Bound state a in thermal gas

Francesco Giacosa
Jan Kochanowski U Kielce – Goethe U Frankfurt

In collaboration with Subhasis Samanta (UJK Kielce)

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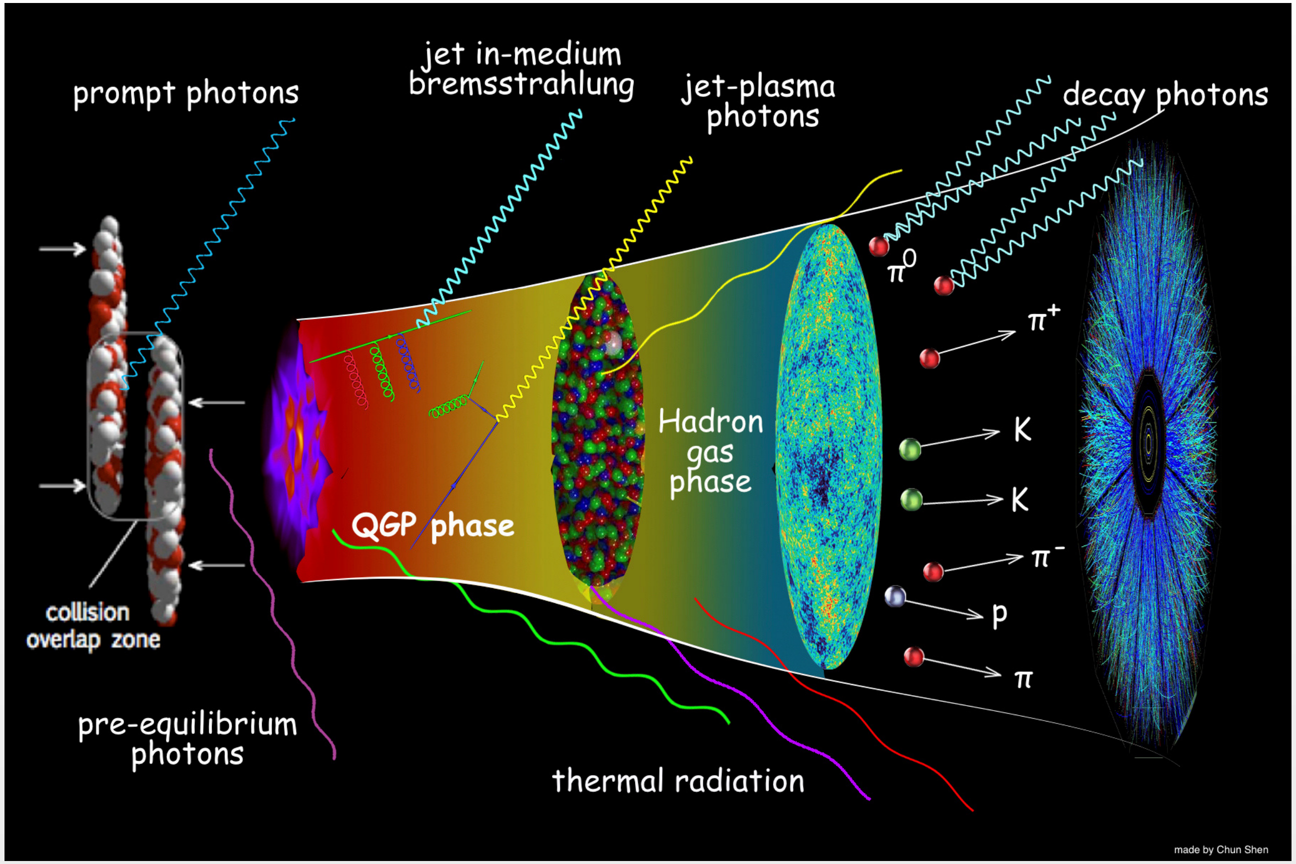
Outline

1. Thermal gas: recall + resonances and phase-shift formula
2. Lee Hamiltonian: a QM model for QFT
   (using a simple example: the rho meson)
3. Two resonances: $f_0(500)$ and $K_0^*(700)$ at nonzero T
4. Bound state at nonzero temperaturę in QFT
5. Conclusions

Part 1: thermal gas + resonances

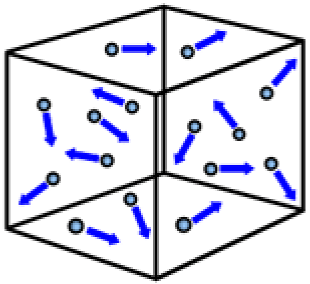
Heavy-ion collisions



At the freeze-out, the emission of hadrons is well described by thermal models.
Question: how to include resonances, such as the rho meson?
Does the $f_0(500)$ and his brother, the light k, play a role?

Theoretical description of a thermal gas

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}$$



The stable part is "easy":

$$\ln Z_k^{\text{stable,}} \quad = f_k V \int \frac{d^3p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

$$P = \frac{T}{V} \ln Z$$

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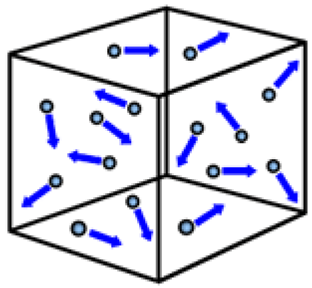

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$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

Yet, how to threat the unstable states (the resonances) ?

(Simplified) Thermal gas in QCD

It is well known that the ‘dominant’ term is given by pions (since these are the lightest mesons).

For simplicity, let us first consider only the pions and the rho meson.

The question is: how to treat the rho meson?

(Simplified) Thermal gas in QCD – the pions

$$\ln Z = \ln Z_\pi + \ln Z_\rho$$

$$\ln Z_\pi = 3V \int \frac{d^3p}{(2\pi)^3} \ln \left[\frac{1}{1 - e^{-\beta E_\pi}} \right]$$

$$E_\pi = \sqrt{\mathbf{p}^2 + M_\pi^2}$$

In the pion gas one performs the sum over all occupation number

(Simplified) Thermal gas in QCD –the rho /1

In the easiest possible approximation, just neglect the fact that the rho is a resonance… treat it as stable

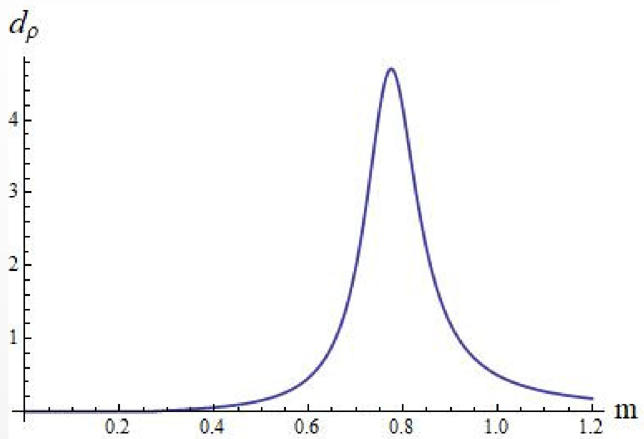
$$\ln Z_\rho = 3 \cdot 3V \int \frac{d^3p}{(2\pi)^3} \ln \left[\frac{1}{1 - e^{-\beta E_\rho}} \right]$$

$$E_\rho = \sqrt{\mathbf{p}^2 + M_\rho^2}$$

Use, for instance, the PDG mass for the rho meson

(Simplified) Thermal gas in QCD –the rho /2

But one can do better. We may take into account that the rho meson is described by a mass distribution



$$\ln Z_\rho = 3 \cdot 3V \int_0^\infty \mathrm{dM} d_\rho(M) \int \frac{d^3p}{(2\pi)^3} \ln\left[\frac{1}{1-e^{-\beta\sqrt{\mathbf{p}^2+M^2}}}\right]$$

As a parameterization of $d_\rho(M)$ one may use BW, relativistic BW, or even more advanced function. (The best is as the imaginary part of the propagator).

$$\int_0^\infty \mathrm{dM} d_\rho(M) = 1$$

$$d_\rho(M) \overset{\text{zero-width limit}}{\longrightarrow} \delta(M - M_\rho)$$

One can go even further

R. Dashen, S.-K. Ma, and H. J. Bernstein, Phys.Rev. **187**, 345 (1969).
R. Dashen and R. Rajaraman, Phys.Rev. **D10**, 694 (1974).

W. Weinhold, B. Friman, and W. Noerenberg, Acta Phys.Polon. **B27**, 3249 (1996).
W. Weinhold, B. Friman, and W. Norenberg, Phys.Lett. **B433**, 236 (1998), arXiv:nucl-th/9710014 [nucl-th].

Result in agreement with the scattering theory of QM and by TD potential in QFT in the low density approximation

Instead of the spectral function or energy distribution, use the (derivative of the) phase-shit.
In the example that we study, consider the pion-pion scattering data in the rho-channel

Phase shift -recall

Next, one has to obtain the partial waves.

$$A(s,t,u) = A(s,\theta) = \sum_{l=0}^{\infty}(2l+1)A_l(s)P_l(\cos\theta)$$

where $P_l(\xi)$ are the Legendre polynomials with

$$\int_{-1}^{+1} d\xi P_l(\xi)P_{l'}(\xi) = \frac{2}{2l+1}\delta_{ll'} \ .$$

For identical particles, one has the following definition of the phase space of the l-th wave

$$\frac{e^{2i\delta_l(s)}-1}{2i} = ka_l(s) = \frac{1}{2}\cdot\frac{k}{8\pi\sqrt{s}}A_l(s)$$

Recall from scattering theory:

$$\frac{e^{2i\delta_k}-1}{2i} = a_k = \frac{-\sqrt{s}\Gamma(\sqrt{s})}{s-m^2+i\sqrt{s}\Gamma(\sqrt{s})}$$

Thermal gas: connection to scattering data

The density function can be directly extracted from two-body scattering data (phase shifts).

$$\ln Z_\rho = \ln Z_{(I=1,J=1)} = 3 \cdot 3V \int_0^\infty \mathrm{dM} \left[\frac{1}{\pi} \frac{d\delta_{\pi\pi,(I=1,J=1)}(M)}{dM} \right] \int \frac{d^3p}{(2\pi)^3} \ln \left[\frac{1}{1 - e^{-\beta\sqrt{\mathbf{p}^2+M^2}}} \right]$$

This is a model-independent way of taking the resonances into account.

Indeed, it is a justification of the validity of thermal gas models (**but**…).

It is actually even more general, since it allows also to include any other reaction and **even repulsions** in some channels.

