The baryon content of the $\pi^{\pm}$

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Based on arXiv:2103.09131, in Coll. with W. Broniowski & E. Ruiz Arriola

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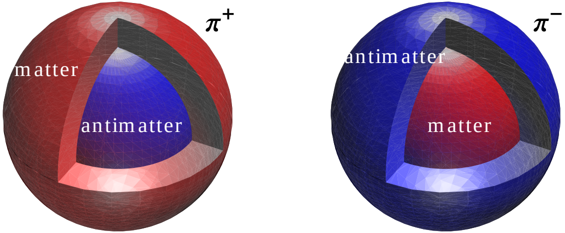
Section 1

Spoiler

Spoiler

- Our final result; recall $\pi^+ \sim u\bar{d}$ and $\pi^- \sim d\bar{u}$



- Matter antimatter (u/d quarks) differently distributed inside the $\pi^{\pm}$
- Admits mechanistic interpretation ($m_d > m_u$)

Outline

- Why form factors? The essentials
- The baryon form factor and symmetries
- Model estimates for the baryon form factor
- Extraction from experimental data
- Further discussions
- Conclusions

Section 2

Why form factors? The essentials

How do we probe charge distributions?

- Recall that, in QCD

$$\partial_\mu \left[\bar{q}_a(x)\gamma^\mu q_b(x)\right] = i(m_a - m_b)\bar{q}_a(x)q_b(x) = 0 \text{ for } a = b$$

- Implies conserved charge and $j_a^0(x) \sim$ charge density operator

$$Q_a = \int d^3x\, j_a^0(x) \sim \int d^3x\, \rho_a(x) = \text{const.} \quad \text{where } j_a^\mu(x) = \bar{q}_a(x)\gamma^\mu q_a(x) \text{ (FNC)}$$

- So the following matrix element contains information on charge distribution

$$\langle \pi^+(p+q)|\, j_a^\mu(x)\, |\pi^+(p)\rangle = e^{iq\cdot x} \langle \pi^+(p+q)|\, j_a^\mu(0)\, |\pi^+(p)\rangle$$

- Indeed, all the structure is encoded in the matrix element

$$\langle \pi^+(p+q)|\, j_a^\mu(0)\, |\pi^+(p)\rangle$$

...that precisely defines the form factor of interest...

Form factors 101: definitions

- Form factors are on-shell matrix elements

$$\langle \pi^+(p+q)| \, j_a^\mu(0) \, |\pi^+(p)\rangle = (2p+q)^\mu F_a(q^2)$$

current is conserved: $\partial_\mu j_a^\mu = 0$ since $q \cdot (2p+q) = m_\pi^2 - m_\pi^2 = 0$

- Back to charge dist., if wavefunction: $\Psi_u \, |u\rangle \otimes \Psi_d \, |\bar{d}\rangle + ...$, we want

$$\rho_a(x) = \langle \Psi_\pi | \, \hat{\rho}_a(x) \, |\Psi_\pi\rangle = \langle \Psi_\pi | \, j_a^0(x) \, |\Psi_\pi\rangle = ... = \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} F_a(-|\vec{q}|^2)$$

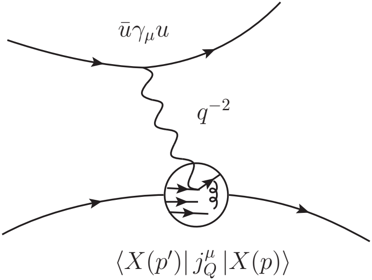
Form factors are fourier-transforms of charge distributions!

- For $F_a(q^2) = 1 \rightarrow \rho(x) = \delta^{(3)}(x) \Rightarrow$ point-like particle!
- $Q_a = \int d^3 r \; \rho(r) = ... = F_a(0)$
- $\langle r^2 \rangle_a = \int d^3 r \; r^2 \rho_a(r) = ... = 6 \frac{dF_a(q^2)}{dq^2}|_{q^2=0}$

$$F_a(q^2) = Q_a(1 + \frac{1}{6}\langle r^2 \rangle_a q^2 + ...)$$

But again, how do we probe charge distributions?

