THERMODYNAMICS OF A COUPLED CHANNEL SYSTEM

POK MAN LO (盧博文)

University of Wroclaw

28 MAY 2021
BIALASOWKA WEBINAR

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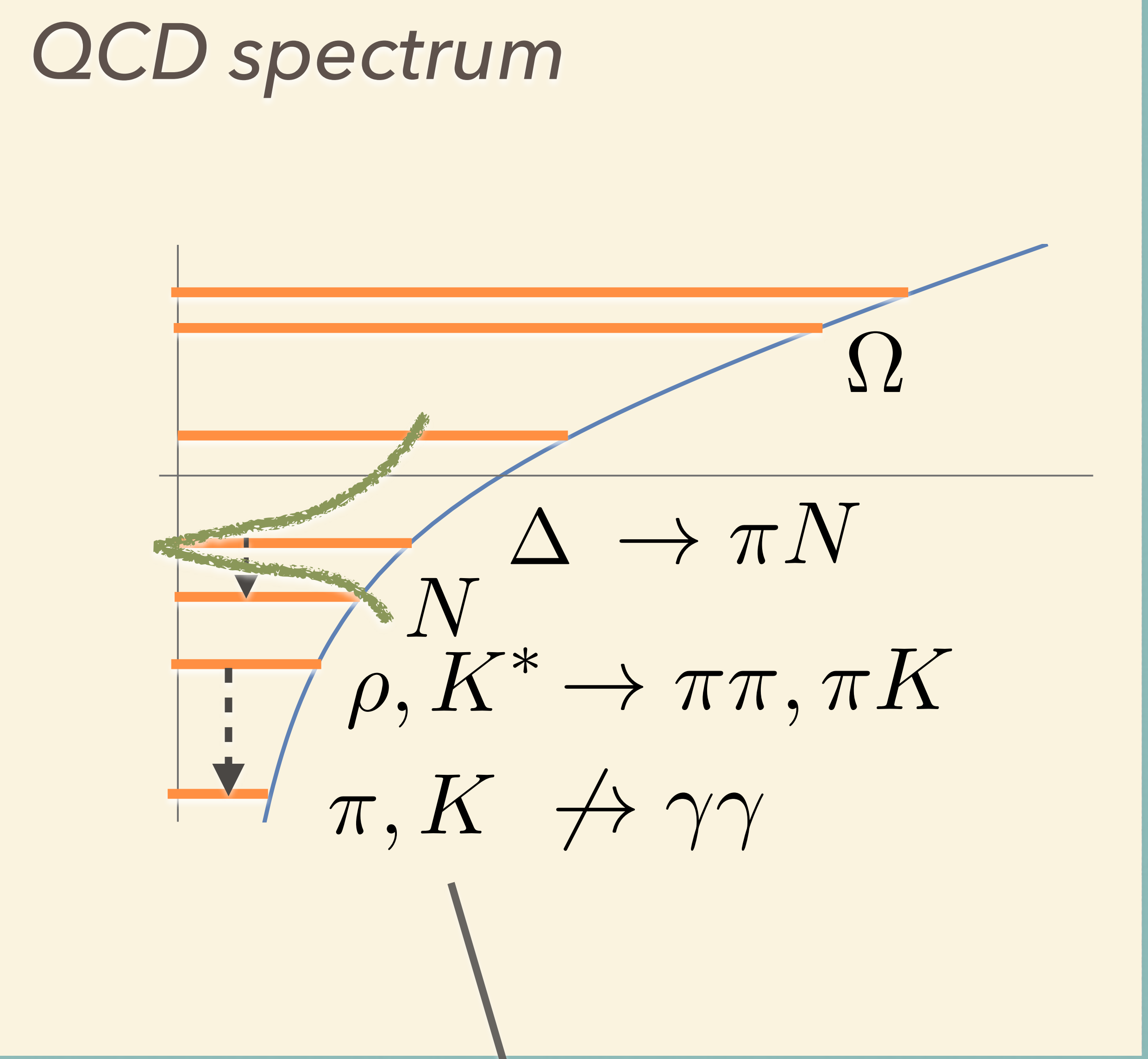
HADRON RESONANCE GAS
&
S-MATRIX FORMULATION

HADRON RESONANCE GAS MODEL

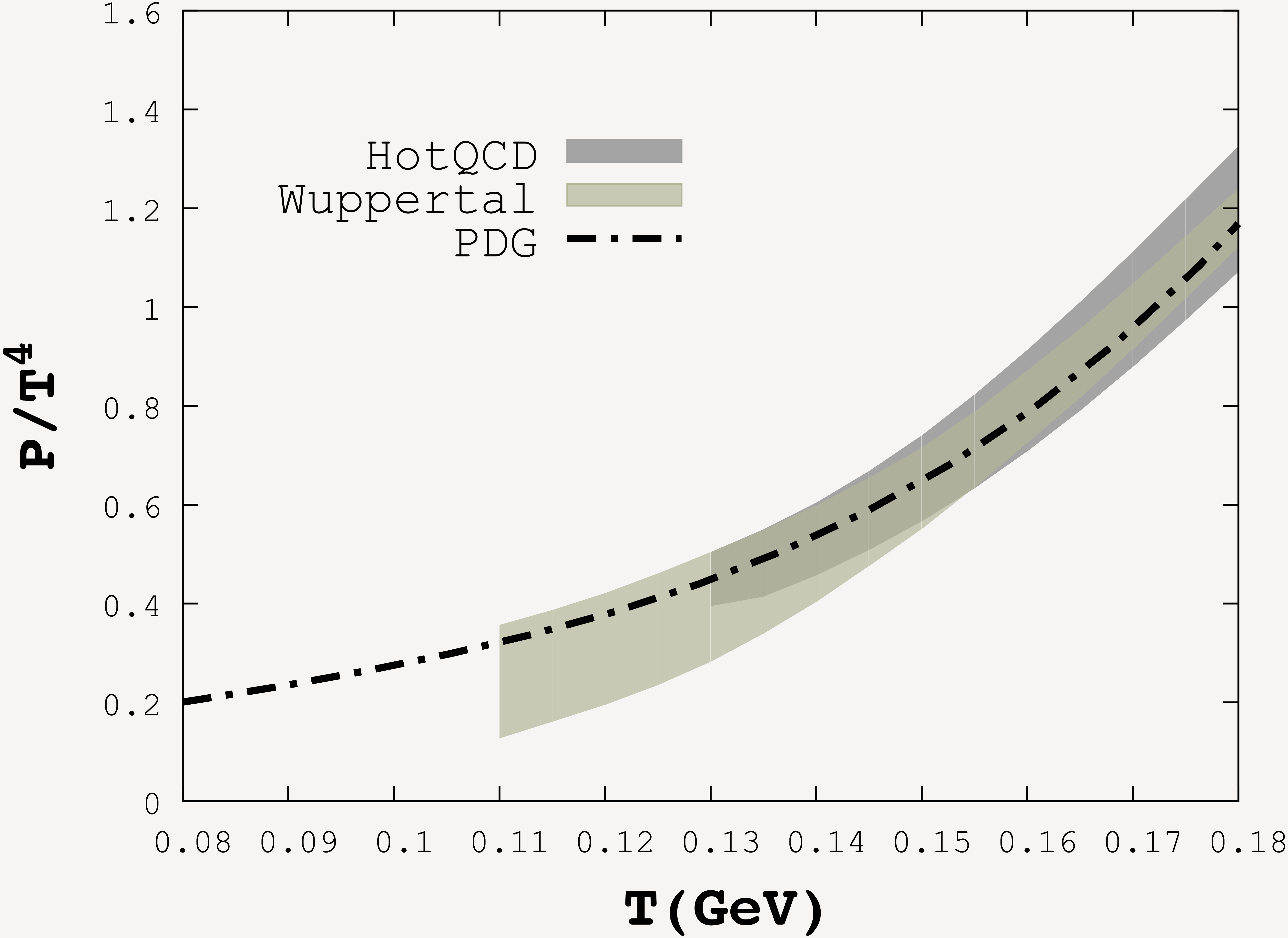
- Confinement

physical quantities



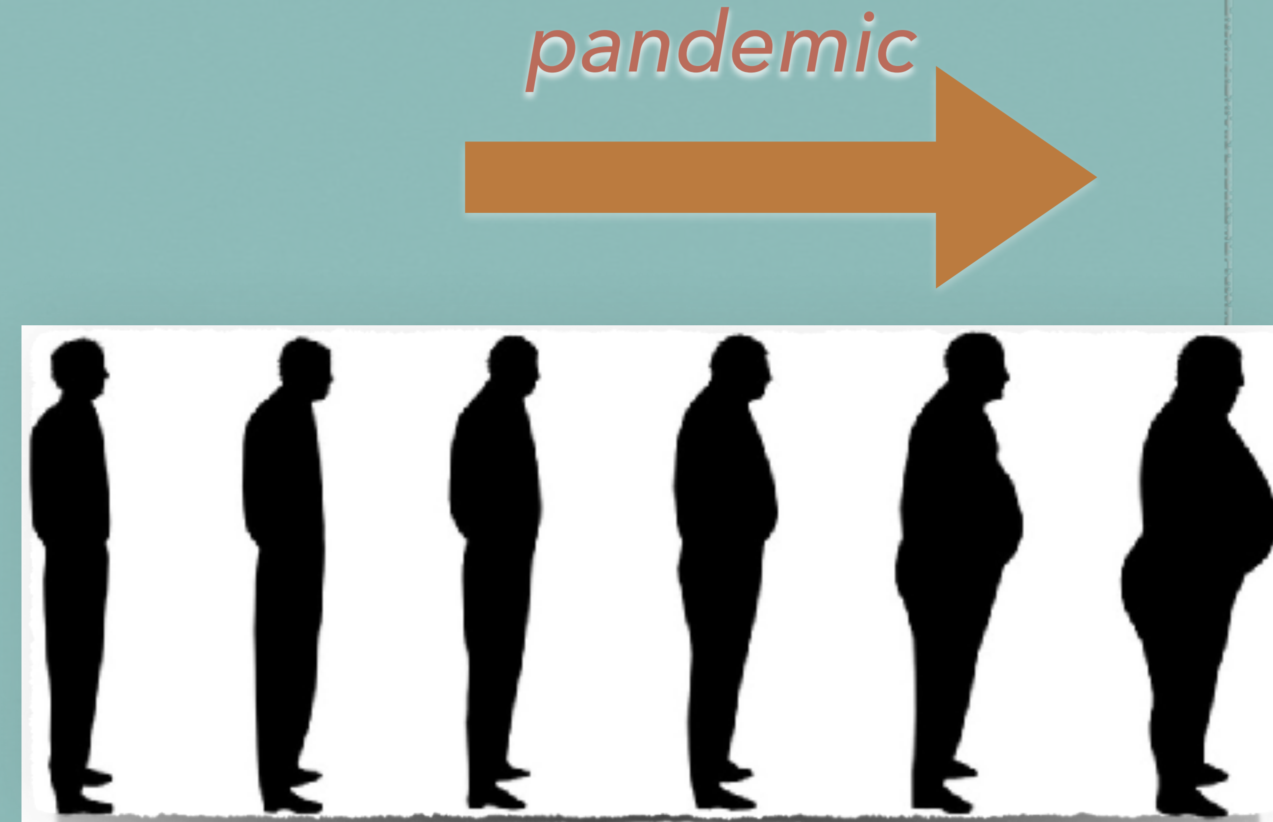
$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

confinement +
spontaneous chiral symmetry breaking



HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

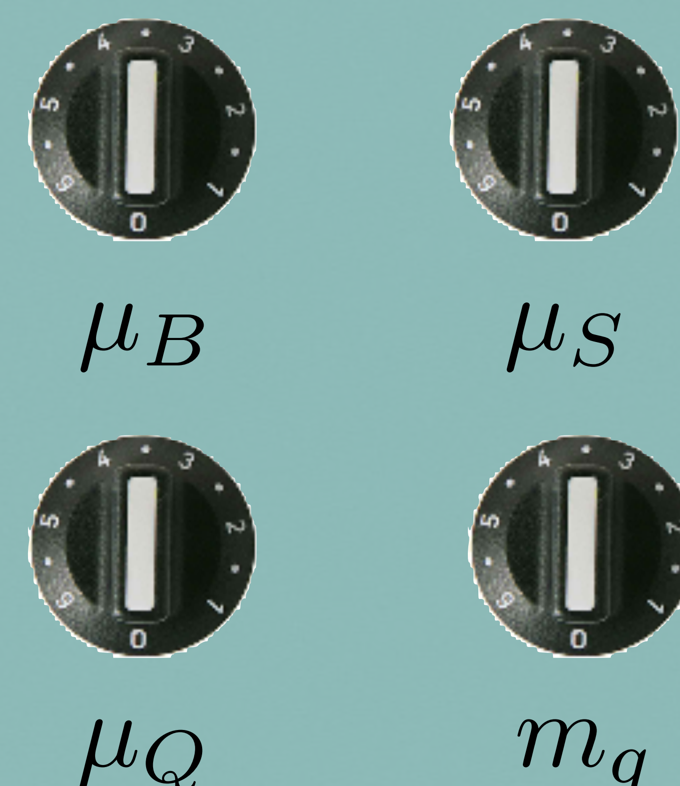
FLUCTUATIONS

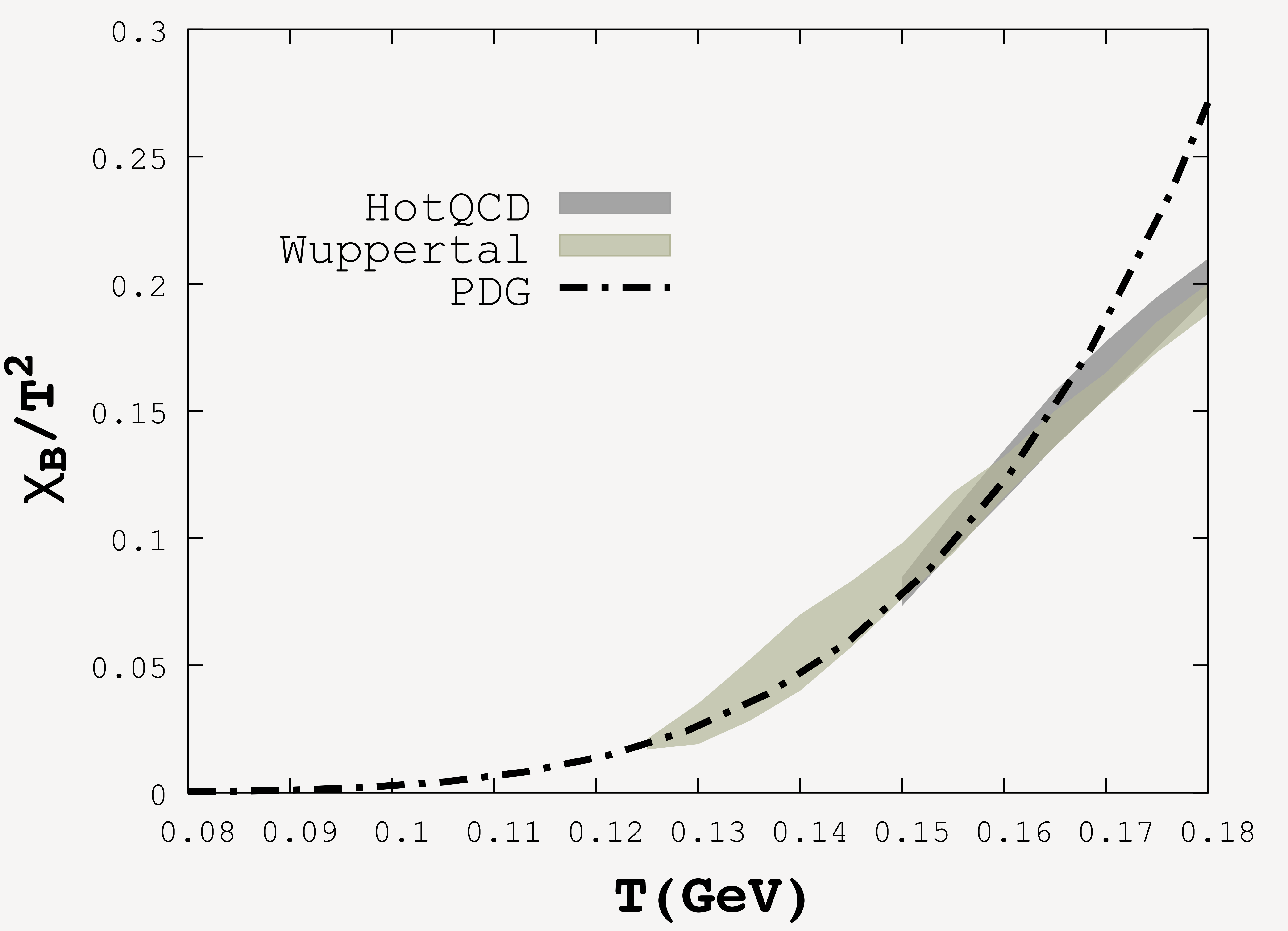


- studying the system by linear response

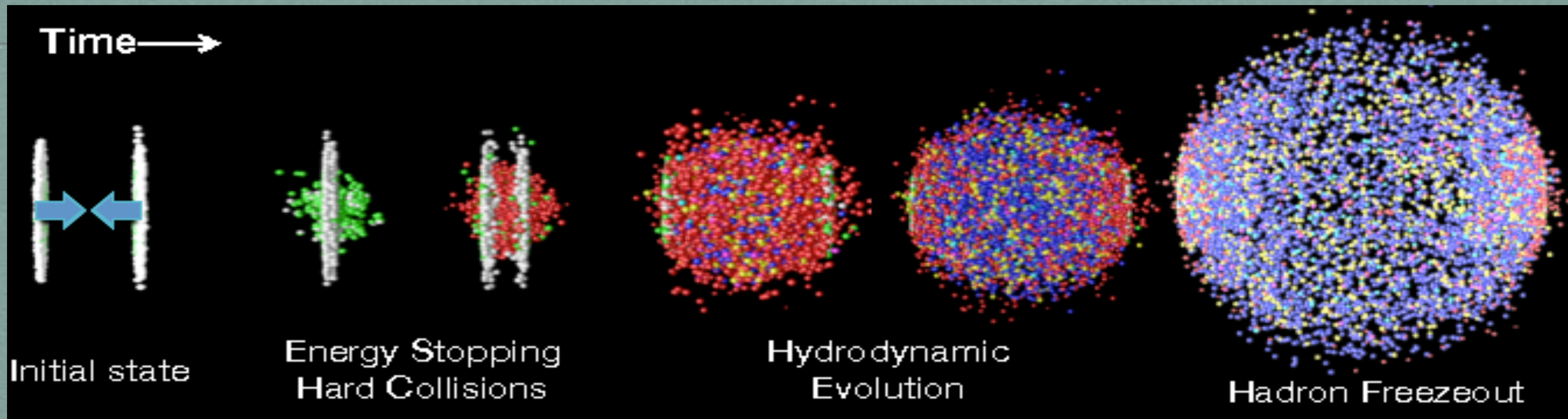
$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

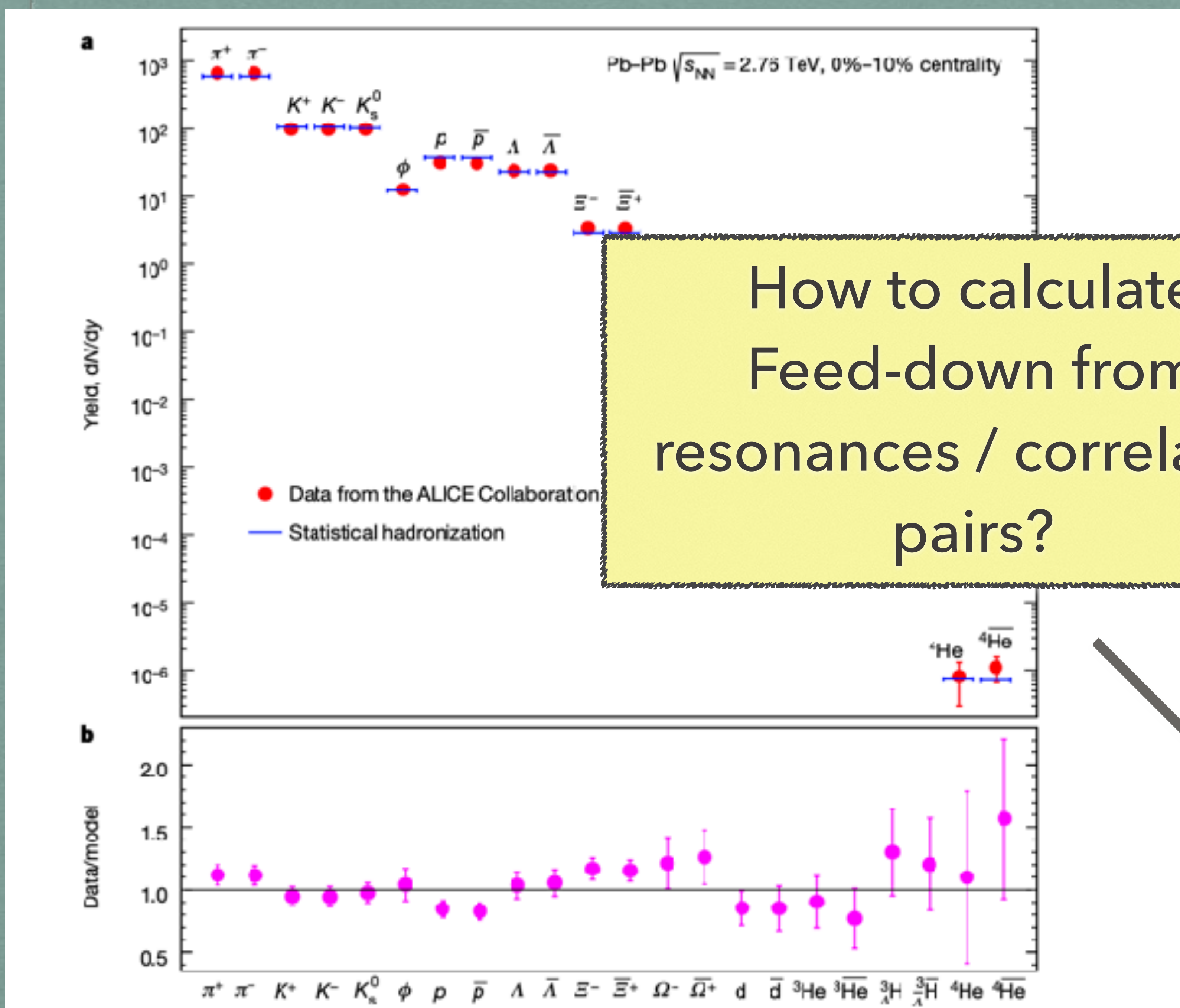
$$\chi_{B,S,\ldots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \ldots} \ln Z$$





HotQCD
Wuppertal
PDG
χ_B/T^2
T(GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18





How to calculate Feed-down from resonances / correlated pairs?

