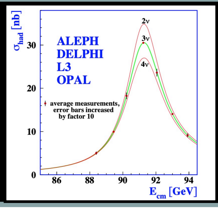
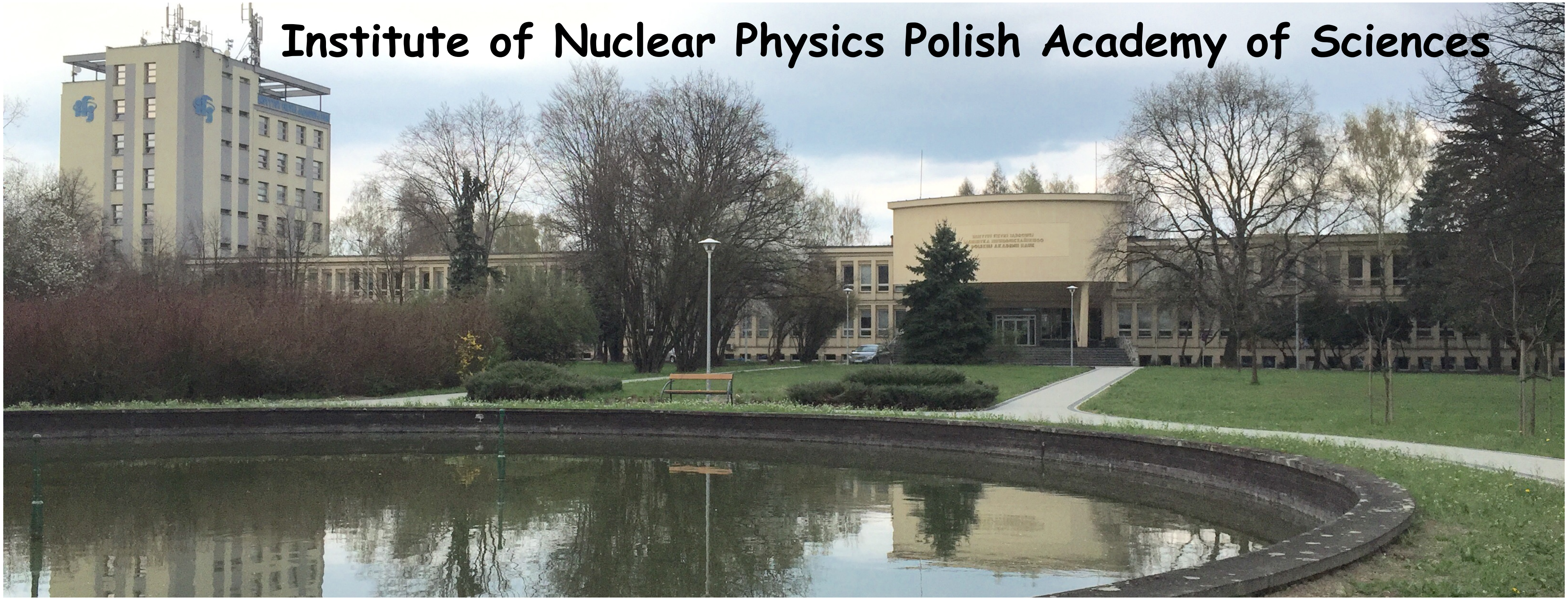


Invisible decays of Z boson: recent improvements and future

Stanisław Jadach

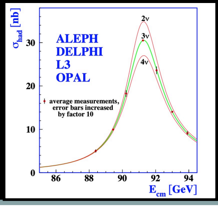
Based on **arXiv:1912.02067** (Phys. Letters B803) by Patrick Janot and S.J.

Institute of Nuclear Physics Polish Academy of Sciences



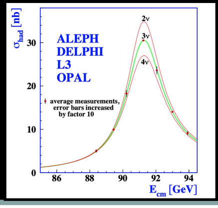
Presented at Institute of Physics AGH, March 06-th, 2020

Partly supported by the Polish National Science Center grant 2016/23/B/ST2/03927 and the CERN FCC Design Study Programme.



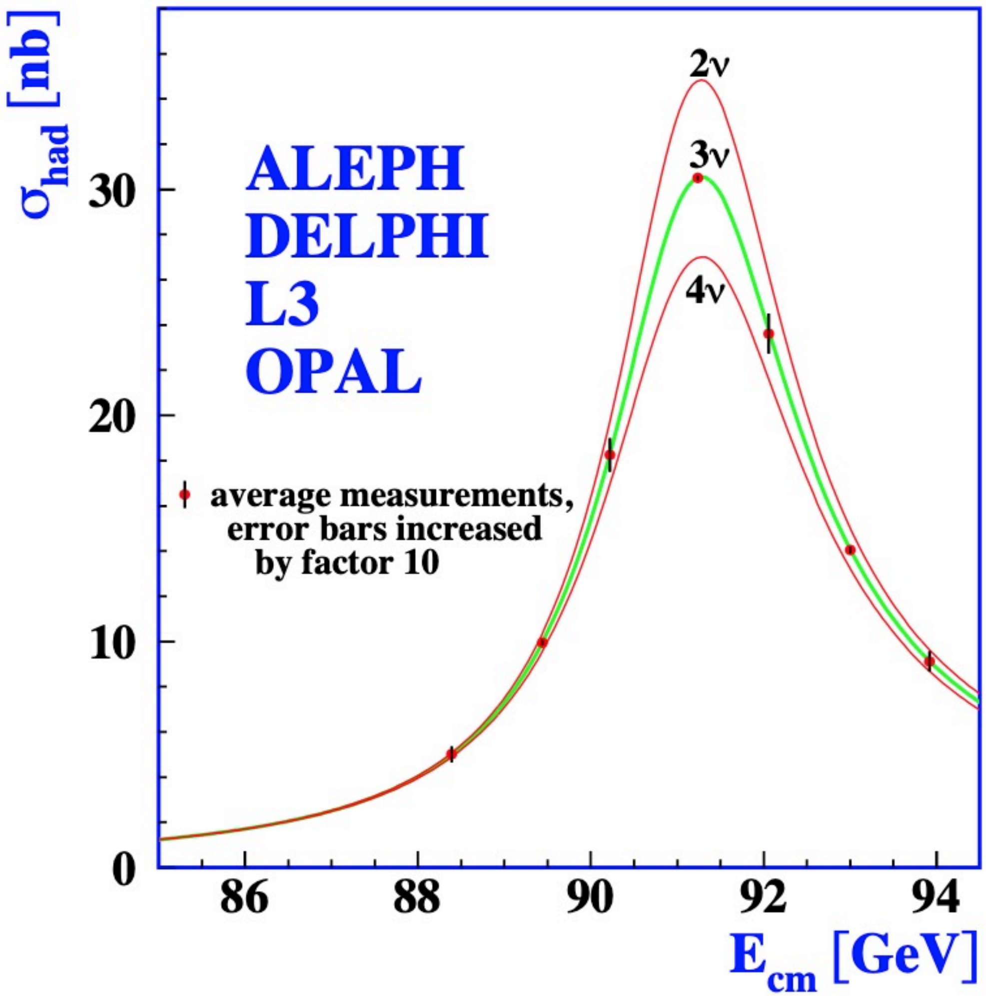
Outline

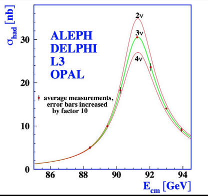
1. Neutrino counting at LEP (ADLO 2005)
2. Improvement 2020 by P.Janot and S.J.
3. Prospects at FCC-ee



Invisible Z width from Z peak cross section at LEP

Very simple basic idea:





Effective no. of neutrinos N_ν parametrises invisible Z decay width $\Gamma_{\nu\nu}$

Combination of measurements of four LEP experiments(2005):

$$N_\nu = 2.9840 \pm 0.0082$$

$\sim 2\sigma$ deviation from the SM value $N_\nu = 3$

N_ν was determined using formula exploiting best measured observables:

$$N_\nu \left(\frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}\right)_{\mathrm{SM}} = \left(\frac{12\pi}{m_Z^2}\frac{R_\ell^0}{\sigma_{\mathrm{had}}^0}\right)^{\frac{1}{2}} - R_\ell^0 - 3 - \delta_\tau$$

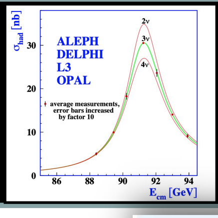
σ_{had}^0= hadronic cross section at the Z peak upon removing QED effects: almost 100% detector acceptance, **luminosity error dominates**.