Important point

$$\frac{1}{\pi}\frac{d\delta_{\pi\pi,(I=1,J=1)}(M)}{dM} \neq d_\rho(M)$$

These two quantities are not equal. They are actually very often similar, Sometimes very similar…but they are not the same. The equal sign only holds in the BW limit.

$$\frac{1}{\pi}\frac{d\delta(M)}{dM} \overset{\text{BW}}{=} \frac{\Gamma}{2\pi}\frac{1}{(M-M_\rho)^2+\Gamma_\rho^2/4} = d_\rho^{\text{BW}}(M)$$

The reason is that the two-pion states are also modified.

$$\ln Z_{tot} = \ln Z_\pi + \ln Z_{(I=1,J=1)}$$

Part 2: the Lee Hamiltonian
and
its application to thermodynamics

P. M. Lo and F. Giacosa, *Thermal contribution of unstable states* Eur. Phys. J. C **79** (2019) no.4, 336 [arXiv:1902.03203 [hep-ph]].

Lee Hamiltonian

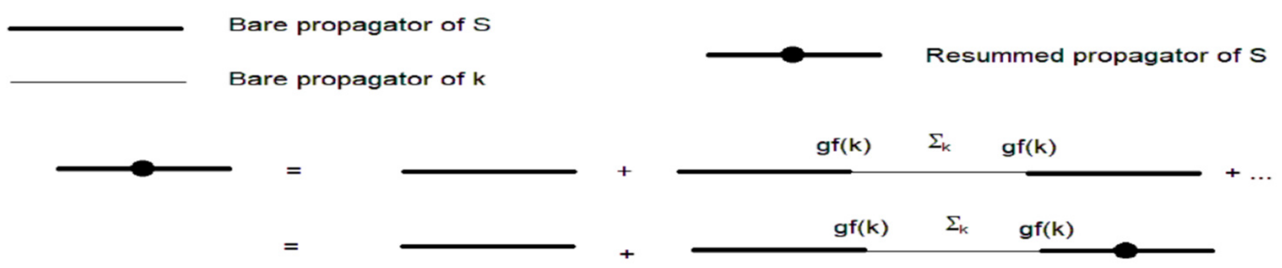
$$H = H_0 + H_1$$

$$H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k|$$

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

|S> is the initial unstable state, coupled to an infinity of final states |k>. (Poincare-time is infinite. Irreversible decay). General approach, similar Hamiltonians used in many areas of Physics. (Jaynes-Cummings approach, Friedrichs model.)

In our example: rho-meson decay. |S> represents a rho meson, |k> represents a two-photon state (flying back-to-back). The quantity k is the three-momentum of one of the outgoing pion.



F.G J.Phys.Conf.Ser. 1612 (2020) 1, 012012, 2001.07781

Figure 1. Schematic presentation of the sum leading to the dressed propagator. In the last part the Bethe-Salpeter resummation is depicted.

Lee Hamiltonian

Used in many areas of physics…also called Jaynes-Cummings Hamiltonian or Friedrichs model

T. D. Lee, *Some Special Examples in Renormalizable Field Theory,* Phys. Rev. **95** (1954) 1329-1334. C. B. Chiu, E. C. G. Sudarshan and G. Bhamathi, *The Cascade model: A Solvable field theory,* Phys. Rev. D **46** (1992) 3508.

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, *Hamiltonian effective field theory study of the N(1535) resonance in lattice QCD* Phys. Rev. Lett. **116** (2016) no.8, 082004 [arXiv:1512.00140 [hep-lat]].

Z. Xiao and Z. Y. Zhou, *On Friedrichs Model with Two Continuum States*, J. Math. Phys. **58** (2017) no.6, 062110 [arXiv:1608.06833 [hep-ph]]. Z. Y. Zhou and Z. Xiao, *Understanding X(3862), X(3872), and X(3930) in a Friedrichs-model-like scheme* Phys. Rev. D **96** (2017) no.5, 054031 Erratum: [Phys. Rev. D **96** (2017) no.9, 099905]

F. Giacosa, *Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels*, Found. Phys. **42**, 1262 (2012).

Propagator and spectral function

$$H = H_0 + H_1 \;;\; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k)\,|k\rangle\langle k| \;;\; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\,(g\cdot f(k))(|S\rangle\langle k| + |k\rangle\langle S|)$$

$$G_S(E) = \left\langle S \left| (E - H + i\varepsilon)^{-1} \right| S \right\rangle = (E - M_0 + \Pi(E) + i\varepsilon)^{-1}$$

$$\Pi(E) = -\int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{g^2 f(k)^2}{E - \omega(k) + i\varepsilon}$$

$$d_S(E) = \tfrac{1}{\pi} \operatorname{Im} G_S(E) = \frac{1}{\pi} \frac{\operatorname{Im}\Pi(E)}{(E - M_0 + \operatorname{Re}\Pi(E))^2 + (\operatorname{Im}\Pi(E))^2};$$

$$a(t) = \left\langle S \left| e^{-iHt} \right| S \right\rangle = \int_{-\infty}^{+\infty} dE\, d_S(E)\, e^{-iEt}$$

Consequences

It follows:

$$\int_{-\infty}^{+\infty} dE d_S(E) = 1 \quad ; \quad G_S(E') = \int_{-\infty}^{+\infty} dE \frac{d_S(E)}{E' - m + i\varepsilon}$$

Fermi golden rule: $\Gamma = \mathrm{Im}[\Pi(M)]/2$ with $M - M_0 + \mathrm{Re}\,\Pi(M) = 0$ (usually: $M < M_0$)

Time evolution and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E)dE = 1$

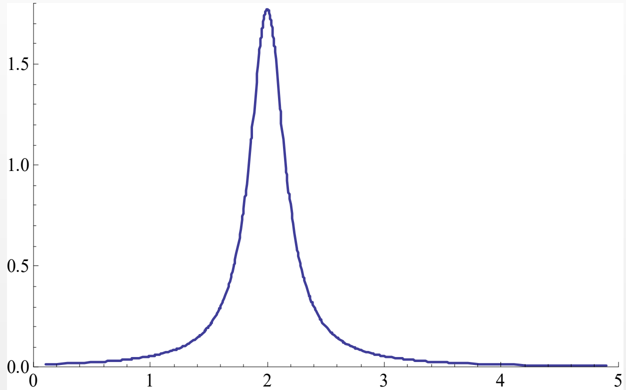
$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE$$

In stable limit : $d_S(E) = \delta(E - M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Breit-Winger (BW) limit

$$H = H_0 + H_1 \; ; \; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\omega(k) |k\rangle\langle k| \; ; \; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$\omega(k) = k \; ; \quad f(k) = 1 \quad \Rightarrow \quad \Pi(E) = ig^2/2 \; ; \; \Gamma = g^2$$



$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma^2/4}$$

$$\Rightarrow a(t) = e^{-i(M_0 - i\Gamma/2)t} \Rightarrow p(t) = e^{-\Gamma t}$$

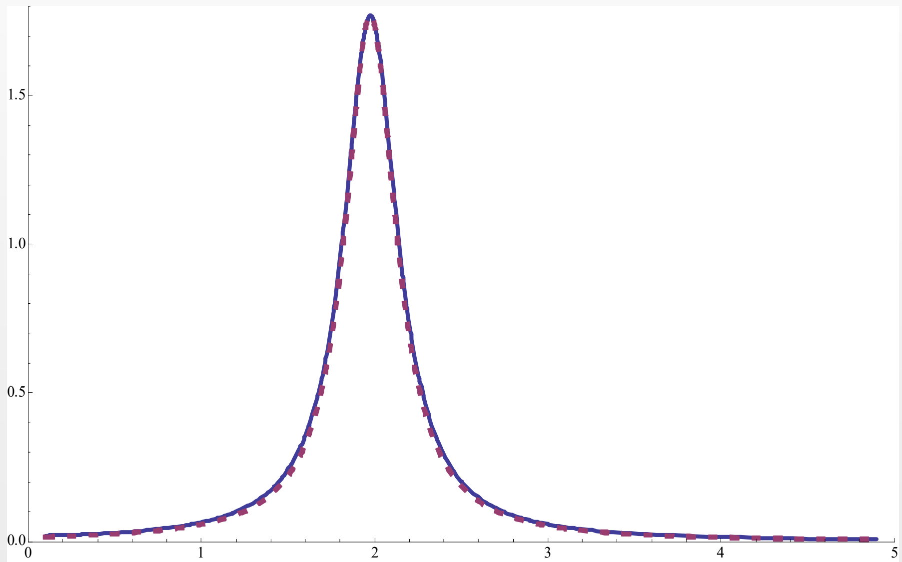
The BWl limit is obtained when the unstable state couples to all
the states of the continuum with the same strength. The decay law is exponential.

Non-exponential case (1)

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$f(k) = \begin{cases} 0 \text{ for } k < E_{min} \\ 1 \text{ for } E_{min} \leq k \leq E_{max} \\ 0 \text{ for } k > E_{max} \end{cases}$$

$d_S(E)$

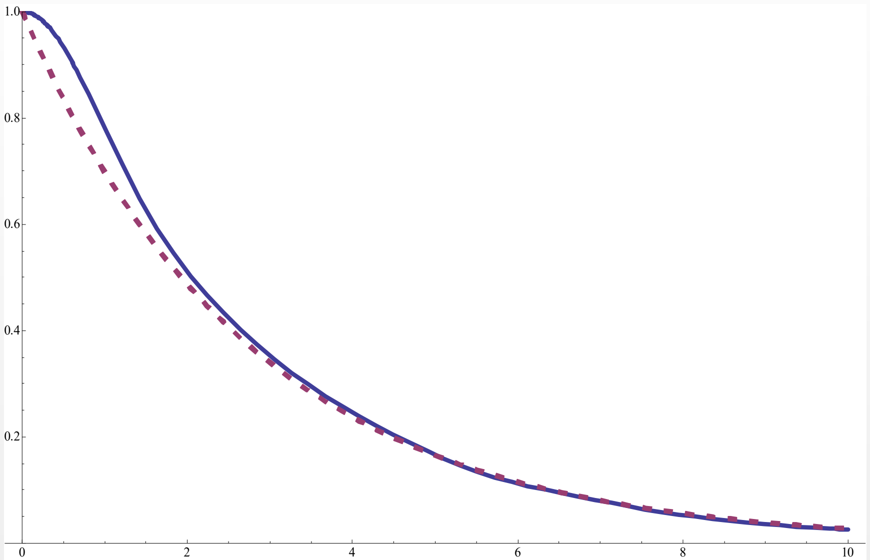


E

$M_0 = 2;\ E_{min} = 0;\ E_{max} = 5;\ g^2 = 0.36$ (all in a.u. of energy)

