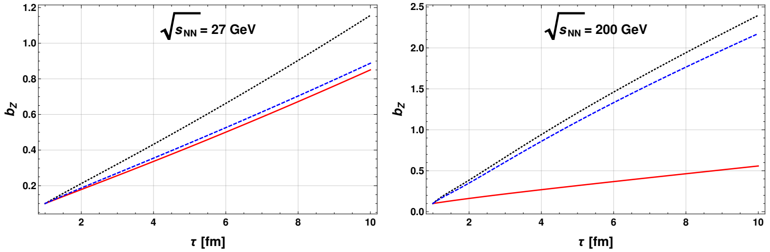
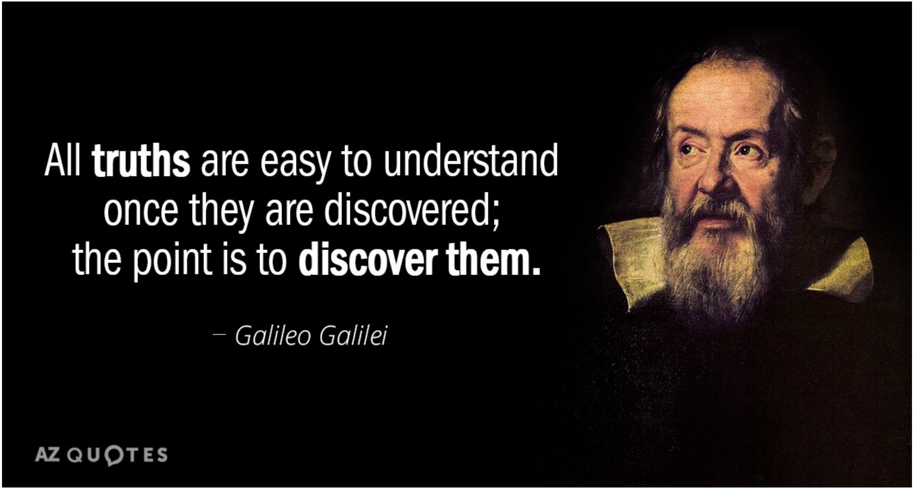


Figure: Spin polarization coefficient b_Z profile for $\sqrt{s_{\rm NN}} = 27\,\mathrm{GeV}$ (left panel) and $\sqrt{s_{\rm NN}} = 200\,\mathrm{GeV}$ (right panel) with initial value $b_Z^0 = 0.1$. The modification of the b_Z evolution slope due to electric field is much more pronounced when μ_0/T_0 is small as can be seen in the right panel. Dotted black line is for $\alpha = -8$, red line is for $\alpha = 0$ and dashed blue line is for $\alpha = 8$.

2103.02592

Summary

- Discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- Showed how our formalism can be compared with the experiments.
- Obtained dynamics of spin polarization coefficients in the Bjorken and Gubser-expanding background.
- Showed the behavior of spin polarization coefficients in the Bjorken-expanding resistive MHD background.
- Results obtained in the Gubser background can be used for description of head-on collisions of initially polarized particles/ions at high energies describing the mixing between polarization components along beam and in the azimuthal direction.
- Incorporation of spin in full 3+1D hydro model required to address the problem of longitudinal polarization.



Thank you for your attention!

Extra Slides

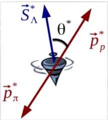
Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}} \mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

P_H: Λ polarization
p_p^*: proton momentum in the Λ rest frame
α_H: Λ decay parameter
($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



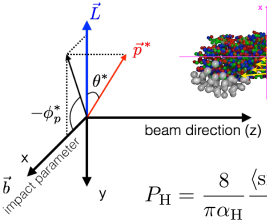
$$\Lambda \to p + \pi^-$$

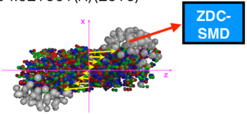
(BR: 63.9%, $c\tau$ ~7.9 cm)

C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)
- S. Voloshin and TN, PRC94.021901(R)(2016)