R_ℓ^0 = ratio of hadronic to leptonic branchings: just ratio of event rates easy to measure, QED corrected.

$(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{\mathrm{SM}} = 1.99125 \pm 0.00083$ Standard Model prediction

$\delta_\tau \simeq -2.263 \times 10^{-3} \pm 0$, well known correction due to finite τ lepton mass

Lumi error $\delta\mathscr{L}/\mathscr{L} \simeq 6.1 \cdot 10^{-4}$ propagates into dominant $\delta N_\nu \simeq 7.5\,(\delta\mathscr{L}/\mathscr{L}) \simeq 0.0046$.

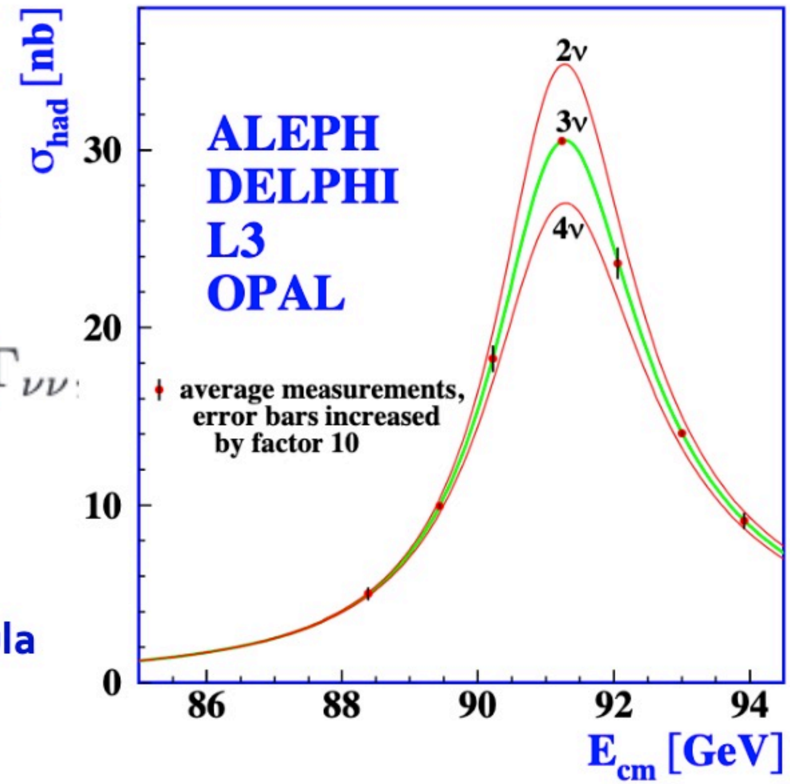


Quick derivation (P. Janot)

- Hadronic peak cross section σ^0_{had} is most sensitive to N_ν
  - Number is about 3, but not quite

$$N_\nu = 2.9840 \pm 0.0082$$

  - 2σ deficit : Dark matter, RHν, … ?



- Practically

$$\Gamma_Z = \underbrace{\Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau}}_{\sim 3+\delta\tau} + \Gamma_{had} + N_\nu \Gamma_{\nu\nu}$$

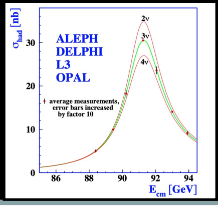
  - Divide by $\Gamma_{\ell\ell}$ ~3+δτ $R^0_l = \frac{\Gamma_{had}}{\Gamma_{ll}}$
  - Take $\Gamma_Z / \Gamma_{\ell\ell}$ from Breit-Wigner formula

$$\sigma^0_{had} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}$$

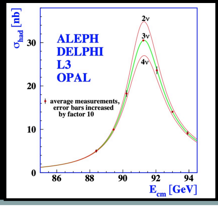
  - And get to the expression

$$N_\nu \left(\frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}\right)_{SM} = \left(\frac{12\pi}{m_Z^2}\frac{R^0_\ell}{\sigma^0_{had}}\right)^{\frac{1}{2}} - R^0_\ell - 3 - \delta_\tau$$

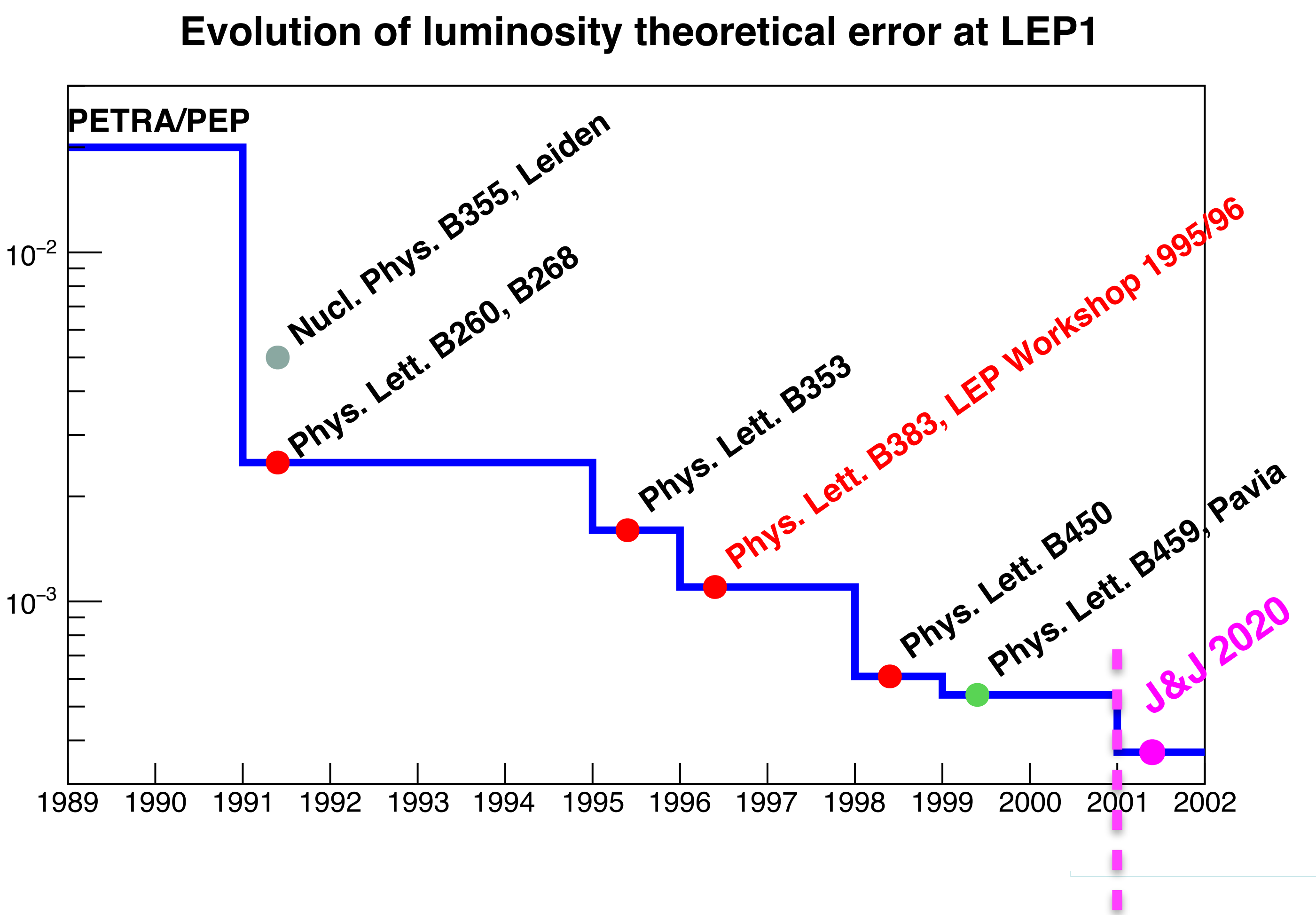
$\left(\frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}\right)_{SM}$ → 1.99125±0.00083 δ_τ → 0.003