Non-exponential case (2)

$p(t)$

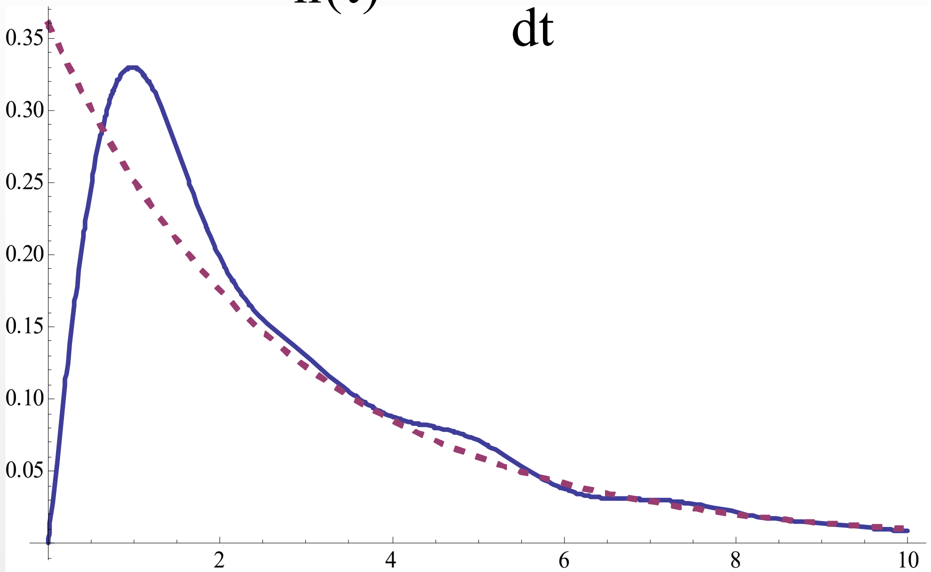


t

Dashed: $p_{BW}(t) = e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

$h(t)$

$$h(t) = -\frac{dp(t)}{dt}$$



t

Dashed: $h_{BW}(t) = \Gamma e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

$$\int_0^t h(u)du = 1 - p(t)$$

Namley: $h(t)dt = p(t) - p(t+dt)$ is the probability that the particles decays between t and t+dt

Prediction   Two-channel case (a)

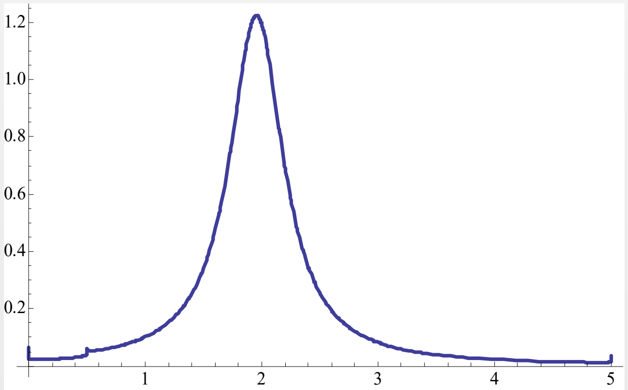
Found Phys (2012) 42:1262–1299
DOI 10.1007/s10701-012-9667-3

Non-exponential Decay in Quantum Field Theory and in Quantum Mechanics: The Case of Two (or More) Decay Channels

Francesco Giacosa

$$H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk (g_1 \cdot f_1(k))(|S\rangle\langle k,1| + |k,1\rangle\langle S|) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk (g_2 \cdot f_2(k))(|S\rangle\langle k,2| + |k,2\rangle\langle S|)$$

$$f_i(k) = \begin{cases} 0 \text{ for } k < E_{i,min} \\ 1 \text{ for } E_{i,min} \leq k \leq E_{i,max} \\ 0 \text{ for } k > E_{i,max} \end{cases}$$



$M_0 = 2;\ E_{1,min} = 0;\ E_{2,min} = 0\ ;\ E_{1,max} = E_{2,max} = 5;$

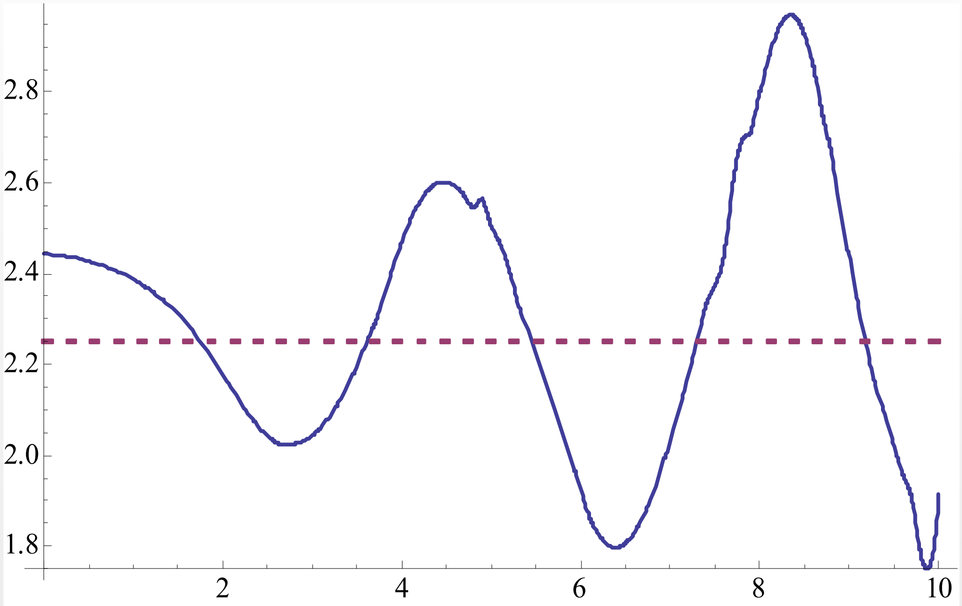
$g_1^2 = 0.36\ ;\ g_2^2 = 0.16$ (all in a.u. of energy)

Prediction Two-channel case (b)

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)

$$\frac{h_1(t)}{h_2(t)}$$



t

Dashed: $\frac{h_{1,BW}(t)}{h_{2,BW}(t)} = \frac{\Gamma_1}{\Gamma_2} = \text{const}$

Measurable effect???

Details in:
F. G., Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels,
Found. Phys. 42 (2012) 1262 [arXiv:1110.5923].

Application to TD

We consider that also the state |k> (the two-pion state) is modified by the presence of the interaction. Not only the rho is modified (quite “expected”), but the pions are not any longer ‘free’.

P. M. Lo and F. Giacosa, *Thermal contribution of unstable states* Eur. Phys. J. C **79** (2019) no.4, 336 [arXiv:1902.03203 [hep-ph]].

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$$P_{\mathrm{KFL}} \approx P_{\pi}^{(0)} + P_{\rho}^{(0)} + \Delta P_{\mathscr{K}}$$
$$= P_{\pi}^{(0)} + P_{\rho}^{(0)} + \Delta P_{\rho} + \Delta P_{2\pi}\,.$$

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Rho spectra function

$$\ln Z_\rho = 3 \cdot 3V \int_0^\infty \mathrm{dM} d_\rho(M) \int \frac{d^3p}{(2\pi)^3} \ln\left[\frac{1}{1-e^{-\beta\sqrt{\mathbf{p}^2+M^2}}}\right]$$

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$$= P_{\pi}^{(0)} + \boxed{P_{\rho}^{(0)} + \Delta P_{\rho} + \Delta P_{2\pi}\,.}$$

Phase-shift’s derivative formula

⇓

$$\ln Z_{\rho} = \ln Z_{(I=1,J=1)} = 3 \cdot 3V \int_0^{\infty} \mathrm{dM} \left[\frac{1}{\pi}\frac{d\delta_{\pi\pi,(I=1,J=1)}(M)}{dM}\right] \int \frac{d^3p}{(2\pi)^3} \ln\left[\frac{1}{1-e^{-\beta\sqrt{\mathbf{p}^2+M^2}}}\right]$$

Additional results with the Lee model

Eur. Phys. J. C (2019) 79:336 Page 7 of 11 **336**

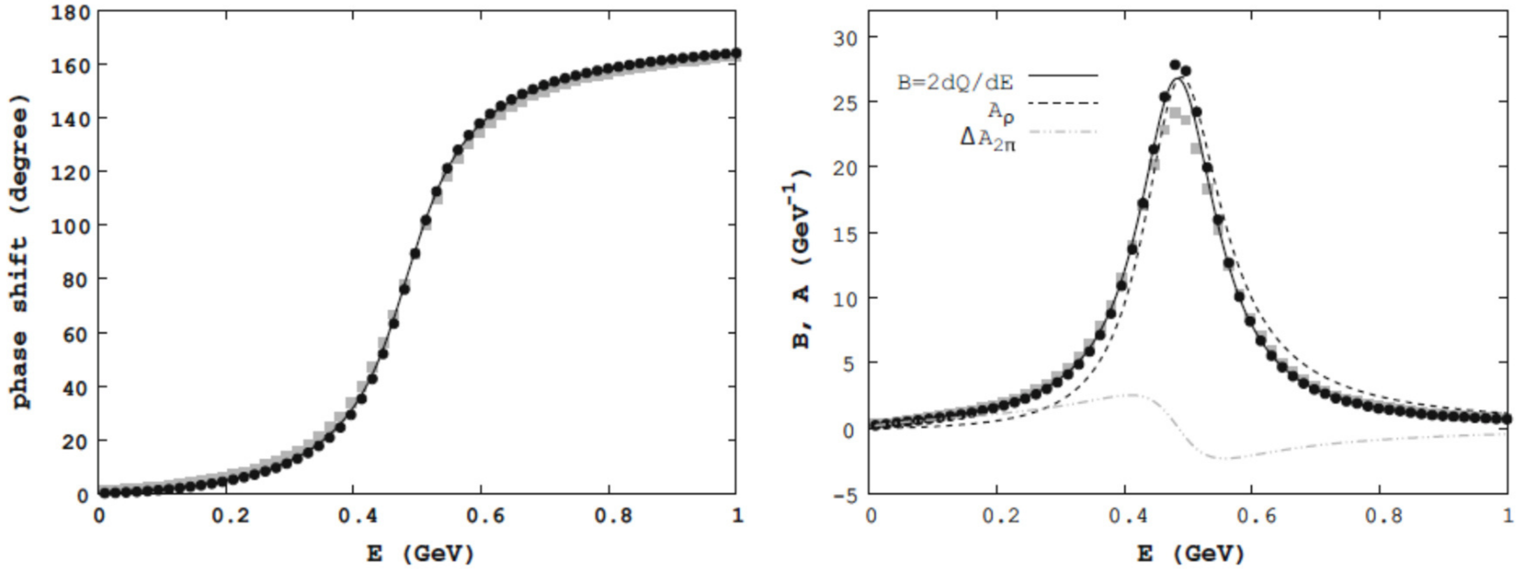


Fig. 1 Phase shift computed from the Lee model (Eq. (36)) and the corresponding effective spectral functions (Eqs. (22), (23)) for the ρ-meson. E is the energy of the relative motion. Note that $B = A_\rho + \Delta A_{2\pi}$. The points are the numerical results obtained from the momentum grid method: Grey squares indicate results based on Eq. (36), while black circles indicate results based on Eq. (48). See text