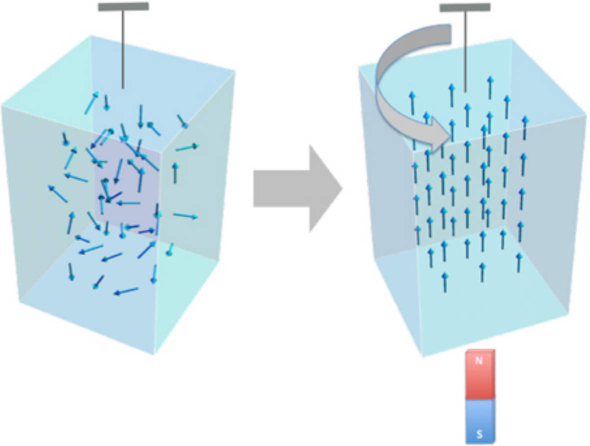


$$P_{\mathrm{H}} = \frac{8}{\pi\alpha_{\mathrm{H}}} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{\mathrm{Res}(\Psi_1)}$$

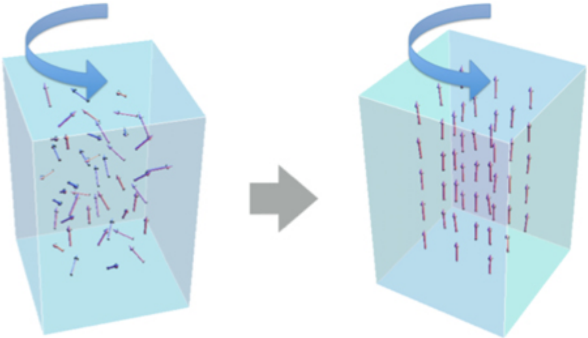
Ψ_1: azimuthal angle of b
ϕ_p^*: ϕ of daughter proton in Λ rest frame
STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

Einstein-De Haas Effect (1915): Rotation induced by Magnetization



Barnett Effect (1915): Magnetization induced by Rotation



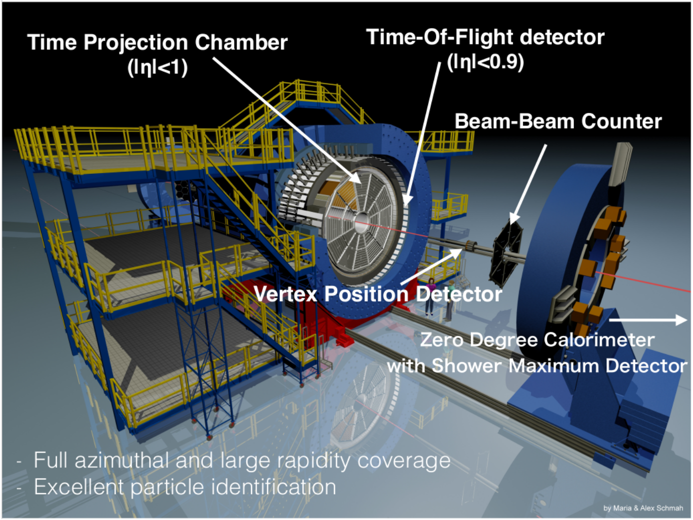


Figure: Schematic view of STAR Detector

Pseudo-gauge transformations

$$
\begin{aligned}
\hat{T}^{\mu\nu} &= \hat{T}^{\mu\nu}_{\mathrm{C}} + \frac{1}{2}\partial_\lambda(\hat{\Phi}^{\lambda,\mu\nu} + \hat{\Phi}^{\nu,\mu\lambda} + \hat{\Phi}^{\mu,\nu\lambda}) \\
\hat{S}^{\lambda,\mu\nu} &= \hat{S}^{\lambda,\mu\nu}_{\mathrm{C}} - \hat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \hat{Z}^{\mu\nu,\lambda\rho}
\end{aligned}
$$

where, $\hat{\Phi}^{\lambda,\mu\nu}$ and $\hat{Z}^{\mu\nu,\lambda\rho}$ are arbitrary differentiable operators called super-potentials satisfying
$\hat{\Phi}^{\lambda,\mu\nu} = -\hat{\Phi}^{\lambda,\nu\mu}$ and $\hat{Z}^{\mu\nu,\lambda\rho} = -\hat{Z}^{\nu\mu,\lambda\rho} = -\hat{Z}^{\mu\nu,\rho\lambda}$

$\rightarrow$ The newly defined tensors preserve the total energy, linear momentum, and angular momentum after integrated over the freeze-out hypersurface.
$\rightarrow$ Conservation laws are unchanged.