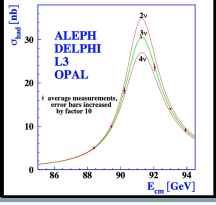


A Little bit of history: Theoretical error of the luminosity at LEP era and its composition



Milestones in LEP lumi TH error





LEP lumi TH milestones

Main TH precision improvements marked in red:

[1] W. Beenakker, F. A. Berends and S. C. van der Marck, Nucl. Phys. B **355** (1991) 281.
Photonic $\mathcal{O}(\alpha^2 L_e^2)$ and vacuum polarization (VP)

[2] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **253** (1991) 469.
Technical precision 0.02% establishe for the "baseline" $\mathcal{O}(\alpha^1)$ MC

[3] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **260** (1991) 438.
First reliable estimate of the precision of $\mathcal{O}(\alpha^1)_{expon}$ multiphoton BHLUMI MC

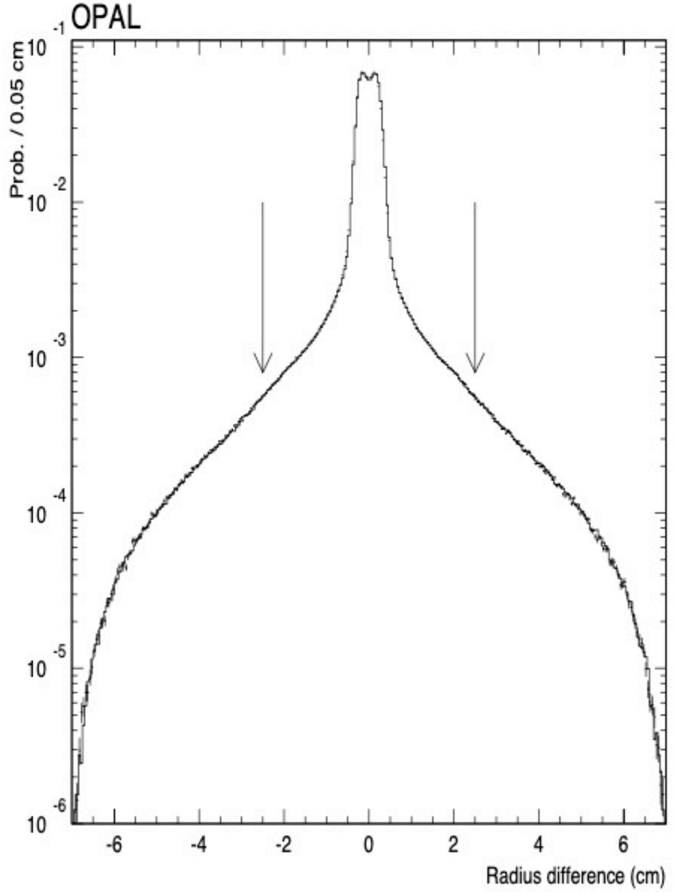
[4] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].
Inclusion of $\mathcal{O}(\alpha^2 L_e^2)$ and $\mathcal{O}(\alpha^3 L_e^3)$, new estimate of $\mathcal{O}(\alpha^2 L)$

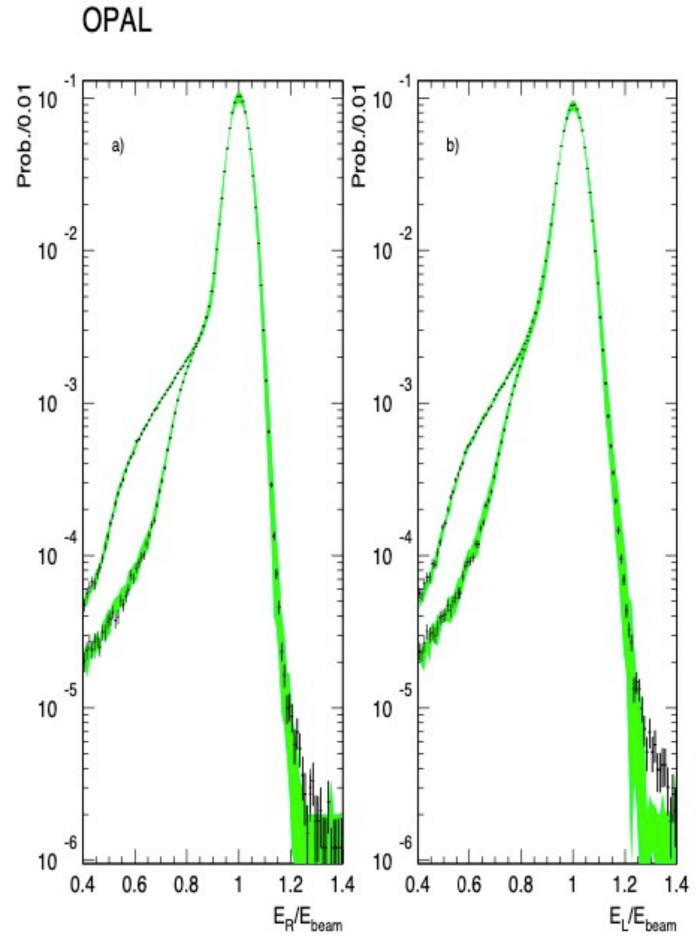
[5] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238
New estimate of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[6] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262
New calculation of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

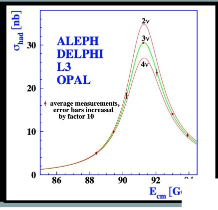
[7] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649
New calculation of missing light real and virtual pairs

Extraordinary agreement of OPAL data vs. BHLUMI MC, collinearity and energy distrs.