Numerical example

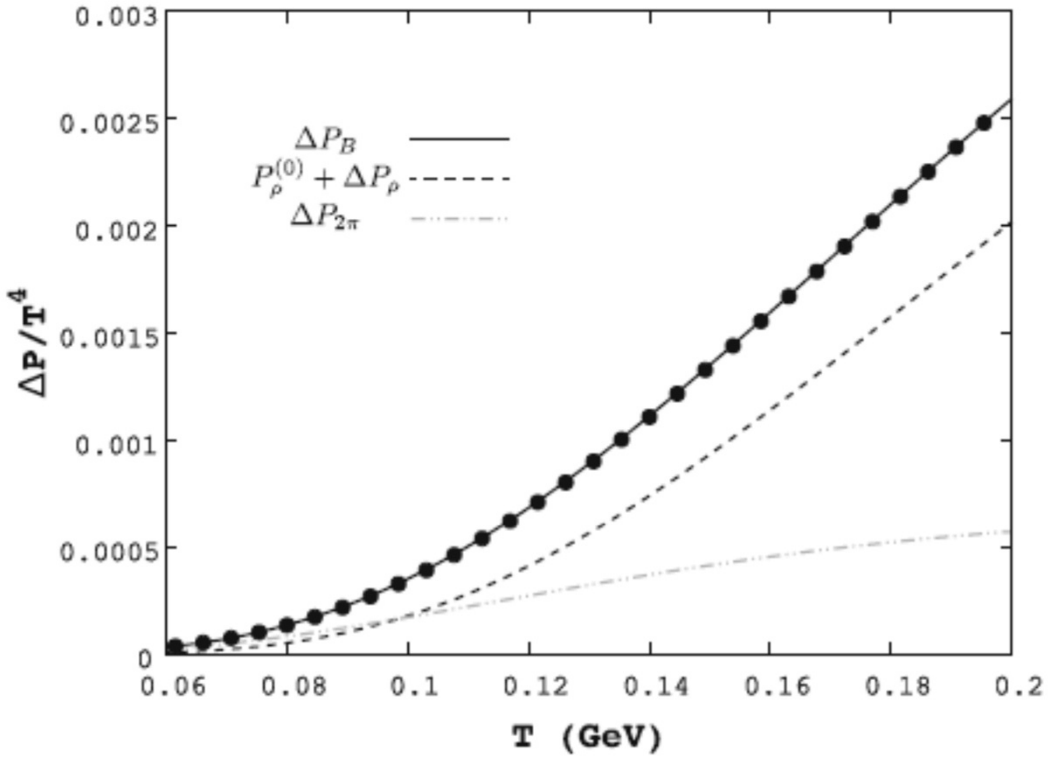


Fig. 2 Interacting contributions to the thermodynamics pressure (normalized to T^4) from different effective spectral functions. The top line (B) is equal to the sum of the contributions from the lower two (see Eq. (69)). The points are the results obtained by directly constructing the partition function from the eigenvalues of the Hamiltonian. See text

Contributions/T^4

$$P_\rho^{(0)} + \Delta P_\rho + \Delta P_{2\pi}$$

$$P_\rho^{(0)} + \Delta P_\rho$$

$$\Delta P_{2\pi}$$

Part 3: Resonances $f_0(500)$ and $K_0^*(700)$

based on W. Broniowski, F.G., V. Begun,

Cancellation of the sigma meson in thermal models
Phys. Rev. C 92 (2015) no.3, 03490
[arXiv:1506.01260 [nucl-th]].

Inclusion fo two additional channels

$$\ln Z = \ln Z_\pi + \ln Z_K + \ln Z_{(1,1--)} + \ln Z_{(0,0++)} + \ln Z_{(2,0++)} + \ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} + \ldots$$

rho-meson f0(500) K0*(700)

$$\ln Z_K = 4V \int \frac{d^3p}{(2\pi)^3} \ln \left[\frac{1}{1 - e^{-\beta E_K}} \right]$$

$$E_K = \sqrt{\mathbf{p}^2 + M_K^2}$$

$$P = \frac{T}{V} \ln Z$$

For the various channels:

Simple and model-independent procedure: **just use scattering data!**

In the scalar channel

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

$f_0(500)$

$I^G(J^{PC}) = 0^+(0^{++})$

also known as σ; was $f_0(600)$

See the related review(s):

Scalar Mesons below 2 GeV

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–550)−*i*(200–350) OUR ESTIMATE			

M. Soltysiak, T. Wolkanowski and F. G., K0*(700) as a companion pole of K0*(1430),'
Nucl. Phys. B 909 (2016) 418
[arXiv:1512.01071 [hep-ph]].

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

$K_0^*(700)$

$I(J^P) = \frac{1}{2}(0^+)$

also known as κ; was $K_0^*(800)$

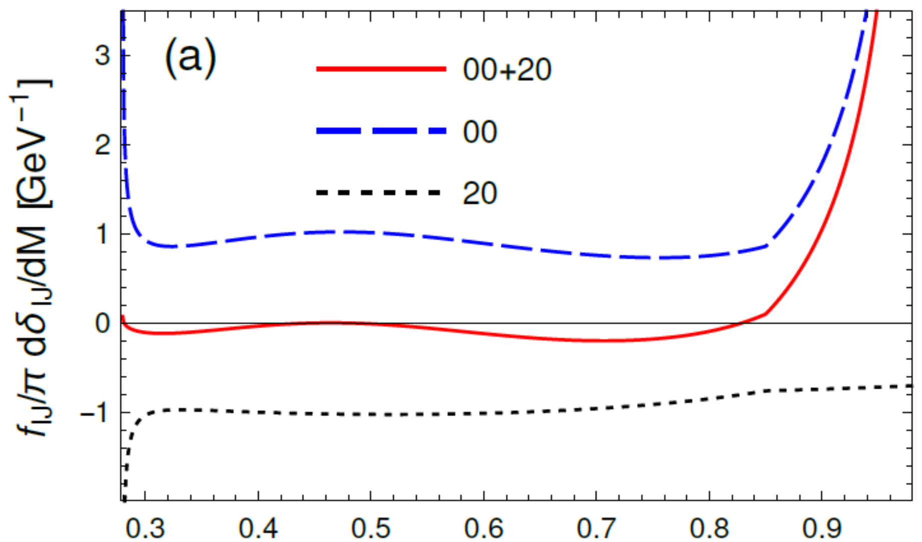
Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(700)$ T-Matrix Pole $\sqrt{s}$

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(630–730) − *i* (260–340) OUR EVALUATION			

What to do?
Simple…phase shifts

The f_0(500) spectral function **and** the isotensor repulsion/1



arxiv: 1506.01260

The total contribution from J=0 is the red curve: $\ln Z_{(0,0)} + \ln Z_{(2,0)}$

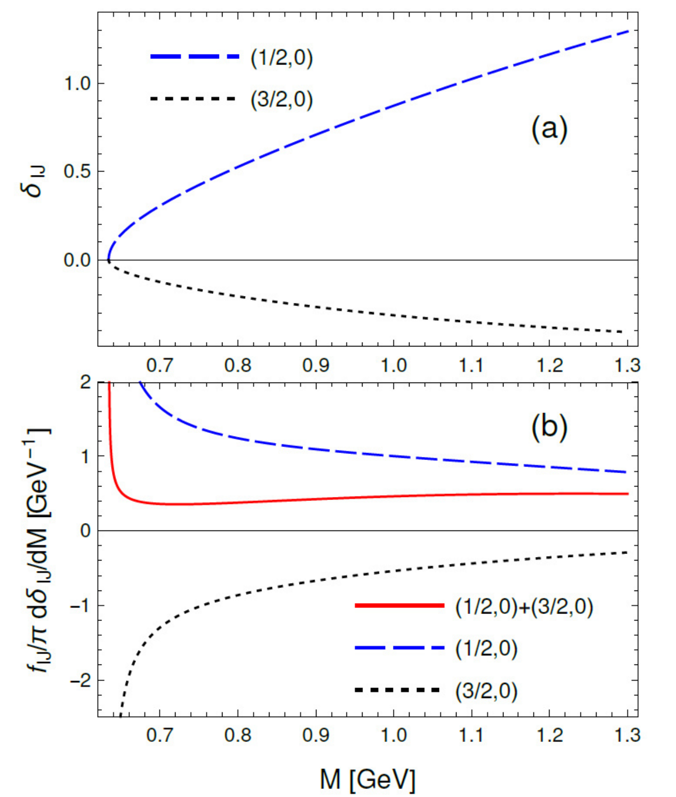
M [GeV]

$\ln Z_{(0,0)}$ is the contribution of f_0(500). It is indeed nonzero and even non-negligible, but it is almost exactly cancelled by the isotensor repulsion. Thermal models however usually neglect repulsions.