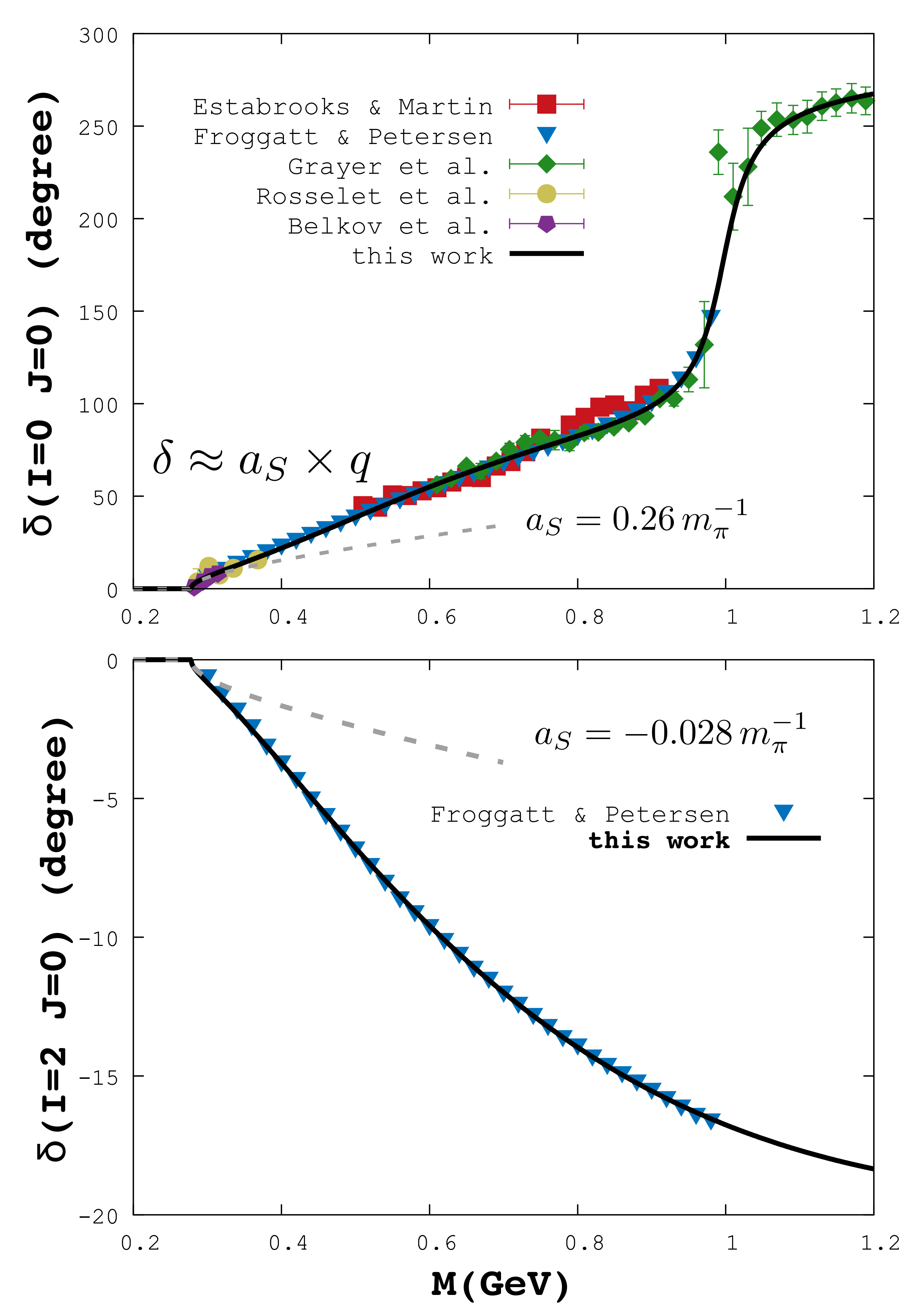
freezeout hadrons yields described by HRG

Freezeout parameters

$$T^f, \mu_B^f, \mu_S^f, \mu_Q^f, \ldots$$

A. Andronic et al 2018 Nature **561,** 321-330 (2018) phase shifts encode hadronic interactions.

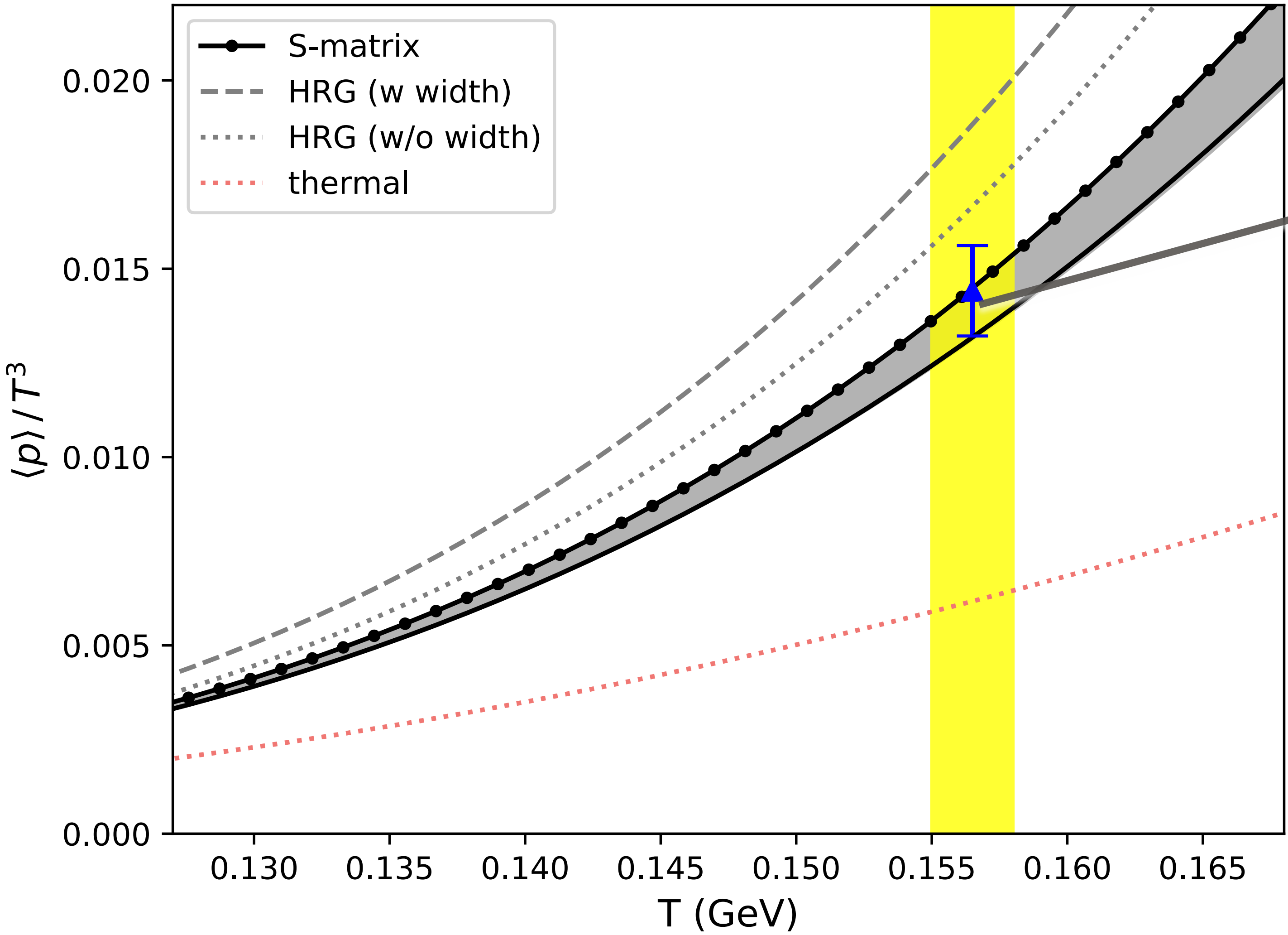
- positive:
  - attractive forces
  - resonances formation
- negative:
  - repulsive forces
  - hard-core



poles and roots effects

W. Broniowski, F. Giacosa and V. Begun, PRC **92**, 034905 (2015)

WHAT IS POSSIBLE WITH S-MATRIX?



thermal model est.

ALICE proton yield

Pb-Pb @ 2.76 TeV

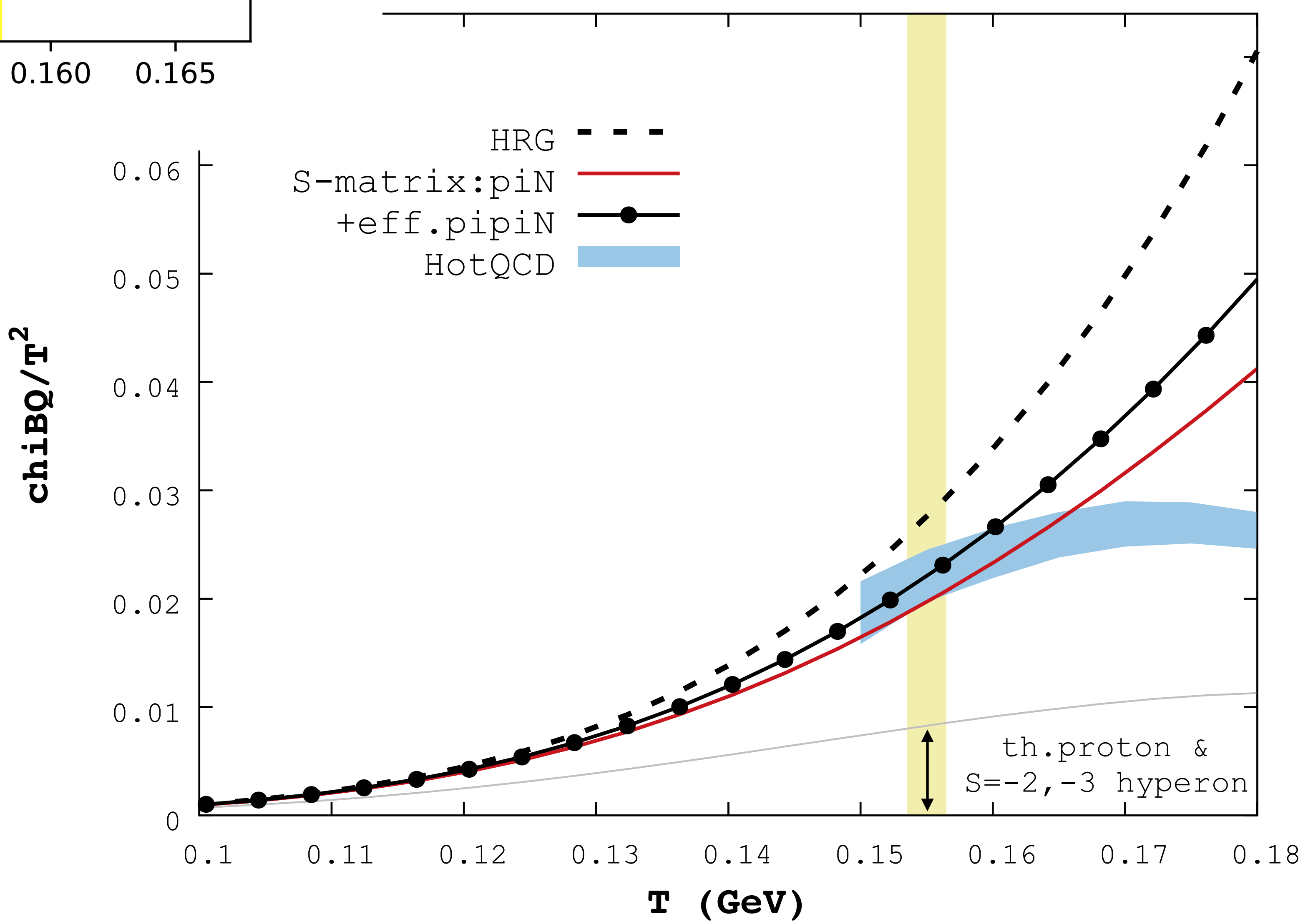
Phys. Lett. B **792**, 304 (2019)

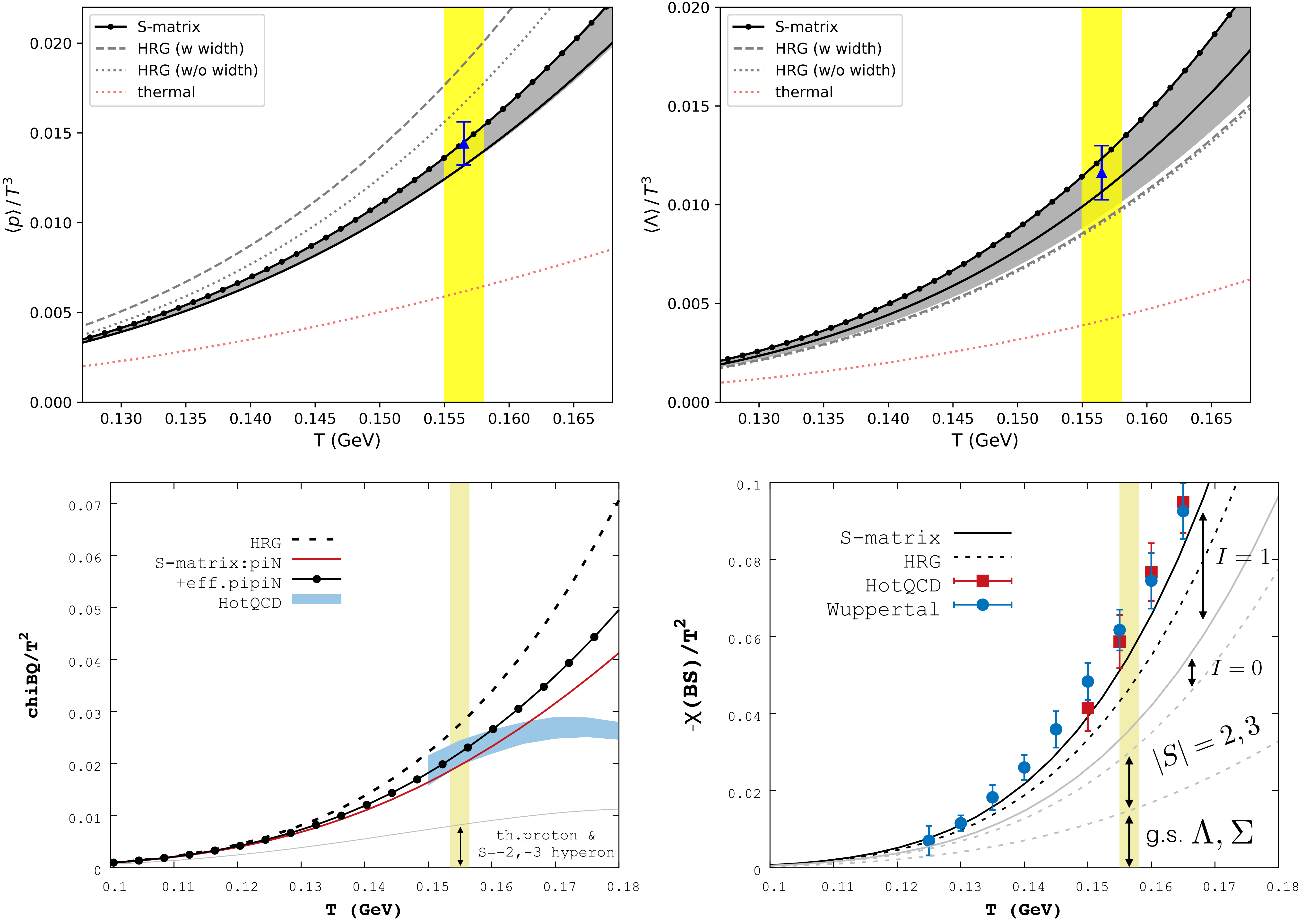
PRC **103**, 014904 (2021)

LQCD result on chiBQ

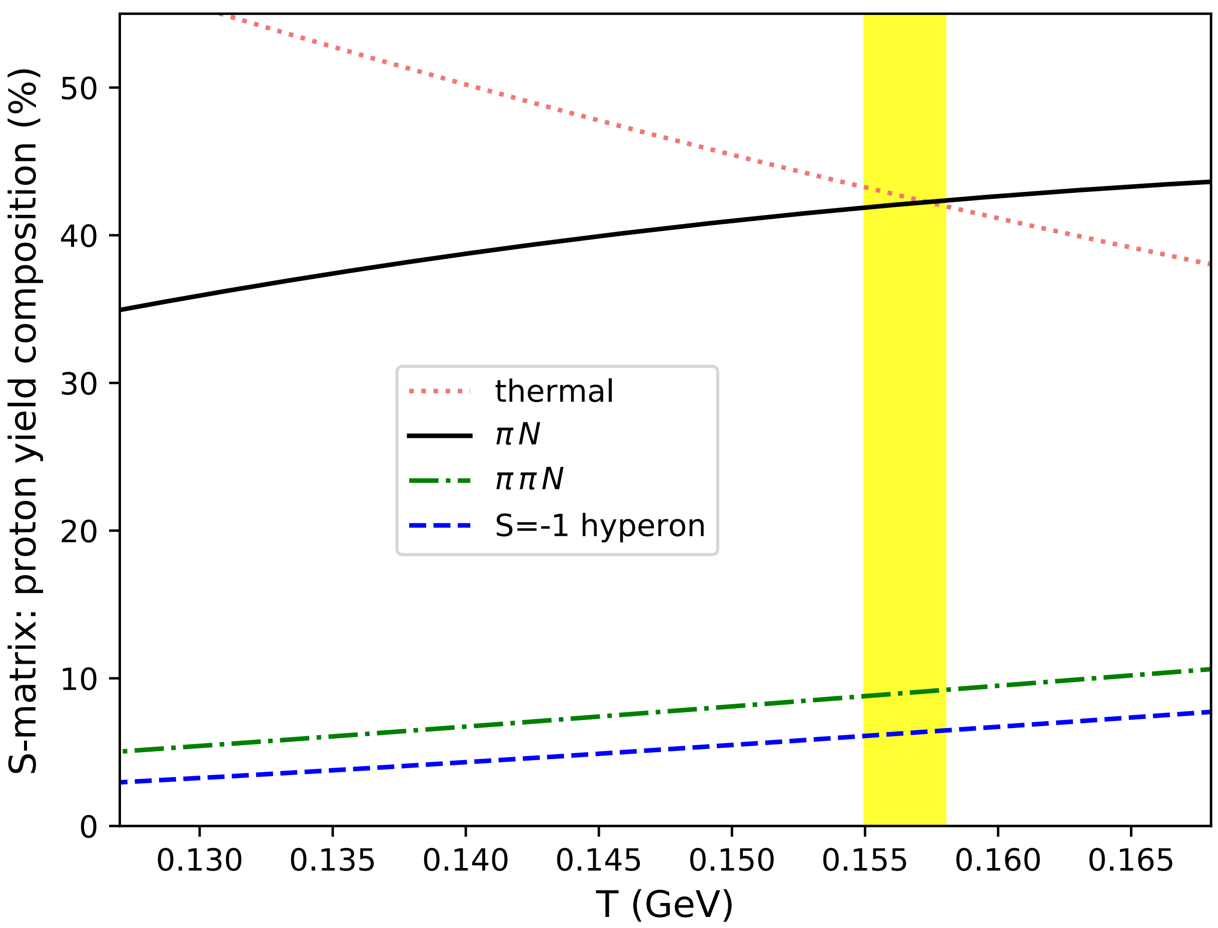
A. Bazavov, et al.,
Phys. Rev. D 86 (2012) 034509.