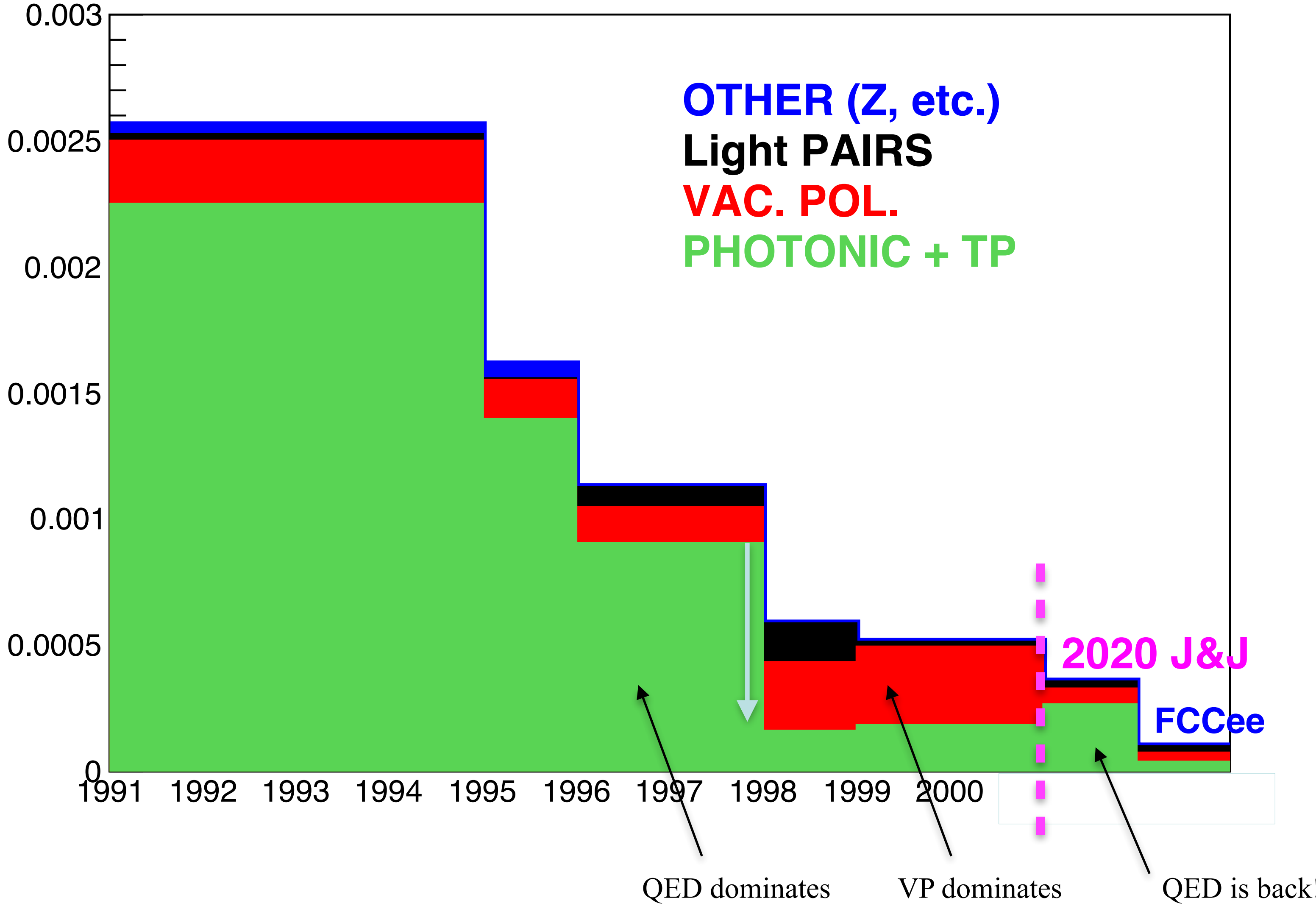


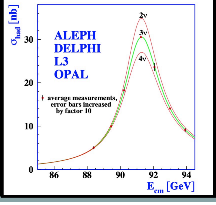
BHLUMI 4.04 Monte Carlo was used by all four LEP experiments. Not only controls luminosity normalization $d\sigma/\sigma = 0.06\%$, but also perfectly agrees with all experimental spectra, with NO “TUNING” to experimental data! (Only one bug in 1995.)



Evolution of components in TH lumi (BHLUMI) error

Components of luminosity theoretical error at LEP1





Recent revival of the interest in Bhabha luminometry and N_ν measurements

Physics Letters B 790 (2019) 314–321

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The path to 0.01% theoretical luminosity precision for the FCC-ee ☆

S. Jadach [a,*], W. Płaczek [b], M. Skrzypek [a], B.F.L. Ward [c,d], S.A. Yost [e]

Physics Letters B 800 (2020) 135068

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Beam-beam effects on the luminosity measurement at LEP and the number of light neutrino species

Georgios Voutsinas [a], Emmanuel Perez [a], Mogens Dam [b], Patrick Janot [a,*]

Physics Letters B 803 (2020) 135319

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Physics Letters B

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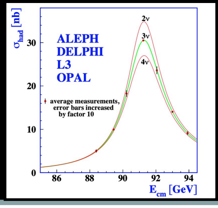
Improved Bhabha cross section at LEP and the number of light neutrino species ☆

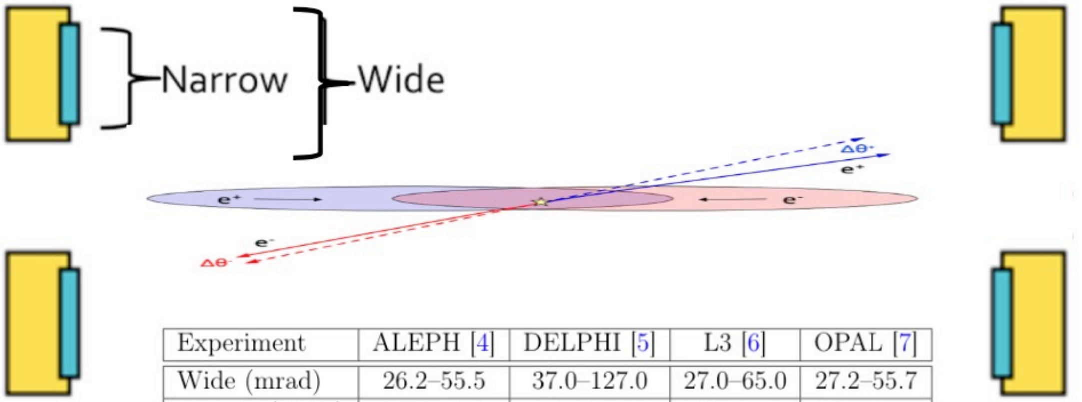
Patrick Janot [a,*], Stanisław Jadach [b]

FCC-ee oriented studies have triggered new interest in re-analysing LEP data



Beam-induced effect on N_ν (qualitatively)

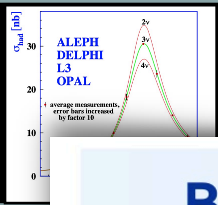
- Luminosity measured from the low-angle Bhabha rate $e^+e^- \rightarrow e^+e^-$



Experiment	ALEPH [4]	DELPHI [5]	L3 [6]	OPAL [7]
Wide (mrad)	26.2–55.5	37.0–127.0	27.0–65.0	27.2–55.7
Narrow (mrad)	30.4–49.5	44.9–113.6	32.0–54.0	31.3–51.6

- Outgoing $e^\pm$ focused by the Lorentz force from the opposite bunch
  - May miss the acceptance
    - → Reduces the accepted Bhabha cross section (wrt published values)
    - → Increases the integrated luminosity (wrt to published values)
    - → Decreases the measured peak cross section (wrt to published values)

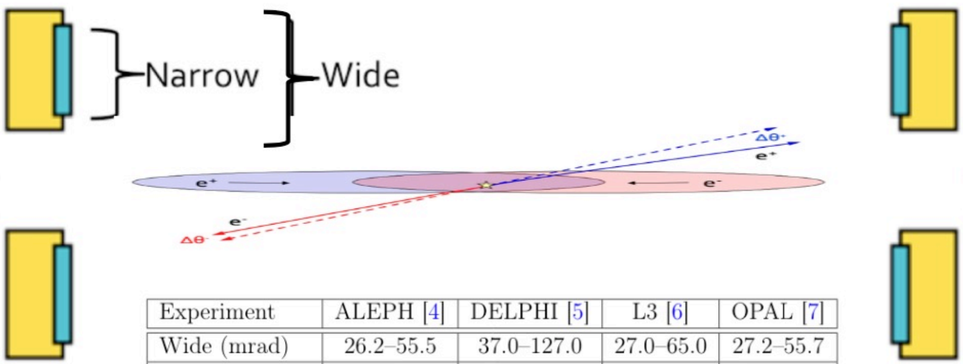
      Note: R_ℓ is not affected by luminosity changes
    - → Increases the number of light neutrino species (wrt …)



Beam-beam effect in LEP data

Beam-induced effect on N_ν (qualitatively)

- Luminosity measured from the low-angle Bhabha rate $e^+e^- \to e^+e^-$



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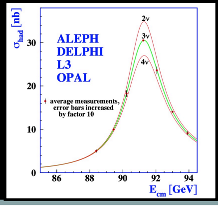
- Outgoing $e^\pm$ focused by the Lorentz force from the opposite bunch
  - May miss the acceptance

$$\delta N_\nu \simeq -(7.465 \pm 0.005) \times \left.\frac{\Delta L}{L}\right|_{91.2\,\mathrm{GeV}} \simeq 0.00720 \pm 0.00041$$

- A more recent / precise prediction of $(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{SM}$ adds 0.00063
  - And reduces very slightly the uncertainty on N_ν

$$N_\nu = 2.9840 \pm 0.0082 \quad \Rightarrow \quad N_\nu = 2.9918 \pm 0.0081$$

Patrick Janot



New study by P.Janot and S.J.