Either you take into account both I=0 and I =2, or –simply- you neglect both of them

$$\ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})} = \int_0^{\Lambda_0} dM \left[\frac{d\delta_{(0,0)}}{\pi dM} + 5 \frac{d\delta_{(2,0)}}{\pi dM} \right] \int_p \ln \left[1 - e^{-\frac{\sqrt{p^2+M^2}}{T}} \right]^{-1}$$

The scalar kaonic resonace K0*(700): (partial) cancellation in thermal models



arxiv: 1506.01260

The total contribution from is the red curve:

$\ln Z_{(1/2,0)} + \ln Z_{(3/2,0)}$

cancellation is evident:
easiest thing to do is to forget about the k.
(Eventually, visible in correlations).

$$\ln Z_{(1/2,0^{++})} + \ln Z_{(3/2,0^{++})} = \int_0^{\Lambda_0} dM \left[2\frac{d\delta_{(1/2,0)}}{\pi dM} + 4\frac{d\delta_{(3/2,0)}}{\pi dM} \right] \int_p \ln\left[1 - e^{-\frac{\sqrt{p^2+M^2}}{T}}\right]^{-1}$$

Rho, pion, and sigma: the trace anomaly

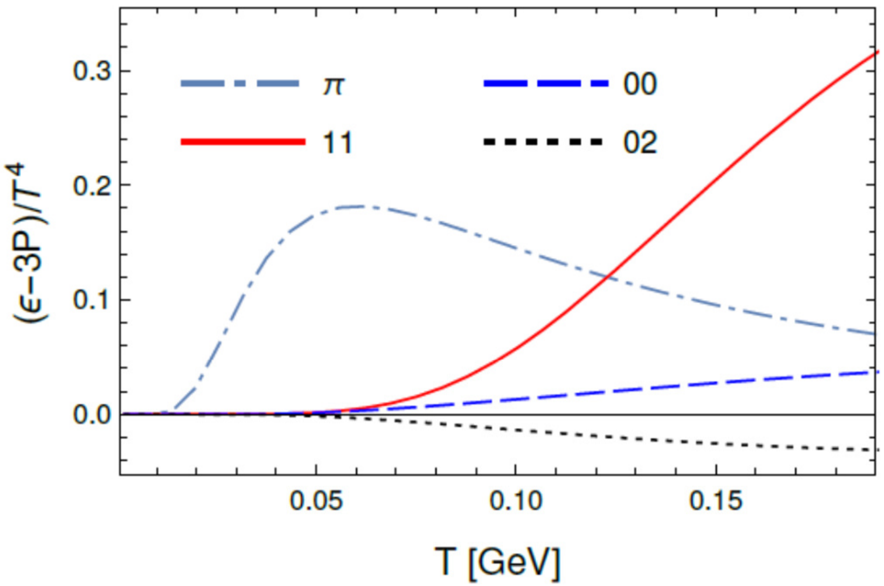


FIG. 3. (color online) Pion, ρ-meson, isoscalar-scalar, and isoscalar-tensor contributions to the volume density of the trace of the energy-momentum tensor divided by T^4, plotted as functions of T.

Part 4:
how to deal with bound states in QFT

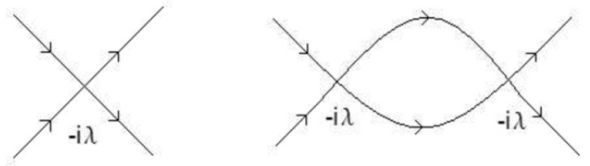
based on S. Samanta, F.G,
QFT treatment of a bound state in a thermal
Phys.Rev.D 102 (2020) 116023
[arXiv:2009.13547 [nucl-ph]]

as well as in an ongoing work

Various states could be hadronic bound states: X(3872), a0(980), …

QFT with quartic interaction term

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$$



+…

Recall:
λ > 0 repulsion
λ < 0 attraction

arxiv: 2009.13547

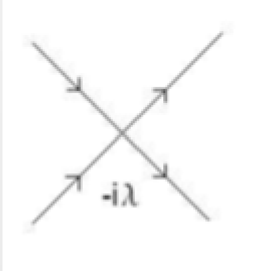
Phi^4 theory, tree-level amplitude

$$A(s,t,u) = A(s,\theta) = \sum_{l=0}^{\infty}(2l+1)A_l(s)P_l(\cos\theta)$$

$$iA(s,t,u) = i(-\lambda) \Rightarrow A(s,t,u) = A(s,\theta) = -\lambda.$$

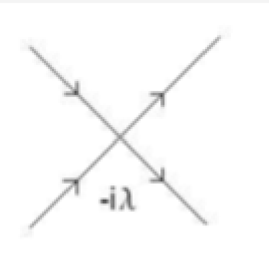
$$A_0(s) = \frac{1}{2}\int_{-1}^{+1} d\xi A(s,\theta) = A(s,\theta) = -\lambda$$

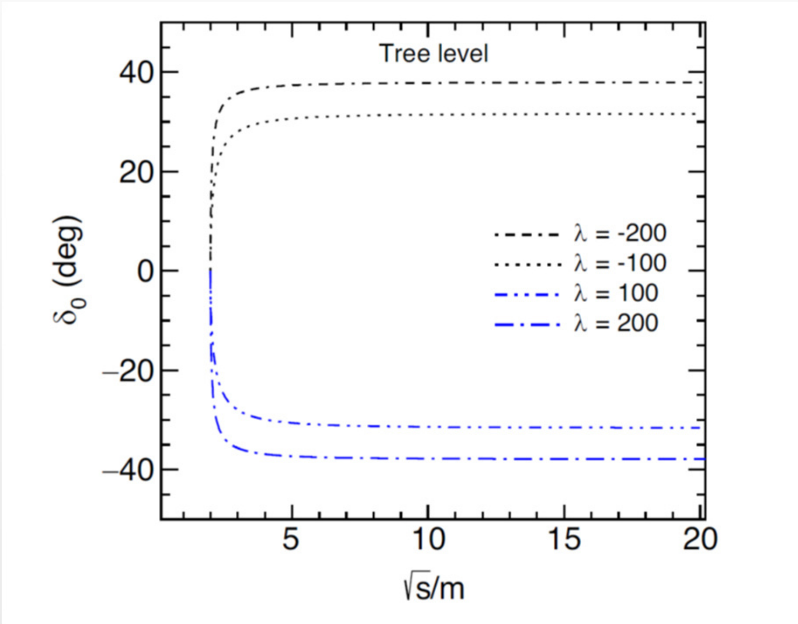
$$a_0^{\mathrm{SL}} = \frac{1}{2}\frac{A_0(s=4m^2)}{8\pi\sqrt{4m^2}} = \frac{1}{2}\frac{-\lambda}{16\pi m}$$



Tree-level phase shift

$$\delta_0(s) = \frac{1}{2}\arg\left[1 - \frac{1}{16\pi}\sqrt{\frac{4m^2}{s} - 1}\, A_0(s)\right]$$





Note: breaking of unitarity!

Pressure of the system

$$P_{\varphi,\text{free}} = -T \int_k \ln\left[1 - e^{-\beta\sqrt{k^2+m^2}}\right]$$

$$P_{\varphi\varphi\text{-int}} = -T \int_{2m}^{\infty} dx \frac{1}{\pi} \frac{d\delta_0(s = x^2)}{dx} \int_k \ln\left[1 - e^{-\beta\sqrt{k^2+x^2}}\right]$$

Interaction contribution

$$P_B = -\theta(\lambda_c - \lambda) T \int_k \ln\left[1 - e^{-\beta\sqrt{k^2+M_B^2}}\right]$$

Bound state contribution

Loop function

One-loop resummed unitarization scheme,
one subtraction with

$$\Sigma(s) = \frac{1}{2}\frac{1}{16\pi}\left(-\frac{1}{\pi}\sqrt{1-\frac{4m^2}{s+i\epsilon}}\ln\frac{\sqrt{1-\frac{4m^2}{s+i\epsilon}}+1}{\sqrt{1-\frac{4m^2}{s+i\epsilon}}-1}\right)+\frac{1}{16\pi^2}$$

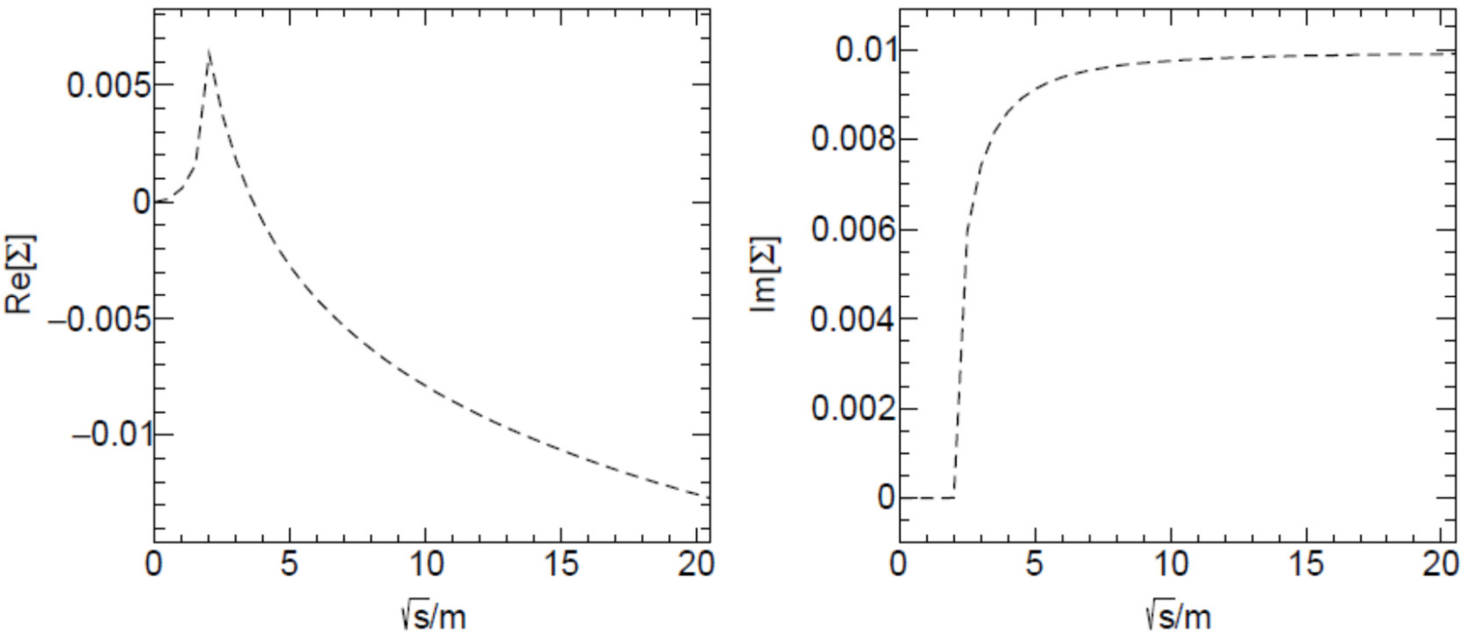
$$\mathrm{Re}\Sigma(s) = \frac{s}{\pi}P\int_{s_{th}}^{\infty}\frac{I(s')}{(s'-s)s'}.$$

$$\mathrm{Im}\Sigma(s) = \begin{cases} \frac{1}{2}\frac{\sqrt{\frac{s}{4}-m^2}}{8\pi\sqrt{s}} & \text{for } s > (2m)^2 \\ \varepsilon & \text{for } s < (2m)^2, \end{cases}$$

