



Statistical hadronization model for heavy-ion collisions in the few-GeV energy regime

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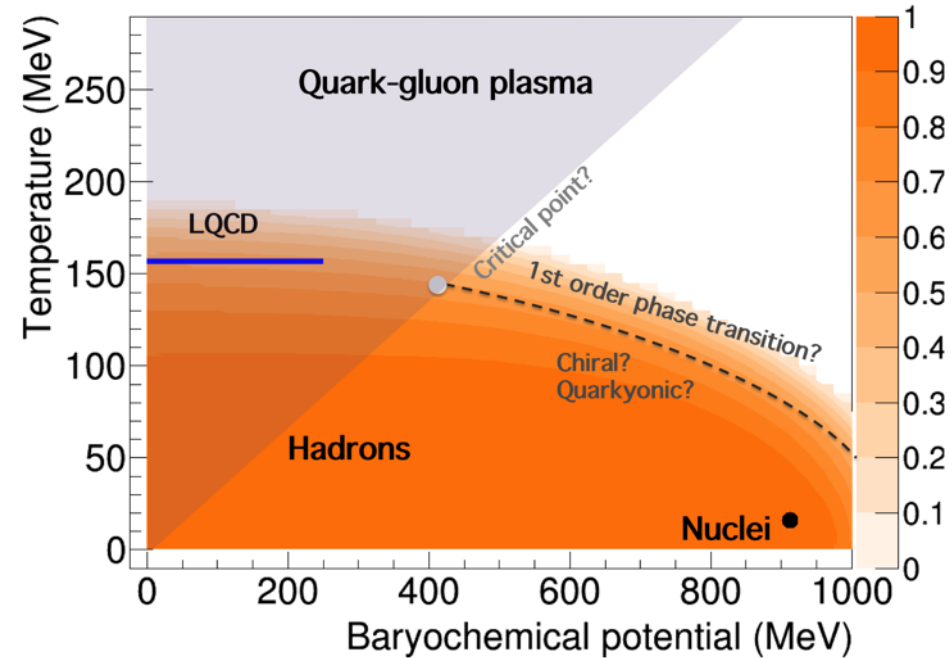
Joachim Stroth

Goethe-University Frankfurt / GSI

basen on:

[PRC 102 \(2020\) 5, 054903](#), [arXiv: 2003.12992 \[nucl-th\]](#)

What is the QCD phase structure?



[P. Steinbrecher (HotQCD), arXiv:1807.05607
Condensate: B.J. Schaefer and J. Wambach]

Vanishing μ_B , high T (lattice QCD)

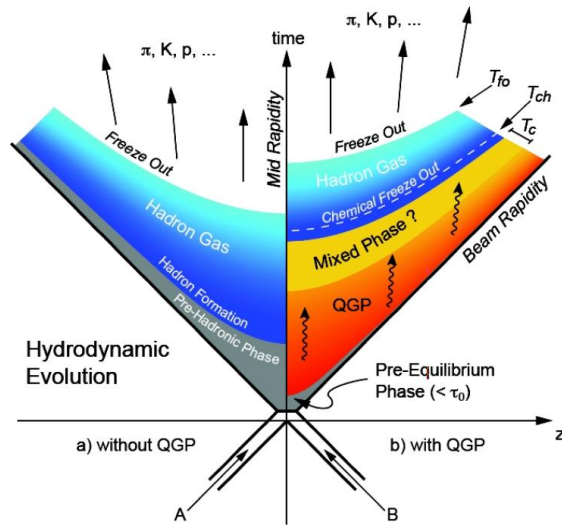
- Crossover, universality
- no CP indicated by lattice QCD at $\mu_B < 400$ MeV, $T > 140$ MeV

Large μ_B moderate T (QCD inspired models)

- Thermal equilibrium?
- 1st order transition?
- QCD critical point?
- Melting of the condensate?

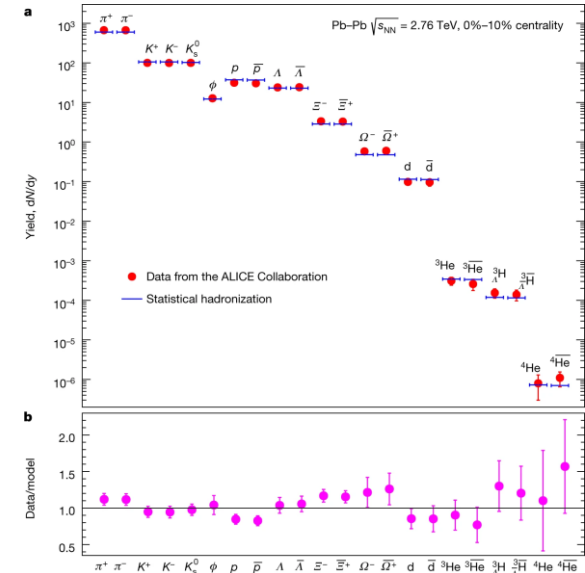
$2 < \sqrt{s_{NN}} < 8$ GeV
Large discovery potential!