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319

Correcting LEP 1990-95 data for low angle Bhabha of ALEPH, DELPHI, L3 and OPAL for realistic event selection, emulating all eight variants of small angle Bhabha LEP detectors.

MC programs used by LEP collaborations

	ALEPH	DELPHI	L3	OPAL
1990	BABAMC (0.320%)	2.01 (0.300%)	2.01 (0.250%)	2.01 (0.300%) later scaled to 4.04
1991-92	2.01 (0.210%)			
Fall 92	2.01 (0.160%)			
1993	4.04 (0.061%)	4.02 (0.170%)	4.04 (0.061%)	4.04 (0.054%)
1994-95		4.03 (0.061%)		

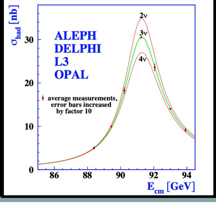
Eight luminosity detectors used at LEP

Expt / LumiCal	Period	Narrow (mrad)	Wide (mrad)
ALEPH LCAL [5]	01/90 → 08/92	57 – 107	43 – 125
DELPHI SAT [6, 7]	01/90 → 12/93	56.0 – 128.6	52.7 – 141.8
L3 BGO [8]	01/90 → 12/92	31.2 – 65.2	25.2 – 71.2
OPAL FD [9]	01/90 → 12/92	65.0 – 105.0	55.0 – 115.0
ALEPH SiCAL [10]	09/92 → 12/95	30.4 – 49.5	26.1 – 55.9
DELPHI STIC [11]	01/94 → 12/95	43.6 – 113.2	37.2 – 126.8
L3 SLUM [12]	01/93 → 12/95	32.0 – 54.0	27.0 – 65.0
OPAL SiW [13]	01/93 → 12/95	31.3 – 51.6	27.2 – 55.7

Bhabha event selections in LEP

Experiment	ALEPH [16]	DELPHI [17, 18, 35]	L3 [8, 36]	OPAL [9, 37, 19]
$E_{1,2}^{min}/E_{beam}$ $E_{1,2}^{max}/E_{beam}$	> 0.44	> 0.65	> 0.40 > 0.80	> 0.45 (→ 92) > 0.50 (93 →)
$\frac{(E_1+E_2)}{2E_{beam}}$	> 0.60 (→ 93) > 0.78 (in 94) > 0.84 (in 95)	–	–	> 0.67 (→ 92) > 0.75 (93 →)
$\Delta\phi$ (mrad)	< 175 (→ 8/92) < 525 (9/92 →)	< 350	< 175	< 350 (→ 92) < 200 (93 →)
$\Delta\theta$ (mrad)	–	–	–	– (→ 92) < 10 (93 →)

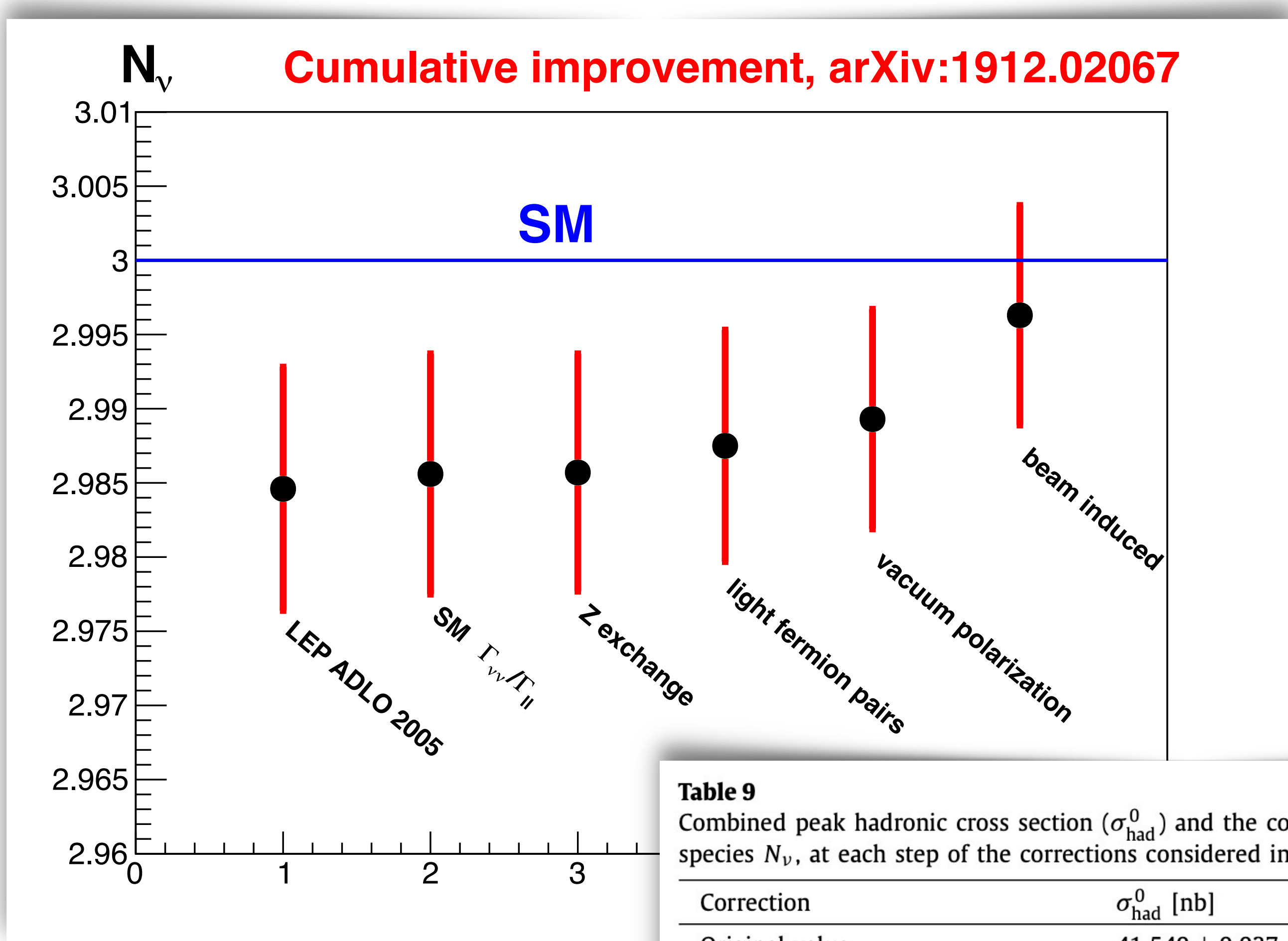
All the above was implemented in the mixture of new C++ and old F77 code on top of BHLUMI 4.x
Corrections due to improvements in the BHLUMI matrix element were calculated using MC samples of 160M events generated at seven energies $\sqrt{s}$ = 88.471, 89.444, 90.216, 91.227, 91.959, 93.00, 93.710 GeV.



New study by P.Janot and S.J.