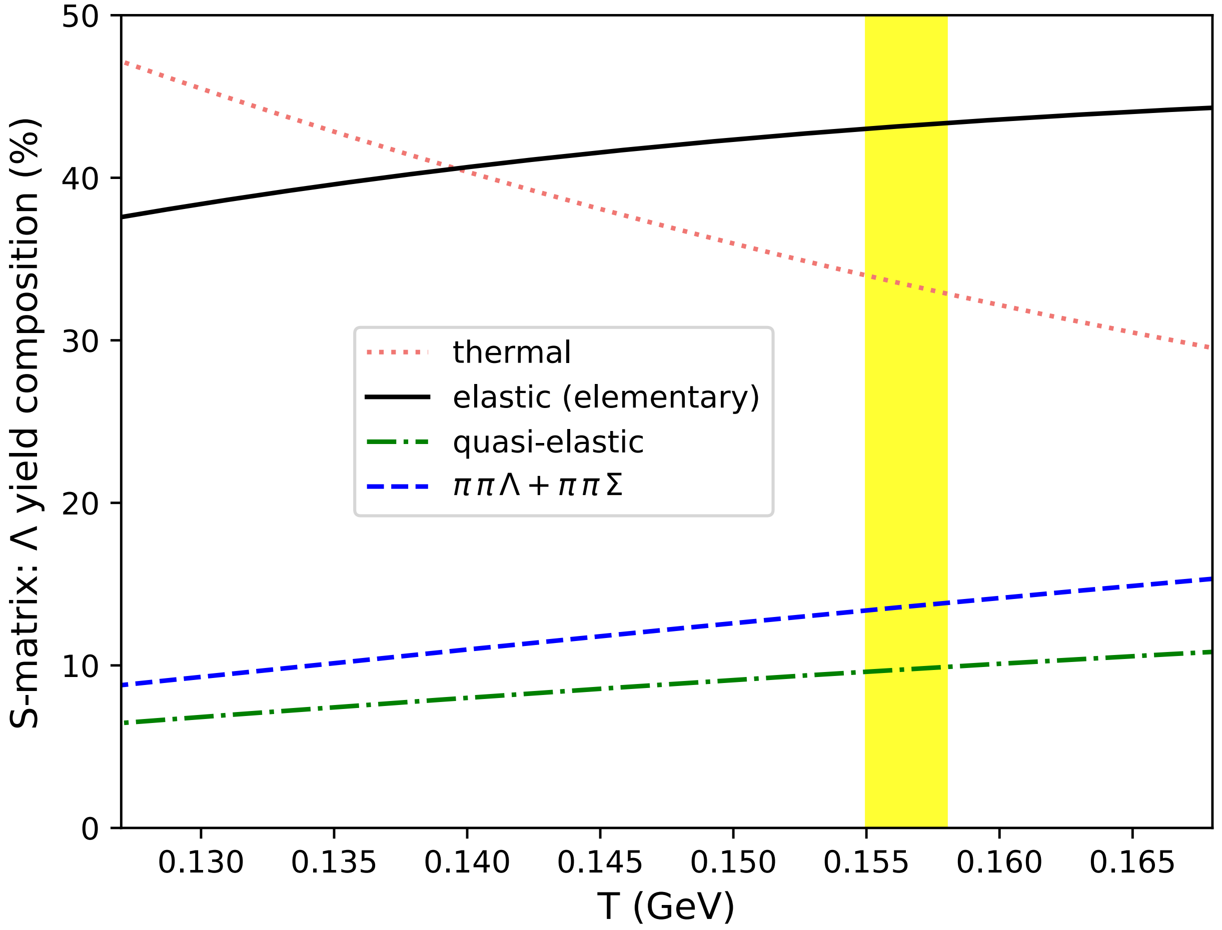
see also
Bellwied et al.
Phys. Rev. D 101, 034506 (2020)





S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩/T^3
T (GeV)
⟨Λ⟩/T^3
T (GeV)
HRG
S-matrix:piN
+eff.pipiN
HotQCD
th.proton &
S=-2,-3 hyperon
chiBQ/T^2
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
-χ(BS)/T^2
T (GeV)





SAID GWU

piN phase shifts
pipiN BGs
hyperons

JPAC

Coupled-Channel system:

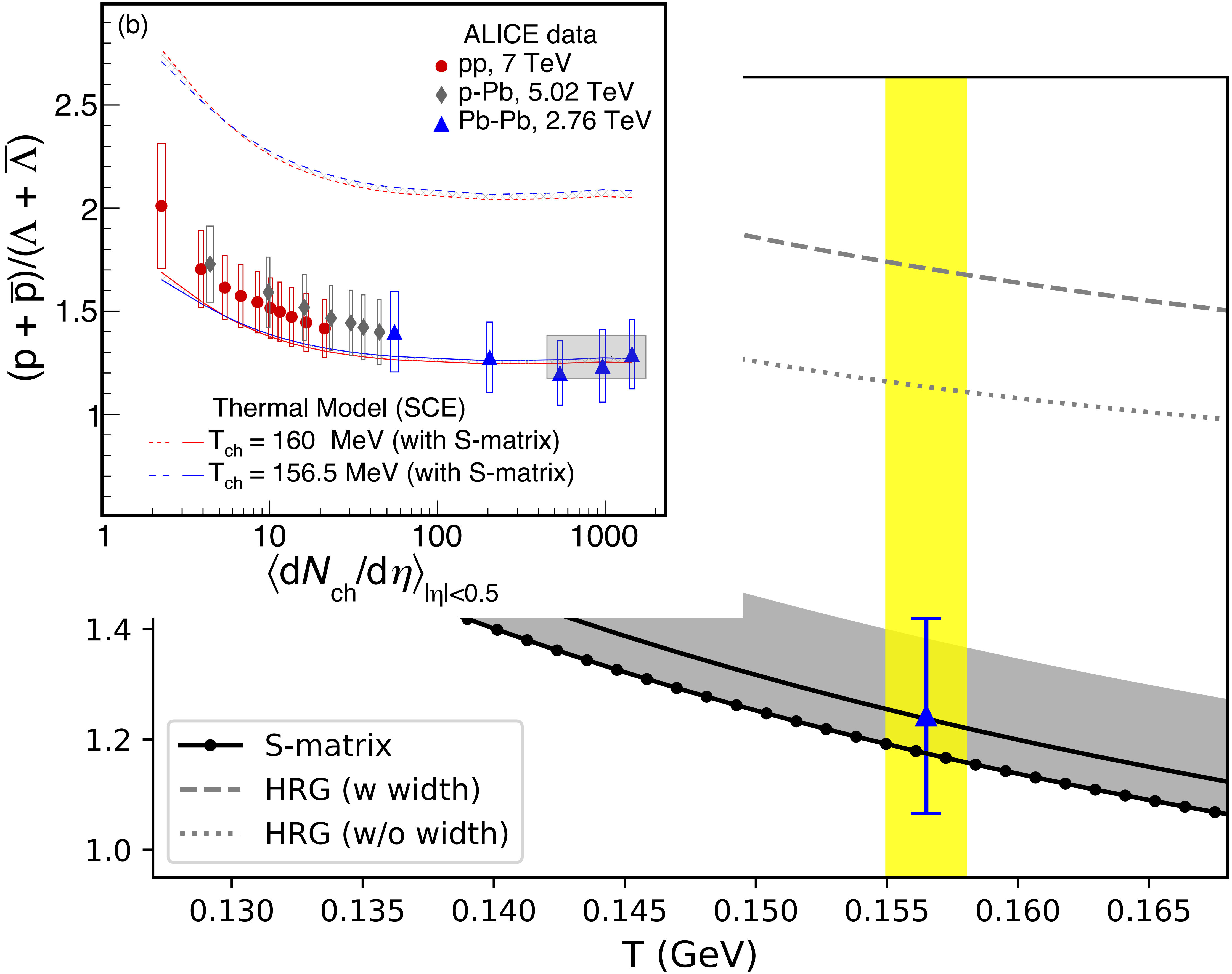
$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$

extra hyperon states beyond PDG

unitarity BGs

consistent treatment of res and non-res. int.

ratio of yields



S-MATRIX APPROACH TO STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

R. Venugopalan and M. Prakash,
Nucl. Phys. **A546**, 718 (1992).

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \dots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

orbital L:
S, P, D, F, etc..

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

DENSITY OF STATES

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr}\left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

single channel, elastic $\frac{1}{\pi}\frac{d}{dE}\delta$

N-body &
Coupled-Channel problem

multi (coupled) channel $\frac{1}{\pi}\frac{d}{dE}\mathcal{Q}$

PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

sum over eigenphases

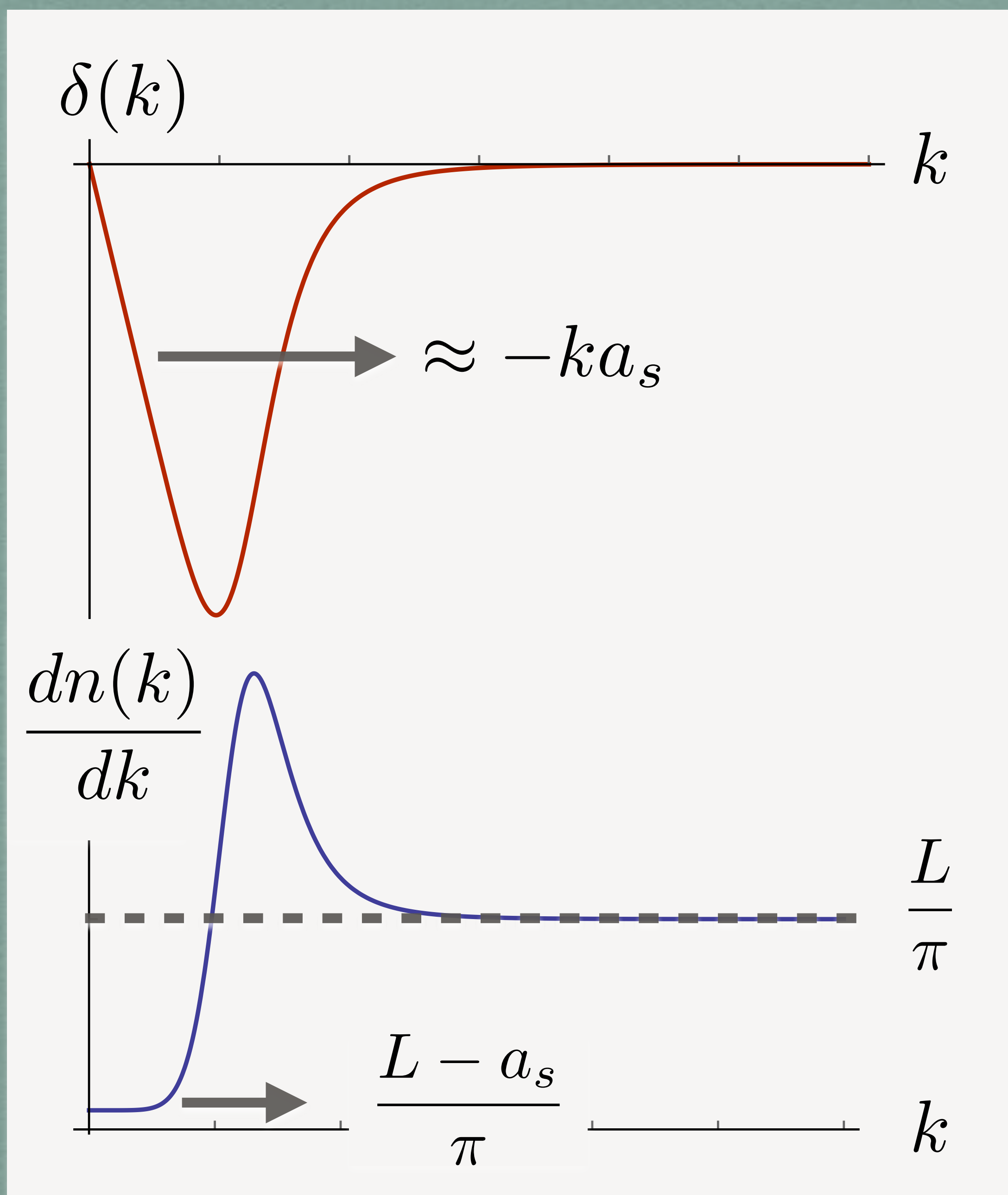
$$\mathcal{Q} = \frac{1}{2}\,\mathrm{Im}\mathrm{Tr}\,\ln S$$

$$= \sum_{\text{channels}} \lambda_i$$

REPULSIONS
&
RESONANCES

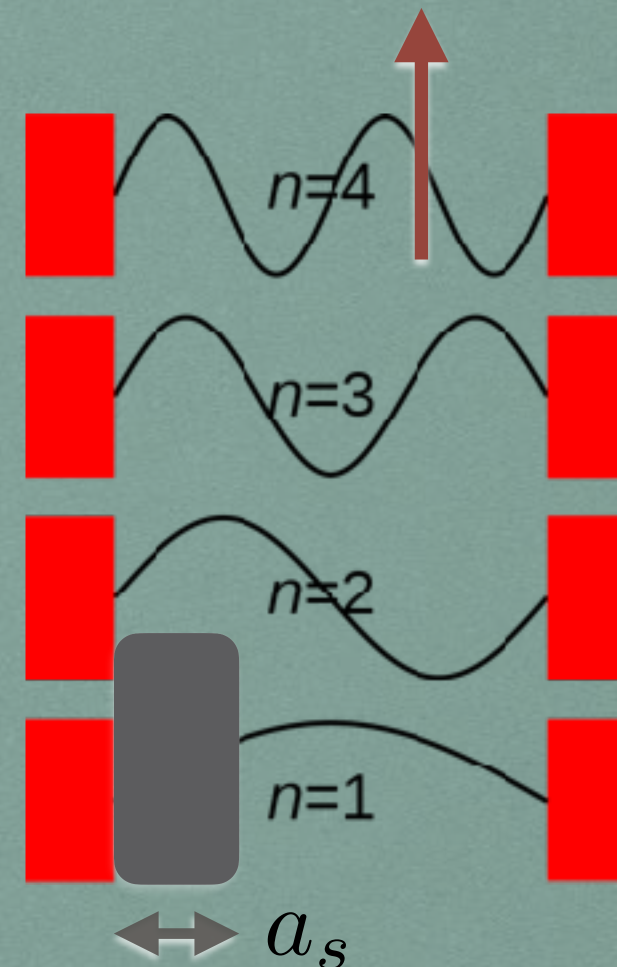
PML, Francesco Giocosa EPJC **79** (2019)

PHASE SHIFT AND DENSITY OF STATES



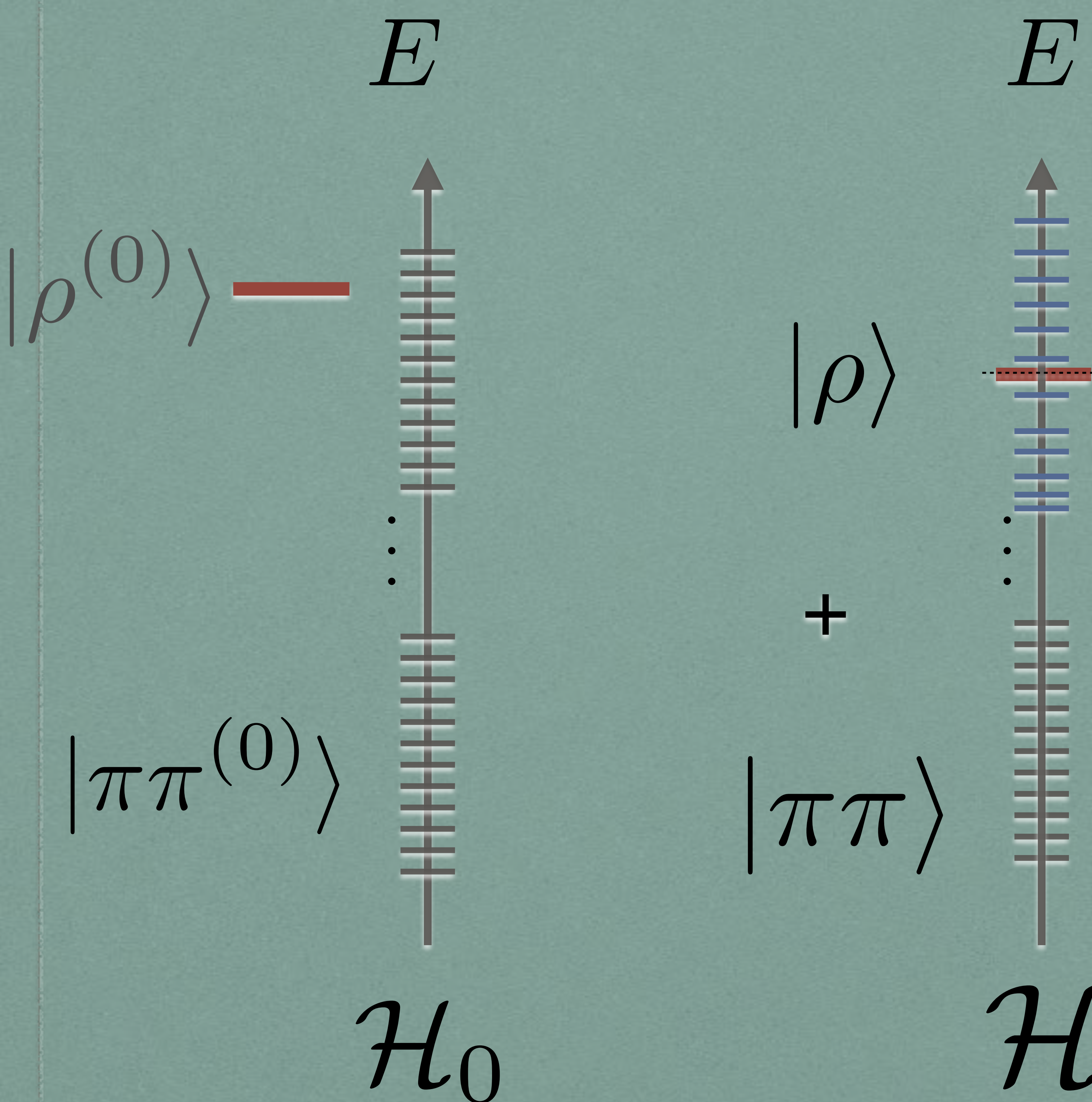
$(0) = \frac{n\pi}{L}$

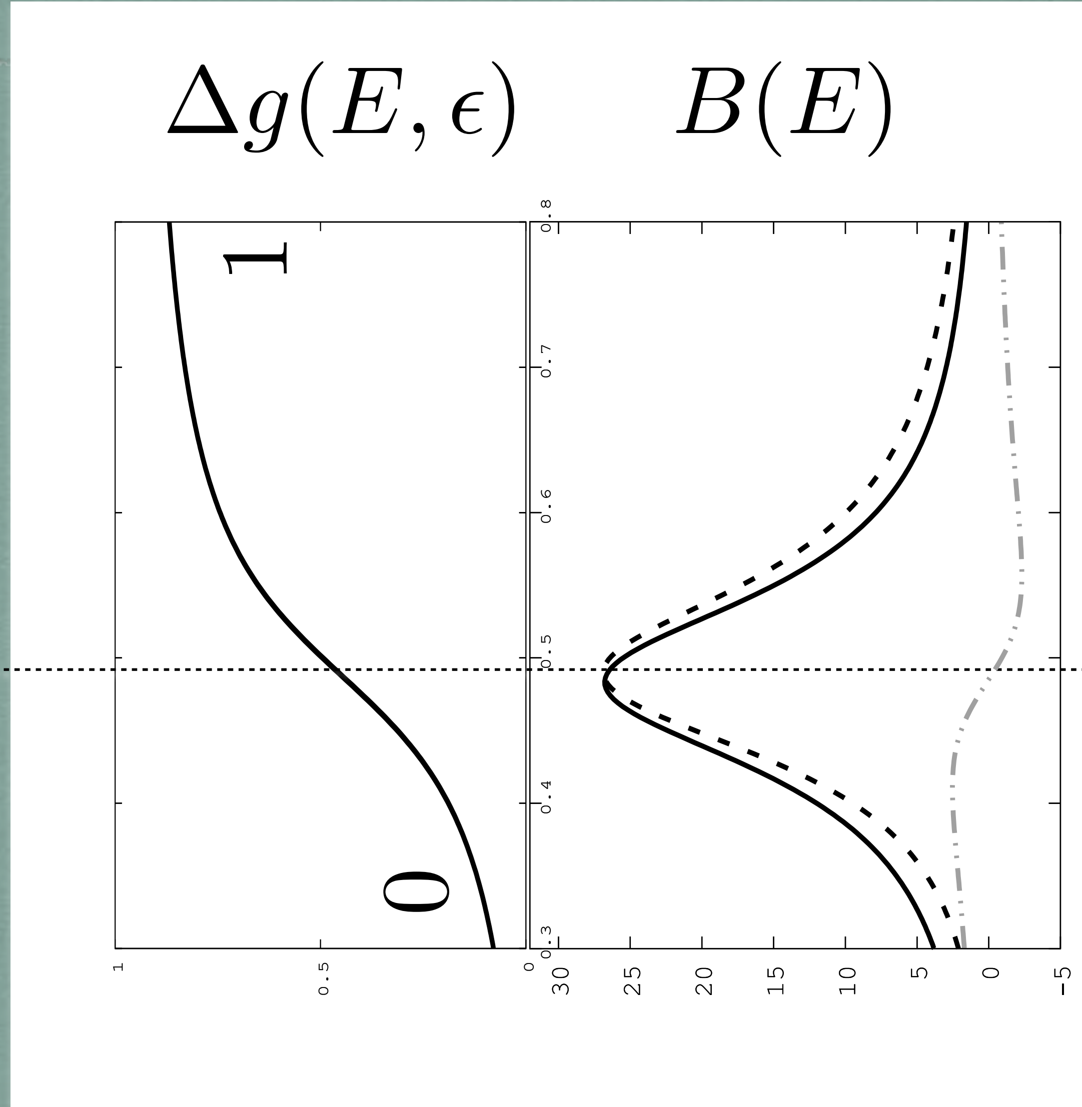
ring potential



density of states

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE}\Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta\mathcal{H}_0}$ vs $\mathrm{Tr}\, e^{-\beta\mathcal{H}}$

for Delta

W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).