Heavy-ion collisions as a tool to study QCD



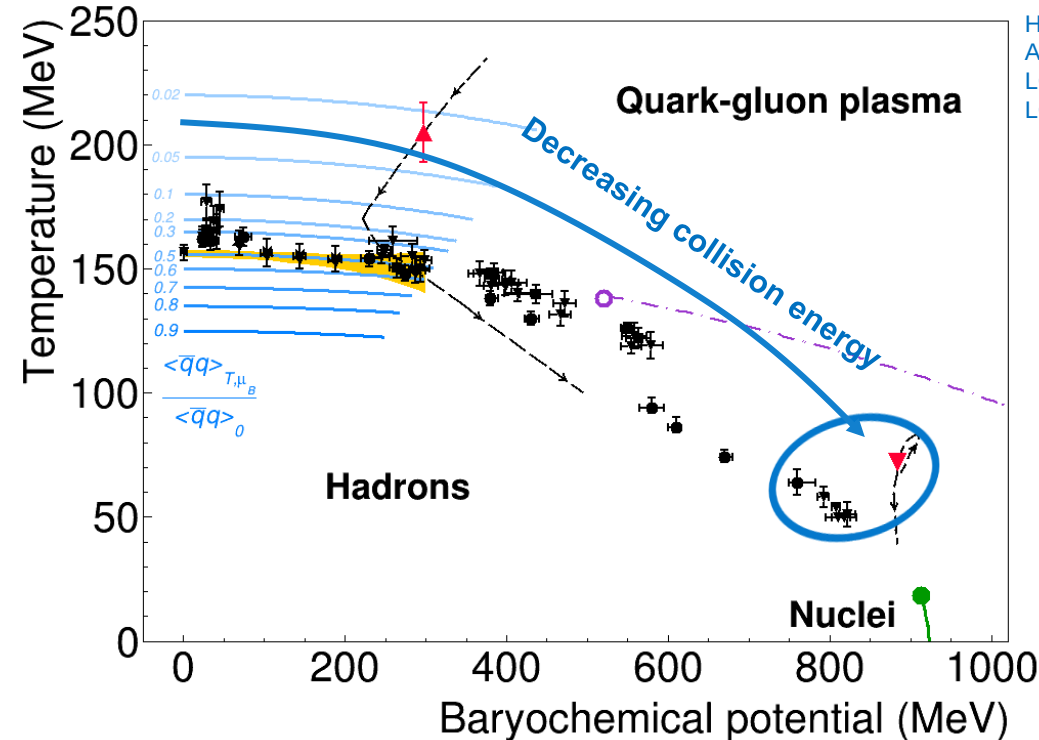
- In chemical equilibrium density of particle i can be written as:

$$n_i = \frac{g_i}{2\pi^2} Y T m^2 K_2\left(\frac{m}{T}\right)$$
 Statistical Hadronization Model (SHM)
- One can fit the ratios of measured particle yields and extract free parameters
 - Location in the phase diagram



A. Andronic et al., Nature 561 (2018) 7723

Mapping the phase diagram with the Statistical Hadronization Model



HADES, *Nature Phys.* 15 (2019) 10, 1040-1045

A. Andronic *et al.*, *Nature* 561 (2018) no.7723

LQCD: S. Borsanyi *et al.* [Wuppertal-Budapest Collab.], *JHEP* 1009 (2010) 073

LQCD: A. Bazavov *et al.*, *PLB* 795 (2019) 15-21

- Is it valid at all to use equilibrium methods at low energies?
 - Particles with strange quarks produced deep below the NN threshold
 - Low number of newly produced particles in the interaction zone: ~40 in central events (mainly pions)
- On the other hand:
 - Original nucleons stopped in the interaction zone (~300 particles in central events)
 - Longer life-time of the system – enough to thermalize

Dynamic description of heavy-ion collisions

Standard prescription at high beam energies (RHIC/LHC):

- Non-equilibrium initial conditions
- Viscous hydrodynamic evolution
 - Equilibrium
 - People often assume: fluid = QGP
- Hadronic final-state rescattering

Standard prescription at "low" beam energies (GSI/FAIR/NICA/...):

- Hadronic transport
- Importance of:
 - Resonance dynamics
 - Nuclear potentials