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319

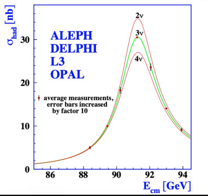
including beam-beam VPDJ effect



Correcting
LEP 1990-95 data for
low angle Bhabha
of ALEPH, DELPHI, L3
and OPAL

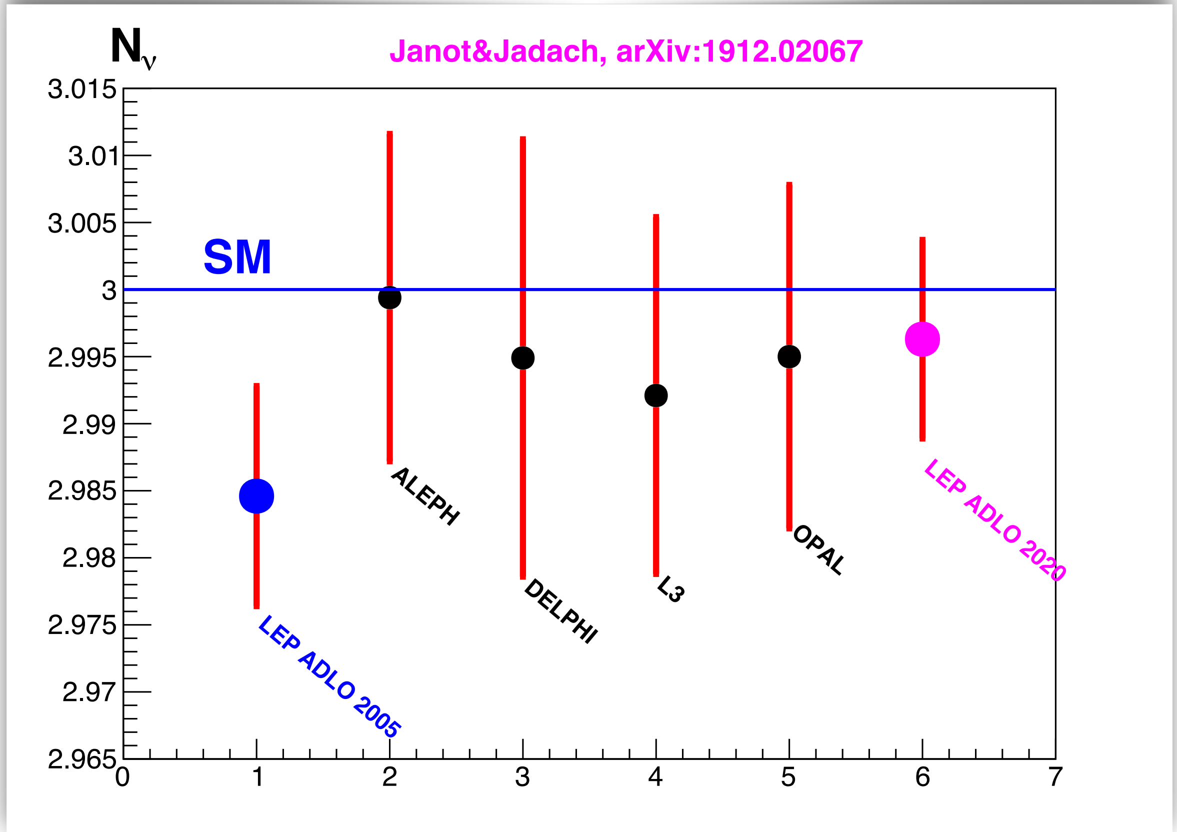
Table 9
Combined peak hadronic cross section (σ^0_{had}) and the corresponding number of light neutrino species N_ν, at each step of the corrections considered in this letter.

Correction	σ^0_{had} [nb]	N_ν
Original value	41.540 ± 0.037	2.9846 ± 0.0082
New $(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{SM}$	41.5400 ± 0.0372	2.9856 ± 0.0081
Z exchange	41.5390 ± 0.0369	2.9857 ± 0.0080
Light fermion-pairs	41.5292 ± 0.0353	2.9875 ± 0.0078
Vacuum polarization	41.5196 ± 0.0324	2.9893 ± 0.0074
Beam-induced	41.4802 ± 0.0325	2.9963 ± 0.0074



New study by P.Janot and S.J. including beam-beam effect of Voutsinas+Perez+Dam+Janot

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319



Old ADLO 2005 value

2.9840 ± 0.0082

Collaborations-wise

ALEPH : $N_\nu = 2.9994 \pm 0.0122,$

DELPHI : $N_\nu = 2.9949 \pm 0.0163,$

L3 : $N_\nu = 2.9921 \pm 0.0133,$

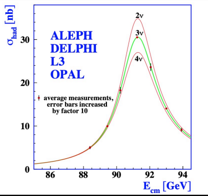
OPAL : $N_\nu = 2.9950 \pm 0.0128,$

Combined new

$$N_\nu = 2.9963 \pm 0.0074,$$

$$\sigma^0_{\mathrm{had}} = 41.4802 \pm 0.0325\,\mathrm{nb},$$

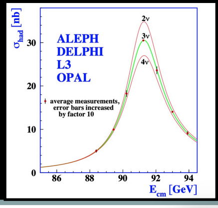
$$\Gamma_Z = 2.4955 \pm 0.0023\,\mathrm{GeV}.$$



New study by P.Janot and S.J.

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319

A litle bit of details

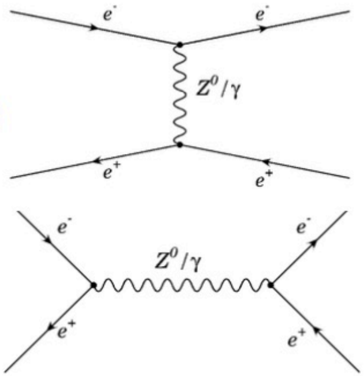


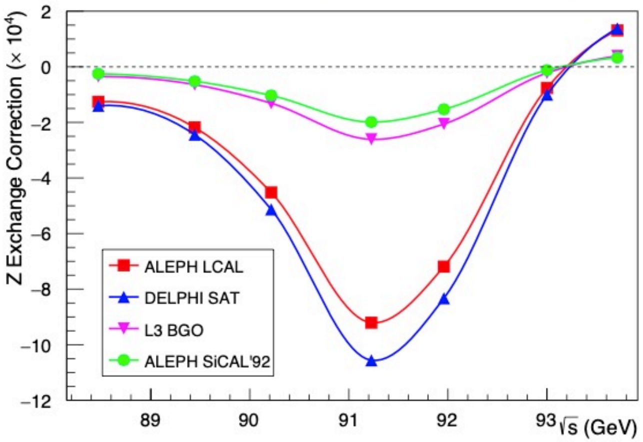
Update of Z-exchange correction

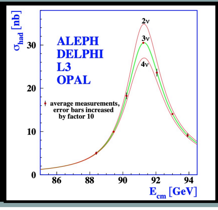
QED corrections to Z exchange

- QED corrections to Z-exchange were not state of the are until 1994
  - BHLUMI 2.01 contained Born-level diagrams
  - O(α) corrections of the order of 50% (implemented in BABAMC)
    - Need higher-orders, implemented in only BHLUMI 4.04
      - → Early LEP data (until 1992) need to be corrected

        Except for OPAL (correction already included)
  - Higher-orders reduce the Bhabha cross section until 1992:







Vacuum polarization

- Appears in t-channel photon propagator
  - Dressed with a loop of lepton or quarks
    - Hadronic part obtained from $\sigma(e^+e^- \to \text{hadrons})$
    - Recent progress from huge data samples with ISR
      - ➔ At B and Φ factories
  - Recent code made available by Fred Jegerlehner in 2019
    - Uncertainty reduced by a factor 4
    - Bhabha cross section reduced too
      - ➔ Reduction in units of 10^{-4} :

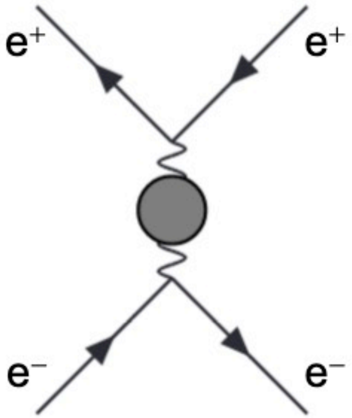
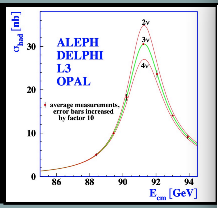