- We know the (basic) theory now, but how to extract them?



$$\mathcal{M} = \frac{e^2}{-q^2}(\bar{u}\gamma_\mu u)\langle X(p')| j_Q^\mu |X(p)\rangle$$

- Carrying these experiments for years with QED! Hofstadter Nobel Prize '61
- Good knowledge for $F_Q(q^2)$, but not accessible probes for $F_a(q^2)$ for $q \neq Q$
- It will require more work for the baryon form factor... let's dive in!

Section 3

The baryon form factor and symmetries

Form factors 102: symmetries and the baryon form factor

- The relevant—conserved—currents in our study (associated charge $Q_{B,3,Q}$)

$$J_B^\mu = \frac{1}{N_c}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d\right), \quad J_3^\mu = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \quad J_Q^\mu = J_3^\mu + \frac{1}{2}J_B^\mu$$

- The relevant form factors ($\partial_\mu J_X^\mu = 0$)

$$\langle \pi^a(p+q) \mid J^\mu_{B,3,Q}(0) \mid \pi^a(p)\rangle = (2p^\mu + q^\mu) F^a_{B,3,Q}(q^2) \qquad a = \{0, \pm\}$$

- Charge conjugation (exact) $\mathcal{C} j_X^\mu \mathcal{C}^\dagger = -j_X^\mu$, $\mathcal{C}\,|\pi^\pm\rangle = |\pi^\mp\rangle$, $\mathcal{C}\,|\pi^0\rangle = |\pi^0\rangle$

$$F_X^0(q^2) = -F_X^0(q^2) = 0 \quad \& \quad F_X^\pm(q^2) = -F_X^\mp(q^2) \quad \text{(opposite charge)}$$

- G-parity $\mathcal{G} = \mathcal{C}e^{i\pi I_2}$ (exact in isospin-symmetric limit $m_u = m_d$)

$$\mathcal{G} j_B^\mu \mathcal{G}^\dagger = -j_B^\mu, \quad \mathcal{G} j_3^\mu \mathcal{G}^\dagger = +j_3^\mu, \quad \mathcal{G}\,|\pi^\pm\rangle = -\,|\pi^\pm\rangle, \quad \mathcal{G}\,|\pi^0\rangle = -\,|\pi^0\rangle$$

$$F_B^\pm(q^2) = -F_B^\pm(q^2) = 0$$

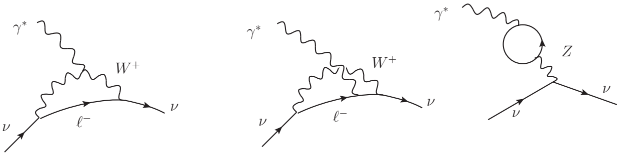
But, in real world $m_u \neq m_d$ so in general $F_B^\pm \neq 0$!

The baryon form factor: final messages

- But isospin is broken! (Davies et al. 2009)

$\Delta m = m_d - m_u = 2.8(2)\ \mathrm{MeV}, m_u = 2.01(14)\ \mathrm{MeV}, m_d = 4.79(16)\ \mathrm{MeV}$

- While ward ids. still require $F_B^{\pm}(0) = 0$ (baryonless), $F_B^{\pm}(q^2)$ not protected



And in QFT, if no symmetry protection, things are generally nonzero!