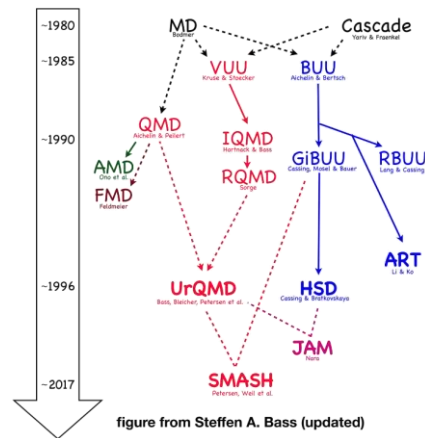
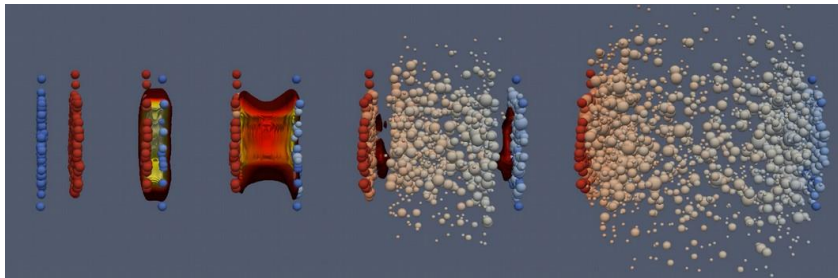
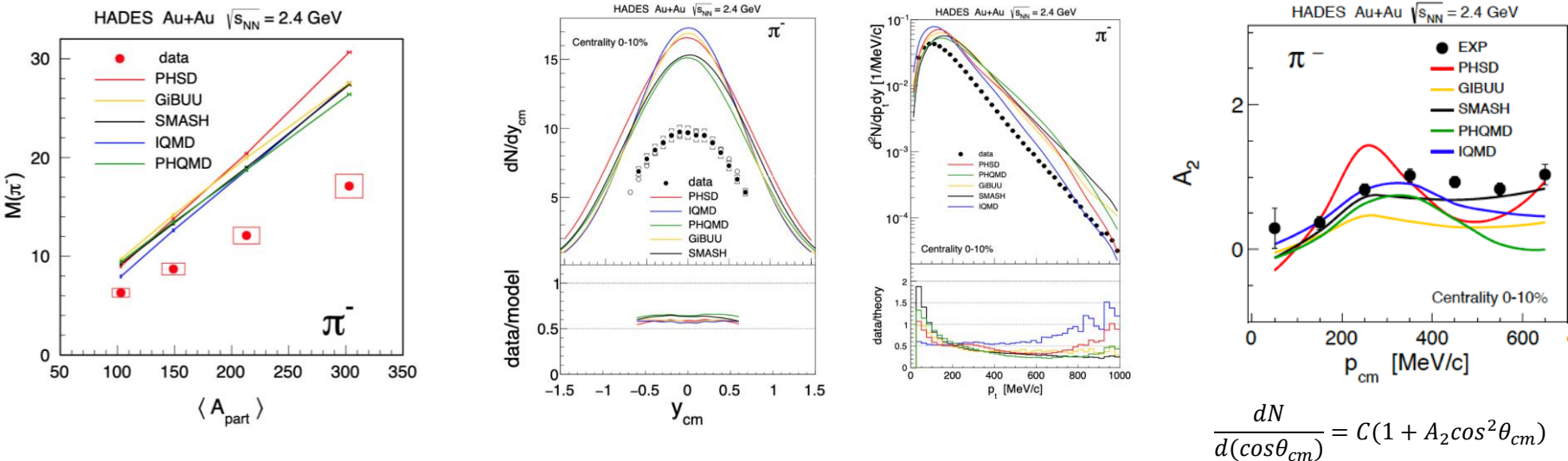


figure from Steffen A. Bass (updated)

Not everything is known yet about few-GeV HIC

Example for π^- , same holds for π^+

HADES Collaboration, EPJA 56 (2020) 10, 259
<https://www.hepdata.net/record/ins1796710>



- Only width of the rapidity distribution is correctly described by the models
- Is there something fundamentally missing?

Pion and Proton "Temperatures" in HIC
R. Brockmann *et al.*, PRL 1984

HADES data vs. other experiments

HADES Collaboration, EPJA 56 (2020) 10, 259
<https://www.hepdata.net/record/ins1796710>

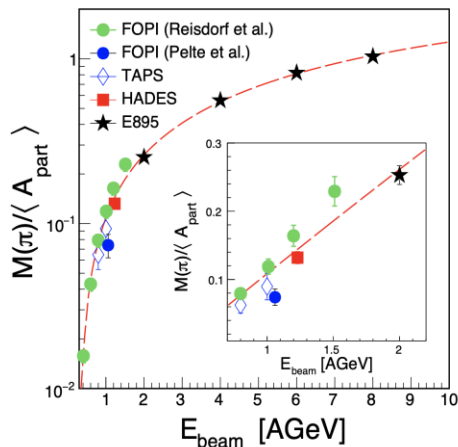


Fig. 8 Pion multiplicity $M(\pi)$ per mean number of participating nucleon $\langle A_{part} \rangle$ as a function of the kinetic beam energy E_{beam} . The dashed curve is a fit to the data points except for the one labeled "FOPI (Pelte et al.)", as suggested in [4]. The inset magnifies the energy region around the HADES point.

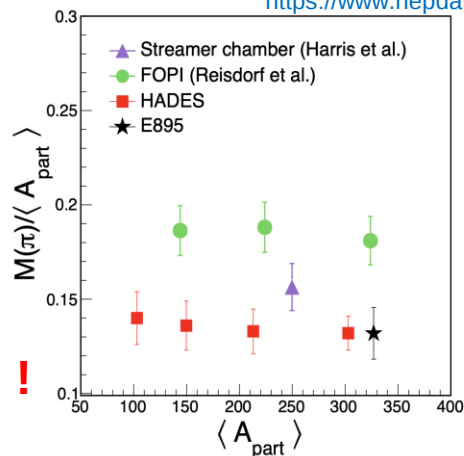


Fig. 9 Comparison of the centrality dependence of $M(\pi)/\langle A_{part} \rangle$ in Au+Au collisions to earlier measurements at similar energies. The results from FOPI, E895, and from the BEVALAC Streamer Chamber (the latter for La+La collisions) have been scaled to 1.23 A GeV; note the suppressed zero on the ordinate.

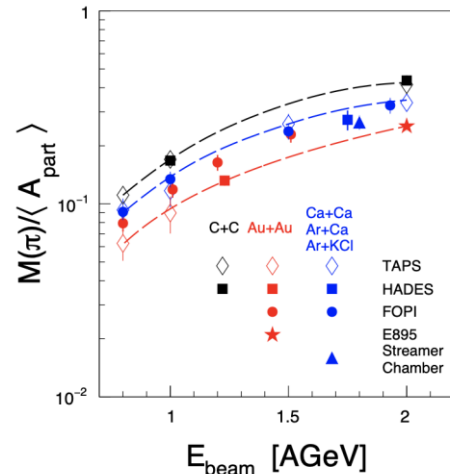


Fig. 10 Pion multiplicity per participating nucleon as a function of beam energy for three different systems: C+C (black) [7, 22, 39], Ar+KCl (blue) [4, 7–9, 40] and Au+Au (red) [4, 6, 7, 11]. The curves are polynomial fits to these data used to interpolate the multiplicities as a function of bombarding energy for corresponding systems.