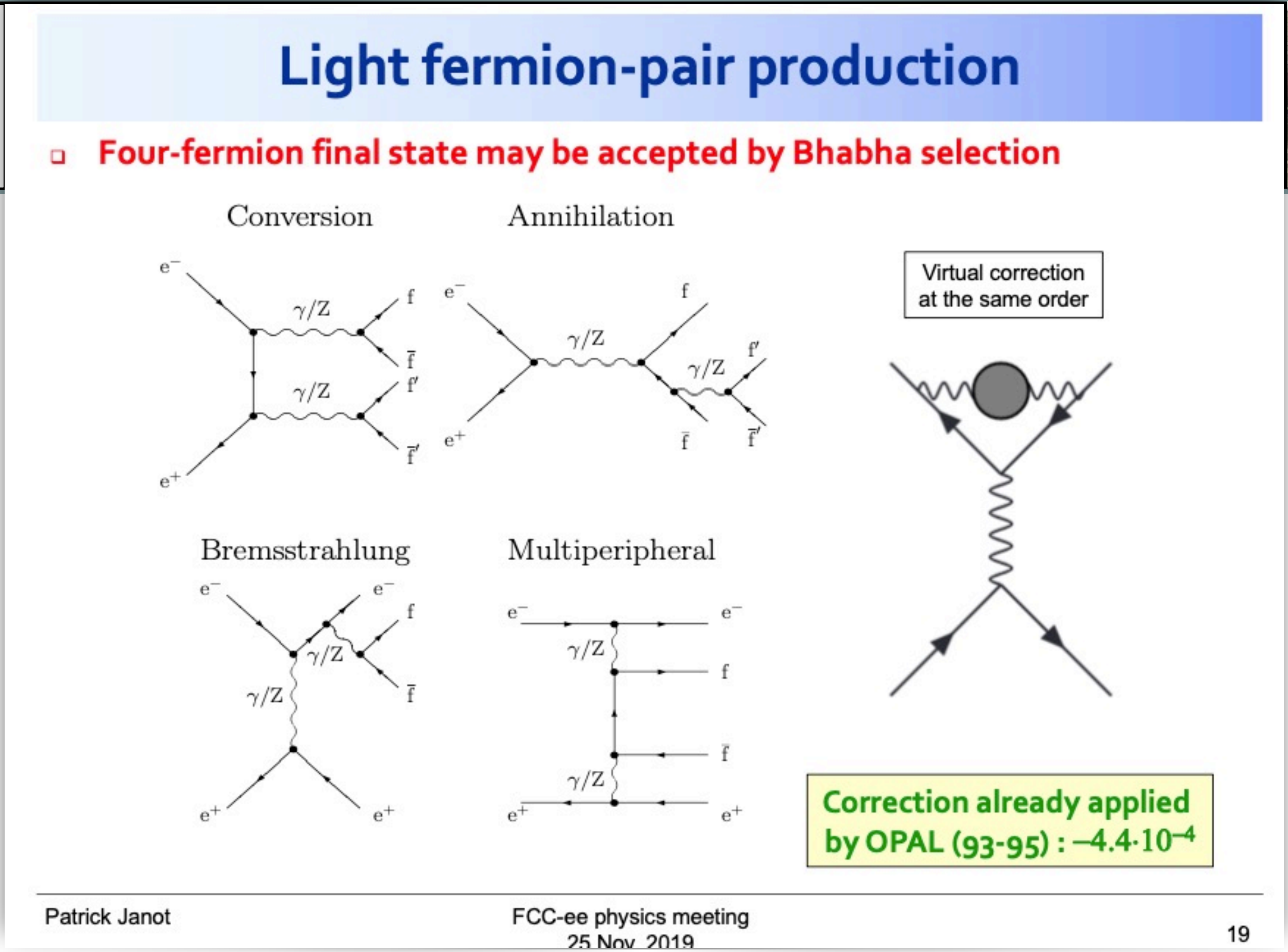
Table 6

Vacuum polarization correction relative to the Bhabha cross section at the Z peak (switching off the Z boson contribution) in the LumiCals of the four LEP experiments, as a function of time. Each entry value, expressed in units of 10^{-4}, corresponds to the change between the 2019 evaluation of the vacuum polarization in the t-channel photon propagator, and the default function proposed in the BHLUMI versions used in the LEP experiments' publications at the time. When relevant, the entries also include non-soft $\mathcal{O}(\alpha^2 L_e^2)$ corrections implemented only in BHLUMI 4.0. Statistical uncertainties are negligible and are not quoted in the table. The numbers display the quasi-linear excursion of the correction when varying $\sqrt{s}$ from 88.471 GeV (subscript) to 93.710 GeV (superscript).

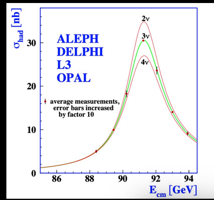
<table>
<tr><th>Experiment</th><th>ALEPH</th><th>DELPHI</th><th>L3</th><th>OPAL</th></tr>
<tr><td>01/90 → 08/92</td><td>$-2.00^{-0.18}_{+0.21}$</td><td rowspan="2">$-1.02^{-0.18}_{+0.18}$</td><td rowspan="2">$+1.57^{-0.09}_{+0.10}$</td><td rowspan="2">$-4.60^{-0.03}_{+0.04}$</td></tr>
<tr><td>09/92 → 12/92</td><td>$+1.22^{-0.12}_{+0.12}$</td></tr>
<tr><td>01/93 → 12/93</td><td rowspan="3">$-2.12^{-0.08}_{+0.09}$</td><td>$-4.62^{-0.06}_{+0.07}$</td><td rowspan="3">$-2.36^{-0.09}_{+0.11}$</td><td rowspan="3">$-2.24^{-0.09}_{+0.10}$</td></tr>
<tr><td>01/94 → 12/94</td><td rowspan="2">$-3.86^{-0.11}_{+0.12}$</td></tr>
<tr><td>01/95 → 12/95</td></tr>
</table>

Vacuum polarisation of Fred Jegerlehner (2019) was cross-checked with the private vacuum-polarisation code from the DHMZ team (Michel Davier, Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang) and the KNT team (Alexander Keshavarzi, Daisuke Nomura, Thomas Teubner)





- Only OPAL corrected its data. Others included fermion pair corrs. in the error budget.
- Now we do the same as OPAL for all luminosity LEP data.
- Real pair emissions using FERMISV MC (Leiden) and KORALW MC (Krakow)
- Virtual pair corrs. added using S. Actis et.al. Phys.Rev.D78 (2008) 085019.



Light fermion pairs

Table 7: Light fermion-pair correction relative to the Bhabha cross section selected by the LumiCals of the four LEP experiments, as a function of time. Entry values sum up real and virtual corrections, and are expressed in units of 10^{-4}. The corrections are found to be independent of the centre-of-mass energy (within statistical uncertainties). The indicated uncertainties combine statistical uncertainties and an estimate of the systematic effect of the detector granularity and the clustering procedure. The latter is taken to be equal to the half the difference between the correction obtained with the detector/clustering emulation and that obtained with the electron and positron exact energies and directions.