PROTON PUZZLE

A. Andronic, P. Braun-Munzinger, B. Friman, PML,
K. Redlich, J. Stachel
Phys. Lett. B **792**, 304 (2019)

proton yields

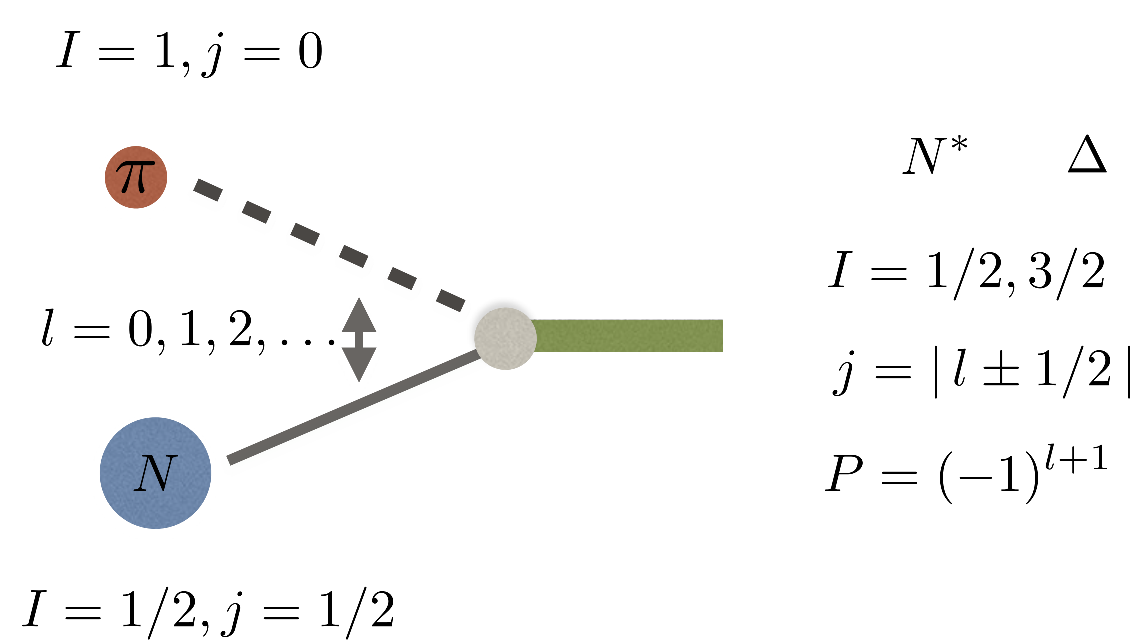
$$\langle p \rangle = \langle p \rangle_{th.} + \langle p \rangle_{N^*} + \langle p \rangle_{\Delta} + \langle p \rangle_{hyp.} + \dots$$

$$\langle p \rangle_{N^*} = \frac{2}{3} \langle N^*_{Q=0} \rangle + \frac{1}{3} \langle N^*_{Q=1} \rangle \approx \frac{1}{2} \langle N^* \rangle$$

$$\langle p \rangle_{\Delta} = \langle \Delta_{Q=2} \rangle + \frac{2}{3} \langle \Delta_{Q=1} \rangle + \frac{1}{3} \langle \Delta_{Q=0} \rangle \approx \frac{1}{2} \langle \Delta \rangle$$

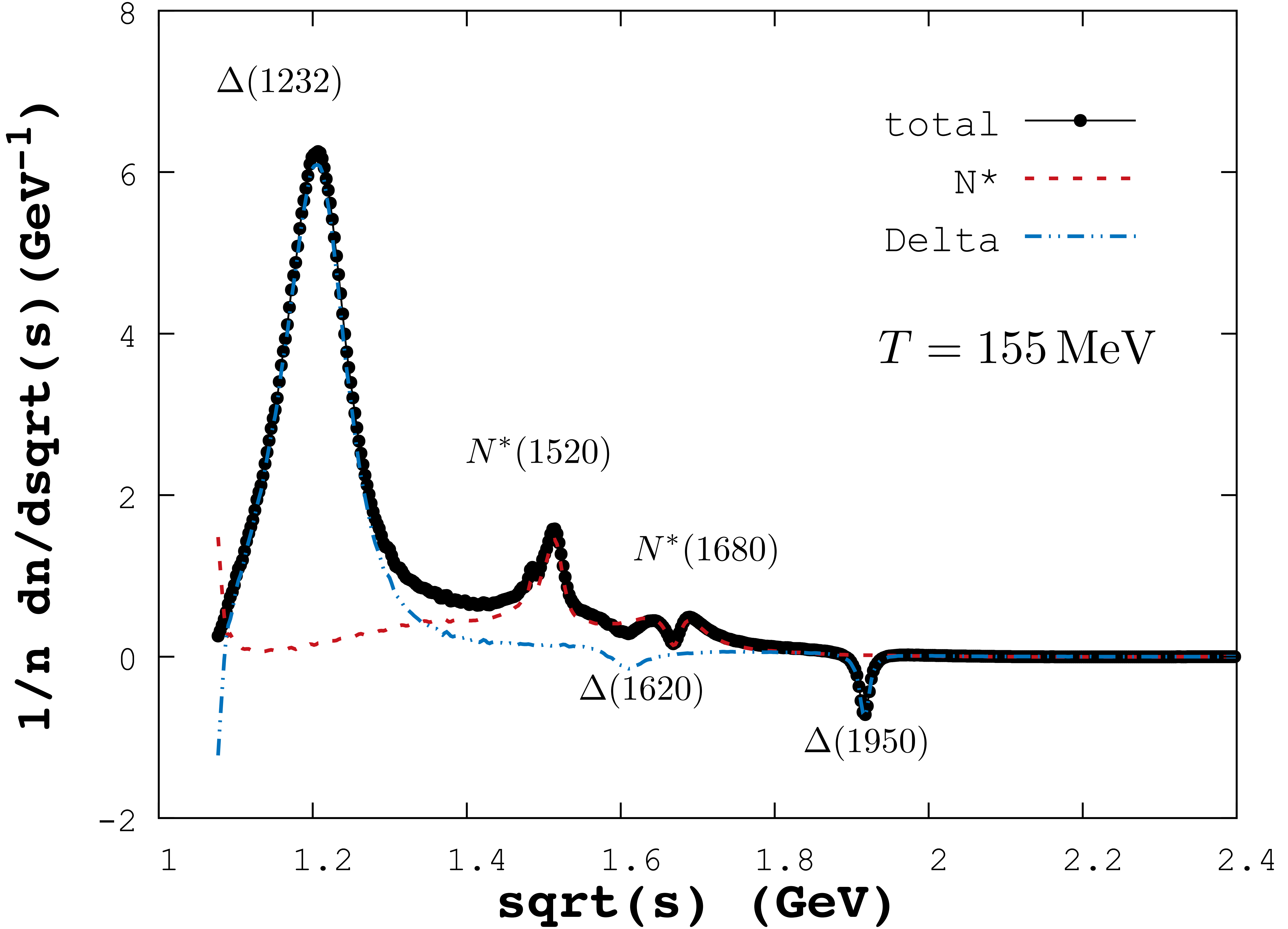
isospin symmetric @ LHC

$$n_a(T) = \int_{m_{th}}^{\infty} \frac{dM}{2\pi} B_a(M)\, n^{(0)}(T, M)$$

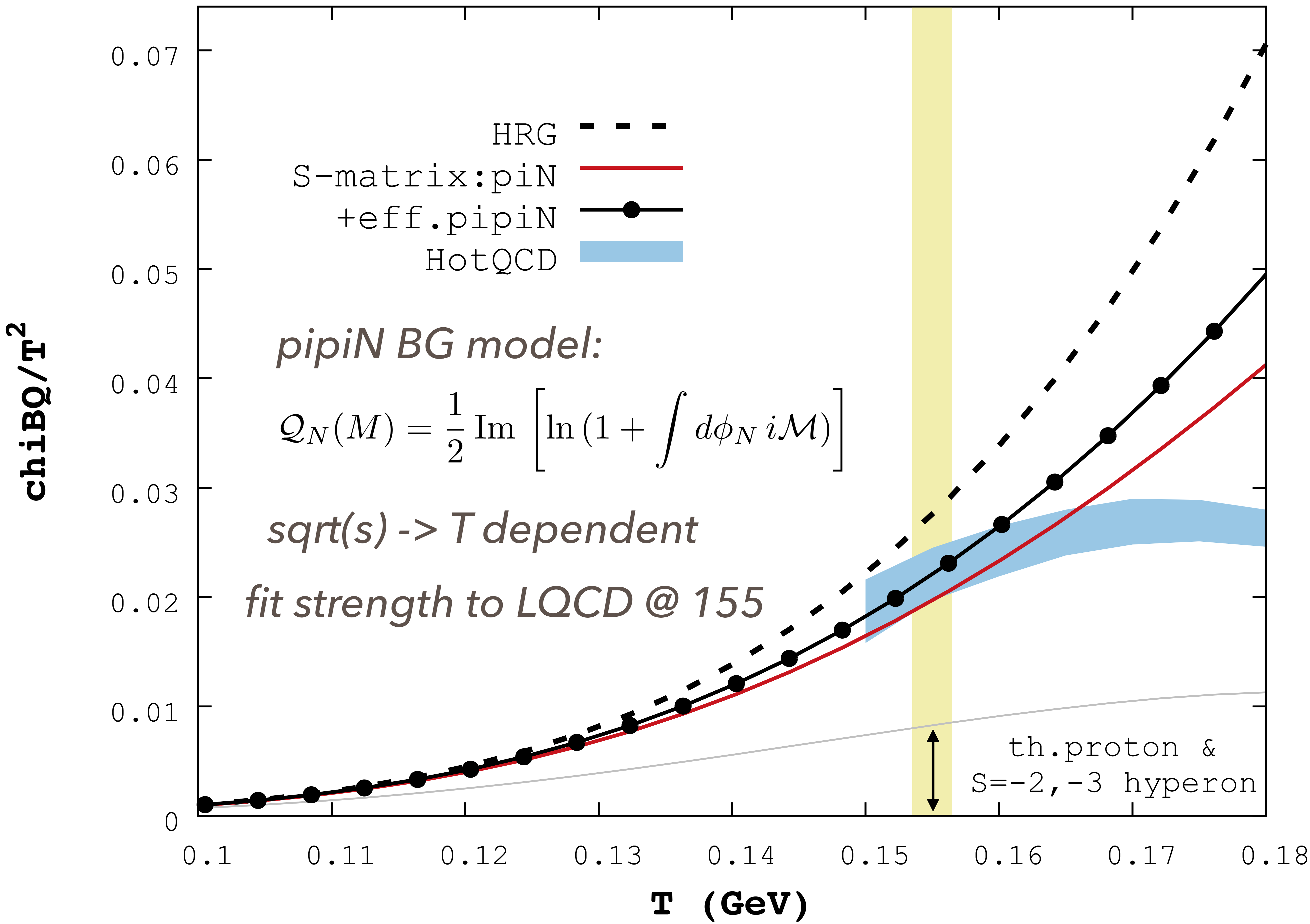


empirical phase shifts

resonant and non-resonant int.



8
6
4
2
0
-2
1
1.2
1.4
1.6
1.8
2
2.2
2.4
sqrt(s) (GeV)
1/n dn/dsqrt(s) (GeV^{-1})
total
N*
Delta
$\Delta(1232)$
$N^*(1520)$
$N^*(1680)$
$\Delta(1620)$
$\Delta(1950)$
$T = 155\,\mathrm{MeV}$


chiBQ/T^2
T (GeV)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
HRG
S-matrix:piN
+eff.pipiN
HotQCD
pipiN BG model:
$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$
sqrt(s) -> T dependent
fit strength to LQCD @ 155
th.proton &
S=-2,-3 hyperon

S = -1 HYPERONS
COUPLED CHANNEL SYSTEM

JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma
PRC **103**, 014904 (2021)

PHASE SHIFT FROM PWA

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Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