HADES results are consistent with the trends established by previous experiments at similar beam energies

Hydro-inspired models

of particle production at the freeze-out

First idea:

[P. J. Siemens and J. O. Rasmussen, PRL 42 \(1979\) 880](#)

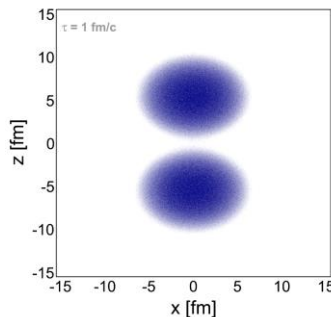
- Used for Ne+NaF at $E_{\text{kin}}/A = 800$ MeV!
- Thermal source of spherical geometry and spherically symmetric expansion
- Constant radial velocity (non-physical for $r = 0$?)

Modification:

[E. Schnedermann, J. Sollfrank, U. W. Heinz, PRC 48 \(1993\) 2462:](#)

- Appropriate for higher-energy collisions (originally S+S at $E_{\text{kin}}/A = 200$ GeV)
- Cylindrically-symmetric geometry and expansion
- Boost invariance in Z direction – "Bjorken scaling"
- Velocity profile: $\beta(r) = \beta_{\text{max}}(r/r_{\text{max}})^n$

Guidance from dynamic models



- Density evolution in Au+Au at $E_{\text{kin}}/A = 1230$ MeV
- Coarse-grained hadronic transport
[T. Galatyuk et al., EPJA 52 \(2016\) 5, 131](#)
- Spherical symmetry clearly more realistic than boost invariance

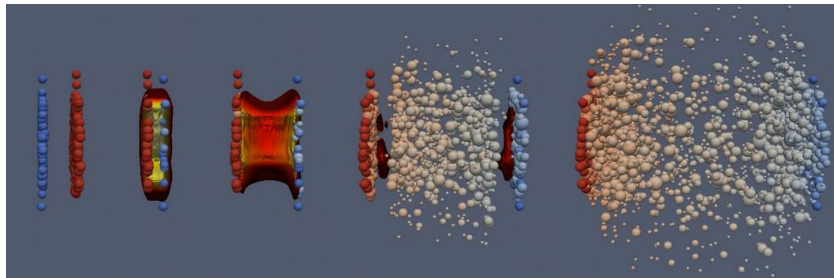
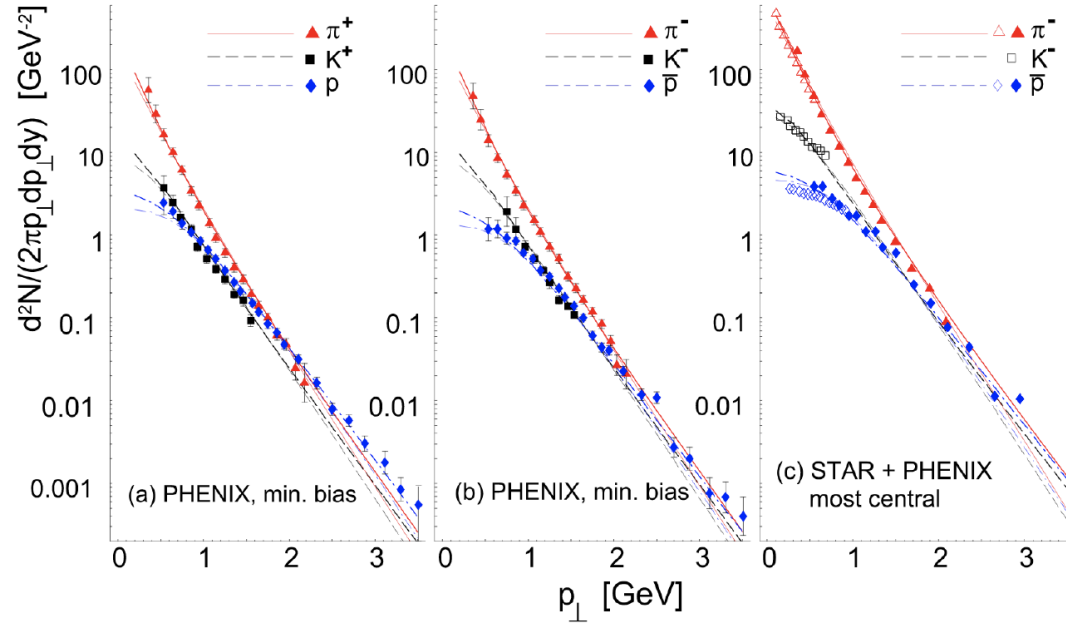


Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard

Single freeze-out scenario

[W. Broniowski and W. Florkowski, PRL 87 \(2001\) 272302](#)

- Chemical freeze-out coincides with kinetic freeze-out
- Hadron yields are given by the integrals of hadron spectra
- Feed-down from resonance decays included
- Successful at RHIC, how it works at SIS18 synchrotron?
- Idea is implemented in the Thermal Event Generator (Therminator 2)