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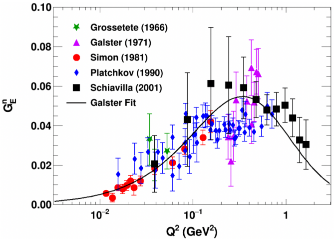

Compilation from Obrecht (2019)

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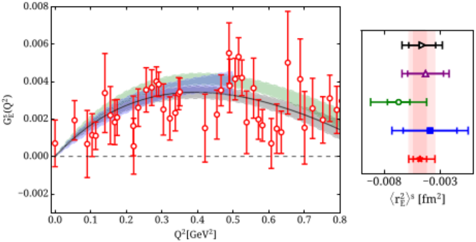

Alexandrou et. al (ETM Coll.)
PRD101 (2020)

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$$F_B^{\pm}(q^2)???$$

$$F_B^{\pm}(0) = 0 \Rightarrow F_B(q^2)^{\pm} = \pm\left[\frac{1}{6}\langle r_B^2\rangle q^2 + \mathcal{O}(q^4)\right]$$

- Must scale as Δm (tiny & challenging!)
- No baryonic currents = no direct access $\Rightarrow$ more work required!
- In the following extract $\langle r_B^2\rangle$! First th. estimates, then exp

Section 4

Model estimates for the baryon form factor

Model I: Effective Lagrangian

- Let's start with an order of magnitude estimate based on dim arguments!

$$J_B^{\mu} = -2i\frac{c\Delta m}{\Lambda^3}\partial_{\nu}\left(\partial^{\mu}\pi^{+}\partial^{\nu}\pi^{-} - \partial^{\nu}\pi^{+}\partial^{\mu}\pi^{-}\right) + \ldots,$$

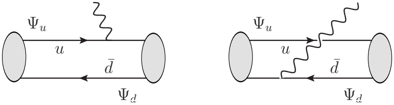
- Essentially the same as χPT would provide
- This implies

$$F_B^{\pm}(q^2) = \pm\frac{c\Delta m}{\Lambda^3}q^2 + \mathcal{O}(q^4) \Rightarrow \langle r^2\rangle_B^{\pi^{\pm}} = \pm 6\frac{c\Delta m}{\Lambda^3}$$

- As an estimate, typically $\Lambda \simeq m_{\rho} \Rightarrow \langle r^2\rangle_B^{\pi^{\pm}} = \pm 6\frac{c\Delta m}{m_{\rho}^3} = c(0.04\ \mathrm{fm})^2$
- We expect c to be an order 1 parameter, while sign not settled (yet)
- To be compared with $\langle r^2\rangle_Q^{\pi} = (0.659(4)\ \mathrm{fm})^2$

Model II: Yukawa model

- This model allows direct "contact" with quarks



- $\rho_B(r) = B_u|\Psi_u(\vec{x})|^2 + B_{\bar{d}}|\Psi_{\bar{d}}(\vec{x})|^2$
- Yukawa distributions $\Psi_q(r) = \frac{M_q^2 e^{-2M_q r}}{\pi r}$ with $M_{u,d} = M - \frac{1}{2}\Delta m$
- For $\Delta m = 0$: $F_Q^{\pi^\pm}(Q^2) = \pm \frac{4M^2}{4M^2+Q^2}$ with $2M \simeq m_\rho \to$ VMD

$$F_B^{\pi^+}(-Q^2) = \frac{1}{N_c}\left[\frac{4M_u^2}{4M_u^2+Q^2} - \frac{4M_d^2}{4M_d^2+Q^2}\right] = \frac{1}{N_c}\frac{8M\Delta m Q^2}{(4M_u^2+Q^2)(4M_d^2+Q^2)}$$

- We obtain $\langle r^2\rangle_B^{\pi^\pm} \simeq \pm 3\Delta m/(N_c M^3) = \pm(0.04\ \mathrm{fm})^2$
- Predicts same magnitude, but also the sign!