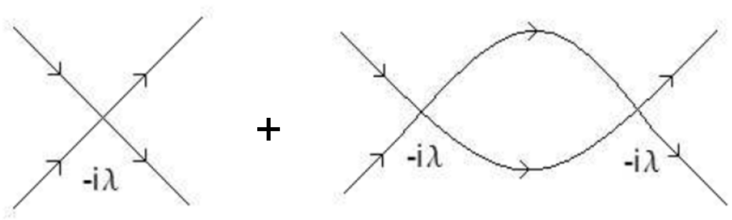
$$\mathrm{Re}\Sigma(s) = \frac{s}{\pi}P\int_{s_{th}}^{\infty}\frac{I(s')}{(s'-s)s'}.$$

$$\Sigma(s\to 0) = 0$$

Loop function/plots



Unitarized phase shifts



$$A_k^U(s) = [A_k^{-1}(s) - \Sigma(s)]^{-1}$$

$$A_0^U(s) = [A_0^{-1}(s) - \Sigma(s)]^{-1} = \frac{-\lambda}{1 + \lambda\Sigma(s)},$$

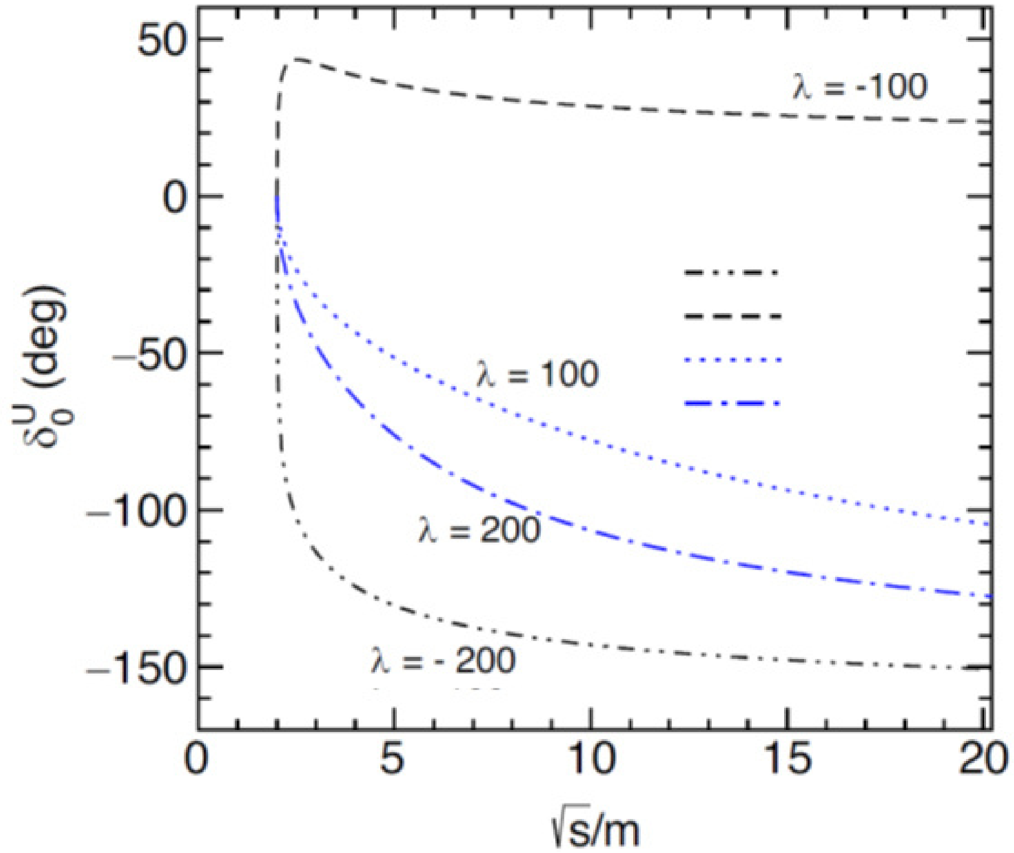
$$\frac{e^{2i\delta_0^U(s)} - 1}{2i} = \frac{1}{2} \cdot \frac{k}{8\pi\sqrt{s}} A_0^U(s).$$

$$a_0^{U,\mathrm{SL}} = \frac{1}{2}\frac{1}{16\pi m}\frac{-\lambda}{1 + \frac{\lambda}{16\pi^2}}.$$