Cooper-Frye formula

F. Cooper and G. Frye, PRD 10 (1974) 186

"Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production"

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x) p^\mu f(x, p)$$

- Spherically symmetric system:

$$x^\mu = (t(r), r\mathbf{e}_r)$$

- Spherical expansion of the "fluid":

$$u^\mu = \frac{1}{\sqrt{1-v^2(r)}} (1, v(r)\mathbf{e}_r)$$

- Sudden freeze-out in the "lab" frame ($t = \text{const}(r)$):

$$d^3\Sigma_\mu \equiv \varepsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial \zeta} \frac{\partial x^\beta}{\partial \phi} \frac{\partial x^\gamma}{\partial \theta} d\zeta d\phi d\theta$$

$$= (r^2 \sin \theta d\theta d\phi dr, 0, 0, 0)$$

Parameter of $\zeta \rightarrow (t(\zeta), r(\zeta))$

Local thermodynamic equilibrium

$$f(x, p) = \frac{g_s}{2\pi} \left[Y^{-1} \exp\left(\frac{p_\mu u^\mu}{T}\right) \pm 1 \right]^{-1}$$

Fugacity factor:

$$Y \equiv \gamma_q^{N_q+N_{\bar{q}}} \gamma_s^{N_s+N_{\bar{s}}} \exp\left(\frac{\mu_B B + \mu_S S + \mu_I I_3}{T}\right)$$

(in this work we assume $\gamma_q = 1$)

- Integrating over the freeze-out hypersurface and phase-space gives back particle multiplicity
- Right sets of assumptions recover the original Siemens-Rasmussen and Schnedermann-Sollfrank-Heinz formulas
- But we assume Hubble-like expansion:
 $v(r) = \tanh(Hr)$

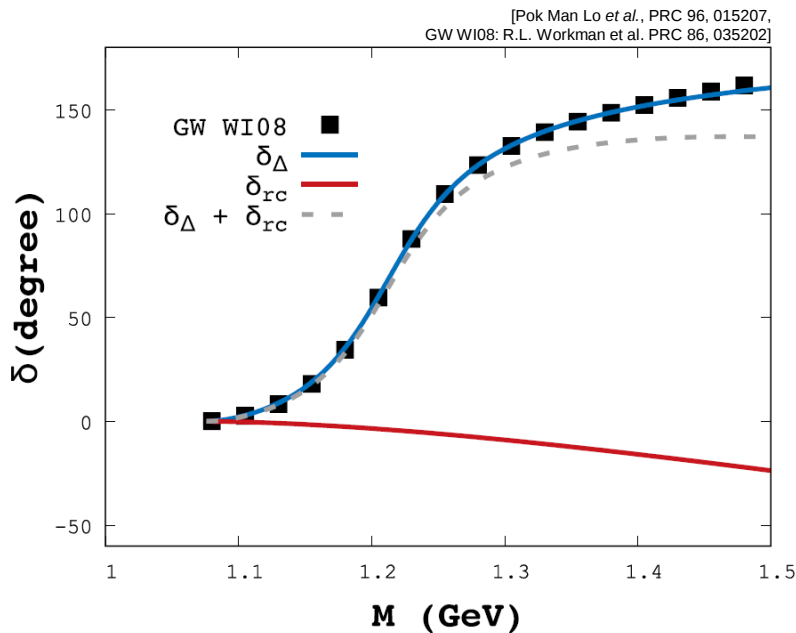
Resonance treatment

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345 (1969)
 R. Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718
 W. Weinhold, and B. Friman, Phys. Lett. B 433 (1998) 236
 Pok Man Lo, Eur. Phys. J. C77 (2017) no.8, 533

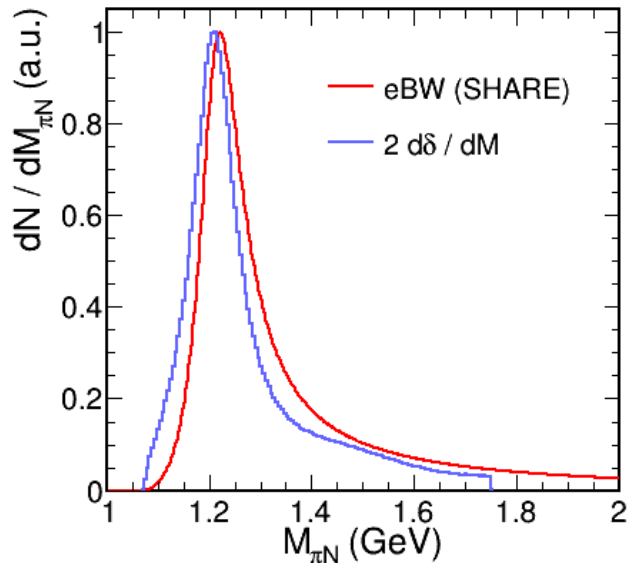


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πN phase shift in the P_{33} channel



Spectral function: $B_l(M) = 2 \frac{d}{dM} \delta_l$



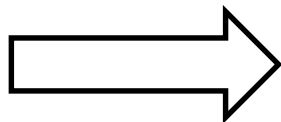
Thermal Event Generator (Therminator 2)

Ingredients of the method:

- Single (chemical and kinetic) freeze-out on a **spherically symmetric hypersurface** (Siemens-Rasmussen blast-wave model)
- Fix thermodynamic parameters with multiplicities of particles:
→ Solve numerically 6 equations for 6 parameters:

protons (incl. those bound in light nuclei): 124.1

π^+ : 9.3
 π^- : 17.1
 K^+ : 0.0598
 K^- : 0.00056
 Λ : 0.0822

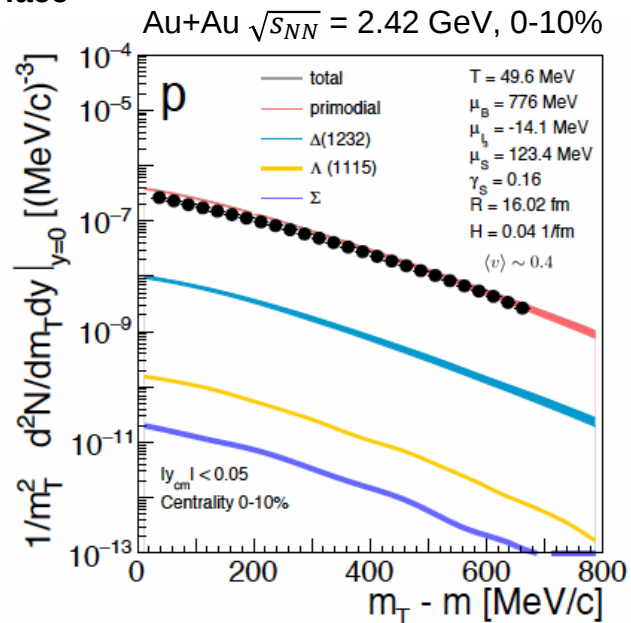


$T = 49.6$ MeV
 $\mu_B = 776$ MeV
 $\mu_{13} = -14.1$ MeV
 $\mu_S = 123.4$ MeV
 $\gamma_S = 0.16$
 $R = 16.02$ fm

- Proton m_t spectrum at mid-y is fitted to get the expansion velocity profile: $H = 0.04$ fm $^{-1}$
- Δ spectral function from πN phase shift

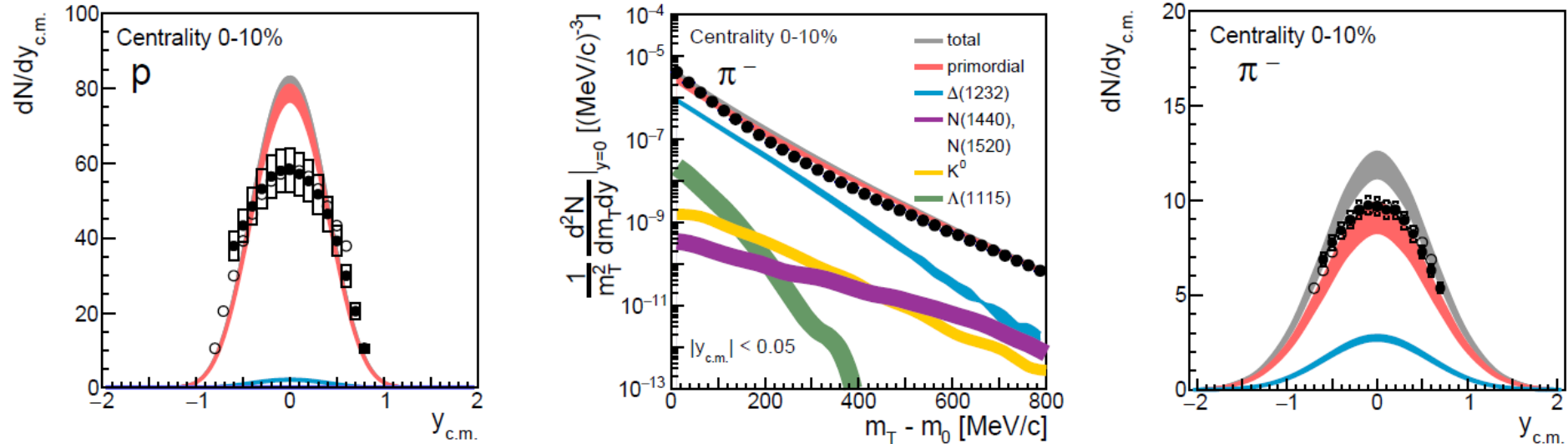
P.M. Lo *et al.*, PRC 96 (2017) 015207

M. Chojnacki *et al.*, Comput. Phys. Comm. 103 (2012) 746-773
SH, W. Florkowski, T. Galatyuk *et al.*, PRC 102 (2020) 5, 054903