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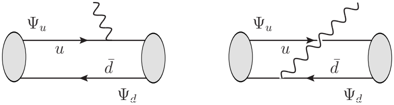

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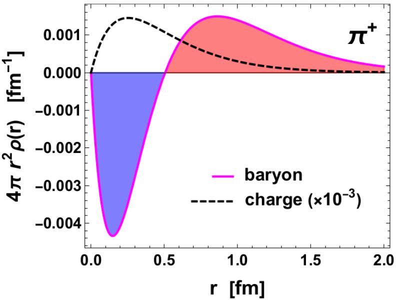
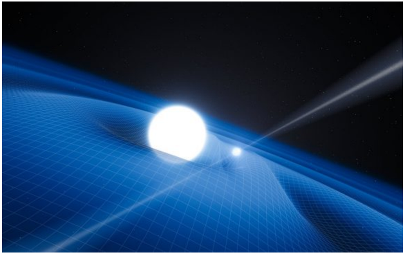
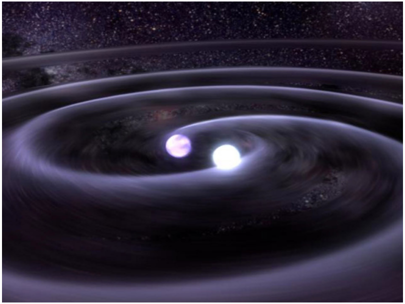


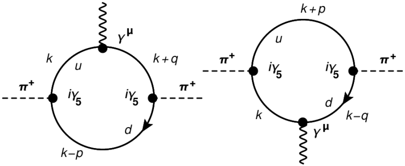


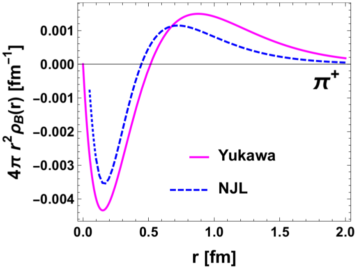


- Mechanistic intuition heavy/light
- "Weighting" 4% mass differences

Model III: Nambu—Jona-Lasinio model

- Better motivated low-energy model of QCD
- Includes chiral symmetry-breaking and constituent quarks $\to \Delta m$!
- Quite succesful at low energies (long distances)



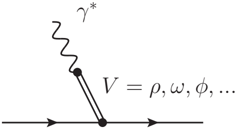


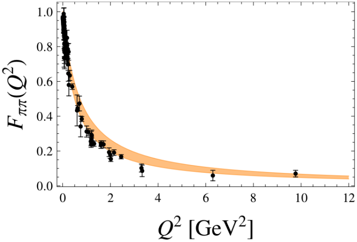
- One obtains quite independent of regularization

$$\langle r^2 \rangle_B^{\pi^\pm} \simeq \pm(0.03\ \mathrm{fm})^2$$

Model IV: $\rho - \omega$ mixing

- So far quark models, move on to hadronic ones for better contact with exp.!
- We follow the idea of VMD, quite successfull in describing $F_Q^{\pm}(Q^2)$





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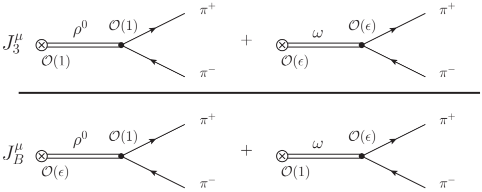

- The states $|\rho^0\rangle = \cos\theta\,|\rho^3\rangle - \sin\theta\,|\omega^0\rangle, \;\; |\omega\rangle = \sin\theta\,|\rho^3\rangle + \cos\theta\,|\omega^0\rangle, \;\; \theta \sim \epsilon$

$$F_3(-Q^2) = \frac{\cos^2\theta\, m_\rho^2}{Q^2 + m_\rho^2} + \frac{\sin^2\theta\, m_\omega^2}{Q^2 + m_\omega^2}, \quad F_B(-Q^2) = \frac{Q^2 \sin(2\theta)(m_\omega^2 - m_\rho^2)}{N_c(Q^2 + m_\rho^2)(Q^2 + m_\omega^2)}$$

- Simplistic model $M_{\rho,\omega}^2 \simeq M_0^2 + B\hat{m}, \;\; M_\phi^2 \simeq M_0^2 + Bm_s$

$\Rightarrow \sin(2\theta)(m_\omega^2 - m_\rho^2) = -2B\Delta m \quad \& \quad B = 2/3(M_\phi^2 + M_{K^*}^2 - 2M_{\rho,\omega}^2)/(m_s - \hat{m})$

$$\langle r^2\rangle_B^{\pi^\pm} \simeq \pm(0.05\ \mathrm{fm})^2$$

Summary

All theoretical models support $\langle r^2 \rangle_B^{\pi^+} \simeq +(0.04 \text{ fm})^2$

But, is it possible to extract it (indirectly) from experiment?