$$\lambda_c = -16\pi^2$$

Unitarized Phase shift

Arxiv: 2009.13547

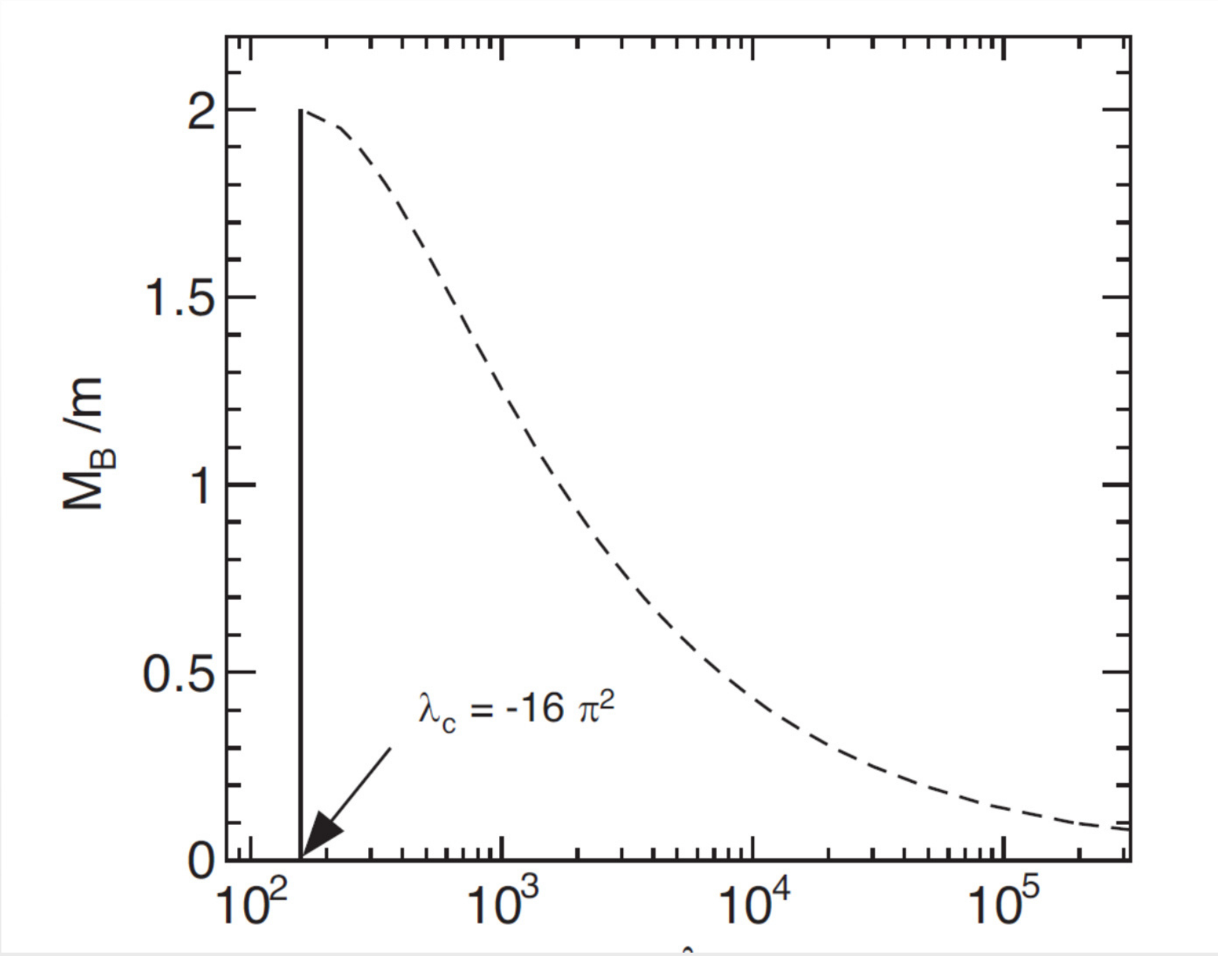


$$\delta_0^U(s \to \infty) = 0 \quad \text{for } \lambda \in (\lambda_c, 0)$$

$$\delta_0^U(s \to \infty) = -\pi \quad \text{for } \lambda < \lambda_c$$

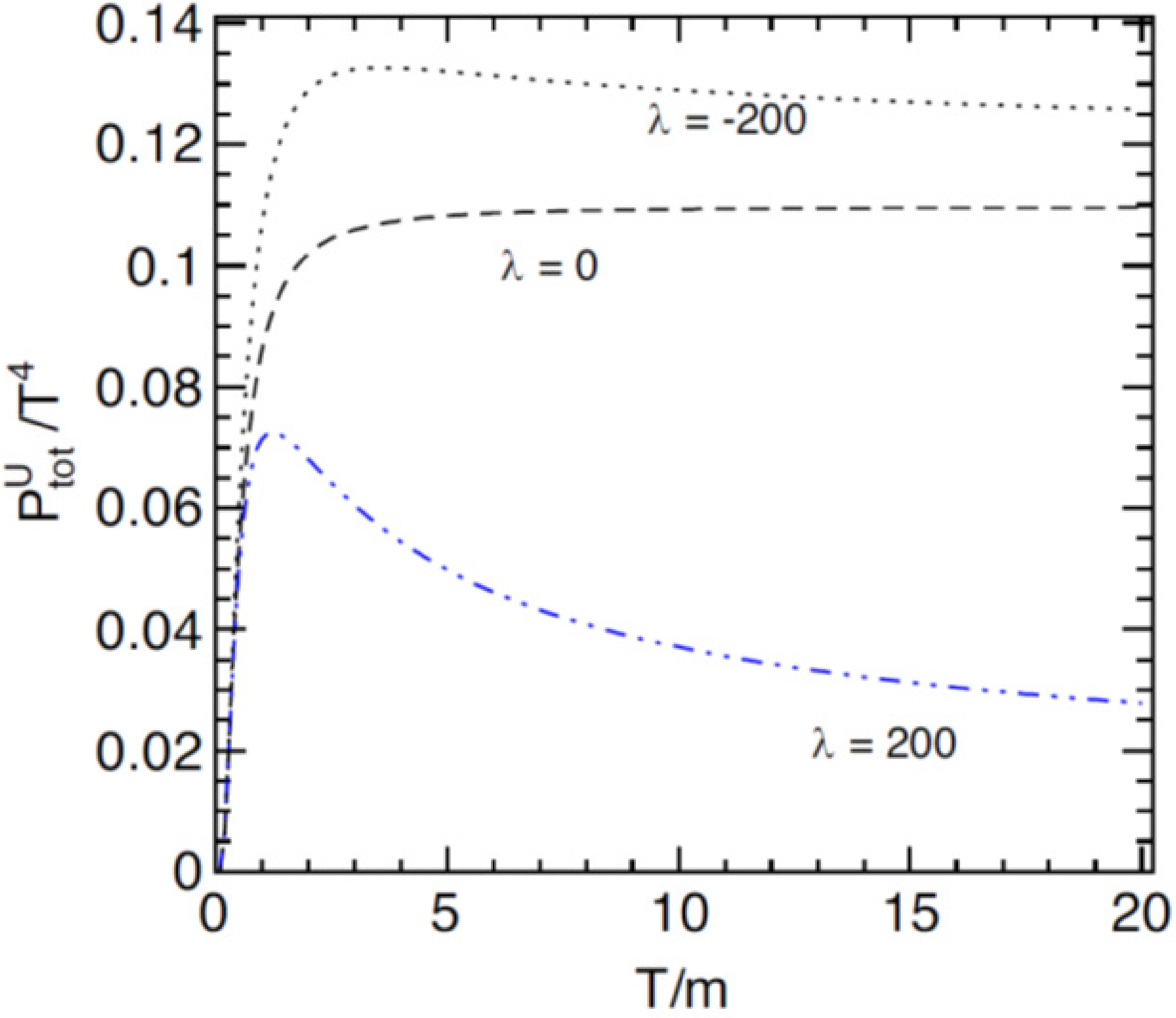
Levinson's theorem

Mass of the bound state



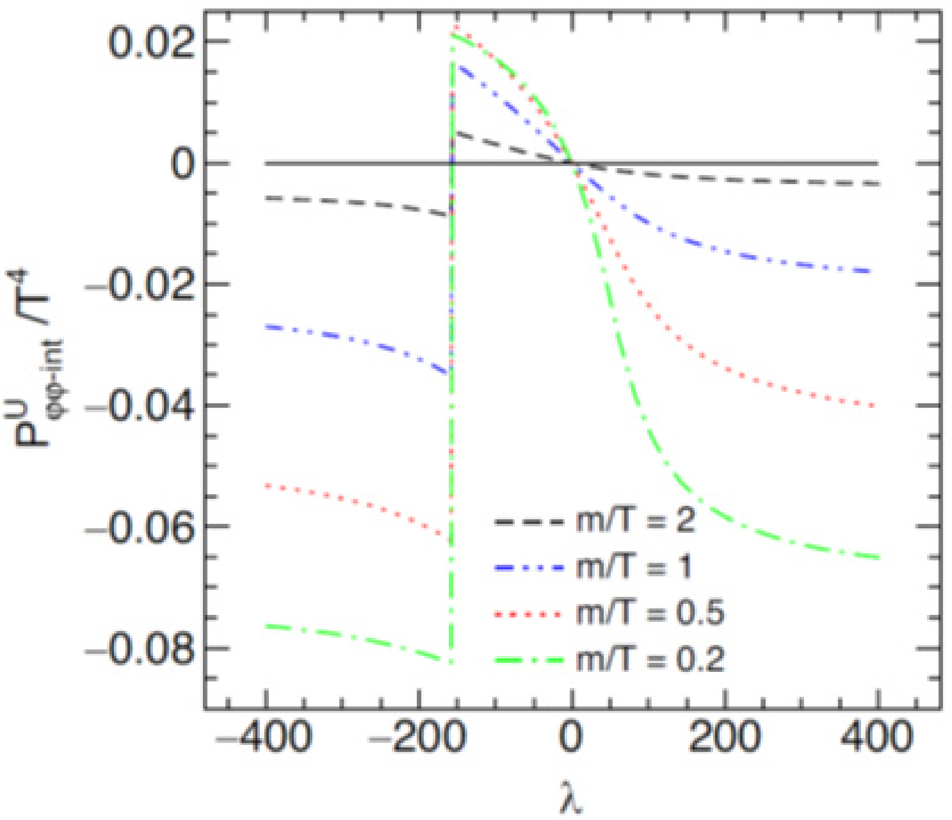
Pressure vs T

arxiv: 2009.13547



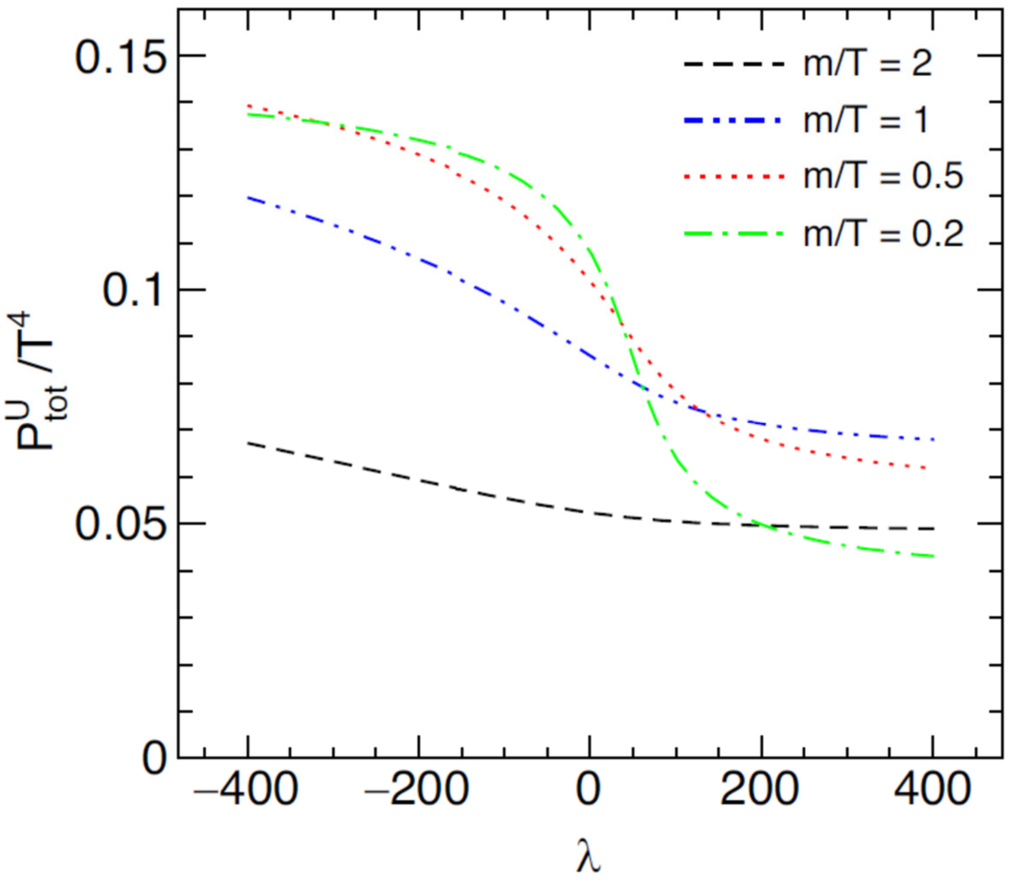
Interacting part of the pressure (without BS) as function of λ >0

arxiv: 2009.13547



Pressure as function of the coupling λ

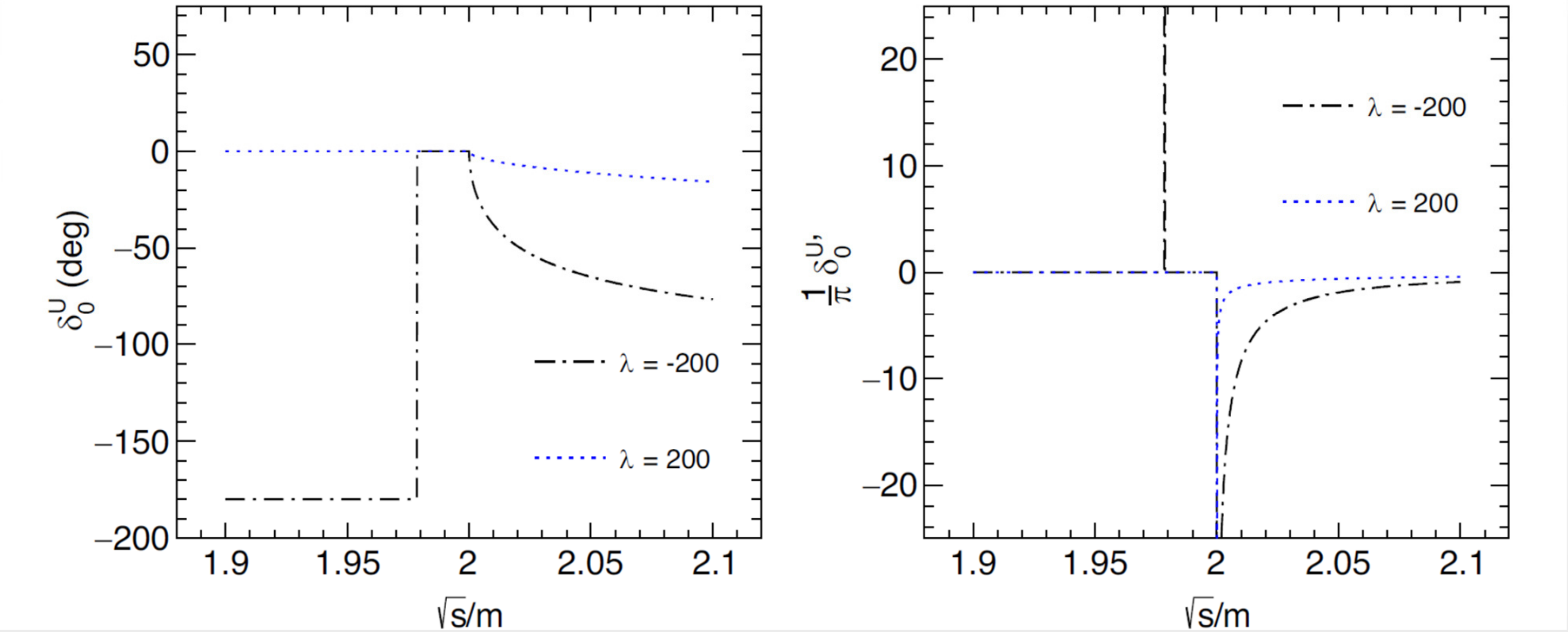
arxiv: 2009.13547



The pressure is continuous also at the critical value!