Spin polarization coefficient evolution equations:

Contracting the spin conservation equation with $U_\beta X_\gamma, U_\beta Y_\gamma, U_\beta Z_\gamma, Y_\beta Z_\gamma, X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$

$\mathcal{P}(\tau) = \mathcal{A}_1,$

$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$

$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$

$$\begin{aligned} \mathcal{A}_1 &= \cosh(\xi)\left(n_{(0)} - \mathcal{B}_{(0)}\right), \\ \mathcal{A}_2 &= \cosh(\xi)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right), \\ \mathcal{A}_3 &= \cosh(\xi)\,\mathcal{B}_{(0)} \end{aligned}$$

Conformal symmetry:

In general, for a system to respect conformal symmetry, its dynamics should be invariant under Weyl rescaling. It implies that the (m, n)-type tensors (including scalars with $(m, n) = (0, 0)$) transform homogeneously, namely

$$A^{\mu_1 \cdots \mu_m}_{\nu_1 \ldots \nu_n}(x) \rightarrow \Omega^{\Delta_A} A^{\mu_1 \cdots \mu_m}_{\nu_1 \ldots \nu_n}(x)$$

where $\Omega \equiv e^{-\varphi(x)}$ with $\varphi(x)$ being function of space-time coordinates and $\Delta_A = [A] + m - n$ is the conformal weight of the quantity A, where $[A]$ is its mass dimension, and m and n being the number of contravariant and covariant indices, respectively.

Transformation rules:

The transformation rules to map the quantities expressed in de Sitter coordinates back to the polar Milne coordinates can be written as

$$
\begin{aligned}
U_\mu(\tau, r) &= \tau \frac{\partial \hat{x}^\nu}{\partial x^\mu} \, \hat{U}_\nu(\rho) \,, \\
\mathcal{E}(\tau, r) = \frac{\hat{\mathcal{E}}(\rho)}{\tau^4}, \quad \mathcal{P}(\tau, r) &= \frac{\hat{\mathcal{P}}(\rho)}{\tau^4}, \quad \mathcal{N}(\tau, r) = \frac{\hat{\mathcal{N}}(\rho)}{\tau^3} \,, \\
T(\tau, r) = \frac{\hat{T}(\rho)}{\tau}, & \qquad \mu(\tau, r) = \frac{\hat{\mu}(\rho)}{\tau}.
\end{aligned}
$$

Conformal transformation of conservation equations:

For the 4D spacetime the conservation law for net baryon number is already conformal-frame independent, i.e. net baryon number is conserved in both Minkowski and de Sitter space-times. In this case, one can write

$$d_\alpha N^\alpha = \Omega^4 \hat{d}_\alpha \hat{N}^\alpha$$

Conservation of energy and linear momentum transforms as

$$d_\alpha T^{\alpha\beta} = \Omega^6 \left[\hat{d}_\alpha \hat{T}^{\alpha\beta} - \hat{T}^\lambda{}_\lambda \hat{g}^{\beta\delta} \partial_\delta \varphi \right]$$

We see that $\hat{T}^{\alpha\beta}$ needs to be traceless in order to be conserved in de Sitter spacetime. Therefore, the breaking of conformal invariance is characterized only by the trace of the energy-momentum tensor

Conformal transformation of the conservation law for spin takes the form

$$d_\alpha S^{\alpha\beta\gamma} \;=\; \Omega^6 \left[\hat{d}_\alpha \hat{S}^{\alpha\beta\gamma} - (\hat{S}_\lambda{}^{\lambda\gamma}\hat{g}^{\beta\sigma} + \hat{S}^{\alpha\beta}{}_\alpha \hat{g}^{\sigma\gamma})\partial_\sigma\varphi\right].$$

We find that the conformal invariance of the spin conservation law requires the spin tensor to satisfy the condition $\hat{S}_\alpha{}^{\alpha\beta} = 0$.