...

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But, is it possible to extract it (indirectly) from experiment?

...

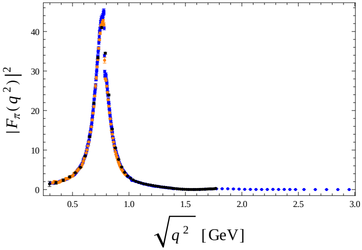
YES!

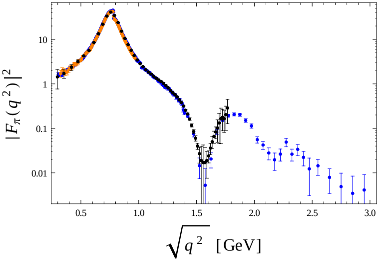
Section 5

Extraction from experimental data

Extraction from experiment

- We have a long tradition of $e^+e^- \to \gamma^* \to \pi^+\pi^-$ measurements!





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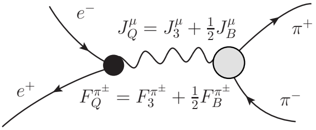

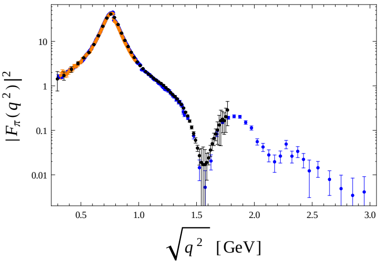

- The form factor decomposes as $F_Q^{\pi^\pm}(q^2) = F_3^{\pi^\pm}(q^2) + \frac{1}{2}F_B^{\pi^\pm}(q^2)$

$$F_3^{\pi^\pm}(q^2) = \frac{D_\rho(q^2) + c_{\rho'}D_{\rho'}(q^2) + \ldots}{1 + c_{\rho'} + \ldots}, \quad F_B^{\pi^\pm}(q^2) = c_{\rho\omega}q^2 D_\rho(q^2)D_\omega(q^2) \xrightarrow{q^2\to 0} 0$$

$$D_V(s) = [m_V^2 - s - im_V\Gamma_V(t)]^{-1}$$

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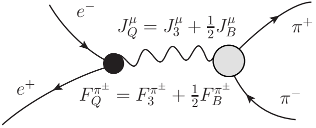

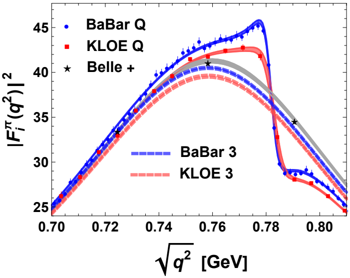

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$$D_V(s) = [m_V^2 - s - i m_V \Gamma_V(t)]^{-1}$$

Fit it!

Extraction from experiment

- The fits in the timelike region $q^2 > 4m_{\pi^{\pm}}^2$
- Can use analytic properties to go to the SL and extract radius

$$F_B^{\pi^{\pm}}(q^2) = \frac{1}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^{\pm}}(s)}{s - q^2} = \frac{q^2}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^{\pm}}(s)}{s(s - q^2)}$$

- Finally, we obtain for the radius

$$\langle r^2\rangle_B^{\pi^+}|_{BaBar} = (0.0411(7)\ \mathrm{fm})^2 \qquad \langle r^2\rangle_B^{\pi^+}|_{KLOE} = (0.0412(12)\ \mathrm{fm})^2$$

- In line with our expectations. Note also $\langle r^2\rangle_B^{\pi^+} = 0.0017\ \mathrm{fm}^2$