Unitarized phase shift (extension below threshold)

Put here the formula an discuss



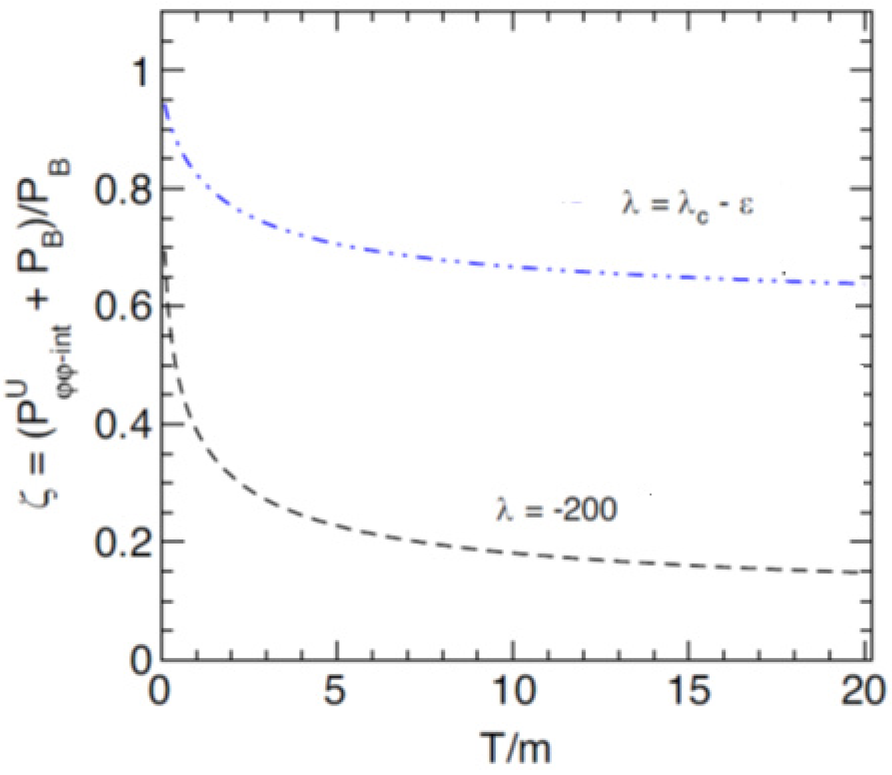
Role of the bound state

The results suggest that for a bound state created close to threshold (thus λ smaller but close to λ_c), the bound state is indeed important

2009.13547

$$P_B + P_{\varphi\varphi\text{-int}} = \xi P_B$$

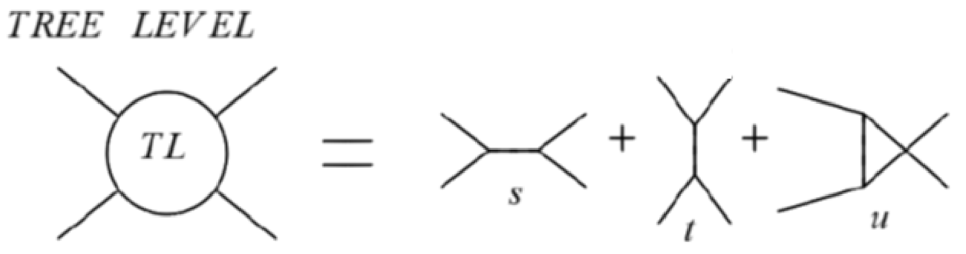
$$P_B = -\theta(\lambda_c - \lambda) T \int_k \ln\left[1 - e^{-\beta\sqrt{k^2+M_B^2}}\right]$$



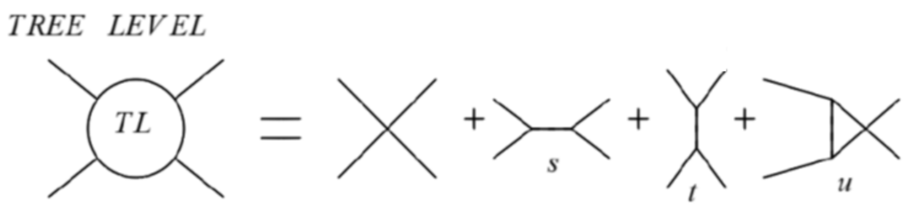
See also:
Ortega et al.,
Counting states and the Hadron Resonance Gas: Does X(3872) count?
PLB 781 (2018) arxiv: 1707.01915

Ongoing work

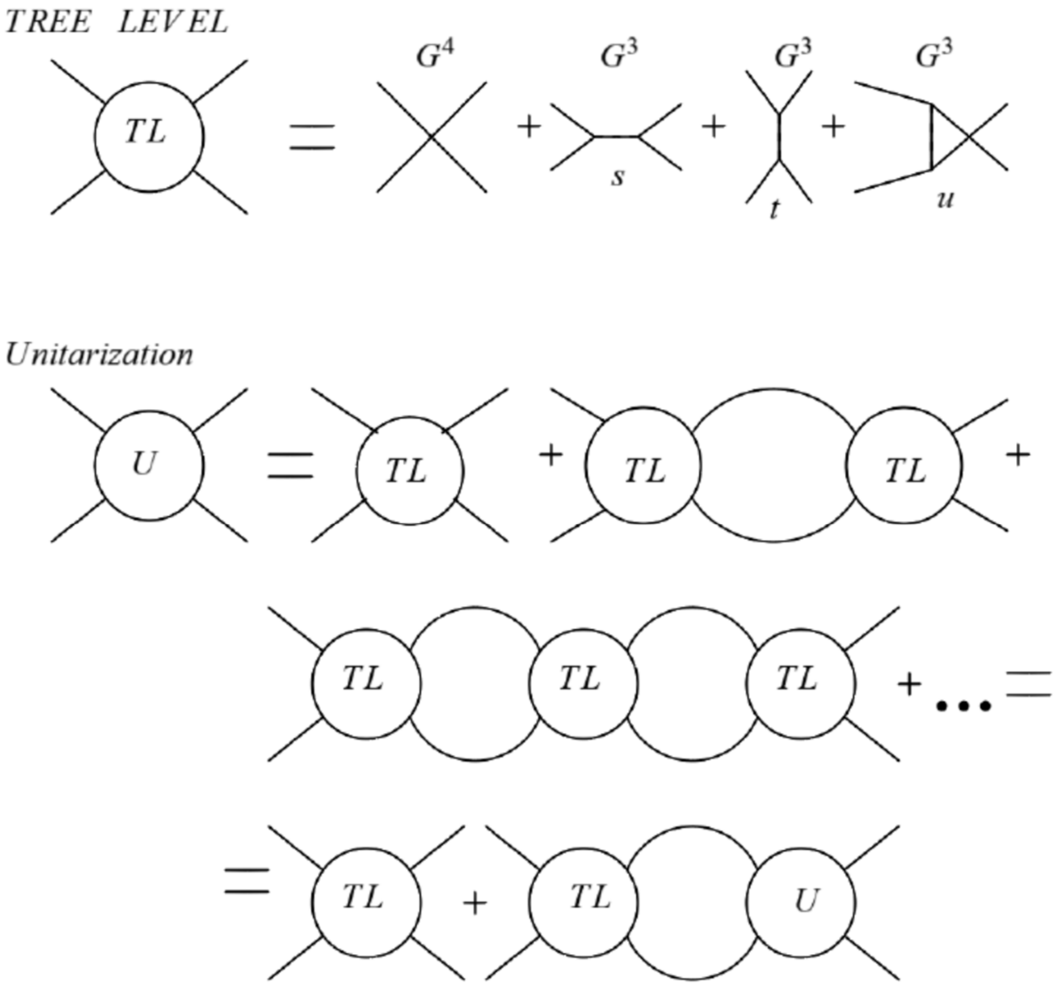
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3 - \frac{\lambda}{4!}\varphi^4;$$



Unitarization: schematic



$U = TL + (TL)\sum U$

Conclusions

For resonances: mass distribution is important, but sometimes not sufficient.

If you have scattering data, easy to use them!

Bound states are relevant, especially if the mass is just below the threshold of constitutents. Yet, there is a partial cancellation ☺

Thank You

QM derivation of the phase-shift formula

As a last step, we discuss a simple way based on Quantum Mechanics. The radial wave function with angular momentum l of a particle scattered by central potential $U(r)$ is

$$\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l] \ , \tag{17}$$

where $k = \left|\vec{k}\right|$ is the length of the three-momentum, and δ_l is the phase shift due the interaction with the potential. If we confine our system into a sphere of radius R, the condition $kR - l\pi/2 + \delta_l = n\pi$ with $n = 0, 1, 2, ...$must be met, since $\psi_l(r)$ has to vanish at the boundary. Conversely, the number of states n_0 that one can have by limiting k in the range $(0, k_0)$ is given by $n_0 = (k_0 R - l\pi/2 + \delta_l)/\pi$. Then, the density of state that one can place between k and $k + dk$ is given by

$$\frac{dn_l}{dk} = \frac{R}{\pi} + \frac{1}{\pi}\frac{d\delta_l}{dk} \ , \tag{18}$$

where the first term describes the density of states $\frac{dn_l^{free}}{dk}$ in absence of interactions, while the second term $\frac{1}{\pi}\frac{d\delta_l}{dk}$ describes the effect of the interacting potential. When translating the discussion from Quantum Mechanics to Quantum Field Theory, we replace the momentum k with the invariant mass M, the angular momentum l with the pair (I, J). Upon summing over the latter, one obtains the full density of states of an interacting pion gas as

$$\frac{dn}{dM} = \delta(M - M_\pi) + \sum_{I,J}\frac{1}{\pi}\frac{d\delta_{(I,J)}(M)}{dM} \ . \tag{19}$$

Phase shift formula - recall

Next, one has to obtain the partial waves.

$$A(s,t,u) = A(s,\theta) = \sum_{l=0}^{\infty}(2l+1)A_l(s)P_l(\cos\theta)$$

where $P_l(\xi)$ are the Legendre polynomials with

$$\int_{-1}^{+1} d\xi P_l(\xi)P_{l'}(\xi) = \frac{2}{2l+1}\delta_{ll'} \ .$$

For identical particles, one has the following definition of the phase space of the l-th wave

$$\frac{e^{2i\delta_l(s)}-1}{2i} = ka_l(s) = \frac{1}{2}\cdot\frac{k}{8\pi\sqrt{s}}A_l(s)$$

Recall from scattering theory:

$$\frac{e^{2i\delta_k}-1}{2i} = a_k = \frac{-\sqrt{s}\Gamma(\sqrt{s})}{s-m^2+i\sqrt{s}\Gamma(\sqrt{s})}$$