-------------------------------------------------------------------
```

- 1 → $\bar{K}N$,
- 2 → $\pi\Sigma$,
- 3 → $\pi\Lambda$,
- 4 → $\eta\Lambda$,
- 5 → $\eta\Sigma$,
- 6 → $\bar{K}_1 N$,
- 7 → $[\bar{K}_3 N]_-$,
- 8 → $[\bar{K}_3 N]_+$,
- 9 → $[\pi\Sigma^*]_-$,
- 10 → $[\pi\Sigma^*]_+$,
- 11 → $[\bar{K}\Delta]_-$,
- 12 → $[\bar{K}\Delta]_+$,
- 13 → $[\pi\Lambda(1520)]_-$,
- 14 → $[\pi\Lambda(1520)]_+$,
- 15 → $\pi\pi\Lambda$,
- 16 → $\pi\pi\Sigma$.

elastic scatterings (elementary)

quasi elastic scatterings

unitarity background

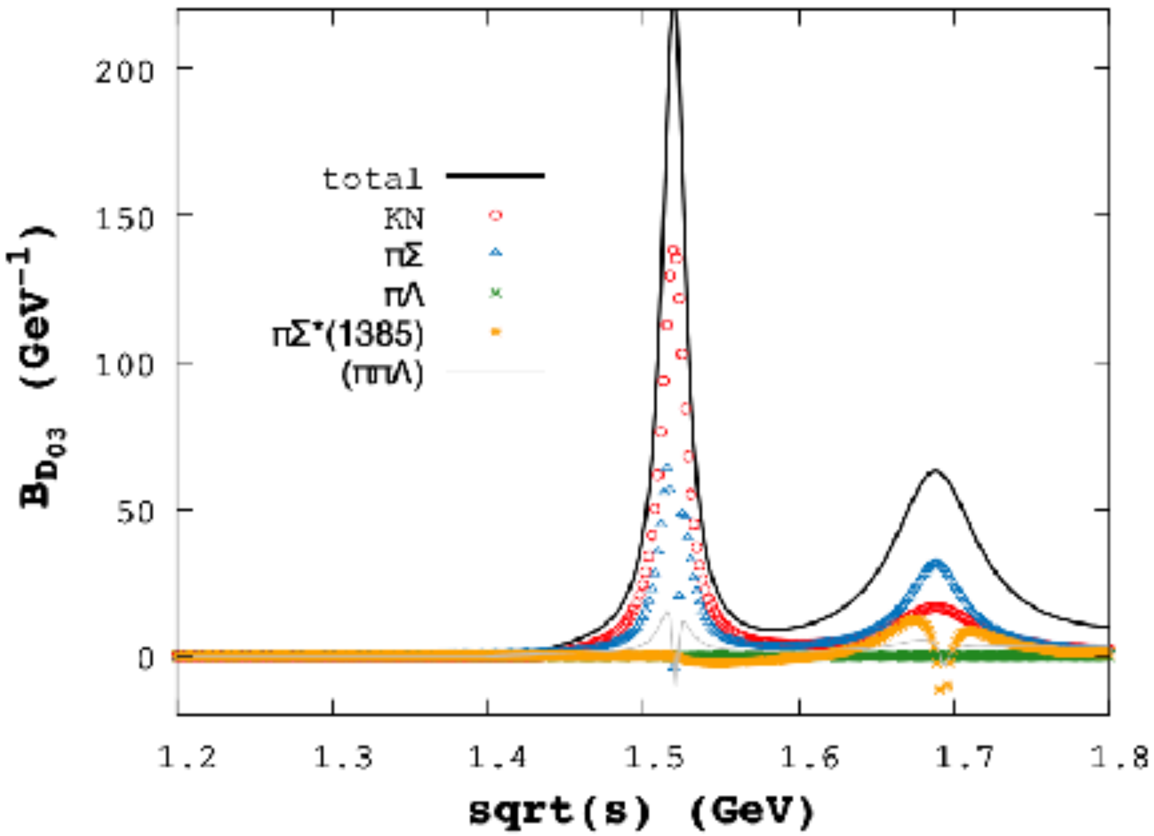
$$B(M) = \frac{1}{2}\,\mathrm{Im}\,\mathrm{Tr}\,\left[S^{-1}\frac{\partial}{\partial M}S - \left(\frac{\partial}{\partial M}S^{-1}\right)S\right].$$

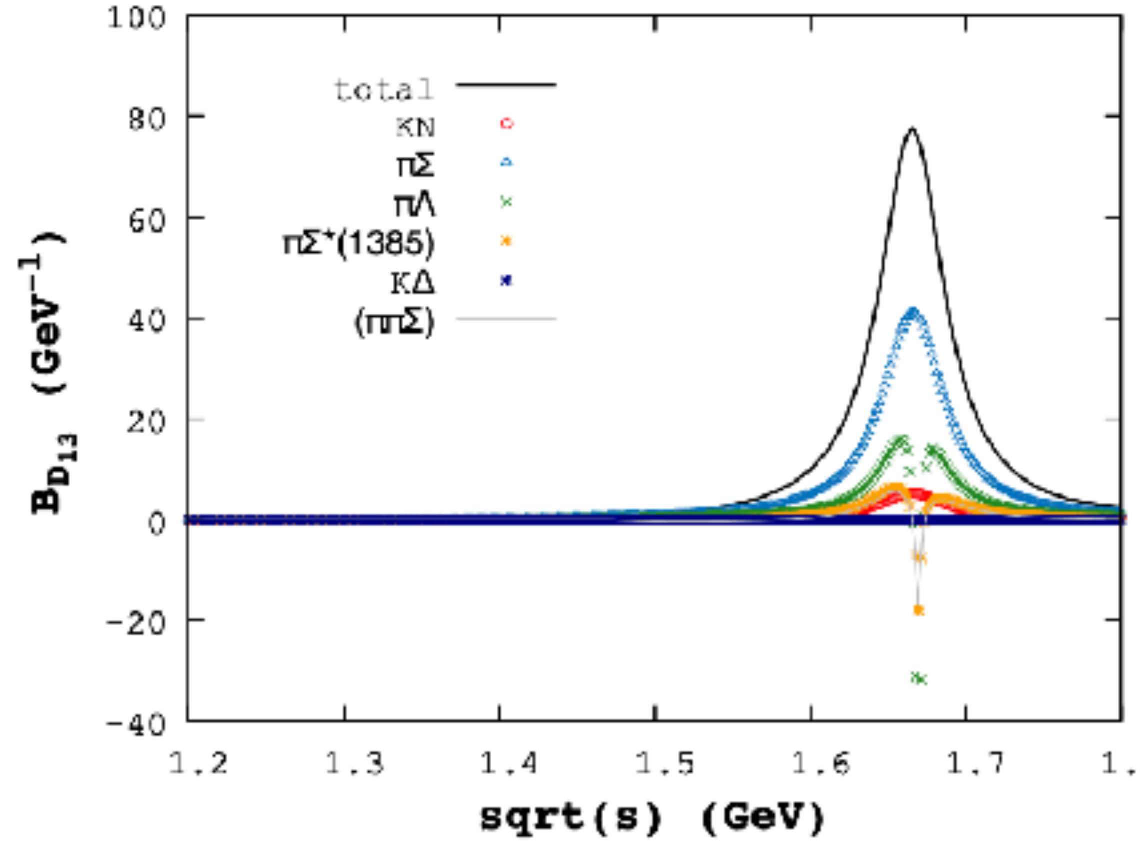
channel	elastic	channel	quasi-elastic	channel	unitarity
1	$\bar{K}N$	6	$\bar{K}_1^*N$	15	$\pi\pi\Lambda$
2	$\pi\Sigma$	7	$[\bar{K}_3^*N]_-$	16	$\pi\pi\Sigma$
3	$\pi\Lambda$	8	$[\bar{K}_3^*N]_+$		
4	$\eta\Lambda$	9	$[\pi\Sigma(1385)]_-$		
5	$\eta\Sigma$	10	$[\pi\Sigma(1385)]_+$		
		11	$[\bar{K}\Delta(1232)]_-$		
		12	$[\bar{K}\Delta(1232)]_+$		
		13	$[\pi\Lambda(1520)]_-$		
		14	$[\pi\Lambda(1520)]_+$		

$$B = \sum_{a=1}^{16} B_a$$

1520, 1690

1670





Λ(1520) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{beam} = 0.39$ GeV/c $4\pi\lambdabar^2 = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	45 ± 1%	243
$\Sigma\pi$	42 ± 1%	268
$\Lambda\pi\pi$	10 ± 1%	259
$\Sigma\pi\pi$	0.9 ± 0.1%	169
$\Lambda\gamma$	0.85 ± 0.15%	350

Λ(1690) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

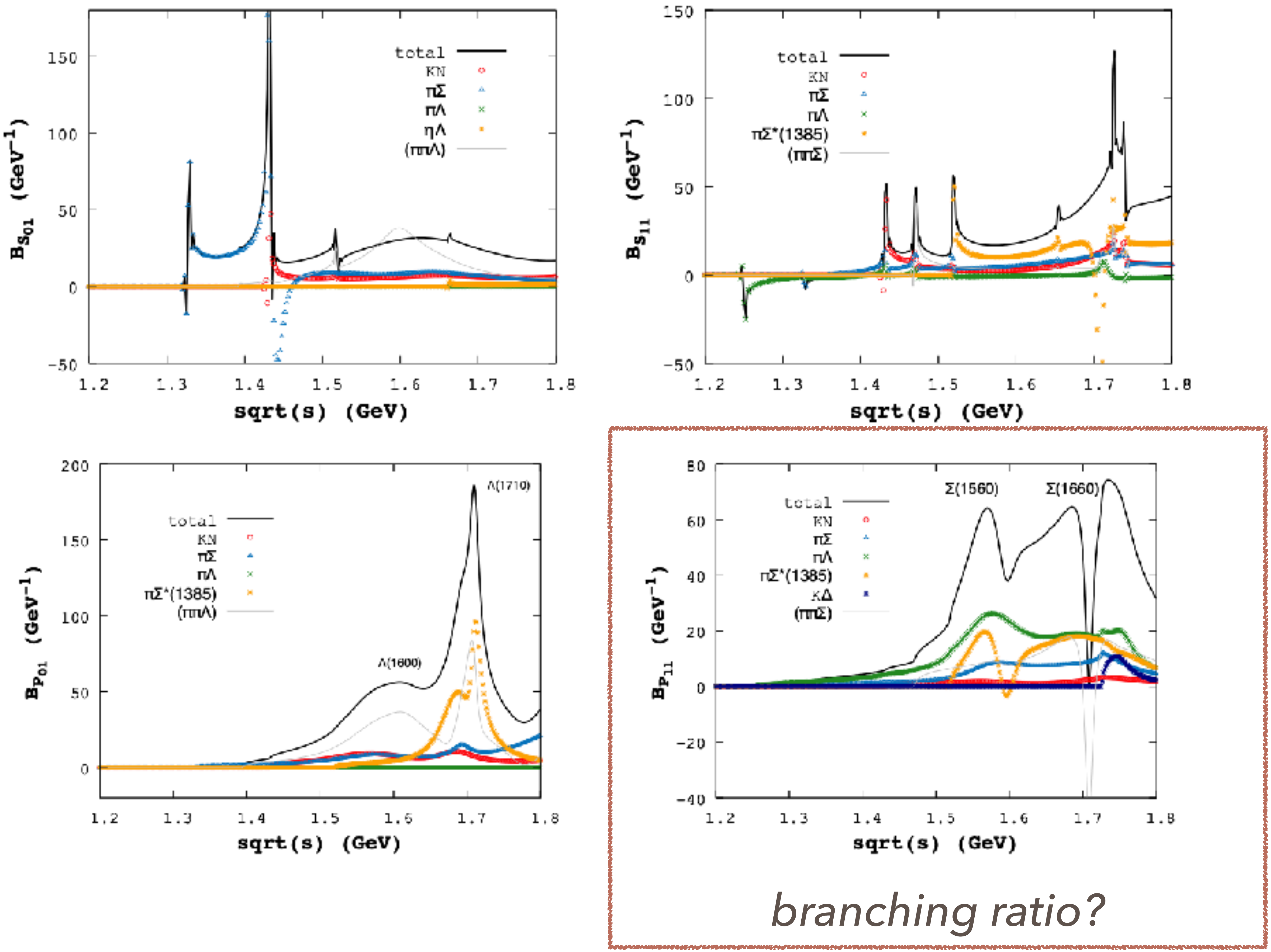
Mass m = 1685 to 1695 (≈ 1690) MeV
Full width Γ = 50 to 70 (≈ 60) MeV
$p_{beam} = 0.78$ GeV/c $4\pi\lambdabar^2 = 26.1$ mb

Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

Σ(1670) 3/2⁻ $I(J^P) = 1(\frac{3}{2}^-)$

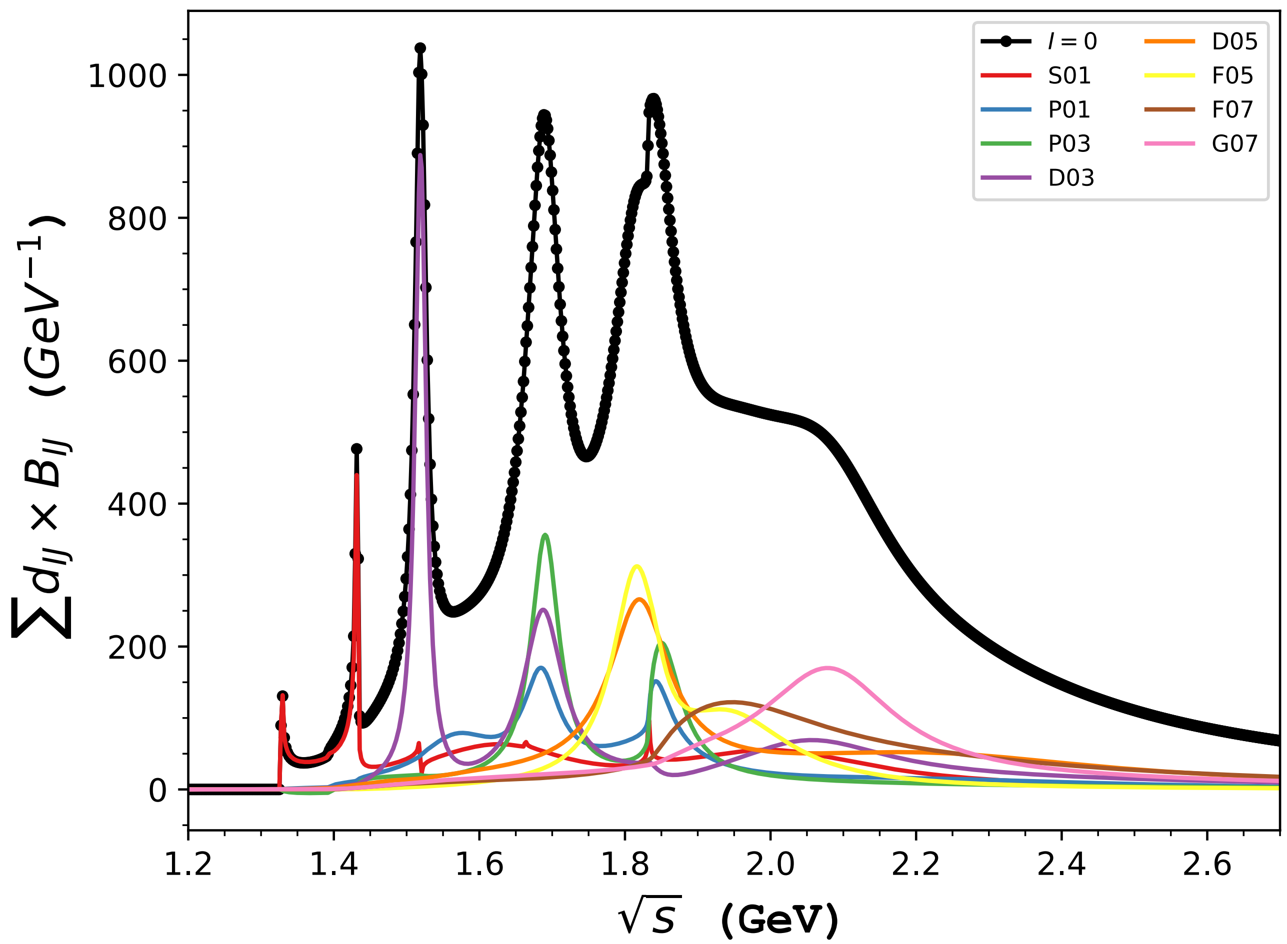
Mass m = 1665 to 1685 (≈ 1670) MeV
Full width Γ = 40 to 80 (≈ 60) MeV
$p_{beam} = 0.74$ GeV/c $4\pi\lambdabar^2 = 28.5$ mb

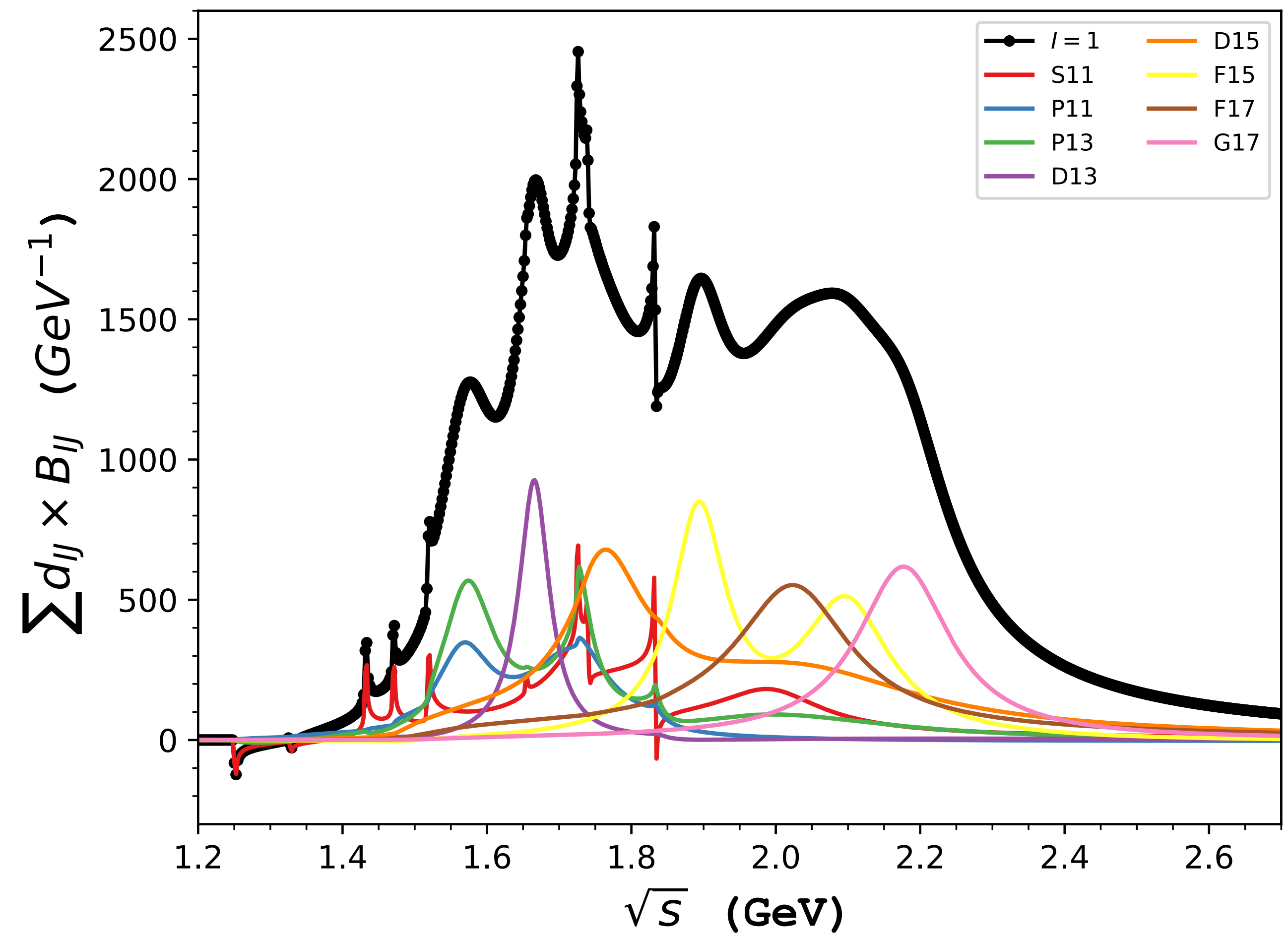
Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394

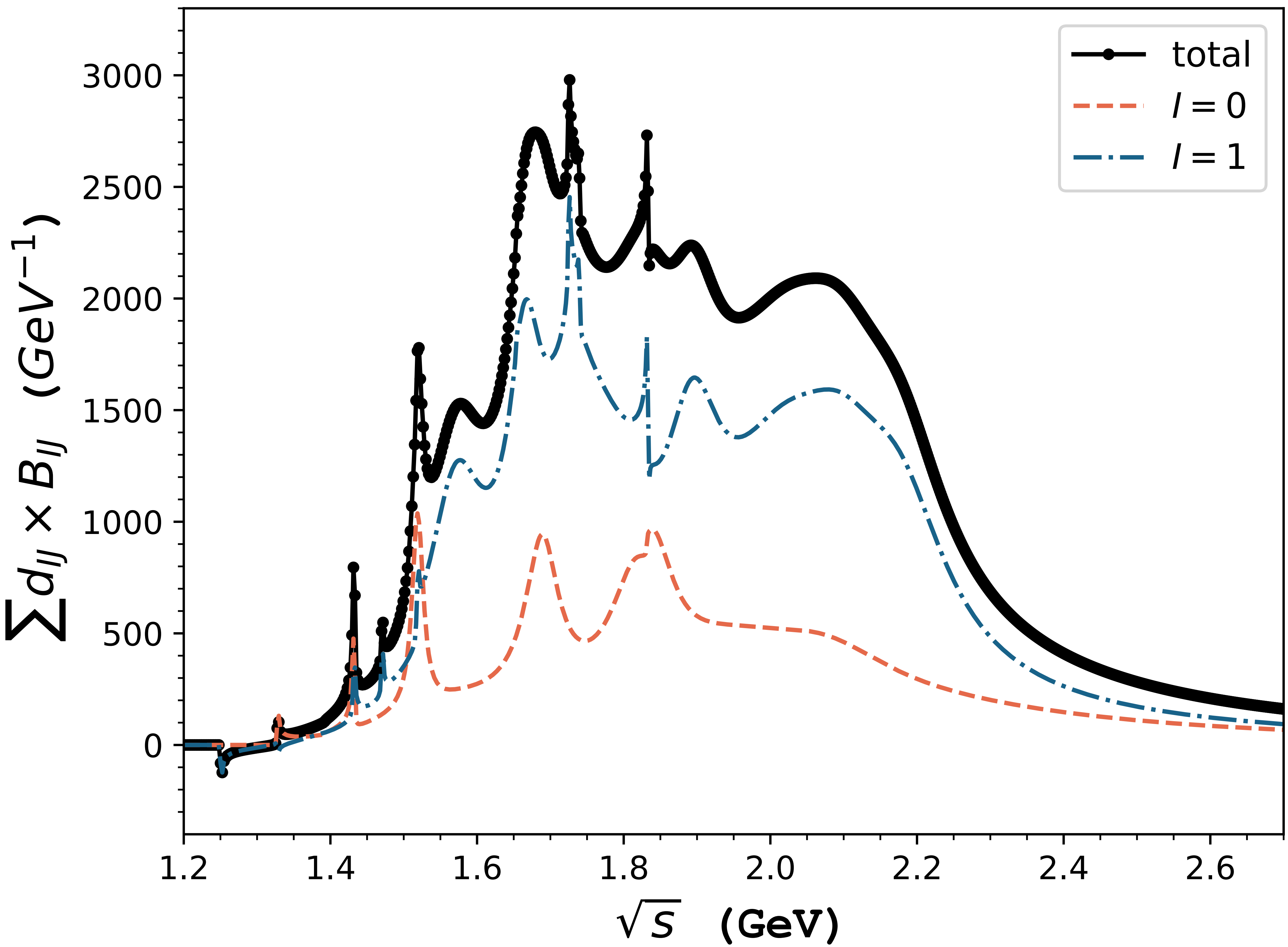


branching ratio?

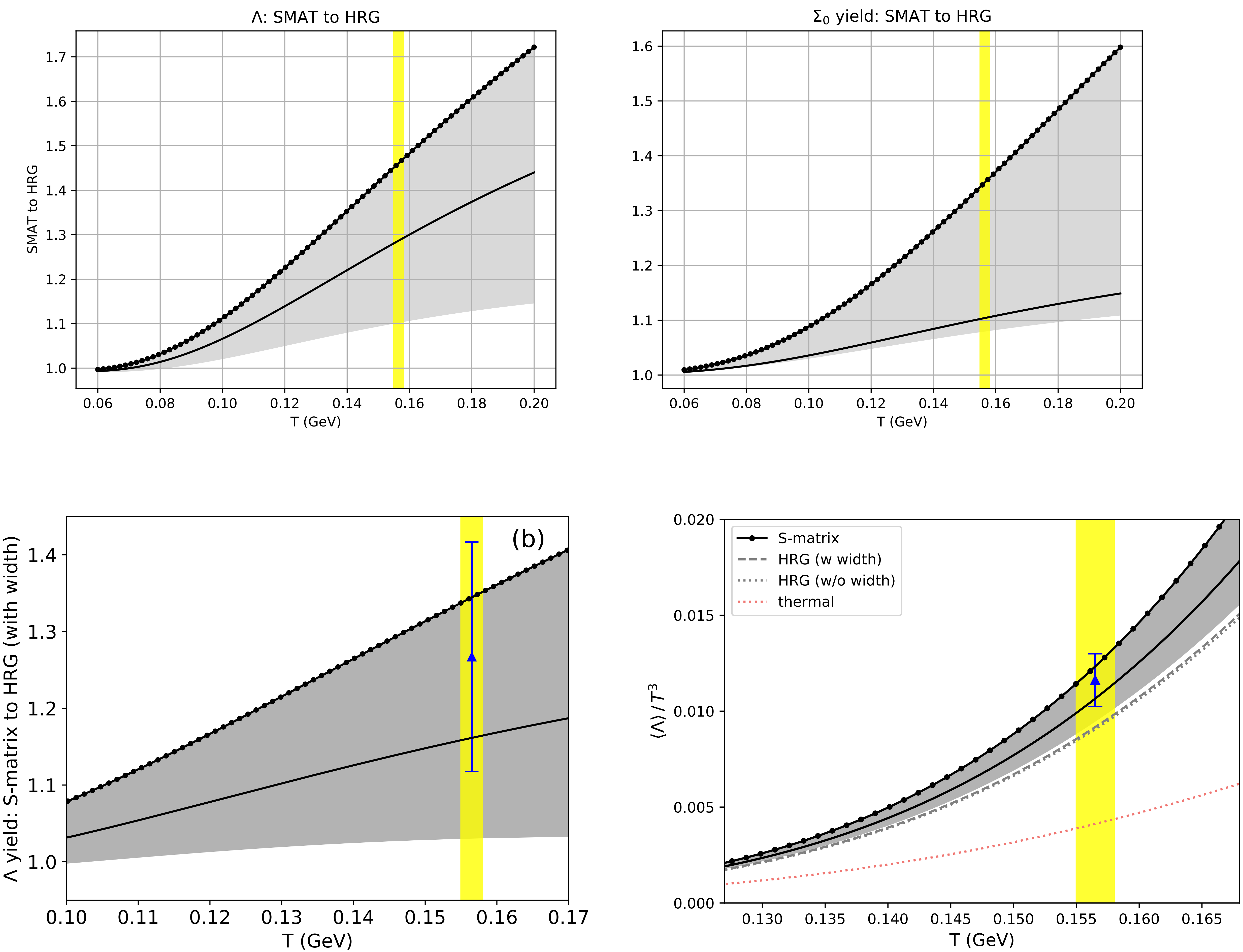
summing all channels…

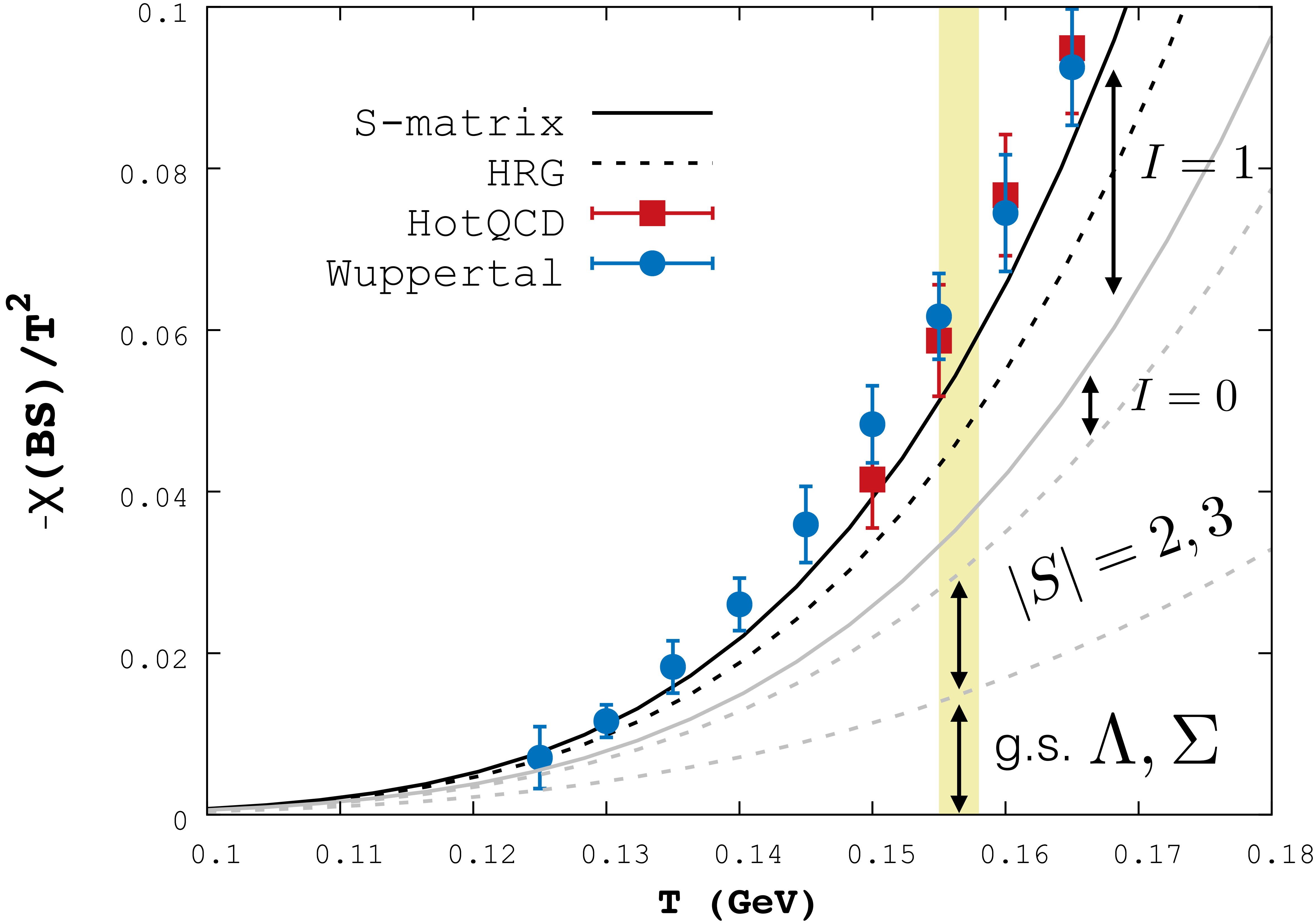






total
$I=0$
$I=1$
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{S}$ (GeV)
3000
2500
2000
1500
1000
500
0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6


Λ: SMAT to HRG
SMAT to HRG
T (GeV)
Σ0 yield: SMAT to HRG
T (GeV)
(b)
Λ yield: S-matrix to HRG (with width)
T (GeV)
S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨Λ⟩/T3
T (GeV)



S-matrix
HRG
HotQCD
Wuppertal
$I = 1$
$I = 0$
$|S| = 2, 3$
g.s. Λ, Σ
$-\chi(BS)/T^2$
T (GeV)