Lattice QCD $\langle r^2\rangle_Q^{\pi^+} = (0.648(15)\ \mathrm{fm})^2 = 0.42(2)\ \mathrm{fm}^2$ (arXiv:2102.06047)
Lattice QCD $\langle r^2\rangle_Q^{\pi^+} = 0.4298(45)_{\mathrm{stat}}(124)_{\mathrm{sys}}\ \mathrm{fm}^2$ (arXiv:2006.05431)

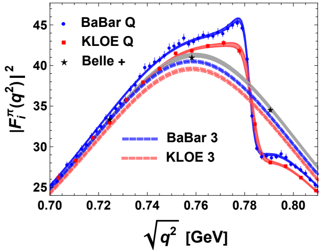
- Might be at reach for lattice in the future!

A word of caution

- In the isospin-symmetric limit (σ = Pauli matrices)

$$\langle \pi^b | \bar{q}\gamma^\mu \sigma^i q | \pi^a \rangle = (p_a + p_b)^\mu F_i^\pi(q^2)$$

- And isospin symmetry implies $F_3^\pi(q^2) = F_\pm^\pi(q^2)$
- Corrections arise at $\mathcal{O}(\Delta m)$ and $F_3(q^2) \neq F_\pm(q^2)$ from τ decays (Belle)



- Unfortunately τ data cannot extract $F_3^\pi(q^2)$

Section 6

Further discussions

Results and dissertation

- Did we just reword the well-known phenomenon of $\rho - \omega$ mixing?

$\Rightarrow$ Asolutely not! We realized about its connection to $F_B^{\pi^{\pm}}(q^2)$

$\Rightarrow$ Actually, many parametrizations $F_B^{\pi^{\pm}}(0) \neq 0$ incorrect

- Not convinced this is more than $\rho - \omega$ mixing?

$\Rightarrow$ Flavor basis $\quad J_u^{\mu} = J_Q^{\mu} + J_B^{\mu} \qquad J_d^{\mu} = 2J_B^{\mu} - J_Q^{\mu}$

$\Rightarrow \langle r^2 \rangle_u^{\pi^+} = \langle r^2 \rangle_Q^{\pi^+} + \langle r^2 \rangle_B^{\pi^+} \qquad \langle r^2 \rangle_d^{\pi^+} = 2\langle r^2 \rangle_B^{\pi^+} - \langle r^2 \rangle_Q^{\pi^+}$

$\Rightarrow \langle r^2 \rangle_u^{\pi^+} - \langle r^2 \rangle_{\bar{\mathbf{d}}}^{\pi^+} = 3\langle r^2 \rangle_B^{\pi^+}$

- Recognizing this property allows for flavor decompostition!

$\Rightarrow$ Proposals for (indirect) extractions; didn't realize it was possible!

Section 7

Conclusions

Conclusions

- New property of the $\pi^{\pm}$ on the table
- Model estimates around $\langle r^2\rangle_B^{\pi^+} \simeq (0.04\ \mathrm{fm})^2$
- Can be indirectly extracted from data despite its tiny size

$$\langle r^2\rangle_B^{\pi^{\pm}} = (0.0411(7)\ \mathrm{fm})^2 = 0.0017\ \mathrm{fm}^2$$
$$\langle r^2\rangle_u^{\pi^+} - \langle r^2\rangle_{\bar{\mathbf{d}}}^{\pi^+} = 3\langle r^2\rangle_B^{\pi^+} = 0.0051\ \mathrm{fm}^2$$

- To be compared to $\langle r^2\rangle_Q^{\pi} = (0.659(4)\ \mathrm{fm})^2 = 0.434(5)\ \mathrm{fm}^2$
- Lattice not there yet, but $\langle r^2\rangle_Q^{\pi^+} = 0.4298(\mathbf{45})_{\mathrm{stat}}(124)_{\mathrm{sys}}\ \mathrm{fm}^2$
- Testing differences in u/d quarks distributions in the $\pi^{\pm}$ **for the first time**