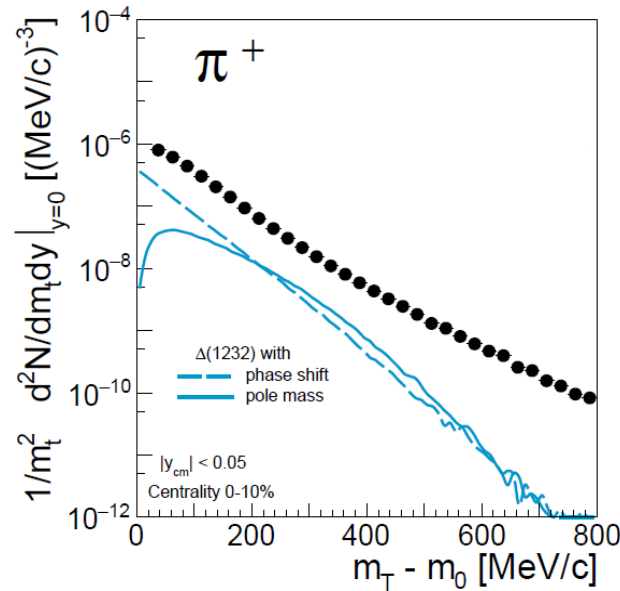
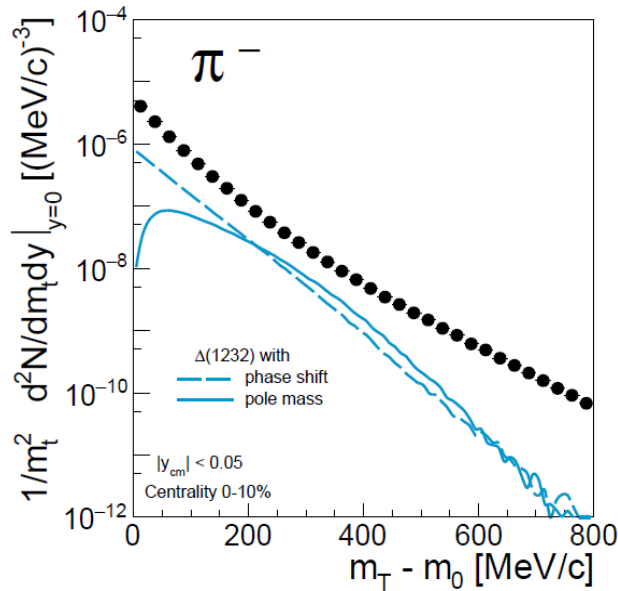
Spectra of bulk particles

Au+Au $\sqrt{s_{NN}} = 2.42$ GeV, 0-10%



- These spectra are **not fitted**, but **predicted** by the model
- Bands: uncertainty from errors on hadron yields
- For π^+ similar level of agreement as for π^-
- Pion slope at high m_T described with $T \sim 50$ MeV and Hubble
- Rapidity too narrow in the model
 - Spherical symmetry is not exactly fulfilled
 - Further improvements are ongoing

Influence of the Δ shape on pion spectra



Transverse mass of pions from Δ decay for different spectral functions:

- Δ with fixed mass of 1.232 GeV
- Spectral function from the πN phase shift in the P_{33} channel

Finite Δ width:
→ more pions at low m_t

Influence of the velocity profile

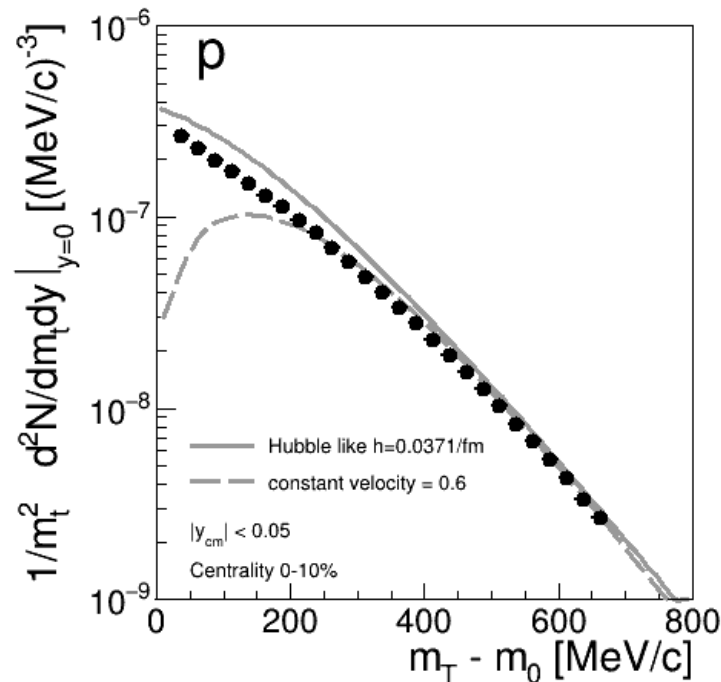
- Hubble-like fireball expansion:
 $v(r) = \tanh(Hr)$
- The parameter H fitted to the proton m_t spectra:
 $H = 0.037 \text{ fm}^{-1}$

- Mean value:

$$\langle v \rangle = \frac{2}{3}HR \left(1 - \frac{1}{5}H^2R^2 \right) \approx 0.4$$