<table>
<tr><th>Experiment</th><th>ALEPH</th><th>DELPHI</th><th>L3</th><th>OPAL</th></tr>
<tr><td>$01/90 \to 08/92$</td><td>-3.58 ± 0.06</td><td rowspan="3">-4.99 ± 0.06</td><td rowspan="2">-3.43 ± 0.04</td><td rowspan="2">-4.51 ± 0.09</td></tr>
<tr><td>$09/92 \to 12/92$</td><td rowspan="2">-3.00 ± 0.06</td></tr>
<tr><td>$01/93 \to 12/93$</td><td rowspan="3">-3.77 ± 0.07</td><td rowspan="3">-4.72 ± 0.17
(-4.40 already applied in [13])</td></tr>
<tr><td>$01/94 \to 12/94$</td><td>-3.52 ± 0.08</td><td rowspan="2">-3.91 ± 0.05</td></tr>
<tr><td>$01/95 \to 12/95$</td><td>-4.38 ± 0.08</td></tr>
</table>

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- Real pair emissions using FERMISV MC (Leiden) and KORALW MC (Krakow)
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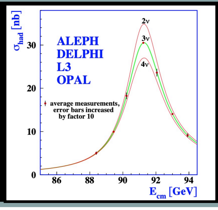


Summary on new LEP luminosity TH error

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319

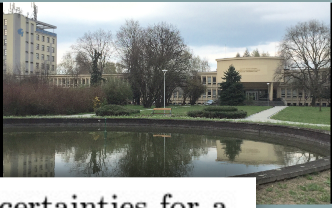
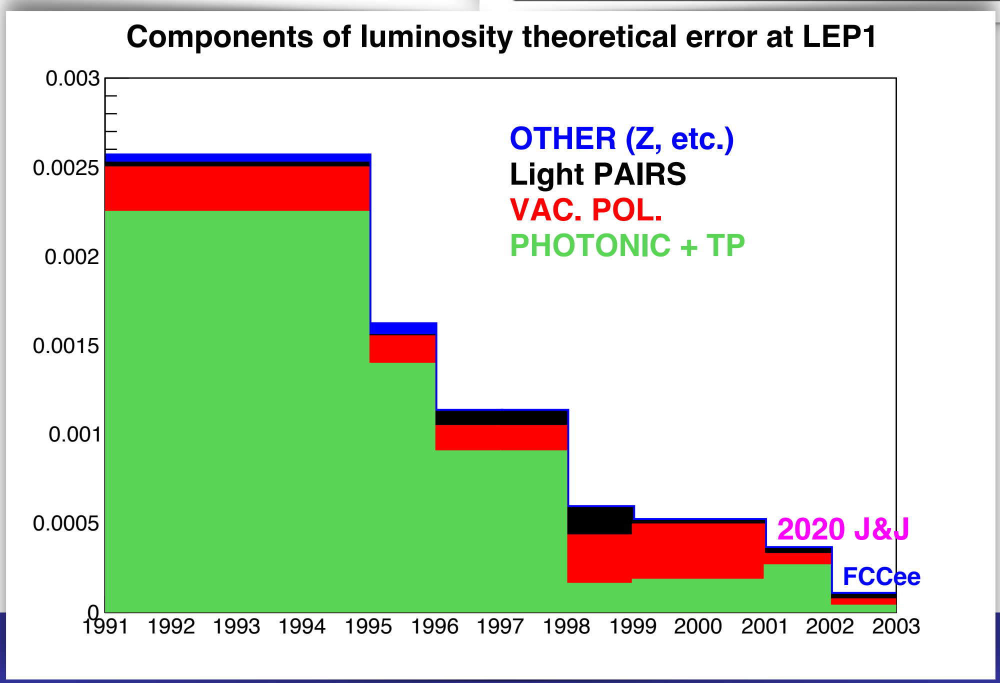
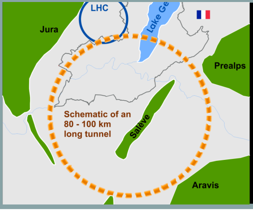


Table 3: Inspired from Refs. [28, 29, 25]: Summary of the theoretical uncertainties for a typical LEP luminosity detector covering the angular range from 58 to 110 mrad (first generation) or from 30 to 50 mrad (second generation). The total uncertainty is the quadratic sum of the individual components.

LEP Publication in:	1994		2000		2019	
LumiCal generation	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}
Photonic $\mathcal{O}(\alpha^2 L_e)$	0.15%	0.15%	0.027%	0.027%	0.027%	0.027%
Photonic $\mathcal{O}(\alpha^3 L_e^3)$	0.09%	0.09%	0.015%	0.015%	0.015%	0.015%
Z exchange	0.11%	0.03%	0.09%	0.015%	0.090%	0.015%
Vacuum polarization	0.10%	0.05%	0.08%	0.040%	0.015%	0.009%
Fermion pairs	0.05%	0.04%	0.05%	0.040%	0.010%	0.010%
Total	0.25%	0.16%	0.13%	0.061%	0.100%	0.037%



Pure QED dominates again as before 1998



Low angle Bhabha (luminosity) at FCCee

arXiv:1902.05912

- LEP legacy, lumi TH error budget

	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $O(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $O(\alpha^3 L^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°-3° (18-52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°-6°. Total uncertainty is taken in quadrature. Technical precision included in (a).

LEP lumi update 2018

Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% [25]
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) s-channel Z-exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038%

- By the time of FCC-ee VP contribution will be merely 0.006%
- QED corrections and Z contrib. come back to front!
- Z contr. easy to master, even if rises at FCC-ee, because (28-58)->(64-86) mrad.

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

- Our FCC-ee forecast is 0.01% provided QED m.e. and VP are improved.

Z invisible width from peak cross section and radiative return

Present (LEP) **FCC-ee**

Peak cross section

QED err. of luminosity $\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\sigma^0_{had}}{\sigma^0_{had}} \simeq 0.06\,\%$

dominates LEP exp. error $N_\nu \simeq 2.984 \pm 0.008\ \{\pm 0.006\}_{QED}$

$$R^0_{\rm inv} = \left(\frac{12\pi R^0_\ell}{\sigma^0_{\rm had} m^2_{\rm Z}}\right)^{\frac{1}{2}} - R^0_\ell - (3+\delta_\tau)\,, \qquad R^0_{\rm inv} = N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\ell\ell}}\right)_{\rm SM}.$$

FCC-ee exp. error (syst.) $\delta N_\nu \simeq 0.001$

Factor ~10 improvement in luminosity is needed!

$\frac{\delta\mathcal{L}}{\mathcal{L}} \simeq 10^{-4} \;\rightarrow\; \delta N_\nu \simeq 8\cdot 10^{-4}$ seems achievable.

Radiative return I

$e^+e^- \rightarrow \nu\bar{\nu}\gamma$

$N_\nu \simeq 2.69 \pm 0.15\ \{\pm 0.06\}_{QED}$

Limited by poor LEP statistics at 161GeV

Expected FCC-ee exp. error of $\sigma_{\nu\bar{\nu}\gamma}$ not yet established,
most likely: $\delta\sigma/\sigma \simeq 0.03\,\% \;\rightarrow\; \delta N_\nu \simeq 0.001$

Future luminosity error 0.01% looks ok.

Estimate of h.o. QED effects using KKMC
is merely 0.02% (unpublished).

Altogether $\delta N_\nu \simeq 0.001$ seems achievable:)
(Factor ~60 improvement in QED rather easy.)

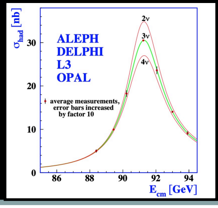
Radiative return II

Measuring ratio $R = \frac{\sigma_{\nu\bar{\nu}\gamma}}{\sigma_{\mu^+\mu^-\gamma}}$

Luminosity error drops out!

QED uncertainty due to FSR in $\sigma_{\mu^+\mu^-\gamma}$ rated at 0.03%
(unpublished study using KKMC).

Again $\delta N_\nu \simeq 0.001$



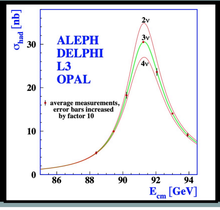
Summary

- **LEP data on luminosity and Z peak cross section are corrected using improved hadronic vacuum polarization, Z-exchange and light pair corrections.**

- **Invisible Z decay width derived from Z peak cross agrees now much better with the SM — 20 years old 2 sigma discrepancy is gone.**

- **Factor 4 or even more precise measurement of Z invisible width will be possible in FCC-ee collider.**

*This work is partly supported by the Polish National Science Center grant 2016/23/B/ST2/03927 and the CERN FCC Design Study Programme.



σhad [nb]
ALEPH
DELPHI
L3
OPAL
2ν
3ν
4ν
average measurements,
error bars increased
by factor 10
0
10
20
30
86
88
90
92
94
Ecm [GeV]

?