0
0.02
0.04
0.06
0.08
0.1
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

TABLE I. The contributions to $|S| = 1$ baryonic pressure from different the partial waves in the S-matrix approach and in HRG approximation in units of $10^{-3}T^4$ at $T = 150$ MeV. In the columns labeled "B-W," the partial pressure contributions obtained from the B-W parametrization of the spectral density are shown.

	$I = 0$				$I = 1$		
	S matrix	HRG	B-W		S matrix	HRG	B-W
S_{01}	0.916	1.139	1.224	S_{11}	1.018	0.282	0.532
P_{01}	0.539	0.607	0.676	P_{11}	1.681	1.275	1.465
P_{03}	0.426	0.403	0.472	P_{13}	1.868	1.857	2.406
D_{03}	1.091	1.127	1.416	D_{13}	0.964	0.995	1.052
D_{05}	0.363	0.221	0.456	D_{15}	1.478	1.219	1.793
F_{05}	0.261	0.308	0.489	F_{15}	0.514	0.503	1.119
F_{07}	0.160	0.085	0.222	F_{17}	0.556	0.238	0.603
G_{07}	0.173	0.057	0.177	G_{17}	0.169	0.095	0.310

not about what resonances should be added,
but more about how they should be added...

NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

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SUSANA COITO

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 25, 2015)

CONTINUUM

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

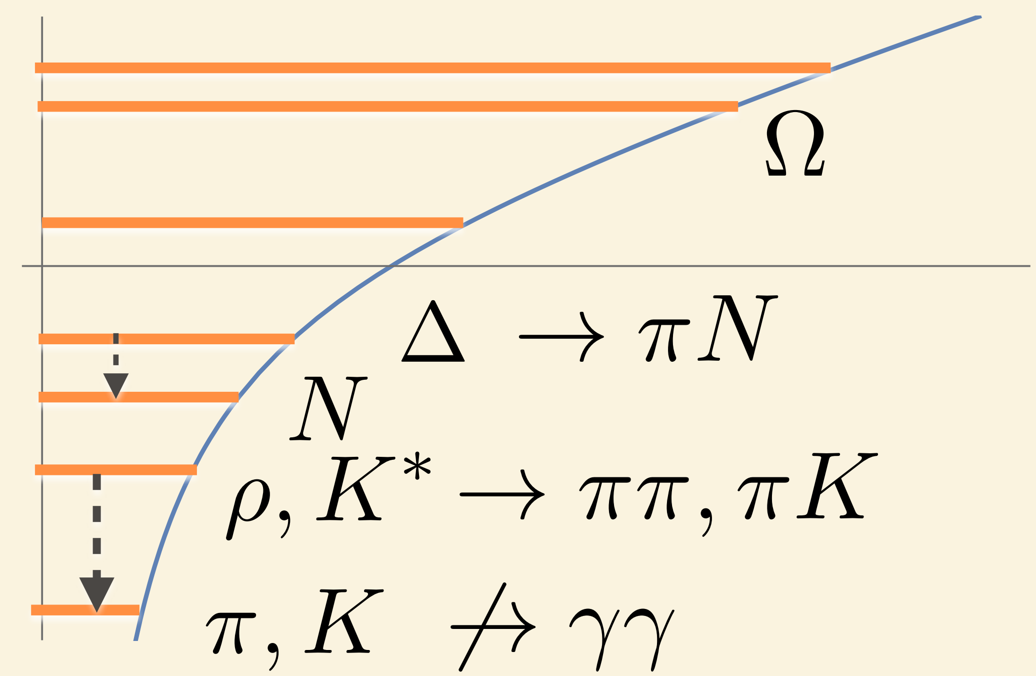
meson loops effects:

shift in hadron masses

S-matrix issues:

how resonances are expressed by the scattering states?
(and by quarks and gluons?)

QCD spectrum

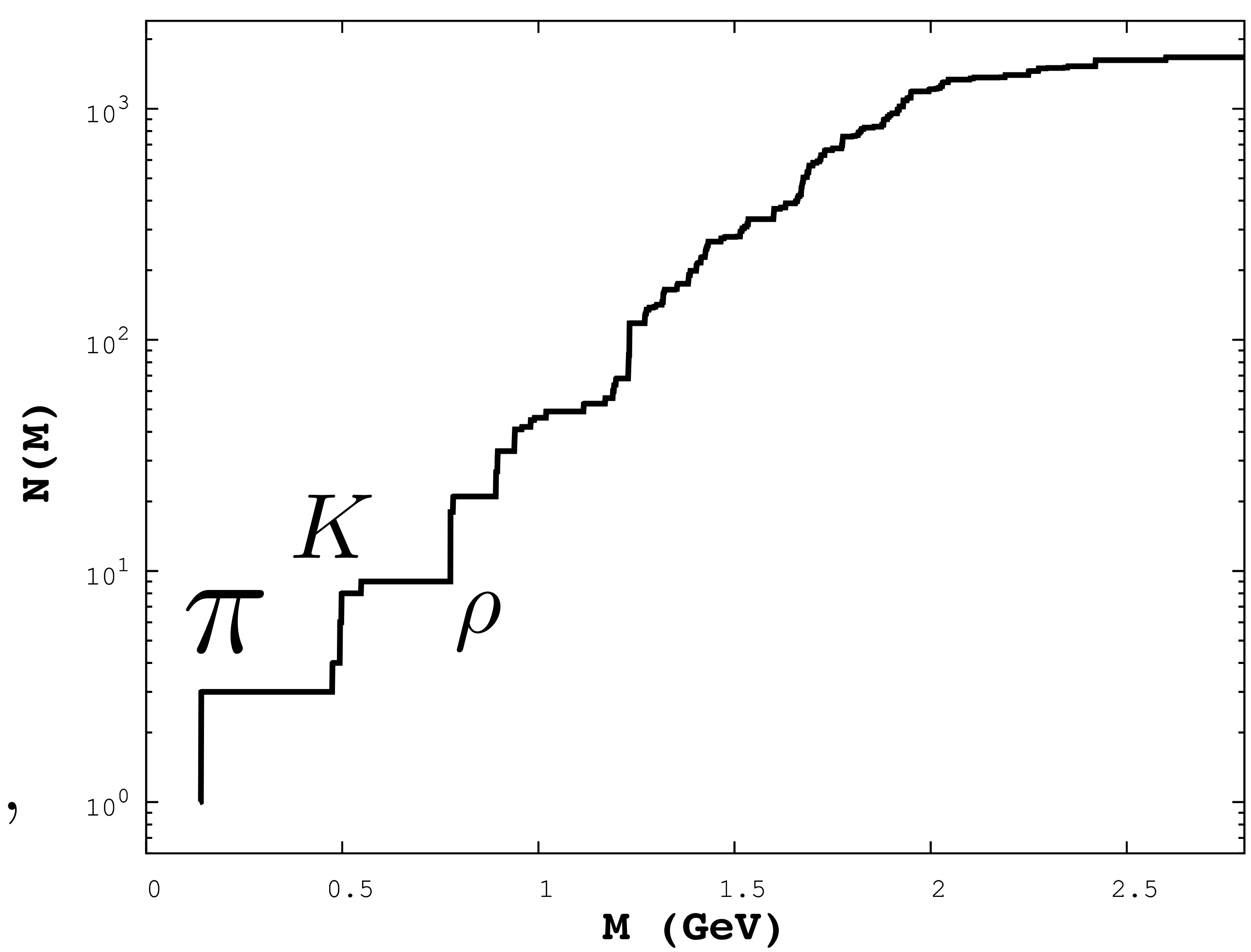


WHY IT’S NOT SUM OF BW

HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z^{\star}_{\text{res}} - E}{z_{\text{res}} - E}, \qquad z_{\text{res}} \approx m_{\text{res}} - i\,0^{+}.$$

$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\text{Im}\,(\text{tr}\ln\,\text{S})$$



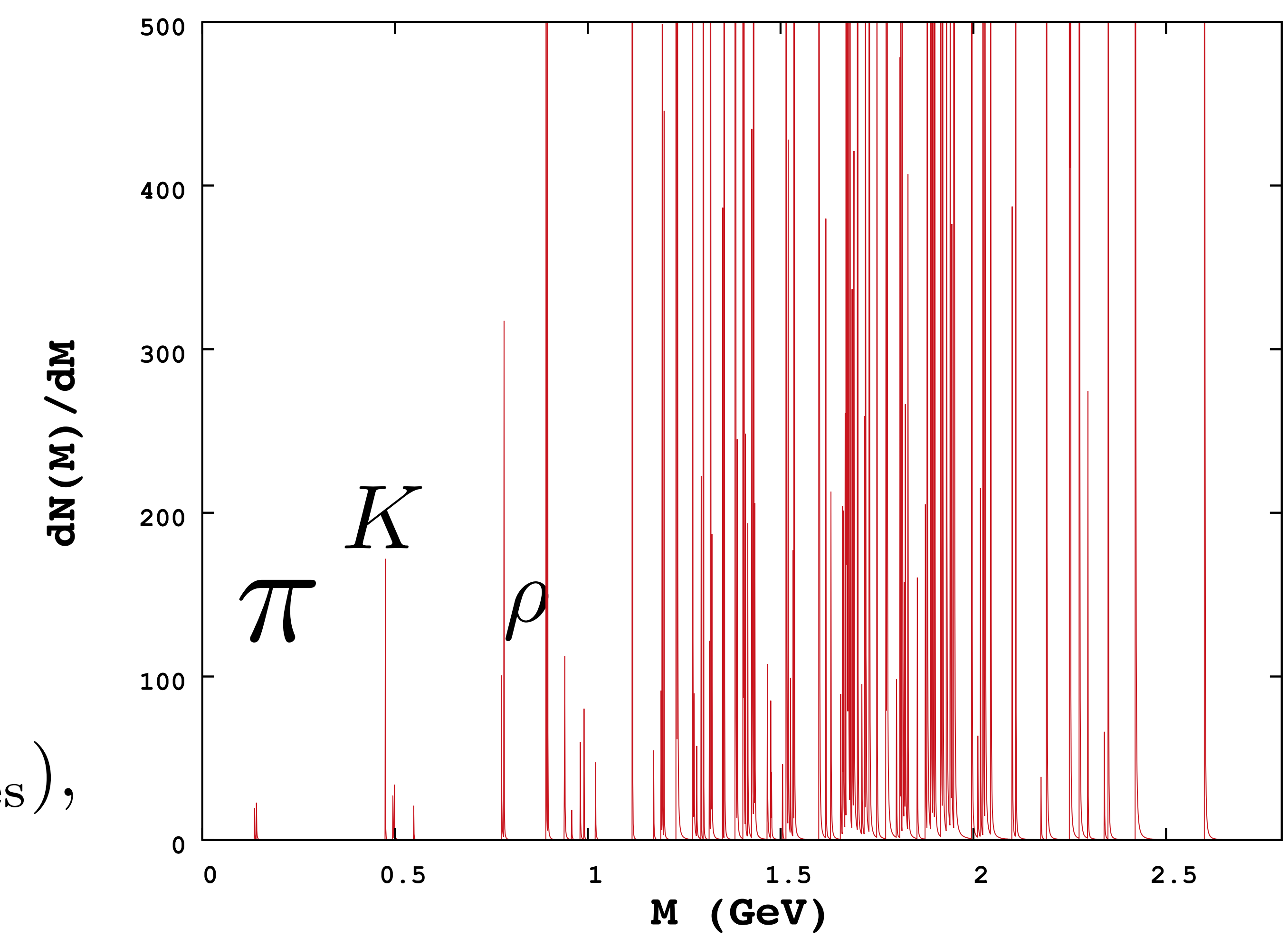
$$\mathcal{Q}_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi\,\theta(E - m_{\text{res}}),$$

HRG AS AN S-MATRIX SCHEME

$$\det\mathrm{S}(E) = \prod_{\{\mathrm{res}\}} \frac{z^{\star}_{\mathrm{res}} - E}{z_{\mathrm{res}} - E}, \qquad z_{\mathrm{res}} \approx m_{\mathrm{res}} - i\,0^{+}.$$

$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S})$$

$$\frac{\partial}{\partial E}$$



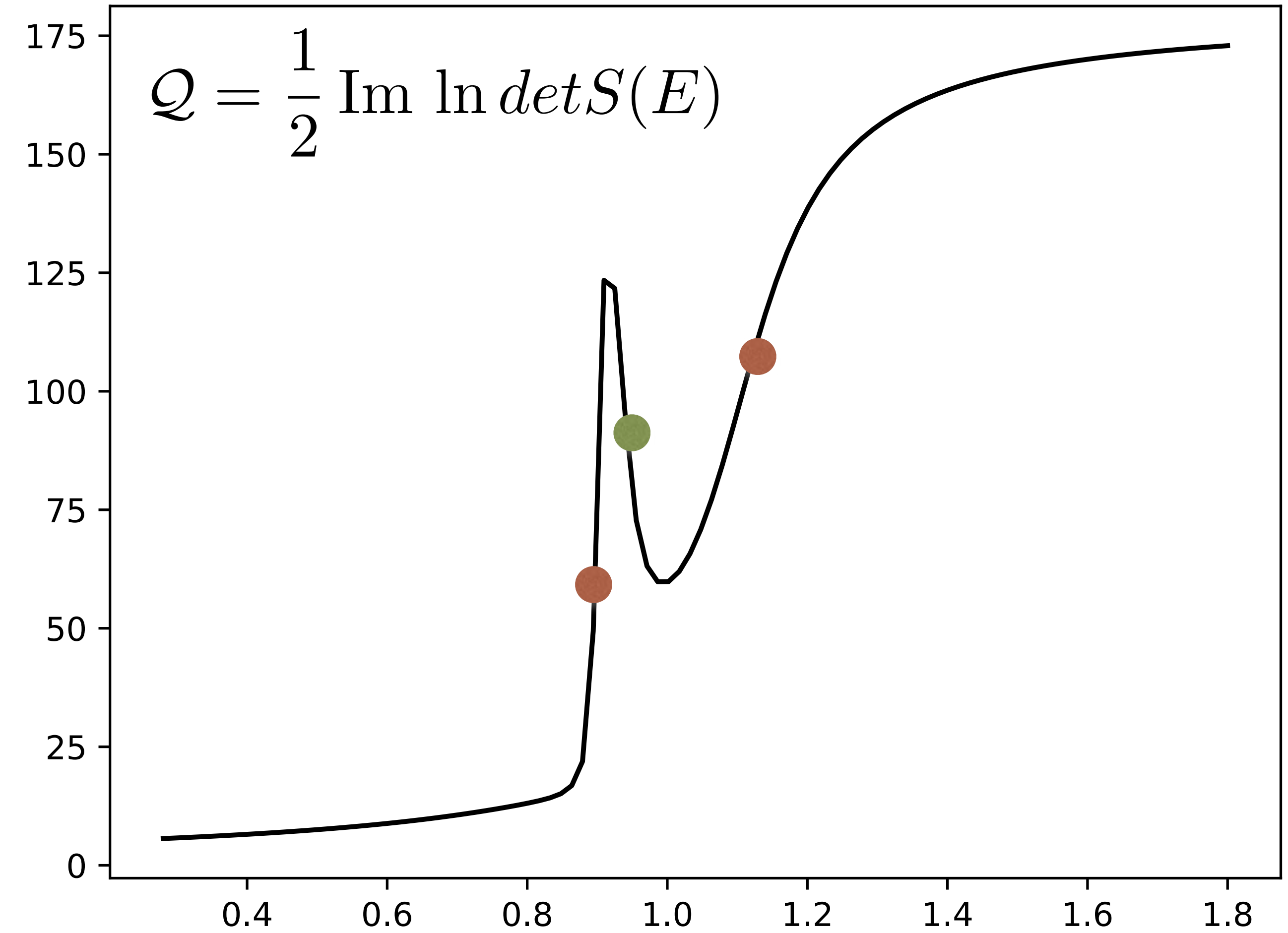
$$\mathcal{Q}_{\mathrm{HRG}}(E) = \sum_{\mathrm{res}} d_{IJ} \times \pi\,\theta(E - m_{\mathrm{res}}),$$

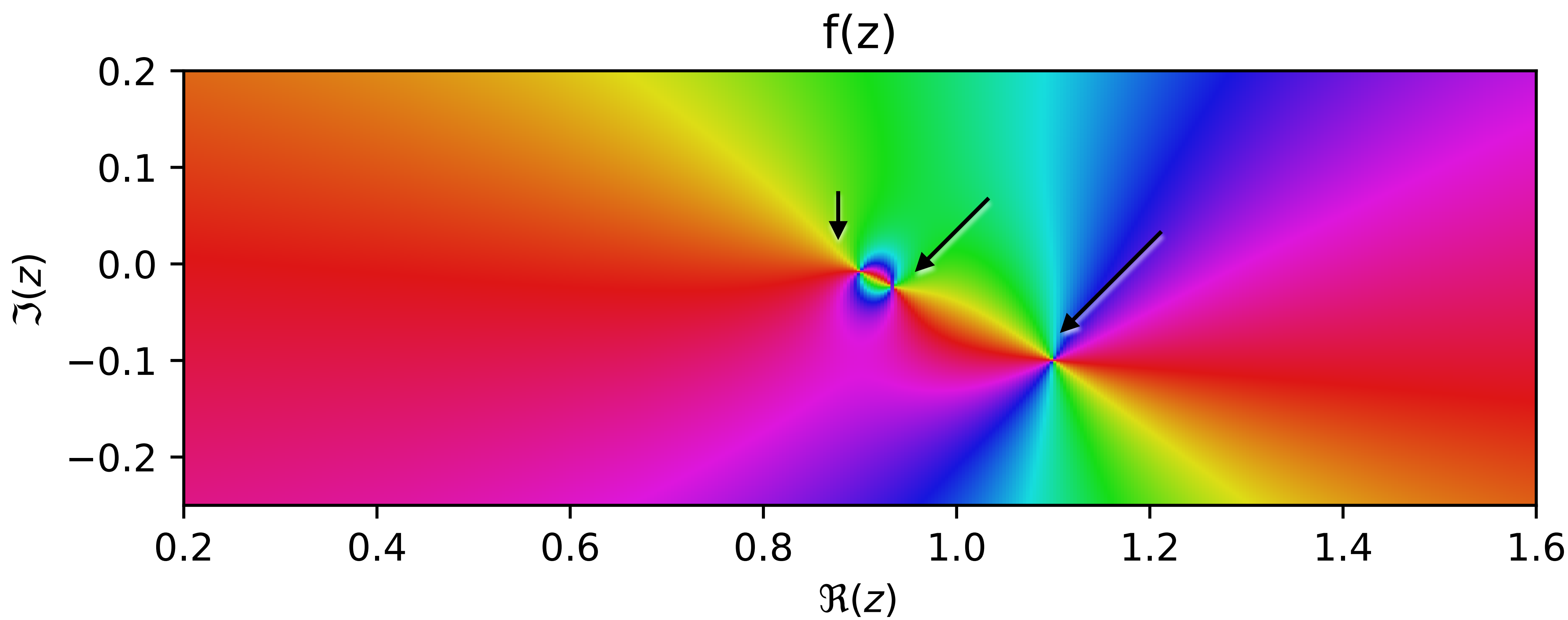
ROOTS IN S-M

2 pole + 1 root

$$detS \propto \frac{g_1}{E-p_1} + \frac{g_2}{E-p_2}$$

$$detS \propto \frac{g_1}{E-p_1} \times \frac{g_2}{E-p_2}$$





DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
  f(980) close to K Kbar threshold
  f(500) dynamically generated
- coupling of open channels: pipi, kkbar with a $|q\bar{q}\rangle$ state