- Best fit with constant velocity

- gives $\langle v \rangle = 0.6$
- fails to describe the data at low m_t

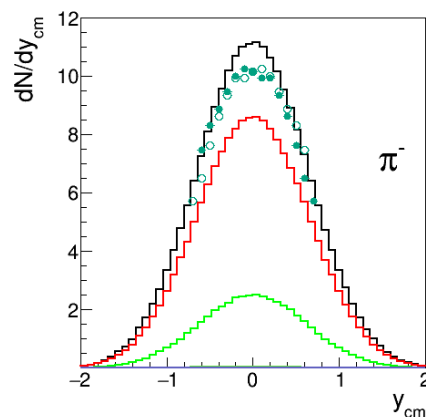
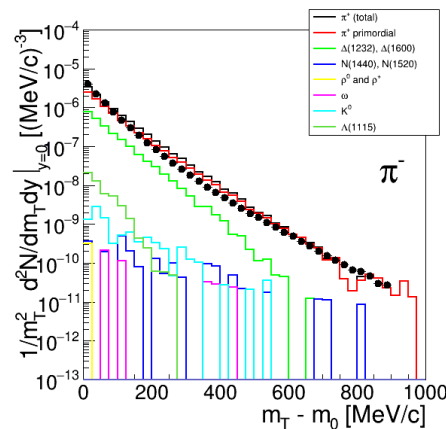
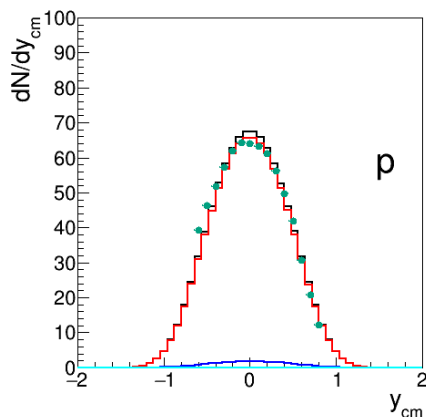
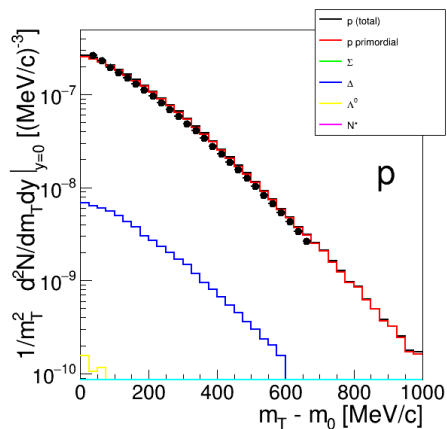


Outlook:

moving from spherical to spheroid symmetry

- Transverse momentum spectra are well described, and
- Rapidity spectra are too narrow compared to experiment
 - Expansion in longitudinal direction should be stronger than in transverse direction
- Guidance from dynamic models
 - Freeze-out hypersurface should be narrower in the longitudinal direction

Ongoing work on systematic fitting the shape parameters



Outlook:

Afterburner for final-state EM interaction

After the freeze-out, particles are propagated according to standard formulas:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{R}{(\mathbf{R} \cdot \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \mathbf{R} \times (\mathbf{u} \times \mathbf{a})]$$

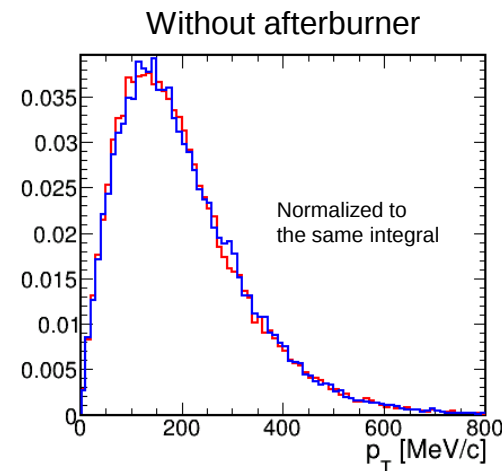
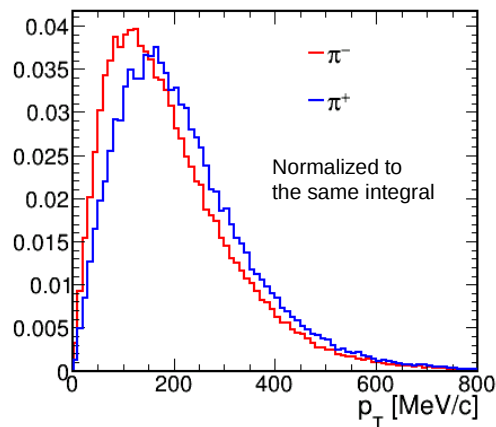
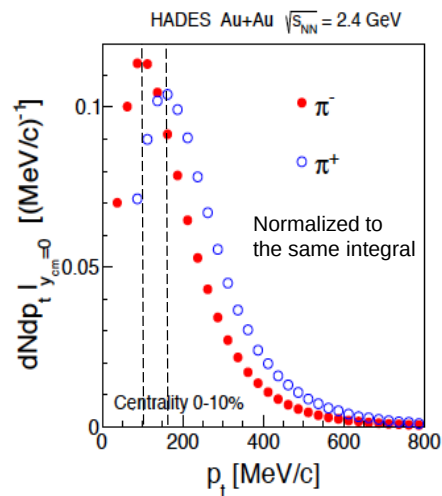
$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\mathbf{R} \equiv \mathbf{r} - \mathbf{w}(t_r),$$

$$\mathbf{u} \equiv c\hat{\mathbf{R}} - \mathbf{v},$$

$$\mathbf{v} \equiv \dot{\mathbf{w}}(t_r)$$

$$|\mathbf{r} - \mathbf{w}(t_r)| = c(t - t_r)$$



Conclusions

- Statistical hadronization model can describe not only multiplicities, but also spectra of bulk particles produced in heavy-ion collisions in $\sqrt{s_{NN}}$ of few GeV
- Ingredients:
 - Spherical, Siemens-Rasmussen-type fireball expansion
 - Hubble-like velocity profile
 - Sudden freeze-out
 - Careful treatment of baryonic resonances

Outlook:

- Spheroidal instead of spherical symmetry
- Final-state EM interactions
- HBT radii, nucleon coalescence, data from STAR fixed-target, FAIR, NICA...

THANK YOU FOR YOUR ATTENTION