Locher, Markushin, Zheng, EPJC 4 (1998)
Kaminski, Lesniak, Loiseau, EPJC 9 (1999)

PML PRD **102,** 034038 (2020)

what you give $\neq$ *what you get* *1 in 5 out!*

$$\frac{1}{E-\mathcal{H}_0} = \begin{matrix} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \; (|q\bar{q}\rangle) \end{matrix} \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \dfrac{1}{E-m^0_{res}} \end{bmatrix}$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

$$G = G_0 + G_0 V_{int} G$$

From Hamiltonian to Scattering Matrix

$$\frac{1}{E - \mathcal{H}_0 \pm i\delta}$$

$$\begin{aligned}
\tilde{S} &= \left(I - G^0_- \, V\right)\left(I + G^0_+ \, T\right) \\
&= I - G^0_- \, V + G^0_+ \, T - G^0_- \, V \, G^0_+ \, T \\
&= I - G^0_- \, V + G^0_+ \, V + G^0_+ \, V \, G^0_+ \, T - G^0_- \, V \, G^0_+ \, T \\
&= I + \left(G^0_+ - G^0_-\right) V + \left(G^0_+ - G^0_-\right) V \, G^0_+ \, T \\
&= I + \left(G^0_+ - G^0_-\right) T \\
&\to I + 2\, i \, \mathrm{Im}\left(G^0_+\right) \times T.
\end{aligned}$$

on-shell limit

TESTING THE ROBUSTNESS

$$\mathcal{Q}(E) = \frac{1}{2}\mathrm{Im}\mathrm{Tr}\{\ln S_E\}$$

Getting
Effective DOS
on
REAL Energy

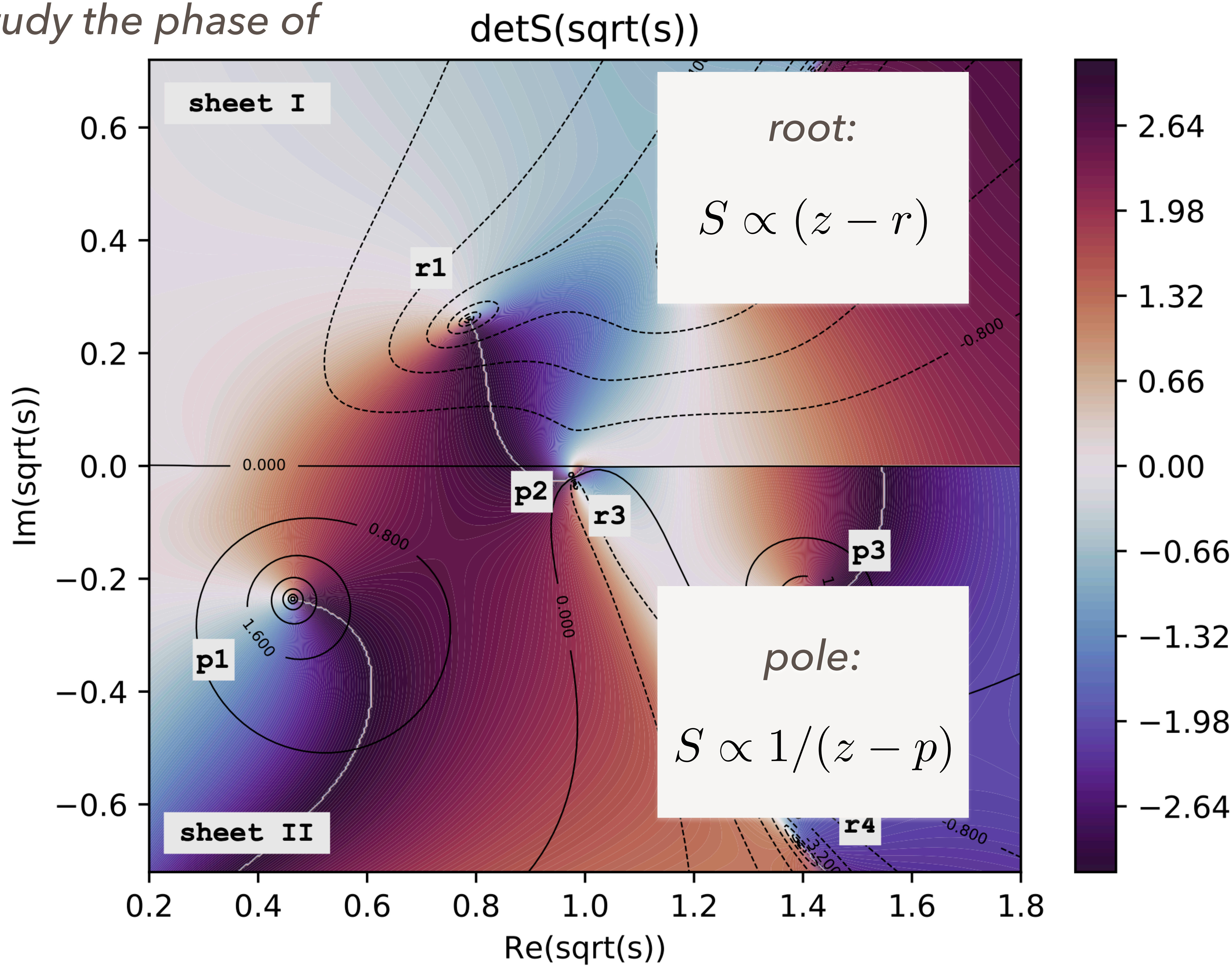
effective DOS

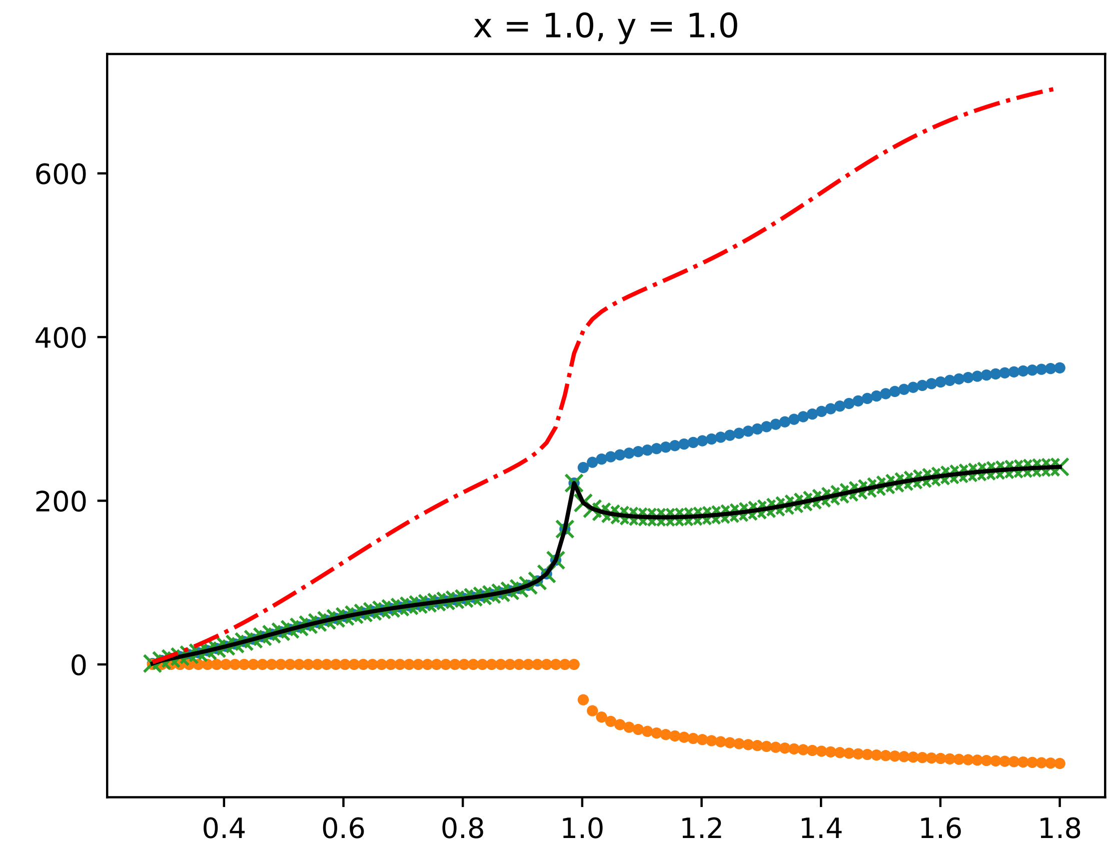
$$B = 2\frac{d}{dE}\mathcal{Q}$$

what is being counted?

can it handle dynamically generated states?

study the phase of

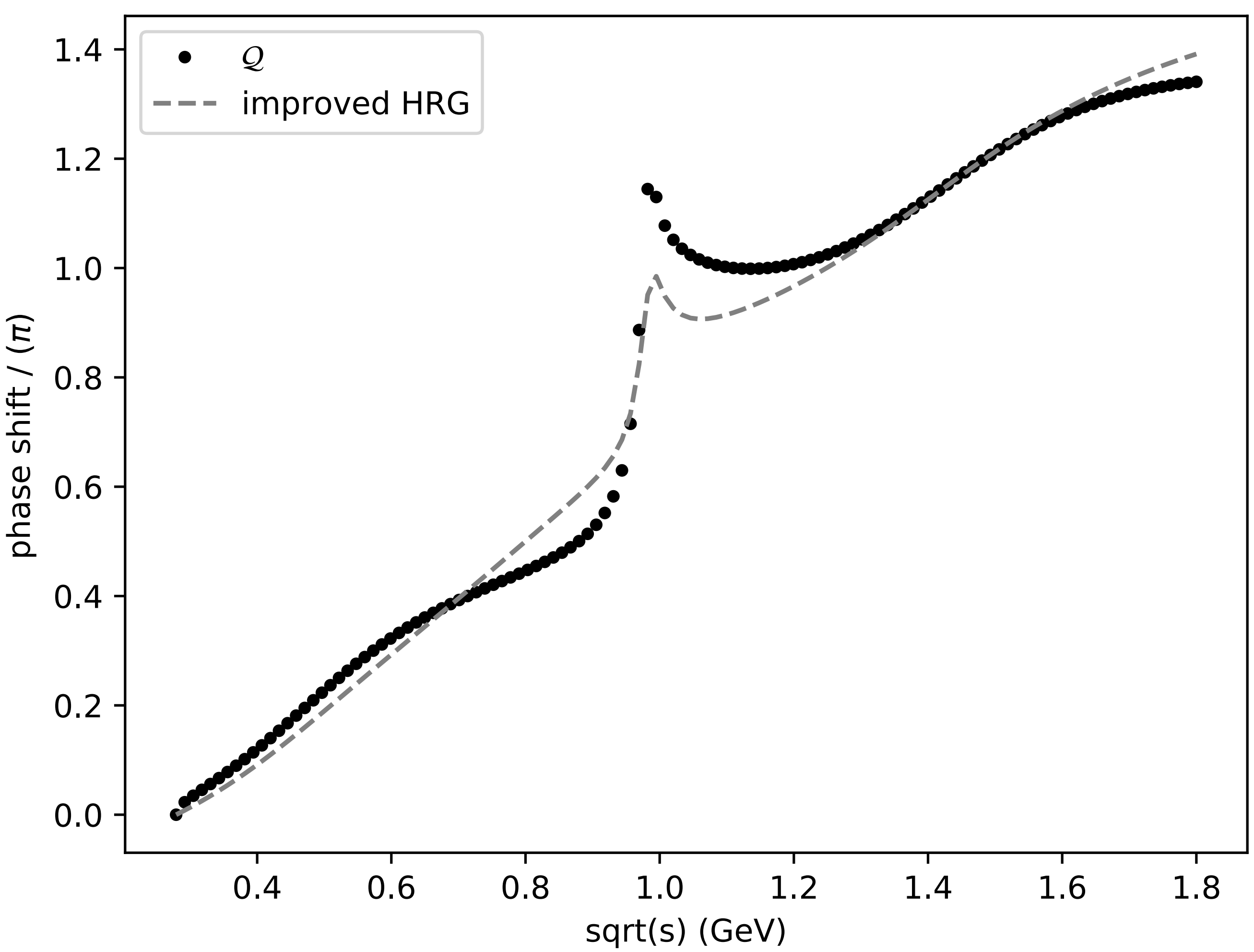




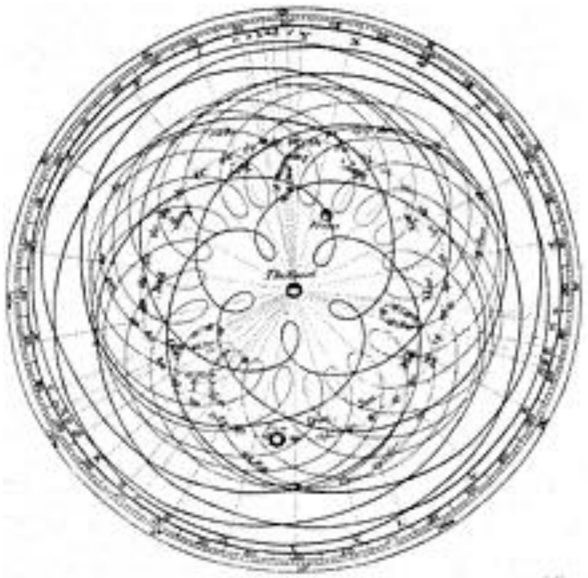
f. [54, 55]

	Re $\sqrt{s}$	Im $\sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

II. Location of resonance poles (p_i) and roots (r_i)
l in the model.



repulsive corrections in
HRG-like scheme:
via roots



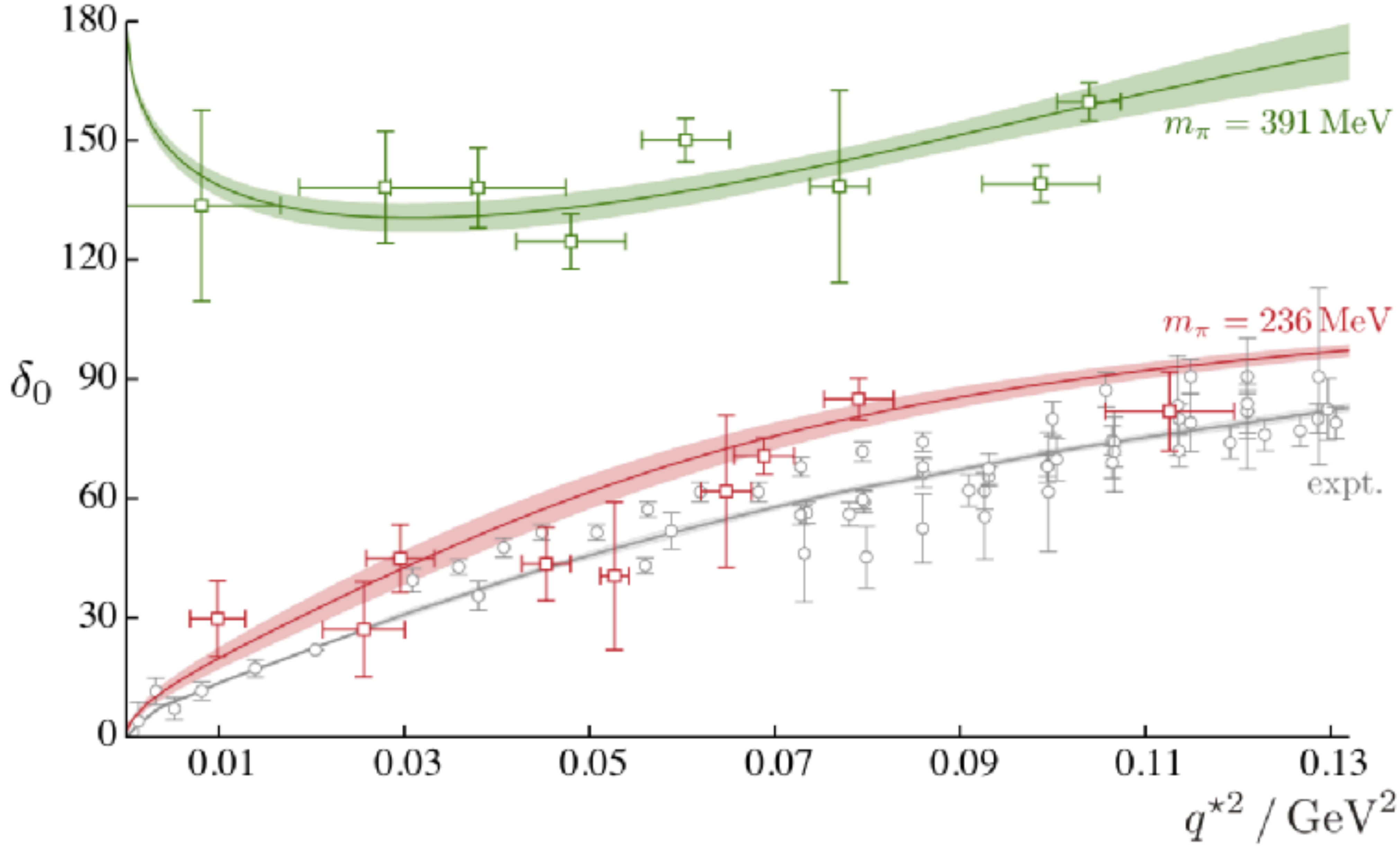
epicycles

solar eclipse

`` Incomplete understanding''

LATTICE COMPUTATIONS ON PHASE SHIFT

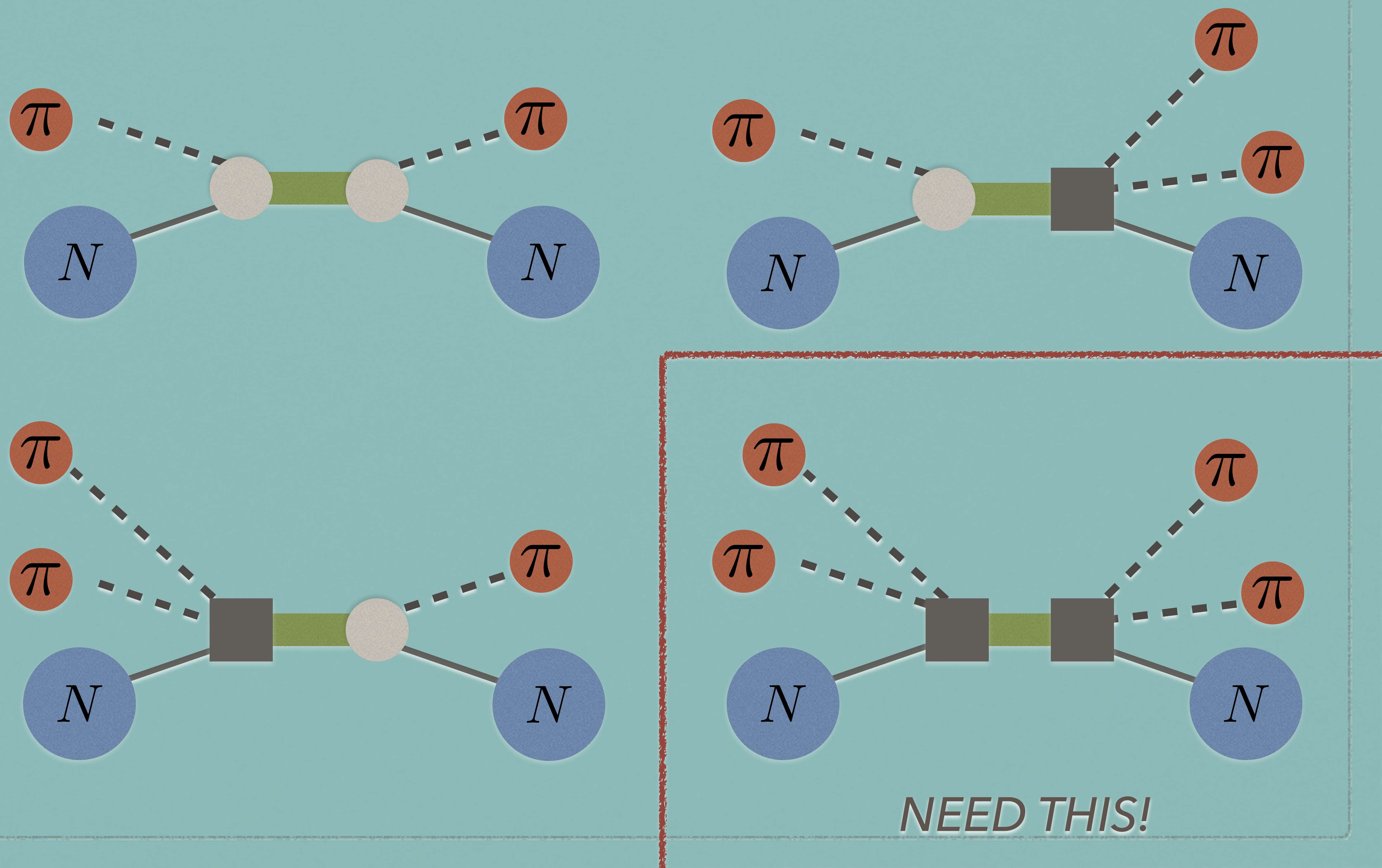
deuteron physics?



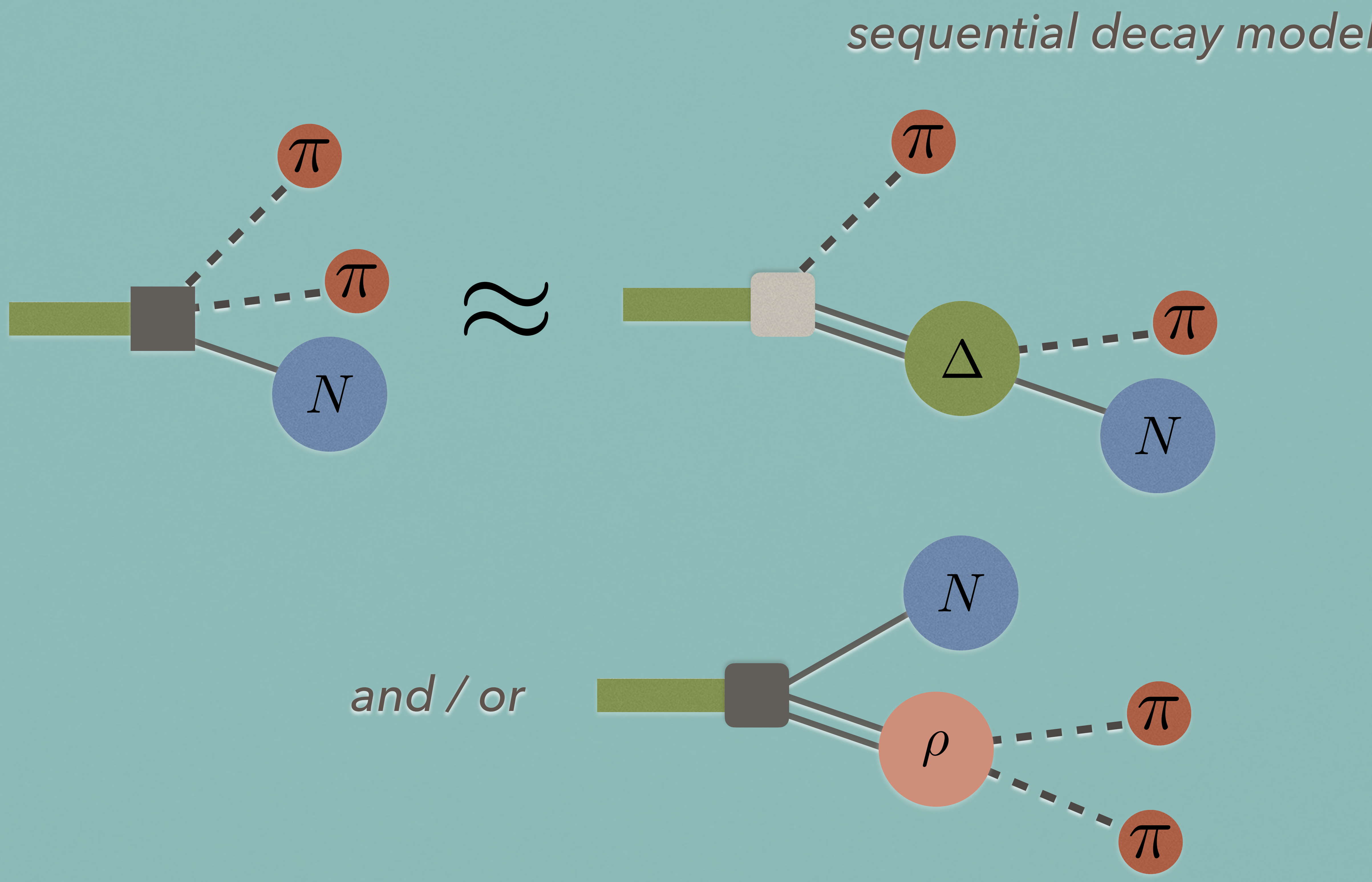
R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

WHAT'S COMING?

ISOBAR MODEL



ISOBAR MODEL



DENSE (R) MEDIUM

RECIPE

Feynman amplitude

- generalized phase shift

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

$$d\phi_N = \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\cdots\frac{d^3p_N}{(2\pi)^3}\frac{1}{2E_N}\times$$
$$(2\pi)^4\,\delta^4(P - \sum_i p_i).$$

phase space approach

PHASE SPACE DOMINANCE

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

- structureless scattering

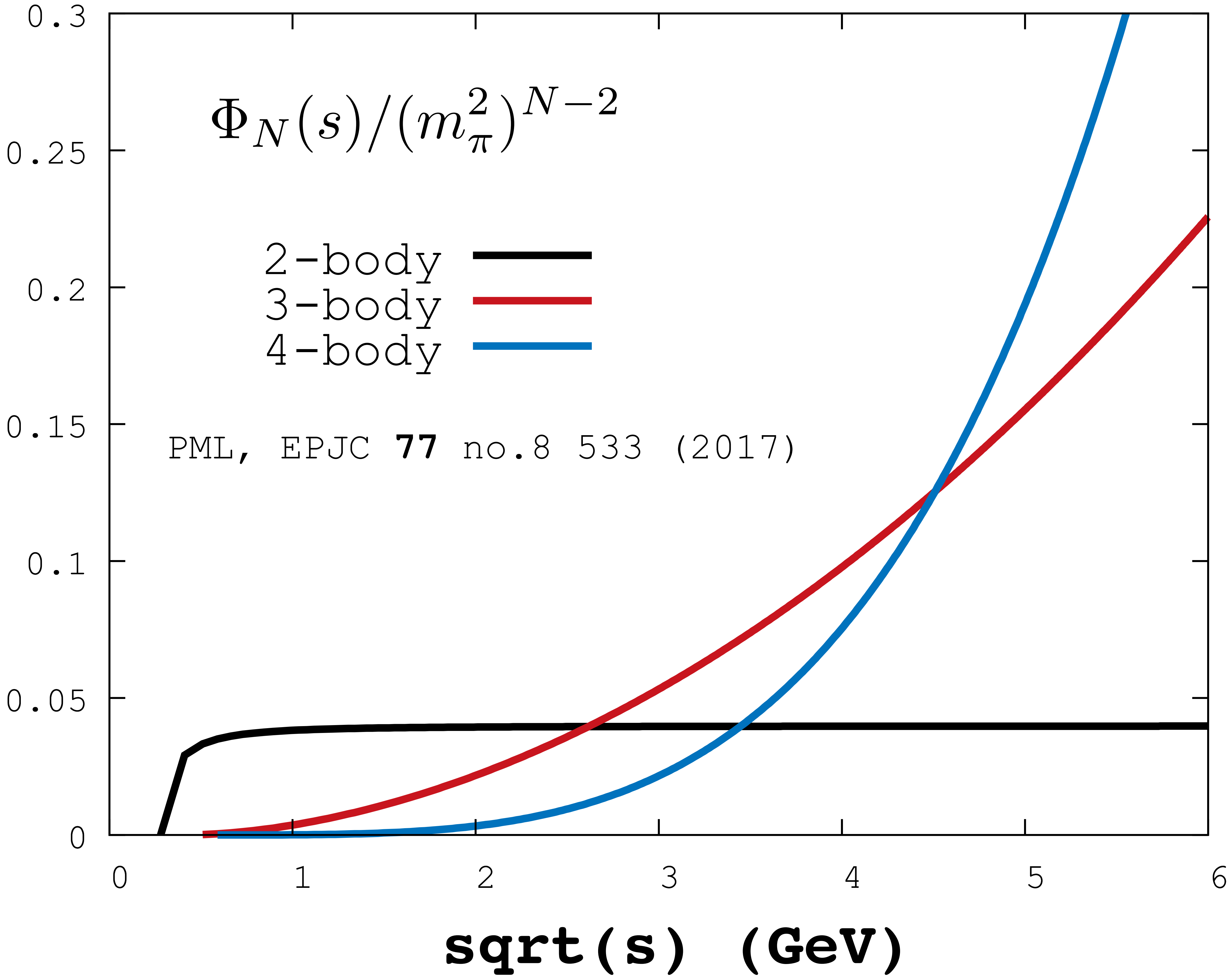
Dimension: $\sim E^{2N-4}$

$$i\mathcal{M} = i\,\lambda_N$$

Källén triangle function

$$\phi_N(s) = \frac{1}{16\,\pi^2 s}\int_{s'_-}^{s'_+} ds'\,\sqrt{\lambda(s,s',m_N^2)}\times$$

$$\phi_{N-1}(s', m_1^2, m_2^2, ..., m_{N-1}^2)$$

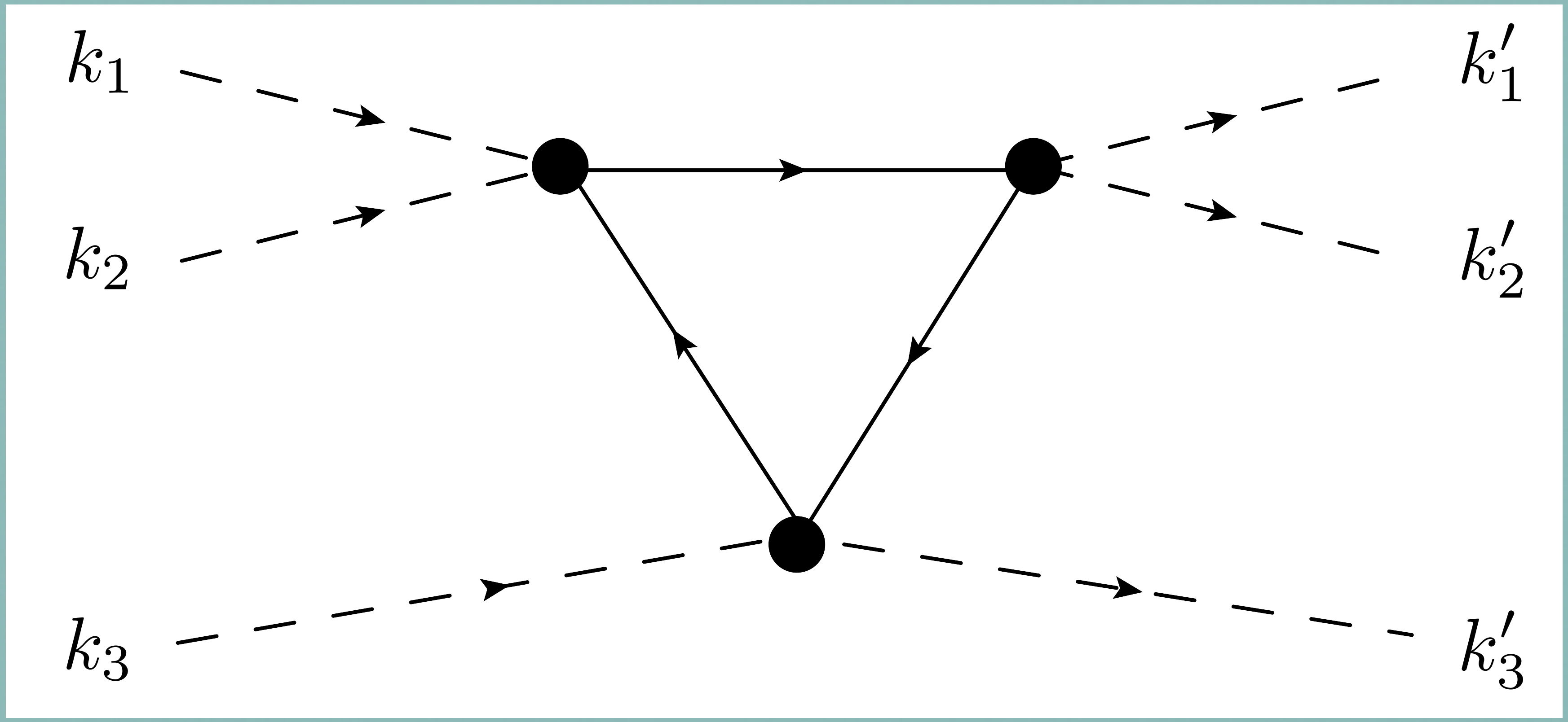


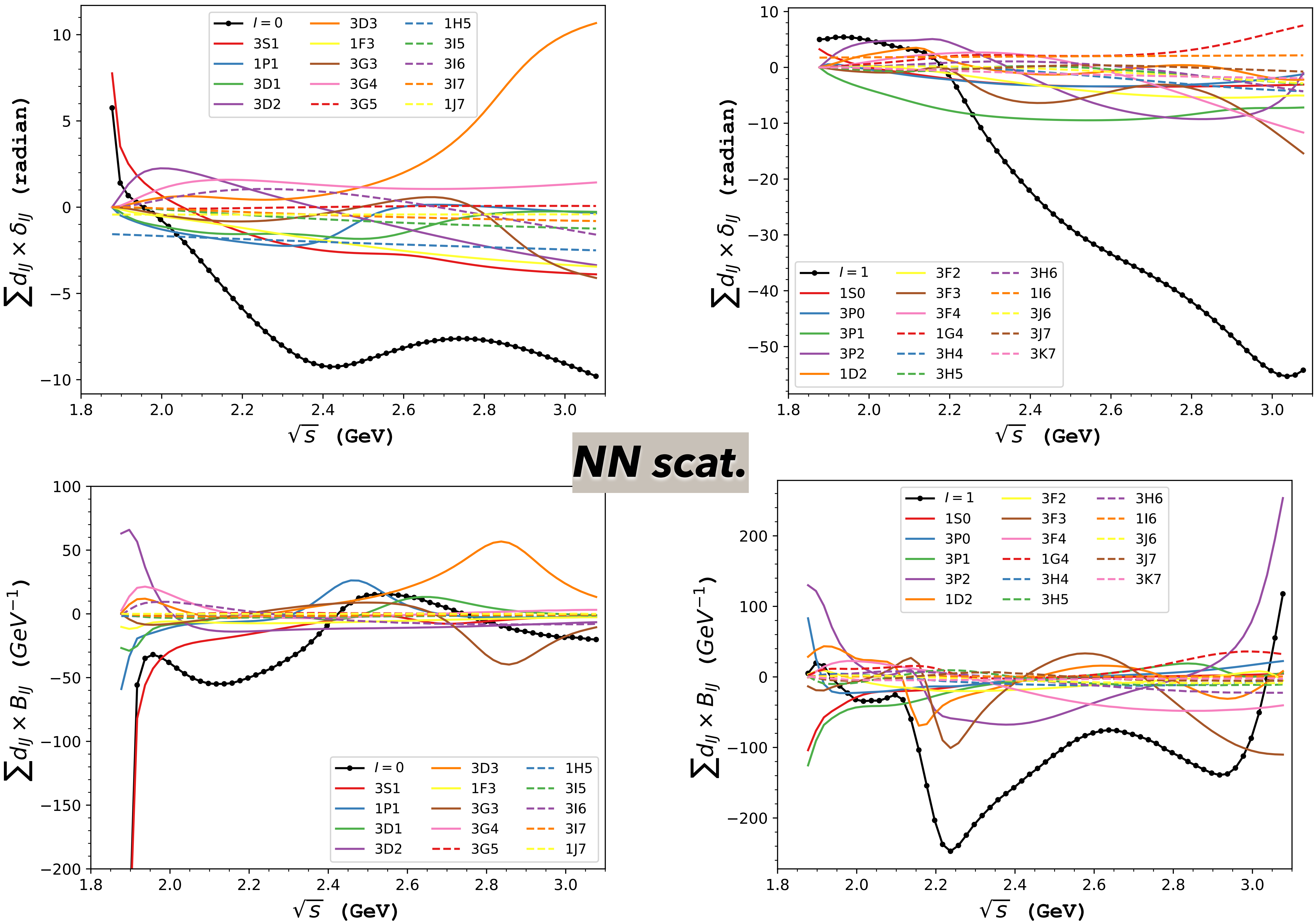
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$\Phi_N(s)/(m_\pi^2)^{N-2}$
2-body
3-body
4-body
PML, EPJC **77** no.8 533 (2017)
sqrt(s) (GeV)

TRIANGLE DIAGRAM

- 3-body diagram

$$b_3 \propto a_S^3$$





$\sum d_{IJ} \times \delta_{IJ}$ (radian)
$\sqrt{s}$ (GeV)
I = 0
3S1
1P1
3D1
3D2
3D3
1F3
3G3
3G4
3G5
1H5
3I5
3I6
3I7
1J7
I = 1
1S0
3P0
3P1
3P2
1D2
3F2
3F3
3F4
1G4
3H4
3H5
3H6
1I6
3J6
3J7
3K7
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
NN scat.

VACUUM PHYSICS?

Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

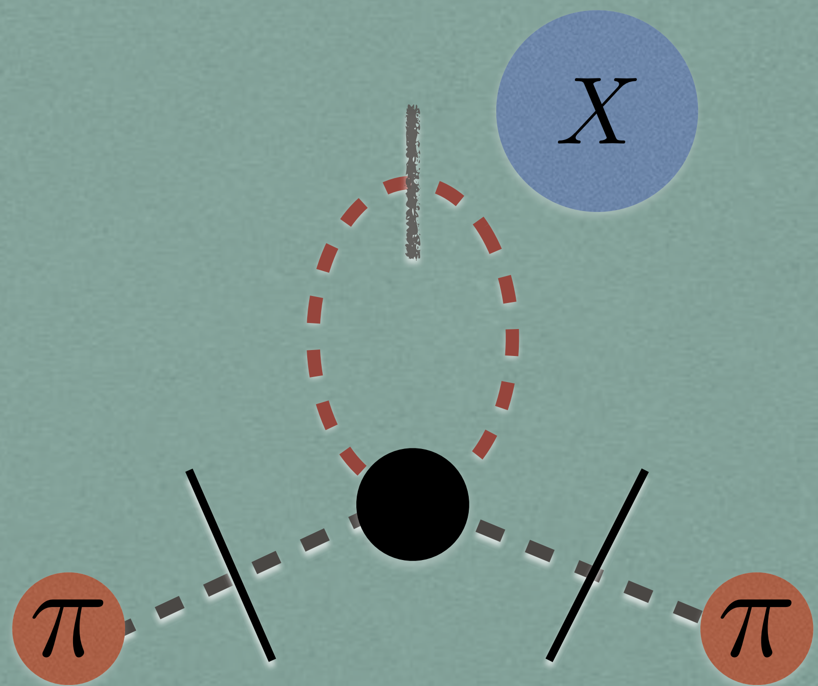
Newman Laboratory, Cornell University, Ithaca, NY, USA

Received 22 November 2006, in final form 3 May 2007
Published 19 July 2007
Online at stacks.iop.org/JPhysA/40/9655

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \mathrm{Tr}(e^{-\beta H})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H. In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,

IN-MEDIUM EFFECTS FROM S-MATRIX

$$\Sigma_\pi =$$



$$\propto \int \frac{d^3q}{\omega_q} n_X T_{\pi X}(s)$$

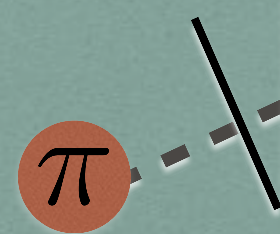
forward amplitude

A. Schenk NPB 363(1991)

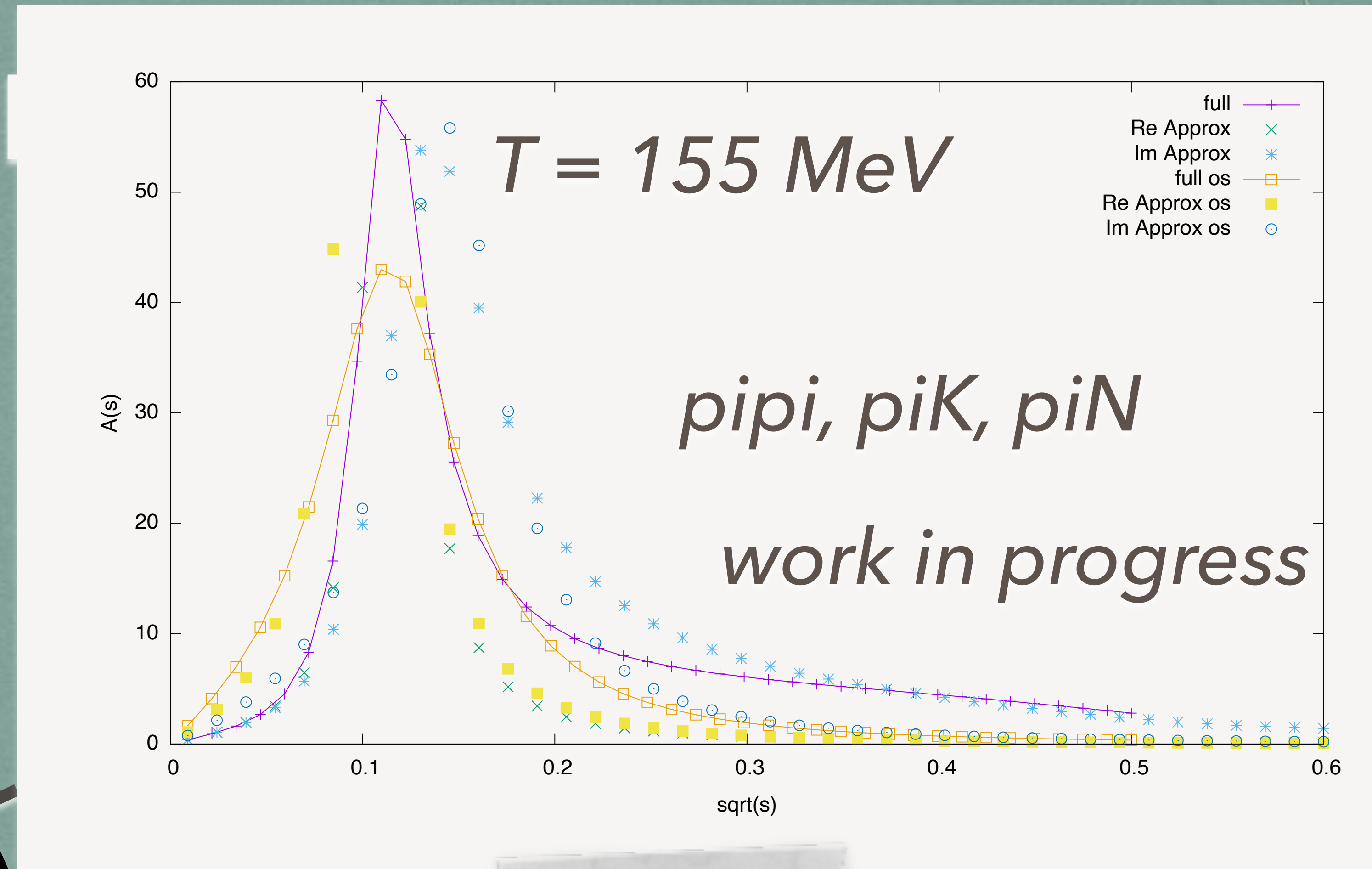
S. Jeon and P. J. Ellis PRD 58 045013 (1998)

IN-MEDIUM EF
S-MATRIX

$$\Sigma_\pi =$$



$$\propto \int \frac{d^3q}{\omega_q} n$$



\+ coupled-channel aspects

A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 5

THANK YOU

CONVERGENCE?

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\rm tot}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-qa_S)$$

$$= -a_S \frac{2\pi}{m_{\rm red}} \xi_N^2 n_N^2$$

$$P^{(0)} = n_N T \xi_N \qquad n_N = (\lambda_T^{-1})^3 = (\frac{m_N T}{2\pi})^{3/2}$$

$$r = \frac{\Delta P}{P^{(0)}} = -2 \times (a_S/\lambda_T) \times \xi_N$$

$$= -2a_S\,(\frac{m_N T}{2\pi})^{1/2}\, e^{(\mu - m_N)/T}$$

CONVERGENCE?

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\text{tot}}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-qa_S)$$

$$= -a_S\frac{2\pi}{m_{\text{red}}}\xi_N^2 n_N^2$$

$$P^{(0)} = n_N T\xi_N$$

$$r = \frac{\Delta P}{P^{(0)}} = -2\times(a_S/\lambda$$

$$= -2a_S\,(\frac{m_N T}{2\pi})^{1/2}\,e^{(\mu-m_N)/T}$$

$$a_S = 20\,fm$$

$$r \approx 0.0727$$ LHC

$$r \approx 0.36$$

$$r \approx 1.92$$

$$T = 60\,MeV$$

$$\mu_B = 700, 800\,MeV$$

RESOLUTION OF PROTON PUZZLE BY EBW ?

- PRC 98, 034906 (2018)
  Vovchenko, Gorenstein, and Stoecker
  Energy dependent Breit Wigner
  as a resolution of proton puzzle
  => Incomplete:
  Reasons:
  1. non-resonant interactions not accounted for

$$B = A + \Delta A$$

  2. overlapping resonances, non-BWs

see Phys. Lett. B **792,** 304 (2019)

W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).